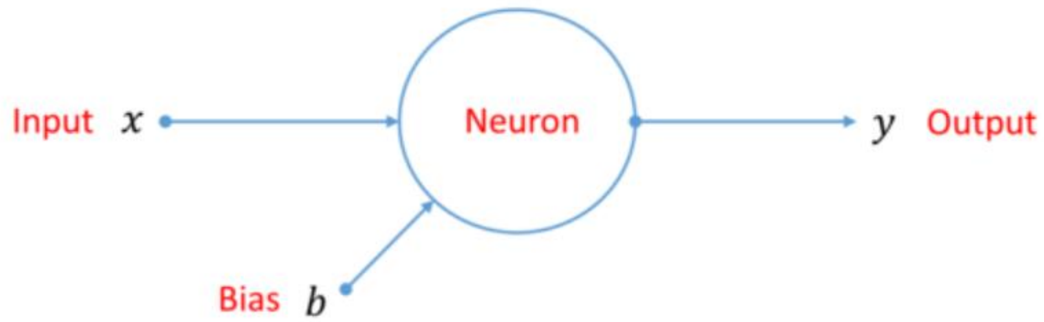
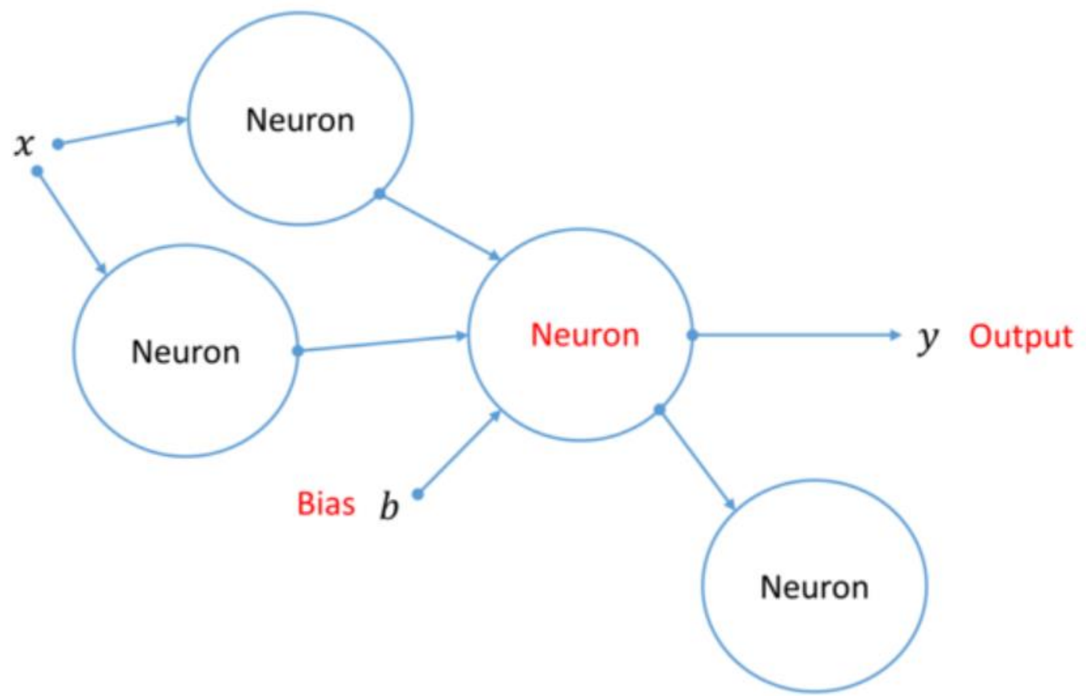
인공 신경망
퍼셉트론의 이해

인공 신경 세포(Artificial Neuron)

- **뉴런**
  - 입력
  - 편향(bias)

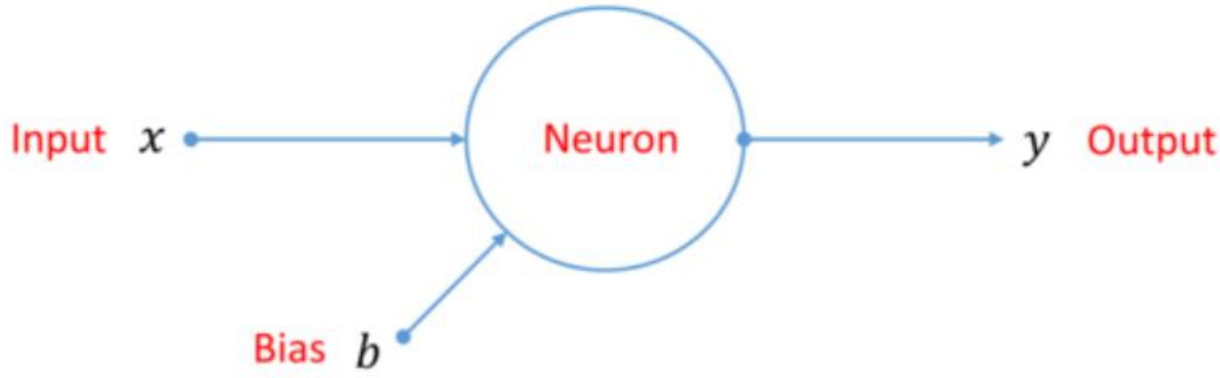


- **신경망(network)**
  - 뉴런의 연결



입력과 출력

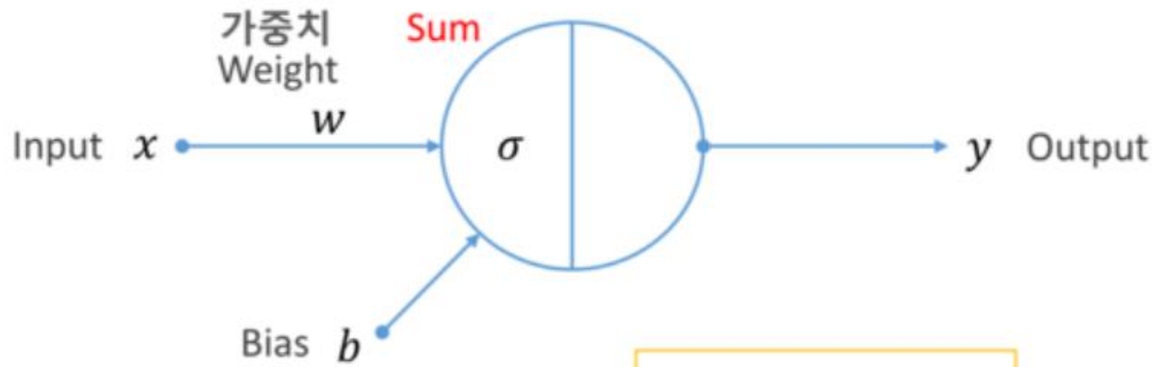
- **편향(bias)**
  - 편향을 조정해 출력을 맞춤



Input x	**Output y**
Size of house	Price
Time spent for studying	Score in exam

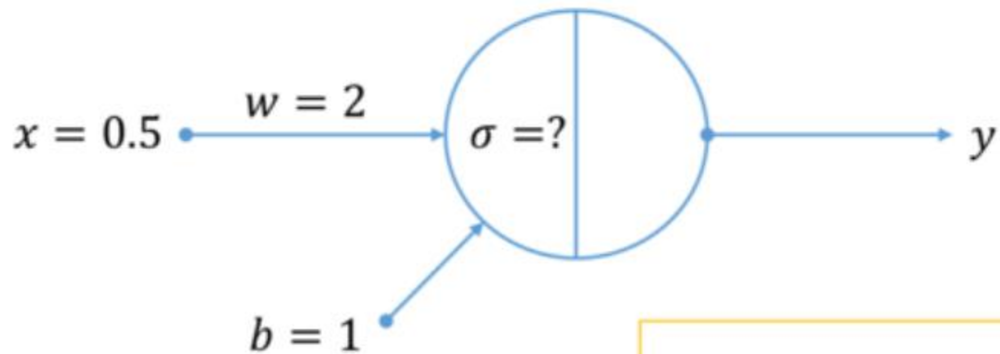
뉴런 연산

- **뉴런 식**



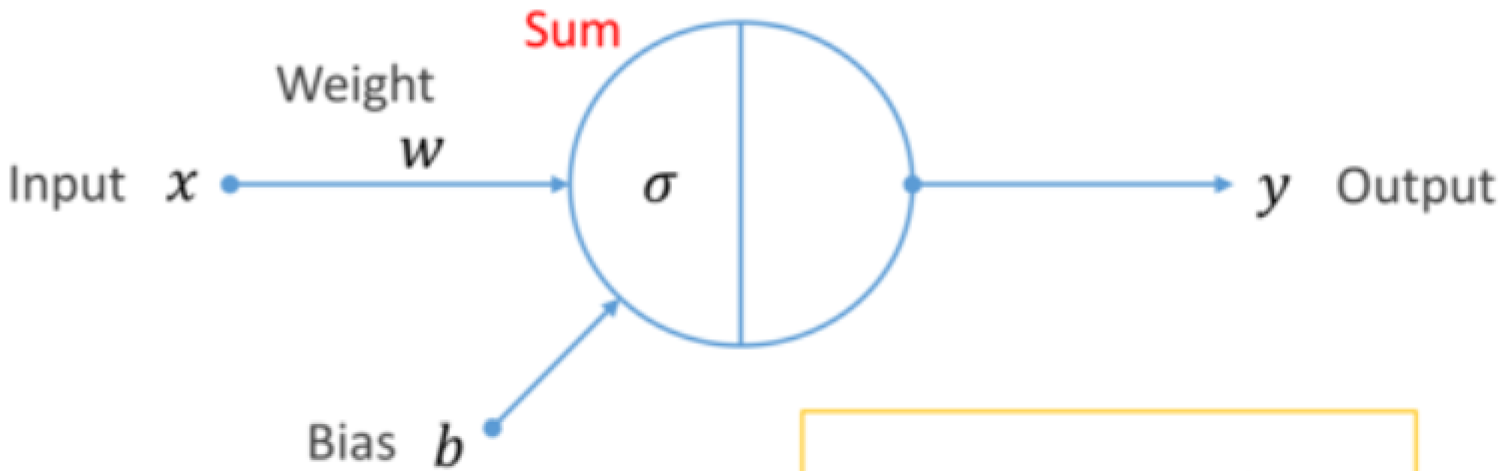
$$\sigma = w \cdot x + b$$

- **가중치와 편향**



$$\begin{aligned}\sigma &= w \cdot x + b \\ &= 2 \cdot 0.5 + 1 \\ &= 2\end{aligned}$$

행렬 곱 연산

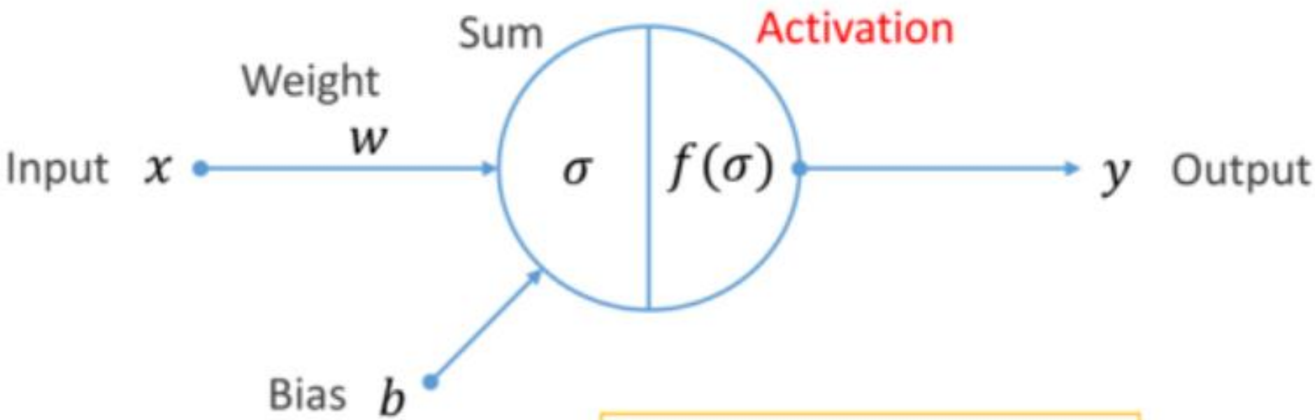


$$\sigma = w \cdot x + b$$
$$= [w\ b]\begin{bmatrix} x \\ 1 \end{bmatrix}$$

행렬의 곱

활성화

- **활성화 함수**
  - 뉴런의 출력 값을 정하는 함수

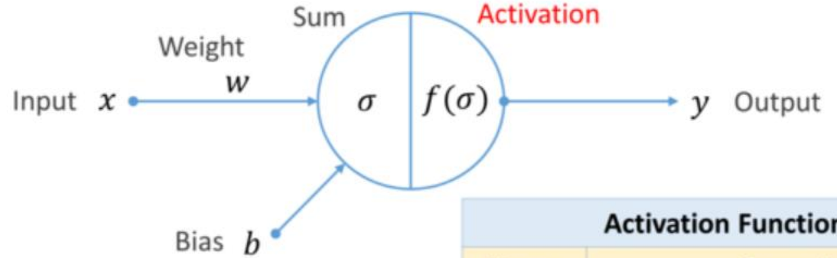


$$\boxed{\sigma = w \cdot x + b}$$

$$\boxed{f(\sigma) = f(w \cdot x + b)}$$

활성화 함수 ReLU, sigmoid

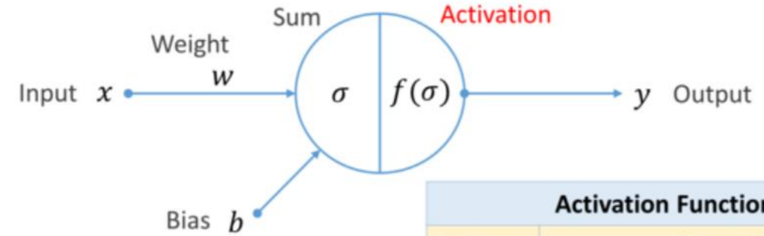
- **ReLU(교재 p43)**
  - Rectified(정류된) Linear Unit(선형 함수, y=x를 의미)
    - **선형 함수를 정류하여 0 이하는 모두 0으로 한 함수**
    - **max(x, 0)**
      - **양수만 사용**
  - 2010년 이후
    - **층이 깊어질수록(deep) 많이 활용**
      - **양수를 그대로 반환하므로 값의 왜곡이 적어지는 효과**
    - **토론토 대학 힌트 교수**
- **Sigmoid**
  - s자 형태의 곡선이라는 의미
    - **예전에 많이 사용**



Activation Functions	
Name	**Formula**
Identity	$f(x) = x$
ReLU	$f(x) = \max(x, 0)$
Sigmoid	$f(x) = \frac{1}{1+e^{-x}}$

*ReLU: Rectified Linear Unit

다양한 활성화 함수 종류



Activation Functions	
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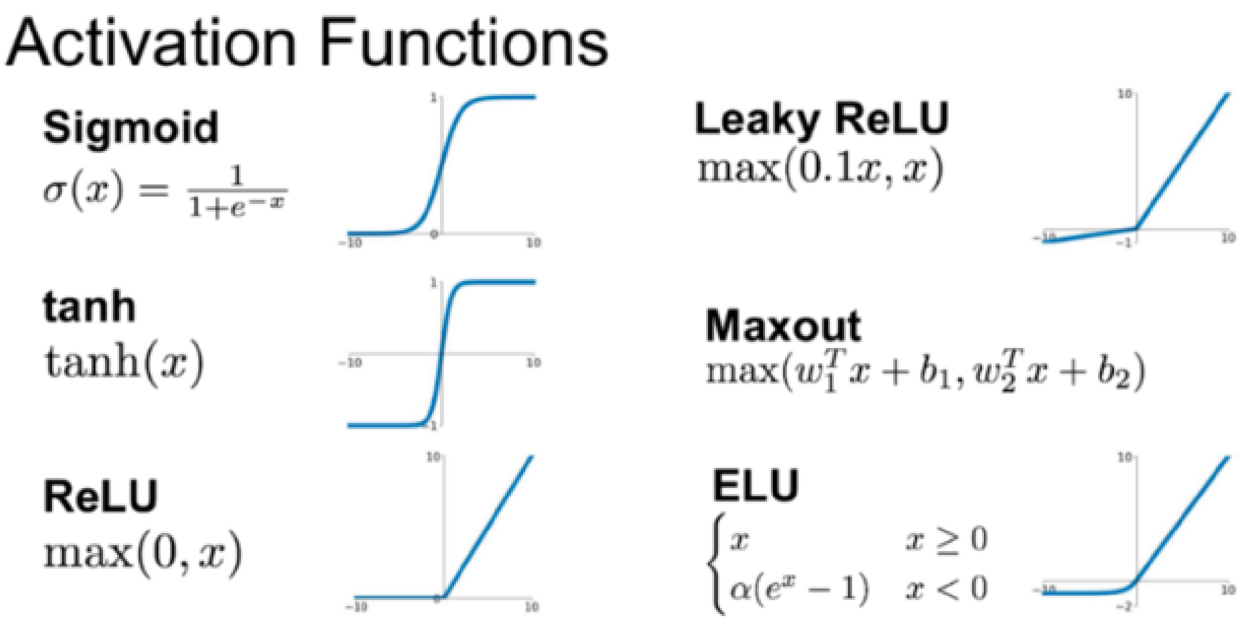
*ReLU: Rectified Linear Unit



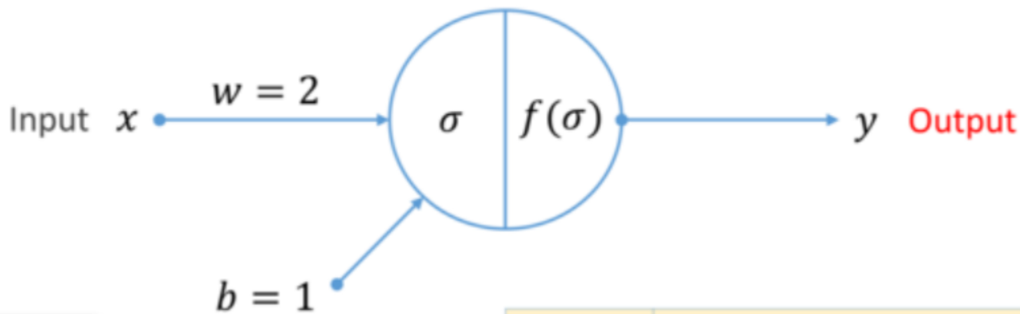
Different Activation Functions and their Graphs

입출력의 예

- **출력 함수로**
  - 동일(identity) 함수(또는 리니어 함수)를 적용

$$y = f(\sigma) = f(w \cdot x + b) = w \cdot x + b$$

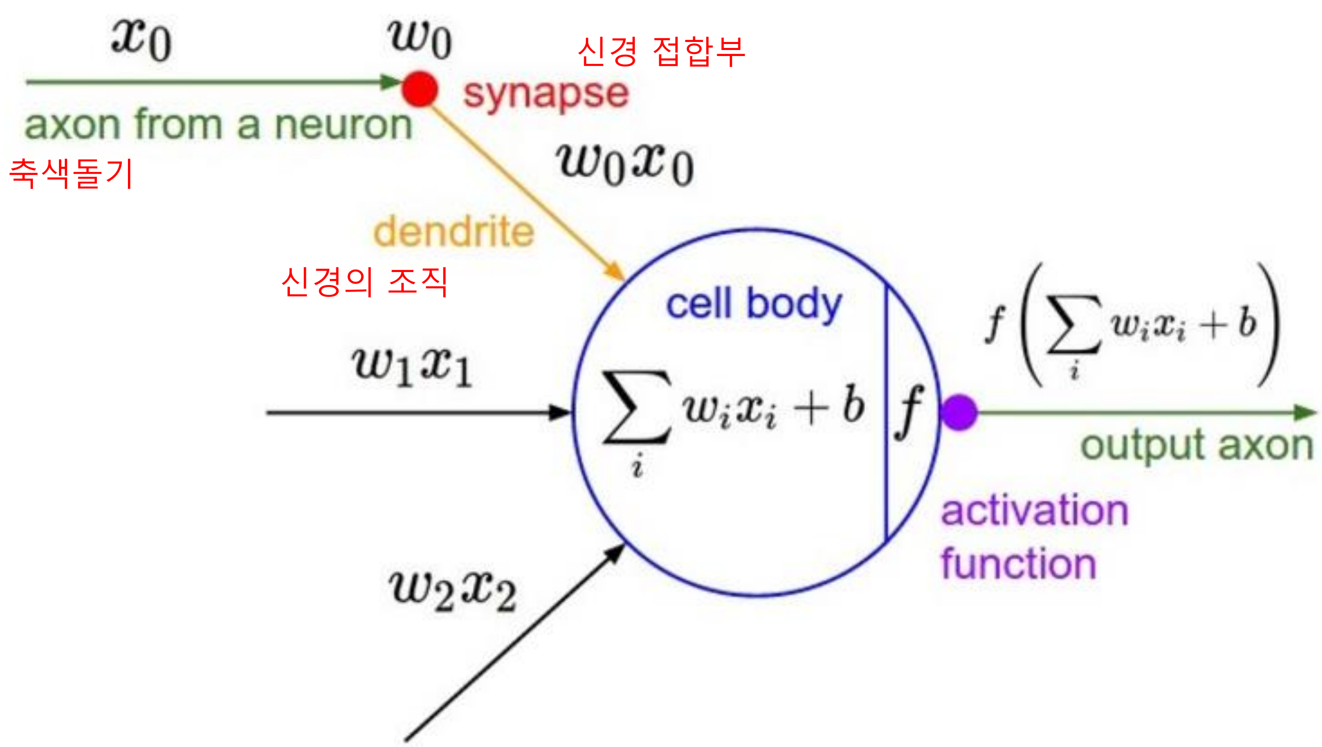
with **identity** (or **linear**) activation functions



$f(x) = x$

Input x	Output y
0	$y = f(2 \cdot 0 + 1) = 1$
1	$y = f(2 \cdot 1 + 1) = 3$
2	$y = f(2 \cdot 2 + 1) = 5$

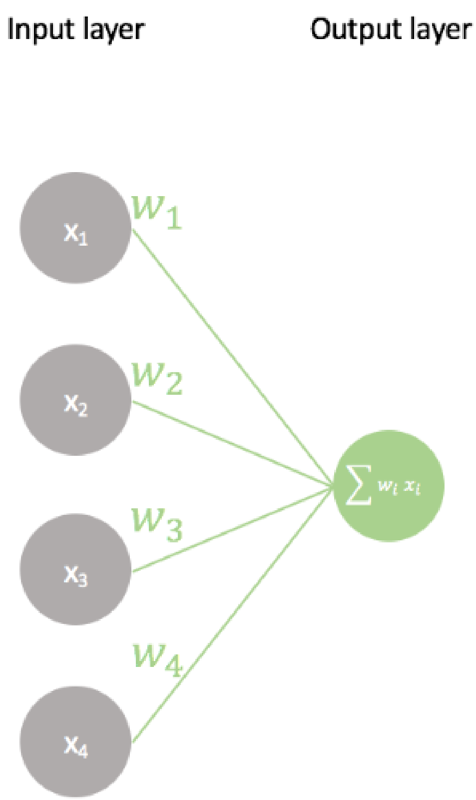
일반화된 인공신경망



x_0
w_0
신경 접합부
synapse
axon from a neuron
축색돌기
$w_0 x_0$
dendrite
신경의 조직
cell body
$\sum_i w_i x_i + b$
f
$f\left(\sum_i w_i x_i + b\right)$
$w_1 x_1$
output axon
activation function
$w_2 x_2$

활성화 함수와 편향

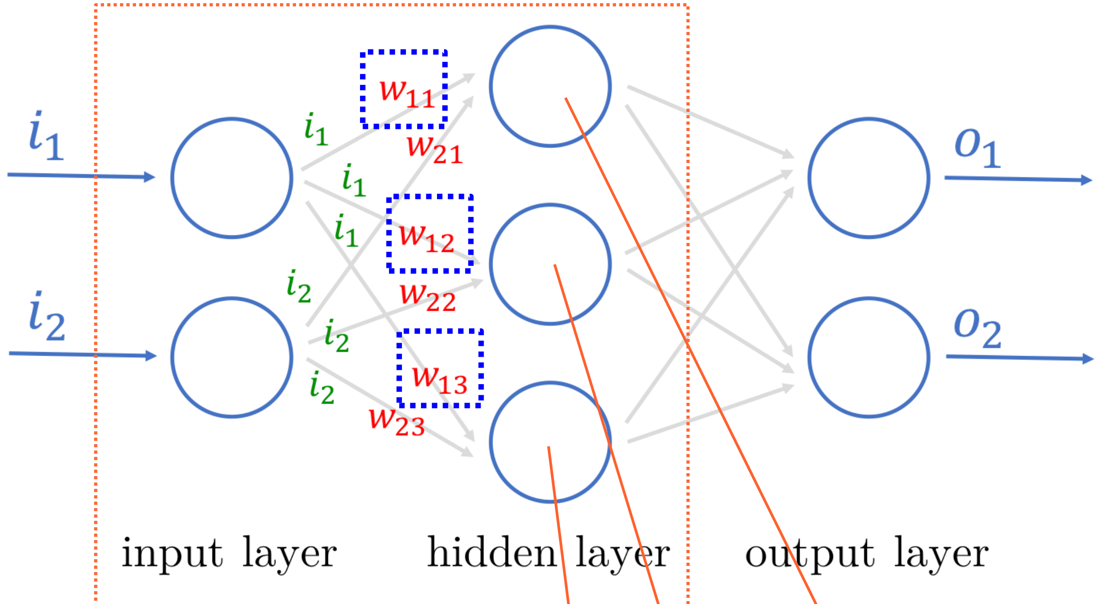
- **결과 값이 임계 값 역할**
  - 결과가 임계 값 이상이면 활성화
  - 결과가 임계 값 미만이면 비활성화



Perceptron Unit

$$f_w(x) = \left\{ \begin{array}{c} \sum w_i x_i \ge \theta \rightarrow \text{neuron fires} \\ \sum w_i x_i < \theta \rightarrow \text{neuron doesn't fire} \end{array} \right\}$$

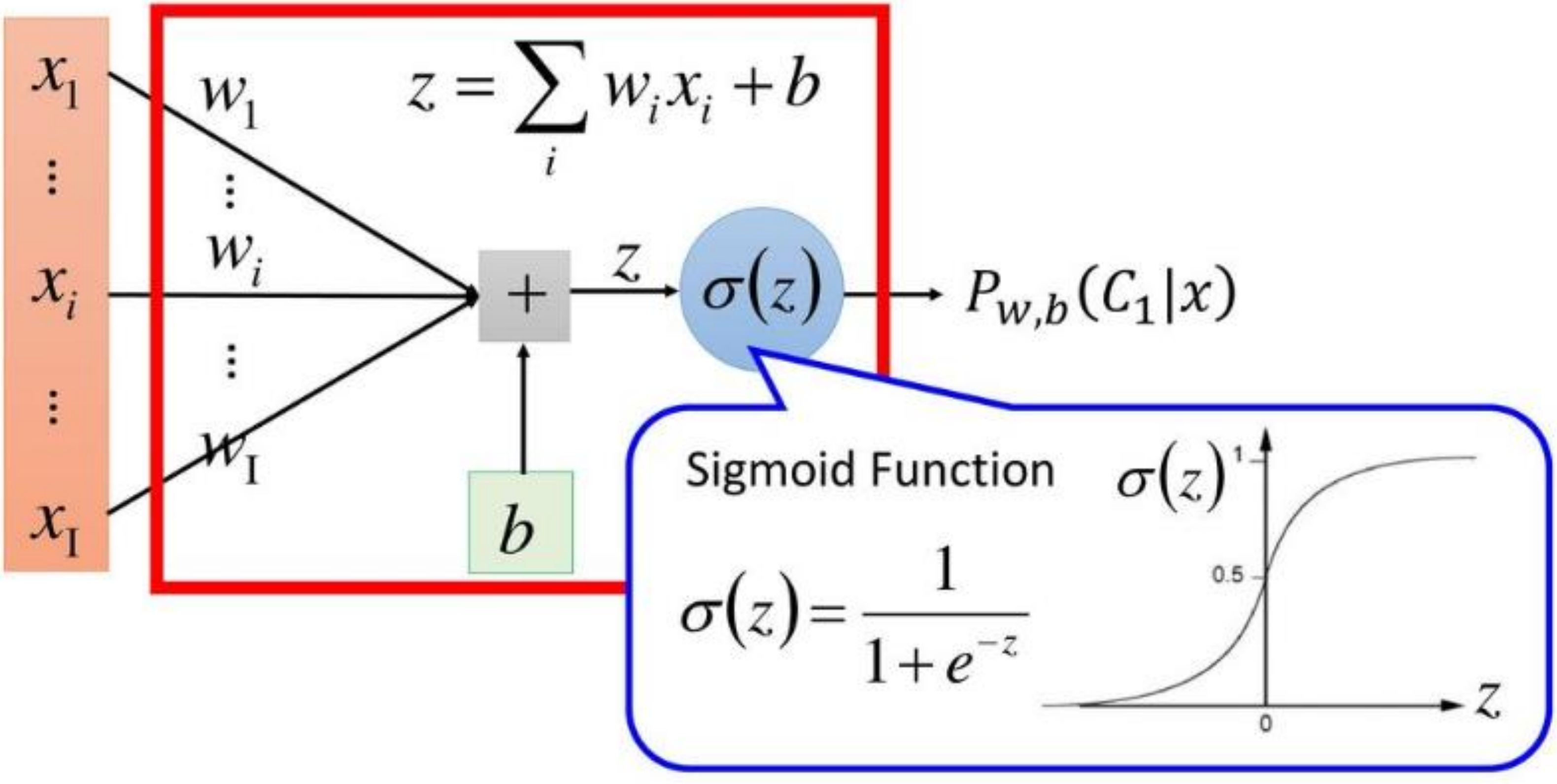
입력 2개, 출력 3개인 신경망 연산



$$\begin{bmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \\ w_{13} & w_{23} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} (w_{11} \times i_1) + (w_{21} \times i_2) \\ (w_{12} \times i_1) + (w_{22} \times i_2) \\ (w_{13} \times i_1) + (w_{23} \times i_2) \end{bmatrix}$$

인공신경망의 시그모이드 함수

- **활성화 함수의 예**
  - 시그모이드 함수
    - **출력 값이 (0~1)**



가중치

- **3 × 3의 가중치 실수**

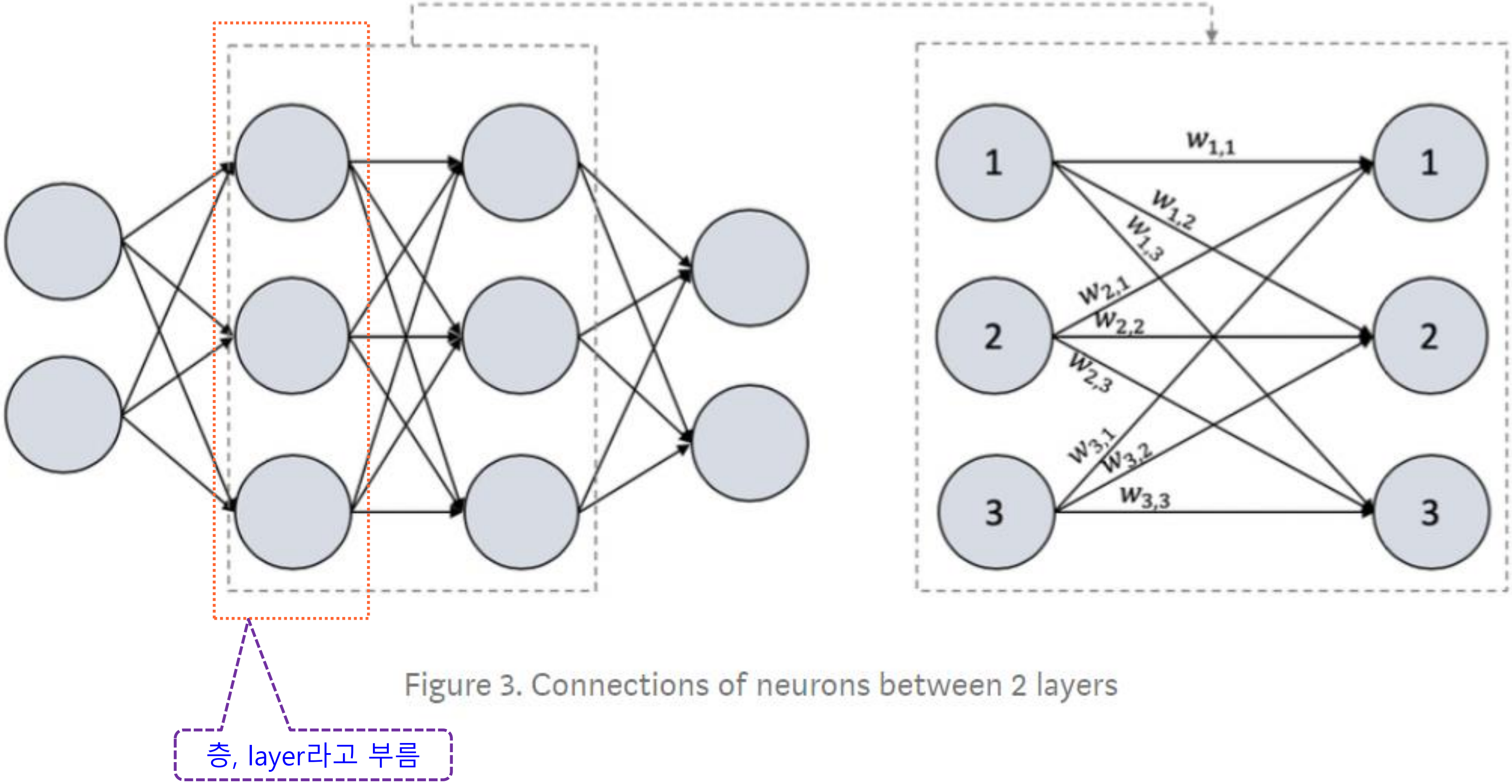


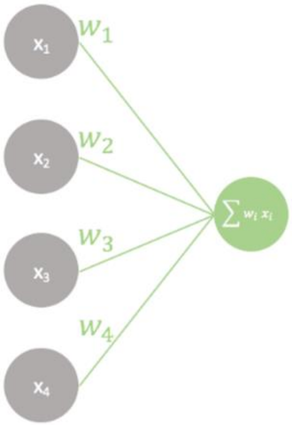
Figure 3. Connections of neurons between 2 layers

https://towardsdatascience.com/neural-networks-i-notation-and-building-blocks-817b1d2ea04b

인공 신경망
행렬 연산

입력의 특징(x_1 x_2 x_3 … x_i)과 입력의 자료 수

- **특징 n개가 있는 뉴런 신경망에서 하나의 출력 계산**



✓ **샘플 수 1개에 대한 계산**

Geektail

$$\boldsymbol{x} = [x_1 \quad x_2 \quad \cdots \quad x_n] \qquad \boldsymbol{w}^T = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

$(1\times n)$ $(n\times 1)$

$$\boldsymbol{x}\boldsymbol{w}^T = w_1x_1 + w_2x_2 + \cdots w_nx_n$$
$$= z \quad \text{스칼라}$$

(1×1)

n: MNIST 손글씨에서 손글씨 이미지 하나의 펙셀 수인 786(28 x 28)을 의미

✓ **샘플 수 s개에 대한 계산**

$$X = \begin{array}{c} n \\ \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{s1} & \cdots & x_{sn} \end{bmatrix} \end{array} s \qquad \boldsymbol{w}^T = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

$(s\times n)$ $(n\times 1)$

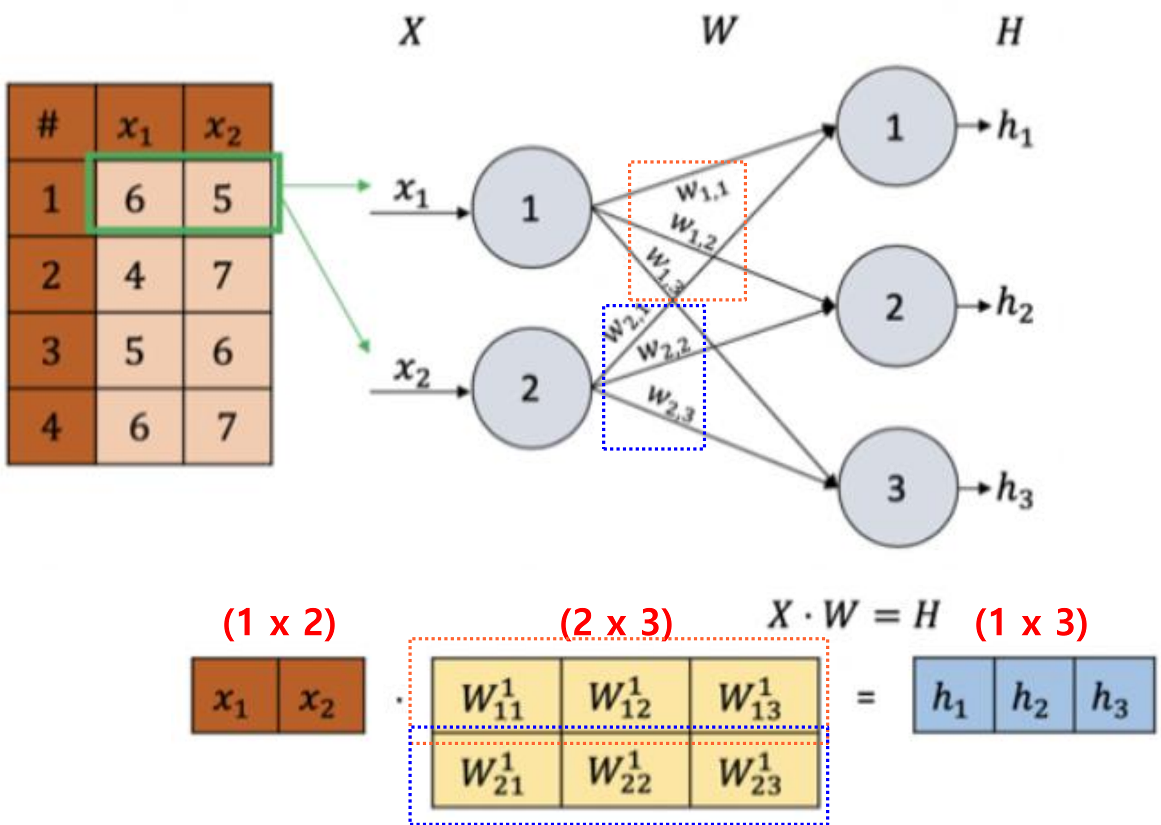
$$X\boldsymbol{w}^T = \begin{bmatrix} w_1x_{11} + w_2x_{12} + \cdots + w_nx_{1n} \\ w_1x_{21} + w_2x_{22} + \cdots + w_nx_{2n} \\ \vdots \\ w_1x_{s1} + w_2x_{s2} + \cdots + w_nx_{sn} \end{bmatrix}$$
$$= \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_s \end{bmatrix} = \boldsymbol{z} \quad \text{벡터}$$

$(s\times 1)$

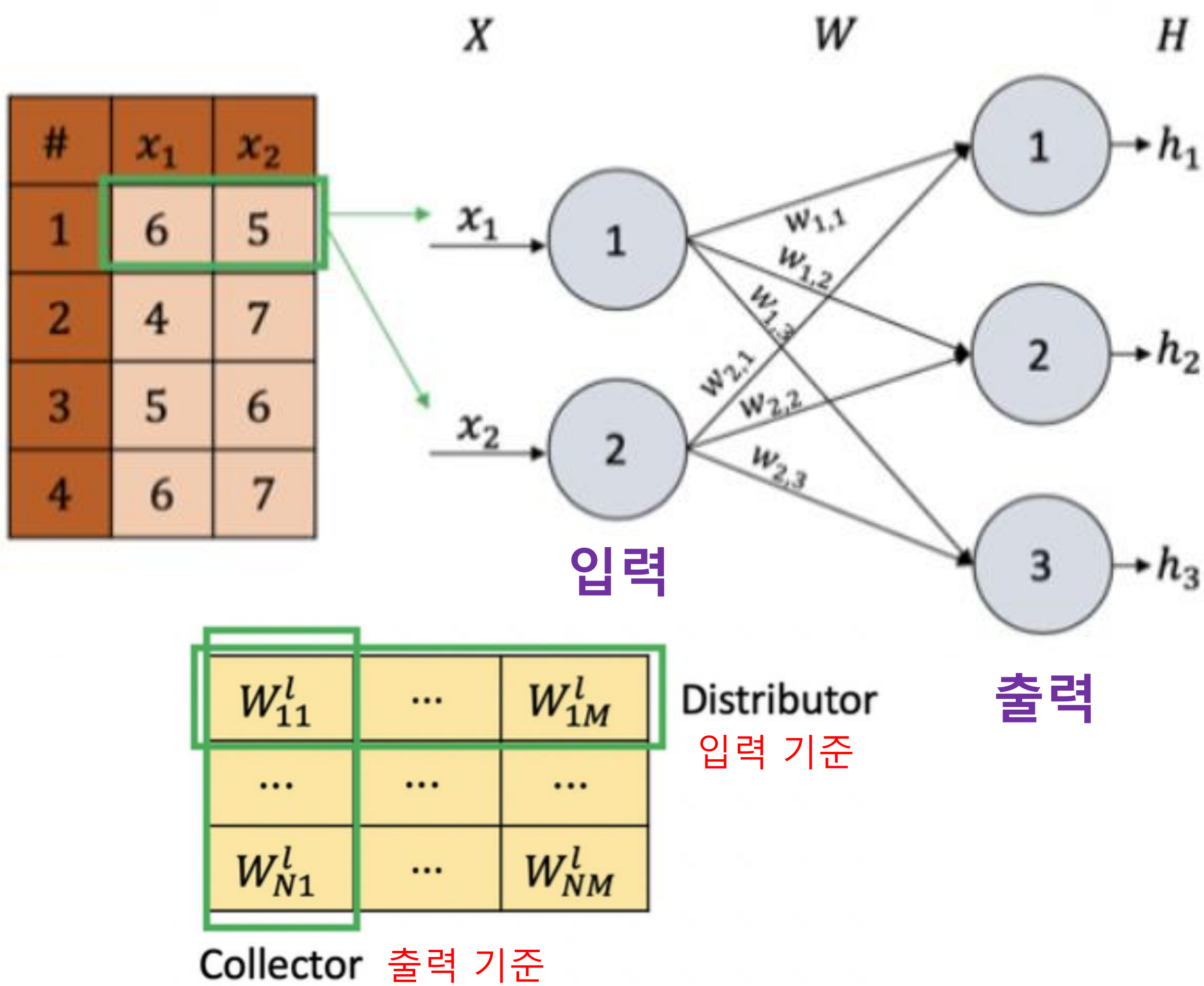
s: MNIST 손글씨에서 훈련 데이터 6만개에서 6만을 의미

신경망 행렬 계산

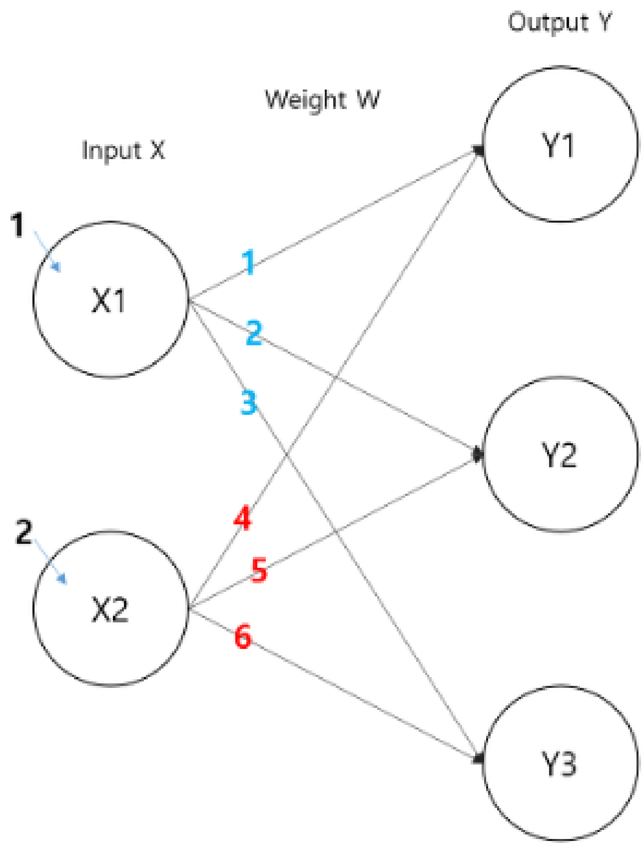
- **특징 2개**
  - 샘플 수 4



뉴런 계산


X
W
H
#
x_1
x_2
1 6 5
2 4 7
3 5 6
4 6 7
x_1
x_2
1
2
$w_{1,1}$
$w_{1,2}$
$w_{1,3}$
$w_{2,1}$
$w_{2,2}$
$w_{2,3}$
1
2
3
h_1
h_2
h_3
입력
출력
W_{11}^l … W_{1M}^l
… … …
W_{N1}^l … W_{NM}^l
Distributor
입력 기준
Collector 출력 기준

계산 사례



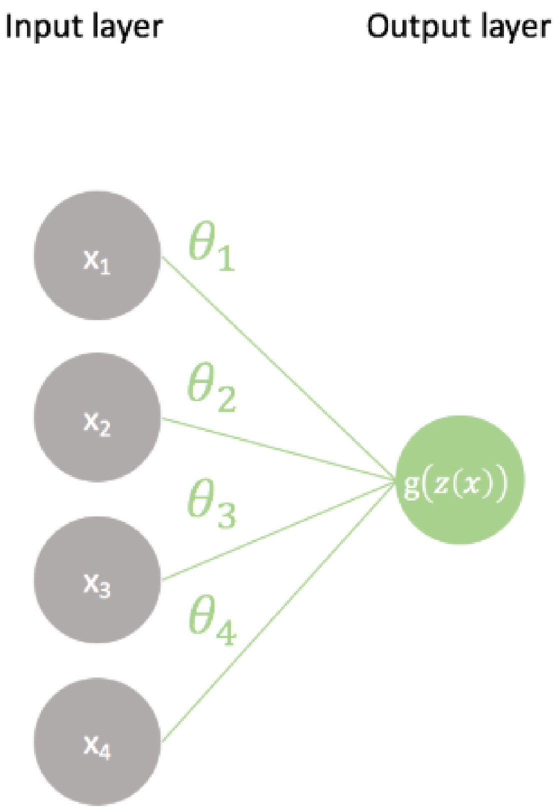
$$X \quad * \quad W \quad = \quad Y$$

$$1 \times 2 \quad * \quad 2 \times 3 \quad = \quad 1 \times 3$$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \quad \begin{pmatrix} 9 & 12 & 15 \end{pmatrix}$$

하나의 출력 뉴런 연산

- **활성화 함수로 시그모이드 함수 적용**



$$\boxed{z(x)} = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + b$$

$$g(z) = \frac{1}{1+e^{\boxed{-z}}}$$

층과 가중치

- **뉴런 층과 가중치 층**

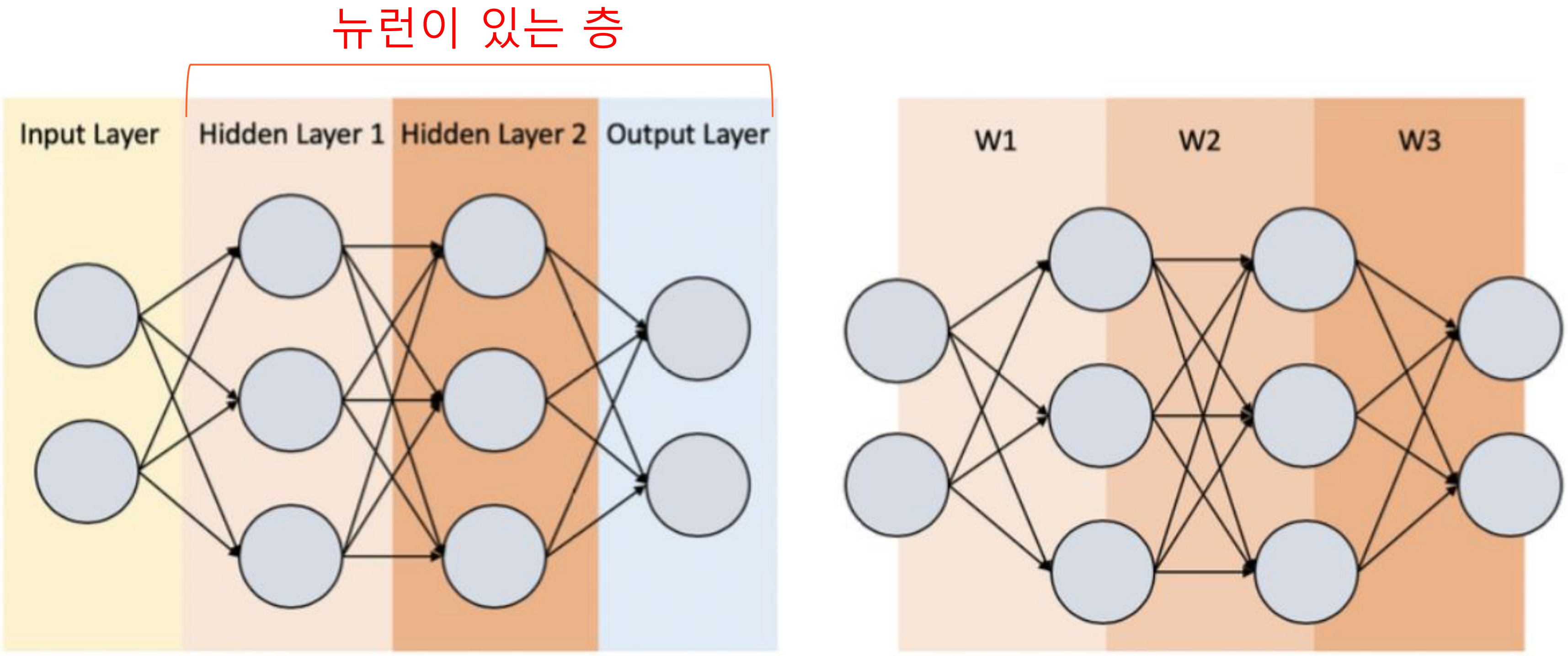
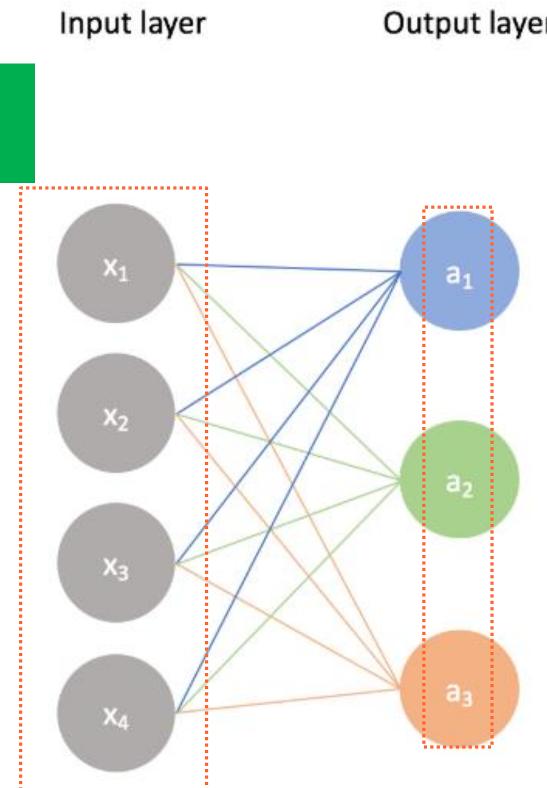


Figure 7. Layers of neuron vs Layers of weights

행렬의 다른 표현

- 입력을 오른쪽 행렬에 배치
- 가중치는 왼쪽 행렬에 배치
- 곱의 순서도 변환



Using multiple observations

$$\begin{bmatrix} w_1 & w_2 & w_3 & w_4 \\ w_1 & w_2 & w_3 & w_4 \\ w_1 & w_2 & w_3 & w_4 \end{bmatrix} \begin{array}{c} \begin{array}{cccc} \text{Observation 1} & \text{Observation 2} & \text{Observation 3} & \text{Observation 4} \end{array} \\ \begin{bmatrix} x_1 & x_1 & x_1 & x_1 \\ x_2 & x_2 & x_2 & x_2 \\ x_3 & x_3 & x_3 & x_3 \\ x_4 & x_4 & x_4 & x_4 \end{bmatrix} \end{array} + \begin{bmatrix} b \\ b \\ b \end{bmatrix} \xrightarrow[\text{activation}]{} \begin{array}{c} \begin{array}{cccc} \text{Observation 1} & \text{Observation 2} & \text{Observation 3} & \text{Observation 4} \end{array} \\ \begin{bmatrix} a_1 & a_1 & a_1 & a_1 \\ a_2 & a_2 & a_2 & a_2 \\ a_3 & a_3 & a_3 & a_3 \end{bmatrix} \end{array}$$

활성화 함수 그리기

실습 파일

- **파일 생성**
  - 06-neuron-activation-dense.ipynb

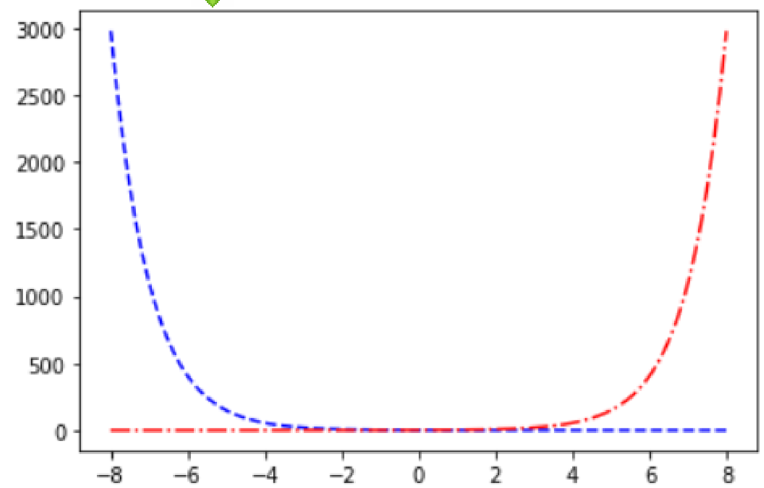
자연수와 자연수의 지수 승

- **e**
  - 자연수, 오일러 수
  - 2.71828
- **y = e^{-x}**
- **y = e^{x}**

```
[76] import numpy as np
     np.e
```

```
2.718281828459045
```

```
[77] import numpy as np
     import matplotlib.pylab as plt

     plt.figure(figsize=(6, 4))
     x = np.linspace(-8, 8, 100)
     plt.plot(x, np.exp(-x), 'b--')
     _ = plt.plot(x, np.exp(x), 'r-.')
```



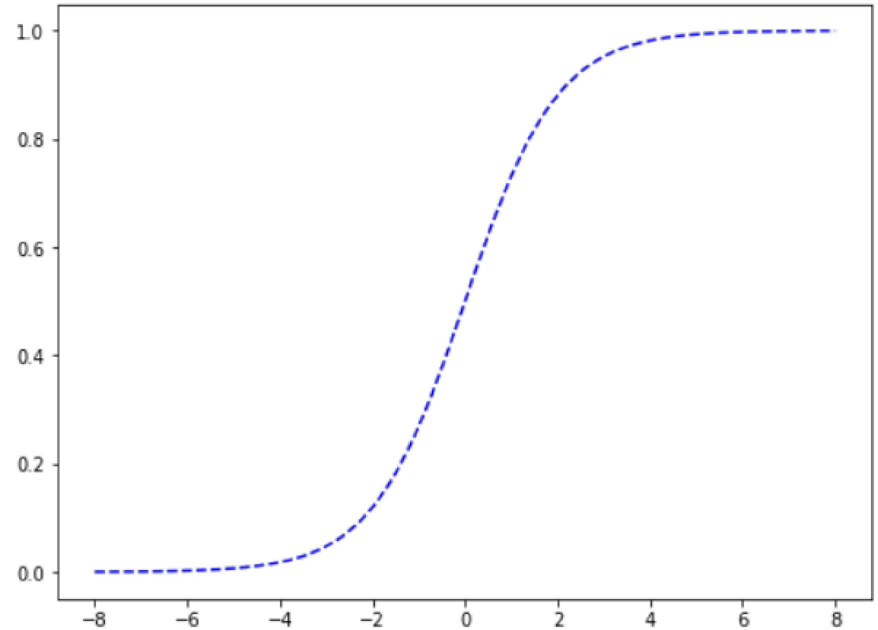
시그모이드 함수

- **S자 곡선**
  - (0, 1) 사이의 값

$$h(x) = \frac{1}{1 + e^{-x}}$$

```
[44]
import numpy as np
import matplotlib.pylab as plt

def sigm_func(x): # sigmoid 함수
    return 1 / (1 + np.exp(-x))

# 시그모이드 함수 그리기
plt.figure(figsize=(8, 6))
x = np.linspace(-8, 8, 100)
plt.plot(x, sigm_func(x), 'b--')
```

```
[<matplotlib.lines.Line2D at 0x7f93b4130cc0>]
```




```
[5]
np.e
```

```
2.718281828459045
```

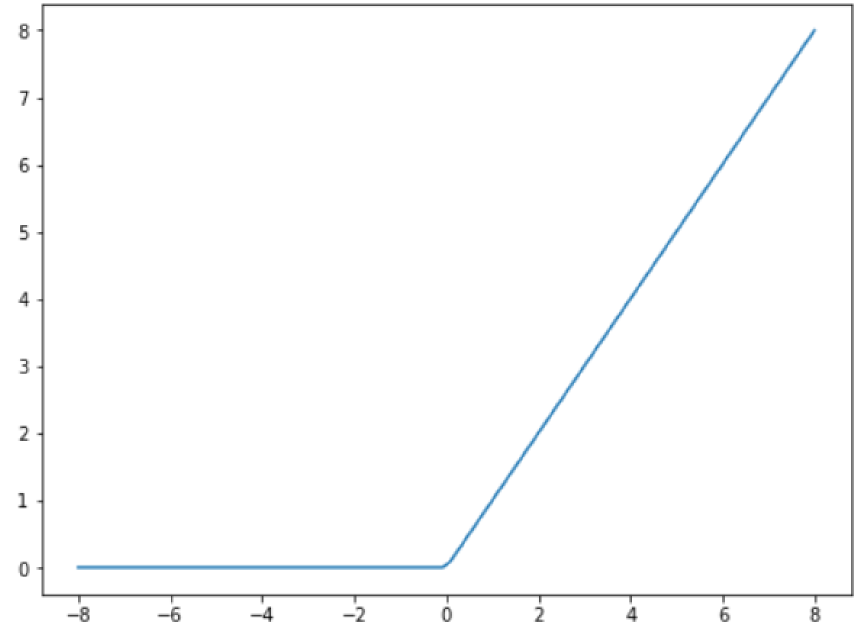
ReLU 함수

- x
  - 0, 음수면 0
  - 양수면 x

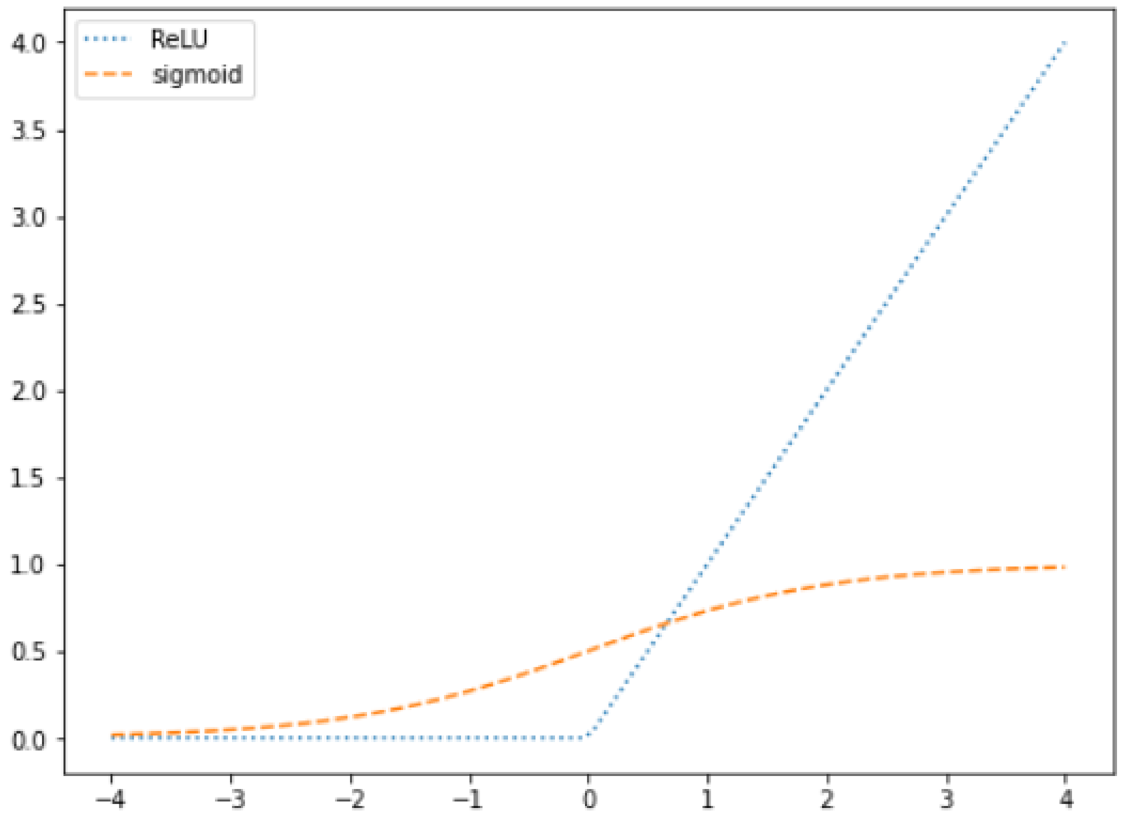
$$h(x) = \begin{cases} 0\ (x \le 0) \\ x(x > 0) \end{cases}$$

[45]
```
import numpy as np
import matplotlib.pylab as plt

def relu_func(x): # ReLU(Rectified Linear Unit, 정류된 선형 유닛) 함수
    return np.maximum(0, x)
    #return (x>0)*x # same

# ReLU 함수 그리기
plt.figure(figsize=(8, 6))
x = np.linspace(-8, 8, 100)
plt.plot(x, relu_func(x))
```

```
[<matplotlib.lines.Line2D at 0x7f93b409b748>]
```



시그모이드 ReLU 함께 그리기

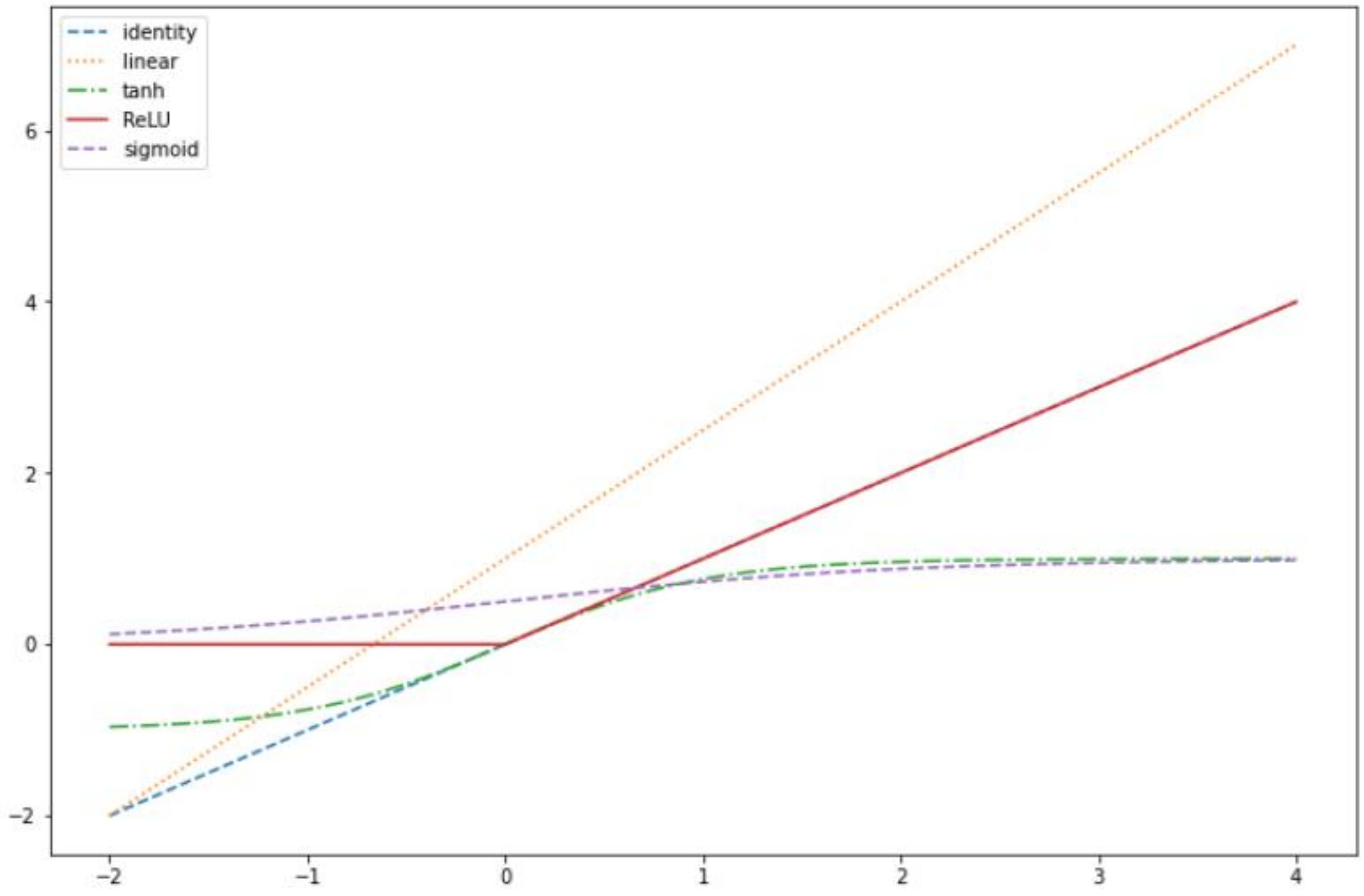



```python
import numpy as np
import matplotlib.pylab as plt

# ReLU(Rectified Linear Unit
# (정류된 선형 유닛) 함수
def relu_func(x):
    return np.maximum(0, x)
    #return (x>0)*x # same

def sigm_func(x): # sigmoid 함수
    return 1 / (1 + np.exp(-x))

# 그래프 그리기
plt.figure(figsize=(8, 6))
x = np.linspace(-4, 4, 100)
y = np.linspace(-0.2, 2, 100)

plt.plot(x, relu_func(x), linestyle=':', label="ReLU")
plt.plot(x, sigm_func(x), linestyle='--', label="sigmoid")
plt.legend(loc='upper left')
```

다양한 활성화 함수

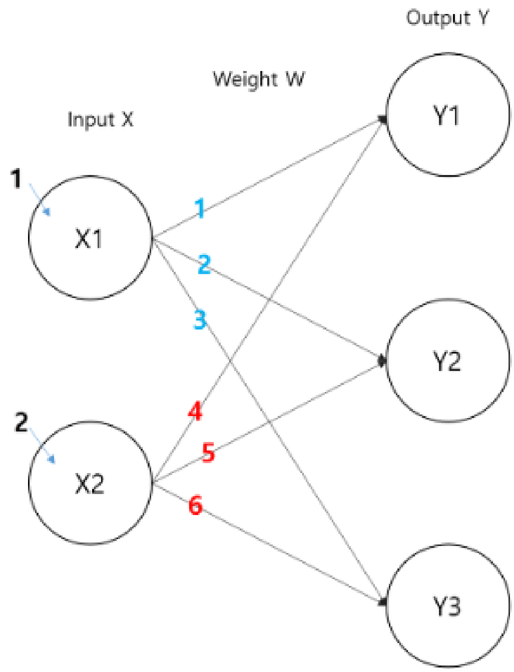
```
import numpy as np
import matplotlib.pylab as plt

def identity_func(x): # 항등함수
    return x

def linear_func(x): # 1차함수
    return 1.5 * x + 1 # a기울기(1.5), Y절편b(1) 조정가능

def tanh_func(x): # TanH 함수
    return np.tanh(x)

def relu_func(x): # ReLU(Rectified Linear Unit, 정류된 선형 유닛) 함수
    return np.maximum(0, x)
    #return (x>0)*x # same

def sigm_func(x): # sigmoid 함수
    return 1 / (1 + np.exp(-x))

# 그래프 그리기
plt.figure(figsize=(12, 8))
x = np.linspace(-2, 4, 100)

plt.plot(x, identity_func(x), linestyle='--', label="identity")
plt.plot(x, linear_func(x), linestyle=':', label="linear")
plt.plot(x, tanh_func(x), linestyle='-.', label="tanh")
plt.plot(x, relu_func(x), linestyle='-', label="ReLU")
plt.plot(x, sigm_func(x), linestyle='--', label="sigmoid")
plt.legend(loc='upper left')
```

활성화 함수 결과



인공 신경망
행렬 연산 코드

계산 사례



X * **W** = **Y**

1 x 2 * 2 x 3 = 1 x 3

$$\begin{pmatrix}1 & 2\end{pmatrix} \quad \begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{pmatrix} \quad \begin{pmatrix}9 & 12 & 15\end{pmatrix}$$

▾ 뉴런의 행렬 연산

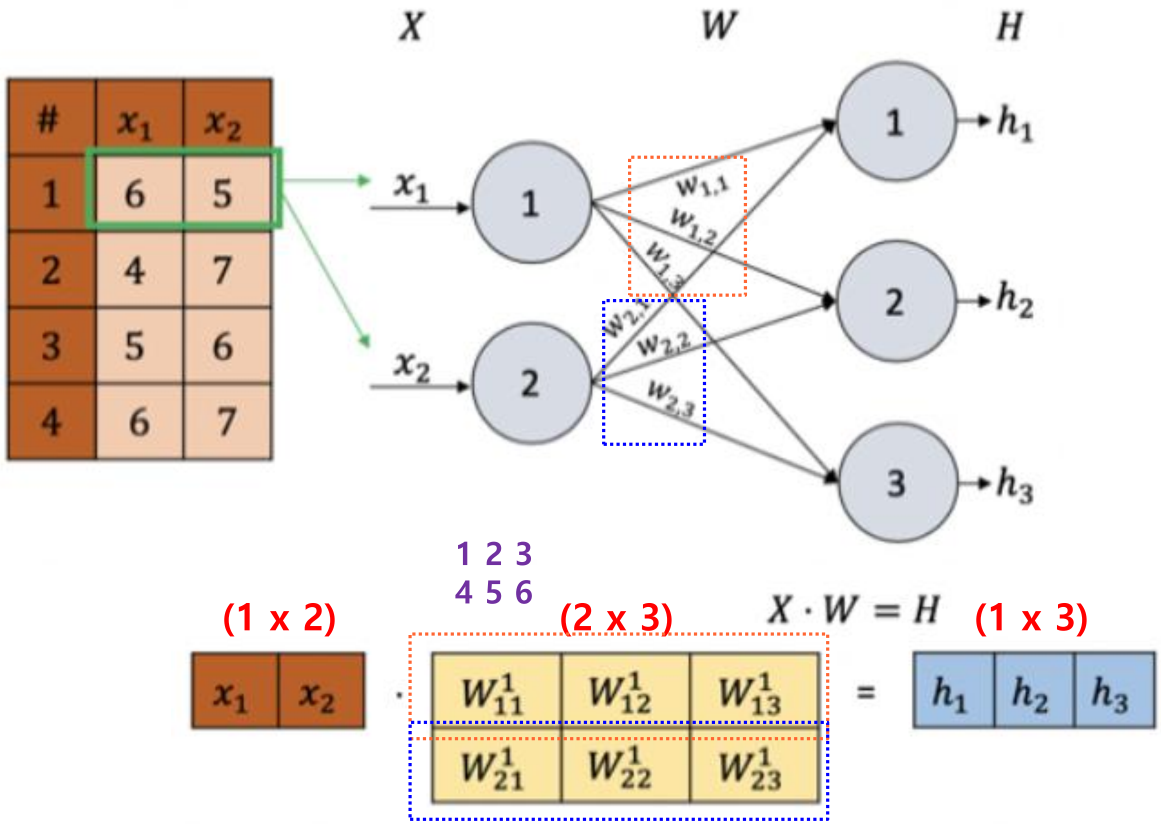
```
[14]  x = [[1, 2]]
      w = [[1, 2, 3], [4, 5, 6]]

      y = tf.matmul(x, w)
      y.numpy()
```

```
array([[ 9, 12, 15]], dtype=int32)
```

신경망 행렬 계산

- **특징 2개**
  - 샘플 수 4

#	x_1	x_2
1	6	5
2	4	7
3	5	6
4	6	7



특징 2, 샘플 수 4개의 행렬 연산

```
[15] x = [[6, 5]]
     w = [[1, 2, 3], [4, 5, 6]]

     y = tf.matmul(x, w)
     y.numpy()
```

```
array([[26, 37, 48]], dtype=int32)
```

```
[16] x = [[6, 5], [4, 7], [5, 6], [6, 7]]
     w = [[1, 2, 3], [4, 5, 6]]

     y = tf.matmul(x, w)
     y.numpy()
```

```
array([[26, 37, 48],
       [32, 43, 54],
       [29, 40, 51],
       [34, 47, 60]], dtype=int32)
```

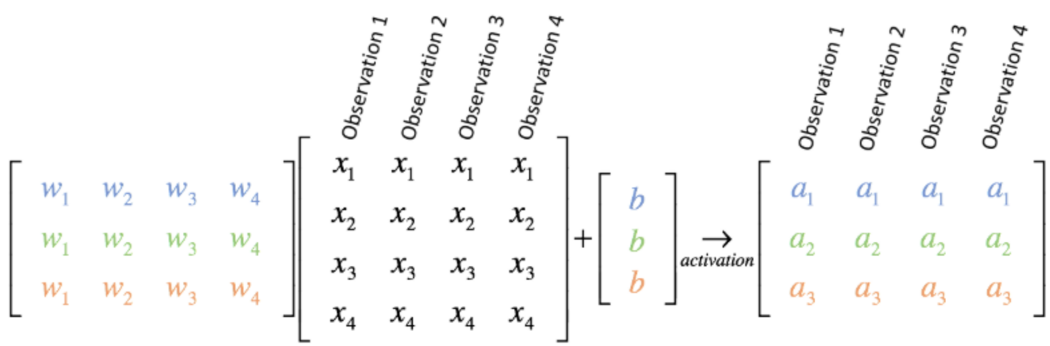
행렬의 순서를 바꾼 계산

- **가중치와 입력 값의 순서도 수정**

```
[78] #x = [[6, 5], [4, 7], [5, 6], [6, 7]]
     #w = [[1, 2, 3], [4, 5, 6]]

     w = [[1, 4], [2, 5], [3, 6]]
     x = [[6, 4, 5, 6], [5, 7, 6, 7]]

     y = tf.matmul(w, x)
     y.numpy()
```

```
array([[26, 32, 29, 34],
       [37, 43, 40, 47],
       [48, 54, 51, 60]], dtype=int32)
```

Using multiple observations

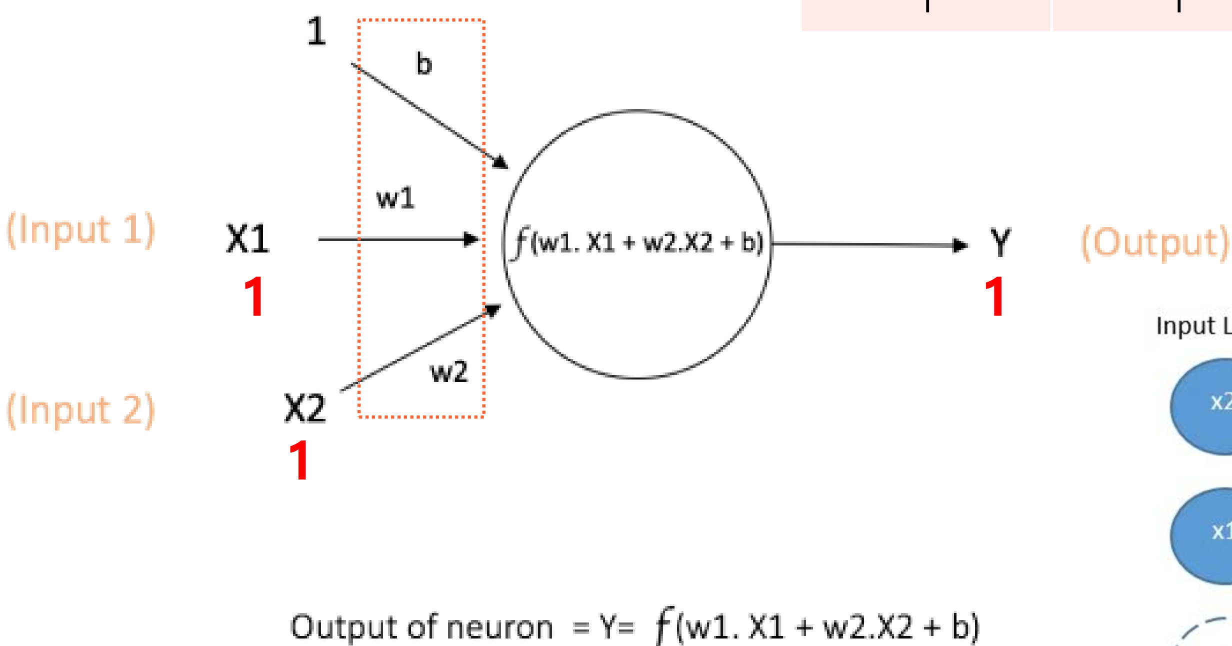


논리 게이트 AND OR XOR 신경망 구현

AND 게이트 구현

- **뉴런 구조**
  - 입력 2개, 편향, 출력 1
  - 구할 값
    - **가중치 2개와 편향 1개**

x1	x2	y
0	0	0
0	1	0
1	0	0
1	1	1



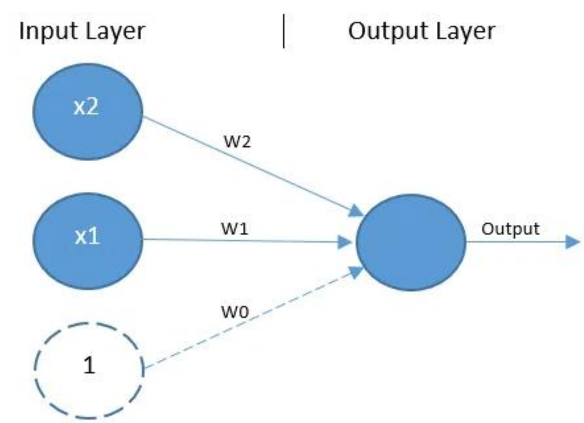


Figure 2: Single Layer Perceptron Network

```
# tf.keras 를 이용한 AND 네트워크 계산
import numpy as np
x = np.array([[1,1], [1,0], [0,1], [0,0]])
y = np.array([[1], [0], [0], [0]])

model = tf.keras.Sequential([
    tf.keras.layers.Dense(units=1, activation='sigmoid', input_shape=(2,)),
])

model.compile(optimizer=tf.keras.optimizers.SGD(lr=0.3), loss='mse')
model.summary()
```

```
Model: "sequential_4"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_6 (Dense)              (None, 1)                 3
=================================================================
Total params: 3
Trainable params: 3
Non-trainable params: 0
_________________________________________________________________
```

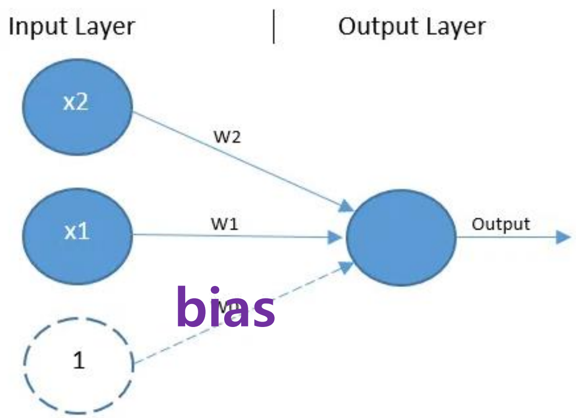


Figure 2: Single Layer Perceptron Network

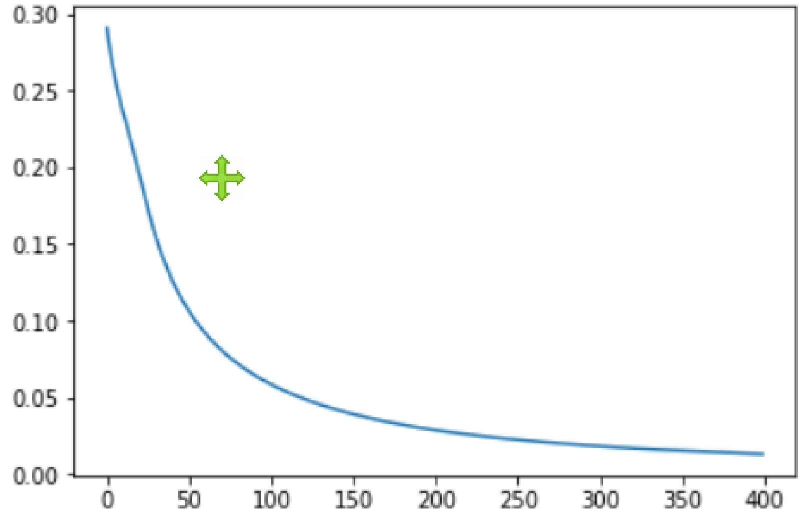
```
history = model.fit(x, y, epochs=400, batch_size=1)
```

```
4/4 [==============================] - 0s 1ms/step - loss: 0.0145
Epoch 372/400
4/4 [==============================] - 0s 2ms/step - loss: 0.0144
Epoch 373/400
4/4 [==============================] - 0s 2ms/step - loss: 0.0144
Epoch 374/400
```

손실 값 그래프와 결과 예측

```
[54] # 3.34 2-레이어 XOR 네트워크의 loss 변화를 선 그래프로 표시
     import matplotlib.pyplot as plt
     plt.plot(history.history['loss'])
```

```
[<matplotlib.lines.Line2D at 0x7efe712955f8>]
```




```
[59] model.predict(x)
```

```
array([[0.8535386 ],
       [0.12342612],
       [0.12364785],
       [0.00339741]], dtype=float32)
```

가중치와 편향 값 알아 보기

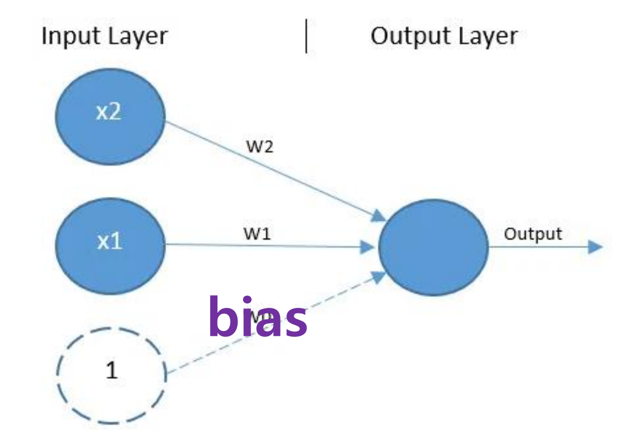


Figure 2: Single Layer Perceptron Network

```
[60] for weight in model.weights:
         print(weight)
```

```
<tf.Variable 'dense_6/kernel:0' shape=(2, 1) dtype=float32, numpy=
array([[3.7209592],
       [3.723007 ]], dtype=float32)>
<tf.Variable 'dense_6/bias:0' shape=(1,) dtype=float32, numpy=array([-5.6813374], dtype=float32)>
```

```
[61] model.weights[0]
```

```
<tf.Variable 'dense_6/kernel:0' shape=(2, 1) dtype=float32, numpy=
array([[3.7209592],
       [3.723007 ]], dtype=float32)>
```

```
[62] model.weights[1]
```

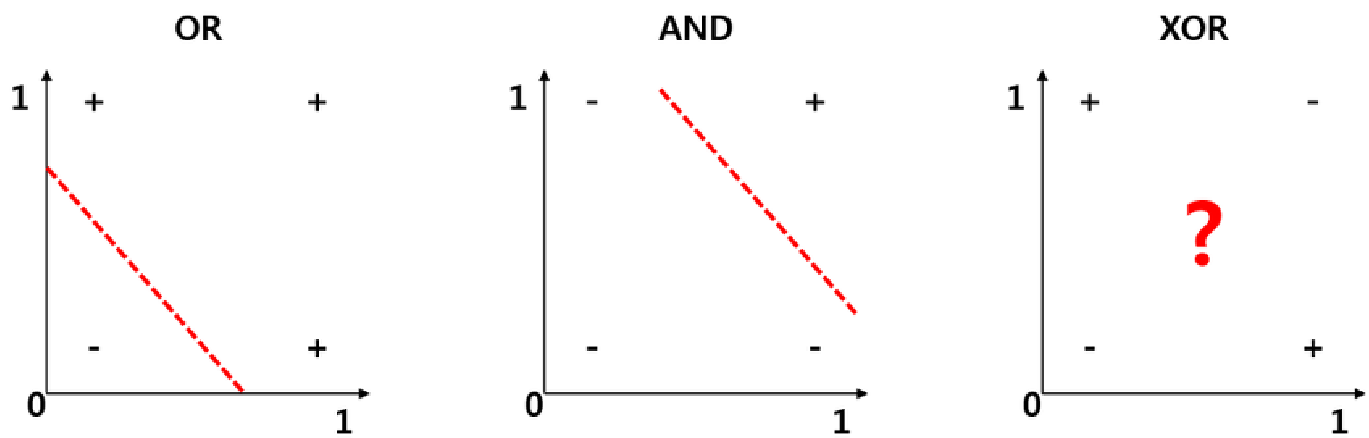
```
<tf.Variable 'dense_6/bias:0' shape=(1,) dtype=float32, numpy=array([-5.6813374], dtype=float32)>
```

OR 게이트 구현

- **여러분이 직접 해 보세요.**

XOR 문제

- **하나의 퍼셉트론으로는 XOR 게이트는 불가능**
  - 마빈 민스키와 시모어 페퍼트가 증명
  - 첫 AI 겨울의 계기



x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

XOR 해결

- **뉴런 3 개의 2층으로 가능**
  - 모델이 구해야 할 총 매개변수(가중치와 편향)
    - **3 * 2 + 3 * 1 = 9개**

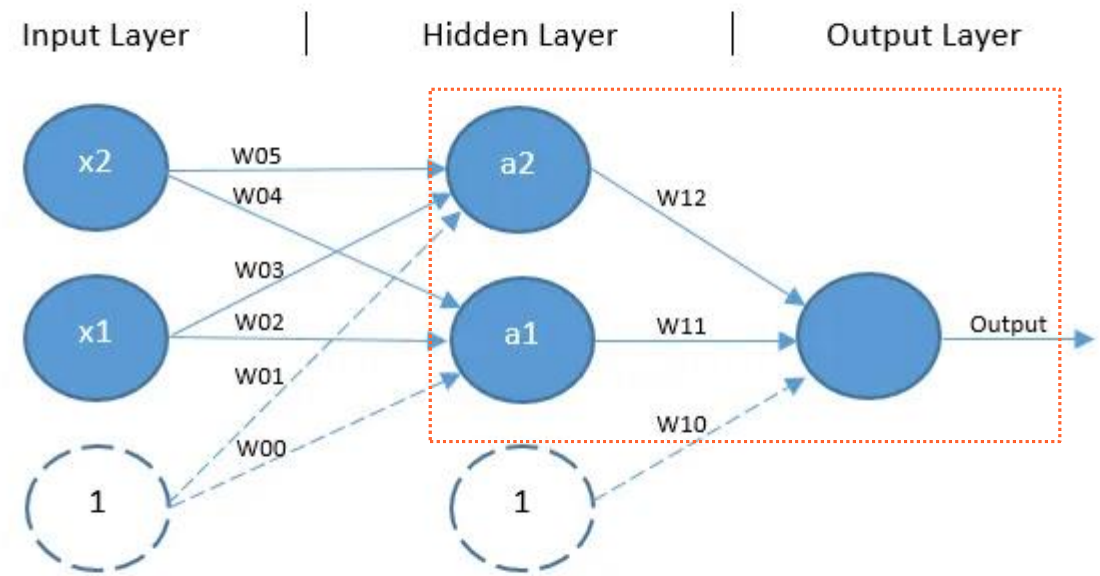


Figure 4: Multilayer Pereceptron Architecture for XOr

Sequential 모델

- **Dense 층**
  - 가장 기본적인 층
  - 인자 units, activation
    - **뉴런 수와 활성화 함수**
  - 인자 input_shape
    - **첫 번째 층에서만 정의**
    - **입력의 차원을 명시**
      - **(2,)**
      - **2개의 입력을 받는 1차원**

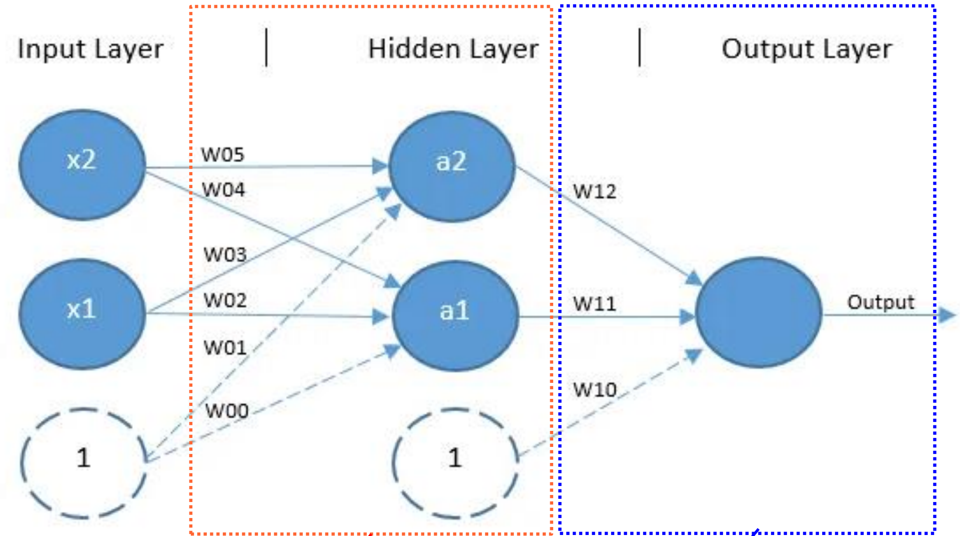


Figure 4: Multilayer Pereceptron Architecture for XOr

```
x = np.array([[1,1], [1,0], [0,1], [0,0]])
y = np.array([[0], [1], [1], [0]])

model = tf.keras.Sequential([
    tf.keras.layers.Dense(units=2, activation='sigmoid', input_shape=(2,)),
    tf.keras.layers.Dense(units=1, activation='sigmoid')
])
```

Sequential 모델과 딥러닝 구조

```
Model: "sequential_1"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_2 (Dense)              (None, 2)                 6
_________________________________________________________________
dense_3 (Dense)              (None, 1)                 3
=================================================================
Total params: 9
Trainable params: 9
Non-trainable params: 0
_________________________________________________________________
```

- **입력, 은닉, 출력 층**
  - 패러미터 수
    - **(입력측 뉴런 수 + 1) * (출력측 뉴런 수)**

```python
x = np.array([[1,1], [1,0], [0,1], [0,0]])
y = np.array([[0], [1], [1], [0]])

model = tf.keras.Sequential([
    tf.keras.layers.Dense(units=2, activation='sigmoid', input_shape=(2,)),
    tf.keras.layers.Dense(units=1, activation='sigmoid')
])
```

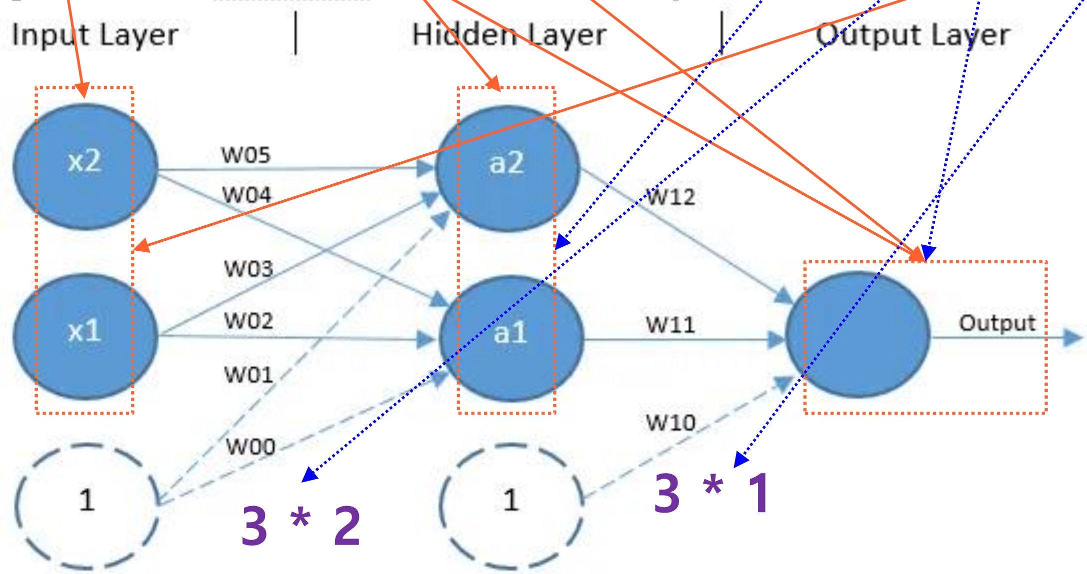


Figure 4: Multilayer Pereceptron Architecture for XOr

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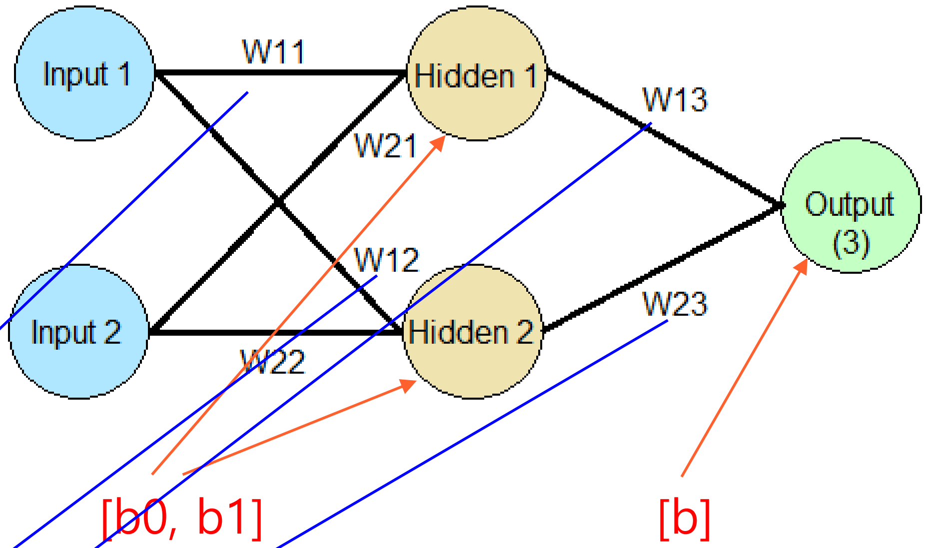
XOR 게이트 구현 소스

```python
# 3.27 tf.keras 를 이용한 XOR 네트워크 계산
import tensorflow as tf
import numpy as np
x = np.array([[1,1], [1,0], [0,1], [0,0]])
y = np.array([[0], [1], [1], [0]])

model = tf.keras.Sequential([
    tf.keras.layers.Dense(units=2, activation='sigmoid', input_shape=(2,)),
    tf.keras.layers.Dense(units=1, activation='sigmoid')
])

model.compile(optimizer=tf.keras.optimizers.SGD(lr=0.3), loss='mse')
model.summary()

# 3.28 tf.keras 를 이용한 XOR 네트워크 학습
history = model.fit(x, y, epochs=2000, batch_size=1)

# 3.29 tf.keras 를 이용한 XOR 네트워크 평가
print(model.predict(x))

# 3.30 XOR 네트워크의 가중치와 편향 확인
for weight in model.weights:
    print(weight)
```

```
Epoch 1999/2000
4/4 [==============================] - 0s 2ms/step - loss: 0.0017
Epoch 2000/2000
4/4 [==============================] - 0s 1ms/step - loss: 0.0017
[[0.04060324]
 [0.9609237 ]
 [0.96031225]
 [0.04571233]]
<tf.Variable 'dense_2/kernel:0' shape=(2, 2) dtype=float32, numpy=
array([[ 5.378626 , -5.299518 ],
       [-5.518024 ,  5.0764313]], dtype=float32)>
<tf.Variable 'dense_2/bias:0' shape=(2,) dtype=float32, numpy=array([-3.0511274, -2.8576972], dtype=float32)>
<tf.Variable 'dense_3/kernel:0' shape=(2, 1) dtype=float32, numpy=
array([[7.6903753],
       [7.7518315]], dtype=float32)>
<tf.Variable 'dense_3/bias:0' shape=(1,) dtype=float32, numpy=array([-3.8067687], dtype=float32)>
```

가중치와 model.weights

- **가중치 결과**
  - 층/kernel
- **편향 결과**
  - 층/bias




```
<tf.Variable 'dense_2/kernel:0' shape=(2, 2) dtype=float32, numpy=
array([[ 5.378626 , -5.299518 ],
       [-5.518024 ,  5.0764313]], dtype=float32)>
<tf.Variable 'dense_2/bias:0' shape=(2,) dtype=float32, numpy=array([-3.0511274, -2.8576972], dtype=float32)>
<tf.Variable 'dense_3/kernel:0' shape=(2, 1) dtype=float32, numpy=
array([[7.6903753],
       [7.7518315]], dtype=float32)>
<tf.Variable 'dense_3/bias:0' shape=(1,) dtype=float32, numpy=array([-3.8067687], dtype=float32)>
```

XOR 모델의 학습 과정 시각화

- **손실(loss) 또는 오류 값의 변화**
  - 가로는 에폭의 수
    - **학습 횟수가 증가하면서 계속 손실은 작아짐**

```
# 3.34 2-레이어 XOR 네트워크의 loss 변화를 선 그래프로 표시
import matplotlib.pyplot as plt

plt.plot(history.history['loss'])
```