Experiment 3

Stress distribution in a Tower bridge

Aim:

Finding principal stresses for a two-dimensional simply supported beam by finding the eigenvalues of the stress matrix. Also, visualizing the Eigen values of stress matrix for a simply supported beam.

Mathematical Background:

The **principal stresses** for a two dimensional simply supported beam **are the eigenvalues** of the stress matrix (say S). The stress matrices (2-D and 3-D) are given by:

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \quad \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

The sigma components are normal stresses and tau components are Shear Stresses. If we change the orientation of the plane, the normal stress component will vary. There exists a **special orientation** where the **normal stresses are maximum**, and these planes are called principal planes and the normal stresses acting on them are called the **principal stresses**. Principal Stress is nothing but maximum normal stress which will happen when Shear Stress are zero, i.e., **the elements other than the diagonal elements in the stress matrix are zero.** Here comes the use of **Diagonalization**. After Diagonalization, the diagonalized matrix will look like:

$$\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

Here, sigma(1, 2 and 3) are the eigen values of the stress matrix which are actually values of principal stresses.

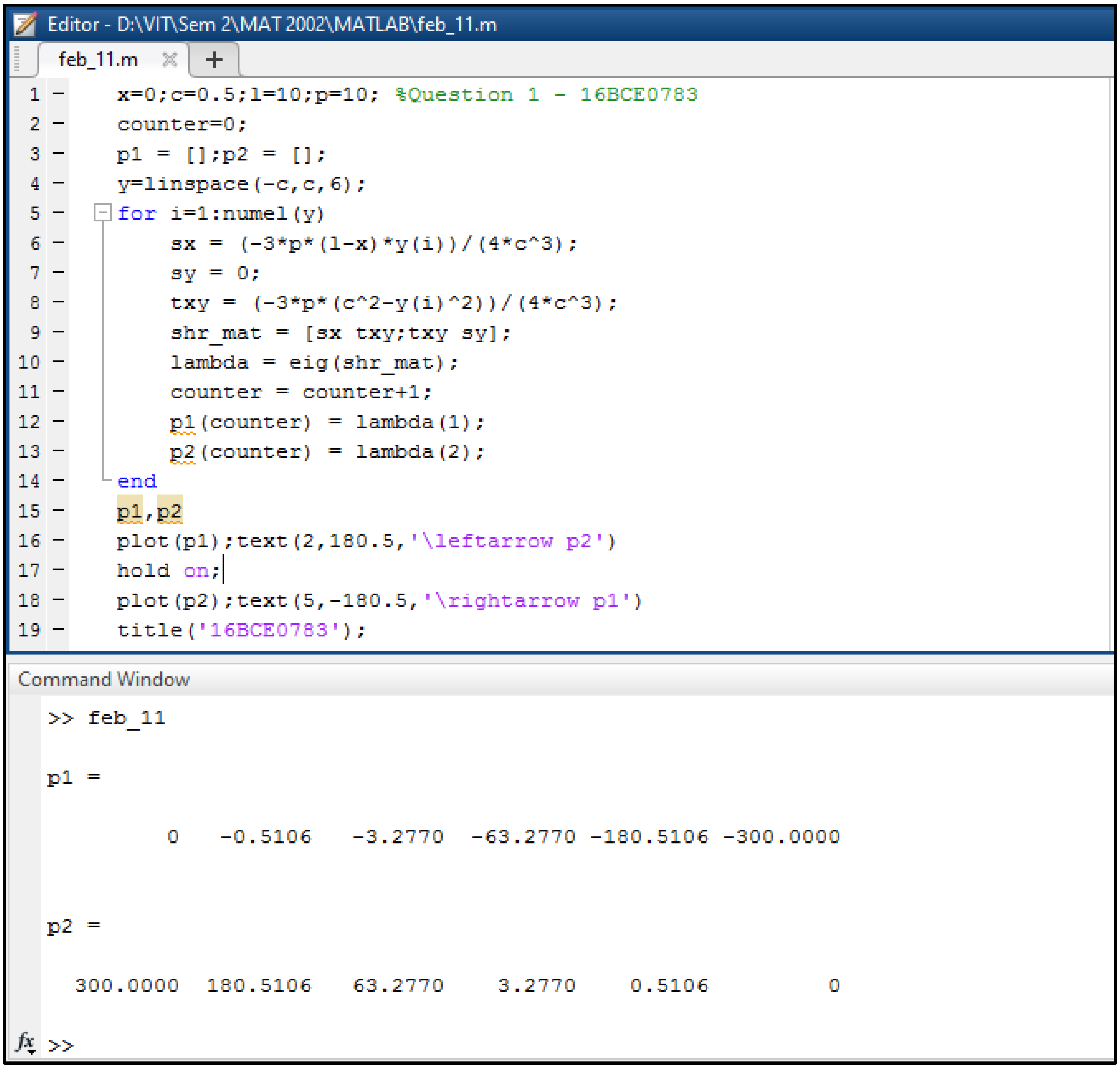
Question 1: Stress distribution in a simply supported beam (normal 2D Plot) (only y varies)

Code:

```
x=0;c=0.5;l=10;p=10; %Question 1 - 16BCE0783
counter=0;
p1 = [];p2 = [];
```

```
y=linspace(-c,c,6);
for i=1:numel(y)
    sx = (-3*p*(l-x)*y(i))/(4*c^3);
    sy = 0;
    txy = (-3*p*(c^2-y(i)^2))/(4*c^3);
    shr_mat = [sx txy;txy sy];
    lambda = eig(shr_mat);
    counter = counter+1;
    p1(counter) = lambda(1);
    p2(counter) = lambda(2);
end
p1,p2
plot(p1);text(2,180.5,'\leftarrow p2')
hold on;
plot(p2);text(5,-180.5,'\rightarrow p1')
title('16BCE0783');
```

Output:



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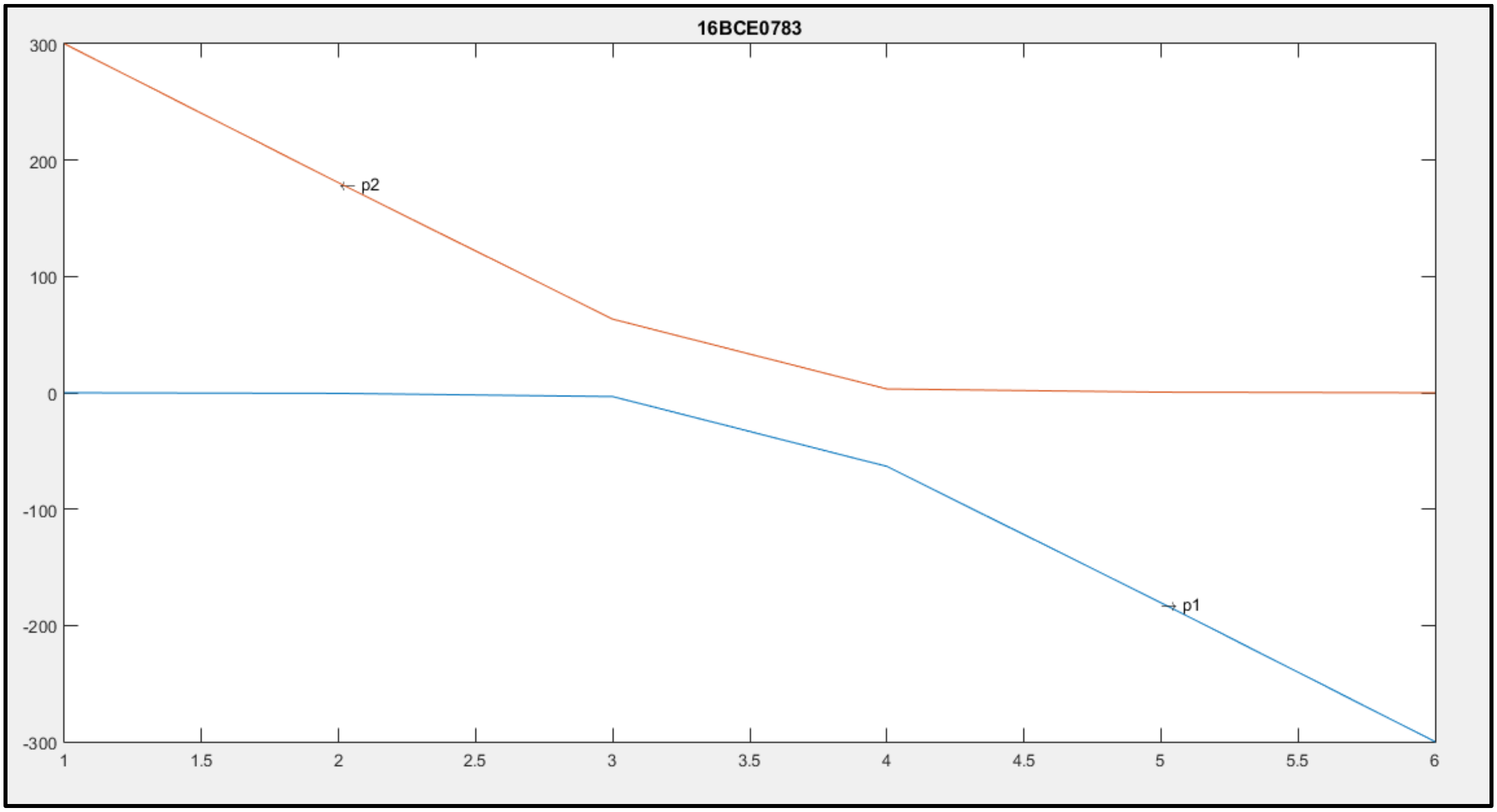
feb_11.m

```
x=0;c=0.5;l=10;p=10; %Question 1 - 16BCE0783
counter=0;
p1 = [];p2 = [];
y=linspace(-c,c,6);
for i=1:numel(y)
    sx = (-3*p*(l-x)*y(i))/(4*c^3);
    sy = 0;
    txy = (-3*p*(c^2-y(i)^2))/(4*c^3);
    shr_mat = [sx txy;txy sy];
    lambda = eig(shr_mat);
    counter = counter+1;
    p1(counter) = lambda(1);
    p2(counter) = lambda(2);
end
p1,p2
plot(p1);text(2,180.5,'\leftarrow p2')
hold on;
plot(p2);text(5,-180.5,'\rightarrow p1')
title('16BCE0783');
```

Command Window

```
>> feb_11

p1 =

         0   -0.5106   -3.2770  -63.2770 -180.5106 -300.0000


p2 =

  300.0000  180.5106   63.2770    3.2770    0.5106         0

>>
```

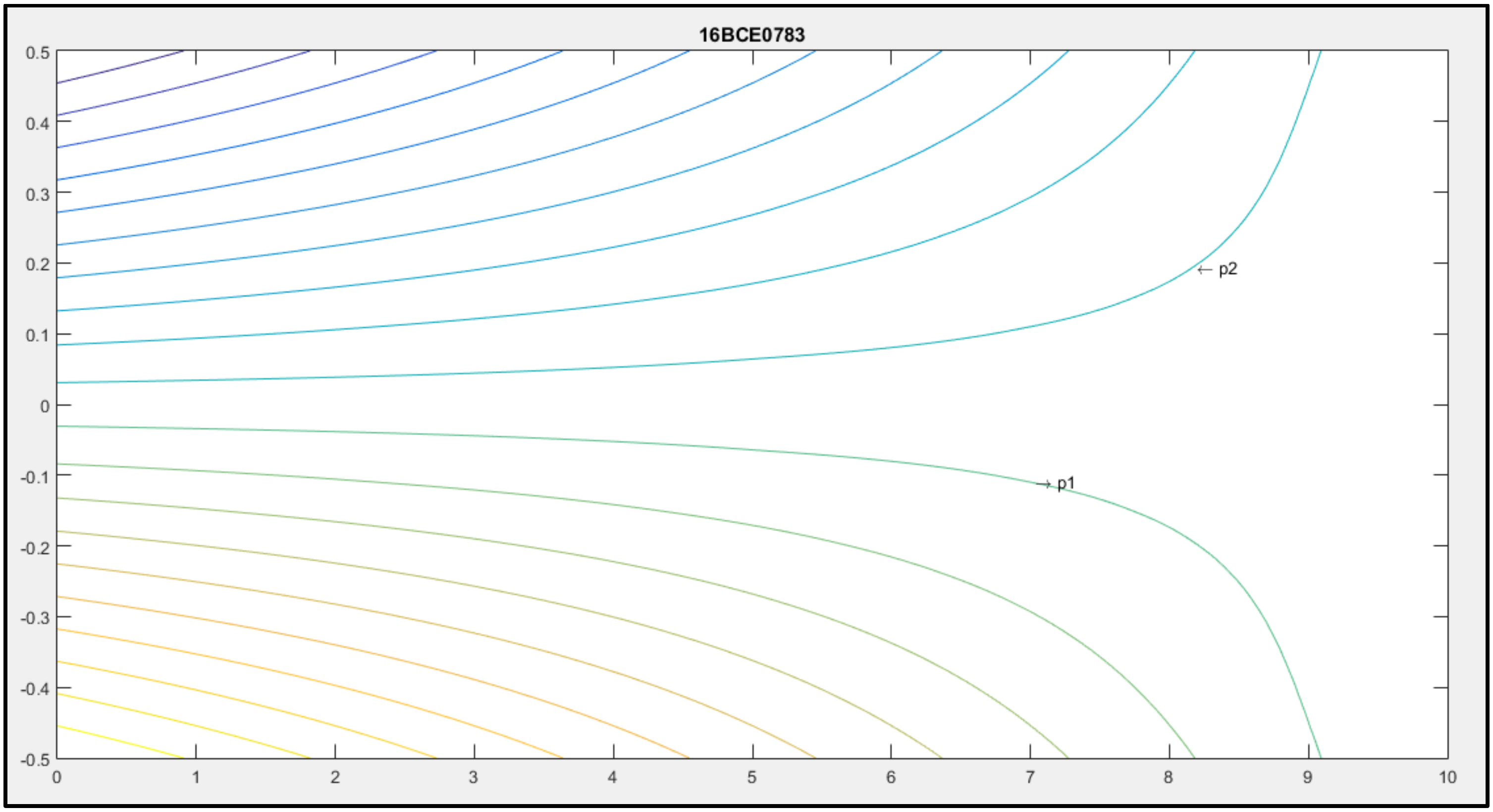


Question 2: Stress distribution in a simply supported beam (contour Plot) (both x and y vary)

Code:

```
c=0.5;l=10;p=10;
p1 = [];p2 = [];
x=linspace(0,l,200);
y=linspace(-c,c,24);
[X,Y] = meshgrid(x,y);
for i=1:numel(x)
    for j=1:numel(y)
        sx = (-3*p*(l-X(j,i)).*Y(j,i))/(4*c^3);
        sy = 0;
        txy = (-3*p*(c^2-Y(j,i).^2))/(4*c^3);
        shr_mat = [sx txy;txy sy];
        lambda = eig(shr_mat);
        p1(j,i) = lambda(1);
        p2(j,i) = lambda(2);
    end
end
contour(X,Y,p2,10);text(8.191,0.1957,'\leftarrow p2')
hold on;
contour(X,Y,p1,10);text(7.035,-0.1087,'\rightarrow p1')
title('16BCE0783')
```

Output:


16BCE0783
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
1
2
3
4
5
6
7
8
9
10
p2
p1