Experiment 5

Laplace Transform - Evaluation and Application

I. Aim

Evaluation of Laplace transform and inverse Laplace transform using MATLAB and applying Laplace transform in solving first order ordinary differential equations.

II. Mathematical Background:

The laplace transform of a function f(t) is defined as:

$$F(s) = \mathcal{L}\left\{f(t)\right\} = \int_0^\infty e^{-st} f(t) dt,$$

If F(s) is laplace transform of f(t), then f(t) is said to be inverse laplace transform of F(s) and is denoted by:

$$f(t) = \mathcal{L}^{-1}\left\{F(s)\right\}.$$

Our code made is to solve a given first order DE using LaPlace Transform.

Let the given DE be: a*y'(t) + b*y(t) = f(t)

Let $y(0) = y_0$ and L{f(t)} = F(s) and L{y(t)} = Y(s)

Now, taking LaPlace on both sides of given DE, we have,

$$Y(s) = \frac{ay_0}{(as+b)} + \frac{F(s)}{(as+b)}$$

Now, taking inverse LaPlace on this equation can give us y(t).

III. MATLAB Code:

Editor - D:\VIT\MATLAB\laplace.m

laplace.m | laplace_final.m | +

```matlab
% 1 is a number but 1 + 0*t and t^0 and 1+t-t are symbols
% laplace(sinh(a*x),x,w) -  output should be in form of w (laplace wrt x)
% ilaplace (1/s) is 1 (but uotput is symbol not number(function of t))
% ilaplace(sinh(a*x),x,w) (same as line 3)
% x = sym('x(t)') to declare x as a function of t.
clc
clear all
syms t s Y
a = input('Enter the coeff of dy/dt: ');
b = input('Enter the coeff of y: ');
c = input('Enter the inhomogeneous term: ');
y0 = input('Enter initial condition: ');
y = sym('y(t)');
y1 = diff(y,t);
deq = a*y1 + b*y - c;
%LTE =
%2*s*laplace(y(t), t, s) - 2*y(0) - 1/s + 3*laplace(y(t), t, s)
%MATLAB cannot solve for laplace(y(t)) so put it in a variable
LTE = laplace(deq,t,s);
% from LTE substitute in respective element in curly braces
LTE = subs(LTE,{'laplace(y(t), t, s)','y(0)'},{Y,y0});
Y = simplify(solve(LTE,Y));
y = ilaplace(Y,s,t);
disp(['The solution of the DE is y(t) = ',char(y)]);
```

Command Window

IV. MATLAB I/O:

```matlab
deq = a*y1 + b*y - c;
%LTE =
%2*s*laplace(y(t), t, s) - 2*y(0) - 1/s + 3*laplace(y(t), t, s)
```

Command Window

```
Enter the coeff of dy/dt: 5
Enter the coeff of y: 2
Enter the inhomogeneous term: t*cos(t)
Enter initial condition: 0
The solution of the DE is y(t) = (105*cos(t))/841 - (105*exp(-(2*t)/5))/841 - (100*sin(t))/841 + (2*t*cos(t))/29 + (5*t*sin(t))/29
>> 
```

V. Question - Answers:

Q1 Answer

Editor - D:\VIT\MATLAB\Refined\laplace_final.m

laplace_final.m

```
clc
clear all
syms t s Y
a = input('Enter the coeff of dy/dt: ');
b = input('Enter the coeff of y: ');
c = input('Enter the inhomogeneous term: ');
y0 = input('Enter initial condition: ');
y = sym('y(t)');
y1 = diff(y,t);
deq = a*y1 + b*y - c;
LTE = laplace(deq,t,s);
LTE = subs(LTE,{'laplace(y(t), t, s)','y(0)'},{Y,y0});
Y = simplify(solve(LTE,Y));
y = ilaplace(Y,s,t);
la1 = laplace(y);
la2 = laplace(c);
transfer = la1/la2;
disp(['The tranfer function is :',char(transfer)]);
```

Command Window

```
Enter the coeff of dy/dt: 4
Enter the coeff of y: 11
Enter the inhomogeneous term: (t^2 - 1)
Enter initial condition: 5
The tranfer function is :-(6744/(1331*(s + 11/4)) - 89/(1331*s) - 8/(121*s^2) + 2/(11*s^3))/(1/s - 2/s^3)
>>
```

Q2 – Answer

A second order DE can be solved by rewriting the second order DE as a System of first order DE