

Experiment 5

Laplace Transform - Evaluation and Application

I. Aim

Evaluation of Laplace transform and inverse Laplace transform using MATLAB and applying Laplace transform in solving first order ordinary differential equations.

II. Mathematical Background:

The laplace transform of a function $f(t)$ is defined as:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt,$$

If $F(s)$ is laplace transform of $f(t)$, then $f(t)$ is said to be inverse laplace transform of $F(s)$ and is denoted by:

$$f(t) = \mathcal{L}^{-1}\{F(s)\}.$$

Our code made is to solve a given first order DE using LaPlace Transform.

Let the given DE be: $a*y'(t) + b*y(t) = f(t)$

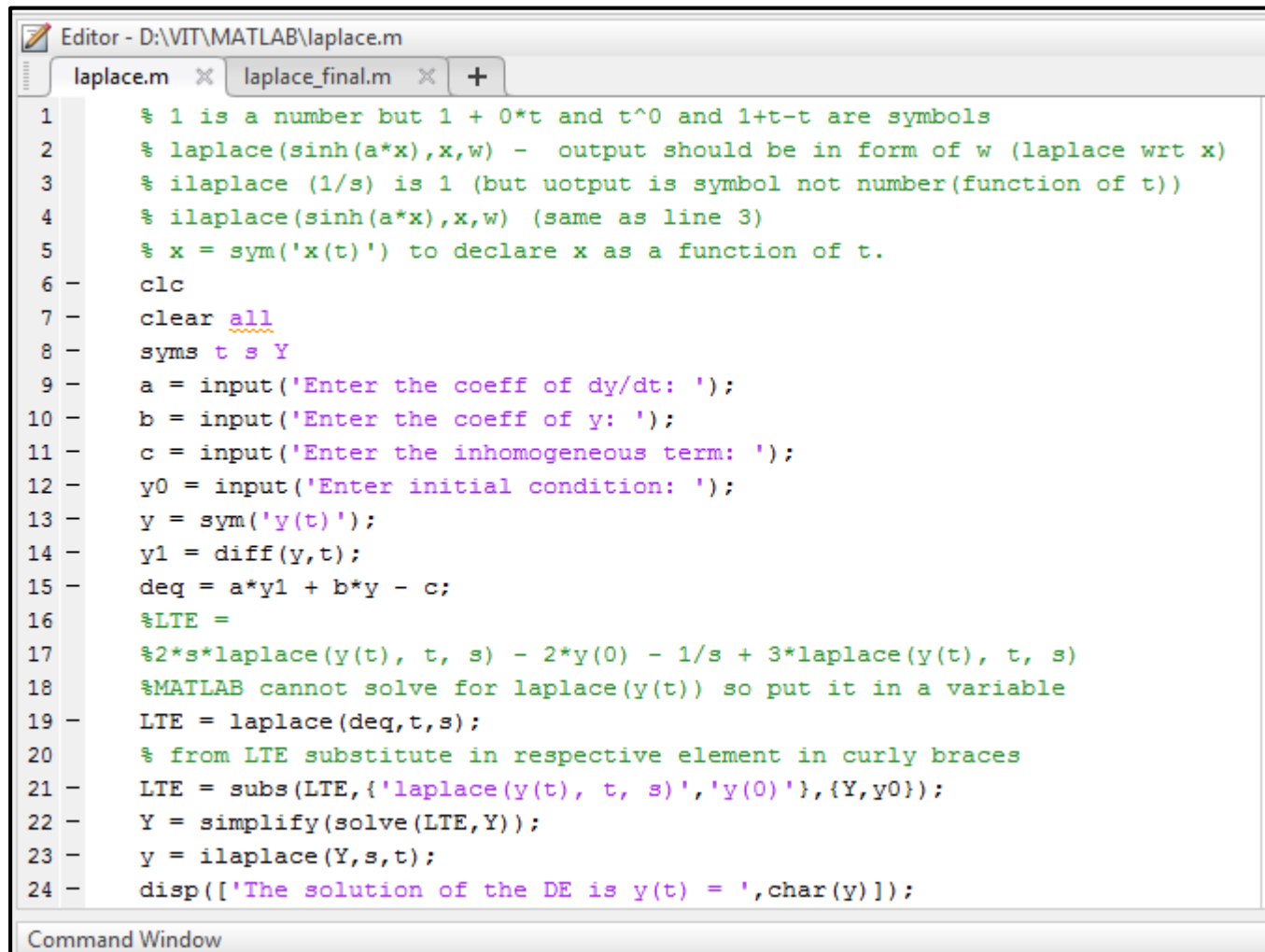
Let $y(0) = y_0$ and $\mathcal{L}\{f(t)\} = F(s)$ and $\mathcal{L}\{y(t)\} = Y(s)$

Now, taking LaPlace on both sides of given DE, we have,

$$Y(s) = \frac{ay_0}{(as + b)} + \frac{F(s)}{(as + b)}$$

Now, taking inverse LaPlace on this equation can give us $y(t)$.

III. MATLAB Code:

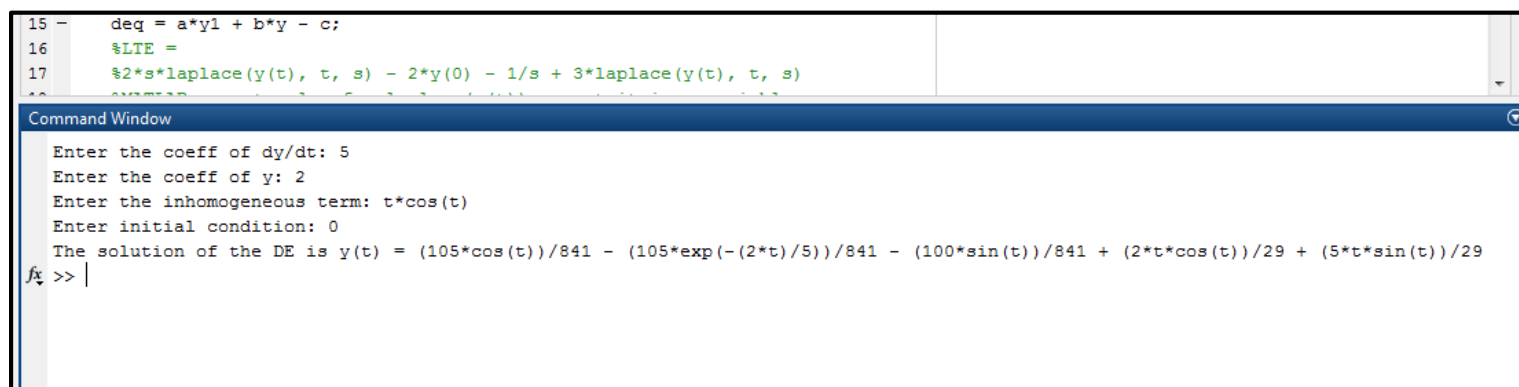


The image shows a MATLAB Editor window with the file 'laplace.m' open. The code is as follows:

```
1 % 1 is a number but 1 + 0*t and t^0 and 1+t-t are symbols
2 % laplace(sinh(a*x),x,w) - output should be in form of w (laplace wrt x)
3 % ilaplace (1/s) is 1 (but uotput is symbol not number(function of t))
4 % ilaplace(sinh(a*x),x,w) (same as line 3)
5 % x = sym('x(t)') to declare x as a function of t.
6 - clc
7 - clear all
8 - syms t s Y
9 - a = input('Enter the coeff of dy/dt: ');
10 - b = input('Enter the coeff of y: ');
11 - c = input('Enter the inhomogeneous term: ');
12 - y0 = input('Enter initial condition: ');
13 - y = sym('y(t)');
14 - y1 = diff(y,t);
15 - deq = a*y1 + b*y - c;
16 %LTE =
17 %2*s*laplace(y(t), t, s) - 2*y(0) - 1/s + 3*laplace(y(t), t, s)
18 %MATLAB cannot solve for laplace(y(t)) so put it in a variable
19 - LTE = laplace(deq,t,s);
20 % from LTE substitute in respective element in curly braces
21 - LTE = subs(LTE,{'laplace(y(t), t, s)','y(0)'},{Y,y0});
22 - Y = simplify(solve(LTE,Y));
23 - y = ilaplace(Y,s,t);
24 - disp(['The solution of the DE is y(t) = ',char(y)]);
```

Below the editor is a Command Window.

IV. MATLAB I/O:



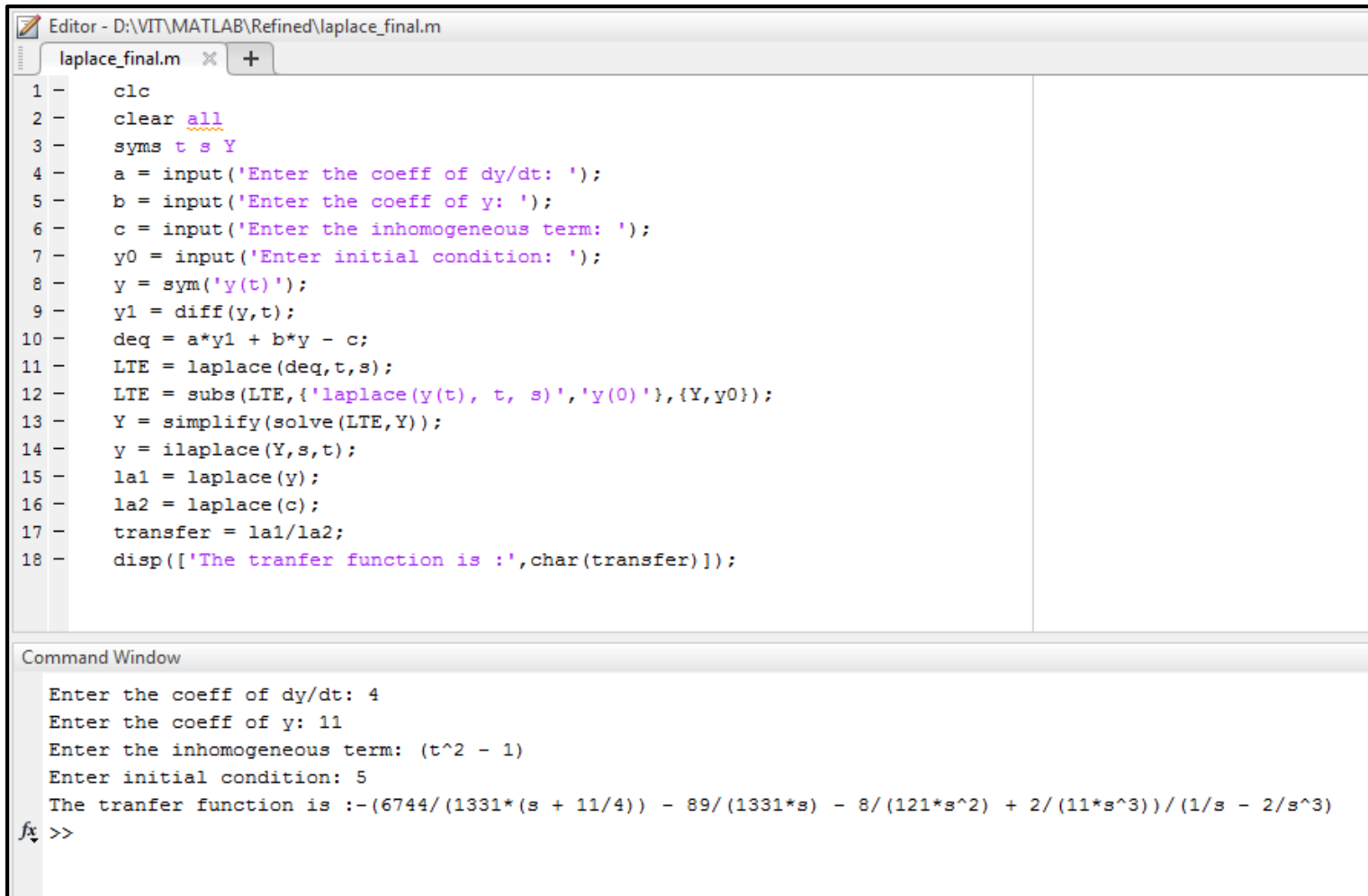
The image shows a MATLAB Command Window with the following input and output:

```
15 - deq = a*y1 + b*y - c;
16 %LTE =
17 %2*s*laplace(y(t), t, s) - 2*y(0) - 1/s + 3*laplace(y(t), t, s)
18 %MATLAB cannot solve for laplace(y(t)) so put it in a variable
19 - LTE = laplace(deq,t,s);
20 % from LTE substitute in respective element in curly braces
21 - LTE = subs(LTE,{'laplace(y(t), t, s)','y(0)'},{Y,y0});
22 - Y = simplify(solve(LTE,Y));
23 - y = ilaplace(Y,s,t);
24 - disp(['The solution of the DE is y(t) = ',char(y)]);

Enter the coeff of dy/dt: 5
Enter the coeff of y: 2
Enter the inhomogeneous term: t*cos(t)
Enter initial condition: 0
The solution of the DE is y(t) = (105*cos(t))/841 - (105*exp(-(2*t)/5))/841 - (100*sin(t))/841 + (2*t*cos(t))/29 + (5*t*sin(t))/29
fx >> |
```

V. Question - Answers:

Q1 Answer



The image shows a MATLAB Editor window with a script named `laplace_final.m` and a Command Window below it. The script defines a second-order differential equation and solves it using Laplace transforms. The Command Window shows the user's inputs and the resulting transfer function.

```
Editor - D:\VIT\MATLAB\Refined\laplace_final.m
laplace_final.m x +
1 -   clc
2 -   clear all
3 -   syms t s Y
4 -   a = input('Enter the coeff of dy/dt: ');
5 -   b = input('Enter the coeff of y: ');
6 -   c = input('Enter the inhomogeneous term: ');
7 -   y0 = input('Enter initial condition: ');
8 -   y = sym('y(t)');
9 -   y1 = diff(y,t);
10 -  deq = a*y1 + b*y - c;
11 -  LTE = laplace(deq,t,s);
12 -  LTE = subs(LTE,{'laplace(y(t), t, s)', 'y(0)'},{Y,y0});
13 -  Y = simplify(solve(LTE,Y));
14 -  y = ilaplace(Y,s,t);
15 -  la1 = laplace(y);
16 -  la2 = laplace(c);
17 -  transfer = la1/la2;
18 -  disp(['The tranfer function is :',char(transfer)]);

Command Window

Enter the coeff of dy/dt: 4
Enter the coeff of y: 11
Enter the inhomogeneous term: (t^2 - 1)
Enter initial condition: 5
The tranfer function is :-(6744/(1331*(s + 11/4)) - 89/(1331*s) - 8/(121*s^2) + 2/(11*s^3))/(1/s - 2/s^3)
fx >>
```

Q2 – Answer

A second order DE can be solved by rewriting the second order DE as a System of first order DE