# Center Weighted Median Filters and Their Applications to Image Enhancement

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*Abstract* —**The center weighted median (CWM) filter, which is a weighted median filter giving more weight only to the central value of each window, is studied. This filter can preserve image details while suppressing additive white and/or impulsive-type noise. The statistical properties of the CWM filter are analyzed. It is shown that the CWM filter can outperform the median filter. Some relationships between CWM and other median-type filters, such as the Winsorizing smoother and the multistage median filter, are derived.**

**In an attempt to improve the performance of CWM filters, an adaptive CWM (ACWM) filter having a space varying central weight is proposed. We show that the ACWM filter is an excellent detail preserving smoother that can suppress signal-dependent noise as well as signal-independent noise.**

## I. Introduction

A VARIETY of adaptive filtering techniques has been proposed for enhancing images degraded by noise [1]–[6]. By adjusting their parameters, depending on local characteristics of the input image, these filters preserve sharp sustained changes (edges) in the image while reducing noise. All of the filters are useful for additive white noise suppression, but are generally ineffective in eliminating impulsive noise that appears as very large spikes of short duration. The adaptive filters in [3] and [6] have been shown to be effective in suppressing signal-dependent noise as well as additive white noise.

The median filter is a simple nonlinear smoothing operation that takes a median value of the data inside a moving window of finite length. This filter has been recognized as a useful image enhancement technique due to its edge preserving smoothing characteristics and its simplicity in implementation [5], [7]. Median filtering preserves edges in images and is particularly effective in suppressing impulsive noise. Application of median filtering to an image, however, requires some caution because median filtering tends to remove image details such as thin lines and corners while reducing noise. Moreover, the performance of median filtering is unsatisfactory in suppressing signal-dependent noise. Recently, in response to these difficulties, several variations of median filters have been introduced. Specifically, the max/median [8], FIR-median hybrid [9], [10], and multistage median [9], [11] filters have been developed for detail preserving smoothing. These filters preserve more image details at the expense of noise suppression. In [12], an adaptive median filter that adjusts the window size depending on the input is proposed to trade between detail preservation and noise suppression. For signal-dependent noise reduction some adaptive median-type filters, such as adaptive double window modified trimmed mean [13] and signal adaptive median [14] filters, have been proposed. The outputs of the adaptive median-type filters are obtained based on the results of median filtering, and as a consequence these adaptive filters tend to remove image details just like median filtering.

The *weighted median* (WM) filter is an extension of the median filter, which gives more weight to some values within the window [15]–[18]. This WM filter allows a degree of control of the smoothing behavior through the weights that can be set, and therefore, it is a promising image enhancement technique. In this paper, we focus our attention on a special case of WM filters called the *center* weighted median (CWM) filter. This filter gives more weight only to the central value of a window, and thus it is easier to design and implement than general WM filters. We shall analyze the properties of CWM filters and observe that CWM filters preserve more details at the expense of less noise suppression like the other nonadaptive detail preserving filters.

In an attempt to improve CWM filters further, an *adaptive* CWM (ACWM) filter having a variable central weight is proposed. We derive this adaptive filter following the approach proposed by Mallows [19]. Specifically, we conceive of an ACWM filter whose "linear" component is close to the adaptive smoothing filter proposed by Lee [3] and Kuan *et al.* [6]. Here the linear component of a nonlinear filter is the FIR filter that operates only on the Gaussian part of the input, and produces the closest output in the mean square error sense to the output of the nonlinear filter [19]. It will be shown that the ACWM filter can offer a more desirable combination of noise suppression and detail preservation properties than can CWM filters. Furthermore, we shall show that ACWM

Manuscript received August 9, 1988; revised November 12, 1990. This work was supported by the National Science Foundation under Grant DCI-8611859 and the Korea Science and Technology Foundation. This paper was recommended by Associate Editor H. Gharavi.

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IEEE Log Number 9101486.

0098-4094/91/0900-0984$01.00 ©1991 IEEE

filters can effectively reduce signal-dependent noise as well as additive white and impulsive noise.

The organization of this paper is as follows. In Section II, we define the CWM filter, derive its deterministic properties, and observe relationships between CWM and some other median-type filters. Statistical properties of CWM filters are analyzed in Section III. In Section IV the ACWM filter is defined. Finally, in Section V, CWM and ACWM filters are applied to enhance images degraded by signal-independent or signal-dependent noise.

## II. CWM Filters

Before defining the CWM filter, it is worthwhile to review the median filter and introduce some common terminology. If we let $\{X(\cdot,\cdot)\}$ and $\{Y(\cdot,\cdot)\}$ be the input and output, respectively, of the median filter, then

$$Y(i,j) = \text{median}\{X(i-s,j-t)|(s,t)\in W\}. \quad (1)$$

Here $W$ is the window that is defined in terms of the image coordinates in the neighborhood of the origin. For example, a $(2N+1)\times(2N+1)$ square window is given by $W=\{(s,t)|-N\leqslant s\leqslant N, -N\leqslant t\leqslant N\}$. The total number of points in a window is called the window size. It is assumed that $W$ includes the origin (0,0) and is symmetric with respect to the origin, i.e., $(s,t)\in W$ implies $(-s,t)\in W$, $(s,-t)\in W$ and $(-s,-t)\in W$. For such a window, an odd window size is guaranteed, and a sample median of the pixel values inside the window can be selected. Throughout, the window size is denoted by $2L+1$.

Consider the WM filter with weights

$$\left\{h(s,t)|(s,t)\in W, \sum_{(s,t)\in W} h(s,t)=c\right\}$$

with $c$ an odd integer greater than or equal to the window size. In obtaining the output $Y(i,j)$ the WM filter generates $h(s,t)$ copies of $X(i-s,j-t)$ for each $(s,t)\in W$, a total of $c$ sample values. Then the median value of the $c$ samples is taken. Thus $Y(i,j)$ is represented as

$$Y(i,j) = \text{median}\{h(s,t) \text{ copies of } X(i-s,j-t)|(s,t)\in W\}. \quad (2)$$

For example, consider the WM filter with $W=\{(-1,0),(0,0),(1,0)\}$ and weights $\{h(-1,0),h(0,0),h(1,0)\}=\{2,3,2\}$. Its output $Y(i,j)$ is given by

$$Y(i,j) = \text{median}\{X(i-1,j),X(i-1,j),X(i,j),X(i,j),X(i,j),X(i+1,j),X(i+1,j)\}.$$

The WM filter with central weight $h(0,0)=2K+1$ and $h(s,t)=1$ for each $(s,t)\neq(0,0)$ is called the CWM filter, where $K$ is a non-negative integer. The output $Y(i,j)$ of the CWM filter is given by

$$Y(i,j) = \text{median}\{X(i-s,j-t), 2K \text{ copies of } X(i,j)|(s,t)\in W\}. \quad (3)$$

When $K=0$, the CWM filter becomes the median filter, and when $2K+1$ is greater than or equal to the window size $2L+1$, it becomes the identity filter (no filtering). Obviously, a CWM filter with a larger central weight performs better in detail preservation but worse in noise suppression than one with a smaller central weight. This is shown statistically in Section III.

There are interesting relationships between CWM and some other median-type filters. These relationships can be observed through the property stated below.

*Property 1:* The output $Y(i,j)$ of a CWM filter with window size $2L+1$ and central weight $2K+1$ is represented by

$$Y(i,j) = \text{median}\{X_{ij}(L+1-K;2L+1), X_{ij}(L+1+K;2L+1), X(i,j)\} \quad (4)$$

where $X_{ij}(r;2L+1)$ is the $r$th smallest one among $2L+1$ samples within the window centered at $(i,j)$, and $X(i,j)$ is the input value at the center.

*Proof:* Consider the extended set having $2L+2K+1$ values, which consist of the $2L+1$ data inside the window and $2K$ copies of $X(i,j)$. Suppose that $X(i,j)<X_{ij}(L+1-K;2L+1)$. Then in the extended set there are $L+K$ values less than or equal to $X_{ij}(L-K;2L+1)$. Thus $X_{ij}(L+1-K;2L+1)$ is the median of the $2L+2K+1$ values in the set and is the output of the CWM filter. Now we assume that $X(i,j)=X_{ij}(L+1-K+m;2L+1)$ with $0\leqslant m\leqslant 2K$. In this case, there exist $L-K+m$ values less than or equal to $X_{ij}(L-K+m;2L+1)$. Note that $L-K\leqslant L-K+m\leqslant L+K$. Since the set contains $2K+1$ copies of $X(i,j)=X_{ij}(L+1-K+m;2L+1)$, $X(i,j)$ is selected as the output. The rest of the proof is trivial. □

Next we shall show that the CWM filter is identical to the Winsorizing smoother, $\text{WS}(2L+1,2a)$, which was proposed by Mallows [19], [20]. Here $2L+1$ is the window size and $a$ is the filter parameter that controls the degree of smoothing. The output $Y(i,j)$ of the Winsorizing smoother $\text{WS}(2L+1,2a)$ is defined as

$$Y(i,j)=\begin{cases} X_{ij}(a;2L+1), & \text{if } X(i,j)\leqslant X_{ij}(a;2L+1)\\ X_{ij}(2L+2-a;2L+1), & \text{if } X(i,j)\geqslant X_{ij}(2L+2-a;2L+1)\\ X(i,j), & \text{otherwise}\end{cases} \quad (5)$$

where $1\leqslant a\leqslant L+1$. The extremes of the Winsorizing smoother are the identity filter (no filtering) with $a=1$, and the median filter with $a=L+1$. Comparison of (4) with (5) shows that the Winsorizing smoother $\text{WS}(2L+1,2a)$ is identical to the CWM filter with central weight $2L+3-2a$ and the same window. Therefore, the properties of CWM filters discussed in this paper are in fact those of Winsorizing smoothers.

Lα=1.0 μm

The CWM filter also has an interesting relationship with a multistage median filter [9], [11], which employs several 1-D median filters and selects either the central sample in the window or one of the filter outputs using a decision logic based on a median filtering algorithm.

*Property 2:* The multistage median filter (unidirectional) with a $3\times3$ square window is identical to the CWM filter with the $3\times3$ square window and the central weight $2K+1=7$.

*Proof:* From the definition of the multistage median filter, it is not difficult to see that $X(i,j)$ is selected as the output $Y(i,j)$ iff it is neither the minimum nor the maximum value in the window, and that

$$Y(i,j)=\begin{cases} X_{ij}(2;9), & \text{iff } X(i,j) \text{ is the minimum}\\ X_{ij}(8;9), & \text{iff } X(i,j) \text{ is the maximum.}\end{cases}$$

Therefore, we get

$$Y(i,j)=\text{median}\left\{X_{ij}(2;9),X_{ij}(8;9),X(i,j)\right\}$$

and from Property 1 the proof is completed. □

This equivalence between CWM and multistage median filters does not hold in general if the window size is greater than $3\times3$; see [21] for an example.

Property 1 is useful for evaluating the outputs of CWM filters. Direct implementation of CWM filtering requires sorting of all samples inside each window. The number of comparisons for the sorting is $O(2L+1)$ when the fast algorithm in [22] is used. Due to Property 1, however, such sorting is not necessary. The values $X_{ij}(L+1-K;2L+1)$ and $X_{ij}(L+1+K;2L+1)$ in (4) can be obtained efficiently by using algorithms that are slight modifications of the fast algorithms for median filtering in [23] and [24]. When such algorithms are applied, the average number of comparisons for CWM filtering with a 2-D square window is reduced to $O(\sqrt{2L+1})$.

## III. Statistical Properties of CWM Filters

In this section the noise suppression and edge and detail preservation characteristics of CWM filters are statistically studied. The noise suppression characteristics of CWM filters are investigated when a constant signal is embedded in additive white noise. The edge and detail preservation properties are examined by considering 2-D inputs with step edges and lines that are also corrupted by additive white noise.

### A. Noise Suppression

First we derive the output distribution of a CWM filter and show its unbiasedness.

*Property 3:* For independently and identically distributed (i.i.d.) inputs, the output distribution function $F_Y(y)$ of the CWM filter with size $2L+1$ and center weight $2K+1$ is given by

$$F_Y(y)=\sum_{j=k_1-1}^{2L}\binom{2L}{j}F_X^{j+1}(y)\left(1-F_X(y)\right)^{2L-j}+\sum_{j=k_2}^{2L}\binom{2L}{j}F_X^{j}(y)\left(1-F_X(y)\right)^{2L+1-j}\quad(6)$$

where $k_1=L+1-K$, $k_2=L+1+K$, and $F_X(\cdot)$ is the input distribution.

*Proof:* Let $S_{ij}=\{X(i-s,j-t)|(s,t)\in W\}$ and $S'_{ij}=S_{ij}-X(i,j)=\{X(i-s,j-t)|(s,t)\in W,\ (s,t)\neq(0,0)\}$. Since $Y(i,j)=\text{median}\{X_{ij}(k_1;2L+1), X_{ij}(k_2;2L+1), X(i,j)\}$ and $X_{ij}(k_2;2L+1)\geqslant X_{ij}(k_1;2L+1)$, we get

$$\begin{aligned}F_Y(y)&=\Pr\left\{X_{ij}(k_1;2L+1)\leqslant y,\ X_{ij}(k_2;2L+1)>y, X(i,j)\leqslant y)\right\}\\&\quad+\Pr\left\{X_{ij}(k_2;2L+1)\leqslant y, X(i,j)\leqslant y)\right\}\\&\quad+\Pr\left\{X_{ij}(k_2;2L+1)\leqslant y, X(i,j)>y\right\}\\&=\Pr\left\{X_{ij}(k_1;2L+1)\leqslant y, X(i,j)\leqslant y\right\}\\&\quad+\Pr\left\{X_{ij}(k_2;2L+1)\leqslant y, X(i,j)>y\right\}\\&=\Pr\left\{\text{at least } k_1 \text{ samples in } S_{ij}\leqslant y, X(i,j)\leqslant y\right\}\\&\quad+\Pr\left\{\text{at least } k_2 \text{ samples in } S_{ij}\leqslant y, X(i,j)>y\right\}\\&=\Pr\left\{\text{at least } k_1-1 \text{ samples in } S'_{ij}\leqslant y\right\}\Pr\left\{X(i,j)\leqslant y\right\}\\&\quad+\Pr\left\{\text{at least } k_2 \text{ samples in } S'_{ij}\leqslant y\right\}\Pr\left\{X(i,j)>y\right\}.\end{aligned}$$

This expression leads to (6). □

It may be noted that if $F_X(x)$ is symmetric about $m$, then $F_Y(y)$ in (6) is also symmetric around $m$. Thus, in this case, the CWM filter is an unbiased estimator of the mean, and $E[Y(i,j)]=E[X(i,j)]=m$.

Using (6), the output variances were computed through numerical integration for i.i.d. Gaussian inputs with mean zero and variance one ($N(0,1)$). The results associated with CWM filters with size $2L+1=9$ are tabulated in Table I. For comparison, the variance of median filters with different window sizes are also shown. As expected, among CWM filters, the one with $2K+1=1$, which is the median filter with a window size of nine, performs the best, and the output variance of the CWM filter increases as the central weight increases. When $2K+1=3$ ($=5$),

TABLE I
Output Variances of CWM Filters of Size 9 and Those of Median Filters of Various Sizes when the Input is i.i.d., $N(0,1)$

| CWM with $2L+1=9$ | | Median | |
|---|---|---|---|
| $2K+1$ | Variance | $2L+1$ | Variance |
| 1 | 0.166 | 3 | 0.449 |
| 3 | 0.237 | 5 | 0.287 |
| 5 | 0.415 | 7 | 0.210 |
| 7 | 0.673 | 9 | 0.166 |

the CWM filter performs better than the median filter with size 5 (3) but worse than that with size 7 (5).

The impulse noise suppression characteristic can be analyzed statistically from Property 3. We evaluated the *breakdown probability* [19] from the output distribution in (6) by assuming a binomial input. Here, roughly speaking, the breakdown probability is the probability of an impulse occurring at the output. This probability is computed from the output distribution in (6) by assuming a binomial input. Table II shows the breakdown probability of CWM filters with $2L+1=9$, and those of median filters. Again it is seen that the CWM filter with $2K+1=3$ $(=5)$ performs better than the median filter with size 5 (3) but worse than that with size 7 (5).

### B. Edge Preservation

We now consider the effects of CWM filters on noisy step edges. The 2-D input sequence representing a noisy step edge is expressed by

$$X(i,j)=\begin{cases} V(i,j), & j\leqslant 0 \\ h+V(i,j), & j\geqslant 1 \end{cases} \tag{7}$$

where $h$ is a constant representing edge height, and $V(i,j)$ is i.i.d. noise with distribution $F_1(x)$. Let the distribution function of $h+V(i,j)$ be $F_2(x)$. Then, obviously, $F_2(x)=F_1(x-h)$.

We will examine the filter behavior near the noisy edge by using the expected value of the output and the root mean squared error (rmse). Here the rmse at $(i,j)$ denoted by $\mathrm{rmse}(i,j)$, is defined as $\mathrm{rmse}(i,j)=\sqrt{E[Y(i,j)-S(i,j)]^2}$ with $Y(i,j)$ as the filtered output, $S(i,j)$ equal to 0 if $j\leqslant 0$, and equal to $h$ if $j\geqslant 1$. In order to compute these quantities, we derive the distribution function $F_{Y_{ij}}(y)$ of the CWM filtered output $Y(i,j)$ which is taken from $m_{ij}$ samples with distribution $F_1(x)$ and $2L+1-m_{ij}$ samples with distribution $F_2(x)$ among $2L+1$ samples within the window centered at $(i,j)$. Note that the number of samples having $F_1(x)$ in the window, $m_{ij}$, depends on the location of the window.

*Property 4:* For the noisy step edge input in (7), the output distribution function $F_{Y_{ij}}(y)$ of the CWM filter with window size $2L+1$ and center weight $2K+1$ is given by

$$\begin{aligned} F_{Y_{ij}}(y)=&\sum_{k=k_1-1}^{2L}\sum_{l=\max(0,k-(2L-d))}^{\min(k,d)}\binom{d}{l}\binom{2L-d}{k-l} \\ &\cdot F_1^l(y)(1-F_1(y))^{d-l}F_2^{k-l}(y) \\ &\cdot(1-F_2(y))^{2L-d-k+l}F_{ij}(y) \\ &+\sum_{k=k_2}^{2L}\sum_{l=\max(0,k-(2L-d))}^{\min(k,d)}\binom{d}{l}\binom{2L-d}{k-l} \\ &\cdot F_1^l(y)(1-F_1(y))^{d-l}F_2^{k-l}(y) \\ &\cdot(1-F_2(y))^{2L-d-k+l}(1-F_{ij}(y)) \end{aligned} \tag{8}$$

where $k_1=L+1-K$, $k_2=L+1+K$,

$$F_{ij}(y)=\begin{cases} F_1(y), & j\leqslant 0 \\ F_2(y), & j\geqslant 1 \end{cases}$$

and

$$d=\begin{cases} m_{ij}-1, & j\leqslant 0 \\ m_{ij}, & j\geqslant 1. \end{cases}$$

*Proof:* From the proof of Property 3, $F_{Y_{ij}}(y)$ can be written as

$$\begin{aligned} F_{Y_{ij}}(y)=&\Pr\{\text{at least } k_1-1 \text{ samples} \\ &\quad \text{in } S'_{ij}\leqslant y\}\Pr\{X(i,j)\leqslant y\} \\ &+\Pr\{\text{at least } k_2 \text{ samples} \\ &\quad \text{in } S'_{ij}\leqslant y\}\Pr\{X(i,j)>y\} \\ =&\sum_{k=k_1-1}^{2L}\sum_{l=\max(0,k-(2L-d))}^{\min(k,d)}P_AF_{ij}(y) \\ &+\sum_{k=k_2}^{2L}\sum_{l=\max(0,k-(2L-d))}^{\min(k,d)}P_A(1-F_{ij}(y)) \end{aligned} \tag{9}$$

where $d$ is the number of samples with distribution $F_1(x)$ in $S'_{ij}$, and

$$\begin{aligned} P_A=&\Pr\{\text{exactly } l \text{ samples with } F_1(x) \text{ in } S'_{ij} \\ &\quad \text{are less than or equal to } y, \\ &\quad \text{exactly } k-l \text{ samples with } F_2(x) \text{ in } S'_{ij} \\ &\quad \text{are less than or equal to } y\} \\ =&\binom{d}{l}\binom{2L-d}{k-l}F_1^l(y)(1-F_1(y))^{d-l} \\ &\cdot F_2^{k-l}(y)(1-F_2(y))^{2L-d-k+l}. \end{aligned} \tag{10}$$

Equation (8) follows directly from (9) and (10). □

Using (8), along with the assumption that $F_1(x)$ is $N(0,1)$, we computed $E[Y(i,j)]$ and $\mathrm{rmse}(i,j)$ along the horizontal filter path (fixed $i$, variable $j$) through numerical integration. Fig. 1(a) and (b) show plots of $E[Y(i,j)]$ and $\mathrm{rmse}(i,j)$, respectively, for the CWM filters with the $3\times 3$ square window, when the step edge with $h=4$ is degraded by a Gaussian $N(0,1)$ noise. It is interesting to see that the CWM filter with $2K+1=7$ has the expected values closest to the ideal step edges, but has the largest rmse values due to its poor noise suppression characteristics. The results show that all of the filters are essentially edge preserving filters.

### C. Line Preservation

The detail preservation characteristic of a CWM filter can be examined by considering binary input images; it is straightforward to see, when the input is a noise-free binary image, that a CWM filter with a $(2N+1)\times(2N+1)$ square window can preserve a straight line of width one if $K\geqslant 2N^2$. This observation can be extended to a noisy

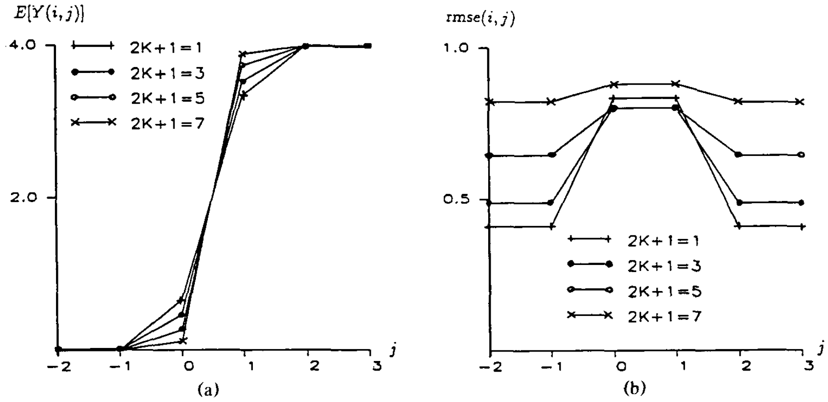


Fig. 1. Results of median and CWM filters, with $3\times3$ square window, for the noisy step edge $h=4$. (a) The output expected values. (b) The rmse.

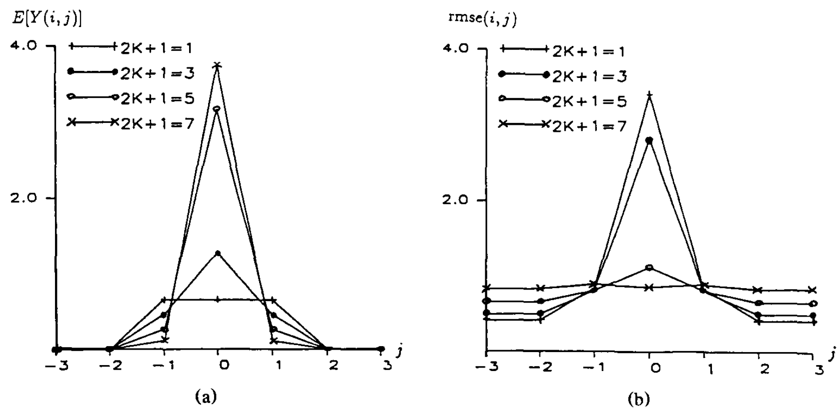


Fig. 2. Results of median and CWM filters, with $3\times3$ square window, for the noisy line $h=4$. (a) The output expected values. (b) The rmse.

line image, as shown below. Consider the noisy line image $X(i,j)$ defined as

$$X(i,j)=\begin{cases} V(i,j), & j\neq 0\\ h+V(i,j), & j=0\end{cases} \tag{11}$$

where $h$ is the height of the line, $V(i,j)$ is i.i.d. with distribution $F_1(x)$.

For the noisy line input in (11), the output distribution function $F_{Y_{ij}}(y)$ of the CWM filter with size $2L+1$ and center weight $2K+1$ is the same as (8) except that

$$F_{ij}(y)=\begin{cases} F_1(y), & j\neq 0\\ F_2(y), & j=0\end{cases} \tag{12}$$

and

$$d=\begin{cases} m_{ij}-1, & j\neq 0\\ m_{ij}, & j=0.\end{cases} \tag{13}$$

Again, under the assumption that $F_1(x)$ is $N(0,1)$, we computed $E[Y(i,j)]$ and $\mathrm{rmse}(i,j)$ for the noisy line along the horizontal filter path through numerical integration. The results associated with the CWM filters with a $3\times3$ square window are plotted in Fig. 2(a) and (b) for $h=4$. It is seen that CWM filters with $2K+1=5$, and 7 preserve the line, while the others remove it.

In summary, the CWM filter can preserve edges and details while reducing noise. However, there exists a clear trade-off between detail preservation and noise suppression properties of this filter. The central weight should be carefully selected depending on both the characteristics of the input image and its noise. The results presented in this section provide some criteria for determining the parameters of CWM filters. For example, suppose that an image is corrupted by impulses that occur with probability 0.0625, and that we wish to remove at least 99% of the impulses. If a CWM filter with a $3\times3$ square window is used, then its central weight should be less than or equal to three (Table II). If the image has lines to be preserved, then none of the CWM filters with a $3\times3$ square window would be satisfactory: the ones having desirable noise suppression characteristics will remove lines. In this case a larger window may be considered (statistical results for

CWM filters with a $5\times 5$ square window are available in [25]).

## IV. Adaptive CWM Filters

In designing a nonlinear filter for simultaneous suppression of Gaussian and impulsive noise, Mallows [19] proposed the following approach of two steps: In the first step, a linear filter appropriate for reducing the Gaussian noise superimposed on the original signal is chosen. In the second, a nonlinear filter whose linear component is close to the linear filter is selected from a set of nonlinear filters that can remove impulses properly. The ACWM filter that will be introduced in this section is based on this approach. It will be shown that the proposed ACWM filter can offer a more desirable combination of detail preservation and noise suppression properties than can CWM filters, and that this adaptive filter can simultaneously suppress impulses, additive white noise, and signal-dependent noise.

We start with the observation that the linear component of a CWM filter is an FIR filter that averages the data inside each window after giving more weight to the central value. When the input is an i.i.d. random process, the linear component of a nonlinear filter selecting one of the data inside each window as its output is given by the FIR filter with the impulse response

$$h(s,t)=\Pr\{Y(i,j)=X(i-s,j-t)\} \tag{14}$$

which is the probability that $X(i-s,j-t)$ is the output at $(i,j)$ [19]. Following from this result and Property 1, it is readily seen that the linear component of the CWM filter is given by

$$h(s,t)=\begin{cases}(2K+1)/(2L+1), & \text{if } (s,t)=(0,0)\\ (1-K/L)/(2L+1), & \text{if } (s,t)\neq(0,0)\end{cases} \tag{15}$$

for each $(s,t)\in W$. Note that $\sum_{(s,t)\in W} h(s,t)=1$. Since $h(s,t)$ are all equal and $h(0,0)\geqslant h(s,t)$ when $(s,t)\neq(0,0)$, then more weight is imposed only on the central value. This filter in (15) will be referred to as the center weighted *average* (CWA) filter.

Next, we make an interesting observation indicating that the adaptive smoother in [3] and [6] is an adaptive filter that gives more weight only to the central value of each window. The output of the adaptive filter for additive white noise suppression is given by

$$Y(i,j)=A(i,j)+R(i,j)[X(i,j)-A(i,j)] \tag{16}$$

where $A(i,j)$ is the average of the values within a window centered at $(i,j)$,

$$R(i,j)=\begin{cases}(\hat\sigma_X^2(i,j)-\sigma_n^2)/(\hat\sigma_X^2(i,j)), & \text{if } \hat\sigma_X^2(i,j)\geqslant\sigma_n^2\\ 0, & \text{o.w.}\end{cases} \tag{17}$$

where $\hat\sigma_X^2(i,j)$ is the sample variance of the data inside the window, and $\sigma_n^2$ is the variance of additive white noise, which is assumed to be known. This filter varies between the average filter and the identity filter depending on local statistics. In (17), roughly speaking, if there exist either edges or impulsive noise components within the window, then $\hat\sigma_X^2(i,j)\gg\sigma_n^2$ and $R(i,j)\simeq 1$. In such a case, $Y(i,j)\simeq X(i,j)$ and this filter can preserve edges but cannot suppress impulses. On the other hand, in gradually varying portions of the input, $\hat\sigma_X^2(i,j)\simeq\sigma_n^2$, and thus $Y(i,j)\simeq A(i,j)$.

TABLE II
Breakdown Probabilities of CWM Filters with Size 9 and Those of Median Filters with Various Sizes when the Input is Binomial, $B(2L+1,p)$

| CWM with $2L+1=9$ | | | Median | | |
|---|---|---|---|---|---|
| $2K+1$ | $p=0.0625$ | $p=0.125$ | $2L+1$ | $p=0.0625$ | $p=0.125$ |
| 1 | 0.00010 | 0.00248 | 3 | 0.01123 | 0.04297 |
| 3 | 0.00067 | 0.00849 | 5 | 0.00222 | 0.01605 |
| 5 | 0.00531 | 0.03296 | 7 | 0.00046 | 0.00624 |
| 7 | 0.02521 | 0.08205 | 9 | 0.00010 | 0.00248 |

The output $Y(i,j)$ in (16) can be rewritten as

$$Y(i,j)=\sum_{(s,t)\in W} h_{ij}(s,t)X(i-s,j-t) \tag{18}$$

where

$$h_{ij}(s,t)=\begin{cases}[2LR(i,j)+1]/(2L+1), & \text{if } (s,t)=(0,0)\\ [1-R(i,j)]/(2L+1), & \text{if } (s,t)\neq(0,0).\end{cases} \tag{19}$$

Note that $\sum_{(s,t)\in W} h_{ij}(s,t)=1$. It is straightforward to see that this filter imposes more weight only on the central value; we will henceforth refer to this particular filter in (18) as the *adaptive* center weighted average (ACWA) filter.

The existence of the ACWA filter and the relation between CWM and CWA filters motivate us to introduce the ACWM filter which adjusts its central weight depending on input data. Consider an ACWM filter with space varying central weight $2K(i,j)+1$. From (15) the linear component of the ACWM filter may be written as

$$h_{ij}(s,t)=\begin{cases}[2K(i,j)+1]/(2L+1), & \text{if } (s,t)=(0,0)\\ [1-K(i,j)/L]/(2L+1), & \text{if } (s,t)\neq(0,0).\end{cases} \tag{20}$$

Comparison of (19) and (20) shows that the linear component of the ACWM filter becomes the same as the ACWA filter if

$$K(i,j)=LR(i,j). \tag{21}$$

Since $K(i,j)$ must be a non-negative integer, $LR(i,j)$ may be rounded.

It may be noticed that the ACWM filter associated with (21) is *not* effective in suppressing impulses, because $K(i,j)$ depends on the sample variance which is sensitive to impulses and because one extreme of this filter is an identity filter. As with ACWA filtering, the ACWM filter tends to become an identity filter if impulses exist within the window. One simple remedy for this is to limit the central weight by precluding any value that would cause

Lα=1.0 μm

the ACWM filter to become an identity filter. The parameter $K(i,j)$ of the proposed ACWM filter is determined by

$$K(i,j) = \lceil (L-T)R(i,j) \rceil \tag{22}$$

where $T$ is an integer, $0 \leqslant T \leqslant L$, and $\lceil x \rceil$ represents rounding of $x$. Since $0 \leqslant R(i,j) \leqslant 1$, we get $0 \leqslant K(i,j) \leqslant L-T$. Therefore, the ACWM filter varies between the median filter and the CWM filter with $2K+1 = 2(L-T)+1$. Generally speaking, it acts like the CWM filter with central weight $2(L-T)+1$ in the neighborhood of edges, lines, and impulses, and like the median filter in gradually varying regions of an image. Thus, as in CWM filtering, the ACWM filter preserves details at the expense of less impulsive noise suppression, but it should be considerably more effective in suppressing nonimpulsive noise than CWM filters. It is reasonable to determine the parameter $T$ in accordance with the central weight of an appropriate CWM filter: for a given image, if a CWM filter with a central weight, say $2K_1+1$, can suppress most impulses while preserving details, then we may set $T$ at $L-K_1$ (the ACWM filter with $T = L-K_1$ acts like the CWM filter with $2K_1+1$ in the vicinity of lines and impulses).

## V. Application of CWM and ACWM Filters to Various Types of Noise

The CWM and ACWM filters are applied to enhance images corrupted by signal-independent or signal-dependent noise. The images under consideration consist of $256 \times 256$ pixels with eight bits of resolution. In order to quantitatively compare the performance of the filters we have discussed, the normalized mean square errors (NMSE) between the original and filtered images are evaluated. The NMSE is given by

$$\mathrm{NMSE} = \frac{\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}[Y(i,j)-S(i,j)]^2}{\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}[X(i,j)-S(i,j)]^2} \tag{23}$$

where $S(i,j)$, $X(i,j)$, and $Y(i,j)$ are the original, noisy input, and filtered images, respectively, and $M = 256$.

In addition, to quantify the error in human visual error criteria, we shall present images showing the differences between the original and the filtered images. In difference images, a zero difference is shown as a black pixel, and a difference of 255 is shown as a white pixel. These images provide information about both the detail-preservation and noise suppression characteristics of filters.

### A. Additive White and Impulsive Noise Suppression

The performance of the filters discussed so far is evaluated by applying them to noisy images degraded by additive white and/or impulsive noise and then by comparing their respective results. The original noise-free image is shown in Fig. 3(a). Three noisy images were generated by adding zero mean i.i.d. Gaussian noise of variance 100, 200, and 400 to the original image, and then were passed through various filters with $5 \times 5$ square window. Fig. 3(b) shows the noisy image with the noise variance 200. In the following, we first compare the NMSE's of the filters, and then visually compare some of the filtered images.

(a) (b)

Fig. 3. (a) Original image. (b) Noisy image (Gaussian noise $\sigma^2 = 200$).

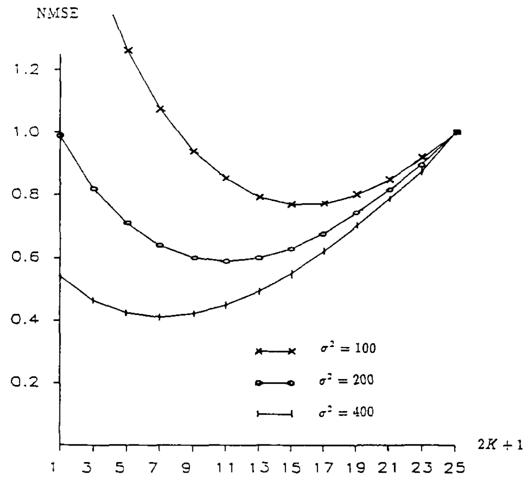


Fig. 4. NMSE's of CWM filters with $5 \times 5$ square window.

Fig. 4 exhibits the NMSE's associated with CWM filters with various central weights. It is interesting to observe that the NMSE curve depicted as a function of the central weight is convex and has a unique minimum value at a certain weight. The central weight that minimizes the NMSE is dependent on the noise variance: it becomes smaller as the noise variance increases. In general, for a given image to be CWM filtered with a certain window, it is most prudent to apply all CWM filters with $1 \leqslant 2K+1 \leqslant 2L+1$, and then to choose one yielding the best result.

Table III summarizes the NMSE's of CWM, ACWM ($T = 2$), ACWA as well as multistage median filters. In each case, the minimal NMSE of a CWM filter is smaller than the NMSE of a median filter. The NMSE's of the multistage median filters, with the exception of the one associated with variance 100, are larger than those measures of the CWM filters with minimal NMSE. The NMSE's associated with ACWM and ACWA filters are always smaller than those of the nonadaptive filters. Between the two adaptive filters, the ACWA filter yielded

TABLE III
NMSE's Associated with Gaussian Noise

| Filter Type (5×5) | | NMSE $\sigma^2 = 100$ | $\sigma^2 = 200$ | $\sigma^2 = 400$ |
|---|---|---|---|---|
| Median | | 1.86 | 0.98 | 0.54 |
| CWM | $2K+1=7$ | 1.07 | 0.64 | 0.41* |
| | $2K+1=11$ | 0.85 | 0.59* | 0.45 |
| | $2K+1=15$ | 0.77* | 0.63 | 0.55 |
| Multistage median | | 0.72 | 0.62 | 0.55 |
| ACWM ($T=2$) | | 0.57 | 0.45 | 0.35 |
| ACWA | | 0.48 | 0.38 | 0.29 |

*indicates the minimal NMSE of CWM filters.

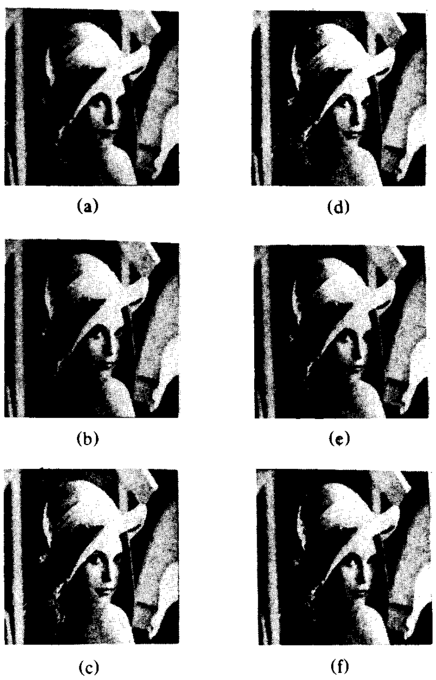

Fig. 5. The results of filtering of the noisy image in Fig. 3(b) with 5×5 square windows. (a) Median filtered. (b) CWM filtered, $2K+1=11$. (c) CWM filtered, $2K+1=13$. (d) Multistage median filtered. (e) ACWM filtered, $T=2$. (f) ACWA filtered.

smaller NMSE's than the ACWM filters. Generally speaking, ACWA filters are superior to ACWM filters in enhancing images corrupted by Gaussian-type noise.

Fig. 5(a)–(f) shows the results of filtering of the noisy image in Fig. 3(b) with a 5×5 window. (The other filtered images are not presented because they lead to a discussion similar to the one stated below.) Comparison of these images clearly indicates that the adaptive filters outperform the nonadaptive ones, while the median filter performs the worst. Visually, the ACWM and ACWA filtered images look similar, and the CWM and multistage median filtered images also appear alike. Fig. 6 shows the difference between the original and the filtered images in Fig. 5. It is readily seen that the median filter caused the most blur. The multistage median filter introduced the least distortion, but its noise suppression characteristic looks somewhat poorer than the other filters.

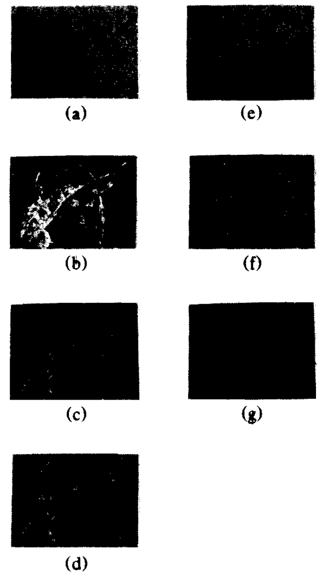

Fig. 6. Difference images between original and (a) noisy image, (b) median filtered, (c) CWM filtered, $2K+1=11$, (d) CWM filtered, $2K+1=13$, (e) multistage median filtered, (f) ACWM filtered, $T=2$, and (g) ACWA filtered.

TABLE IV
NMSE's Associated with Both Gaussian and Impulsive Noise

| Filter Type (5×5) | | NMSE $p = 0.02$ and $\sigma^2 = 200$ | $p = 0.1$ and $\sigma^2 = 200$ |
|---|---|---|---|
| Median | | 0.51 | 0.173 |
| CWM | $2K+1=7$ | 0.33 | 0.119* |
| | $2K+1=11$ | 0.31* | 0.123 |
| | $2K+1=15$ | 0.32 | 0.148 |
| Multistage median | | 0.34 | 0.153 |
| ACWM ($T=2$) | | 0.28+ | 0.243 |
| ACWM ($T=8$) | | 0.30 | 0.115+ |
| ACWA | | 0.62 | 0.812 |

* and +, respectively, indicate the mininal NMSE's of CWM and ACWM filters.

Next we compare the performance of the filters in reducing impulsive noise. The noisy image in Fig. 3(b) was further corrupted by both positive and negative impulses having values 255 and 0, respectively. By considering two different probabilities of an impulse occurring, $p = 0.02$ and $p = 0.1$, two noisy images were generated, and then passed through the filters. (The probabilities of occurrence of a positive and a negative impulse are the same.) Table IV shows the resulting NMSE's. The ACWM filters best attenuated the noise, while the ACWA filter per-

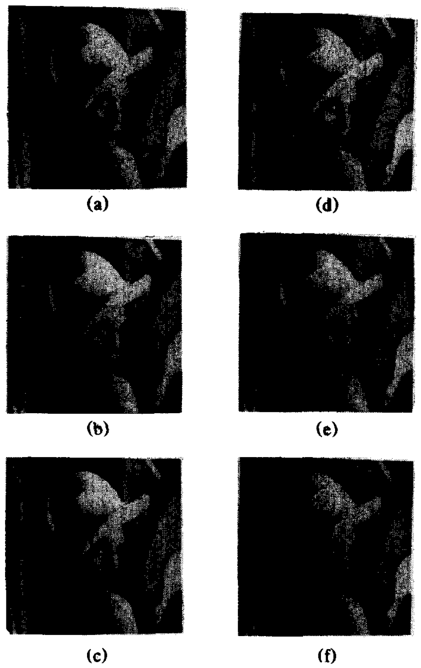

Fig. 7. (a) Noisy image corrupted by Gaussian noise $\sigma^2 = 200$ and impulses $p = 0.02$, (b) CWM filtered, $2K+1 = 11$, (c) CWM filtered, $2K+1 = 13$, (d) multistage median filtered, (e) ACWM filtered, $T = 2$, and (f) ACWA filtered.

formed the worst. As expected, in ACWM filtering a larger value of $T$ is required to suppress impulses that occur with higher probability. It should be noted that when $p = 0.1$, the performance of the CWM filter with $2K+1 = 7$ is very close to that of the ACWM filter with $T = 8$. This is because the ACWM filter with the parameter $T$ performs like the CWM filter with central weight $2(L-T)+1$ in the neighborhood of impulses; in this case $L = 12$ and $T = 8$. (When $p = 0.1$, the $5 \times 5$ window of the filter contains impulses at almost every window position.) The noisy image degraded by both Gaussian noise $\sigma^2 = 200$ and impulses $p = 0.02$ and the filtered images are shown in Fig. 7. The difference images corresponding to the images in Fig. 7 are presented in Fig. 8. It is clearly seen that the ACWA filter cannot suppress impulses while the others can.

### *B. Multiplicative Noise Suppression*

In [3] and [6], it has been shown that ACWA filters are effective in suppressing signal dependent noise such as multiplicative noise. Thus it is expected that ACWM filters are also useful for signal dependent noise reduction. In this subsection, we shall show that the ACWM filter is effective for multiplicative noise suppression.

A useful representation of the image corrupted by multiplicative noise [26], [27] is given by

$$X(i,j) = S(i,j) + bS(i,j)V(i,j) \qquad (24)$$

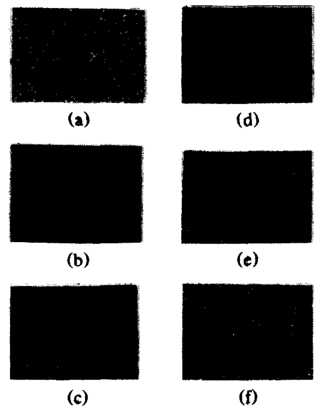

Fig. 8. Difference images between original and (a) noisy image, (b) CWM filtered, $2K+1 = 11$, (c) CWM filtered, $2K+1 = 13$, (d) multistage median filtered, (e) ACWM filtered, $T = 2$, and (f) ACWA filtered.

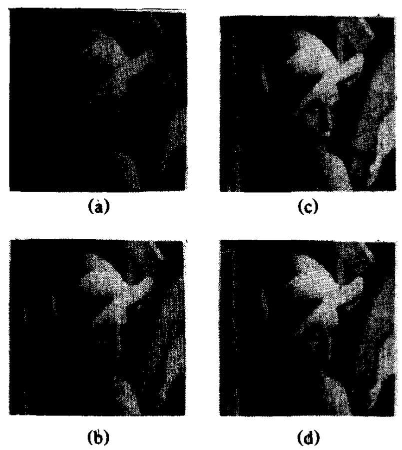

Fig. 9. (a) Noisy image corrupted by multiplicative noise with $b = 0.15$, (b) median filtered, (c) ACWM filtered, $T = 2$, and (d) ACWA filtered.

where $S(i,j)$ is the original image, $V(i,j)$ is zero mean i.i.d. random noise with variance one, and $b$ is a constant. (This model is essentially the same as the additive form of the multiplicative noise model in [6].) The ACWA filter for the multiplicative noise suppression is given by (16) and (17), except that in (17) $\sigma_n^2$ is replaced by

$$\hat{\sigma}_n^2(i,j) = b^2\left[\hat{\sigma}_X^2(i,j) + \hat{u}_X^2(i,j)\right]/(1+b^2) \qquad (25)$$

where $\hat{u}_X(i,j)$ is the sample mean [3]. Therefore, $K(i,j)$ of the ACWM filter is determined by (22) with $R(i,j)$ having $\hat{\sigma}_n^2(i,j)$ in place of $\sigma_n^2$.

Fig. 9(a) shows a noisy version of the original image in Fig. 3(a), which has been degraded by multiplicative noise with $b = 0.15$. The median, ACWA, and ACWM filters, all with $5 \times 5$ square windows, were applied to the noisy image. The filtered images are shown in Fig. 9(b)–(d). The corresponding NMSE's are the following: the median filter (NMSE = 0.75), the ACWM ($T = 2$) filter (NMSE = 0.034), and the ACWA filter (NMSE = 0.28). Obviously, the adaptive filters outperformed the median filter and suppressed the multiplicative noise reasonably well. The ACWA and ACWM filtered images look similar, but the NMSE of the ACWA filter is smaller than that of the ACWM filter. It appears that both ACWM and ACWA filters are useful for multiplicative noise suppression.

The results in this section indicate that the ACWM filter is an effective detail preserving smoother that can suppress additive white, impulsive, and multiplicative noise.

## VII. Conclusions

The CWM and ACWM filters, which are useful detail preserving smoothers, were proposed and their properties were analyzed. Interesting relationships among CWM, multistage median filters, and Winsorizing smoothers were pointed out. It was shown that ACWM filters can suppress multiplicative noise as well as additive white and impulsive noise.

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