MC125: Project

Pb 1) The carrying capacity of an organism in a given environment is defined to be the maximum population of that organism that the environment can sustain indefinitely. Let K represent the carrying capacity for a particular organism in a given environment and let r be a real number that represents the growth rate. The function $N(t)$ represents the population of this organism as a function of time t, and the constant N_0 represents the initial population (population of the organism at time t = 0). Then the logistic differential equation is

$$\frac{\dot{N}}{N} = r\left(1 - \frac{N}{K}\right).$$

Find out a) Existence and Uniqueness of Solution. b) Analytic Solution. c) Series Solution. d) Sketch the solutions $N(t)$ for different initial conditions.

Pb 2) (Allee Effect) For certain species of organisms, the effective growth rate $\frac{\dot{N}}{N}$ is highest at intermediate N. This is called the Allee effect (Edelstein-Keshet 1988). For example, imagine that it is too hard to find mates when N is very small, and there is too much competition for food and other resources when N is large.

a) Show that $\frac{\dot{N}}{N} = r - a(N - b)^2$ provides an example of the Allee effect, if r, a, and b satisfy certain constraints, to be determined.

b) Find out a) Existence and Uniqueness of Solution. b) Analytic Solution. c) Series Solution.

b) Sketch the solutions $N(t)$ for different initial conditions.

c) Compare the solutions $N(t)$ to those found for the logistic equation. What are the qualitative differences, if any.