Tutorial - 6

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Q.1. What do you mean by Minimum Spanning Tree? What are applications of MST?

→ Minimum spanning Tree is a subset of edges of connected edge-weighted undirected graph that connects all vertices together without any cycle & with minimum possible edge weighted.

Applications →

(i) Consider n stations are to be linked using a communication network and lying of communication link b/w any two stations involves a cost. The ideal solution would be to extract a subgraph termed as minimum cost spanning tree.

(ii) Designing LAN.

(iii) Suppose you want to construct highways or railroad spanning several cities, then we can use concept of MST.

(iv) Laying pipeline connecting offshore drilling site, refineries & consumer markets.

Q.2. Analyze time and space complexity of Prime, Kruskal, Dijstra and Bellman Ford Algorithm.

→ Prism's Algo

Time :- $O(|E| \log |V|)$

space :- $O|V|$

Kruskal's Algo

Time :- $O|E| \log |E|$

space :- $O|V|$

Dijkstra's Algo

Time :- $O(V^2)$

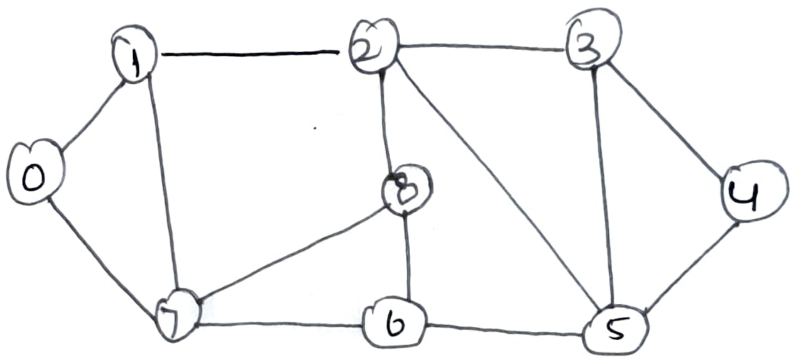
space :- $O(V^2)$

Bellman Ford's Algo

Time :- $O(VE)$

space :- $O(E)$

Q.3. Apply Kruskal and Prism's Algorithm on given graph to compute MST and its weight.

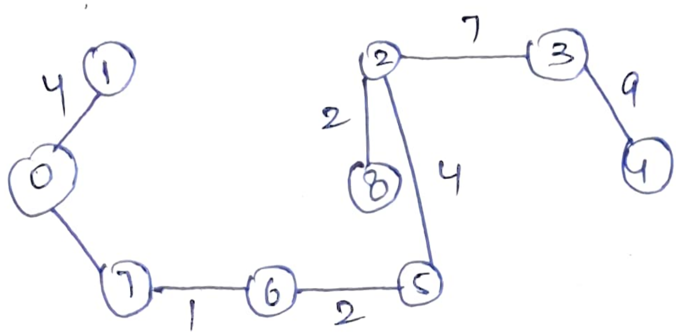


→ Kruskal's Algo

U	V	W	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	×
2	3	7	✓
7	8	7	×
0	7	8	✓
1	2	8	×
4	3	9	✓
4	5	10	×
1	7	11	×
3	5	14	×

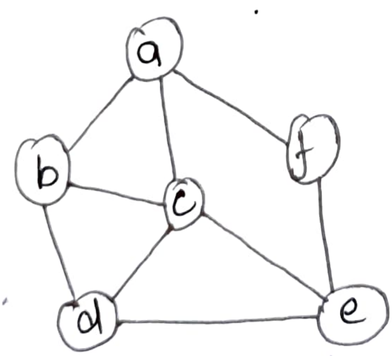
Prim's Algo

weight :- $4+8+2+4+2+$
$7+9+3$
$\Rightarrow 37$



weight :- 1+2+2+4+4+
7+8+9
⇒ 37

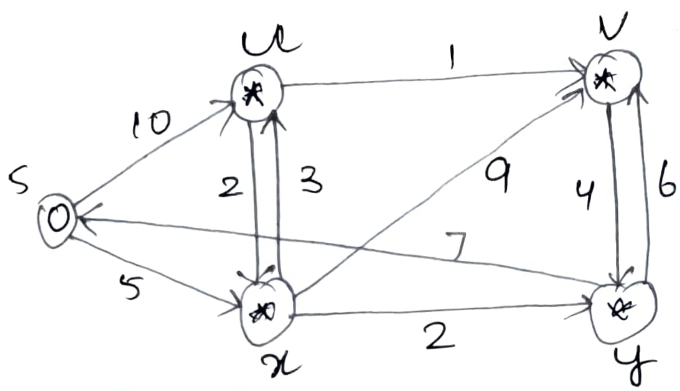
Q.4. Given a directed graph. You are also given the shortest path from source vertex 's' to destination vertex 't'. Does the shortest path remain same in following cases:
(i) If weight of every edge is increased by 10 units.
(ii) If weight of every edge is multiplied by 10 units.



→ (i) The shortest path may change. The reason is that there may be different no. of edge in different paths from 's' to 't'. For eg:- let shortest path of weight 15 and has edges 5. Let there we another path with 2 edges and total weight 25. The weight of shortest path is increased by 5*10 and becomes 15+50. Weight of other path is increased by 2*10 & becomes 25+20. So, the shortest path changes to other path with weight as 45.

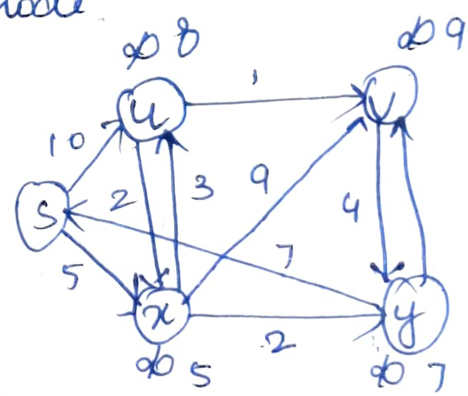
(ii) If we multiply all edges weight by 10, the shortest path doesnot change. the reason is that weights of all path from 's' to 't' gets multiplied by same unit the number of edges on path doesn't matter.

Q.5. Apply Dijkstra & Bellman Ford Algo. on graph given right side to compute shortest path to all nodes from node s.



→ Dijkstra's Algo

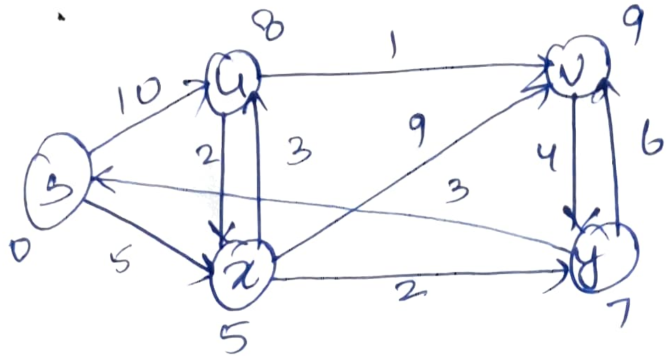
Node	shortest Dist. from source node
u	8
x	5
v	9
y	7



Bellman Ford Algorithm

	s	u	v	x	y
1st →	0	~~∞~~ ∞	∞	~~∞~~ 5	∞
2nd →	0	10	~~∞~~ 11	~~∞~~ 5	∞
3rd →	0	~~10~~ 8	~~11~~ 9	5	~~∞~~ 7
4th →	0	8	9	5	7

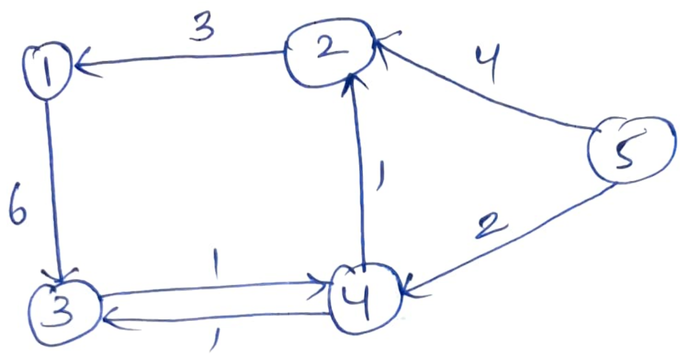
graph does not have -ve cycle.



$\Rightarrow$ Final Graph

Q.6. Apply all pair shortest path algo - Floyd Warshall on below mentioned graph. Also analyze space & time complexity of it.

$\rightarrow$



$$
\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & \infty & 6 & 3 & \infty \\
2 & 2 & 0 & \infty & \infty & \infty \\
3 & \infty & \infty & 0 & 2 & \infty \\
4 & \infty & 1 & 1 & 0 & \infty \\
5 & \infty & 4 & \infty & 2 & 0 \\
\hline
\end{array}
$$

$$
\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & \infty & 6 & 3 & \infty \\
2 & 2 & 0 & 8 & 5 & \infty \\
3 & \infty & \infty & 0 & 2 & \infty \\
4 & \infty & 1 & 1 & 0 & \infty \\
5 & \infty & 4 & \infty & 2 & \infty \\
\hline
\end{array}
$$

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & \infty & 6 & 3 & \infty \\
2 & 2 & 0 & 8 & 5 & \infty \\
3 & \infty & \infty & 0 & 2 & \infty \\
4 & 3 & 1 & 1 & 0 & \infty \\
5 & 6 & 4 & 12 & 2 & \infty
\end{array}
$$

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & \infty & 6 & 3 & \infty \\
2 & 2 & 0 & 8 & 5 & \infty \\
3 & \infty & \infty & 0 & 2 & \infty \\
4 & 3 & 1 & 1 & 0 & \infty \\
5 & 6 & 4 & 12 & 2 & 0
\end{array}
$$

Time complexity $\Rightarrow O(|V|^3)$

space complexity $\Rightarrow O(|V|^2)$