Tutorial-4

Name :- Daksh Goel

Section :- F

Roll no. :- 51

Univ. Roll no :- 2016713

① $T(n) = 3T(n/2) + n^2$

$\Rightarrow$ $a = 3$, $b = 2$, $f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (case 3)

$\therefore$ according to master Theorem

$T(n) = \theta(n^2)$

② $T(n) = 4T(n/2) + n^2$.

$\Rightarrow$ $a = 4$, $b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ (case 2)

$\therefore$ according to masters theorem $T(n) = \theta(n^2 \log n)$

③ $T(n) = T(n/2) + 2^n$.

$\Rightarrow$ $a = 1$, $b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (case 3)

$\therefore$ according to master theorem $T(n) = \theta(2^n)$

④ $T(n) = 2^n T(n/2) + n^n$

$\therefore$ Master's theorem is not applicable as a is function of n.

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5. $T(n) = 16T(n/4) + n$.

$a = 16,\ b = 4,\ f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > f(n)$ (case 1)

$\therefore T(n) = \theta(n^2)$.

6. $T(n) = 2T(n/2) + (n \log n)$

$\Rightarrow a = 2,\ b = 2,\ f(n) = n \log n$

$n^{\log_b a} = n^{\log_2 2} = n$

Now $f(n) > n$

$\therefore$ According to mastom $T(n) = \theta(n \log n)$

7. $T(n) = 2T(\frac{n}{2}) + \frac{n}{\log n}$

$\Rightarrow a = 2,\ b = 2,\ f(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > f(n)$

$\therefore$ According to master theorem $T(n) = \theta(n)$

8. $T(n) = 2T(n/4) + n^{0.51}$

$\Rightarrow a = 2,\ b = 4,\ f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.51}$

$n^{0.51} < f(n)$

$\therefore$ According master Theorem $T(n) = \theta(n^{0.51})$

9. $T(n) = 0.5T(n/2) + \frac{1}{n}$

$\therefore$ Master's Not applicable as $a < 1$.

10. $T(n) = 16T(n/4) + n!$

$\Rightarrow a = 16,\ b = 4,\ f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 < n!$

$\therefore$ According ~~mo~~ to maston, $T(n) = \theta(n!)$

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(11) $T(n) = 4T(n/2) + \log n$

$\Rightarrow$ $a=4$, $b=2$, $f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

$\therefore$ According to master's, $T(n) = \Theta(n^2)$.

(12) $T(n) = \text{sqrt}(n) + (n/2) + \log n$

$\Rightarrow$ $\therefore$ Master's Not Applicable as a is not content.

(13) $T(n) = 3T(n/2) + n$

$\Rightarrow$ $a=3$, $b=2$, $f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

$\therefore$ According to master's theorem, $T(n) = O(n^{\log_2 3})$

(14) $T(n) = 3T(n/3) + \sqrt{n}$

$\Rightarrow$ $a=3$, $b=3$, $f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

$\therefore$ According to master theorem, $T(n) = \Theta(n)$

(15) $T(n) = 4T(n/2) + cn$

$\Rightarrow$ $a=4$, $b=2$, $f(n) = c * n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c * n$

$\therefore$ According to master's theorem, $T(n) = \Theta(n^2)$

(16) $T(n) = 3T(n/4) + n \log n$

$\Rightarrow$ $a=3$, $b=4$, $f(n) = n \log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n \log n$

$\therefore$ According to master's theorem, $T(n) = \Theta(n \log n)$

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(17) $T(n) = 3T(n/3) + n/2$

$\Rightarrow$ $a=3,\ b=3$, $f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$\Theta(n) = \Theta(n/2)$

$\therefore$ According to master's theorem
$T(n) = \Theta(n \log n)$

(18) $T(n) = 6T(n/3) + n^2 \log n$

$\Rightarrow$ $a=6,\ b=3,\ f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2 \log n$

$\therefore$ According to master's theorem
$T(n) = \Theta(n^2 \log n)$

(19) $T(n) = 4T(n/2) + n/\log n$

$\Rightarrow$ $a=4,\ b=2,\ f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n$

$\therefore$ According to master's theorem $T(n) = \Theta(n^2)$

(20) $T(n) = 64T(n/8) - n^2 \log n$

$\Rightarrow$ Master's theorem not applicable as $f(n)$ is not increasing function.

(21) $T(n) = 7T(n/3) + n^2$

$\Rightarrow$ $a=7,\ b=3,\ f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^7$

$n^7 < n^2$

$\therefore$ According to masters', $T(n) = \Theta(n^2)$.

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(22) $T(n) = T(n/2) + n(2-\cos n)$

$\Rightarrow$ Master's Theorem is not applicable since regularity condition is isolated in Case 3.