Test Booklet Code

00

Aakash

Medical | IIT-JEE | Foundations

(Divisions of Aakash Educational Services Limited)

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Answers & Solutions

Time : 1 hrs. **M.M. : 40**

for

GUJCET-2019

(Mathematics)

Important Instructions :

1. The Mathematics test consists of 40 question. Each question carries 1 marks. For correct response, the candidate will get 1 marks. For each incorrect response 1/4 mark will be deducted. The maximum marks are 40.
2. This test is of 1 hours duration.
3. Use **Black Ball Point Pen only** for writing particulars on OMR Answer Sheet and marking answers by darkening the circle.
4. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
5. **On completion of the test, the candidate must handover the Answer Sheet to the Invigilator in the Room/Hall. The candidates are allowed to take away this Test Bookle with them.**
6. The Set No. for this Booklet is 00. Make sure that the Set No. Printed on the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
7. The candidate should ensure that the Answer Sheet is not folded. Do not make any stray marks on the Answer Sheet.
8. Do not write your Seat No. anywhere else, except in the specified space in the Test Booklet/Answer Sheet.
9. Use of White fluid for correction is not permissible on the Answer Sheet.
10. Each candidate must show on demand his/her Admission Card to the Invigilator.
11. No candidate, without special permission of the Superindent or Invigilator, should leave his/her seat.
12. Use of manual Calculator is permissible.
13. The candidate should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and must sign the Attendance Sheet (Patrak-01). Cases where a candidate has not signed the Attendance Sheet (Patrak-01) will be deemed not to have handed over the Answer Sheet and will be dealt with as an unfair means case.
14. The candidates are governed by all Rules and Regulations of the Board with regards to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.
15. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.
16. The candidates will write the Correct Test Booklet Set No. as given in the Test Booklet/Answer Sheet in the Attendance Sheet. (Patrak-01)

PART–C : MATHEMATICS

1. If the rate of change of area of rhombus with respect to it's side is equal to the side of rhombus, then the angles of rhombus are................

(1) $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ (2) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

(3) $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ (4) $\frac{5\pi}{12}$ and $\frac{7\pi}{12}$

Answer (2)

Sol. Area = $a^2 \sin\theta = A$

$= \frac{dA}{Da} = 2a \sin\theta = a$

$= \sin\theta = \frac{1}{2}$

$\therefore$ Angles are

$\frac{\pi}{6}$ and $\frac{5\pi}{6}$

2. The approximate value of 5^{201} is ...., where, $(\log_e 5 = 1.6095)$.

(1) 25.4125 (2) 25.5025

(3) 25.2525 (4) 25.4024

Answer (4)

Sol. $f(x) = 5^x,\ x = 2,\ \Delta x = .01$

Now $f(x+\Delta x) = f(x) + f'(x)\Delta x$

$f(5^{2.01}) = 25 + ((5^2 \log^5) x.01)$

$= 25.40237 \approx 25.4024$

3. $f(x) = \frac{x}{\log x^e}$ is increasing on the interval ....; where $x \in R^+ - \{1\}$

(1) $(-e, \infty)$ (2) $(0, \infty) - \{1\}$

(3) $\left(\frac{1}{e}, 1\right) \cup (1, \infty)$ (4) $\left(\frac{1}{e}, \infty\right)$

Answer (3)

Sol. $f(x) = \frac{x}{\log x^e} = x \log_e x (x \neq 1)$

$f'(x) > 0 \Rightarrow \log x + 1 > 0$

$\Rightarrow \log_e x > -1$

$\Rightarrow x > \frac{1}{e}; (x \neq 1)$

$\Rightarrow \left(\frac{1}{e}, 1\right) \cup (1, \infty)$

4. $\int (2 + \log x)(ex)^x dx = ... + C; x > 1$.

(1) $(ex)^x$ (2) $(ex)^{-x}$

(3) x^x (4) e^{x^x}

Answer (1)

Sol. $\int (2 + \log x)(ex)^x dx$

Now, $(ex)^x = t$

$\Rightarrow x \log(ex) = \log t$

$\Rightarrow x(1 + \log x) = \log t$

$\Rightarrow (1 + (\log x + 1)) dx = \frac{1}{t} dt$

$\Rightarrow (2 + \log x)(ex)^x dx = dt$

$\therefore \int dt = t + c = (ex)^x + c$

5. $\int e^{\sqrt{x}} dx$ = ...+C; x > 0

(1) $2(\sqrt{x} - 1)e^{\sqrt{x}}$ (2) $2(1 - \sqrt{x})e^{\sqrt{x}}$

(3) $(1 - \sqrt{x})e^{\sqrt{x}}$ (4) $(\sqrt{x} - 1)e^{\sqrt{x}}$

Answer (1)

Sol. $\int e^{\sqrt{x}} dx \quad \sqrt{x} = t \ ; \quad dx = 2tdt$

$\Rightarrow 2\int e^t t dt$

$\Rightarrow 2[e^t t - e^t]$

$\Rightarrow 2e^{\sqrt{x}}[\sqrt{x} - 1] + c$

6. If $\int \frac{\sin x}{\sin(x-\alpha)}dx = px - q\log|\sin(x-\alpha)| + C$, then pq = ...

(1) $-\frac{1}{2}\sin 2\alpha$ (2) $\frac{1}{2}\sin 2\alpha$

(3) $\sin 2\alpha$ (4) $-\sin 2\alpha$

Answer (1)

Sol. $\int \frac{\sin x}{\sin(x-\alpha)}dx$

$\Rightarrow \int \frac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)}dx$

$\Rightarrow \int (\cos\alpha + \sin\alpha\cot(x-\alpha))dx$

$\Rightarrow x(\cos\alpha) + \sin\alpha.\log(|\sin(x-\alpha)|) + c$

p = $\cos\alpha$

q = $-\sin\alpha$

$\therefore pq = \frac{-1}{2}\sin 2\alpha$

7. $\int_1^3 \left(\frac{x^2+1}{4x}\right)^{-1} dx$ = ...

(1) log 5 (2) log 25

(3) $\frac{1}{2}\log 5$ (4) log 100

Answer (2)

Sol. $\int_1^3 \left(\frac{x^2+1}{4x}\right)^{-1}$

$\Rightarrow \int_1^3 \left(\frac{4x}{x^2+1}\right)dx$

$\Rightarrow x^2+1 = t \Rightarrow 2xdx = dt$

$\int_2^{10} \frac{2dt}{t} \Rightarrow 2(\log 10 - \log 2)$

$\Rightarrow$ log 25

8. If $\int_1^K (2x-3) = 12$, then K = ...

(1) –2 and 5 (2) 2

(3) 5 (4) –5

Answer (1)

Sol. $\int_1^K (2x-3) = 12$

$\Rightarrow \left[x^2-3x\right]_1^K = 12$

$\Rightarrow (K^2-3K)-(1-3) = 12$

$\Rightarrow K^2-3K+2-12 = 0$

$\Rightarrow K^2-3K-10 = 0$

$\Rightarrow$ K = 5, –2

("dx" is not mention in question)

9. $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 2x}{1+25^x}dx$ = ...

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$

(3) $-\frac{\pi}{2}$ (4) $-\frac{\pi}{4}$

Answer (1)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 2x}{1+25^x}dx$

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2(2x)}{1+25^{-x}}dx$

$\left[\int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$

add,

$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2(2x)dx$

$2I = 2\int_0^{\frac{\pi}{2}} \cos^2(2x)dx$

$I = \int_0^{\frac{\pi}{2}} \left(\frac{1+\cos 4x}{2}\right)dx = \frac{\pi}{4}$

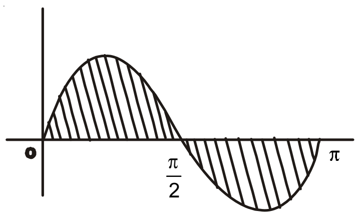
10. The area bounded by curve $y=\sin 2x\,(x=0 \text{ to } x=\pi)$ and $X-$axis is ....

(1) 4 (2) 1

(3) 2 (4) $\frac{3}{2}$

Answer (3)

Sol. $y=\sin 2x \quad (x=0 \text{ to } x)$



$$\Rightarrow \text{Area} = 2\int_0^{\frac{\pi}{2}} \sin(2x)\,dx$$

$$= 2\left[-\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= -[-1-1]$$

$$= 2$$

11. Area bounded by the ellipse $2x^2+3y^2=1$ is ....

(1) $\frac{\pi}{6}$ (2) 6π

(3) $\frac{\pi}{\sqrt{6}}$ (4) $\sqrt{6}\,\pi$

Answer (3)

Sol. Area of ellipse is π ab

$$\Rightarrow \frac{x^2}{\left(\frac{1}{\sqrt{2}}\right)^2}+\frac{y^2}{\left(\frac{1}{\sqrt{3}}\right)^2}=1$$

$$\Rightarrow \text{Area} = \pi\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{\sqrt{6}}$$

12. The integrating factor (I.F.) of differential equation $\frac{dy}{dx}(1+x)-xy=1-x$ is ....

(1) $(1+x)e^{x}$ (2) $(1+x)e^{-x}$

(3) $(x-1)e^{-x}$ (4) $(1-x)e^{-x}$

Answer (2)

Sol. $\frac{dy}{dx}(1+x)-xy=1-x$

$$\Rightarrow \frac{dy}{dx}-\frac{x}{1+x}y=(1-x)$$

$$\text{I.F.} = e^{-\int\frac{x}{1+x}dx}$$

$$= (1+x)e^{-x}$$

13. If the general solution of some differential equation is $y=a_1(a_2+a_3).\cos(x+a_4)-a_5.e^{x+a_6}$ then order of differential equation is ....

(1) 6 (2) 4

(3) 5 (4) 3

Answer (4)

Sol. $y=a_1(a_2+a_3)\,.\cos(x+a_4)-a_5e^{x+a_6}$

$$\Rightarrow y=A_1\cos(x+a_4)-A_2e^{x}$$

Order is 3

14. If the length of the subnormal at any point of the curve is constant, then the eccentricity of this curve is ....

(1) $e=\sqrt{2}$ (2) $0<e<1$

(3) $e>1$ (4) $e=1$

Answer (4)

Sol. For parabola, length of subnormal is constant $\quad\therefore e=1$

15. If $|\bar{x}|=|\bar{y}|=|\bar{x}+\bar{y}|=1$, then $|\bar{x}-\bar{y}|=$.............

(1) $\sqrt{2}$ (2) 1

(3) $\sqrt{3}$ (4) 3

Answer (3)

Sol. $|\vec{x}+\vec{y}|^2+|\vec{x}-\vec{y}|^2=2\left(|\vec{x}|^2+|\vec{y}|^2\right)$

$$\therefore |\vec{x}-\vec{y}|=\sqrt{3}$$

16. If $\bar{x}$ is a vector in the direction of $(2,-2,1)$ of magnitude 6 and $\bar{y}$ is a vector in the direction of $(1,1,-1)$ of magnitude $\sqrt{3}$, then $|\bar{x}+2\bar{y}|=$............

(1) 40 (2) $\sqrt{17}$

(3) $\sqrt{35}$ (4) $2\sqrt{10}$

Answer (4)

Sol. Now $\vec{x} = (4, -4, 2)$

$\vec{y} = (1, 1, -1)$

$\Rightarrow$ 36 + 4 (3) + 4 (4 - 4 -2) = $\left|\vec{x} + 2\vec{y}\right|^2$

$\Rightarrow$ $\left|\vec{x} + 2\vec{y}\right| = 2\sqrt{10}$

17. The angle between two adjacent sides $\bar{a}$ and $\bar{b}$ of parallelogram is $\frac{\pi}{6}$. If $\bar{a} = (2, -2, 1)$ and $\bar{b} = 2|\bar{a}|$, then area of this parallelogram is ....

(1) 9 (2) $\frac{9}{2}$

(3) 18 (4) $\frac{3}{4}$

Answer (1)

Sol. Area of parallelogram $= |a \times b|$

$= |a|\,|b|\,|\sin\theta|$

$= 3 \times (2 \times 3) \times \frac{1}{2}$

$= 9$

18. The perpendicular distance from the point of intersection of line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z}{-1}$ and plane $2x - y + z = 0$ to the Z-axis is.............

(1) 1 (2) 2

(3) $\sqrt{5}$ (4) 5

Answer (?)

Sol. Question is incorrect, as line is lying on plane

$\therefore$ perpendicular distance can be multiple values

19. The measure of the angle between the line $\bar{r} = (2, -3, 1) + k(2, 2, 1);\, k \in R$ and the plane $2x - 2y + z + 7 = 0$ is ....

(1) $\cos^{-1}\frac{1}{9}$ (2) $\sin^{-1}\frac{1}{3}$

(3) $\tan^{-1}\frac{1}{4\sqrt{5}}$ (4) $\frac{\pi}{2}$

Answer (3)

Sol. $\sin\theta = \frac{4-4+1}{3\times 3} = \frac{1}{9}$

$= \tan\theta = \frac{1}{\sqrt{80}} = \frac{1}{4\sqrt{5}}$

$\theta = \tan^{-1}\frac{1}{4\sqrt{5}}$

20. The image of the point A(1, 2, 3) relative to the plane π is $B(3, 6, -1)$, the equation of plane π is ......

(1) $x + 2y + 3z - 1 = 0$ (2) $x - 2y + 2z - 8 = 0$

(3) $x + 2y - 2z + 8 = 0$ (4) $x + 2y - 2z - 8 = 0$

Answer (4)

Sol. Midpoint will lie on plane

= (2, 4, 1)

And D. R. of Normal to the plane is $(1, 2, -2)$

Now equation of pane is given by

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$= 1(x-2) + 2(y-4) - 2(z-1) = 0$

$= x - 2 + 2y - 8 - 2z + 2 = 0$

$= x + 2y - 2z - 8 = 0$

21. $f: R \to R,\, f(x) = x^2 + 3x + 4$ is ....

(1) one-one and onto

(2) many-one and not onto

(3) one-one and not onto

(4) not one-one and onto

Answer (2)

Sol. $f'(x) = 2x + 3$, As this depends on x $\Rightarrow$ many-one

Here Range = $\left[\frac{7}{4}, \infty\right) \neq \text{co} - \text{domain} \Rightarrow \text{Not onto}$

Alternative method

A parabola is many-one and not onto from $R \to R$

22. If $a * b = \frac{ab}{10}; a, b \in Q^+$, then $(5 * 8)^{-1}$=............

(1) 4 (2) 10

(3) $\frac{1}{25}$ (4) 25

Answer (4)

Sol. $a*b = \dfrac{ab}{10}$

Identity element = 10

Now Inverse $= \dfrac{100}{a}$

$5*8 = \dfrac{5\times 8}{10} = 4 \;\; = a$

$\therefore$ inverse $= \dfrac{100}{4} = 25$

23. If $f:N\to N$, $f(x)=x+3$, then $f^{-1}(x)=$..........

(1) $x+3$ (2) $x-3$

(3) does not exists (4) $3-x$

Answer (3)

Sol. Function is not bijective

$\Rightarrow$ Inverse does not exists

24. $\sin^2\left(\sin^{-1}\dfrac{1}{2}\right)+\tan^2\left(\sec^{-1}2\right)+\cot^2\left(\mathrm{cosec}^{-1}4\right)=$.......

(1) $\dfrac{73}{4}$ (2) $\dfrac{89}{4}$

(3) $\dfrac{37}{2}$ (4) 19

Answer (1)

Sol. $\sin^2\left(\sin^{-1}\left(\dfrac{1}{2}\right)\right)+\tan^2\left(\sec^{-1}2\right)+\cot^2\left(\mathrm{cosec}^{-1}4\right)$

$= \dfrac{1}{4}+3+15$

$= \dfrac{73}{4}$

25. $\tan\left(\cos^{-1}\dfrac{4}{5}+\tan^{-1}\dfrac{2}{3}\right)$ =.................

(1) $\dfrac{3}{17}$ (2) $\dfrac{17}{4}$

(3) $\dfrac{17}{6}$ (4) $\dfrac{6}{17}$

Answer (3)

Sol. $\tan\left(\tan^{-1}\left(\dfrac{3}{4}\right)+\tan^{-1}\left(\dfrac{2}{3}\right)\right)$

$= \tan\left(\tan^{-1}\left(\dfrac{\dfrac{3}{4}+\dfrac{2}{3}}{1-\dfrac{1}{2}}\right)\right)$

$= \tan\left(\tan^{-1}\left(\dfrac{17}{6}\right)\right) = \dfrac{17}{6}$

26. $\cos\left(\cot^{-1}\left(\mathrm{cosec}\left(\cos^{-1}a\right)\right)\right)=$........ (Where, $0<a<1$)

(1) $\dfrac{1}{\sqrt{2-a^2}}$ (2) $\sqrt{2-a^2}$

(3) $\sqrt{3-a^2}$ (4) $\dfrac{1}{\sqrt{2+a^2}}$

Answer (1)

Sol. $\cos\left(\cot^{-1}\left(\mathrm{cosec}\left(\cos^{-1}a\right)\right)\right)$

$= \cos\left(\cot^{-1}\left(\dfrac{1}{\sqrt{1-a^2}}\right)\right)$

$= \cos\cos^{-1}\left(\dfrac{1}{\sqrt{2-a^2}}\right)$

$= \dfrac{1}{\sqrt{2-a^2}}$

27. $\begin{vmatrix}\sin^2\theta & \cos^2\theta\\ -\cos^2\theta & \sin^2\theta\end{vmatrix}$ = ...

(1) $\cos 2\theta$ (2) $\dfrac{1}{2}\left(1-\sin^2 2\theta\right)$

(3) $\dfrac{1}{2}\left(1+\cos^2 2\theta\right)$ (4) $\dfrac{1}{2}\sin^2 2\theta$

Answer (3)

Sol. $\begin{vmatrix}\sin^2\theta & \cos^2\theta\\ -\cos^2\theta & \sin^2\theta\end{vmatrix}$

$= \sin^4\theta+\cos^4\theta$

$= 1-2\sin^2\theta\cos^2\theta$

$= 1-\dfrac{\sin^2 2\theta}{2}$

$= 1-\left(\dfrac{1-\cos^2 2\theta}{2}\right)$

$= \dfrac{1}{2}\left(1+\cos^2 2\theta\right)$

28. If $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} = 2016K$, then K = ...

(1) 24 (2) $\frac{1}{24}$

(3) 84 (4) $\frac{1}{84}$

Answer (4)

Sol. $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} = 12\begin{vmatrix} 1 & 2 & 6 \\ 1 & 3 & 12 \\ 1 & 4 & 20 \end{vmatrix}$

= 24 = 2016 k

$k = \frac{1}{84}$

29. If $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+x & 1+z & 1+3z \end{vmatrix} = 10K\,xyz\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

then K=........... $\left(\text{Where } xyz \neq 0;\ 3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} \neq 0\right)$.

(1) $\frac{1}{5}$ (2) 5

(3) 2 (4) 1

Answer (1)

Sol. $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix}$

$= (10k)xyz\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

$xyz\begin{vmatrix} \frac{1}{x}+1 & \frac{1}{x} & \frac{1}{x} \\ \frac{1}{y}+1 & \frac{1}{y}+2 & \frac{1}{y} \\ \frac{1}{z}+1 & \frac{1}{z}+1 & \frac{1}{z}+3 \end{vmatrix}$

$= xyz\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{y}+1 & \frac{1}{y}+2 & \frac{1}{y} \\ \frac{1}{z}+1 & \frac{1}{z}+1 & \frac{1}{z}+3 \end{vmatrix} (R_1 \to R_1+R_2+R_3)$

$= 2xyz\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)(C_2 \to C_2 - C_1 \text{ and } C_3 \to C_3 - C_1)$

$\Rightarrow k = \frac{1}{5}$

30. If the inverse of the matrix = $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is

$\frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & \alpha \\ 2 & 2 & -3 \end{bmatrix}$ then, a = ...

(1) 3 (2) 2

(3) 4 (4) –2

Answer (2)

Sol. $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & \alpha \\ 2 & 2 & -3 \end{bmatrix}$

$\Rightarrow \alpha = 2$

31. Matrix $A_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$;

$r = 1, 2, 3, \ldots\ldots$ If $\sum_{r=1}^{100} |A_r| = \left(\sqrt{10}\right)^K$, then K= ....;

$\left(|A_r| = \det(A_r)\right)$.

(1) 2 (2) 4

(3) 6 (4) 8

Answer (4)

Sol. $|A_r| = (2r-1)$

$= \sum_{r=1}^{100}(2r-1)$

$= 10000 = 10^4 = \left(\sqrt{10}\right)^8 = \left(\sqrt{10}\right)^k$

= k = 8

32. $\frac{d}{dx}\left(3\cos\left(\pi/6 + x^0\right) - 4\cos^3\left(\pi/6 + x^0\right)\right) = \ldots\ldots\ldots$

(1) $\cos\left(3x^0\right)$ (2) $\frac{\pi}{60}\cos\left(3x^0\right)$

(3) $\frac{\pi}{60}\sin\left(3x^0\right)$ (4) $\frac{-\pi}{60}\sin\left(3x^0\right)$

Answer (2)

Sol. $\frac{d}{dx}\left(-\cos\left(\frac{\pi}{2}+3x^0\right)\right)$

$\frac{d}{dx}\left(\sin\left(3x^0\right)\right)$

$= \frac{\pi}{60}\cos\left(3x^0\right)$

33. If $f(x)=1+x+x^2+....+x^{1000}$, then $f'(-1)=$.........

(1) -50 (2) -100

(3) -500 (4) 500500

Answer (3)

Sol. $f'(x)=1+2x+3x^2+........1000x^{999}$

$= f'(-1)=1-2+3-4+5.............-1000$

$= -500$

34. Applying mean value theorem on $f(x)=\log x;\ x\in[1, e]$ the value of e =........

(1) $\log\ (e-1)$ (2) $1-e$

(3) $e-1$ (4) 2

Answer (3)

Sol. $f'(c)=\frac{f(b)-f(a)}{b-a}$ a = 1

b = e

$= \frac{1}{x}=\frac{1-0}{e-1}$

$= x = \ e-1$

35. If $\int\sin^{13}x\cos^3 x\,dx = A\sin^{14}x+B\sin^{16}x+C$, then A + B =..........

(1) $\frac{1}{110}$ (2) $\frac{15}{112}$

(3) $\frac{17}{112}$ (4) $\frac{1}{112}$

Answer (4)

Sol. $I=\int\sin^{13}x\cos^3 x dx$

$= \int\sin^{13}x\left(1-\sin^2 x\right)\cos x\ dx$

$= \int\left(\sin^{13}x-\sin^{15}x\right)\cos x\ dx$

let sin x = t

$\Rightarrow \cos\ x\ dx\ =\ dt$

$\Rightarrow\ I=\int\left(t^{13}-t^{15}\right)\ dt$

$= \frac{t^{14}}{14}-\frac{t^{16}}{16}+C$

$= \frac{\sin^{14}x}{14}-\frac{\sin^{16}x}{16}+C$

$\therefore A=\frac{1}{14}\ B=\frac{-1}{16}$

$\therefore A+B=\frac{1}{112}$

36. If $\int\frac{1+\cos x}{\cos x-\cos^2 x}dx=\log|\sec x+\tan x|-2f'(x)+C$, then $f(x)=$.............

(1) $2\cot\left(\frac{x}{2}\right)$ (2) $-2\cot\left(\frac{x}{2}\right)$

(3) $2\log\left|\sin\frac{x}{2}\right|$ (4) $-2\log\left|\sin\frac{x}{2}\right|$

Answer (3)

Sol. $\int\frac{1+\cos x}{\cos x(1-\cos x)}..........dx$

$= \int\left(\frac{1}{\cos x}+\frac{2}{1-\cos x}\right)dx$

$\int\left((\sec x)+2\left(\text{cosec}^2 x+\cot x\cdot\text{cosec} x\right)\right)dx$

$= \log(|\sec x+\tan x|)\ -2\cot x-2\,\text{cosec} x+c$

$\therefore f'(x)=\cot x+\text{cosec}\, x$

$f'(x)=\cot\left(\frac{x}{2}\right)$

$\therefore f(x)=2\log\left(|\sin\left(\frac{x}{2}\right)|\right)$

37. The probability that an event A occurs in a single trial of an experiment is 0.6. In the first three independent trials of the experiment, the probability that A occurs atleast once is ......

(1) 0.930 (2) 0.925

(3) 0.936 (4) 0.927

Answer (3)

Sol. $1-(.4)^3$

= 0.936

38. If $6P(A)=8P(B)=14P(A\cap B)=1$, then $P\left(\frac{A'}{B}\right)=$....

(1) $\frac{3}{7}$ (2) $\frac{3}{5}$

(3) $\frac{4}{7}$ (4) $\frac{2}{5}$

Answer (1)

Sol. $P\left(\frac{A'}{B}\right)=\frac{P(A'\cap B)}{P(B)}$

$=\frac{P(B)-P(A\cap B)}{P(B)}$

$==1-\frac{P(A\cap B)}{P(B)}$

Now $P(A)=\frac{1}{6}$

$P(B)=\frac{1}{8}$

$P(A\cap B)=\frac{1}{14}$

$\therefore 1-\frac{\frac{1}{14}}{\frac{1}{8}}$

$=\frac{3}{7}$

39. The mean and variance of a random variable X having a binomial distribution are 6 and 3 respectively. The probability of variable X less than 2 is ....

(1) $\frac{13}{2048}$ (2) $\frac{15}{4096}$

(3) $\frac{13}{4096}$ (4) $\frac{25}{2048}$

Answer (3)

Sol. $\text{mean}=np=6$; $\text{varience}=npq=3$

$\therefore q=\frac{1}{2}$

$\therefore p=\frac{1}{2}$

$\therefore n=12$

$\sum_{r=0}^{1}{}^{12}C_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{12-r}$

$=\left(\frac{1}{2}\right)^{12}+12\times\frac{1}{2}\times\left(\frac{1}{2}\right)^{11}$

$=\left(\frac{1}{2}\right)^{12}[1+12]$

$=\frac{13}{4096}$

40. The coordinates of the corner points of the bounded feasible region are $(10,0),(2,4),(1,5)$ and $(0,8)$. The maximum of objective function $z=60x+10y$ is ....

(1) 700 (2) 600

(3) 800 (4) 110

Answer (2)

Sol. Maximum of objective functions

z= 60 x + 10 y is obtained at (10, 0)

$\therefore (z)_{maximum}$

= 600

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