GUJCET 2022 SET: 08

MATHEMATICS

1) If $2\sin^{-1}x = \sin^{-1}2x\sqrt{1-x^2}$ then $x \in$ ________.

(A) $\left[-\frac{1}{\sqrt{2}},\ 1\right]$ (B) $[0,\ 1]$

(C) $\left[\frac{1}{\sqrt{2}},\ 1\right]$ (D) $\left[-\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]$

2) $\cos\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=0$ then $x =$ ________.

(A) 1 (B) $\frac{1}{5}$

(C) 0 (D) 5

3) Binary operation $*$ on R is given by $a*b=\frac{a+b}{2}$. Then $*$ is ________.

(A) not commutative but associative

(B) commutative and associative

(C) commutative but not associative

(D) not commutative and not associative

(Space for Rough Work) $\frac{a+b}{2} = \frac{b+a}{2}$

4) Let $A=\{-1,-2,3,4\}$. Number of all one-one functions from the set A to itself is ________.

(A) 24

(B) 16

(C) 4

(D) 256

5) If functions f and g are defined as:

$$f:\left[0,\frac{\pi}{2}\right]\rightarrow R,\quad f(x)=\sin x \text{ and}$$

$$g:\left[0,\frac{\pi}{2}\right]\rightarrow R,\quad g(x)=\cos x$$

then ____________.

(A) $f+g$ is one-one and fg is not one-one

(B) $f+g$ is not one-one and fg is one-one

(C) $f+g$ is not one-one and fg is not one-one

(D) $f+g$ is one-one and fg is one-one

6) If $y=100e^{2x}+200e^{-2x}$ and $\frac{d^2y}{dx^2}=ay$ then $a=$ ________.

(A) 4

(B) −4

(C) 2

(D) 0

(Space for Rough Work)

Function $f:[1.2, 1.9] \to R, f(x) = [x]$, where $[x]$ denotes the greatest int
less than or equal to x. Then __________.

(A) $f'(x) = 1$

(B) f is not differentiative

(C) $f'(x) = 0$

(D) f is not continuous function

8) If $x = \sqrt{10^{\sin^{-1} t}}$, $y = \sqrt{10^{\cos^{-1} t}}$ then $\frac{dy}{dx} =$ __________.

(A) $-\frac{x}{y}$

(B) $\frac{y}{x}$

(C) 0

(D) $-\frac{y}{x}$

9) The interval in which $y = x^2 e^{-x}$ is increasing is __________.

(A) $(0, 2)$

(B) $(-2, 0)$

(C) $(2, \infty)$

(D) $(-\infty, \infty)$

10) Equation of tangent line to $16x^2 + 25y^2 = 1$, which is parallel to Y-axis is __________

(A) $5y - 1 = 0$

(B) $5x - 1 = 0$

(C) $4y + 1 = 0$

(D) $4x - 1 = 0$

7) Function $f:[1.2, 1.9] \rightarrow R, f(x)=[x]$, where $[x]$ denotes the greatest integer less than or equal to x. Then ________.

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(D) $4x-1=0$

$\Delta V / \Delta t$

d

r

11) A cylindrical tank of diameter 20 m is being filled with wheat at the rate of 314 cubic meter per hour. Then the depth of the wheat is increasing at the rate of ______.

(A) 0.5 m/h (B) 0.1 m/h

(C) 1.1 m/h (D) 1 m/h

12) $\int e^{\sin x} \sin 2x \, dx =$ ________ $+ C$.

(A) $e^{\sin x}(\sin x + 1)$ (B) $2e^{\sin x}(\sin x - 1)$

(C) $2e^{\sin x}(\sin x + 1)$ (D) $e^{\sin x}(\sin x - 1)$

13) $\int \sqrt{\dfrac{\cos x - \cos^3 x}{1 - \cos^3 x}} \, dx =$ __________ $+ C$.

(A) $-\dfrac{3}{2}\cos^{-1}\left(\cos^{3/2} x\right)$ (B) $-\dfrac{2}{3}\cos^{-1}\left(\cos^{3/2} x\right)$

(C) $\dfrac{3}{2}\cos^{-1}\left(\cos^{3/2} x\right)$ (D) $\dfrac{2}{3}\cos^{-1}\left(\cos^{3/2} x\right)$

14) $\int (x+1)(x+3)(x+2)^7 \, dx =$ __________ $+ C$.

(A) $\dfrac{(x+3)^{10}}{10} + \dfrac{(x+3)^8}{8}$ (B) $\dfrac{(x+2)^{10}}{10} + \dfrac{(x+2)^8}{8}$

(C) $\dfrac{(x+3)^{10}}{10} - \dfrac{(x+3)^8}{8}$ (D) $\dfrac{(x+2)^{10}}{10} - \dfrac{(x+2)^8}{8}$

(Space for Rough Work)

15) $\int \frac{x}{(x-1)(x-2)}dx =$ ________ $+ C$.

(A) $\log|(x-1)(x-2)|$

(B) $\log\left|\frac{(x-2)^2}{x-1}\right|$

(C) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|$

(D) $\log\left|\frac{(x-1)^2}{x-2}\right|$

16) $\int_{-\pi/4}^{\pi/4} \sin^2 x \, dx =$ ________.

even

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{4} - \frac{1}{2}$

(C) $\frac{\pi}{4} - 1$

(D) $\frac{\pi}{4} + \frac{1}{2}$

17) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\cancel{x^{13}} + \cancel{x\cos x} + \cancel{\tan^{15} x} + 1\right) dx =$ ________.

(A) 1

(B) 2

(C) π

(D) 0

18) If $f(a+b-x)=f(x)$ then $\int_a^b x\, f(x)dx =$ ______.

(A) $\frac{a+b}{2}\int_a^b f(x)dx$

(B) $\frac{a+b}{2}\int_a^b f(b+x)dx$

(C) $\frac{b-a}{2}\int_a^b f(x)dx$

(D) $\frac{a+b}{2}\int_a^b f(b-x)dx$

19) $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx =$ ______.

(A) $\frac{\pi}{4}$

(B) 0

(C) -1

(D) 1

20) The area of the region bounded by the two parabolas $y=x^2$ and $y^2=x$ is ______.

(A) $\frac{3}{4}$

(B) 3

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

21) The area of the parabola $x^2 = 12y$ bounded by its latus rectum is ________.

(A) 3

(B) $\frac{24}{3}$

(C) 24

(D) $\frac{8}{3}$

22) The area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$ is ________.

(A) $3\sqrt{3}$

(B) $3\sqrt{8}$

(C) 8

(D) $8\sqrt{3}$

23) If length of subnormal at any point of a curve is always constant then that curve represents a ________.

(A) Parabola

(B) Hyperbola

(C) Ellipse

(D) Rectangular hyperbola

24) The integrating factor of the differential equation $x\frac{dy}{dx} - y = x^2$ is ________.

(A) e^{-x}

(B) $\frac{1}{x}$

(C) e^{x}

(D) x

25) If the vectors $\hat{i}-\hat{j}+\hat{k}$, $3\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+\lambda\hat{j}-3\hat{k}$ are coplanar then $\lambda =$ ______.

(A) 15
(B) -15
(C) 5
(D) $\frac{5}{3}$

26) Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$. If $\vec{a}\times\vec{b}$ is a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is ______.

(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{6}$

27) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=$ ______.

(A) $-\frac{1}{2}$
(B) $\frac{3}{2}$
(C) $\frac{1}{2}$
(D) $-\frac{3}{2}$

28) The angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x+2y-11z=3$ is ______.

(A) $\cos^{-1}\left(\frac{1}{8}\right)$
(B) $\cos^{-1}\left(\frac{8}{21}\right)$
(C) $\sin^{-1}\left(\frac{8}{21}\right)$
(D) $\sin^{-1}\left(\frac{1}{8}\right)$

(Space for Rough Work)

29) The area of a triangle having the points $A(1,1,1)$, $B(1,2,3)$ and $C(2,3,1)$ as its vertices is ______.

(A) $\frac{\sqrt{19}}{2}$
(B) $\frac{\sqrt{21}}{2}$
(C) $\frac{19}{2}$
(D) $\frac{21}{2}$

30) The lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles then value of p is ______.

(A) $\frac{11}{7}$
(B) 7
(C) $\frac{70}{11}$
(D) $\frac{7}{11}$

31) The mean number of heads in three tosses of a fair coin is ______.

(A) 3.5
(B) 0.5
(C) 15
(D) 1.5

32) If for Bernoulli distribution $B\left(10,\frac{1}{2}\right)$, it is given that $P(X\leq 2)=m\left(\frac{1}{2}\right)^{10}$ then $m=$ ______.

(A) 101
(B) 55
(C) 56
(D) 46

(Space for Rough Work)

33) Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is ________.

(A) $2/5$ (B) $4/5$

(C) $1/5$ (D) $1/2$

34) Corner points of the feasible region of objective function $Z = 3x + 9y$ of a linear programming problem are (0, 10), (5, 5), (15, 15) and (0, 20). Minimum value of Z is ________.

(A) 70 (B) 90

(C) 50 (D) 60

35) If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then $A^3 =$ ________.

(A) $\begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\cos 3\theta & \sin 3\theta \end{bmatrix}$

(B) $\begin{bmatrix} -\cos 3\theta & \sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{bmatrix}$

(C) $\begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$

(D) $\begin{bmatrix} \cos 3\theta & -\sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$

(Space for Rough Work)

36) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ and B is inverse of A then $\alpha =$ ____.

(A) 10

(B) 9

(C) 3

(D) 5

37) For real numbers x, y, z such that $x \neq y \neq z$, $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ and $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} \neq 0$ then $xyz =$ ______.

(A) 2

(B) -1

(C) 0

(D) 1

38) If a, b, c are measurements of sides of ΔABC and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then

$\sin^2 A + \sin^2 B + \sin^2 C =$ ________.

(A) $\frac{13}{4}$ (B) $\frac{9}{4}$

(C) $\frac{15}{4}$ (D) $\frac{11}{4}$

39) If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ then sum of all the elements of $A^{-1} =$ ________.

(A) 6 (B) -6

(C) 0 (D) $\frac{11}{6}$

40) If $\sin^{-1} a = \alpha + \beta$, $\sin^{-1} b = \alpha - \beta$ then $\sin^2 \alpha + \cos^2 \beta =$ ________.

(A) ab (B) $1 - ab$

(C) $ab - 1$ (D) $1 + ab$

(Space for Rough Work)