Creating Second-Ordered Bootstrap Confidence Intervals for Effect Size Estimates

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Introduction / Background

Our project will aim to develop a second-ordered bootstrap confidence interval for effect size estimates. Bootstrapping is to take a dataset and resample the dataset with replacement (this means that for example, if you had a bag with 4 numbers (1, 2, 3, 4), and took a number out, then put it back, then chose from the bag again 4 times in total, a resample could be 1, 2, 2, 4 as you pulled a 1, then a 2, then 2 again, then 4) many times to create many simulated samples. The bootstrapping allows us to take many different measures of our datasets such as confidence intervals and standard errors. Bootstrapping can be applied in a variety of ways using many different formulas on our statistics taken from the samples. I would like to create a bootstrapping technique that is of the second-order for our confidence intervals, which would allow it to converge faster. By faster, I mean that we would want to have a n in the denominator (second-order) rather than a square root (first-order) as this would create smaller fractions faster (i.e. 1, 1/2, 1/3, 1/4, …, 1/n versus 1, $\frac{1}{\sqrt{2}} = \frac{1}{1.4142}, \frac{1}{\sqrt{3}} = \frac{1}{1.7321}, \frac{1}{\sqrt{4}} = \frac{1}{2}, \ldots, \frac{1}{\sqrt{n}}$) which eventually get extremely small and closer to 0 without actually hitting that number (convergence). Not only this, but I would also like to get those confidence intervals for different measures of effect size estimates, of which I will use two, Cohen's d and delta MAD (median absolute difference). Effect sizes are used to measure the strength of relationships between variables. For parametric distributions aka normally distributed distributions (this means that the data is symmetrical, 50% of the data is greater than and less than the mean, and the mean is equal to the median) we would use Cohen's d (which uses means to calculate relationships). However, in the case of non-parametric or non-normal distributions, we would use delta MAD as median is not affected by skewed data (asymmetric data) like how means are (a mean can be pulled up (increased) or pulled down (decreased) by outliers, points that are significantly far away from the rest of the data, or a few higher values that would make our distribution skewed). The reason that we would want to calculate confidence intervals of such effect sizes is that there will always be some level of error in an estimate of a statistic, no matter how much is known to form that estimate. Confidence intervals allow us to create a range of possible values that our test statistic (what we are trying to find) is expected to be in.

Research Question

How can I bootstrap confidence intervals of effect size estimates to get them to converge faster?

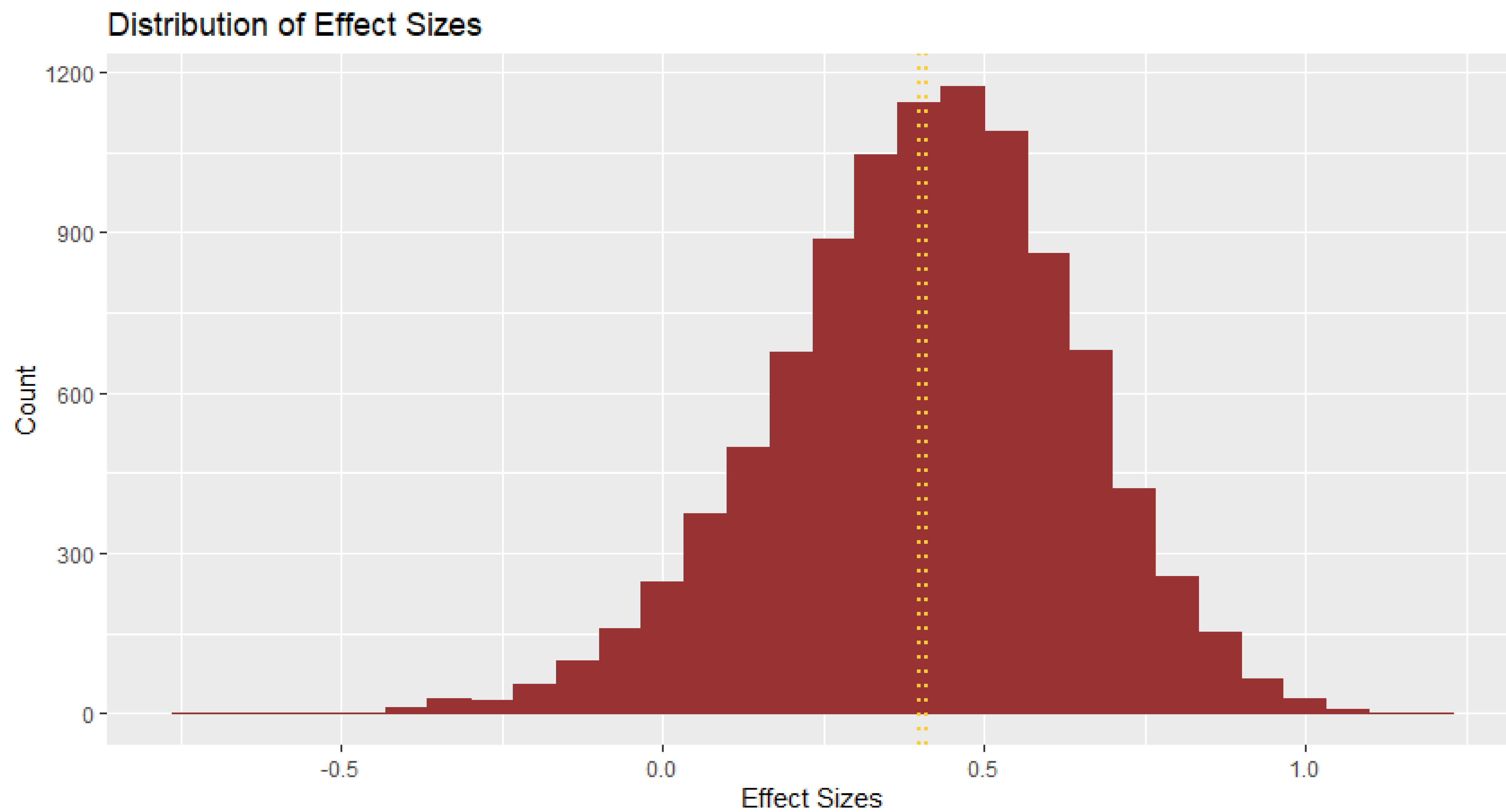


Figure 1

This is a graph of the distribution of Cohen's d effect sizes. To create this, I used an example of a class of 20 students before and after some lesson. The mean for "before" was created using a random normal distribution with a mean of 0, and a standard deviation of 1. Then, I created 2 more random normal distributions to be added together to make a random distribution for the "after". This was 80% (16 students) did not change so they still had a mean of 0 and a standard deviation of 1, while 20% (4 students) had a mean of 3 and a standard deviation of 2. I then calculated Cohen's d with $d = \frac{M_1 - M_2}{S}$. This was then done 10000 times more to get a distribution of the effect sizes. A confidence interval around the mean was created, which you can see in the gold dotted lines.

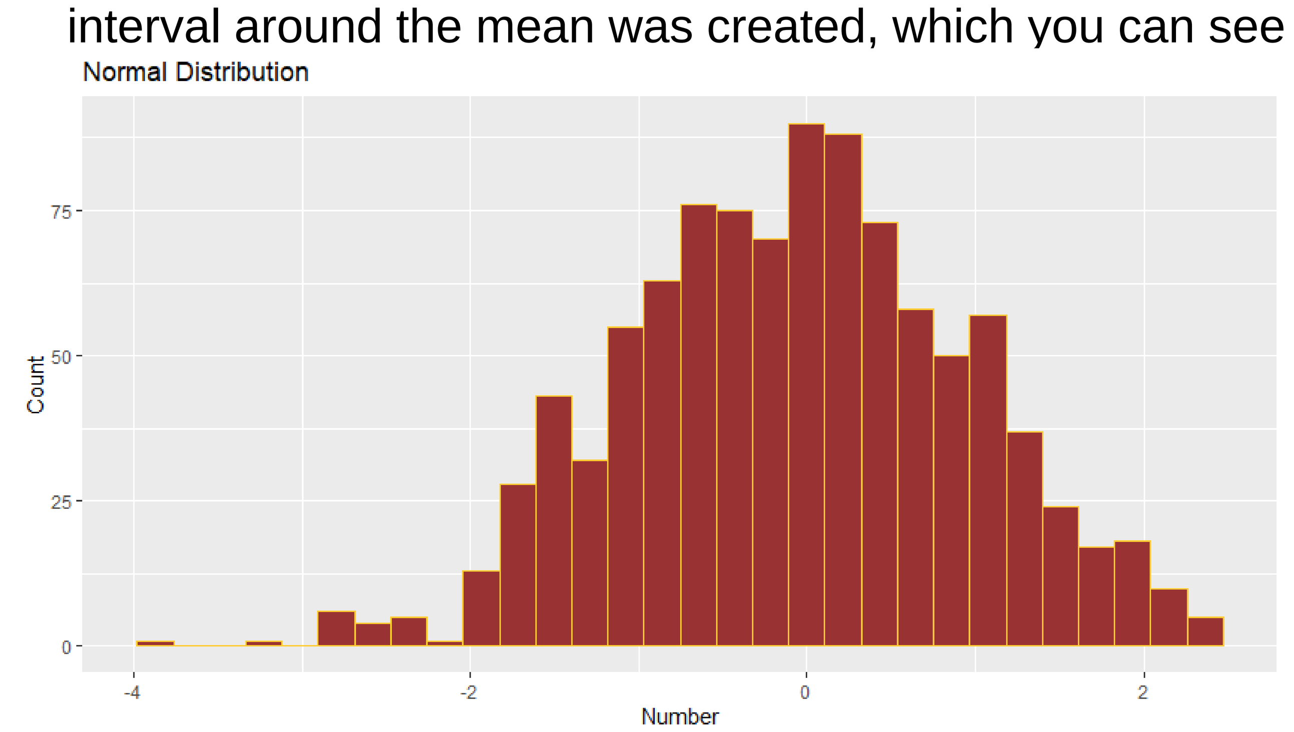
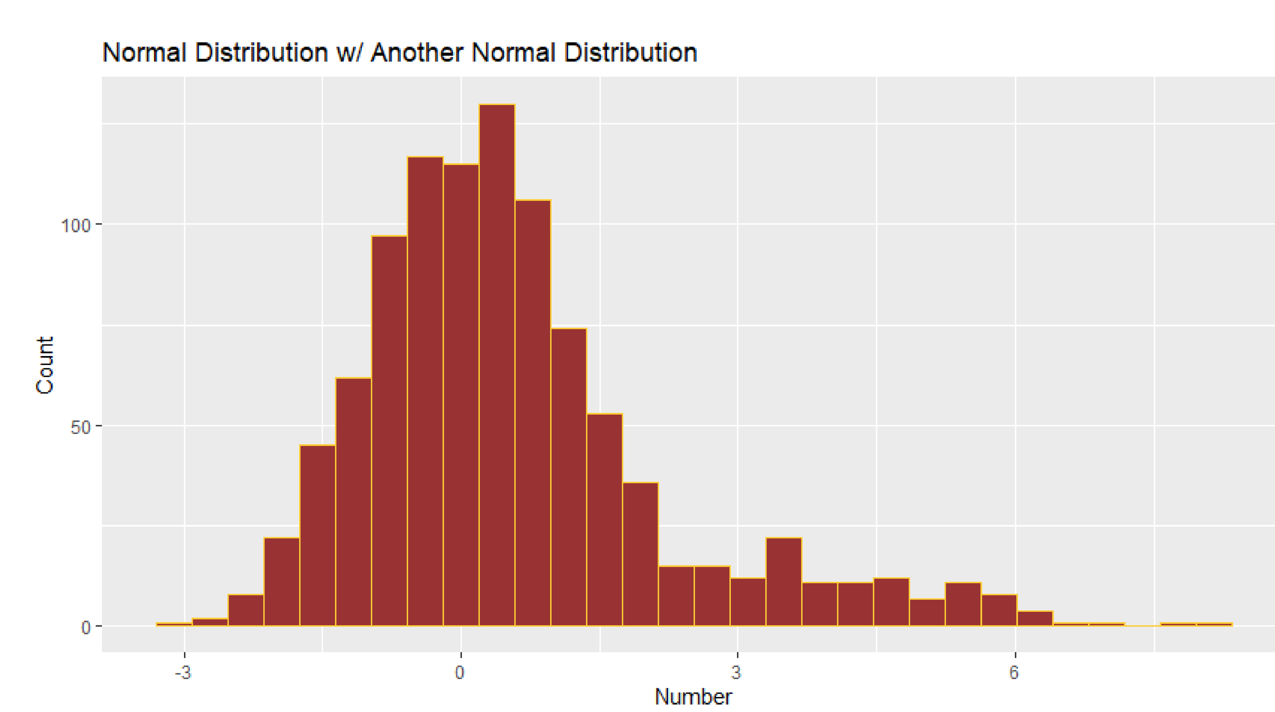


Figure 2

This figure is an example of a random normal distribution with 1000 data points, a mean of 0, and a standard deviation of 1.

Figure 3

This figure is an example of a random normal distribution with 1000 datapoints. In this case, 80% of the 1000 data points (800) had a mean of 0 and a standard deviation of 1, whole 20% (200) had a mean of 3 and a standard deviation of 2. As you can see, even adding a different normal distribution can quickly add a skew to the data.



Methods

I will use simulated samples from different types of datasets created using bootstrap confidence intervals and compare the rate of convergences for each different bootstrapping method used.

- Create random populations
  - Normally distributed data contaminated with normally distributed data
  - Normally distributed data contaminated with non-normally distributed data
  - Normally distributed data contaminated with log-normally distributed data
- Do different simulations with bootstrapped confidence intervals on our datasets
- Work on changing things in the bootstrap method to develop an answer to creating a confidence interval of effect sizes that converges faster

Significance

From surveying prior literature, there is not much to be known about second-order convergence. It is not yet a major topic and there is much research to be done. There has been some work done in bootstrap confidence intervals, however much of that has been specific to mean, median, and other statistical tools. Effect sizes have become a major type of statistic used in recent years, and as such, there should be appropriate ways of producing confidence intervals for effect sizes to allow more accuracy in our estimates of such effect sizes. Not only building confidence intervals, but also building confidence intervals to converge faster. Even though computer processing power is as not much of a barrier as it once was, faster is always better. We should always reach for new ways to solve things and gain the ability to do things we could not before.

References

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