3. Theoritical limit of Gaussian Classification

Following things are given
Classes – x_1 & x_2

x_1

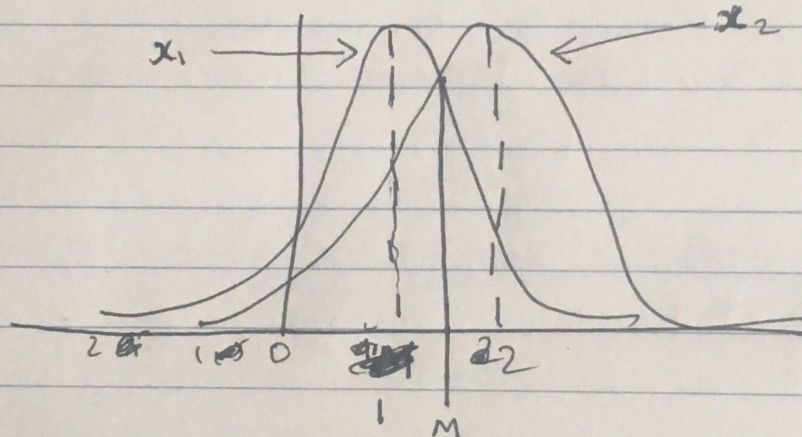
Mean (μ_1) = 1
Variance (σ^2) = 1

x_2

Mean (μ_2) = 2
Variance (σ^2) = 1

~~Next, Variance (σ^2~~

Wher σ = standard deviation



From the above graph, we can see that two ~~different~~ classification intersect each other at point M. The value for M is near about 1.5 which is quite clear from the graph too.

Now, the equation to calculate the theoritical limit is :

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \, dx$$

where, σ = standard deviation
μ = Mean
σ^2 = variance.

But, we have two classification: (x_1 & x

For x_1,

$$F(x_1) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{1.5} e^{-\frac{(x-\mu_1)^2}{\sigma^2}} dx$$

Substituting value of $\sigma = 1$, $\mu = 1$ & $\sigma^2 = 1$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{1.5} e^{-\frac{(x-1)^2}{2}} dx$$

Now, Substitution method can help to integrate

$$u = \frac{x-1}{2}$$

$$\therefore du = \left(\frac{1}{2} - 1\right) dx = \frac{1}{2} dx$$

$$\therefore dx = 2du$$

$$= \lim_{t \to -\infty} \frac{1}{\sqrt{2\pi}} \int_{t}^{1.5} 2e^{-u} du = \frac{\sqrt{2}}{\sqrt{\pi}} \left[e^{-\left(\frac{t-1}{2}\right)} - e^{-\left(\frac{1.5-1}{2}\right)} \right]$$

by integrating term we will get,

$F(x_1) \approx 0.69$

$\therefore \quad F(x_1) \approx 0.69$

Now, similarly for classification x_2,

$$F(x_2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{1.5}^{\infty} e^{-\frac{(x-\mu_2)}{2\sigma^2}} \, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{1.5}^{\infty} e^{-\frac{(x-2)}{2}} \, dx \qquad (\mu_2 = 2 \;\&\; \sigma = 1)$$

Now, similar calculation can be perform as previous

$$= \frac{1}{\sqrt{2\pi}} \left[-2 e^{\frac{-(x-2)}{2}} \right]_{1.5}^{\infty}$$

$$\approx 0.308$$

Now,

$$F(x_1) + F(x_2) = 1$$

$$\therefore 0.691 + 0.308 \approx 1$$

Here,

$F(x_1) = 0.691$

$F(x_2) = 0.308$