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A Robust Algorithm based on Spatial Differencing Matrix for Source Number Detection and DOA Estimation in Multipath Environment

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Abstract

In this paper, a robust algorithm based on spatial differencing matrix is proposed for source number detection and directions-of-arrival (DOAs) estimation of a number of coherent signals simultaneously impinging on the far field of a uniform linear array (ULA). In this proposed approach, the number and DOAs of incident coherent signals can be simultaneously estimated via the eigenvectors of the defined spatial differencing matrix. Compare with the previous works, the proposed method can improve the source number detection and DOA estimation accuracy, as well increase robustness to SNR change.

Keywords:Direction of arrival (DOA), uniform linear array (ULA), source number detection, spatial differencing.

1.Introduction

In array signal processing, DOA estimation of narrowband planewave signals has been intensively studied in the past few decades. Several high-resolution algorithms, such as multiple signal classification (MUSIC) [1] and estimation of signal parameter via rotation invariance techniques (ESPRIT) [2], have been proposed to resolve the uncorrelated signals. It is well-known that the performances of these high-resolution methods largely depend on the successful determination of the number of sources. Thus, several source number detection methods [3], [4] have been suggested. AIC, MDL, Gerschgorin radii approach etc are well-known algorithms for estimating the number of sources. In general, in the case of high signal-to-noise ratio (SNR) and large snapshots, the above methods can correctly determine the number of sources. But for low SNR, small snapshots or the perturbation in array response, misestimate

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often happen. However, there are often highly correlated or coherent sources in multipath propagation environments caused by various reflective surfaces or in military scenarios when smart jammers are present. Those high-resolution methods will fail in such environments since they inherently require the signals to be uncorrelated or lowly correlated. In order to decorrelate the coherent signals, many effective methods have been proposed, of which the spatial smoothing technique [5] is especially noteworthy. Based on the spatial sample of the data, the idea of the matrix pencil (MP) is utilized in [6], [7]. Unlike the conventional covariance matrix techniques, MP method can find the DOA easily in the presence of coherent signals without additional processing of spatial smoothing. Unfortunately, the required SNR is too high to put the method into application. In [8], an effective method that exploits the property of oblique projection is proposed. A DOA estimation method with fewer sensors is presented in [9]. But it is computationally not very attractive, too. Another efficient method is presented in [10]. In this method, the information of uncorrelated signals can be eliminated from the signal subspace after estimating the DOAs and power of uncorrelated signals, and then a C-matrix which only contains the information of coherent signals can be obtained. Finally, the DOAs of coherent signals are estimated with a new matrix constructed by the C-matrix. A deflation approach is proposed in [11], but the number of coherent signals it can resolve is less than half of the number of array sensors. In [12], a non-Toeplitz matrix is constructed to resolve more signals by exploiting the symmetric configuration of a ULA. However, the computational cost is high when using all of the constructed matrices, whereas, the performance degrades with utilizing only one constructed matrix. A kind of differencing method is introduced in [13]~[16]. The DOAs of uncorrelated and coherent sources are estimated separately. The differencing technique is firstly introduced in [13] and [14], which utilize the property that the covariance matrix of uncorrelated sources is a Toeplitz matrix for a ULA. But they can only resolve two coherent sources in each group (a group of coherent signals are generated by the multipath of a far field source). A spatial smoothing-differencing method is presented in [15], which can resolve more sources. It utilizes the forward smoothing matrix and the backward one to eliminate the contribution of uncorrelated sources. Unfortunately, the differencing matrix is rank deficient for odd number of coherent sources after the covariance matrix of uncorrelated sources is subtracted. Thus, it needs extra processing to recover the rank, which is justified only by simulation and lacks theoretical proof. In [16], the covariance matrix of uncorrelated sources needs to be constructed, which may be difficult in realization.

In this paper, a robust algorithm based on spatial differencing matrix is presented. In this proposed approach, the number and DOAs of incident coherent signals can be estimated simultaneously by the eigenvectors of the defined spatial differencing matrix. The paper is organized as follows. Section II briefly introduces the data model. The proposed method is developed in detail in Section III. In Section IV, simulation results are presented to verify the performance of the proposed approach. Section V provides a concluding remark to summarize the paper.

2.Data model

Consider a ULA of M isotropic sensors equispaced by d . Let θ be the DOA of a narrowband far field signal. Employing the first element of the ULA as the phase reference, the steering vector can be given by $\boldsymbol{a}(\theta)=[1,v,\cdots,v^{M-1}]^{\mathrm{T}}$ in which $v=\exp\{j2\pi d/\lambda\sin\theta\}$ where λ denoting the carrier wavelength of the signal. The superscript $(\cdot)^{\mathrm{T}}$ stands for the transpose operator.

Suppose that there are K coherent narrowband signals with distinct DOAs impinging on the array. Without the loss of generality, assume that these signals are L groups of K coherent signals which come from L statistically independent far field sources $s_k(t)$ with power $\sigma_k^2,\ k=1,\cdots,L$ and with p_k multipath signals for each source. Assume that the coherent signal coming from the direction θ_{kl} is corresponding to the l th multipath propagation of source $s_k(t)$, for $k=1,\cdots,L$ and $l=1,\cdots,p_k$.

The $M\times 1$ array output vector $\boldsymbol{x}(t)$ at time t is given by

$$\begin{aligned}\boldsymbol{x}(t) &= [x_1(t),\cdots,x_M(t)]^{\mathrm{T}}\\ &=\sum_{i=1}^{L}\sum_{l=1}^{p_k}\boldsymbol{a}(\theta_{il})\rho_{il}s_i(t)+\boldsymbol{n}(t)\\ &=\boldsymbol{Gs}(t)+\boldsymbol{n}(t)\end{aligned} \tag{1}$$

where $\boldsymbol{a}(\theta_k)=[1,e^{-j\frac{2\pi d}{\lambda}\sin(\theta_k)}\cdots,e^{-j\frac{2\pi(M-1)d}{\lambda}\sin(\theta_k)}]^{\mathrm{T}}$ is the steering vector. ρ_{il} is the complex fading coefficient of the l th multipath propagation corresponding to the i th source $\boldsymbol{\rho}_i=[\rho_{i1},\cdots,\rho_{ip_i}]^{\mathrm{T}}$, $\boldsymbol{G}=[\boldsymbol{A}_1\boldsymbol{\rho}_1,\cdots,\boldsymbol{A}_L\boldsymbol{\rho}_L]$ with $\boldsymbol{A}_i=[\boldsymbol{a}(\theta_{i1}),\cdots,\boldsymbol{a}(\theta_{ip_i})]$. $\boldsymbol{s}(t)=[s_1(t),\cdots,s_L(t)]^{\mathrm{T}}$. $\boldsymbol{n}(t)=[n_1(t),\cdots,n_M(t)]^{\mathrm{T}}$ with $n_i(t)$ denoting the additive noise of the i th sensor.

The common assumptions are listed as follows.

(A1): All sources $s_1,\cdots,s_L$ are uncorrelated to each other.

(A2): $n_i(t)(i=1,\cdots,M)$ is a complex Gaussian random process with zero-mean and equal variance σ_n^2 and uncorrelated with $s_i(t)(i=1,\cdots,L)$.

Under the above assumptions, we can obtain the array covariance matrix

$$\boldsymbol{R}=\mathrm{E}\{\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)\}=\boldsymbol{GR}_s\boldsymbol{G}^{\mathrm{H}}+\sigma_n^2\boldsymbol{I}_M \tag{2}$$

where $\mathrm{E}\{\cdot\}$ and $(\cdot)^{\mathrm{H}}$ denote expectation and Hermitia transpose, respectively. $\boldsymbol{R}_s=\mathrm{diag}\{\sigma_1^2,\cdots,\sigma_L^2\}$ being the covariance matrix of $\boldsymbol{s}(t)$. $\boldsymbol{I}_M$ stands for the $M\times M$ identity matrix.

3.Algorithm formulation

3.1.The Spatial Differencing Matrix

All of the developments in this paper rely on a spatial differencing matrix. Employing the defined spatial differencing matrix, the number of incident coherent signals and DOA can be estimated effectively.

The spatial differencing matrix is defined as follows.

Definition 3.1*:* An $M\times M$ matrix $\boldsymbol{D}_p$ is called the p order spatial differencing matrix of the $M\times M$ matrix $\boldsymbol{R}$, if it is given by

$$\boldsymbol{D}_p=\frac{1}{p}\sum\nolimits_{k=1}^{p}\{\boldsymbol{R}_1-\boldsymbol{J}_{M-p+1}\boldsymbol{R}_k^*\boldsymbol{J}_{M-p+1}\} \tag{3}$$

where $\boldsymbol{R}_k=\boldsymbol{K}_k\boldsymbol{R}\boldsymbol{K}_k^{\mathrm{H}}(k=1,\cdots,p)$ in which $\boldsymbol{K}_k$ is defined in (11). $\boldsymbol{J}_m$ denotes the exchange matrix with ones on its antidiagonal and zeros elsewhere. The superscript $(\cdot)^*$ stands for the complex conjugation without transposition.

The selection matrix $\boldsymbol{K}_m\in\Re^{(M-p+1)\times M}$ is defined as follows

$$\boldsymbol{K}_m=[\boldsymbol{0}_{(M-p+1)\times(m-1)}\boldsymbol{I}_{(M-p+1)}\boldsymbol{0}_{(M-p+1)\times(p-m)}] \tag{4}$$

in which $\boldsymbol{I}_m$ denotes the $m \times m$ identity matrix.

Based on the above definition, we can prove that the p order spatial differencing matrix $\boldsymbol{D}_p$ of the array covariance matrix $\boldsymbol{R}$ given in (3) has the following important properties.

***Theorem 3.1**:* Assume that there are K coherent narrowband signals with distinct DOAs impinging on a ULA which consists of M isotropic sensors. Without the loss of generality, assume that the signals are L groups of K coherent signals, which come from L statistically independent far field sources $s_k(t)$ with power σ_k^2, for $k = 1, \cdots, L$ with p_k multipath signals for each source. The matrix $\boldsymbol{D}_p$ is the p order spatial differencing matrix of the array covariance matrix $\boldsymbol{R}$. If $p \ge \max_k p_k (k \in \{1, \cdots, L\})$ and $K = \sum_{i=1}^{L} p_i < M - p + 1$, then the rank of $\boldsymbol{D}_p$ is equal to the number of the coherent signal sources, namely, $\operatorname{rank}(\boldsymbol{D}_p) = K$.

Proof: From Theorem 3.1, the spatial differencing matrix $\boldsymbol{D}_p$ can be rewritten as

$$\boldsymbol{D}_p = \frac{1}{p}\sum_{k=1}^{p} \boldsymbol{F}_k = \frac{1}{p}\sum_{k=1}^{p} (\boldsymbol{G}_1 \boldsymbol{B} \boldsymbol{R}_s \boldsymbol{B}^{\mathrm{H}} \boldsymbol{G}_1^{\mathrm{H}} - \boldsymbol{J}_{M-p+1})(\boldsymbol{G}_1 \Phi^{k-1} \boldsymbol{B} \boldsymbol{R}_s \boldsymbol{B}^{\mathrm{H}} (\Phi^{k-1})^{\mathrm{H}} \boldsymbol{G}_1^{\mathrm{H}})^* \boldsymbol{J}_{M-p+1} \quad (5)$$

where $\boldsymbol{G}_1 = \boldsymbol{K}_1 \boldsymbol{G} \boldsymbol{K}_1$ is defined by (4) $\boldsymbol{B} = [\rho_1, \cdots, \rho_L]$ with $\rho_i = [\rho_{i1}, \cdots, \rho_{ip_i}]^{\mathrm{T}}$ in which ρ_{ik} being the complex fading coefficient of the *k*th multipath propagation corresponding to *i*th source. $\Phi = \operatorname{blkdiag}\{\Phi_1, \cdots, \Phi_L\}$. $\Phi_k = \operatorname{diag}\{e^{-j\frac{2\pi d}{\lambda}\sin\theta_{k1}}, \cdots, e^{-j\frac{2\pi d}{\lambda}\sin\theta_{kp_k}}\}$ with θ_{km} denoting the DOA of the *m*th multipath propagation corresponding to the *k*th source $s_k(t)$.

Notice that $\boldsymbol{J}_{M-p+1} \boldsymbol{G}_1^* = \boldsymbol{G}_1 \boldsymbol{\Phi}^{M-p+1}$, thus the equation (5) can be rearranged as

$$\boldsymbol{D}_p = \frac{1}{p}\sum_{k=1}^{p} (\boldsymbol{G}_1 \boldsymbol{B} \boldsymbol{R}_s \boldsymbol{B}^{\mathrm{H}} \boldsymbol{G}_1^{\mathrm{H}} - \boldsymbol{G}_1 \boldsymbol{\Phi}^{M+2-p-k} \mathrm{B}^* \boldsymbol{R}_s \mathrm{B}^{*\mathrm{H}} (\boldsymbol{\Phi}^{M+2-p-k})^{\mathrm{H}} \boldsymbol{G}_1^{\mathrm{H}} = \boldsymbol{G}_1 \boldsymbol{R}_{sp} \boldsymbol{G}_1^{\mathrm{H}} \quad (6)$$

where $\boldsymbol{R}_{sp} = \sum_{k=1}^{p} (\boldsymbol{B} \boldsymbol{R}_s \boldsymbol{B}^{\mathrm{H}} - \boldsymbol{\Phi}^{M+2-p-k} \mathrm{B}^* \boldsymbol{R}_s \mathrm{B}^{*\mathrm{H}} (\boldsymbol{\Phi}^{M+2-p-k})^{\mathrm{H}}) / p$. $\boldsymbol{G}_1 = \boldsymbol{K}_1 [\boldsymbol{A}_1, \cdots, \boldsymbol{A}_L]$, which is full-column rank since $M > K$.

Clearly, the rank of $\boldsymbol{D}_p$ is equal to that of $\boldsymbol{R}_{sp}$ .Thus, our task is to prove that $\boldsymbol{R}_{sp}$ has rank K, namely, $\operatorname{rank}(\boldsymbol{R}_{sp}) = K$ .Using $B = [\boldsymbol{\rho}_1, \cdots, \boldsymbol{\rho}_L]$, $\boldsymbol{R}_s = \operatorname{diag}\{\sigma_1^2, \cdots, \sigma_L^2\}$, and $\boldsymbol{\Phi} = \operatorname{blkdiag}\{\boldsymbol{\Phi}_1, \cdots, \boldsymbol{\Phi}_L\}$, $\boldsymbol{R}_{sp}$ can be expressed as

$$\boldsymbol{R}_{sp} = \operatorname{blkdiag}\{\boldsymbol{R}_1, \cdots, \boldsymbol{R}_L\} \quad (7)$$

where $\boldsymbol{R}_m = \sigma_m^2 \dfrac{1}{p} \sum_{k=1}^{p} (\boldsymbol{\rho}_m \boldsymbol{\rho}_m^{\mathrm{H}} - \boldsymbol{\Phi}_m^{M+2-p-k} (\boldsymbol{\rho}_m \boldsymbol{\rho}_m^{\mathrm{H}})^* (\boldsymbol{\Phi}_m^{M+2-p-k})^{\mathrm{H}}), \sum_{k=1}^{L} p_m = K$.

The equation (7) means that $\operatorname{rank}(\boldsymbol{R}_{sp}) = K$, if and only if $\operatorname{rank}(\boldsymbol{R}_m) = p_m \forall m \in (1, \cdots, L)$. Thus, our task is to prove that $\operatorname{rank}(\boldsymbol{R}_m) = p_m$, when $p \ge \max_k p_k \; k \in (1, \cdots, L)$.

In matrix form, $\boldsymbol{R}_m$ can be written as follows

$$\boldsymbol{R}_m = \boldsymbol{F}_m \boldsymbol{V}_m \boldsymbol{F}_m^{\mathrm{H}} \quad (8)$$

where $\boldsymbol{F}_m = [\boldsymbol{\rho}_m, \boldsymbol{\Phi}_m^{M+2-p-1}\boldsymbol{\rho}_m^*, \cdots, \boldsymbol{\Phi}_m^{M+2-2p}\boldsymbol{\rho}_m^*]$. $\boldsymbol{V}_m = \mathrm{diag}\{\frac{\sigma_m^2}{p}, -\frac{\sigma_m^2}{p}, \cdots, \frac{\sigma_m^2}{p}, -\frac{\sigma_m^2}{p}\}$.

From (8), it's not difficult to prove that $\mathrm{rank}(\boldsymbol{R}_m) = \mathrm{rank}(\boldsymbol{F}_m)$. Recalling that the rank of a matrix is unchanged by a permutation of its columns, it can be easily verified that

$$\mathrm{rank}(\boldsymbol{F}_m) = \mathrm{rank}([\boldsymbol{\rho}_m^*, \boldsymbol{\Phi}_m\boldsymbol{\rho}_m^*, \cdots, \boldsymbol{\Phi}_m^{p-1}\boldsymbol{\rho}_m^*]) = \mathrm{rank}(\boldsymbol{\Omega}_m\boldsymbol{\Gamma}_m) \tag{9}$$

where $\boldsymbol{\Omega}_m = \mathrm{diag}\{\boldsymbol{\rho}_m^*\}$. $\boldsymbol{\Gamma}_m = [\boldsymbol{b}_1^{\mathrm{T}}, \cdots, \boldsymbol{b}_{pm}^{\mathrm{T}}]^{\mathrm{T}}$, in which $\boldsymbol{b}_k = [1, e^{-j\frac{2\pi d}{\lambda}\sin\theta_{mk}} \cdots, e^{-j(p-1)\frac{2\pi d}{\lambda}\sin\theta_{mk}}]$ for $k = 1, \cdots, p_m$.

To show that the matrix $\boldsymbol{F}_m$ is of rank p_m, namely, it is full row rank, it suffices to show that each element of $\boldsymbol{\rho}_m$ is nonzero element and that $\boldsymbol{\Gamma}_m$ is full row rank. The first fact follows by the *k*th signal has nonzero energy for $\forall k \in (1, \cdots, p_m)$. Since $\boldsymbol{\Gamma}_m$ with $e^{-j\frac{2\pi d}{\lambda}\sin\theta_{mk}} \neq e^{-j\frac{2\pi d}{\lambda}\sin\theta_{ml}}$ for $\forall k \neq l$,k , $l \in (1, \cdots, p_m)$ is a $p_m \times p$ Vandermonde matrix, we have $\mathrm{rank}(\boldsymbol{\Gamma}_m) = \min(p_m, p)$ Thus, the second fact follows by $p \geq \max_k p_k$ $k \in (1, \cdots, L)$. From this, it follows that the spatial differencing matrix $\boldsymbol{D}_p$ is of rank K, if $p \geq \max_k p_k$.

This concludes the proof.

3.2.Source number detection and DOA estimation

We here present a brief development of a effective method based on the spatial differencing matrix for source number detection and DOA estimation in multipath environment.

Based on Theorem 3.1, the eigendecomposition of $\boldsymbol{D}_p$ can be expressed as

$$\boldsymbol{D}_p = \boldsymbol{G}_1\boldsymbol{R}_{sp}\boldsymbol{G}_1^{\mathrm{H}} = \boldsymbol{U}_p \sum\nolimits_p \boldsymbol{U}_p^{\mathrm{H}} = \boldsymbol{U}_{ps} \sum\nolimits_{ps} \boldsymbol{U}_{ps}^{\mathrm{H}} + \boldsymbol{U}_{pn} \sum\nolimits_{pn} \boldsymbol{U}_{pn}^{\mathrm{H}} \tag{10}$$

where $\boldsymbol{G}_1 = \boldsymbol{K}_1\boldsymbol{G}$ in which $\boldsymbol{K}_1 = [\boldsymbol{I}_{(M-p+1)} \quad \boldsymbol{0}_{(M-p+1)\times(p-1)}]$ and $\boldsymbol{G} = [\boldsymbol{A}_1 \quad \boldsymbol{A}_2 \quad \cdots \quad \boldsymbol{A}_L]$. $\sum_p = \mathrm{diag}\{\lambda_1, \cdots, \lambda_{M-p+1}\}$ with $|\lambda_1| \geq \cdots \geq |\lambda_k| > \quad |\lambda_{k+1}| = \cdots = |\lambda_{M-p+1}| = 0$. $\boldsymbol{U}_p = [\boldsymbol{u}_1, \cdots, \boldsymbol{u}_{M-p+1}]$ is a unitary matrix. $\boldsymbol{U}_{ps} = [\boldsymbol{u}_1, \cdots, \boldsymbol{u}_K]$, $\sum_{ps} = \mathrm{diag}\{\lambda_1, \cdots, \lambda_K\}$, $\boldsymbol{U}_{pn} = [\boldsymbol{u}_{K+1}, \cdots, \boldsymbol{u}_{M-p+1}]$, and $\sum_{pn} = \mathrm{diag}\{\lambda_{K+1}, \cdots, \lambda_{M-p+1}\}$.

The columns of $\boldsymbol{U}_{ps}$ span the signal subspace which is spanned also by the columns of $\boldsymbol{G}_1 = \boldsymbol{K}_1\boldsymbol{G}$. In other words, the signal subspace is spanned by the columns of $\boldsymbol{G}_1 = [\boldsymbol{A}_{11} \cdots \boldsymbol{A}_{L1}]$ in which $\boldsymbol{A}_{i1} = \boldsymbol{K}_1\boldsymbol{A}_i$ for $i = 1 \cdots L$ .Since $\mathrm{span}\{\boldsymbol{U}_{ps}\} = \mathrm{span}\{\boldsymbol{G}_1\}$, there must exist a unique nonsingular matrix $\boldsymbol{T}$ such that $\boldsymbol{U}_{ps} = \boldsymbol{G}_1\boldsymbol{T}$.

Let

$$\begin{cases} \boldsymbol{E}_1 = \boldsymbol{K}_{c1}\boldsymbol{U}_p = \boldsymbol{K}_{c1}[\boldsymbol{U}_{ps} \quad \boldsymbol{U}_{pn}] \\ \boldsymbol{E}_2 = \boldsymbol{K}_{c2}\boldsymbol{U}_p = \boldsymbol{K}_{c2}[\boldsymbol{U}_{ps} \quad \boldsymbol{U}_{pn}] \end{cases} \tag{11}$$

where $\boldsymbol{K}_{c1} = [\boldsymbol{I}_{M-p} \quad \boldsymbol{0}] \in \Re^{(M-p)\times(M-p+1)}$ and $\boldsymbol{K}_{c2} = [\boldsymbol{0} \quad \boldsymbol{I}_{M-p}] \in \Re^{(M-p)\times(M-p+1)}$.

Using $\boldsymbol{U}_{ps} = \boldsymbol{G}_1\boldsymbol{T} = \boldsymbol{K}_1[\boldsymbol{A}_1 \cdots \boldsymbol{A}_L]\boldsymbol{T}$, the equation (11) can be written as

$$\begin{cases} \boldsymbol{E}_1 = \boldsymbol{K}_{c1}[\boldsymbol{G}_1\boldsymbol{T} \quad \boldsymbol{U}_{pn}] = [\boldsymbol{V}_1 \quad \boldsymbol{K}_{c1}\boldsymbol{U}_{pn}] \\ \boldsymbol{E}_2 = \boldsymbol{K}_{c2}[\boldsymbol{G}_1\boldsymbol{T} \quad \boldsymbol{U}_{pn}] = [\boldsymbol{V}_2 \quad \boldsymbol{K}_{c2}\boldsymbol{U}_{pn}] \end{cases} \tag{12}$$

where $\boldsymbol{V}_1 = \boldsymbol{K}_{c1}[\boldsymbol{A}_{11} \cdots \boldsymbol{A}_{L1}]\boldsymbol{T}$ and $\boldsymbol{V}_2 = \boldsymbol{K}_{c2}[\boldsymbol{A}_{11} \cdots \boldsymbol{A}_{L1}]\boldsymbol{T}$.

Invoking the structure of the antenna array, it is easily seen that $\boldsymbol{V}_2 = \boldsymbol{V}_1\boldsymbol{\Theta}$ where $\boldsymbol{\Theta} = \boldsymbol{T}^{-1}\boldsymbol{\Phi}\boldsymbol{T}$ in which $\boldsymbol{\Phi} = \mathrm{diag}\{[\mathrm{e}^{-j\frac{2\pi d}{\lambda}\sin(\theta_{11})} \cdots \mathrm{e}^{-j\frac{2\pi d}{\lambda}\sin(\theta_{1p1})} \cdots \mathrm{e}^{-j\frac{2\pi d}{\lambda}\sin(\theta_{L1})} \cdots \mathrm{e}^{-j\frac{2\pi d}{\lambda}\sin(\theta_{Lp_L})}]\}$.

Substituting $\boldsymbol{V}_2 = \boldsymbol{V}_1\boldsymbol{\Theta}$ into (12) yields the signal eigenvector relations

$$\boldsymbol{E}_2 = [\boldsymbol{V}_1 \quad \boldsymbol{K}_{c1}\boldsymbol{U}_{pn}]\begin{bmatrix} \boldsymbol{\Theta} & \boldsymbol{\Gamma}_1 \\ \boldsymbol{0} & \boldsymbol{\Gamma}_2 \end{bmatrix} = \boldsymbol{E}_1\boldsymbol{\Psi} \tag{13}$$

where $\boldsymbol{\Psi} = [\psi_{k,j}] = \begin{bmatrix} \boldsymbol{\Theta} & \boldsymbol{\Gamma}_1 \\ \boldsymbol{0} & \boldsymbol{\Gamma}_2 \end{bmatrix} \in C^{(M-p+1)\times(M-p+1)}$.

Define

$$\Delta_k = \begin{bmatrix} \psi_{k+1,1} & \psi_{k+1,2} & \cdots & \psi_{k+1,k} \\ \psi_{k+2,1} & \psi_{k+2,2} & \cdots & \psi_{k+2,k} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{M-p,1} & \psi_{M-p,2} & \cdots & \psi_{M-p,k} \end{bmatrix} \tag{14}$$

When $k = K = \sum_{k=1}^{L} p_k$, $\Delta_k = 0$. When $k \neq K$, Δ_k contains nonzero components. Therefore, we may select the number that minimizes the variance $\rho(k)$ of the components of Δ_k as the detected number of coherent sources

$$\hat{K} = \min_k \rho(k) = \frac{\|\Delta_k\|^2}{(M-p-k)k} \tag{15}$$

where $k = 1, 2, \cdots, M-p-1$, $\|\cdot\|$ is the Frobenius norm.

For joint source number detection and DOA estimation in multipath environment, the algorithm based on this development is summarized below.

Summary of the proposed method

1) Collect data and estimate the auto-covariance matrix $\hat{\boldsymbol{R}}$

2) Calculate the spatial differencing matrix $\boldsymbol{D}_p$, according to definition 3.1.

3) Compute the eigendecompositon of the spatial differencing matrix $\boldsymbol{D}_p$.

4) Solve $\boldsymbol{E}_2 = \boldsymbol{E}_1\boldsymbol{\Psi}$ based on LS or TLS criterion, where $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are given by (11).

5) Estimate the number of coherent signals $\hat{K}$ by calculating (15).

6) Compute $\hat{\lambda}_k$ $(k = 1, \cdots, K)$ as the eigenvalue of $\hat{\boldsymbol{\Theta}} = \boldsymbol{E}_{1K}\boldsymbol{E}_{2K}$ with $\boldsymbol{E}_{1K}$ and $\boldsymbol{E}_{2K}$ being the first $\hat{K}$ columns of $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$, respectively.

4.Simulation Results

In this section, we construct several simulations to validate the proposed algorithm. Consider a nine-sensors ULA with interelement spacing $d = \lambda/2$ is employed. Suppose the array manifold $\boldsymbol{G}$ is perturbed by the perturbation $\Delta\boldsymbol{G} = |\Delta g_{ij}|$ with Δg_{ij} being a complex Gaussion distributed random variable with zero-mean and variance $\sigma^2 = 1$. We use $P \triangleq \max_{i,j}\{\Delta g_{ij}\}$ as the performance measure. All results provided are based on 200 independent runs.

In the first simulation, two algorithms are carried out for comparing the detection right rate, including the Gerschgorin radii approach based on spatial smoothing method [4,5], in multipath propagation environment. For convenience, we named the Gerschgorin radii approach based on spatial smoothing method as SS-GDE. The input SNR of the *k*th source is defined as $10\log_{10}(\sigma_k^2/\sigma_n^2)$. Assume that all sources are of equal power. The detection right rate of signal number is defined as $v = r_n/N$ where r_n standing for the number of the successful detection tests and N denoting the total number of all tests.

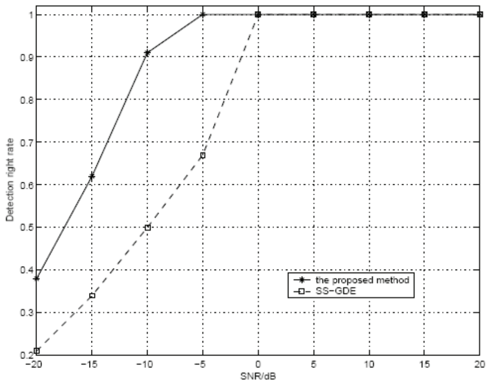

Fig.1 The detection right rate of source number with P=0.15

Assume that two groups of coherent signals come from [0°, 30°] and[-40°, 10°,20°] The fading amplitudes of the two groups of coherent signals are [0.8,0.7] and [0.9,0.8,0.7], while the phase are [35°, 80°] and [97°, 135°,250°], respectively. Fig.1 gives the detection right ratio of the two aforementioned algorithms with P = 0.15 and the SNRs ranging from -20dB to 20dB. As shown in this figure, the proposed method can provide more better detection capability and superior noise-resistant capability. Table 1 gives the detection right rate of the proposed approach with different perturbation. *d* presents the source number. From this table, we can see that the proposed method has better robustness to perturbation change.

Table 1THE PERFORMANCE OF THE PROPOSED METHOD VERSUS THE PERTURBATION OF ARRAY RESPONSE

d \ P	0.17	0.25	0.5	0.72
2	100%	100%	100%	5%
3	100%	100%	100%	3%
4	100%	100%	100%	2%
5	100%	100%	100%	1%

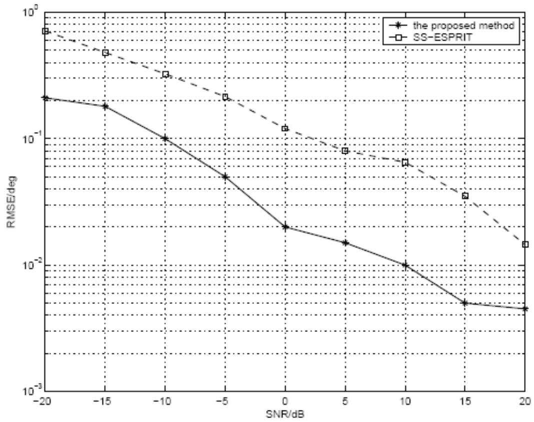

Fig.2 RMSE curves versus SNR

The second simulation studies the DOA estimation performance of the proposed method and the ESPRIT based on spatial smoothing method [2,5] (i.e., named as SS-ESPRIT). Consider L = 3 groups of six coherent signals coming from [-60°, 10°,-30°,20°, 0°,30°]. The fading amplitudes and phase of the coherent signals are [0.7,0.8,0.9,0.75,0.85,0.9] and [120°, 230°,70°,112°, 40°,130°], respectively. The number of snapshots is 1024. We use root-meansquare- error (RMSE) of the DOA estimates as the the performance measure. RMSE is given by $\mathrm{RMSE}=\sqrt{\sum_{n=1}^{N}\sum_{k=1}^{K}(\hat{\theta}_k(n)-\theta_k)^2/NK}$ where $\hat{\theta}_k(n)$ is the estimate of θ_k for the nth Monte Carlo trial. N and K are the number of all tests and all incident signals, respectively. The RMSE curves of the DOA estimates versus the input SNR are shown in Fig.2. This figure shows our proposed method has better estimation performance than the SS-ESPRIT.

5.Conclusions

In this paper, a robust algorithm is presented for joint source number detection and DOA estimation in multipath propagation environment. The number and DOAs of incident signals can be estimated by the eigenvectors of the defined spatial differencing matrix. Compared with the previous works, the proposed method produces considerably fewer estimation errors and better robustness to SNR change for source number detection and DOA estimation. Computer simulations verify that the proposed method is practical, effective.

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