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Covariance vector sparsity-aware DOA estimation for monostatic MIMO radar with unknown mutual coupling

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In this paper, a covariance vector sparsity-aware DOA estimation method is proposed for monostatic multiple-input multiple-output (MIMO) radar with unknown mutual coupling. The new method firstly utilizes the banded symmetric Toeplitz structure of the mutual coupling matrix (MCM) in both of the transmit and receive arrays to eliminate the unknown mutual coupling. Then a sparse representation framework of the array covariance vector is formulated for obtaining the coarse DOA estimation. Finally, a refined maximum likelihood estimation procedure is introduced to estimate the DOA based on the recovered result. Compared with conventional algorithms, the proposed method provides higher angular resolution and better angle estimation performance. Furthermore, the computational complexity of the proposed method is reasonable, because it only involves single measurement vector (SMV) problem and does not require a dense discretized sampling grid for the recovered procedure. Simulation results are used to verify the effectiveness and advantages of the proposed method.

1. Introduction

Multiple-input multiple-output (MIMO) radar has drawn considerable attention and becomes a hot research topic in the field of radar [1–3]. According to the configuration of transmit array and receive array, MIMO radar can be grouped into two classes: One is named as statistical MIMO radar [2], and the other is named as colocated MIMO radar [3] (including bistatic MIMO radar and monostatic MIMO radar). The antennas for both transmit array and receive array are separated away from each other in statistical MIMO radar, whereas they are close to each other in the monostatic MIMO radar. In this paper, we investigate the DOA estimation problem in monostatic MIMO radar.

Direction of arrival (DOA) estimation is a fundamental aspect in array signal processing [4] and MIMO radar applications in practice [5–7]. In MIMO radar, the subspace-based methods, such as multiple signal classification (MUSIC) algorithm [8], estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm [9], have been proposed for angle estimation. In addition, by exploiting the special structure of the virtual array in monostatic MIMO radar with uniform linear array, the reduced dimensional ESPRIT (RD-ESPRIT) [10] and reduced dimensional Capon (RD-Capon) algorithm [11] are proposed to estimate DOA. Unfortunately, these methods strongly depend on the array manifold of MIMO radar, which is often perturbed by mutual coupling in practical situations. In the presence of unknown mutual coupling, the above methods have angle estimation performance degradation or even fail to work. In order to solve this problem, the ESPRIT-Like algorithm [12] is presented for angle estimation by exploiting the Toeplitz structure of the mutual coupling matrix (MCM). On the other hand, the emerging field of sparse representation-based methods have attracted a lot of attention recently, which provides a new perspective for DOA estimation. Some sparse representation-based methods, such

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as l_1-SVD [13], l_1-SRACV [14] and CMSR [15], have been proposed for DOA estimation. Compared with subspace-based methods, the simulation results have verified that these sparse representation based methods provide higher angle resolution, lower sensitivity to the estimated number of the targets, and adapt better to the case of low SNR. In addition, a revised l_1-SVD algorithm has been proposed for angle estimation in passive array [16] with unknown mutual coupling, which leads the angle estimation performance degradation. On the other hand, some sparse representation-based methods have presented for angle estimation in MIMO radar [17,18]. However, to the best of our knowledge, under the sparse representation framework, there are no literatures about the angle estimation for MIMO radar in the presence of unknown mutual coupling.

In this paper, we propose a covariance vector sparsity-aware DOA estimation method for monostatic MIMO radar with unknown mutual coupling. Firstly, utilizing the banded symmetric Toeplitz structure of the mutual coupling matrix (MCM) in both of the transmit and receive arrays, a linear transformation can be designed to eliminate the effect of the unknown mutual coupling. Then the received data matrix can be turned into a low dimensional one by exploiting the reduced transformation technique, and a covariance vector-based sparse representation framework is formulated to estimate the coarse DOA. Finally, the refined DOA estimation can be obtained based on recovered result via maximum likelihood estimation method. The proposed method has higher resolution and better angle estimation performance than conventional algorithms. Additionally, the computational complexity of the proposed method is reasonable, because it only involves single measurement vector (SMV) problem and does not require a dense discretized sampling grid for the recovered procedure.

Notation: $(\cdot)^{\mathrm{H}}$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{-1}$, $(\cdot)^*$, $(\cdot)^+$ and $\mathrm{Re}\{\cdot\}$ denote conjugate-transpose, transpose, inverse, conjugate, pseudo-inverse and real part operator, respectively. $\otimes$, $\odot$ and $\mathrm{Tr}(\cdot)$ denote the Kronecker product, the Khatri–Rao product and the trace, respectively. $\mathbf{I}_K$ denotes a $K \times K$ dimensional unit matrix. $\mathbf{A}^{(l_2)}$ denotes a column vector whose qth element equals to the l_2 norm of the qth row of $\mathbf{A}$. $\|\cdot\|_1$ and $\|\cdot\|_F$ denote the l_1 norm and Frobenius norm, respectively.

2. Data model

Consider a narrowband monostatic MIMO radar system equipped with M transmit antennas and N receive antennas, and both of them are half-wavelength spaced uniform linear arrays (ULA). The transmit array uses M transmit antennas to transmit M different narrowband waveforms which have identical bandwidth and centre frequency but are orthogonal. In monostatic MIMO radar, the transmit and receive arrays are assumed to be closely located. Thus, a target located in the far-field can be seen by both of them at the same angle (i.e. direction of arrival (DOA)). Considering the effect of unknown mutual coupling in both of the transmit and receive arrays, the mutual coupling matrices of the transmit and receive arrays are denoted as $\mathbf{C}_t$ and $\mathbf{C}_r$, which can be expressed as [12]

$$\begin{aligned}\mathbf{C}_t &= \mathrm{toeplitz}\{[c_{t0}, c_{t1}, \ldots, c_{tK}, 0, \ldots, 0]\} \in \mathbb{C}^{M\times M}\\ \mathbf{C}_r &= \mathrm{toeplitz}\{[c_{r0}, c_{r1}, \ldots, c_{rK}, 0, \ldots, 0]\} \in \mathbb{C}^{N\times N}\end{aligned} \tag{1}$$

where c_{ik} $(i=r,t,k=0,1,\ldots,K)$ is the mutual coupling coefficient, which is a factor concerned with the distance between two elements [12] and satisfied with $0<|c_{tK}|<\cdots<|c_{t1}|<|c_{t0}|=1$. Assuming that there exists P targets in the far-field, θ_p $(p=1,2,\ldots,P)$ denotes the DOA of the pth target with respect to the normals of transmit and receive arrays. In the presence of unknown mutual coupling, the output of the matched filter at the receive array can be expressed as [16,17]

$$\mathbf{x}(t)=\mathbf{CAs}(t)+\mathbf{n}(t) \tag{2}$$

where $\mathbf{C}=\mathbf{C}_t \otimes \mathbf{C}_r \in \mathbb{C}^{MN\times MN}$ is the MCM for the transmit–receive array. $\mathbf{A}=\mathbf{A}_t \odot \mathbf{A}_r \in \mathbb{C}^{MN\times P}$ is the transmit–receive steering matrix, where $\mathbf{A}_t=[\mathbf{a}_t(\theta_1),\ldots,\mathbf{a}_t(\theta_P)] \in \mathbb{C}^{M\times P}$ is the transmit steering matrix composed with the transmit steering vector $\mathbf{a}_t(\theta_p)=[1,\exp(j\pi \sin\theta_p),\ldots,\exp(j\pi(M-1)\sin\theta_p)]^{\mathrm{T}}$, and $\mathbf{A}_r=[\mathbf{a}_r(\theta_1),\ldots,\mathbf{a}_r(\theta_P)] \in \mathbb{C}^{N\times P}$ is the receive steering matrix composed with the receive steering vector $\mathbf{a}_r(\theta_p)=[1,\exp(j\pi\sin\theta_p),\ldots,\exp(j\pi(N-1)\sin\theta_p)]^{\mathrm{T}}$. $\mathbf{s}(t)=[s_1(t),s_2(t),\ldots,s_P(t)]^{\mathrm{T}} \in \mathbb{C}^{P\times 1}$ is the signal vector, and $s_p(t)=\beta_p(t)e^{j2\pi f_p t}$ with $\beta_p(t)$ and f_p being the reflection coefficient and Doppler frequency, respectively. $\mathbf{n}(t)$ is a Gaussian white noise vector with zero-mean and covariance matrix $\sigma^2\mathbf{I}_{MN}$. By collecting J snapshots, the received data matrix can be written as

$$\mathbf{X}=[\mathbf{x}(t_1),\ldots,\mathbf{x}(t_J)]=\mathbf{CAS}+\mathbf{N} \tag{3}$$

where $\mathbf{S}=[\mathbf{s}(t_1),\ldots,\mathbf{s}(t_J)] \in \mathbb{C}^{P\times J}$ is the signal matrix and $\mathbf{N}=[\mathbf{n}(t_1),\ldots,\mathbf{n}(t_J)] \in \mathbb{C}^{MN\times J}$ is the Gaussian white noise matrix. Some statistical assumptions on the target signals and noises are made as follows:

(i) The target signals $\mathbf{S}$ are spatially uncorrelated, temporally white and zero-mean, and the noise matrix $\mathbf{N}$ is zero-mean, complex circular Gaussian.
(ii) The noises are statistically independent of all target signals.

With these assumptions, the covariance matrix of the received data $\mathbf{X}$ can be written as

$$\mathbf{R}_{\mathbf{x}}=\mathbf{CAR}_s(\mathbf{CA})^{\mathrm{H}}+\mathbf{R}_n \tag{4}$$

where $\mathbf{R}_s=\mathrm{E}(\mathbf{SS}^{\mathrm{H}})=\mathrm{diag}([\rho_1^2,\ldots,\rho_P^2])$ with ρ_p^2 being the power of the pth target signal. The variance of the noises is $\mathbf{R}_n=\mathrm{E}(\mathbf{NN}^{\mathrm{H}})=\mathrm{diag}([\sigma_1^2,\ldots,\sigma_{MN}^2])$ with $\{\sigma_i^2\}_{i=1}^{NM}$ being the noise power. It has been indicated in [16] that the sparse representation manners [17,18] cannot be applied straightforwardly to estimate the DOAs due to the unknown mutual coupling. In addition, when the l_1-SVD algorithm in [16] is used for DOA estimation based on Eq. (3), it leads to angle estimation performance degradation due to the aperture loss and high dimensional complete dictionary. Therefore, the work in this paper is to propose an accurate DOA estimation method based on the sparse representation of the covariance vector without any knowledge of the mutual coupling matrix $\mathbf{C}$.

3. Covariance vector sparsity-aware DOA estimation in the presence of mutual coupling

It has been mentioned above that the sparse representation manner [17,18] is invalid due to the unknown mutual coupling. In order to formulate an efficient sparse representation framework, the effect of mutual coupling in both of the transmit and receive arrays must be eliminated firstly. Noting the special structure of the mutual coupling matrices $\mathbf{C}_t$ and $\mathbf{C}_r$ in Eq. (1), two selection matrices are defined as [12]

$$\boldsymbol{\Gamma}_1 = [\mathbf{0}_{(M-2K)\times K} \quad \mathbf{I}_{(M-2K)\times(M-2K)} \quad \mathbf{0}_{(M-2K)\times K}] \in \mathbb{C}^{(M-2K)\times M} \tag{5a}$$

$$\boldsymbol{\Gamma}_2 = [\mathbf{0}_{(N-2K)\times K} \quad \mathbf{I}_{(N-2K)\times(N-2K)} \quad \mathbf{0}_{(N-2K)\times K}] \in \mathbb{C}^{(N-2K)\times N} \tag{5b}$$

Then we have

$$\boldsymbol{\Gamma}_1\mathbf{C}_t\mathbf{A}_t = \overline{\mathbf{A}}_t\boldsymbol{\Phi}_t, \quad \boldsymbol{\Gamma}_2\mathbf{C}_r\mathbf{A}_r = \overline{\mathbf{A}}_r\boldsymbol{\Phi}_r \tag{6}$$

where $\overline{\mathbf{A}}_t = [\overline{\mathbf{a}}_t(\theta_1), \dots, \overline{\mathbf{a}}_t(\theta_P)] \in \mathbb{C}^{\overline{M}\times P}$ is the new transmit steering matrix composed with $\overline{\mathbf{a}}_t(\theta_p) = [1, \exp(j\pi \sin\theta_p), \dots, \exp(j\pi(\overline{M}-1)\sin\theta_p)]^{\mathrm{T}}$, $\overline{\mathbf{A}}_r = [\overline{\mathbf{a}}_r(\theta_1), \dots, \overline{\mathbf{a}}_r(\theta_P)] \in \mathbb{C}^{\overline{N}\times P}$ is the new receive steering matrix composed with $\overline{\mathbf{a}}_r(\theta_p) = [1, \exp(j\pi \sin\theta_p), \dots, \exp(j\pi(\overline{N}-1)\sin\theta_p)]^{\mathrm{T}}$, $\overline{M} = M-2K$ and $\overline{N} = N-2K$. $\boldsymbol{\Phi}_t = \mathrm{diag}([\nu_{t1}, \nu_{t2}, \dots, \nu_{tP}]) \in \mathbb{C}^{P\times P}$ is a diagonal matrix composed with $\nu_{tp} = \sum_{k=-K}^{K} c_{t|k|} e^{j\pi(k+K)\sin(\theta_p)} (p = 1, 2, \dots, P)$, and $\boldsymbol{\Phi}_r = \mathrm{diag}([\nu_{r1}, \nu_{r2}, \dots, \nu_{rP}]) \in \mathbb{C}^{P\times P}$ is a diagonal matrix composed with $\nu_{rp} = \sum_{k=-K}^{K} c_{r|k|} e^{j\pi(k+K)\sin(\theta_p)}$ $(p = 1, 2, \dots, P)$, where $\nu_{tp}$and ν_{tp} are non-zero scalars [12]. According to Eq. (6), it is indicated that the effect of mutual coupling can be eliminated after using the selection matrix. Then a selection matrix can be constructed as $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}_1 \otimes \boldsymbol{\Gamma}_2 \in \mathbb{C}^{\overline{MN}\times MN}$, and a linear transformation is exploited for the signal model in Eq. (3), which is expressed as

$$\begin{aligned}\overline{\mathbf{X}} &= \boldsymbol{\Gamma}\mathbf{C}\mathbf{A}\mathbf{S} + \boldsymbol{\Gamma}\mathbf{N} \\ &= \overline{\mathbf{A}}\overline{\mathbf{S}} + \overline{\mathbf{N}}\end{aligned} \tag{7}$$

where $\overline{\mathbf{A}} = \overline{\mathbf{A}}_t \odot \overline{\mathbf{A}}_r \in \mathbb{C}^{\overline{MN}\times P}$, $\overline{\mathbf{S}} = \boldsymbol{\Phi}\mathbf{S} \in \mathbb{C}^{P\times J}$ with $\boldsymbol{\Phi} = \mathrm{diag}([\nu_{t1}\nu_{r1}, \dots, \nu_{tP}\nu_{rP}])$, and $\overline{\mathbf{N}} = \boldsymbol{\Gamma}\mathbf{N}$, where $\nu_{tp}\nu_{rp}$ $(p = 1, \dots, P)$ is a non-zero scalar. It is indicated in Eq. (7) that after the linear transformation, the effect of mutual coupling is eliminated, and the received data $\overline{\mathbf{X}}$ corresponds to a monostatic MIMO radar with $\overline{M}$ transmit antennas and $\overline{N}$ receive antennas.

According to Eq. (7), the linear transformation brings some aperture loss compared with the received data in Eq. (3). Thus, the angle estimation performance of l_1-SVD algorithm [17] is degraded remarkably, especially with a large value K. In order to compensate the aperture loss, an enlarged aperture sparse representation framework for DOA estimation via vectorizing the covariance matrix is proposed in the following section. After eliminating the influence of the unknown mutual coupling, it is noted that there are only $\overline{M}+\overline{N}-1$ distinct elements in the steering vector $\overline{\mathbf{a}}_t(\theta) \otimes \overline{\mathbf{a}}_r(\theta)$, which can be expressed as

$$\overline{\mathbf{a}}_t(\theta) \otimes \overline{\mathbf{a}}_r(\theta) = \mathbf{G}\mathbf{b}(\theta) \tag{8}$$

where $\mathbf{G}$ and $\mathbf{b}(\theta)$ are the reduced-dimensional transformation matrix and the steering vector, which can be written as

$$\begin{aligned}&\mathbf{G} = [\mathbf{J}_0^{\mathrm{T}}, \mathbf{J}_1^{\mathrm{T}}, \dots, \mathbf{J}_{\overline{M}-1}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{\overline{MN}\times(\overline{M}+\overline{N}-1)} \\ &\mathbf{b}(\theta) = [1, \exp(j\pi \sin\theta_p), \dots, \exp(j\pi(\overline{M}+\overline{N}-2)\sin\theta_p)]^{\mathrm{T}} \in \mathbb{C}^{(\overline{M}+\overline{N}-1)\times 1}\end{aligned} \tag{9}$$

where

$$\mathbf{J}_m = [\mathbf{0}_{\overline{N}\times m}, \mathbf{I}_{\overline{N}}, \mathbf{0}_{\overline{N}\times(\overline{M}-m-1)}] \in \mathbb{C}^{\overline{N}\times(\overline{M}+\overline{N}-1)}, \quad m = 0, 1, \dots, \overline{M}-1 \tag{10}$$

According to Eq. (8), it can be observed that the steering vector $\overline{\mathbf{a}}_t(\theta) \otimes \overline{\mathbf{a}}_r(\theta)$ can be turned into a lower dimensional one by exploiting reduced dimensional transformation. Then the reduced dimensional matrix can be constructed as $\mathbf{D} = \mathbf{F}^{-(1/2)}\mathbf{G}^{\mathbf{H}} \in \mathbb{C}^{(\overline{M}+\overline{N}-1)\times\overline{MN}}$, which avoids the additional spatial colored noise [10], where $\mathbf{F} \in \mathbb{C}^{(\overline{M}+\overline{N}-1)\times(\overline{M}+\overline{N}-1)}$ is defined as

$$\begin{aligned}\mathbf{F} &= \mathbf{G}^{\mathrm{H}}\mathbf{G} \\ &= \mathrm{diag}[1, 2, \dots, \underbrace{\min(\overline{M},\overline{N}), \dots, \min(\overline{M},\overline{N})}_{|\overline{M}-\overline{N}|+1}, \dots, 2, 1]\end{aligned} \tag{11}$$

Then multiplying $\mathbf{D}$ with the data matrix $\overline{\mathbf{X}}$, we have

$$\mathbf{Y} = \mathbf{D}\overline{\mathbf{X}} = \mathbf{F}^{(1/2)}\mathbf{B}\overline{\mathbf{S}} + \mathbf{D}\overline{\mathbf{N}} \tag{12}$$

where $\mathbf{B} = [\mathbf{b}(\theta_1), \dots, \mathbf{b}(\theta_P)]$ is the steering matrix. According to Eq. (12), it is indicated that the data matrix $\mathbf{Y} \in \mathbb{C}^{(\overline{M}+\overline{N}-1)\times J}$ corresponds to a virtual array with a weight matrix $\mathbf{F}^{(1/2)}$ without the unknown mutual coupling. According to the statistical assumptions (i), (ii) and Eq. (4), the covariance matrix of $\mathbf{Y}$ can be expressed as

$$\mathbf{R}_Y = \mathrm{E}(\mathbf{Y}\mathbf{Y}^{\mathrm{H}}) = \mathbf{F}^{(1/2)}\mathbf{B}\mathbf{R}_{\overline{S}}(\mathbf{F}^{(1/2)}\mathbf{B})^{\mathrm{H}} + \mathbf{R}_{\overline{N}} \tag{13}$$

where $\mathbf{R}_{\overline{S}} = \boldsymbol{\Phi}\mathbf{R}_S\boldsymbol{\Phi}^* = \mathrm{diag}([\overline{\rho}_1^2, \dots, \overline{\rho}_P^2])$ is a diagonal matrix with $\{\nu_{tp}\nu_{rp}\rho_p^2\nu_{tp}^*\nu_{rp}^*\}_{p=1}^{P}$. $\mathbf{R}_{\overline{N}} = \mathrm{E}(\overline{\mathbf{N}}\overline{\mathbf{N}}^{\mathrm{H}}) = \mathrm{diag}([\sigma_1^2, \cdots, \sigma_{\overline{M}+\overline{N}-1}^2])$ is a diagonal matrix with $\{\sigma_i^2\}_{i=1}^{\overline{M}+\overline{N}-1}$. According to the covariance matrix $\mathbf{R}_Y$ in Eq. (13), a new way for array augmentation by the covariance matrix vectorization is presented in [19], which can be adopted to enlarge the array aperture in the sparse representation framework. Then vectorizing the covariance matrix $\mathbf{R}_Y$, we have

$$\mathbf{R}_v = \mathrm{vec}(\mathbf{R}_Y) = [\mathbf{F}^{(1/2)}\mathbf{B}^* \odot \mathbf{F}^{(1/2)}\mathbf{B}]\mathbf{z} + \mathbf{n}_v \tag{14}$$

where $\mathbf{z} = [\overline{\rho}_1^2, \dots, \overline{\rho}_P^2]^{\mathrm{T}}$, $\mathbf{n}_v = \mathrm{vec}(\mathbf{R}_{\overline{N}})$. It can be observed that after vectorizing the covariance matrix $\mathbf{R}_Y$, the data vector $\mathbf{R}_v$ corresponds to a virtual array with single snapshot, and the steering matrix is $\mathbf{F}^{(1/2)}\mathbf{B}^* \odot \mathbf{F}^{(1/2)}\mathbf{B} \in \mathbb{C}^{(\overline{M}+\overline{N}-1)^2\times P}$ with its dimension $(\overline{M}+\overline{N}-1)^2$. Compared with Eqs. (7) and (12), the aperture of virtual array in Eq. (14) is enlarged remarkably, which can improve both the spatial resolution and the angle estimation performance. Based on Eq. (14), a sparse representation framework with a single measurement vector is proposed for DOA estimation. Let $\{\overline{\theta}_l\}_{l=1}^{L}(L \gg P)$ be a discretized sampling grid of all DOAs of interest, the complete dictionary can be formulated as $\mathbf{\Omega}_{\overline{\theta}} = \mathbf{F}^{(1/2)}\mathbf{B}_{\overline{\theta}}^* \odot \mathbf{F}^{(1/2)}\mathbf{B}_{\overline{\theta}} \in \mathbb{C}^{(\overline{M}+\overline{N}-1)^2 \times L}$, where $\mathbf{B}_{\overline{\theta}} = [\mathbf{b}(\overline{\theta}_1), \ldots, \mathbf{b}(\overline{\theta}_L)]$. Exploiting the sparse representation viewpoint, the data vector in Eq. (14) can be written as

$$\mathbf{R}_v = \mathbf{\Omega}_{\overline{\theta}}\mathbf{z}_{\overline{\theta}} + \mathbf{n}_v \tag{15}$$

where $\mathbf{z}_{\overline{\theta}}$ is a $L \times 1$ vector. Due to that $\mathbf{z}_{\overline{\theta}}$ and $\mathbf{z}$ have the same row support, the vector $\mathbf{z}_{\overline{\theta}}$ is a K-sparse vector, whose nonzero elements are equal to $\{\overline{\rho}_p^2\}_{p=1}^P$ and correspond to DOAs in the complete dictionary $\mathbf{\Omega}_{\overline{\theta}}$. Thus, the DOA estimation can be turned into the detection of the nonzero elements of $\mathbf{z}_{\overline{\theta}}$. For measuring the smallest number of the non-zero elements of $\mathbf{z}_{\overline{\theta}}$, a direct sparse metric is the l_0 norm penalty. However, the l_0 norm minimization problem is nonconvex and NP-hard. Thereby it cannot be solved. Referencing to [16–18], the l_1 norm penalty is adopted to solve this problem, and the l_1 norm minimization problem can be considered as

$$\min \|\mathbf{z}_{\overline{\theta}}\|_1, \quad \text{s.t.} \|\mathbf{R}_v - \mathbf{\Omega}_{\overline{\theta}}\mathbf{z}_{\overline{\theta}}\|_F \le \sqrt{\eta} \tag{16}$$

where η is the regularization parameter, which balances the sparsity and the estimation error and plays an important role in sparse representation framework. In practice, the covariance matrix can be estimated by $\hat{\mathbf{R}}_Y = \mathbf{Y}\mathbf{Y}^{\mathrm{H}}/J$. The estimated error between $\hat{\mathbf{R}}_Y$ and $\mathbf{R}_Y$ is written as $\Delta\mathbf{R} = \hat{\mathbf{R}}_Y - \mathbf{R}_Y$, whose vectorized form satisfies [20]

$$\mathrm{vec}(\Delta\mathbf{R}) \sim \mathrm{AsN}\left(\mathbf{0}, \frac{1}{J}\mathbf{R}_Y^{\mathrm{T}} \otimes \mathbf{R}_Y\right) \tag{17}$$

where $\mathrm{AsN}(u, \sigma^2)$ denotes the asymptotic normal distribution with mean u and variance σ^2. Then a weighted matrix can be defined as $\mathbf{W}^{-1/2} = \sqrt{J}\mathbf{R}_Y^{-T/2} \otimes \mathbf{R}_Y^{-1/2}$, and we can obtain

$$\|\mathbf{W}^{-1/2}\,\mathrm{vec}(\Delta\mathbf{R})\|_2^2 \sim \mathrm{As}\,\chi^2((\overline{M}+\overline{N}-1)^2) \tag{18}$$

where $\mathrm{As}\,\chi^2(M^2)$ denotes the asymptotic chi-square distribution with M^2 degrees of freedom. According to Eqs. (16) and (18), the covariance vector-based sparse representation framework can be formulated as

$$\min \|\mathbf{z}_{\overline{\theta}}\|_1, \quad \text{s.t.} \|\mathbf{W}^{-1/2}[\hat{\mathbf{R}}_v - \hat{\mathbf{n}}_v - \mathbf{\Omega}_{\overline{\theta}}\mathbf{z}_{\overline{\theta}}]\|_F \le \sqrt{\overline{\eta}} \tag{19}$$

where $\hat{\mathbf{R}}_v = \mathrm{vec}(\hat{\mathbf{R}}_Y)$ is the estimated covariance vector. The noise vector $\hat{\mathbf{n}}_v$ can be estimated as $\hat{\mathbf{n}}_v = \overline{\sigma}^2\mathrm{vec}(\mathbf{I}_{\overline{M}+\overline{N}-1})$, where $\overline{\sigma}^2$ is the noise power, and $\overline{\eta}$ is the regulation parameter. Estimating $\mathbf{z}_{\overline{\theta}}$ in Eq. (19) can be calculated by SOC programming software packages such as SeDuMi [21] and CVX [22]. Then the estimated DOAs $\boldsymbol{\vartheta} = [\vartheta_1, \ldots, \vartheta_P]$ are obtained with P peaks by plotting $\mathbf{z}_{\overline{\theta}}$. For obtaining accurate DOA estimation, a dense discretized sampling grid is necessary. However, a dense discretized sampling grid leads to high computational complexity of the recovered procedure and a high coherent complete dictionary that violates the condition for the signal sparse reconstruction. In order to avoid this problem, a refined DOA estimation procedure based on maximum likelihood method is introduced by utilizing the coarse DOA estimated in Eq. (19). According to References [23–25], the deterministic maximum likelihood (MDL) DOA estimator can be shown as follows:

$$\hat{\boldsymbol{\vartheta}} = \arg\min_{\boldsymbol{\vartheta}} \mathrm{tr}(\mathbf{V}^{\perp}\mathbf{R}_Y) \tag{20}$$

where $\mathbf{V}^{\perp} = \mathbf{I}_{\overline{M}+\overline{N}-1} - \mathbf{V}$, $\mathbf{V} = \hat{\mathbf{B}}\hat{\mathbf{B}}^{+}$, in which the steering matrix $\hat{\mathbf{B}} = \mathbf{F}^{(1/2)}[\mathbf{b}(\vartheta_1), \ldots \mathbf{b}(\vartheta_P)]$ corresponding to the DOA estimation $\boldsymbol{\vartheta} = [\vartheta_1, \ldots, \vartheta_P]$. It is obvious that Eq. (20) is a nonlinear multidimensional minimization problem, and a Newton-type method can be exploited to solve it. The DOA estimation is iteratively calculated as

$$\boldsymbol{\vartheta}_{i+1} = \boldsymbol{\vartheta}_i - \mathbf{H}^{-1}\nabla \tag{21}$$

where $\boldsymbol{\vartheta}_i$ is the DOA estimation at the ith iteration. $\mathbf{H}$ and ∇ denote the Hessian matrix and the gradient of the criterion function in each iteration evaluated in $\boldsymbol{\vartheta}_i$, respectively, and they are given as [23–25]

$$\nabla = -2\,\mathrm{Re}\{\mathrm{diag}(\hat{\mathbf{B}}^{+}\mathbf{R}_Y\mathbf{V}^{\perp}\overline{\mathbf{B}})\} \tag{22}$$

and

$$\mathbf{H} = 2\,\mathrm{Re}\{(\overline{\mathbf{B}}^{\mathrm{H}}\mathbf{V}^{\perp}\overline{\mathbf{B}}) \odot (\hat{\mathbf{B}}^{+}\mathbf{R}_Y\hat{\mathbf{B}}^{+\mathrm{H}})^{\mathrm{T}}\} \tag{23}$$

where $\overline{\mathbf{B}} = \mathbf{F}^{(1/2)}[\overline{\mathbf{b}}(\vartheta_1), \ldots, \overline{\mathbf{b}}(\vartheta_P)]$, and $\overline{\mathbf{b}}(\vartheta)$ is the 1st-order derivative of $\mathbf{b}(\vartheta)$ with respect to ϑ. After obtaining the DOAs estimated in Eq. (19) as the initial parameter for the Newton iteration, Eqs. (21)–(23) are used to update the DOA estimation values. Then the ith iteration procedure can be seen as convergence when $\|\boldsymbol{\vartheta}_{i+1} - \boldsymbol{\vartheta}_i\|_2 \le \tau$ or it achieves the maximum number of iterations, where τ is a predefined small value, and the accurate DOAs can be obtained.

4. Related remarks and Cramer–Rao bound

Remark 1. In Eq. (19), the noise power $\overline{\sigma}^2$ and the regulation parameter $\overline{\eta}$ are needed to estimate for recovering the sparse vector $\mathbf{z}_{\overline{\theta}}$. According to Eq. (18), the estimated error in Eq. (19) satisfies the asymptotic chi-square distribution with $(\overline{M}+\overline{N}-1)^2$ degrees of freedom. Thus, the parameter $\overline{\eta}$ can be selected as the upper value of the estimated error with a high probability $1-\varepsilon$ confidence interval, i.e., $\varepsilon = 0.001$ is enough. By using Matlab software, the function $chi2inv(1-\xi, (\overline{M}+\overline{N}-1)^2)$ can be used to calculate $\overline{\eta}$. On the other hand, the noise power $\overline{\sigma}^2$ can be estimated by the average of $\overline{M}+\overline{N}-1-P$ smallest eigenvalues of the covariance matrix $\hat{\mathbf{R}}_Y$.

Remark 2. It is well known that the Newton method gives an ultimate quadratic convergence to $\boldsymbol{\vartheta}$ when the initialized value is close enough. In the proposed method, the covariance matrix is vectorized in Eq. (14), which enlarges the aperture of virtual array and improves the spatial angular resolution. Thus, the coarse DOAs are obtained by Eq. (19) that are close enough for the quadratic convergence in Newton method, which means that the Newton method can converge quickly and provide accurate DOA estimation. On the other hand, the reconstruction procedure in Eq. (19) only involves single measurement vector (SMV) problem through vectorizing the covariance matrix. The computational complexity of solving Eq. (19) is $O(L^3)$ [13], and each Newton iteration needs $O((\overline{M}+\overline{N}-1)^2)P$. The main computational complexity of the proposed method is $O(L^3+\upsilon(\overline{M}+\overline{N}-1)^2P)$, where υ is the total number of iteration. The computational complexity of the revised l_1-SVD in [16] is $O(L_1^3P)$, where L_1 is the total number of the discretized sampling grid. Due to the fact that the proposed method does not require a dense discretized sampling grid, i.e., $L_1 \gg L \gg \overline{M}+\overline{N}-1>P$, the computational complexity of the proposed method is more reasonable than l_1-SVD algorithm.

Remark 3. The maximal number of identified targets is an important aspect that can be considered for the sparse representation-based algorithms. After eliminating the effect of mutual coupling and using reduced dimensional transformation, the element number of the steering vector is $\overline{M}+\overline{N}-1$. Then according to [19], any set of $2(\overline{M}+\overline{N}-1)-1$ columns of the complete dictionary $\boldsymbol{\Omega}_{\overline{\theta}}$ is independent, which leads to $\mathrm{Spark}[\boldsymbol{\Omega}_{\overline{\theta}}]=2(\overline{M}+\overline{N}-1)$, where $\mathrm{Spark}[\boldsymbol{\Omega}_{\overline{\theta}}]$ denotes the smallest integer of columns of the complete dictionary $\boldsymbol{\Omega}_{\overline{\theta}}$ that are linearly dependent. Under the sparse representation framework, a necessary and sufficient condition for determining unique P-sparse vector in Eq. (15) is $P<\mathrm{Spark}[\boldsymbol{\Omega}_{\overline{\theta}}]/2=\overline{M}+\overline{N}-1$ [26], which indicates that the maximal number of identified targets is $\overline{M}+\overline{N}-2$ in the proposed method.

Cramer–Rao bound (CRB): According to [27], the CRB of DOA estimation in monostatic MIMO radar with unknown mutual coupling can be derived as

$$CRB=\frac{\delta^2}{2}\left\{\mathrm{Re}(\hat{\mathbf{D}}^{\mathrm{H}}\mathbf{P}_V^{\perp}\hat{\mathbf{D}})\right\}^{-1} \tag{24}$$

where $\mathbf{P}_V^{\perp}=\mathbf{I}_J\otimes(\mathbf{I}_{MN}-\tilde{\mathbf{A}}(\tilde{\mathbf{A}}^{\mathrm{H}}\tilde{\mathbf{A}})^{-1}\tilde{\mathbf{A}}^{\mathrm{H}})$ with $\tilde{\mathbf{A}}=\mathbf{CA}$, and δ^2 is the mean of $\delta_i^2(i=1,\ldots,MN)$. $\hat{\mathbf{D}}=[\partial\hat{\mathbf{d}}/\partial\theta_1,\ldots,\partial\hat{\mathbf{d}}/\partial\theta_P, \partial\hat{\mathbf{d}}/\partial c_{t0},\ldots,\partial\hat{\mathbf{d}}/\partial c_{tK},\partial\hat{\mathbf{d}}/\partial c_{r0},\ldots,\partial\hat{\mathbf{d}}/\partial c_{rK}]$ with $\hat{\mathbf{d}}=\{(\tilde{\mathbf{A}}\mathbf{s}(t_1))^{\mathrm{T}},\ldots,(\tilde{\mathbf{A}}\mathbf{s}(t_J))^{\mathrm{T}}\}^T$, where $\partial\hat{\mathbf{d}}/\partial r$ denotes the 1st-order derivative of $\hat{\mathbf{d}}$ with respect to r.

5. Simulation results

In this section, some simulation results are presented to verify the advantages of the proposed method. The ESPRIT-Like algorithm [12], l_1-SVD algorithm in [16] based on Eq. (12), and CRB are used to compare with the proposed method. The root mean square error (RMSE) is used to evaluate the angle estimation performance, which is defined as

$$\mathrm{RMSE}=(1/P)\sum_{p=1}^{P}\sqrt{(1/Q)\sum_{i=1}^{Q}(\vartheta_{p,i}-\theta_p)^2} \tag{25}$$

where $\vartheta_{p,i}$ is the estimation of DOA θ_p for the ith Monte Carlo trial, Q is the number of the Monte Carlo trials. A narrowband monositatic MIMO radar system equipped with M transmit antennas and N receive antennas is considered and both of the transmit and receive arrays are half-wavelength spaced uniform linear arrays (ULA). In the most of cases, it is assumed that the number of targets is known as $P=3$, and DOAs of uncorrelated three targets are $\theta_1=-10^\circ$, $\theta_2=0^\circ$ and $\theta_3=10^\circ$, respectively. Two mutual coupling cases for the transmit and the received array are considered in the following simulations: (1) $K=1$ with $[c_{t0},c_{t1}]=$

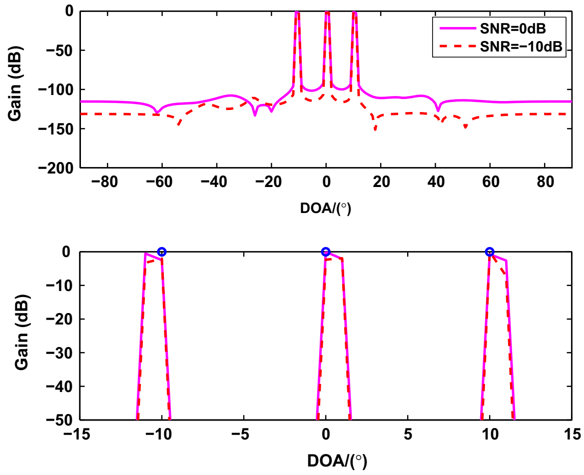

Fig. 1. The spatial spectrum of the proposed method for coarse DOA estimation when $K=1$.

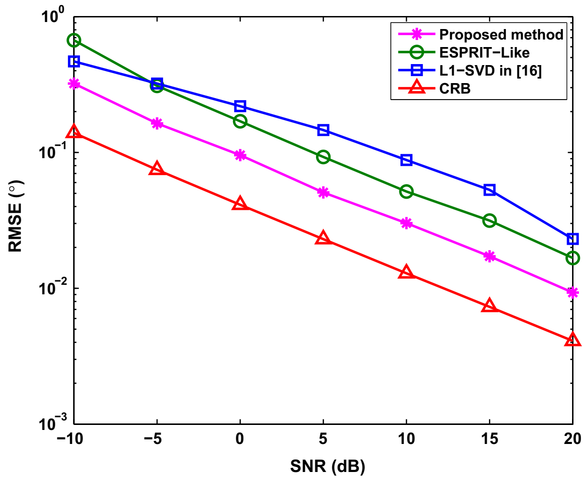

Fig. 2. RMSE versus SNR with 3 targets when $K=1$.

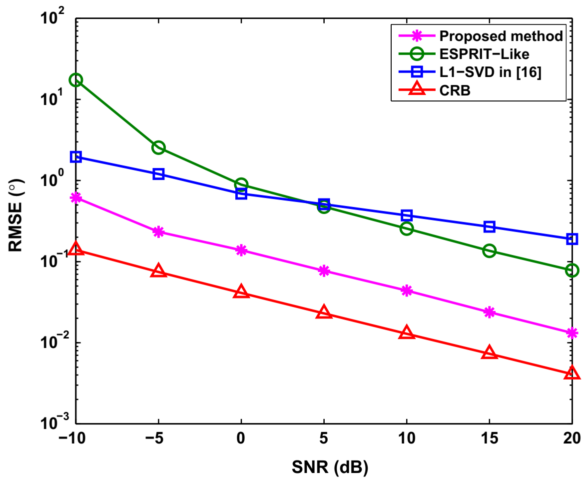

Fig. 3. RMSE versus SNR with 3 targets when $K=2$.

$[1, 0.0617 + j0.0203]$ and $[c_{r0}, c_{r1}] = [1, 0.1021 - j0.1038]$. (2) $K=2$ with $[c_{t0}, c_{t1}, c_{t2}] = [1, 0.5 + j0.002, 0.2 + j0.061]$ and $[c_{r0}, c_{r1}, c_{r2}] = [1, 0.4 + j0.0121, 0.15 + j0.0251]$. The input signal-to-noise ratio (SNR) is defined as $\text{SNR} = 10\log_{10}(\|\mathbf{CAS}\|_F^2 / \|\mathbf{N}\|_F^2)$. The discretized sampling grids are uniform with 0.1° and 1° from −90° to 90° for the l_1-SVD algorithm and the proposed method, respectively. The number of the Monte Carlo is $Q=200$.

Fig. 1 shows the spatial spectrum of the proposed method with different SNR, where $M=N=8$, $J=200$, and mutual coupling case (1) are considered. As seen in Fig. 1, the proposed method has sufficient spatial peaks for the coarse DOA estimation which are near to the true values. It is indicated that the coarse DOAs are close enough for the quadratic convergence in Newton method. Thus, the proposed method can provide accurate DOA estimation.

Figs. 2 and 3 show the RMSE versus SNR with different mutual coupling cases, where $M=N=8$, $J=200$, and mutual coupling case (1) for Fig. 2 and mutual coupling case (2) for Fig. 3 are considered. It is indicated in both of Figs. 2 and 3 that the l_1-SVD algorithm provides better angle estimation performance than the ESPRIT-Like algorithm in low SNR region. On the other hand, the proposed method provides better angle estimation performance than the l_1-SVD and ESPRIT-Like algorithms, particularly with a large value K. This is because that the proposed method enlarges the array aperture and sufficiently provides initial DOA estimation to the Newton iteration for the accurate DOA estimation.

Fig. 4 shows the RMSE versus snapshots, where $M=N=8$, SNR$=5$ dB, and mutual coupling case (1) are considered. It can be seen from Fig. 4 that the proposed method provides better angle estimation than l_1-SVD and ESPRIT-Like algorithms, and works well with lower snapshots.

Fig. 5 shows that the target resolution probability versus SNR, where $M=N=8$, $J=200$, and mutual coupling case (1) are considered. All targets can be seen as successful detection when the absolute estimation errors of the DOAs for all targets are within 0.1°. As seen in Fig. 5, all the methods provide 100% target resolution probability when the SNR is high enough. As the SNR decreases, the resolution probability of each method begins to drop at a certain point, which is defined as SNR threshold. The proposed method provides lower SNR threshold than both l_1-SVD and ESPRIT-Like algorithms, which indicates that the proposed method has higher target resolution probability than both of them.

Fig. 6 shows the RMSE versus angle separation, where $M=N=8$, SNR $=0$ dB and mutual coupling case (1) are considered. Two uncorrelated targets with different DOAs as $\theta_1 = 2°$ and $\theta_2 = \theta_1 + \Delta\theta°$ are used, where the angle $\Delta\theta°$ varies from 4° to 16°. As seen in Fig. 6, compared with l_1-SVD and ESPRIT-Like algorithms, the proposed method shows the best angle estimation performance for closely spaced targets, which means that the proposed method has higher spatial angular resolution than both of them. The reason is that the virtual aperture is remarkably enlarged in the proposed method, then the better angle

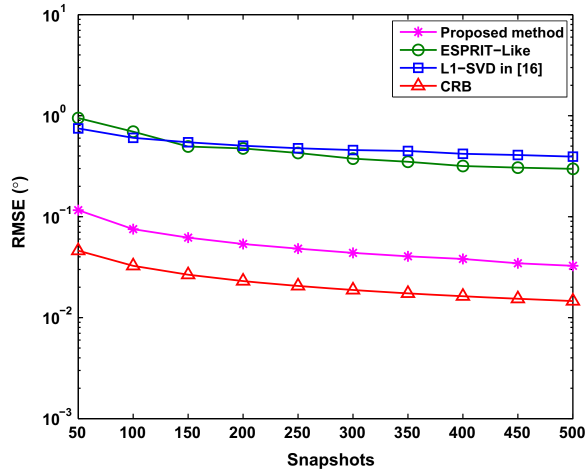

Fig. 4. RMSE versus snapshots with 3 targets when $K=1$.

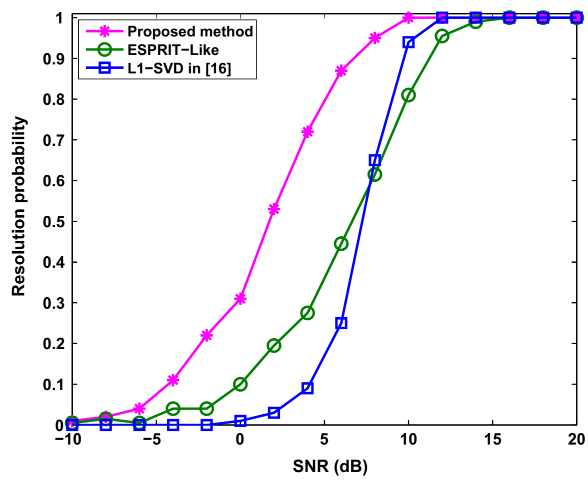

Fig. 5. Target resolution probability versus SNR with 3 targets when $K=1$.

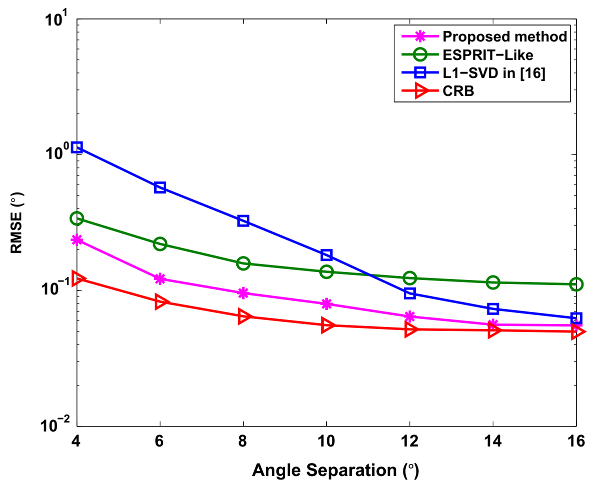

Fig. 6. RMSE versus angle separation with 2 targets when $K=1$.

estimation performance is obtained by Newton iteration with the efficient DOA initialization.

6. Conclusion

In this paper, we have proposed a covariance vector sparsity-aware DOA estimation method for monostatic MIMO radar with unknown mutual coupling. In the proposed method, the coarse DOAs are estimated by using the sparse construction of covariance vector in the presence of unknown mutual coupling, then a maximum likelihood estimation procedure based on the recovered results is exploited for the accurate DOA estimation. The computational complexity of the proposed method is analyzed, and the simulation results show that the proposed method provides better angle estimation performance and higher spatial resolution than both l_1-SVD and ESPRIT-Like algorithms.

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