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High Resolution Multipath Time Delay Estimation Based on FLOCCS-ESPRIT

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Abstract: The non-Gaussian α -stable distribution is introduced to model impulsive noise. Based on the theory of fractional lower order statistics (FLOS), the fractional lower order cross-covariance (FLOCC) sequence for two received signals is obtained and the fractional lower order cross-covariance spectrum (FLOCCS) can be approached by taking a Fourier transform for the FLOCC sequence. When the FLOCCS is treated as a sequence in the time domain, the problem of multipath time delay estimation (TDE) may be converted into one on multi-frequencies estimation or directions of arrival estimation. Accordingly, the high resolution multipath TDE can be realized with the ESPRIT technology. This idea on multipath TDE is referred to as FLOCCS-ESPRIT in this paper. Computer simulations show that this method has good performance both in a Gaussian noise and in an impulsive noise environment.
Key words: signal processing; multipath time delay estimation; α -stable distribution; FLOCC; FLOCCS; estimation signal parameter via rotational invariance techniques (ESPRIT)
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1 Introduction

Time delay estimation (TDE) is a fundamental issue in many signal processing areas, such as wireless positioning, radar, sonar and seismic exploration, etc. Signal multipath propagation is often encountered in these areas. The correlation-based TDE methods can be easily implemented. However, its resolution capability is limited to the reciprocal of the bandwidth of the resource signal. Therefore, in the literature, more attention is paid to high resolution estimation techniques. Relaxation-based algorithms such as WRELAX[1–2] and EM[3] decouple the multidimensional optimization problem into a sequence of one-dimensional optimization problems. However, these approaches are generally applied to the active TDE with known transmitted signals. Another class of methods is to take the cross power spectrum (CPS) sequence of received signals as an equivalent time sequence, with which a high resolution CPS can be estimated by modern spectrum estimation approaches[4]. This converts the TDE problem into sinusoidal parameters estimation or directions of arrival estimation, so that high resolution harmonic decomposition approaches (such as MUSIC[5] and ESPRIT[6]) can be used to obtain the passive TDE.

Noises are omnipresent. In a multipath environment, the received signal is the superposition of source signals arriving from different paths under background noises. The Gaussian (normal) distribution plays a predominant role in traditional signal processing. Previous studies on high resolution TDE are mainly based on second-order statistics under the Gaussian assumption of noise. However, many noises encountered in practice are impulsive and do not obey a Gaussian distribution. The idea based on the Gaussian assumption may cause the performance of an algorithm to degenerate and the results are unacceptable[7–8]. Impulsive noise can be modeled as the α -stable distribution[7]. Signals and noises in this class tend to produce large-amplitude samples far from the average value and occur more frequently than Gaussian signals. Compared with the Gaussian distribution density, the tail of the α -stable density is heavier. The generalized central limit theorem states that if the sum of independent and identically

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distributed (i.i.d) random variables converges to a distribution by increasing the number of variables, the limit distribution must be α-stable distribution. α-stable distribution contains the Gaussian distribution and the Cauchy distribution and can therefore be broadly applied in a variety of areas. Reference [8] proposed several robust α-stable model-based TDE approaches. However, the resolution of these methods is similar to that of correlation methods. Therefore, it is necessary to develop an algorithm for a robust high resolution multipath TDE.

In our investigation, we modelled the noise in received signals as α-stable distribution noise. Based on the theory of fractional lower order statistics (FLOS), a fractional lower order cross-covariance (FLOCC) sequence for two received signals is transformed with a Fourier transform into a frequency sequence, referred to as the fractional lower order cross-covariance spectrum (FLOCCS)[9]. Thus, a passive multipath TDE can be converted into sinusoidal frequencies estimation or directions of arrival estimation if the FLOCCS is treated as a time sequence. Accordingly, the high resolution multipath TDE can be realized by performing the signal parameter estimation via rotational invariance techniques (ESPRIT)[10–11], referred to in our paper as a FLOCCS-ESPRIT algorithm. Computer simulations show that the proposed approach is robust both in a Gaussian noise and in an impulsive noise environment and has a high resolution multipath TDE when the differences in time delay are less than the reciprocal of the bandwidth of the source signal.

2 α-Stable Distribution and FLOCCS

α-stable distribution is a sort of generalized Gaussian distribution. α-stable distribution does not have closed-form of probability density functions (pdf) except for a few known cases. Therefore, it is described by the characteristic function[7] as

$$\phi(t) = \exp\{ j\mu t - \gamma |t|^{\alpha} [1 + j\beta \operatorname{sgn}(t)\omega(t,\alpha)]\} \tag{1}$$

where $\omega(t,\alpha) = \tan(\pi\alpha/2)$ when $\alpha \neq 1$ and $\omega(t,\alpha) = (2/\pi)\lg|t|$ when $\alpha = 1$. α-stable distribution is completely determined by its four parameters. The most important parameter is the characteristic exponent α, measuring the thickness of the tail of the distribution. A small value of α implies considerable probability mass in the tails of the distribution. Gaussian distribution ($\alpha = 2$) is a special case of α-stable distribution. When $0 < \alpha < 2$, α-stable distribution is called fractional lower order α-stable (FLOA) distribution. β is the symmetry parameter (the distribution is called symmetric α-stable distribution ($S\alpha S$) when $\beta = 0$), γ is the dispersion, and μ is the location parameter.

FLOA does not have finite second or higher order moments. Therefore, FLOS becomes a new and useful tool under FLOA noise for multipath TDE. The FLOCC plays a role analogous to the cross-covariance in second order moments. The FLOCC of $x_1(n)$ and $x_2(n)$, denoted as $R^{(F)}_{x_1x_2}(m)$, is defined as[8]

$$\begin{gathered} R^{(F)}_{x_1x_2}(m) \overset{\Delta}{=} E\left\{[x_2(n)]^{<A>}[x_1(n+m)]^{<B>}\right\} \\ 0 \le A < \frac{\alpha}{2}, \quad 0 \le B < \frac{\alpha}{2} \end{gathered} \tag{2}$$

for all values of α.

Similar to the CPS of random signals, which is a Fourier transform of the cross-correlation function, the FLOCCS $P^{(F)}_{x_1x_2}(\omega)$ is defined as a Fourier transform of the FLOCC $R^{(F)}_{x_1x_2}(m)$ as[9]

$$P^{(F)}_{x_1x_2}(\omega) = \sum_{m=-\infty}^{+\infty} R^{(F)}_{x_1x_2}(m)\, \mathrm{e}^{-j\omega m} \tag{3}$$

The FLOCCS has characteristics similar to the CPS.

The FLOCC and the FLOCCS are a pair of Fourier transforms. The FLOCC can be obtained by inverse Fourier transform of FLOCCS as follows

$$R^{(F)}_{x_1x_2}(m) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} P^{(F)}_{x_1x_2}(\omega)\, \mathrm{e}^{jm\omega}\mathrm{d}\omega \tag{4}$$

3 Multipath TDE Approach Based on FLOCCS-ESPRIT

3.1 Data model

For the sake of convenience in the study of passive multipath TDE, the number of multipaths p is set as 2. The received signals can then be modeled as

$$\begin{cases} x_1(n) = s(n) + v_1(n) \\ x_2(n) = s(n-D_1) + s\left(n-D_2\right) + v_2(n) \end{cases} \tag{5}$$

where $x_1(n)$ and $x_2(n)$ are two received signals from two sensors, respectively. $v_1(n)$ and $v_2(n)$ are $S\alpha S$ noise processes, which are assumed to be independent from each other. $s(n)$ is the signal of interest independent of $v_1(n)$ and $v_2(n)$. All the pro- cesses are assumed to be stationary. D_1 and D_2 are the time delays of $s(n)$ for two paths.

3.2 Problem of the traditional approach based on second order statistics

In a Gaussian noise environment, the limit of resolution of correlation-based multipath TDE can be exceeded by using high resolution sinusoidal parameters estimation methods. The basic idea of these methods is that the CPS sequence for two received signals is regarded as a complex-valued time sequence. After taking a Fourier transform for the across-relation sequence, the CPS sequence is as follows:

$$P_{x_1x_2}(\omega)=F[R_{x_1x_2}(m)]=G_{ss}(\omega)(\mathrm{e}^{-j\omega D_1}+\mathrm{e}^{-j\omega D_2}) \quad (6)$$

Thus, the multipath TDE, i.e. estimating D_1 and D_2, is converted into the estimation of sinusoidal frequencies. In array signal processing, the estimation of directions of arrival is identical to the estimation of harmonic frequencies. The ESPRIT method is applied to equivalent time sequences in order to obtain high resolution frequencies estimation. Consequently, a high resolution multipath TDE is obtained. This method is the cross power spectrum ESPRIT algorithm, referred to as CPS-ESPRIT in the following simulations.

However, the noise is often impulsive in engineering practice. It does not have a finite cross-correlation function and a CPS. The performance of algorithms based on second order statistics will degrade significantly in such noise conditions. Therefore, it is necessary to develop robust approaches for high resolution multipath TDE.

3.3 Robust high resolution multipath TDE based on FLOS

In order to obtain robust high resolution multipath TDE in an impulsive noise environment, the FLOCCS can be obtained by taking a Fourier transform for the FLOCC sequence as follows

$$P_{x_1x_2}^{(F)}(\omega)=F[R_{x_1x_2}^{(F)}(m)]=P_{ss}^{(F)}(\omega)(\mathrm{e}^{-j\omega D_1}+\mathrm{e}^{-j\omega D_2})= P_{ss}^{(F)}(\omega)\psi(\omega) \quad (7)$$

where $P_{x_1x_2}^{(F)}(\omega)$ is the FLOCCS for two received signals. $P_{ss}^{(F)}(\omega)$ the fractional lower order autocovariance spectrum for the source signal. $\psi(\omega)$ the phase function of $P_{x_1x_2}^{(F)}(\omega)$. Furthermore $P_{x_1x_2}^{(F)}(\omega)$ normalized and added to the term of noise $U(k)$, because, in practice, it is difficult to obtain that the signal and noise are strictly statistically independent. Thus $R_{x_1x_2}^{(F)}(m)$ is obtained by means of equation (2) with parameter $A=B$. The FLOCCS sequence is as follows

$$C(k)=\frac{P_{x_1x_2}^{(F)}(k)}{\left|P_{x_1x_2}^{(F)}(k)\right|}+U(k)=\exp(-j\Delta\omega kD_1)+ \exp(-j\Delta\omega kD_2)+U(k)\quad 0\leq k\leq 2\pi/\Delta\omega \quad (8)$$

where $\Delta\omega$ is the sampling interval in the frequency domain. Regardless of the physical meaning, equation (8) may express a complex-valued sequence in the time domain, which is not impulsive. The variable k of frequency is equivalent to the variable of time. $2\pi/\Delta\omega$ is equivalent to the length of the time sequence. D_1 and D_2 are identical to the frequencies of the sinusoidal signals in the time domain. As a result, $C(k)$ may be regarded as a combination of two sinusoidal signals under a Gaussian noise in the time domain.

The estimation of FLOCCS can be obtained directly[9]. First, $X_2^{(F)}(\omega)=\sum_{n=0}^{N-1}\left\{[x_2(n)]^{<A>}\right\}\mathrm{e}^{-jn\omega}$ and $X_1^{(F)}(\omega)=\sum_{n=0}^{N-1}\left\{[x_1(n)]^{<B>}\right\}\mathrm{e}^{-jn\omega}$ are obtained by taking the N points Fourier transform of $[x_2(n)]^{<A>}$ and $[x_1(n)]^{<B>}$, respectively. Then, the estimated FLOCCS $\hat{P}_{x_1x_2}^{(F)}(\omega)$ is the conjugate product for $X_2^{(F)}(\omega)$ and $X_1^{(F)}(\omega)$ as follows

$$\hat{P}_{x_1x_2}^{(F)}(\omega)=\frac{1}{N}X_2^{(F)}(\omega)[X_1^{(F)}(\omega)]^* \quad (9)$$

The problem of estimating sinusoidal frequencies in the time domain is equivalent to the problem of estimating directions of arrival for multiple signals with an array signal processing Therefore, the multipath TDE is obtained by means of the ESPRIT approach, which is applied in the multiple signals directions of arrival estimation in the array signal processing. Although ESPRIT does not use the concept of any spectrum, it can, all the same, estimate harmonic frequencies[10–12].

The proposed FLOCCS-ESPRIT approach is as follows:

Step 1: Estimate the FLOCCS for $x_1(n)$ and $x_2(n)$ with parameter $A=B$ by using a direct method and normalize the estimated FLOCCS amplitude. The length of $x_1(n)$ or $x_2(n)$ is N points. Thus, the normalized FLOCCS sequence $C(k)$ can be obtained.

Step 2: Regard $C(k)$ as a time sequence. Then, estimate the autocorrelation function $\hat{R}_{cc}(0)$, $\hat{R}_{cc}(1)$, $\cdots$, $\hat{R}_{cc}(m)$ for this sequence. Choose $m>p$, where p is the number of multipaths.

Step 3: Construct the autocovariance matrix

$$\hat{\boldsymbol{R}}_{cc}=\begin{pmatrix} R_{cc}(0) & \cdots & R_{cc}^*(m-1) \\ \vdots & & \vdots \\ R_{cc}(m-1) & \cdots & R_{cc}(0)\end{pmatrix}$$

from the estimated autocorrelation function.

Step 4: Compute the eigenvalue decomposition of matrix $\hat{\boldsymbol{R}}_{cc}=\hat{\boldsymbol{U}}\Sigma\hat{\boldsymbol{U}}^{\boldsymbol{H}}$. The matrix $\hat{\boldsymbol{U}}_s$ is that part of the matrix $\hat{\boldsymbol{U}}$ retained eigen vectors associated with the main eigenvalue of matrix $\hat{\boldsymbol{R}}_{cc}$. The number of the main eigenvalues of matrix $\hat{\boldsymbol{R}}_{cc}$ equals the number of multipaths p.

Step 5: Obtain the matrix $\hat{\boldsymbol{U}}_1$ by cancelling the last row of the matrix $\hat{\boldsymbol{U}}_s$ and obtain the matrix $\hat{\boldsymbol{U}}_2$ by getting omitting the first row of the matrix $\hat{\boldsymbol{U}}_s$. The relation of $\hat{\boldsymbol{U}}_1$ and $\hat{\boldsymbol{U}}_2$ is $\hat{\boldsymbol{U}}_2=\hat{\boldsymbol{U}}_1\hat{\boldsymbol{\Psi}}$ [12].

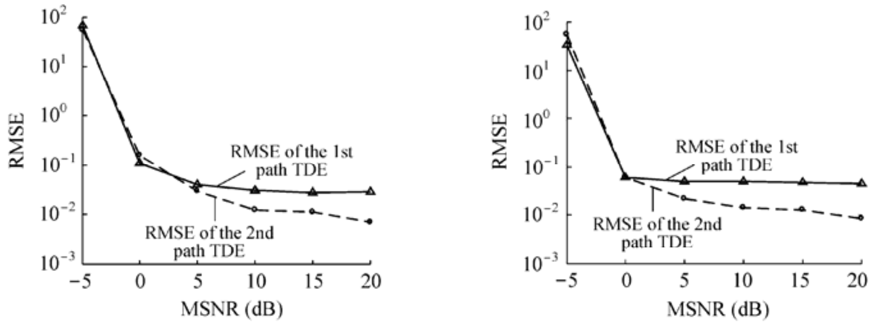
Step 6: Compute the eigenvalue decomposition of matrix $\hat{\boldsymbol{\Psi}} = \left(\hat{\boldsymbol{U}}_1{}^H \hat{\boldsymbol{U}}_1\right)^{-1} \hat{\boldsymbol{U}}_1{}^H \hat{\boldsymbol{U}}_2$. The eigenvalues $\hat{\lambda}_i$ of the matrix $\hat{\boldsymbol{\Psi}}$ are $\mathrm{e}^{-j2\pi \hat{D}_i / N}$, $i = 1, \cdots, p$. Therefore, the estimated time delay is $\hat{D}_i = -(N \angle \hat{\lambda}_i)/(2\pi)$, where N is the number of received signals.

4 Simulation Results

The chirp signal is adopted as the complex-valued envelope of $s(t)$ as

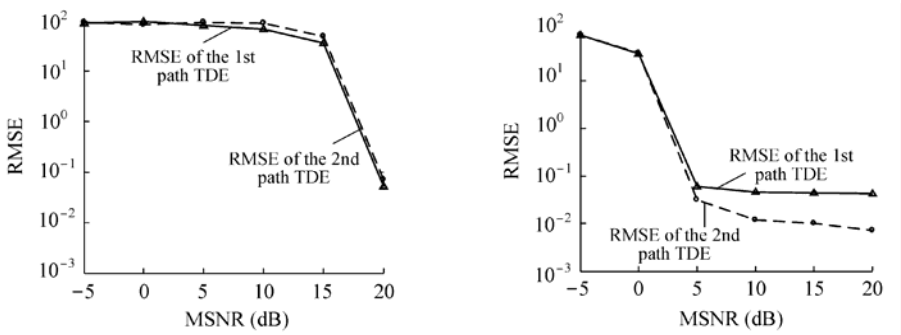
$$s(t) = \mathrm{e}^{j2\pi m_f t^2}, \quad 0 \leq t \leq T \tag{10}$$

where $m_f = 10^5$ represents the chirp rate and $T = 10$ ms is the duration. The signal $s(t)$ is equally digitized into $M = 80$ samples with the sampling frequency $F_s = 8$ kHz. The signal bandwidth is $B_s = 2m_f T = 2 \text{ kHz} = Fs/4$. The sequence length of the received signal is set as $N = 320$. The delayed version of the received signal contains two paths, i.e. $p = 2$. The limited resolution of correlation-based algorithms between neighboring multipaths is $D = 1/B_s = 4T_s$. In order to express the high resolution of ESPRIT-based algorithms for multipath TDE, both true time delays are set as $D_1 = 8T_s$ and $D_2 = 7.5T_s$, respectively. Under the fractional lower order $S\alpha S$ noise environment, the mixed-SNR (MSNR)[6] is set for $\mathrm{MSNR_{dB}}$ to be $10\lg(\sigma_s^2/\gamma_n)$, where σ_s^2 is the variance of $s(t)$. γ_n is the dispersion of $v_1(t)$ or $v_2(t)$ and equals half the variance in the Gaussian case. The root mean square error (RMSE) of time delay estimations between CPS-ESPRIT-based and FLOCCS-ESPRIT-based (set parameter $A = B = 0.1$) is compared with the computer simulations. Each experiment is performed 20 times independently. 20 trials are averaged to form convergence curves. Fig. 1 shows the curves of the RMSE for the two approaches in Gaussian noise. Figs. 2 and 3 show the curves of the RMSE for the two approaches in impulsive noise, $\alpha = 1.2$ and $\alpha = 0.8$, respectively.



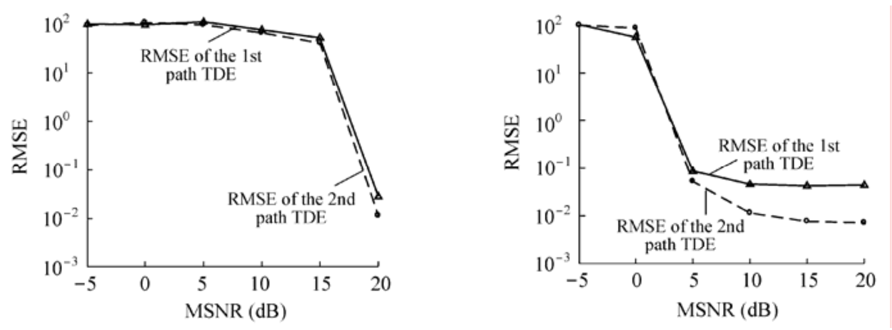
(a) Based on CPS-ESPRIT approach (b) Based on FLOCCS-ESPRIT approach

Fig. 1 Relation between RMSE and MSNR in a Gaussian noise



(a) Based on CPS-ESPRIT approach (b) Based on FLOCCS-ESPRIT approach

Fig. 2 Relation between RMSE and MSNR in an impulsive noise ($\alpha = 1.2$)



(a) Based on CPS-ESPRIT approach (b) Based on FLOCCS-ESPRIT approach

Fig. 3 Relation between RMSE and MSNR in an impulsive noise ($\alpha = 0.8$)

In Gaussian noise, both the CPS-ESPRIT and the FLOCCS-ESPRIT have low RMSE for multipath TDE and high estimation precision when MSNR exceeds 5 dB. In impulsive noise, such as $\alpha = 1.2$ and $\alpha = 0.8$, the RMSE of FLOCCS-ESPRIT-based is still low when MSNR is 5 dB; however, the CPS-ESPRIT-based algorithm can only be used when MSNR exceeds 15 dB. These simulations show that the proposed approach is robust.

5 Conclusions

Based on the FLOS theory, we applied the concept of FLOCCS, which can be obtained by taking a Fourier transform to the FLOCC sequence for two received signals. The high-resolution TDE is approached by means of high-resolution sinusoidal frequencies estimations or directions of arrival estimations if the FLOCCS sequence is treated as a time sequence. Compared with the second-order statistics-based CPS-ESPRIT method, the proposed FLOCCS-ESPRIT algorithm can be applied more broadly. It works well not only in Gaussian noise but also in impulsive noise environments.

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