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Application of Artificial Bee Colony Algorithm to Maximum Likelihood DOA Estimation

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Abstract

Maximum Likelihood (ML) method has an excellent performance for Direction-Of-Arrival (DOA) estimation, but a multidimensional nonlinear solution search is required which complicates the computation and prevents the method from practical use. To reduce the high computational burden of ML method and make it more suitable to engineering applications, we apply the Artificial Bee Colony (ABC) algorithm to maximize the likelihood function for DOA estimation. As a recently proposed bio-inspired computing algorithm, ABC algorithm is originally used to optimize multivariable functions by imitating the behavior of bee colony finding excellent nectar sources in the nature environment. It offers an excellent alternative to the conventional methods in ML-DOA estimation. The performance of ABC-based ML and other popular meta-heuristic-based ML methods for DOA estimation are compared for various scenarios of convergence, Signal-to-Noise Ratio (SNR), and number of iterations. The computation loads of ABC-based ML and the conventional ML methods for DOA estimation are also investigated. Simulation results demonstrate that the proposed ABC based method is more efficient in computation and statistical performance than other ML-based DOA estimation methods.

Keywords: DOA estimation, maximum likelihood, artificial bee colony algorithm, bio-inspired computing

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1 Introduction

The estimation of Direction-Of-Arrival (DOA) is an important problem in array signal processing, which can be widely used in the areas of radar, sonar, seismology, wireless communication, *etc*.. It has attracted great amount of interest for decades, and many useful estimation methods have been proposed and analyzed, including the Maximum Likelihood (ML) methods[1], the Multiple Signal Classification (MUSIC) methods[2], the Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT)[3], *etc*.. The ML method is an excellent statistically effective and robust estimation technique. Its performance is better than the subspace decomposition class methods such as MUSIC and ESPRIT, especially under the conditions of lower Signal-to-Noise Ratio (*SNR*) or smaller snapshot number. Furthermore, ML method can estimate the parameters effectively when the sources are coherent signals, in which condition the subspace decomposition class methods will lose efficiency. While we can get the optimal DOA angles through the ML method theoretically, but the ML estimator requires the maximization of a nonlinear multimodal likelihood function. Since it requires multidimensional solution search which makes the operation much more complicated, the application of the ML method is restricted.

Due to the characteristics of the ML method, many alternative multidimensional searching methods have been proposed in the past decades to reduce the the computational complexity, such as the Alternating Projection (AP) method[4], Space-Alternating Generalized Expectation-maximization (SAGE) method[5], Method Of Direction Estimation (MODE)[6], MODEX[7], and modified MODEX[8], *etc*.. Unfortunately, these methods still have some drawbacks that restrict their applications. The AP method converts the multidimensional searching to unidimensional searching, but its convergence becomes rather slow when the source number increases. The SAGE method requires detailed knowledge of the

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response of the measurement system in order to return reliable and accurate estimates, and the computational complexity may still be high due to the iteration processes. Most of the MODE-based methods require the eigen decomposition of data covariance matrix and there is a threshold restricting all the MODE-based methods. When the *SNR* or snapshot number is below the threshold value, the performance of these methods decreases greatly.

Recently, bio-inspired computing methods have been used in ML-DOA estimation, including Genetic Algorithm (GA)[9,10], Particle Swarm Optimization (PSO)[11–13], Differential Evolution (DE) method[13], and Clone selection algorithm (CLONALG)[13], *etc*.. The GA is one of the most powerful and popular global optimization tools. However, its application is somewhat limited by the precocity and slow convergence. To improve the performance of GA, several modified GA methods[14,15] were proposed, but their performances are not satisfied either. With the development of bio-inspired computing, more and more new methods were proposed. Boccato *et al*. applied some new meta-heuristic methods to the ML-DOA estimation[13], including PSO, DE and CLONALG. They demonstrated that these methods are capable to estimate the DOAs according to the ML criterion. They also compared the performance of each method with other conventional methods. They found that most meta-heuristic methods were efficient for ML-DOA estimation, and performed better than other classes of methods especially in lower SNR conditions. Moreover, there are many other efficient bio-inspired computing methods proposed in recent years, and it is worthy to test the performance of these methods applied to ML-DOA estimation and find which one is better.

Artificial Bee Colony (ABC) algorithm[17–20] is a newly proposed bio-inspired optimization method which simulates the behavior of bee colony finding excellent nectar sources. The most outstanding advantage of ABC algorithm is that it makes global and local optimized searching in each iteration, which can increase the probability of finding the global optimal solution and avoid the local optimal solutions. These characteristics contribute to the improvement of convergence and the reduction of iteration for the application of ABC algorithm to the ML-DOA estimation. The ML-DOA estimation based on ABC algorithm was first mentioned in Ref. [21], but it did not prove the applicability and the superiority of ABC algorithm when it was used in DOA estimation. In this work, we apply ABC algorithm to the ML-DOA estimation with a modified probability function of onlooker bees becoming to employed bees, and compare its performance with other popular bio-inspired computing methods mentioned in Ref. [13] to demonstrate its convergence property, accuracy and efficiency for solving the problem of DOA estimation.

The organization of this paper is as follows: Section **2** presents the data model and the ML estimator. Section **3** provides the description of ABC algorithm and the ABC-ML estimator. Section **4** gives the simulation results to demonstrate convergence property and statistical performance of ABC-ML estimator, and compares it with DE-ML, PSO-ML and CLONALG-ML estimator. Section **5** concludes the paper.

2 Data model and maximum likelihood estimation

2.1 Data model

We assume that N narrow-band far-field signal sources impinge on an antenna array with M ($M > N$) isotropic sensors. Under classical assumptions, the array output vector at sampling time-instant k can be written as

$$\boldsymbol{y}(k) = \boldsymbol{A}(\theta)\boldsymbol{s}(k) + \boldsymbol{v}(k),\ k = 1, 2, \cdots, K \tag{1}$$

where $\boldsymbol{y}(k) \in \mathbb{C}^{M\times 1}$ represents the snapshot data vector, $\boldsymbol{s}(k) \in \mathbb{C}^{N\times 1}$ represents the unknown vector of signal waveforms, $\boldsymbol{v}(k) \in \mathbb{C}^{M\times 1}$ represents an additive noise data vector. K denotes the number of data samples (snapshots). The array transfer matrix $\boldsymbol{A}(\theta) \in \mathbb{C}^{M\times N}$ has the following special structure as

$$\boldsymbol{A}(\theta) = \left[\boldsymbol{a}(\theta_1) \quad \boldsymbol{a}(\theta_2) \quad \cdots \quad \boldsymbol{a}(\theta_N)\right], \tag{2}$$

where $\left\{\boldsymbol{a}(\theta_n)\right\}_{n=1}^{N} \in \mathbb{C}^{M\times 1}$ is called steering vector and $[\theta_1\ \theta_2 \ldots \theta_N]^{\mathrm{T}}$ are the parameters of interest. The exact form of $\left\{\boldsymbol{a}(\theta_n)\right\}_{n=1}^{N}$ depends on the position of the nodes in sensor network. For the Uniform Linear Array (ULA), the steering vector has the form as

$$\boldsymbol{a}(\theta_n) = \begin{bmatrix} 1 & \mathrm{e}^{-j\omega_0 \frac{d}{c}\sin\theta_n} & \cdots & \mathrm{e}^{-j\omega_0 (M-1)\frac{d}{c}\sin\theta_n} \end{bmatrix}^{\mathrm{T}}, \tag{3}$$

where $\omega_0 = 2\pi c/\lambda$, c is wave propagation speed and λ is the wave-length. $(\bullet)^{\mathrm{T}}$ denotes the transpose of matrix

$(\bullet)$. d is the distance between each sensor, it satisfies $0 < d \le \lambda/2$ for avoiding angle ambiguity problems.

Furthermore, we assume that the number of signal sources is known or estimated by other methods. The vectors of signals and noise are assumed to be uncorrelated, stationary, white and zero-mean complex Gaussian processes with second-order moments given as follows

$$\begin{cases} E\left[\boldsymbol{s}(k)\boldsymbol{s}(i)^{\mathrm{H}}\right] = \boldsymbol{S}\delta_{k,i}, \\ E\left[\boldsymbol{s}(k)\boldsymbol{s}(i)^{\mathrm{T}}\right] = 0, \\ E\left[\boldsymbol{v}(k)\boldsymbol{v}(i)^{\mathrm{H}}\right] = \sigma^2\boldsymbol{I}\delta_{k,i}, \\ E\left[\boldsymbol{v}(k)\boldsymbol{v}(i)^{\mathrm{T}}\right] = 0, \end{cases} \tag{4}$$

where $E[\bullet]$ represents the statistical expectation operator, $\boldsymbol{S}$ is the unknown signal covariance matrix, $\delta_{k,i}$ is the Kronecker delta operator, σ^2 is the unknown noise power, $\boldsymbol{I}$ denotes the identity matrix and $(\bullet)^{\mathrm{H}}$ denotes the complex conjugate transpose of matrix $(\bullet)$.

Under the assumption above, the spatial covariance matrix of $\boldsymbol{y}(k)$ is given by

$$\begin{aligned} \boldsymbol{R}_y &= E\left[\boldsymbol{y}(k)\boldsymbol{y}^{\mathrm{H}}(k)\right] \\ &= \boldsymbol{A}(\theta)E\left[\boldsymbol{s}(k)\boldsymbol{s}^{\mathrm{H}}(k)\right]\boldsymbol{A}^{\mathrm{H}}(\theta) + \sigma^2\boldsymbol{I} \\ &= \boldsymbol{A}(\theta)\boldsymbol{S}\boldsymbol{A}^{\mathrm{H}}(\theta) + \sigma^2\boldsymbol{I}. \end{aligned} \tag{5}$$

In practice, the snapshot number of the received array data is finite, so we usually use the sample covariance matrix $\hat{\boldsymbol{R}}_y$ which is obtained by K snapshots of $\boldsymbol{y}(k)$ as

$$\hat{\boldsymbol{R}}_y = \frac{1}{K}\sum_{i=1}^{K}\boldsymbol{y}(i)\boldsymbol{y}^{\mathrm{H}}(i). \tag{6}$$

2.2 Maximum likelihood estimation

Considering the hypothesis presented in the previous section and the Deterministic ML (DML), we make homogeneous sampling of the array to the spatial N signal sources in K uniformly-spaced snapshots. Since the sampling is independent, the joint probability density function of sample data is given by

$$\boldsymbol{f}\left(\boldsymbol{y}_1,\cdots,\boldsymbol{y}_K\right) = \prod_{i=1}^{K}\frac{\exp\left(-\dfrac{1}{\sigma^2}\left|\boldsymbol{y}_i - \boldsymbol{A}(\theta)\boldsymbol{s}(i)\right|\right)}{\det\left(\pi\sigma^2\boldsymbol{I}\right)}, \tag{7}$$

where $\det(\bullet)$ denotes the determinant of matrix $(\bullet)$. Then the Maximum Likelihood Estimator (MLE) of the parameter θ can be written as below by maximizing the function above

$$\hat{\theta} = \arg\max_{\theta} tr\left\{\left[\boldsymbol{A}(\theta)\left(\boldsymbol{A}^{\mathrm{H}}(\theta)\boldsymbol{A}(\theta)\right)^{-1}\boldsymbol{A}^{\mathrm{H}}(\theta)\right]\hat{\boldsymbol{R}}_y\right\}, \tag{8}$$

where $tr(\bullet)$ denotes the trace of matrix $(\bullet)$.

Fig. 1 is an example of the likelihood spectrum of two signal sources impinge on a 10-sensor ULA with *SNR* being 0 dB and −15 dB. The DOAs are 20° and 30°, and 100 snapshots are applied. It is obvious to notice in Fig. 1 that as the *SNR* decreases, not only the global optima move apart from the true DOAs, but also the number of local maximums increases. This characteristic complicates the optimization of the likelihood function, especially for lower *SNR*.

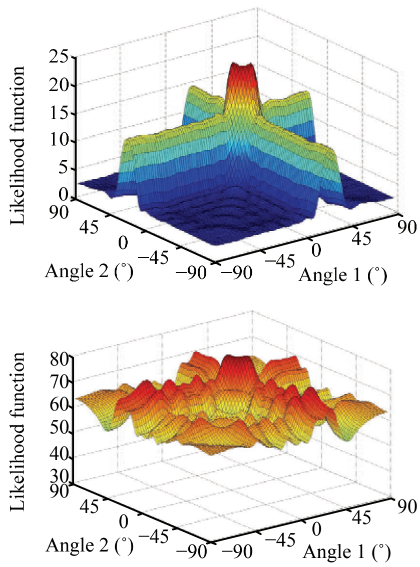


Fig. 1 The spectrum of likelihood function. (a) *SNR*= 0 dB; (b) *SNR* = −15 dB.

Grid search is one of the most accurate methods to find the optima of the likelihood function. Its computational load depends on not only the grid size and searching range we select, but also the source number we want to estimate. We can approximate the number for calculating the likelihood function over a grid search as below

$$n_s = \left(\frac{s_{\max} - s_{\min}}{p_s} + 1\right)^N, \tag{9}$$

where $(s_{\min}, s_{\max})$ is the searching range, p_s is the distance between each grid point. It is obvious that with the increasing of the source number, the computational load will increase exponentially, which restricts the application of this approach.

3 ABC-ML DOA estimation

3.1 Artificial bee colony algorithm

ABC algorithm was proposed by Karaboga and Basturk in 2005. It is a bio-inspired optimization method which simulates the behavior of bee colony finding excellent nectar sources. ABC algorithm is an excellent swarm based meta-heuristic global optimization algorithm, which has many advantages such as low computational load, fast convergence, and little adjustment terms, *etc*.. So it has been widely used to solve various kinds of optimization problem.

In ABC algorithm, the colony of artificial bees contains three groups of bees: employed bees, onlooker bees and scout bees. Both onlookers and scouts are also called unemployed bees. They play different but important roles in ABC algorithm. Employed bees are closely bound up with specific food sources where they are gathering nectar. Onlooker bees stay at the hive and watch the dance of employed bees. Then they choose a food source to gather nectar. Scout bees search for food sources randomly. Initially, all food source positions are unknown and searched by scouts randomly. Thereafter, the scouts become employed bees and sort the food sources according to their nectar amounts. Each employed bee corresponds to one food source, and makes a neighborhood search of the specific food source in each iteration. After the return of each iteration, employed bees share the information of food sources to onlookers through dancing in the hive. Then onlookers choose a target from different food sources and go on searching. The neighborhood search is controlled by a parameter called 'limit'. If a food source is not improved by a predetermined number of trials, the food source is abandoned and the associated employed bee becomes to a scout bee to search a new food source randomly. The number of trials for abandoning a food source is equal to the value of 'limit' which is an important parameter for ABC algorithm.

The ABC algorithm is a process of searching for the food source with most nectar amounts. After each iteration, the best food source is memorized. Onlookers can increase the artificial bee number associated with excellent food sources, which increase the rate of convergence of the algorithm. Scouts search for new food sources, which help the algorithm avoiding local optima. The main steps of ABC algorithm is given below:

Step 1: Send the scout bees onto the initial food sources.

Step 2: Divide the artificial bee colony into two parts: employed bees and onlooker bees (Usually, the first half of the colony consists of the employed bees and the second half includes the onlookers).

Step 3: Each employed bee searches for a new food source having more nectar with in the neighborhood of the food source. If a better food source is found, the new food source replaces the old one.

Step 4: Each onlooker chooses a food source depending on its nectar amount and searches for a new food source in its neighborhood. If a better food source is found, the onlooker becomes to an employed bee and the new food source replaces the old one.

Step 5: If the better food source is not found when the number of searching trials reach the 'limit' number, the old food source is abandoned and the associated employed bee becomes to a scout to go on searching randomly.

Step 6: Memorize the best food source found so far. Go to Step 2 until the end requirements are met.

3.2 Optimization of MLE based on ABC algorithm

In this section, we apply ABC algorithm to the optimization of MLE for DOA estimation. In ABC algorithm, the position of a food source represents a possible solution to the optimization problem and the nectar amount of a food source corresponds to the quality (fitness) of the associated solution. The number of employed bees is equal to the number of solutions since each employed bee is associated with one and only one solution. To apply ABC algorithm, the considered optimization problem is first converted to the problem of finding the best parameter vector which maximizes a fitness function. The artificial bees randomly discover a population of initial solution vectors and then iteratively improve them by employing the strategies: moving towards better solutions by means of a neighborhood search mechanism while abandoning poor solutions.

Suppose the population size of the artificial bee colony is L_s, the population size of employed bees and onlooker bees are L_e and L_0 (we assume $L_e = L_0$). The searching dimension equals to the number of signal sources N, and the searching range is $[X_{\min}, X_{\max}]$, where $X_{\min} = -90°$ and $X_{\max} = 90°$ are respectively the lower and upper band of the DOAs. Each food source $\boldsymbol{X}_i^N$ is an N-dimensional solution vector to the optimization problem which is to maximize the likelihood function. All the vectors of the solutions are initialized by scout bees and the following definition is used for initialization purposes:

$$X_i^j = X_{\min}^j + rand(0,1)(X_{\max} - X_{\min}), \tag{10}$$

where $j = 1,2,\ldots,N$; $i = 1,2,\ldots,L_e$. Then we get the initial solution vectors $\boldsymbol{X_N}$ of ML-DOA.

Employed bees search for new solutions which make the value of fitness function larger in the neighborhood of the old solutions. We define the fitness function equaling to the likelihood function of ML-DOA as

$$\begin{aligned} &fit_i(\boldsymbol{X}_i^N) \\ &= tr\left\{\left[\boldsymbol{A}(\boldsymbol{X}_i^N)\left[\boldsymbol{A}^{\mathrm{H}}(\boldsymbol{X}_i^N)\boldsymbol{A}(\boldsymbol{X}_i^N)\right]^{-1}\boldsymbol{A}^{\mathrm{H}}(\boldsymbol{X}_i^N)\right]\boldsymbol{R}_y\right\}. \end{aligned} \tag{11}$$

Employed bees can get neighborhood solutions using the equation as below

$$W_i^j = X_i^j + \varphi_i^j(X_i^j - X_q^j), \tag{12}$$

where $j = 1,2,\ldots,N$; $i = 1,2,\ldots,L_e$, $\boldsymbol{W}_i^N$ is a neighborhood solution, φ_i^j is a random number within the range $[-1,1]$, $\boldsymbol{X}_i^N$ and $\boldsymbol{X}_q^N$ are randomly selected solutions with the condition $i \neq q$. In a certain sense, this kind of solution search is a mapping from one individual space to another individual space. Its probability distribution is only concerned with the current solution state $\boldsymbol{X}_i^N$, but unconcerned with previous solution state.

Then the greedy selection is applied between the current solution $\boldsymbol{X}_i^N$ and the neighborhood solution $\boldsymbol{W}_i^N$ to select the one with better fitness value for the next iteration. The greedy selection guarantees the artificial bee population persisting excellent solutions, which makes the evolution without degeneration.

For onlooker bees, a fitness function based selection technique is used. An onlooker chooses a solution according to the probability values calculated from the fitness values which are provided by employed bees. The probability value of the ith solution is calculated using the equation as

$$P_i(\boldsymbol{X}_i^N) = c_a \frac{fit_i(\boldsymbol{X}_i^N)}{\max\limits_i fit_i(\boldsymbol{X}_i^N)} + c_b,\ (i = 1,2,\cdots,L_e) \tag{13}$$

where c_a, c_b are probability coefficients within the range [0, 1] which determine the probability value that the onlookers become employed bees. More onlookers are recruited to the solutions with higher probability values. After choosing the solutions, onlookers search for new solutions within the neighborhood of the old ones similar to the employed bees. In this way, a positive feedback behavior appears.

Employed bees whose solutions are not improved through a number of trials, while the trials number reaches the value of ‘limit’, become scout bees and these solutions are abandoned. After that, the new scouts go on searching for new solutions randomly. This kind of operation is used to enhance the diversity of population and avoid the solutions falling into local optima, which also improves the probability to find global optimum. It is the most important difference between ABC algorithm and other bio-inspired computing algorithms.

Thus, through several iterations of the operations above, we can finally find the maximization of the likelihood function and the DOAs concerned accurately.

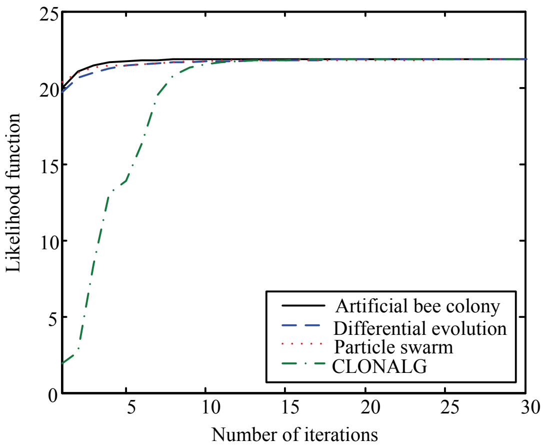
4 Simulation and discussion

In this section, we demonstrate the simulation results of the convergence properties of the proposed method. Then, we compare the proposed method with grid search method and other popular bio-inspired computing algorithms in statistical performance and computational load. We choose three meta-heuristic methods including DE, PSO and CLONALG representatively to compare with the proposed method for ML-DOA estimation. For the sample data, the receiver array is supposed to be a 10 sensor uniform linear array, and 100 data snapshots are used.

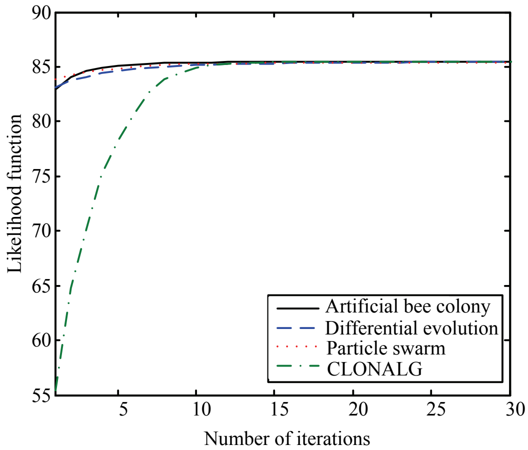
4.1 Convergence properties

Convergence rate and speed are important factors to evaluate the efficiency of an algorithm. In order to demonstrate accurate and reliable simulation results, the maximization process curves of likelihood function are obtained over an average of 100 times of the experiment. The curves are the maximum values of likelihood function found by the algorithm for each iteration versus the increasing number of iterations. The simulation experiment takes two signals with the directions of $\theta = [20^\circ\ 30^\circ]$. The population sizes of the algorithms to be compared are all 100, and other parameters of the algorithms are tuned properly.

Fig. 2 shows the convergence properties of ABC, DE, PSO and CLONALG with *SNR* = 0 dB and *SNR* = −15 dB respectively. As we can observe, ABC algorithm finds global optimum faster than the other three algorithms no matter *SNR*=0 dB or *SNR* = −15 dB. That means ABC algorithm can always find the maximum of the ML estimator with fewer iterations, which influence the computational load of the algorithms.



(a)



(b)

Fig. 2 The maximization process curves of likelihood function with two signal sources. (a) *SNR* = 0 dB; (b) *SNR* = −15 dB.

It is known that the ML method can estimate the parameters more effectively than some other popular methods such as MUSIC and ESPRIT especially when the sources are coherent signals. Therefore, the convergence properties of bio-inspired computing algorithms based ML-DOA estimation with coherent signal sources are worthy to analyze. Fig. 3 shows the convergence properties of the four algorithms mentioned above when the sources are two coherent signals with *SNR* = 0 dB. It is possible to observe that all the four algorithms are able to find the optima of the MLE, while ABC algorithm requires fewer iterations.

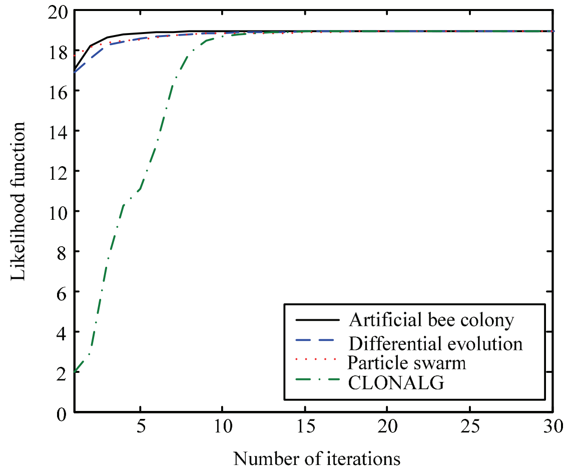


Fig. 3 The maximization process curves of likelihood function with two coherent signal sources when *SNR* = 0 dB.

It is also necessary to examine the convergence properties when the number of signal sources increases. Fig. 4 shows the simulation results when three signal sources with the direction of $\theta = [20^\circ\ 30^\circ\ 45^\circ]$ impinge on the sensor array with *SNR* = 0 dB. It is obvious that the four algorithms need more iterations to locate the global optimum of the ML estimator when more signal sources are considered. Nevertheless, ABC algorithm still estimates the parameters faster than the other three algorithms.

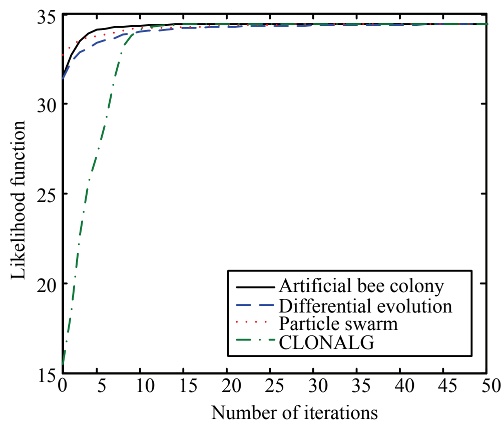


Fig. 4 The maximization process curves of likelihood function with three signal sources when *SNR* = 0 dB.

From the above simulation results we can conclude that the ABC algorithm has faster convergence than DE, PSO and CLONALG algorithms for the ML-DOA estimation problem. This characteristic makes ABC algorithm estimate the DOAs with smaller computational load than the other three bio-inspired computing algorithms.

4.2 Statistical performance

For statistical performance comparison, we carry out 100 Monte Carlo experiments, and take Root-Mean-Square Error (*RMSE*) as the standard to judge the performances of the algorithms. The population sizes and the maximum iteration numbers are all selected to be 100. The *RMSE* is calculated as

$$RMSE = \sqrt{\frac{1}{NN_{\text{run}}} \sum_{l=1}^{N_{\text{run}}} \sum_{i=1}^{N} \left[\hat{\theta}_i(l) - \theta_i \right]^2}, \tag{14}$$

where N_{run} is the number of experiments, N is the number of sources, θ_i is the DOA of the ith signal source, $\hat{\theta}_i(l)$ denotes the estimate of the ith DOA achieved in the lth experiment.

Fig. 5 shows the simulation results of the *RMSE*s curves for DOA estimations versus *SNR* with two signal sources and three signal sources respectively. We can observe that *RMSE* curve of ABC is very close to the curve of DE, which performs a little bit better than the *RMSE* curves of PSO and CLONALG. This phenomenon exists in both two signal sources situation and three signal sources situation. Moreover, we can not get high resolution estimates by using PSO and CLONALG as *SNR* increases. This is mainly because these two algorithms are not able to find the global optima all the time. The spectral peak of ML becomes sharp when *SNR* increases, but sometimes PSO and CLONALG only converge to the solutions beside the spectral peak point. This feature can be improved by increasing the population size or the iteration number, which means PSO and CLONALG require more computational loads to get acceptable estimation result than ABC and DE in the ML-DOA problem. In addition, the *RMSE*s of ABC and DE based ML-DOA are close to that of the Cramer-Rao Bound (CRB).

Fig. 6 shows the simulation results of the *RMSE*s curves for DOA estimation versus *SNR* with two coherent signal sources. It is obvious that the four algorithms are effective to estimate the DOAs under the situation of coherent signals. ABC and DE are able to get better performances than PSO and CLONALG, especially when *SNR* increases.

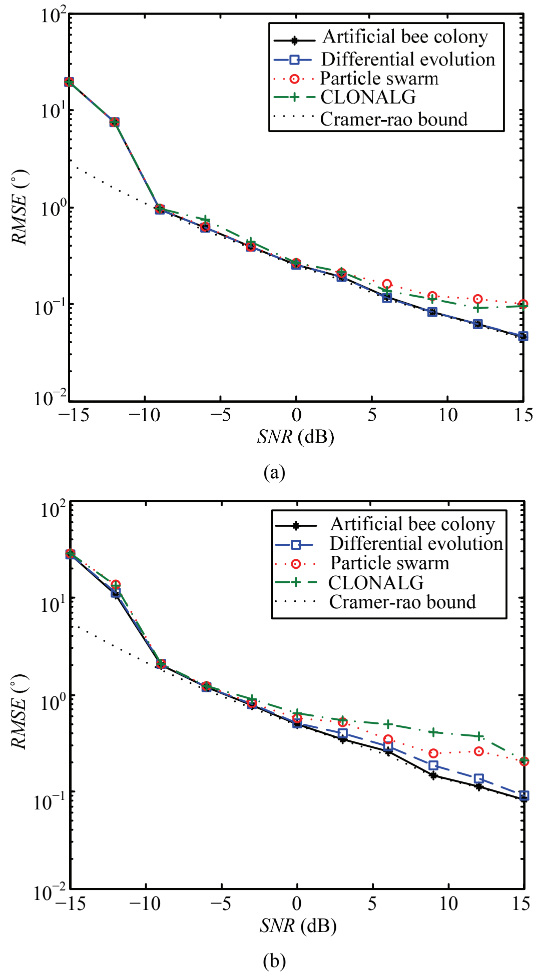


Fig. 5 DOA estimation *RMSE*s versus *SNR* (a) with two signal sources; (b) with three signal sources.

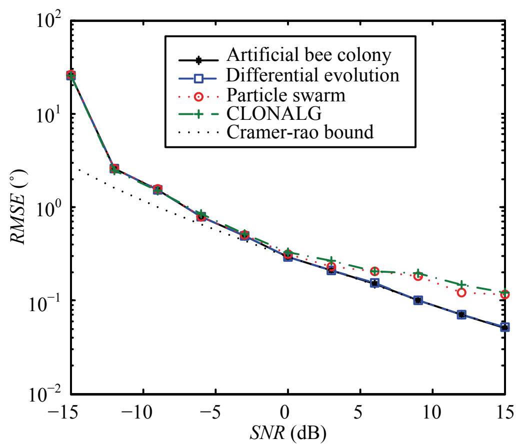


Fig. 6 DOA estimation *RMSE*s versus *SNR* with two coherent signal sources.

Furthermore, it is necessary to evaluate the performance of ABC based ML-DOA method compared with other popular conventional methods. Consider the situation of two independent signal sources impinging on the array, and the computing grid is selected to be 0.05°. Fig. 7 shows the simulation results of *RMSE*s curves for ABC, grid search, MODEX and AP algorithms versus *SNR*. It is possible to observe that the performance of ABC based method is very close to the results from grid search and CRB, whose performance is better than MODEX and AP algorithms under the situations of lower *SNR*.

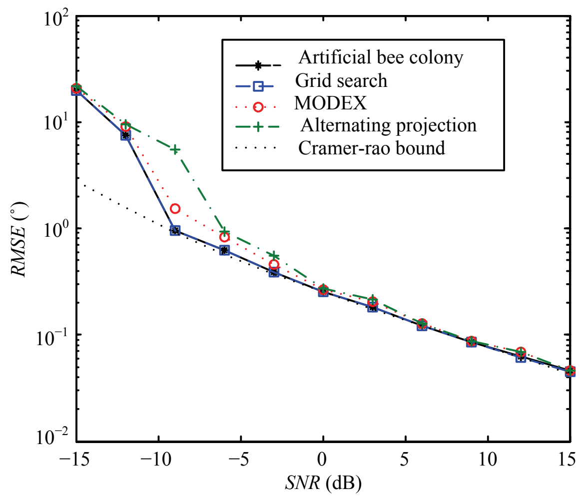


Fig. 7 DOA estimation *RMSE*s versus *SNR* for ABC and conventional ML-DOA methods with two signal sources.

As the simulation results demonstrated above, it is possible to summarize that ABC based ML-DOA method performs better than most other methods which are popular to be used. The *RMSE* of DOA estimation obtained by ABC algorithm is close to the result from grid search and close to CRB.

4.3 Computational loads

The population size is one of the most important parameters for bio-inspired computing algorithms, which influences the computational load of the algorithm directly. For the ML-DOA problem, the population size determines the number of candidate solutions, which is obtained by calculating the maximum likelihood function in each iteration. Therefore, the algorithm with high accuracy and low population size is wanted.

Consider the situation of two signal sources and three signal sources impinging on the array with $SNR = 0$ dB separately. Fig. 8 shows the curves of the *RMSE*s for DOA estimation versus the population size of ABC, DE, PSO and CLONALG. As shown in Fig. 8a, for the situation of two independent signal sources, ABC based ML-DOA method can get acceptable estimates when the population size is larger than 20. While DE and PSO need more population size (more than 50) to get accurate estimates and CLONALG needs population with more than 100 individuals to get acceptable estimates. Considering the situation of three independent signal sources, all four algorithms require more population sizes to get accurate estimation than that of two signals, as shown in Fig. 8b. However, ABC algorithm still uses the smallest population size in these algorithms to estimate the DOAs accurately. In this sense, ABC algorithm is able to solve the ML-DOA estimator with the lowest computation burden in these four bio-inspired computing algorithms.

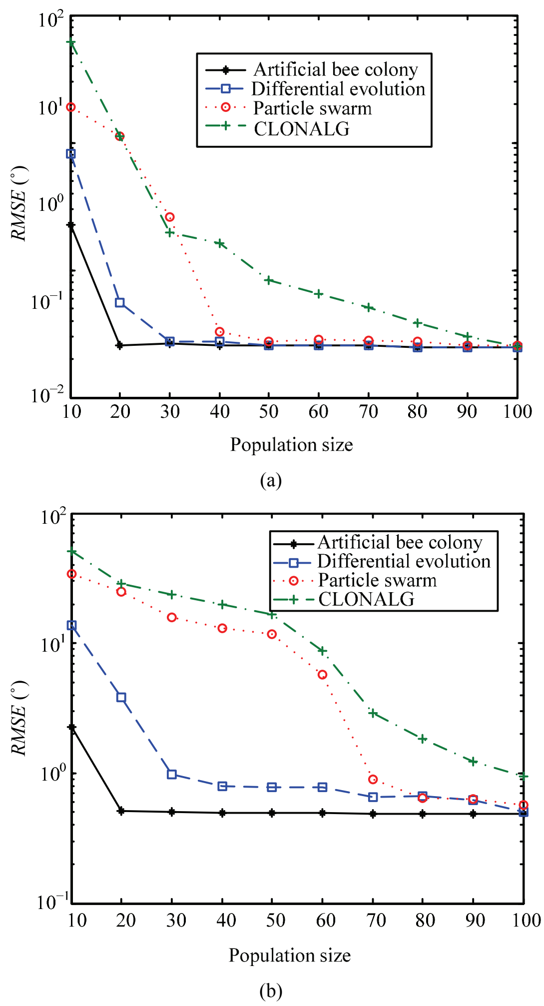


Fig. 8 DOA estimation *RMSE*s versus population size for the algorithms (a) with two signal sources; (b) with three signal sources.

For the situation of two coherent signal sources, Fig. 9 shows the curves of *RMSE*s for DOA estimation versus the population size of the four algorithms mentioned above. It is obvious that ABC algorithm still can find global optima of the MLE using the lowest population size.

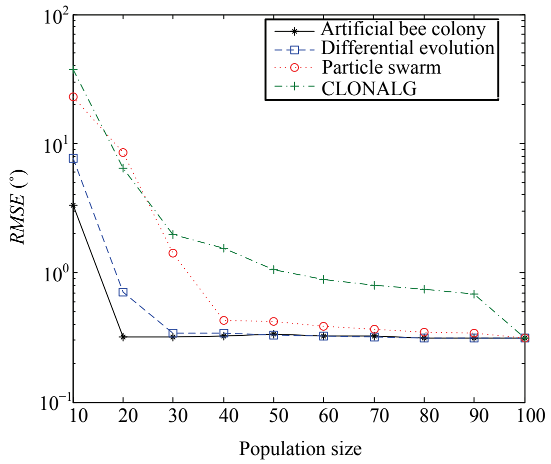


Fig. 9 DOA estimation *RMSE*s versus population size for the algorithms with two coherent signal sources.

Besides, it is also necessary to compare the computation load of ABC based ML-DOA method with other popular conventional algorithms. We compare the computing time of ABC, AP and MODEX algorithms in this part. The computing time is obtained by an average of 100 runs, which are carried out on a Pentium Dual-Core E6700 PC using Matlab 2010b. We consider the uncorrelated signal sources impinging on the array with *SNR*= 0 dB. The computing grid is selected to be 0.05°, the population size and the iteration number of ABC algorithm are all selected to be 50. Table 1 shows the simulation results. It is obvious to observe that ABC-ML method is able to get high resolution DOA estimates with lower computing cost than the other two algorithms.

Table 1 The computing time of different algorithms

Algorithms	ABC-ML		AP-ML		MODEX	
Number of signal sources	N=2	N=3	N=2	N=3	N=2	N=3
Computing time (s)	0.6229	0.6263	0.7357	1.0196	0.6271	0.8746
RMSE (°)	0.1295	0.1889	0.1618	0.2312	0.1411	0.2096

5 Conclusion

DOA estimation is a widely used fundamental technique in array signal processing for locating the target in the air space, under the ground or under the water. Many efficient methods have been proposed but few have been used in practice for various defects. Among these kinds of methods, the ML-DOA estimation is a theoretical optimal method with high resolution, but the large computation burden has blocked it from practical use. In this paper, we applied ABC algorithm, a newly proposed bio-inspired computing method, to optimize the maximum likelihood function with the intention to reduce the computational load. We used a modified probability function to determine the probability for onlooker bees becoming to employed bees, and compared the performance of ABC based ML-DOA estimation with the methods based on DE, PSO and CLONALG. Simulation results verified the better convergence property of ABC-ML estimator as compared with other three popular bio-inspired computing methods. Besides, the statistical performance of ABC-ML estimator was also analyzed. The proposed method was compared with other popular conventional ML based methods such as AP and MODEX. Simulation results demonstrated that ABC based ML-DOA estimation is better in both statistical performance and computational cost. As a result, the ABC-ML estimator is an accurate and computational efficient method to estimate the DOAs, which is more appropriate to engineering applications and worth to be researched deeply.

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