Short communication

Direction of Arrival estimation by modified Orthogonal Propagator Method with linear prediction in low SNR scenarios

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ARTICLE INFO

ABSTRACT

In propagator based methods for Direction of Arrival (DoA) finding, conventional approaches for eliminating the noise impact are based on signal enhancement or noise power estimation. In this paper, a new method is introduced from another point of view. When the noise is an additive spatially and temporally white Gaussian noise, theoretically, it only impacts the diagonal elements of the data covariance matrix. Firstly, the proposed method utilizes the principle of Linear Prediction (LP) to rebuild the denoised diagonal elements of the data covariance matrix. Then, the Orthogonal Propagator Method (OPM) is directly applied for DoA estimation. Compared with the conventional OPM-based methods, the proposed method is more robust to the noise, especially in low Signal to Noise Ratio (SNR) scenarios. Simulation examples are provided to demonstrate the performance of the proposed method.

1. Introduction

Direction of Arrival (DoA) estimation is of practical importance in many engineering applications, for example, radar, sonar, and wireless communications [1,2]. A variety of methods have been proposed to estimate DoA. MUSIC [3] and ESPRIT [4] have asymptotically infinite resolution but high computational burden, since they are based on the EigenValue Decomposition (EVD) of the data covariance matrix. Propagator-based methods, for example, Propagator Method (PM) [5,6] and Orthogonal Propagator Method (OPM) [7], have high resolution and accuracy by using only linear operations without any EVD operation, which make real time processing possible.

Nevertheless, propagator-based methods are developed with signal models under ideal condition and without the consideration of noise. Thus, their performance is degraded in low SNR scenarios. There are three possible solutions to solve this problem. Firstly, the least squares method can be applied to estimate the propagator (the principle of PM and OPM). However, this method only reduces the noise impact but does not eliminate it [5]. The other two possible solutions are either to estimate the power of the noise [8,9] or to apply signal enhancement techniques [10]. In [10], Li *et al.* propose to enhance the received signals by using Karhunen–Loève Transformation (KLT). Stoica *et al.* [8] propose an eigenvalue-based method to estimate the power of the noise. However, these two methods require EVD. In addition, Marcos and Sanchez-Araujo [6] and Stoica *et al.* [8] also propose two different linear operation methods for estimating the power of the noise without EVD.

In this paper, we propose a new way to eliminate the noise impact. When the noise is an additive spatially and temporally white Gaussian noise, theoretically, it only affects the main diagonal elements of the data covariance matrix. In this case, the noise impact can be removed by restoring the denoised main diagonal elements. In the configuration of Uniform Linear Array (ULA), the data covariance matrix of totally uncorrelated incoming signals is a Toeplitz and Hermitian matrix. In theory, the elements in the first column (line) of the data covariance matrix are linear combinations of complex exponentials, which can be modeled by a Linear Prediction (LP) like model [11]. Therefore, we can restore the denoised main diagonal elements by using LP and then reconstruct a denoised covariance matrix.

The rest of this paper is organised as follows: the received signal model is presented in Section 2. Section 3 describes the proposed method. Simulation results and a discussion on the performance of the conventional OPM, OPM-MPEN (method of propagator with elimination of noise) [6], OPM-EVD [8], OPM-KLT [10] and the proposed method are given in Section 4. Conclusions and perspectives are drawn in Section 5.

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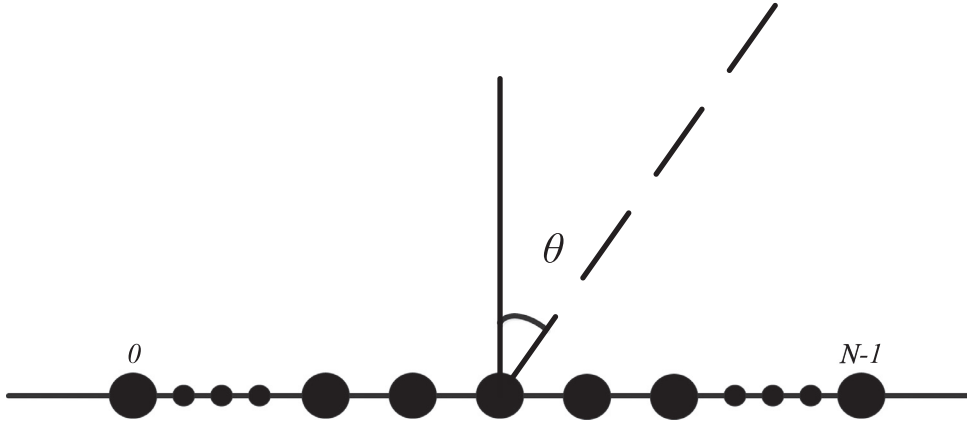*E-mail addresses:* msun@shmtu.edu.cn (M. Sun), jingjing.pan1@etu.univ-nantes.fr (J. Pan).

Fig. 1. Uniform Linear Array configuration.

2. Signal model

Consider K far-field narrow-band signals impinging on a ULA with N isotropic antenna elements, as shown in Fig. 1. The received signals are supposed totally uncorrelated. The indexes of the antennas are set to $0, 1, \ldots, N-1$. Then, the received signal at the mth element can be written as

$$r_m(t) = \sum_{k=1}^{K} s_k(t) \exp\left(j2m\pi \frac{d}{\lambda} \sin\theta_k\right) + n_m(t) \qquad (1)$$

where $s_k(t)$ is the complex envelope of the kth signal, with DOA θ_k, received at the 0th element; $n_m(t)$ is an additive spatially and temporally white Gaussian noise at the mth element with zero mean and variance σ^2; d denotes the distance between two adjacent elements and λ is the wavelength of the received signals. (1) can be written in the following vector form:

$$\mathbf{r}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \qquad (2)$$

with the following notation definitions: $\mathbf{r}(t) = [r_0(t) \ r_1(t) \ \ldots \ r_{N-1}(t)]^T$ is the $(N \times 1)$ received signal vector, the superscript T denotes the transpose operation; $\mathbf{A} = [\mathbf{a}(\theta_1) \ \mathbf{a}(\theta_2) \ \ldots \ \mathbf{a}(\theta_K)]$ is the $(N \times K)$ directional matrix; $\mathbf{a}(\theta_k) = [1 \ \exp(j2\pi \frac{d}{\lambda} \sin\theta_k) \ \ldots \ \exp(j2(N-1)\pi \frac{d}{\lambda} \sin\theta_k)]^T$ is the directional vector; $\mathbf{s}(t) = [s_1(t) \ s_2(t) \ \ldots \ s_K(t)]^T$ is the $(K \times 1)$ source vector; $\mathbf{n}(t) = [n_0(t) \ n_1(t) \ \ldots \ n_{N-1}(t)]^T$ is the $(N \times 1)$ noise vector, with zero mean and covariance matrix $\sigma^2\mathbf{I}$, $\mathbf{I}$ is the $(N \times N)$ identity matrix.

According to signal model (2) and assuming that the noise is independent from the signals, the covariance matrix $\mathbf{R}$ can be written as

$$\mathbf{R} = E\{\mathbf{r}(t)\mathbf{r}^H(t)\} = \mathbf{A}\mathbf{\Gamma}\mathbf{A}^H + \sigma^2\mathbf{I} \qquad (3)$$

where $E\{:\}$ denotes the expectation, $\mathbf{\Gamma} = E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$ is the $(K \times K)$ dimensional covariance matrix of the source vector $\mathbf{s}(t)$ and the superscript H denotes the conjugate transpose operation. Therefore, the elements of $\mathbf{R}$ can be expressed as

$$R(m-n) = \mathbf{R}(m, n)$$
$$= \begin{cases} \sum_{k=1}^{K} P_{s_k} + \sigma^2 & m = n \\ \sum_{k=1}^{K} P_{s_k} \exp\left(j2\pi(m-n)\frac{d}{\lambda}\sin\theta_k\right) & m \neq n \end{cases} \qquad (4)$$

where $m, n \in [1, \ldots N]$, P_{s_k} is the power of the kth signal. As shown in (4), the noise is presented only in the diagonal elements of $\mathbf{R}$,

which can be rewritten as

$$\mathbf{R} = \begin{pmatrix} R(0) & R(-1) & \ldots & R(1-N) \\ R(1) & R(0) & \ldots & R(2-N) \\ \ldots & \ldots & \ldots & \ldots \\ R(N-1) & R(N-2) & \ldots & R(0) \end{pmatrix} \qquad (5)$$

3. Modified OPM (OPM-LP) for DoA estimation

The objective of this work is to build a denoised covariance matrix for the implementation of OPM. Since $\mathbf{R}$ is a Toeplitz and Hermitian matrix, the main diagonal elements are the same (equal to $R(0)$). According to (4), the elements in $\mathbf{R}$ are a linear combination of exponentials. Therefore, we can use LP for restoring $R(0)$. A Toeplitz and Hermitian matrix is uniquely determined by its first line (column). In the proposed method, the information of the first column is used in the estimation of $R(0)$. The first column of $\mathbf{R}$ is partitioned into several overlapping sub-sequences and each sub-sequence contains L elements, as shown in Fig. 2. The LP prediction $\hat{R}(n)$ for $R(n)$ has the following form:

$$\hat{R}(n) = \sum_{l=1}^{L} \omega_l R(n+l), n = 0, \ldots, N-L-1 \qquad (6)$$

where ω_l $(l = 1, \ldots, L)$ is the prediction coefficient. Therefore, we can obtain the denoised element $R(0)$ from the rest of the elements of the first column with the predictions coefficients. The prediction coefficients can be estimated using the sub-sequences $\mathbf{r}_2, \mathbf{r}_3, \ldots \mathbf{r}_{N-L}$. In this case, the LP expression can be formulated as

$$\underbrace{\begin{bmatrix} R(2) & R(3) & \ldots & R(L+1) \\ \vdots & \vdots & \vdots & \vdots \\ R(N-L) & R(N-L+1) & \ldots & R(N-1) \end{bmatrix}}_{\mathbf{X}} \underbrace{\begin{bmatrix} \omega_1 \\ \vdots \\ \omega_L \end{bmatrix}}_{\boldsymbol{\omega}}$$
$$= \underbrace{\begin{bmatrix} R(1) \\ \vdots \\ R(N-L-1) \end{bmatrix}}_{\mathbf{y}}. \qquad (7)$$

Since $\mathbf{X}$ and $\mathbf{y}$ are known, the coefficient vector $\boldsymbol{\omega}$ can be estimated by the least-squares approach:

$$\boldsymbol{\omega} = (\mathbf{X}^H\mathbf{X})^{-1}\mathbf{X}^H\mathbf{y} = \mathbf{X}^+\mathbf{y}. \qquad (8)$$

where $\mathbf{X}^+$ is the Moore–Penrose inverse of $\mathbf{X}$. One should notice that $\boldsymbol{\omega}$ has a unique solution only when $N-L-1 \geq L$. Therefore, in
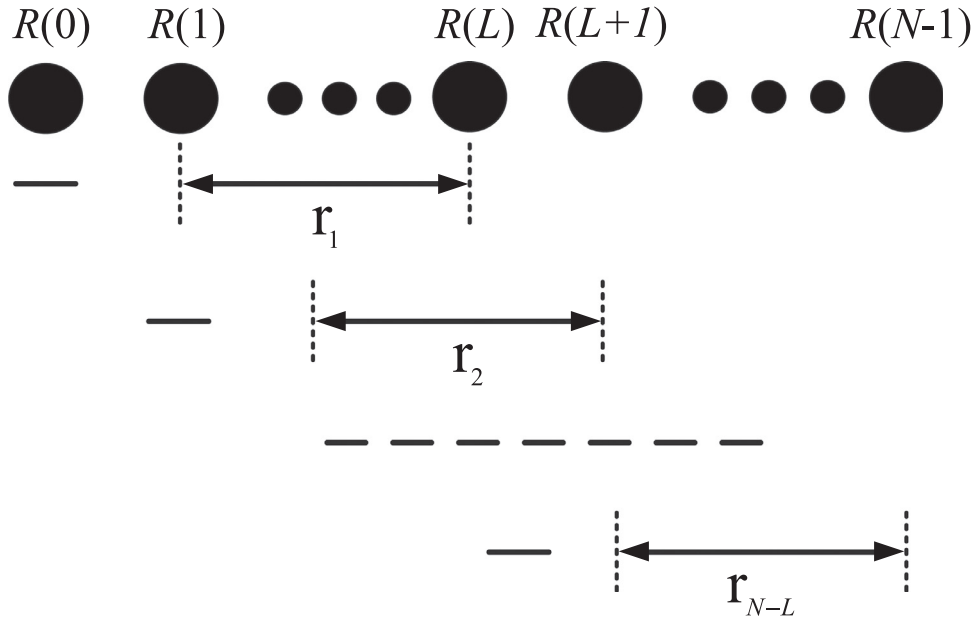
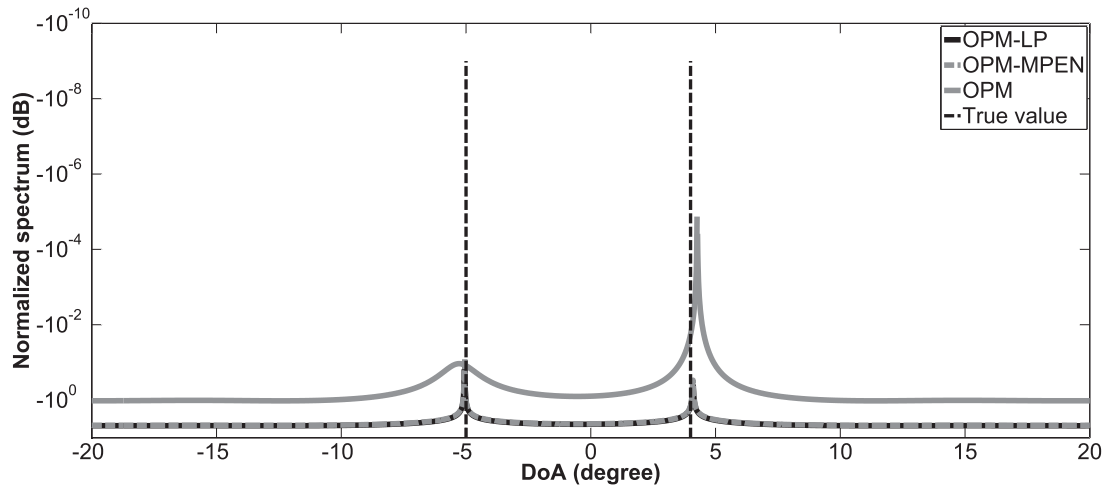**Fig. 2.** Set of the overlapping sub-sequences of the first line of **R**.

Fig. 3. Case *a*, pseudo-spectrums of the OPM-LP, conventional OPM and OPM-MPEN for DoA estimation, the two DoAs are $\theta_1 = -5°$ and $\theta_2 = 4°$.
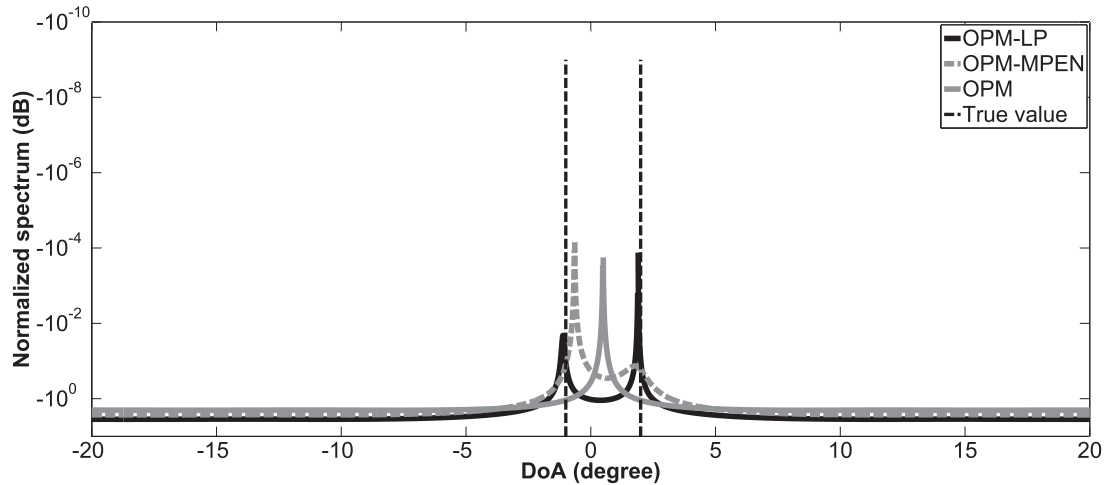

Fig. 4. Case *b*, pseudo-spectrums of the OPM-LP, conventional OPM and OPM-MPEN for DoA estimation, the two DoAs are $\theta_1 = -1°$ and $\theta_2 = 2°$.

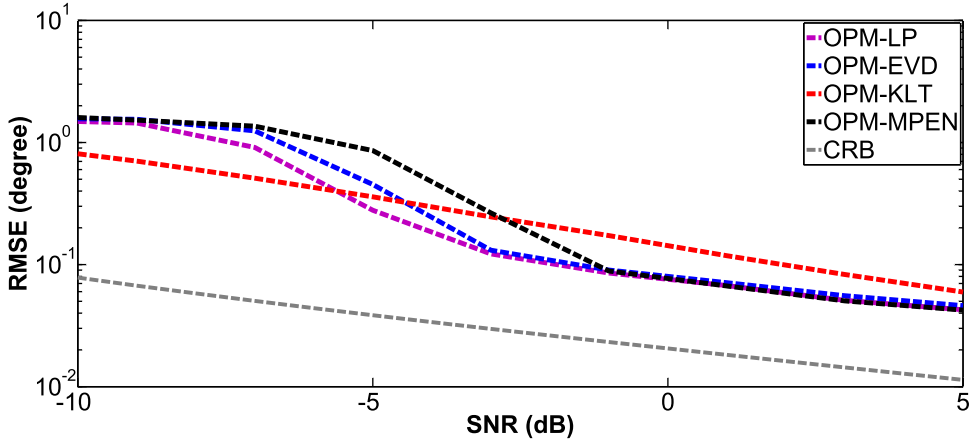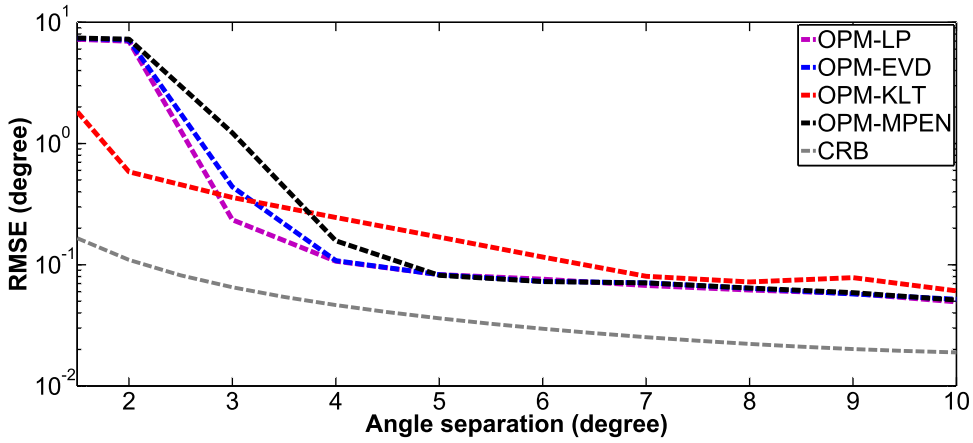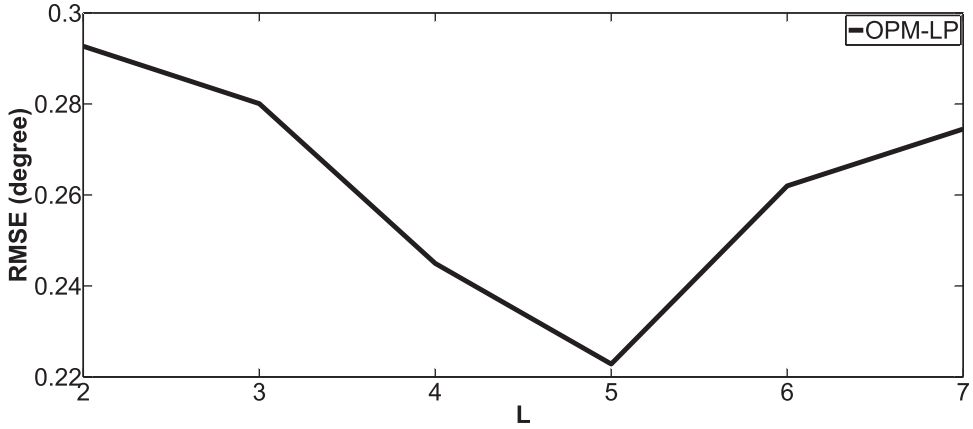**Fig. 5.** Case *b*, RMSE of DoA estimation versus SNR.

Fig. 6. RMSE of DoA estimation in terms of angle separation.

Fig. 7. RMSE of DoA estimation as a function of *L*.

order to resolve K sources, the constraint for L is $K \leq L \leq \frac{1}{2}(N-1)$. Then, we can calculate $\hat{R}(0)$ with the estimated $\boldsymbol{\omega}$:

$$\hat{R}(0) = \sum_{l=1}^{L} \omega_l R(l). \tag{9}$$

Since the noise is removed, OPM [7] can be directly applied on the denoised data covariance matrix for DoA estimation.

4. Simulations and discussion

In this section, the performance of OPM-LP is tested on 4 different numerical simulations. A ULA with 15 isotropic elements is considered, along with two far-field uncorrelated narrow-band signals with equal power, that is $N = 15$, $K = 2$. The number of elements in each sub-sequence L is equal to 6. The distance between two adjacent elements is half of the wavelength.

In the first experiment, the pseudo-spectrums of the proposed method, conventional OPM and OPM-MPEN are estimated. Two

cases are taken into account with different DoAs of the incoming signals: Case *a*. $\theta_1 = -5°$ and $\theta_2 = 4°$; Case *b*. $\theta_1 = -1°$ and $\theta_2 = 2°$. SNR is fixed at -5 dB. The data covariance matrix is estimated from 1000 independent snapshots. The pseudo-spectrum search is performed over $[-20°, 20°]$ with step size $0.001°$. Figs. 3 and 4 show the pseudo-spectrums of the proposed method and the conventional OPM and OPM-MPEN for Cases *a* and *b*, respectively. For Case *a*, both the proposed method and OPM-MPEN can detect the true DOAs of the incoming signals, as shown in Fig. 3. The result of the conventional OPM is biased. When the angle separation becomes small, in Case *b*, the conventional OPM fails to estimate the DoAs while OPM-MPEN estimates the DoA with bias. However, the proposed method remains accurate, even when the angle separation is small.

In the second simulation, the performance of the proposed method versus SNR is assessed with a Monte-Carlo process of 500 independent runs. The Root Mean Square Error (RMSE) of the estimated DoA is defined as follows:

$$\text{RMSE} = \sqrt{\frac{1}{KJ} \sum_{j=1}^{J} \sum_{k=1}^{K} \left(\hat{\theta}_{kj} - \theta_k\right)^2} \quad (10)$$

where $\hat{\theta}_{kj}$ denotes the estimated DoA of the *k*th incoming signal for the *j*th run, and θ_k the true value. SNR varies from -10 to 5 dB. The angles in Case *b* are applied here. The proposed method is compared with OPM-MPEN, OPM-EVD [8], OPM-KLT [10], and also Cramér–Rao Bound (CRB). Fig. 5 plots the RMSE of DoA estimation with respect to SNR. For all the compared methods, the RMSE is continuously decreasing when SNR increases. It can be seen from Fig. 5 that the RMSE of the proposed method is smaller than that of OPM-MPEN and OPM-EVD, especially in low SNR scenarios. Although OPM-KLT has smaller RMSE than the proposed method at lower SNR, the proposed method has a more significant decrease of RMSE as SNR increases. Moreover, the computational complexity of the proposed denoising procedure (LP) is $O(L^2(N - L - 1))$, which is similar to MPEN ($O(N^2(K + 1) + NK^2)$) and is much smaller than that of EVD and KLT ($O(N^3)$).

In the third simulation, the statistical performance of the proposed method versus the angle separation between two incoming signals is assessed with 500 independent runs of the algorithm. One of the incoming signals is fixed at $\theta_1 = 0°$ while the other comes from $\theta_2 = \theta_1 + \Delta\theta$. SNR is fixed at -5 dB and $\Delta\theta$ varies from $1°$ to $10°$. As shown in Fig. 6, when the angle separation is small ($1°$ to $2°$), all methods fail to detect the true DoAs of the incoming signals. Moreover, the RMSE continuously decreases when the angle separation increases. Similar to the second simulation, the proposed method offers better performance than that of OPM-MPEN, OPM-KLT and OPM-EVD.

In the last simulation, the performance of the proposed method versus the length of sub-sequence *L* is studied with 500 independent runs. The angles in Case *b* are applied, SNR $= -5$ dB. $L \in [2, 7]$ due to $K \leq L \leq \frac{1}{2}(N - 1)$. It can be seen from Fig. 7, when $L = 5$, the RMSE reaches its minimum. Therefore, in this situation, $L = 5$ is optimal value for DoA estimation. However, the RMSE shows similar value with different *L*.

5. Conclusion

This paper proposes a new method OPM-LP for DoA estimation in low SNR scenarios. This new technique utilizes the principle of LP, which offers a new way to eliminate the noise impact. Simulation results show the stability, robustness and efficiency of the proposed method. In perspective, the proposed method will be tested with real measurements.

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