Computers and Electrical Engineering 40 (2014) 1731–1740

Contents lists available at SciVerse ScienceDirect

Computers and Electrical Engineering

journal homepage: www.elsevier.com/locate/compeleceng

A coherent direction of arrival estimation method using a single pulse ☆

Chen Chen [a,*], Xiaofei Zhang [a,b]

[a] College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
[b] Hubei Key Laboratory of Intelligent Wireless Communications, South-Central University for Nationalities, Wuhan 430074, China

ARTICLE INFO

Article history:
Available online 9 July 2013

ABSTRACT

This paper addresses the problem of coherent direction of arrival (DOA) estimation in monostatic multi-input multi-output (MIMO) radar using a single pulse, and links the trilinear model to derive a coherent DOA estimation method. We use the received data to construct a set of Toeplitz matrices through which a trilinear model is formed, and then the trilinear decomposition is used to attain the DOAs of sources. The proposed algorithm is effective for a single pulse. Compared to the forward backward spatial smoothing estimation method of signal parameters via rotational invariance techniques (ESPRIT), the ESPRIT-like of Han, and the ESPRIT-like of Li algorithms, our method has better angle estimation performance. Numerical simulations present the effectiveness and improvement of our approach.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

The potential advantages of multiple-input multiple-output (MIMO) radars over conventional phased-array based ones has been verified for flexible beam pattern, angular diversity and more degrees of freedom (DOF), to name a few [1,2]. Utilizing multiple antennas to propagate diverse waveforms and reflect parallel-processed signals, several algorithms for MIMO radar have been established for angle estimation [3–9]. Multiple signal classification (MUSIC) algorithm has been discussed in [3,4], which requires a spectral peak searching with a high computation cost, while estimation method of signal parameters via rotational invariance techniques (ESPRIT) algorithm, which has exploited the invariance property for angle estimation in MIMO radar in [5–7], does not require the spectral peak searching. Propagator method (PM) algorithm for angle estimation in MIMO radar has been investigated in [8,9], which does not require eigen-decomposition of covariance matrix of the received data. Because of this, PM algorithm has lower complexity than ESPRIT. Besides, the angle estimation performance of PM algorithm is close to that of ESPRIT in high signal to noise ratio (SNR). A trilinear alternating least square (TALS) algorithm has been exploited to estimate direction of arrival (DOA) in MIMO radar without additional pair matching [10], which has a better performance than conventional algorithms.

The DOA estimation algorithms listed above are all proposed for the incoherent sources, this paper considers coherent angle estimation for MIMO radar. Coherent signals are often encountered in multipath or some other scenarios. Since multipath signals arise from the different propagation paths of the same target, their Doppler frequencies are identical and their radar cross sections are correlated random variables. Regretfully, the algorithmic performance in [3–10] displays degradation at different levels in the presence of coherent sources. Regarding to the coherent signal models, the coherency of sources will bring about rank loss in covariance matrix of received data. To solve this problem, some typical technologies, such as the

☆ Reviews processed and approved for publication by Editor-in-Chief Dr. Manu Malek.

* Corresponding author. Tel.: +86 13952072665.
E-mail address: chen-chen_cc@163.com (C. Chen).

0045-7906/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compeleceng.2013.06.008

forward spatial smoothing (FSS), the backward spatial smoothing (BSS), forward–backward spatial smoothing (FBSS), and their improved approaches have been investigated in [11–14]. Han and Zhang [15] proposed an ESPRIT-like algorithm for coherent DOA estimation, which finds the signal subspace and noise subspace by reconstructing a Toeplitz matrix. Recently, Li et al. [16] proposed an ESPRIT-like algorithm in which data matrix decomposition method is employed to reconstruct a special matrix, and singular value decomposition (SVD) based ESPRIT algorithm is used to resolve the coherent DOA estimation in monostatic MIMO radar. The angle performance analysis [17,18] based on a single pulse has been researched recently, which has potential advantages over conventional phased-array radar. However, the methods in [11–16] are all based on multiple pulses. When we consider a single pulse, the angle estimation performance of them will seriously degrade, and the performance and the robustness of that still have much room for improvement. The approaches designed specifically for angle estimation with a single pulse need further study.

In this paper, a trilinear decomposition-based coherent DOA estimation algorithm for monostatic MIMO radar using a single pulse is proposed. We use the same symmetrical uniform linear array mode for its transmit/receive array. By reconstructing the received data and then utilizing it to construct a set of Toeplitz matrices, a trilinear model is formed. After that, trilinear decomposition is performed to get the estimation of the direction matrix. Finally, least square (LS) principle is employed to attain the DOAs of sources. The proposed algorithm is effective for a single pulse, and it has better angle estimation performance than the FBSS-ESPRIT algorithm, the ESPRIT-like of Han [15], and the ESPRIT-like of Li et al. [16] algorithms. Numerical simulations present the effectiveness and improvement of our approach.

The trilinear decomposition, also known as parallel factor (PARAFAC) analysis, has been naturally related to angle estimation in MIMO radar in [10]. However, PARAFAC solution for angle estimation in MIMO radar usually is non-unique when the coherent sources exist, and it cannot work robustly for a single pulse. Our work links trilinear model to derive a coherent angle estimation method in monostatic MIMO radar using a single pulse, which can be regarded as an extension of the work in [10].

The remainder of this paper is structured as follows. Section 2 develops the data model for the monostatic MIMO radar using a single pulse; Section 3 establishes our coherent DOA estimation algorithm in addition with the complexity analysis. In Section 4, numerical simulations are presented to verify the effectiveness and improvement of the proposed algorithm, while the final conclusions are made in Section 5.

Notation. $(\bullet)^T$, $(\bullet)^H$, $(\bullet)^*$, $(\bullet)^{-1}$ and $(\bullet)^+$ denote transpose, conjugate transpose, conjugate, matrix inversion and pseudo-inverse operations, respectively. $\mathbf{I}_P$ stands for a $P \times P$ identity matrix. $\circ$, $\otimes$ denote Khatri–Rao product and Kronecker product, respectively. $\mathbf{D}_n(\bullet)$ is to take the nth row of the matrix to construct a diagonal matrix.

2. Data model

We consider a monostatic MIMO radar system equipped with uniform linear arrays (ULAs) spaced d between adjacent antennas for its transmit array and receive array, and we assume that the transmit and receive arrays both have $2N + 1$ antennas which is clearly shown in Fig. 1. The transmit antennas transmit orthogonal waveforms with identical bandwidth B_w and central frequency f_0. $\mathbf{s}_n = [s_n(1), s_n(2), \ldots, s_n(P)]^T$ denotes the sampled baseband coded signal of the nth transmit antenna within one repetition interval with P being the number of waveform sequence, so the transmit orthogonal waveforms can be expressed as $\mathbf{S} = [\mathbf{s}_{-N}, \ldots, \mathbf{s}_0, \ldots, \mathbf{s}_N]^T$. We also assume that there are K far-field sources which include both coherent sources and incoherent sources. Define θ_k as the DOA of kth source. Therefore, the received data matrix of a single pulse can be expressed as [16].

$$\mathbf{Q} = \sum_{k=1}^{K} \mathbf{a}(\theta_k)\beta_k e^{j2\pi f_{dk} t} \mathbf{a}^T(\theta_k)\mathbf{S} + \mathbf{V} \tag{1}$$

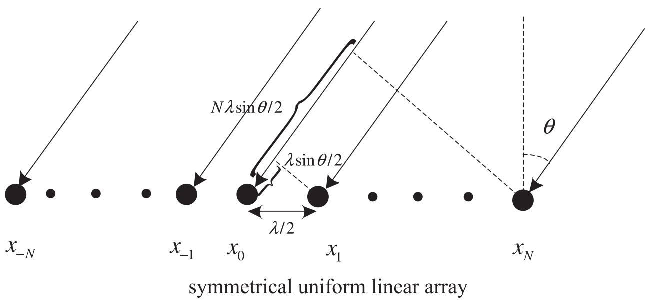

Fig. 1. The structure of transmit/receive array.

where β_k and f_{dk} are the radar cross section (RCS) fading coefficient and Doppler frequency of the kth sources, respectively. Assume that β_k submits to Gaussian distribution of zero mean and $\mathbf{V} \in \mathbf{C}^{(2N+1)\times P}$ is the additive Gaussian white noise matrix. $\mathbf{a}(\theta_k) \in \mathbf{C}^{(2N+1)\times 1}$ is the steering vector of the receive/transmit array. Due to the steering vector of ULA, we have

$$\mathbf{a}(\theta_k) = \left[e^{j2N\pi d\sin\theta_k/\lambda_0}, \ldots, 1, \ldots, e^{-j2N\pi\sin\theta_k/\lambda_0}\right]^T \tag{2}$$

where $\lambda_0 = c/f_0$ is the wavelength. According to the orthogonal property of the transmit waveforms, we have the covariance matrix of $\mathbf{SR}_{SS} = \mathbf{SS}^H/P = \mathbf{I}_{2N+1}$. After range-compression using $\mathbf{S}^H/P$, range-compressed data matrix $\mathbf{Y}$ can be denoted as

$$\mathbf{Y} = \mathbf{QS}^H/P = \sum_{k=1}^{K}\mathbf{a}(\theta_k)\beta_k e^{j2\pi f_{dk}t}\mathbf{a}^T(\theta_k)\mathbf{SS}^H/P + \mathbf{VS}^H/P = \sum_{k=1}^{K}\mathbf{a}(\theta_k)\beta_k e^{j2\pi f_{dk}t}\mathbf{a}^T(\theta_k) + \mathbf{VS}^H/P = \mathbf{A\Lambda A}^T + \mathbf{N} \tag{3}$$

where $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K)]$ and $\mathbf{\Lambda} = diag(\mathbf{d})$ with $\mathbf{d} = [\beta_1 e^{j2\pi f_{d1}t}, \beta_2 e^{j2\pi f_{d2}t}, \ldots, \beta_K e^{j2\pi f_{dK}t}]^T$ being a column vector consisting of amplitudes and phases of the K sources in the single pulse. $\mathbf{N} = \mathbf{VS}^H/P$ is the noise matrix. Coherent signals are often encountered in multipath or some other scenarios, their Doppler frequencies and their radar cross sections are assumed to be identical random variables. It means that the entries of vector $\mathbf{d}$ corresponding to the coherent sources are identical. In order to insure the uniqueness of angles, we must have $d \leqslant \lambda_0/2$. For ease of calculation in the following section, we assume the distance between adjacent transmit/receive antennas $d = \lambda_0/2$.

3. Coherent DOA estimation in Monostatic MIMO radar using a single pulse

3.1. Establish a set of Toeplitz matrices

The (n,m) element of signal matrix $\mathbf{Y}$ described in (3) can be expressed as

$$y(n,m) = \sum_{k=1}^{K} d_k e^{j(N-m-n+2)\pi\sin\theta_k} + \delta_{n,m}, \quad n,m = 1,2,\ldots,2N+1 \tag{4}$$

where $d_k = \beta_k e^{j2\pi f_{dk}t}$ is the kth elements of vector $\mathbf{d}$, and $\delta_{n,m}$ is the corresponding entry of noise matrix $\mathbf{N}$. We can establish a Toeplitz matrix via the entries of nth row of matrix $\mathbf{Y}$, which can be denoted as

$$\mathbf{R}_n = \begin{bmatrix} y(n,N+1) & y(n,N+2) & \cdots & y(n,2N+1) \\ y(n,N) & y(n,N+1) & \cdots & y(n,2N) \\ \vdots & \vdots & \ddots & \vdots \\ y(n,1) & y(n,2) & \cdots & y(n,N+1) \end{bmatrix} = \mathbf{A}_0\mathbf{D}_n(\mathbf{H})\mathbf{A}_0^H + \mathbf{N}_n, \quad n = 1,2,\ldots,2N+1 \tag{5}$$

where $\mathbf{A}_0 = [\mathbf{a}_0(\theta_1), \mathbf{a}_0(\theta_2), \ldots, \mathbf{a}_0(\theta_K)] \in \mathbf{C}^{(N+1)\times K}$ with $\mathbf{a}_0(\theta_k) = [1, e^{j\pi\sin\theta_k}, \ldots, e^{jN\pi\sin\theta_k}]^T$, $\mathbf{D}_n(\bullet)$ is to take the nth row of the matrix to construct a diagonal matrix. $\mathbf{H} \in \mathbf{C}^{(2N+1)\times K}$ can be expressed as

$$\mathbf{H} = \begin{bmatrix} d_1 & d_2 & \cdots\cdots & d_K \\ d_1 e^{-j\pi\sin\theta_1} & d_2 e^{-j\pi\sin\theta_2} & \cdots\cdots & d_K e^{-j\pi\sin\theta_K} \\ \vdots & \vdots & \cdots\cdots & \vdots \\ d_1 e^{-j2N\pi\sin\theta_1} & d_2 e^{-j2N\pi\sin\theta_2} & \cdots\cdots & d_K e^{-j2N\pi\sin\theta_K} \end{bmatrix}$$

We find that the rank of diagonal matrix $\mathbf{D}_n(\mathbf{H})$ $(n = 1, 2, \ldots, 2N + 1)$ equals to the number of sources, so the rank of $\mathbf{R}_m$ equals to the number of sources. Therefore, we can establish $2N + 1$ Toeplitz matrices through (5) as a set, noted as $\{\mathbf{R}_1, \mathbf{R}_2, \ldots, \mathbf{R}_{2N+1}\}$. Then we stack a large matrix expressed as

$$\mathbf{R}_x = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \\ \vdots \\ \mathbf{R}_{2N+1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_0\mathbf{D}_1(\mathbf{H}) \\ \mathbf{A}_0\mathbf{D}_2(\mathbf{H}) \\ \vdots \\ \mathbf{A}_0\mathbf{D}_{2N+1}(\mathbf{H}) \end{bmatrix} \mathbf{A}_0^H + \begin{bmatrix} \mathbf{N}_1 \\ \mathbf{N}_2 \\ \vdots \\ \mathbf{N}_{2N+1} \end{bmatrix} \tag{6}$$

3.2. Trilinear decomposition

Without considering the noise, the entries of the matrix $\mathbf{R}_x$ shown in (6) can be denoted as a trilinear model,

$$r^x_{m,n,j} = \sum_{k=1}^{K} a_{m,k} a^*_{n,k} h_{j,k}, \quad m,n = 1,\ldots,N+1; \quad j = 1,2,\ldots,2N+1 \tag{7}$$

where $a_{m,k}$ and $a^{*}_{n,k}$ are the (m,k) element and the conjugate of that in $\mathbf{A}_0$, respectively. $h_{j,k}$ denotes the (j,k) element in $\mathbf{H}$. According to the symmetric of the trilinear model, by matrix transformation, we have the other two matrices formed by slices in the corresponding directions as

$$\mathbf{R}_y = \begin{bmatrix} \mathbf{A}_0^{*}\mathbf{D}_1(\mathbf{A}_0) \\ \mathbf{A}_0^{*}\mathbf{D}_2(\mathbf{A}_0) \\ \vdots \\ \mathbf{A}_0^{*}\mathbf{D}_{N+1}(\mathbf{A}_0) \end{bmatrix}\mathbf{H}^T + \begin{bmatrix} \mathbf{N}_1^y \\ \mathbf{N}_2^y \\ \vdots \\ \mathbf{N}_{N+1}^y \end{bmatrix} \tag{8a}$$

$$\mathbf{R}_z = \begin{bmatrix} \mathbf{H}\mathbf{D}_1(\mathbf{A}_0^{*}) \\ \mathbf{H}\mathbf{D}_2(\mathbf{A}_0^{*}) \\ \vdots \\ \mathbf{H}\mathbf{D}_{N+1}(\mathbf{A}_0^{*}) \end{bmatrix}\mathbf{A}_0^T + \begin{bmatrix} \mathbf{N}_1^z \\ \mathbf{N}_2^z \\ \vdots \\ \mathbf{N}_{N+1}^z \end{bmatrix} \tag{8b}$$

According to the model shown in (7), we can perform trilinear decomposition to estimate the matrix $\mathbf{A}_0$. Trilinear alternating least square (TALS) algorithm is the common data detection method for trilinear model [19]. The principle of TALS can be adopted to fit low rank trilinear models on the basis of noisy observations. We concisely show the basic idea behind TALS for three major steps: (1) Update one matrix each time using LS, which is conditioned on previously obtained estimates for the remaining matrices; (2) Proceed to update the other matrices; (3) Repeat until convergence of the LS cost function. The TALS algorithm applied in the trilinear model established in this paper is discussed in detail as follows.

Eq. (6) can be rewritten as

$$\mathbf{R}_x = (\mathbf{A}_0 \circ \mathbf{H})\mathbf{A}_0^H + \mathbf{N}^x \tag{9}$$

LS fitting is $\min_{\mathbf{A}_0,\mathbf{H}}\|\widetilde{\mathbf{R}}_x - (\mathbf{A}_0 \circ \mathbf{H})\mathbf{A}_0^H\|_F$, where $\widetilde{\mathbf{R}}_x$ is the noisy signal. The LS update for $\mathbf{A}_0^H$ is

$$\widehat{\mathbf{A}}_0^H = (\widehat{\mathbf{A}}_0 \circ \widehat{\mathbf{H}})^{+}\widetilde{\mathbf{R}}_x \tag{10}$$

where $\widehat{\mathbf{A}}_{0_0}$ and $\widehat{\mathbf{H}}_0$ are the previously obtained estimates of $\mathbf{A}_0$ and $\mathbf{H}$, respectively. Similarly, the (8a) can be expressed as

$$\mathbf{R}_y = (\mathbf{A}_0^{*} \circ \mathbf{A}_0)\mathbf{H}^T + \mathbf{N}^y \tag{11}$$

According to (11), LS fitting is $\min_{\mathbf{A}_0,\mathbf{H}}\|\widetilde{\mathbf{R}}_y - (\mathbf{A}_0^{*} \circ \mathbf{A}_0)\mathbf{H}^T\|_F$, where $\widetilde{\mathbf{R}}_y$ is the noisy signal. Then the LS update for $\mathbf{H}^T$ is given by

$$\widehat{\mathbf{H}}^T = (\widehat{\mathbf{A}}_0^{*} \circ \widehat{\mathbf{A}}_0)^{+}\widetilde{\mathbf{R}}_y \tag{12}$$

where $\widehat{\mathbf{A}}_0^{*}$and $\widehat{\mathbf{A}}_0$ denote previously obtained estimates of $\mathbf{A}_0^{*}$ and $\mathbf{A}_0$, respectively. Finally, (8b) can be described as

$$\mathbf{R}_z = (\mathbf{H} \circ \mathbf{A}_0^{*})\mathbf{A}_0^T + \mathbf{N}^z \tag{13}$$

LS fitting is $\min_{\mathbf{A}_0,\mathbf{H}}\|\widetilde{\mathbf{R}}_z - (\mathbf{H} \circ \mathbf{A}_0^{*})\mathbf{A}_0^T\|_F$, so the LS update for $\mathbf{A}_0^T$ is shown as

$$\widehat{\mathbf{A}}_0^T = (\widehat{\mathbf{H}} \circ \widehat{\mathbf{A}}_0^{*})^{+}\widetilde{\mathbf{R}}_z \tag{14}$$

where $\widehat{\mathbf{H}}$ and $\widehat{\mathbf{A}}_0^{*}$ denote previously obtained estimates of $\mathbf{H}$ and $\mathbf{A}_0^{*}$, respectively, and $\widetilde{\mathbf{R}}_z$ stands for the noisy matrix.

According to (10), (12), and (14), the matrices $\mathbf{A}_0, \mathbf{H}$, and $\mathbf{A}_0^{*}$ are updated with the conditioned LS, respectively. For zero-mean white Gaussian noise, TALS yields maximum likelihood (ML) estimates provided that the global minimum has been achieved [20]. The major shortcomings of TALS algorithm lie on the occasional slowness of the convergence process [21] through TALS is quite easy to implement and guaranteed to converge, initialized randomly, or initialized by eigen decomposition method to accelerate convergence. In this paper, for improving computational efficiency, we use the complex parallel factor (COMFAC) analysis algorithm [22] for trilinear decomposition. COMFAC algorithm is essentially a fast implementation of TALS which speeds up the LS fitting. As for the improvement, COMFAC compresses the three-way data into a smaller three-way data. After fitting the model in the condensed space, the solution can be recovered to the original space within a few TALS steps. Normally, smaller TALS steps are sufficient for this refining stage because the recovered model is close to the LS solution.

Theorem 1 [23]. *$\boldsymbol{R}_{y,n} = \boldsymbol{A}_0^{*}D_n(\boldsymbol{A}_0)\boldsymbol{H}^T, n = 0,\cdots,N,$ where $\boldsymbol{A}_0^{*} \in \boldsymbol{C}^{(N+1)\times K}, (\boldsymbol{H} \in \boldsymbol{C}^{(2N+1)\times K}, \boldsymbol{A}_0 \in \boldsymbol{C}^{(N+1)\times K}$. Consider that matrices are full k-rank [21] and the matrix $\boldsymbol{A}_0$ is with Vandermonde characteristic, if*

$$2k_{\mathbf{A}_0} + k_{\mathbf{H}_0} \geqslant 2K + 2 \tag{15}$$

then $\boldsymbol{A}_0, \boldsymbol{A}_0^{*}$, and $\boldsymbol{H}$ are unique up to permutation and scaling of columns, that is to say, any other matrices $\overline{\boldsymbol{A}}_0, \overline{\boldsymbol{A}}_0^{*}$, and $\overline{\boldsymbol{H}}$ that construct $\boldsymbol{R}_{y,n}, n = 0,\cdots,N$ can be related to $\boldsymbol{A}_0, \boldsymbol{A}_0^{*}$, and $\boldsymbol{H}$ through $\widehat{\boldsymbol{A}}_0 = \boldsymbol{A}_0\boldsymbol{\Pi}\boldsymbol{\Delta}_1, \widehat{\boldsymbol{A}}_0^{*} = \boldsymbol{A}_0^{*}\boldsymbol{\Pi}\boldsymbol{\Delta}_2$, and $\widehat{\boldsymbol{H}} = \boldsymbol{H}\boldsymbol{\Pi}\boldsymbol{\Delta}_3$, where $\boldsymbol{\Pi}$ is a permutation matrix, and Δ_1, Δ_2, Δ_3 notes for the diagonal scaling matrices satisfying $\Delta_1\Delta_2\Delta_3 = \boldsymbol{I}_K$.

Generally, we have $k_{\mathbf{A}_0} = N+1, k_{\mathbf{H}} = \min(2N+1, K) = K$ in the trilinear model established in this paper, then the inequality in (15) becomes $2(N+1)+K \geqslant 2K+2$. The identifiable condition is rewritten as $K \leqslant 2N$, hence the maximum number of sources that can be identified is $2N$.

3.3. DOA estimation

By imposing COMFAC algorithm for trilinear decomposition, we get the estimation of direction matrix $\mathbf{A}_0$ as $\widehat{\mathbf{A}}_0 = \mathbf{A}_0\mathbf{\Pi}\mathbf{\Delta}_1 + \mathbf{W}_0$, where $\mathbf{\Pi}$ is a permutation matrix, and $\mathbf{\Delta}_1$ denotes the diagonal scaling matrix. $\mathbf{W}_0$ is the estimation error. Within COMFAC algorithm, permutation ambiguity and scale ambiguity are inherent. However, the scale ambiguity can be resolved easily by normalization, while the existence of permutation ambiguity does not affect the DOA estimation.

As discussed in (5), the kth column of the matrix $\mathbf{A}_0$ is $\mathbf{a}_0(\theta_k)$, which can be denoted by $\mathbf{a}_0(\theta_k) = [1, e^{j\pi\sin\theta_k}, \ldots, e^{jN\pi\sin\theta_k}]^T$. Define

$$\hat{\gamma}_k = angle(\hat{\mathbf{a}}_0(\theta_k)) = [0, \pi\sin\hat{\theta}_k, \ldots, N\pi\sin\hat{\theta}_k)]^T + \mathbf{n}_r \tag{16}$$

where $\hat{\mathbf{a}}_0(\theta_k)$ is the normalized kth column vector of the estimate of $\mathbf{A}_0$ and $\mathbf{n}_r$ denotes the corresponding noise vector. Impose least squares (LS) principle to estimate the DOA, define $\mathbf{c}_0$ as the LS solution, the least fitting amounts to

$$\min_{\mathbf{c}_0}\|\mathbf{D}\mathbf{c}_0 - \hat{\gamma}_k\|_2 \tag{17}$$

where

$$\mathbf{D} = \begin{bmatrix} 1 & 0 \\ 1 & \pi \\ \vdots & \vdots \\ 1 & N\pi \end{bmatrix} \tag{18}$$

and

$$\mathbf{c}_0 = [c_{01}, c_{02}]^T \tag{19}$$

The LS solution $\mathbf{c}_0$ is given by

$$[\hat{c}_{01}, \hat{c}_{02}]^T = \mathbf{D}^+\hat{\gamma}_0 \tag{20}$$

Extract $\hat{c}_{02}$, which is the estimation of $\sin\theta_k$, and furthermore, we can obtain the estimation of θ_k via

$$\hat{\theta}_k = sin^{-1}(\hat{c}_{02}), \quad k = 1, 2, \ldots, K \tag{21}$$

where $\hat{\theta}_k$ is the DOA estimation of the kth source.

3.4. The procedures of the proposed algorithm

Till now, we have achieved the proposal for the proposed coherent DOA estimation algorithm in monostatic MIMO radar using a single pulse. We show major steps of the proposed algorithm as follows.

Step 1. Establish $2N+1$ Toeplitz matrices from elements in received data $\mathbf{Y}$ by (5).
Step 2. According to (6), stack $\mathbf{R}_x$ via the matrices established in Step.1, the entries of $\mathbf{R}_x$ can be denoted as a trilinear model, and then establish $\mathbf{R}_y$ and $\mathbf{R}_z$ based on slices of the other two directions in the trilinear model via (8a) and (8b).
Step 3. Initialize $\mathbf{A}_0$ and $\mathbf{H}$ randomly, and impose COMFAC algorithm to estimate the matrix $\mathbf{A}_0$, and then utilize the LS principle to estimate the DOAs through (16)–(21).

3.5. Complexity analysis

In contrast to the FBSS-ESPRIT, the ESPRIT-like of Han [15] and the ESPRIT-like of Li et al. [16], our algorithm has a heavier computational load. In our algorithm, the complexity of each iteration is $O(3K^3 + K^2(5N^2 + 6N+1)+3KN^2 + 6KN^3)$. The average number of TALS iterations required for trilinear decomposition with COMFAC algorithm is about a dozen. However, the FBSS-ESPRIT algorithm costs $O(2(2N+1)^3 + 6NK^2 + 2K^3)$, the ESPRIT-like of Han requires $O((2N+1)((N+1)^3 + 3NK^2 + 2K^3))$, and the ESPRIT-like of Li costs $O(2(2N+1)^3+(2N+1)(K^2 + 2NK) + K^3)$. The complexity of these algorithms listed above is clearly presented in the following Table 1.

3.6. Advantages of the proposed algorithm

The advantages of the proposed algorithm can be presented as follows.

Table 1
Complexity of listed algorithms.

Algorithms	Complexity
The proposed algorithm	$O(3K^3 + K^2(5N^2 + 6N+1)+3KN^2 + 6KN^3)$
FBSS-ESPRIT	$O(2(2N + 1)^3 + 6NK^2 + 2K^3)$
ESPRIT of Han [15]	$O((2N + 1)((N + 1)^3 + 3NK^2 + 2K^3))$
ESPRIT of Li [16]	$O(2(2N + 1)^3+(2N + 1)(K^2 + 2NK) + K^3)$

(1) The proposed algorithm is effective and robust for a single pulse.
(2) The proposed angle estimation algorithm is effective for coherent sources and incoherent sources.
(3) The proposed algorithm has better angle estimation performance than the FBSS-ESPRIT algorithm, the ESPRIT-like of Han [15], and the ESPRIT-like of Li et al. [16] for monostatic MIMO radar.

4. Numerical Simulations

We firstly investigate the convergence performance of our proposed algorithm in this simulation. The sum of squared residuals (SSR) in the trilinear fitting is defined as $SSR = \sum_{j=1}^{2N+1}\sum_{m=1}^{N+1}\sum_{n=1}^{N+1}[\tilde{r}^x_{m,n,j} - \hat{a}_{m,k}\hat{a}^*_{n,k}\hat{h}_{j,k}]^2$, where $\hat{a}_{m,k}$ and $\hat{a}^*_{n,k}$ are the (m,k) element and the conjugate of that in $\widehat{\mathbf{A}}_0$, respectively, $\hat{h}_{j,k}$ denotes the (j,k) element in $\widehat{\mathbf{H}}$, and $\tilde{r}^x_{m,n,j}$ is the corresponding noisy data. Define $DSSR = SSR_i - SSR_0$, where SSR_i is the SSR of the ith iteration, and SSR_0 is the SSR in the convergence condition. Initialize $\mathbf{A}_0$ and $\mathbf{H}$ randomly. Fig. 2 presents the algorithmic convergence performance between COMFAC (Our proposed algorithm uses COMFAC.) and TALS with $N = 7$, $K = 3$, and SNR = 5 dB. From Fig. 2, we find that COMFAC has much better convergence performance than TALS.

In the following simulations, we assume that the numerical simulation results converge when the $SSR \leqslant 10^{-8}$. $2N + 1$ denotes the number of transmit/receive antennas, and K denotes the number of sources. Suppose that the central frequency $f_0 = 600$ MHz and bandwidth $B_w = 4$ MHz. We present 1000 Monte Carlo simulations to assess the angle estimation performance of the proposed algorithm. Define root mean squared error (RMSE) as follows.

$$\mathrm{RMSE} = \frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{1000}\sum_{n=1}^{1000}\left(\hat{\theta}_{k,n} - \theta_k\right)^2} \tag{22}$$

where θ_k denotes the perfect DOA of the kth source, $\hat{\theta}_{k,n}$ is the estimation of θ_k in the nth Monte Carlo trail. In Figs. 3–7, we assume that there are three sources in which two sources are incoherent and the other one is coherent with another two, and they located at angles $\theta_1 = 10°$, $\theta_2 = 30°$, and $\theta_3 = 50°$, respectively.

Fig. 3 presents the DOA estimation of the proposed algorithm in monostatic MIMO radar with $N = 7$, $K = 3$, and SNR = 5 dB. Fig. 4 depicts the DOA estimation with $N = 7$, $K = 3$, and SNR = 20 dB. Figs. 3 and 4 illustrate that our algorithm is effective for coherent DOA estimation in monostatic MIMO radar.

Fig. 5 shows the coherent DOA estimation performance comparison of the proposed algorithm, the FBSS-ESPRIT algorithm, the ESPRIT-like of Han [15], the ESPRIT-like of Li et al. [16], and the Cramér–Rao Bound (CRB) for monostatic MIMO radar with $N = 7$, and $K = 3$, while Fig. 6 depicts the DOA estimation performance comparison with $N = 8$, and $K = 3$,. It is indicated that our algorithm has better angle estimation performance than the FBSS-ESPRIT algorithm, the ESPRIT-like of Han, and the ESPRIT-like of Li for monostatic MIMO radar.

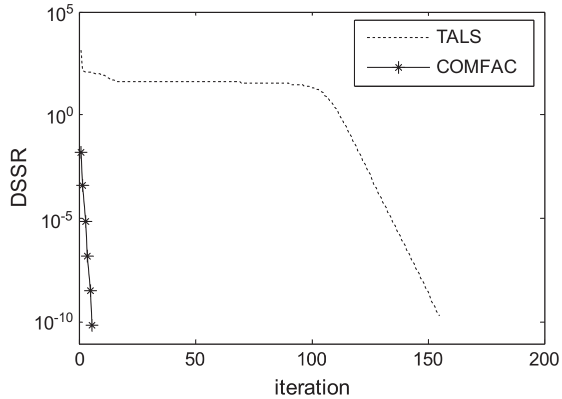

Fig. 2. Comparison of algorithmic convergence performance ($N = 7$, $K = 3$, SNR = 5 dB).

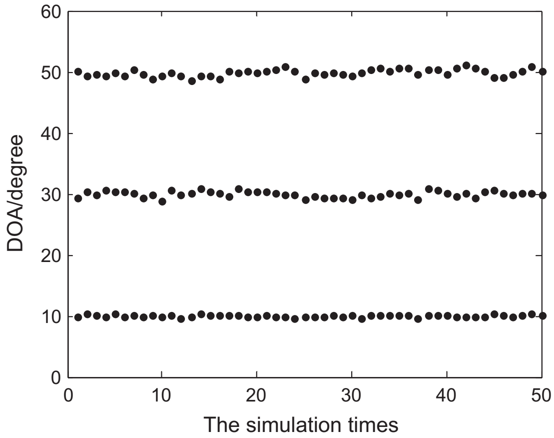

Fig. 3. DOA estimation of our algorithm in SNR = 5 dB ($N = 7$, $K = 3$).

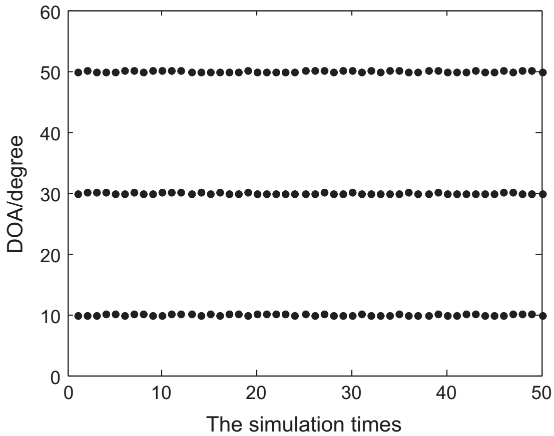

Fig. 4. DOA estimation of our algorithm in SNR = 20 dB ($N = 7$, $K = 3$).

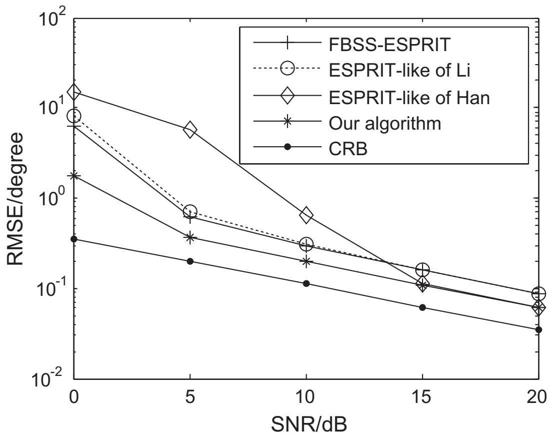

Fig. 5. DOA estimation performance comparison ($N = 7$, $K = 3$).

Fig. 7 depicts the DOA estimation performance of the proposed algorithm with different N ($K = 3$). It is clear shown that the angle estimation performance of our algorithm is gradually improving with the number of antennas increasing. Multiple antennas improve estimation performance because of diversity gain.

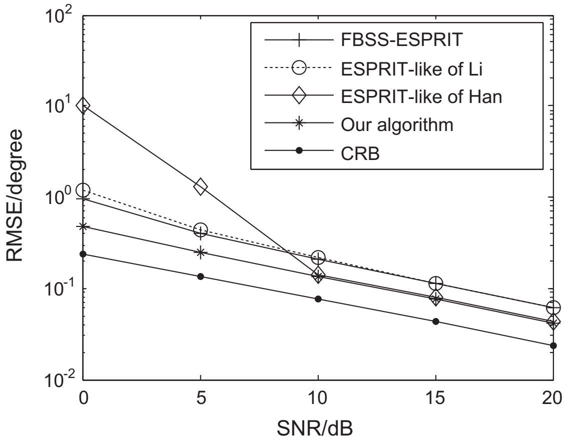

Fig. 6. DOA estimation performance comparison ($N = 8$, $K = 3$).

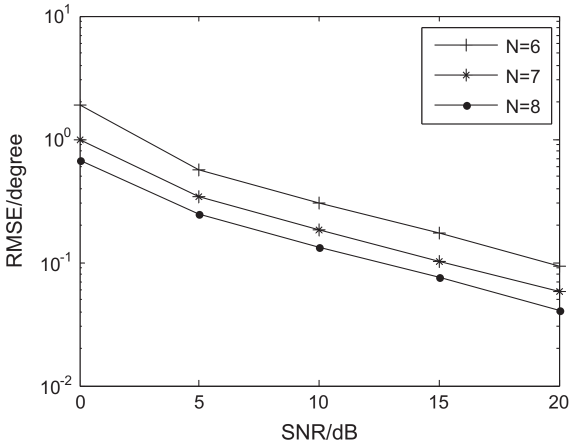

Fig. 7. DOA estimation performance of our algorithm with different N ($K = 3$).

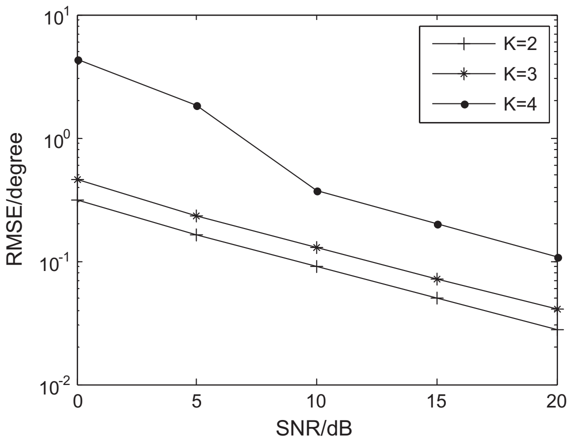

Fig. 8. DOA estimation performance of our algorithm with different K ($N = 8$).

Fig. 8 shows the DOA estimation performance of the proposed algorithm with different K ($N = 8$). We assume two coherent sources located at angles $\theta_1 = 10^\circ$ and $\theta_2 = 20^\circ$, three coherent sources located at angles $\theta_1 = 10^\circ$, $\theta_2 = 30^\circ$, and $\theta_3 = 50^\circ$, four coherent sources located at angles $\theta_1 = 10^\circ$, $\theta_2 = 20^\circ$, $\theta_3 = 30^\circ$, and $\theta_4 = 50^\circ$. It is clear shown that the DOA estimation performance of the proposed algorithm is gradually improving with the number of sources reducing because fewer sources bring lower interference of each other.

5. Conclusion

In this paper, we investigated the topic of coherent DOA estimation in monostatic MIMO radar, and a trilinear decomposition-based coherent DOA estimation algorithm using a single pulse was presented. A trilinear model was derived by reconstructing the received data of monostatic MIMO radar for a set of Toeplitz matrices, after that, trilinear decomposition performing via COMFAC algorithm was employed to get a direction matrix, and the uniqueness of the estimated direction matrix was proved through Theorem 1 in our paper. Finally, LS principle was used to attain the DOAs of sources from the normalized estimated direction matrix. Our approach utilized COMFAC algorithm instead of conventional TALS algorithm to speed up the LS fitting and improve the convergence performance of our method, which is displayed in Fig. 2. The proposed coherent DOA estimation algorithm is effective and robust for monostatic MIMO radar using a single pulse, and it is applicable to angle estimation for coherent or incoherent sources, which is clearly presented by Fig. 3 and Fig. 4. Moreover, our algorithm has better angle estimation performance than the FBSS-ESPRIT algorithm, the ESPRIT-like of Han [15], and the ESPRIT-like of Li et al. [16] for the case of a single pulse, which is shown in Fig. 5 and Fig. 6. Figs. 7 and 8 present the universal conditions which impact angle estimation performance in array signal processing, that is the number of antennas in transmit/receive array and the number of located sources. Our work is a further extension of the conventional trilinear decomposition work [10] for coherent angle estimation using a single pulse.

Acknowledgements

This work is supported by A Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD), China NSF Grants (61201208, 61271327, 61071164), Jiangsu Planned Projects for Postdoctoral Research Funds (1201039C), China Postdoctoral Science Foundation (2012M521099), Open project of key laboratory of underwater acoustic communication and marine information technology (Xiamen University), Hubei Key Laboratory of Intelligent Wire1ess Communications (IWC2012002), Open project of Key Laboratory of Nondestructive Testing (Nanchang Hangkong University), Open project of Key Laboratory of modern acoustic of Ministry of Education (Nanjing University), the Aeronautical Science Foundation of China (20120152001), by Qing Lan Project and the Fundamental Research Funds for the Central Universities (NZ2012010, kfjj120115, kfjj20110215).

References

[1] Li J, Stoica P. MIMO radar—diversity means superiority. In: Proc. 14th adaptive sensor array process. Workshop (ASAP'06). Mass, USA: Lincoln Lab; December 2006. p. 2–6.
[2] Haimovich AM, Blum RS, Cimini LJ. MIMO radar with widely separated antennas. IEEE Signal Process Mag 2008;25(1):116–29.
[3] Gao X, Zhang XF, Feng GP. On the MUSIC-derived approaches of angle estimation for bistatic MIMO radar. In: International conference on wireless networks and information systems; 2009. p. 343–6.
[4] Zahernia A, Dehghani MJ, Javidan R. MUSIC algorithm for DOA estimation using MIMO arrays. In: 6th International conference on telecommunication systems, services, and applications; 2011. p. 149–53.
[5] Jin M, Liao GS, Li J. Joint DOD and DOA estimation for bistatic MIMO radar. Signal Process 2009;89(2):244–51.
[6] Duofang C, Baixiao C, Guodong Q. Angle estimation using ESPRIT in MIMO radar. Electron Lett 2008;44(12):770–1.
[7] Chen JL, Gu H, Su WM. A new method for joint DOD and DOA estimation in bistatic MIMO radar. Signal Process 2010;90(2):714–8.
[8] Zheng ZD, Zhang JY. Fast method for multi-target localization in bistatic MIMO radar. Electron Lett 2011;47(2):138–9.
[9] Li A, Wang S. Propagator method for DOA estimation using fourth-order cumulate. In: 7th International conference on wireless communications, networking and mobile computing; 2011. p. 1–4.
[10] Zhang XF, Xu ZY, Xu LY. Trilinear decomposition-based transmit angle and receive angle estimation for MIMO radar. IET Radar Sonar Navig 2011;5(6):626–31.
[11] Shan TJ, Wax M, Kailath T. On spatial smoothing for direction-of-arrival estimation of coherent signals. IEEE Trans ASSP 1985;33(4):806–11.
[12] Pillai SU, Kwon BH. Forward/backward spatial smoothing techniques for the coherent signal identification. IEEE Trans ASSP 1989;37(1):8–15.
[13] Weixiu D, Kirlin RL. Improved spatial smoothing techniques for DOA estimation of coherent signals. IEEE Trans Signal Process 1991;39(5):1208–10.
[14] Wang BH, Wang YL, Chen H. Weighted spatial smoothing for direction-of-arrival estimation of coherent signals. In: Proceedings of IEEE antennas and propagation society international symposium, San Antonio, Texas. vol. 2; 2002. p. 668–71.
[15] Han FM, Zhang XD. An ESPRIT-like algorithm for coherent DOA estimation. Antenn Wireless Propag Lett 2005;4:443–6.
[16] Li CC, Liao GS, Zhu SQ, Wu SY. An ESPRIT-like algorithm for coherent DOA estimation based on data matrix decomposition in MIMO radar. Signal Process 2011;91:1803–11.
[17] Zhou Q, Chen BX, Yao SY. Performance analysis of monopulse angle measurement technology in different MIMO radar arrays. Fire Control Radar Technol 2010;39(3):1–6.
[18] Hu TL, Li XG. Performance analysis of sum and difference patterns of amplitude dual-plane monopulse radar. Modern Radar. 2006;28(8):11–21.
[19] Kruskal JB. Three-way arrays: rank and uniqueness of trilinear decompositions with application to arithmetic complexity and statistics. Linear Algebra Appl 1977;18:95–138.
[20] Vorobyov SA, Rong Y, Sidiropoulos ND, Gershman AB. Robust iterative, fitting of multilinear models. IEEE Trans Signal Process 2005;53(8):2678–89.
[21] Tomasi G, Bro R. A comparison of algorithms for fitting the PARAFAC model. Comput Stat Data Anal 2006;50:1700–34.
[22] Bro R, Sidiropoulos ND, Giannakis GB. A fast least squares algorithm for separating trilinear mixtures. In: Proc. inter. workshop independent component analysis and blind signal separation, Aussois, France; January 1999. p. 289–94.

[23] Sidiropoulos ND, Liu X. Identifiability results for blind beamforming in incoherent multipath with small delay spread. IEEE Trans Signal Process 2001;49(1):228–36.

Chen Chen is now a graduate student in Department of Electronic Engineering, Nanjing University of Aeronautics & Astronautics (NUAA). His research is focused on array signal processing and communication signal processing.

Xiaofei Zhang received M.S. degree in electrical engineering from Wuhan University, Wuhan, China, in 2001. He received Ph.D. degree in communication and information systems from Nanjing University of Aeronautics & Astronautics (NUAA) in 2005. Now, he is a Professor in Electronic Engineering Department, NUAA, Nanjing, China. His research is focused on array signal processing and communication signal processing.