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A sparse representation scheme for angle estimation in monostatic MIMO radar

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ABSTRACT

In this paper, the problem of direction of arrival (DOA) estimation for monostatic multiple-input multiple-output (MIMO) radar is addressed, and a sparse representation scheme for DOA estimation is proposed. Firstly, the reduced-dimensional transformation matrix and SVD-technique are utilized to reduce the computational complexity of the sparse signal reconstruction. Then the coefficients of the reduced-dimensional Capon (RD-Capon) spectrum are exploited to design a weight matrix for reweighting l_1 norm constraint minimization to enhance the sparsity of the solution, and the DOA can be estimated by finding the non-zero rows in the recovered matrix. The angle estimation performance of the proposed method is better than RD-ESPRIT and RD-Capon algorithms. Furthermore, the proposed method works well for coherence targets without any decorrelation procedure, and has low sensitivity to the priori information of the target number. Simulation results verify the effectiveness of the proposed method.

1. Introduction

Multiple-input multiple-output (MIMO) radar has drawn considerable attention and becomes a hot research topic in the field of radar, and it has been verified that MIMO radar has a lot of potential advantages over conventional phased-array radar [1–3]. Based on the configuration of the transmit array and receive array, MIMO radar can be divided into two classes. One is named as statistical MIMO radar, in which the transmit antennas and receive antennas are widely spaced to obtain coherent processing gain for solving target scintillation problem [2]. The other is named as colocated MIMO radar [3], including bistatic MIMO radar and monostatic MIMO radar, whose transmit antennas and receive antennas are closed spaced. The colocated MIMO radar can obtain virtual aperture which is larger than real aperture, so it can achieve unambiguous angle estimation. In bistatic MIMO radar, the transmit array and receive array are separated away from each other but they are close to each other in the monostatic counterpart. Thus, the direction of departure (DOD) and direction of arrival (DOA) are different in bistatic MIMO radar but they are same in monostatic MIMO radar. In this paper, we focus on the angle estimation in monostatic MIMO radar.

Angle estimation for MIMO radar has attracted more and more attention over the past few years [4–6]. A number of angle estimation algorithms have been developed for MIMO radar, which contain 2-D Capon algorithm [7], 2-D multiple signal classification (MUSIC) [8], estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm [9] and parallel factor analysis algorithm [10]. In [11,12], RD-ESPRIT and RD-Capon are proposed for DOA estimation in monostatic MIMO radar by exploiting the especial configuration of the virtual array, and the angle estimation performance is improved. In [13], the transmit beamspace energy focusing

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technique for DOA estimation is presented, and the SNR gain at each receive antenna is maximized. Thus, the angle estimation performance is improved obviously. However, most of the method mentioned above are based on subspace technique, which rely heavily on accurate covariance matrix estimation and the priori information of the target number, and fail to work with coherent targets.

Recently, sparse representation has drawn much interest in the areas of statistical signal analysis and parameter estimation. In [14], l_1-SVD (singular value decomposition) algorithm uses l_1-norm penalty to enforce sparsity and SVD technique to reduce the computational complexity, which contributes much to the development of DOA estimation method via sparse recovery. Some recently proposed methods include sparse representation of covariance matrix [15] and covariance matrix vector [16], in which the DOA is estimated in the correlation domain instead of the data domain. The methods mentioned above are based on l_1 norm constrained minimization, which has a drawback that larger coefficients are penalized more heavily than smaller coefficients. In [17], an iterative algorithm for reweighting l_1 norm minimization is proposed for single measurement vector (SMV) problem, in which large weights are used to punish those entries who are more likely to be zero in recovered signal, whereas small weights are used to reserve larger entries. However, the DOA estimation problem in MIMO radar is usually encountered with multiple measurement vector (MMV) problem, and the iterative algorithm is not suitable any more. On the other hand, the two-dimensional dictionary matrix is needed for recovering the sparse matrix in MIMO radar, which leads very high computational complexity and perhaps fail to recover the sparse matrix.

In this paper, we propose a sparse representation scheme for angle estimation in monostatic MIMO radar. Firstly, the reduced-dimensional transformation matrix and SVD-technique are utilized to deduce the dimensional of the received signal (i.e. the dimensional of the dictionary matrix and recovered matrix). Then exploiting the relationship between RD-Capon algorithm and dictionary matrix, a weight matrix is formulated for the MMV problem in monostatic MIMO radar, which reweighs l_1 norm constraint minimization to enhance the sparsity of the solution. The DOA can be estimated by finding the non-zero rows in the recovered matrix. The proposed method provides better angle estimation performance than RD-ESPRIT and RD-Capon algorithms. Furthermore, the proposed method is effective with coherence targets and can be performed well without the correct determination of target number.

Notation: $(\cdot)^{\mathrm{H}}$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{-1}$, $(\cdot)^{*}$ and $(\cdot)^{+}$ denote conjugate-transpose, transpose, inverse, conjugate, pseudo-inverse, respectively. $\otimes$ denotes the Kronecker product operator. $\mathbf{I}_K$ denotes a $K \times K$ dimensional unit matrix. $\mathbf{A}^{(l_2)}$ denotes a column vector whose qth element equal to the l_2 norm of the qth row of $\mathbf{A}$. $\|\cdot\|_1$ and $\|\cdot\|_F$ denote the l_1 norm and Frobenius norm, respectively.

2. Data model

Consider a narrowband monostatic MIMO radar system equipped with M transmit antennas and N receive antennas, and both of which are half-wavelength spaced uniform linear arrays (ULA). The transmit and receive arrays are adopted to be closely located in monostatic MIMO radar so that a target located in the far-field can be seen by both of them at the same angle (i.e. direction of arrival (DOA)). At transmit array, M transmit antennas are used to transmit M different narrowband waveforms which have identical bandwidth and centre frequency but are orthogonal. Let P denote the number of uncorrelated targets. θ_p $(p = 1, 2, \ldots, P)$ denotes the DOA of the pth target with respect to the normals of transmit and receive arrays. According to [11,12], the output of the matched filter at the receive array can be expressed as

$$\mathbf{x}(t) = [\mathbf{a}_t(\theta_1) \otimes \mathbf{a}_r(\theta_1), \ldots, \mathbf{a}_t(\theta_P) \otimes \mathbf{a}_r(\theta_P)]\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_P(t)]^{\mathrm{T}} \in \mathbb{C}^{P\times 1}$, and $s_p(t) = \beta_p(t) e^{j2\pi f_p t}$ with $\beta_p(t)$ and f_p being the reflection coefficient and Doppler frequency, respectively. $\mathbf{a}_t(\theta_p) = [1, \exp(j\pi \sin\theta_p), \ldots, \exp(j\pi(M-1)\sin\theta_p)]^{\mathrm{T}}$ is the transmit steering vector and $\mathbf{a}_r(\theta_p) = [1, \exp(j\pi \sin\theta_p), \ldots, \exp(j\pi(N-1)\sin\theta_p)]^{\mathrm{T}}$ is the receive steering vector. $\mathbf{n}(t) \in \mathbb{C}^{MN\times 1}$ is a Gaussian white noise vector with zero-mean and covariance matrix $\sigma^2\mathbf{I}_{MN}$. Defining $\mathbf{A} = [\mathbf{a}_t(\theta_1) \otimes \mathbf{a}_r(\theta_1), \ldots, \mathbf{a}_t(\theta_P) \otimes \mathbf{a}_r(\theta_P)]$, then the received signal in Eq. (1) is rewritten as $\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$. In the presence of multiple snapshots, by collecting J snapshots, the received data becomes

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N}, \tag{2}$$

where $\mathbf{X} = [\mathbf{x}(t_1), \ldots, \mathbf{x}(t_J)]$, $\mathbf{S} = [\mathbf{s}(t_1), \ldots, \mathbf{s}(t_J)]$ and $\mathbf{N} = [\mathbf{n}(t_1), \ldots, \mathbf{n}(t_J)]$ is the Gaussian white noise matrix.

3. Sparse representation scheme for DOA estimation

3.1. l_1-SVD algorithm for DOA estimation in monostatic MIMO radar

In this section, we introduce the l_1-SVD algorithm [14] to DOA estimation in monostatic MIMO radar. By collecting J snapshots, the singular value decomposition (SVD) of $\mathbf{X}$ can be written in the form as

$$\mathbf{X} = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{V}_s^{\mathbf{H}} + \mathbf{U}_n\mathbf{\Lambda}_n\mathbf{V}_n^{\mathbf{H}} \tag{3}$$

where $\mathbf{U}_s \in \mathbb{C}^{MN\times P}$ and $\mathbf{V}_s \in \mathbb{C}^{J\times P}$ are composed with the singular vectors corresponding to the P largest singular values. $\mathbf{U}_n \in \mathbb{C}^{MN\times(MN-P)}$ and $\mathbf{V}_n \in \mathbb{C}^{J\times(MN-P)}$ are composed with the singular vectors corresponding to the residual *MN-P* singular values. $\mathbf{\Lambda}_s$ and $\mathbf{\Lambda}_n$ are diagonal matrices whose diagonal elements are the P largest singular values and the residual *MN-P* singular values, respectively.

Multiplying the received signal $\mathbf{X}$ by $\mathbf{V}_s$, we have

$$\mathbf{X}_{\mathbf{SV}} = \mathbf{A}\mathbf{S}_{SV} + \mathbf{N}_{SV} \tag{4}$$

where $\mathbf{S}_{SV} = \mathbf{S}\mathbf{V}_s$ and $\mathbf{N}_{SV} = \mathbf{N}\mathbf{V}_s$. In order to use the sparse representation scheme to estimate DOA, let $\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_L$ be the discretized sampling grid of all DOAs of interest. The number of the potential DOAs will be much greater than the number of targets, i.e. $L \gg P$. Then a two-dimensional transmit–receive dictionary is constructed for the potential DOAs, which can be written as $\mathbf{A}_{\hat{\theta}} = \mathbf{A}_{\mathbf{t}}^{\hat{\mathbf{\theta}}} \otimes \mathbf{A}_{\mathbf{r}}^{\hat{\mathbf{\theta}}}$, where $\mathbf{A}_{\mathbf{t}}^{\hat{\mathbf{\theta}}} = [\mathbf{a}_t(\hat{\theta}_1) \cdots, \mathbf{a}_t(\hat{\theta}_L)]$ and $\mathbf{A}_{\mathbf{r}}^{\hat{\mathbf{\theta}}} = [\mathbf{a}_r(\hat{\theta}_1) \cdots, \mathbf{a}_r(\hat{\theta}_L)]$. If the true DOAs are on (or close to) the discretized sampling grid, under the sparse representation framework, Eq. (4) can be represented with the two-dimensional transmit–receive dictionary $\mathbf{A}_{\hat{\theta}}$ as

$$\mathbf{X_{SV}} = \mathbf{A}_{\hat{\theta}}\mathbf{S}_{\hat{\theta}} + \mathbf{N}_{SV} \tag{5}$$

where $\mathbf{S}_{\hat{\theta}} \in \mathbb{C}^{L^2 \times P}$ is a matrix whose lth row corresponding to a potential DOA $\hat{\theta}_l$. It is easy to verify that $\mathbf{S}_{\hat{\theta}}$ and $\mathbf{S}_{SV}$ have the same row support. Since the sample size $L \gg P$, the matrix $\mathbf{S}_{\hat{\theta}}$ has a few nonzero rows, i.e., the spatial spectrum of $\mathbf{S}_{\hat{\theta}}$ is sparse. According the principle of l_1-SVD algorithm [14], the DOA estimation problem can be turned into the following constrained minimization problem:

$$\min \|\hat{\mathbf{S}}_{\hat{\theta}}^{(l_2)}\|_1 \quad \text{s.t.} \|\mathbf{X_{SV}} - \mathbf{A}_{\hat{\theta}}\mathbf{S}_{\hat{\theta}}\|_F \le \eta \tag{6}$$

where η is the regularization parameter. Note that the way to choose η has been described in [14]. The upper value of $\|\mathbf{N}_{SV}\|_F$ with a 99% confidence interval is used as a choice for η. After obtaining the matrix $\mathbf{S}_{\hat{\theta}}$, the DOA can be estimated by exploiting the position of nonzero rows of $\mathbf{S}_{\hat{\theta}}$.

Remark 1. The l_1-SVD algorithm in Eq. (6) exploits two-dimensional transmit–receive dictionary matrix, which is very large. For example, if there are 50 potential DOAs and $M=N=10$, the transmit–receive dictionary will have 100 rows and 2500 columns. It leads very high computational complexity and perhaps fail to recover the matrix $\mathbf{S}_{\hat{\theta}}$. In the next section, a reduced-dimensional transformation is used to reduce the computational complexity, which only needs one-dimensional dictionary.

Remark 2. According to Eq. (6), the l_1 norm constrained minimization problem is encountered with multiple measurement vector (MMV) problem, and the iterative algorithm in [17] is not suitable to this issue. In the next section, a reweighted l_1 norm minimization algorithm is developed for MMV problem by using the relationship between the dictionary matrix and Capon algorithm.

3.2. A reweighted l_1-SVD scheme for DOA estimation

(1) *Reduced-dimensional transformation for received data*: According to the structure of the transmit–receive steering vector $\mathbf{a}_t(\theta) \otimes \mathbf{a}_r(\theta)$, we have

$$\mathbf{a}_t(\theta) \otimes \mathbf{a}_r(\theta) = [\underbrace{1, z, \ldots, z^{N-1}}_{N}, \underbrace{z, z^2, \ldots, z^N}_{N}, \ldots, \underbrace{z^{M-1}, z^M, \ldots, z^{N+M-2}}_{N}]^{\mathrm{T}} \tag{7}$$

where $z = \exp(j\pi \sin \theta)$. According to Eq. (7), it is indicated that the transmit–receive steering vector only has $M+N-1$ distinct elements. Thus, the transmit–receive steering vector can also be expressed as

$$\mathbf{a}_t(\theta) \otimes \mathbf{a}_r(\theta) = \mathbf{G}\mathbf{b}(\theta) \tag{8}$$

where $\mathbf{G}$ and $\mathbf{b}(\theta)$ are the reduced-dimensional transformation matrix and the steering vector, which can be written as

$$\mathbf{G} = \begin{bmatrix} \mathbf{J}_0 \\ \mathbf{J}_1 \\ \vdots \\ \mathbf{J}_{M-1} \end{bmatrix} \in \mathbb{C}^{MN \times (M+N-1)} \tag{9}$$

$$\mathbf{b}(\theta) = [1, \exp(j\pi \sin \theta_p), \ldots, \exp(j\pi(M+N-2) \sin \theta_p)]^{\mathrm{T}}$$

where

$$\mathbf{J}_m = [\mathbf{0}_{N\times m}, \mathbf{I}_N, \mathbf{0}_{N\times(M-m-1)}], \quad m = 0, 1, \ldots, M-1 \tag{10}$$

According to Eq. (9), we define a matrix as $\mathbf{F} = \mathbf{G}^{\mathrm{H}}\mathbf{G}$, which is shown as

$$\mathbf{F} = \mathrm{diag}[1, 2, \ldots, \underbrace{\min(M,N), \ldots, \min(M,N)}_{|M-N|+1}, \ldots, 2, 1] \tag{11}$$

In order to avoid the colored noise, the reduced-dimensional transformation matrix is defined as $\mathbf{W} = \mathbf{F}^{-(1/2)}\mathbf{G^H}$, which is satisfied with $\mathbf{WW^H} = \mathbf{I}_{M+N-1}$. Then using the reduced-dimensional transformation matrix $\mathbf{W}$ for the received data $\mathbf{X}$, we have

$$\begin{aligned}\mathbf{Y} = \mathbf{WX} &= \mathbf{F}^{-(1/2)}\mathbf{FBS} + \mathbf{WN} \\ &= \mathbf{F}^{(1/2)}\mathbf{BS} + \mathbf{WN}\end{aligned} \tag{12}$$

where $\mathbf{B} = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \ldots, \mathbf{b}(\theta_P)]$. According to Eq. (12), it is indicated that the reduced-dimensional received data corresponds to a virtue array with weight matrix $\mathbf{F}^{(1/2)}$.

(2) *A reweighted l_1-SVD scheme for DOA estimation*: After reduced-dimensional transformation, the steering matrix for $\mathbf{Y}$ is $\mathbf{F}^{(1/2)}\mathbf{B}$. Then the one-dimensional dictionary matrix is constructed for the potential DOAs, which can be written as $\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}} = \mathbf{F}^{(1/2)}[\mathbf{b}(\hat{\theta}_1), \mathbf{b}(\hat{\theta}_2), \ldots, \mathbf{b}(\hat{\theta}_L)]$. Then let the SVD of $\mathbf{Y}$ be

$$\mathbf{Y} = \hat{\mathbf{U}}_s\hat{\mathbf{\Lambda}}_s\hat{\mathbf{V}}_s^{\mathbf{H}} + \hat{\mathbf{U}}_n\hat{\mathbf{\Lambda}}_n\hat{\mathbf{V}}_n^{\mathbf{H}} \tag{13}$$

where $\hat{\mathbf{U}}_s \in \mathbb{C}^{(M+N-1)\times P}$, $\hat{\mathbf{\Lambda}}_s \in \mathbb{C}^{P\times P}$, $\hat{\mathbf{V}}_s \in \mathbb{C}^{J\times P}$, $\hat{\mathbf{U}}_n \in \mathbb{C}^{(M+N-1)\times(M+N-1-P)}$, $\hat{\mathbf{\Lambda}}_n \in \mathbb{C}^{(M+N-1-P)\times(M+N-1-P)}$, $\hat{\mathbf{V}}_s \in \mathbb{C}^{J\times(M+N-1-P)}$. Similar to Eq. (5), a new sparse representation framework for the reduced-dimensional data can be constructed, which is shown as

$$\mathbf{Y_{SV}} = \mathbf{Y}\hat{\mathbf{V}}_s = \mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}}\hat{\mathbf{S}}_{\hat{\theta}} + \mathbf{W}\hat{\mathbf{N}}_{SV} \tag{14}$$

where $\hat{\mathbf{S}}_{\hat{\theta}} = \mathbf{S}\hat{\mathbf{V}}_s$ and $\hat{\mathbf{N}}_{SV} = \mathbf{N}\hat{\mathbf{V}}_s$. Similar to $\mathbf{S}_{\hat{\theta}}$, $\hat{\mathbf{S}}_{\hat{\theta}}$ only has a few nonzero rows corresponds to DOAs of the possible targets. Thus, the spatial spectrum of $\hat{\mathbf{S}}_{\hat{\theta}}$ is also sparse. In order to obtain the matrix $\hat{\mathbf{S}}_{\hat{\theta}}$, a constrained minimization problem is considered as follows:

$$\min \|\hat{\mathbf{S}}_{\hat{\theta}}^{(l_2)}\|_1 \quad \text{s.t.} \|\mathbf{Y_{SV}} - \mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}}\hat{\mathbf{S}}_{\hat{\theta}}\|_F \le \hat{\eta} \tag{15}$$

where $\hat{\eta}$ is the regularization parameter. After obtaining the recovered matrix $\hat{\mathbf{S}}_{\hat{\theta}}^{(l_2)}$, the DOAs can be estimated by finding the nonzero elements of $\hat{\mathbf{S}}_{\hat{\theta}}^{(l_2)}$. However, it has been pointed out that larger coefficients are penalized more heavily than smaller coefficients in l_1 norm constrained minimization problem. Thus, the exact recovery in signal recover processing cannot be obtained. Then an iterative reweighted formulation of l_1 norm constrained minimization is proposed to enhance the sparse solution. But it is only suitable for SMV problem. Now, a reweighted formulation of l_1 norm constrained minimization for MMV problem is proposed by using the relationship between the dictionary matrix $\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}}$ and the RD-Capon spatial spectrum. The dictionary matrix $\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}}$ can be divided into two parts, which is expressed as $\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}} = \mathbf{F}^{(1/2)}[\mathbf{B}_{\hat{\theta}}^{(1)}, \mathbf{B}_{\hat{\theta}}^{(2)}]$, where $\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}}^{(1)}$ is composed with the steering vector $\mathbf{F}^{(1/2)}\mathbf{b}(\theta_p)$ $(p=1,2,\ldots,P)$ corresponding to the possible targets and $\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}}^{(2)}$ is composed with the residual steering vectors of the dictionary matrix $\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}}$. Then we have the following equation:

$$\begin{aligned}(\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}})^{\mathrm{H}}\mathbf{R}_Y^{-1}\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}} &= [((\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}}^{1})^{\mathrm{H}}\mathbf{R}_Y^{-1}\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}})^{\mathrm{T}},\\ &\quad ((\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}}^{2})^{\mathrm{H}}\mathbf{R}_Y^{-1}\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}})^{\mathrm{T}}]\\ &= [\mathbf{W}_{(1)}^{\mathrm{T}},\mathbf{W}_{(2)}^{\mathrm{T}}] \end{aligned} \tag{16}$$

where $\mathbf{R}_Y=(1/J)\mathbf{Y}\mathbf{Y}^{\mathrm{H}}$ is the covariance matrix of the received data $\mathbf{Y}$. Let $\mathbf{W}_{(1)}^{(l_2)}$ and $\mathbf{W}_{(2)}^{(l_2)}$ be the column vectors which denote l_2 norm of each row of $\mathbf{W}_{(1)}$ and $\mathbf{W}_{(2)}$, respectively. Then we formulate the weighted vector as follows:

$$\hat{\mathbf{W}}=[(\mathbf{W}_{(1)}^{(l_2)})^{\mathrm{T}},(\mathbf{W}_{(2)}^{(l_2)})^{\mathrm{T}}]/\max(\mathbf{W}_{(2)}^{(l_2)}) \tag{17}$$

when $J\to\infty$, based on the ideal of the Capon algorithm, it is indicated that $\mathbf{W}_{(1),i}^{(l_2)}/\max(\mathbf{W}_{(2)}^{(l_2)})\to 0$ and $0<\mathbf{W}_{(2,i)}^{(l_2)}/\max(\mathbf{W}_{(2)}^{(l_2)})\le 1$, where $\mathbf{W}_{(1,i)}^{(l_2)}$ and $\mathbf{W}_{(2,i)}^{(l_2)}$ are the ith entry of $\mathbf{W}_{(1)}^{(l_2)}$ and $\mathbf{W}_{(2)}^{(l_2)}$, respectively. Then the entries of $\mathbf{W}_{(1)}^{(l_2)}/\max(\mathbf{W}_{(2)}^{(l_2)})$ are much smaller than those of $\mathbf{W}_{(2)}^{(l_2)}/\max(\mathbf{W}_{(2)}^{(l_2)})$. We define the weighted matrix as follows:

$$\mathbf{D}=\mathrm{diag}(\hat{\mathbf{W}}) \tag{18}$$

In MMV problem, this weighted matrix $\mathbf{D}$ is used to achieve the idea that large weights are used to punish those entries who are more likely to be zero in recovered signal, whereas small weights are used to reserve larger entries, which is consistent with the methodology of the iterative reweighted l_1 norm minimization. Finally, we can formulate the reweighted l_1 norm constrained minimization problem as follows:

$$\min\|\mathbf{D}\hat{\mathbf{S}}_{\hat{\theta}}^{(l_2)}\|_1 \quad \text{s.t.}\ \|\mathbf{Y}_{\mathbf{SV}}-\mathbf{F}^{(1/2)}\mathbf{B}_{\hat{\theta}}\hat{\mathbf{S}}_{\hat{\theta}}\|_F\le\hat{\eta} \tag{19}$$

Eq. (19) can be calculated by SOC (second order cone) programming software packages such as SeDuMi [18] and CVX [19]. The DOA estimates are then obtained by plotting $\hat{\mathbf{S}}_{\hat{\theta}}^{(l_2)}$, solved from (19).

Remark 3. In the l_1 norm constrained minimization problem, the choice of regularization parameter plays an important role, which balances the fit of the solution to the data versus the sparsity prior. In Eq. (12), due to the fact that $\mathbf{W}$ is an orthogonal matrix, $\mathbf{W}\mathbf{N}$ is also complex Gaussian distribution with covariance matrix $\sigma^2\mathbf{I}_{M+N-1}$. Then $\mathbf{W}\hat{\mathbf{N}}_{SV}=\mathbf{W}\mathbf{N}\hat{\mathbf{V}}_s$ is satisfied with approximate complex Gaussian distribution because $\hat{\mathbf{V}}_s$ is only a function of $\mathbf{W}\mathbf{N}$ [see Eqs. (13) and (14)]. Thus, $\|\mathbf{W}\hat{\mathbf{N}}_{SV}\|_F^2$ has approximately a χ^2 distribution with $(M+N-1)P$ degrees of freedom upon normalization by variance σ_v^2. Then the regularization parameter $\hat{\eta}$ of the proposed method is chosen by the upper value of $\|\mathbf{W}\hat{\mathbf{N}}_{SV}\|_F$ with a 99% confidence interval [14].

Remark 4. In the proposed method, the weighted matrix $\mathbf{D}$ is formulated by using the methodology of Capon algorithm, which does not rely on the prior information of the target number. Thus, the proposed method is not very sensitivity to the prior information of the target number, which is consistent with the l_1-SVD algorithm. On the other hand, the main computational complexity of the proposed method is in formulating the weighted matrix $\mathbf{D}$ and solving Eq. (19). The formulation of the weighted matrix $\mathbf{D}$ requires $O\{(M+N-1)^2J+(M+N-1)^3+L[(M+N)(M+N-1-P)]\}$, and solving Eq. (19) through SOC programming requires $O\{P^3L^3\}$. The computational complexity of the RD-Capon and RD-ESPRIT is $O\{(M+N-1)^2J+(M+N-1)^3+L[(M+N)(M+N-1-P)]\}$ and $(M+N-1)^2J+(M+N-1)^3$, respectively. Although the proposed method exploits the reduced-dimensional transformation and SVD-technique to reduce the complexity of sparsity signal reconstruction, the computational cost of the proposed method is higher than RD-ESPRIT and RD-Capon algorithms. However, compared with RD-ESPRIT and RD-Capon algorithms, the advantages of the proposed method outweigh the cost of additional computation: the proposed method provides better angle estimation performance, and can be performed well without the correct determination of the target number, and can handle coherent targets without any decorrelation preprocessing.

4. Simulation results

In this section, we present some numerical simulations to demonstrate the effectiveness and advantages of the proposed method algorithm. The RD-ESPRIT [11], RD-Capon algorithm [12] and CRB [13] are used to compare with the proposed method. In this paper, the root mean square error (RMSE) is used for DOA estimation performance evaluation, which is calculated by the formula

$$\mathrm{RMSE}=(1/P)\sum_{p=1}^{P}\sqrt{(1/Q)\sum_{i=1}^{Q}(\hat{\theta}_{p,i}-\theta_p)^2} \tag{20}$$

where $\hat{\theta}_{p,i}$ is the estimation of DOA θ_p for the ith Monte Carlo trial, Q is the number of the Monte Carlo trials. In most cases, three uncorrelated targets are assumed to be with angles as $\theta_1=5.6^\circ$, $\theta_2=12.6^\circ$ and $\theta_3=30.6^\circ$. The input signal-to-noise ratio (SNR) is defined as $\mathrm{SNR}=10\log_{10}(\|\mathbf{AS}\|_F^2/\|\mathbf{N}\|_F^2)$. The number of the Monte Carlo trials $Q=200$ is used, and the direction grid is uniform with 0.01° sampling from -90° to 90° for the proposed method and RD-Capon method.

Fig. 1 shows the spatial spectra of the proposed method and RD-Capon for three uncorrelated targets, where $M=N=8$, $J=300$ and SNR$=0$ dB are used. From Fig. 1, it can be seen that the proposed method provides more sharp peaks than RD-Capon, which means that the proposed method has higher resolution than RD-Capon, though the weight matrix for the proposed method is based on RD-Capon algorithm.

Fig. 2 shows that the spatial spectra of the proposed method and RD-Capon for three coherent targets, where $M=N=8$, $J=300$, SNR$=0$ dB and three targets are coherent. In order to compare the decorrelation ability of the proposed method and RD-Capon, the forward/backward spatial smoothing [20] technique is also added to the RD-Capon (denoted as RD-Capon (FBSS)), using 6-antennas

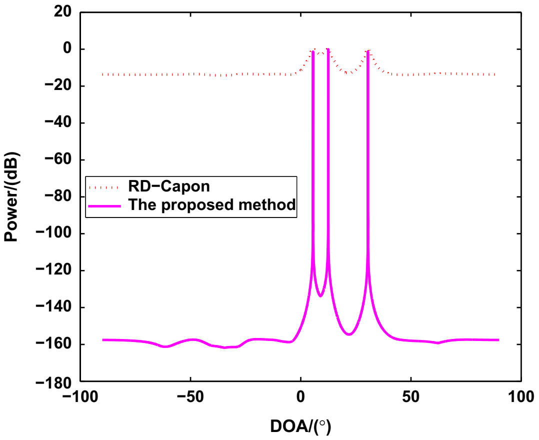

Fig. 1. Spatial spectra of the proposed method and RD-Capon for three uncorrelated targets.

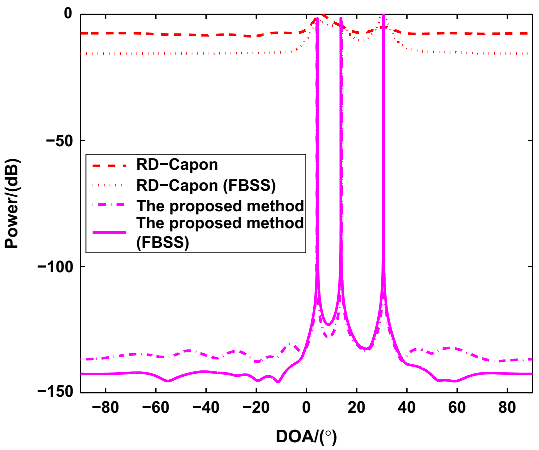

Fig. 2. Spatial spectra of the proposed method and RD-Capon for three coherent targets.

smoothing subarray for both transmit array and receive array. Then the RD-Capon (FBSS) algorithm is applied to construct the weight matrix for the proposed method (denoted as the proposed method (FBSS)). It can be seen from Fig. 2 that the proposed method works well for coherent targets without any decorrelation procedure while RD-Capon algorithm fails to work. On the other hand, after using forward/backward spatial smoothing technique, the proposed method provides more sharp peaks than RD-Capon (FBSS), and the performance of the proposed method can be improved.

Fig. 3 shows the sensitivity of the proposed method to the priori information of the target number, where $M=N=8$, $J=300$ and SNR$=0$ dB are used. As seen in Fig. 3, the incorrect determination of the target number in the proposed method has no catastrophic consequences, i.e., the proposed method has low sensitivity to the priori information of the target number. The main reason is that the weighted matrix does not rely on the priori information of the target number. Furthermore, the weighted matrix is used for enhancing sparsity and achieving more accurate DOA estimation.

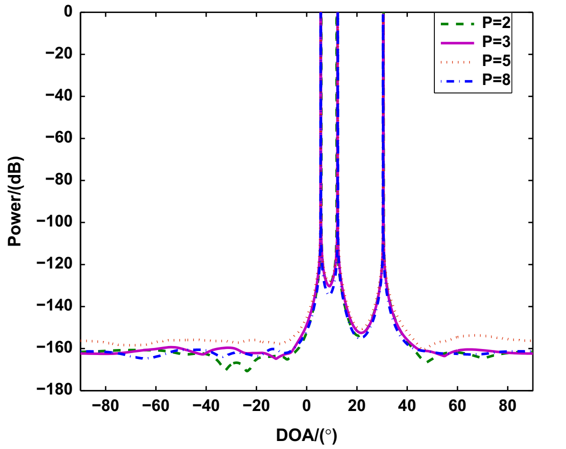

Fig. 3. Sensitivity of the proposed method to the priori information of the target number.

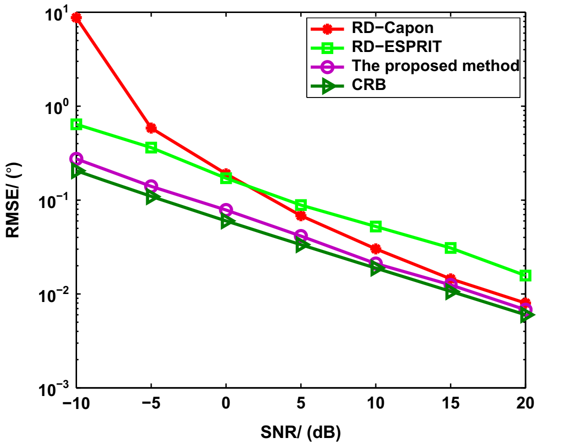

Fig. 4. RMSE versus SNR with three uncorrelated targets.

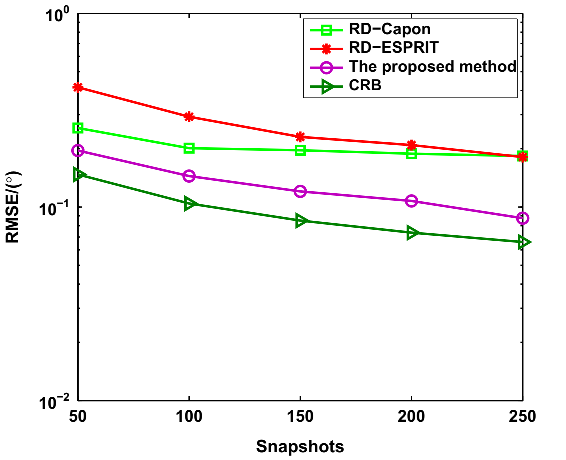

Fig. 5. RMSE versus snapshots with three uncorrelated targets.

Fig. 4 depicts the RMSE of the DOA estimation versus SNR with different methods, where $M=N=8$ and $J=300$ are used. As can be seen from Fig. 4, the proposed method

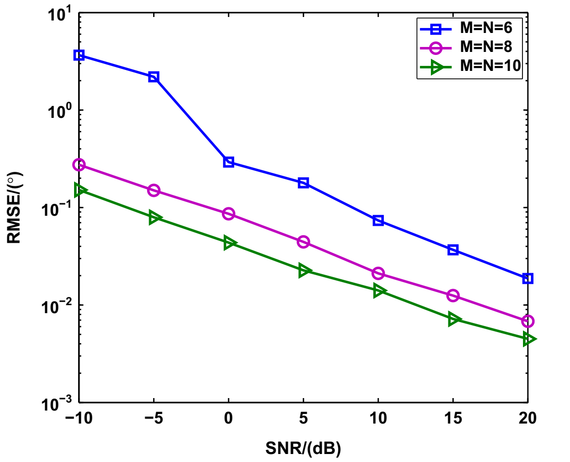

Fig. 6. RMSE versus different transmit and receive antennas with three uncorrelated targets.

has a lower RMSE than that of RD-ESPRIT and RD-Capon, especially in low SNR region. Furthermore, the RMSE of the proposed method is very close to CRB. Owing to use the reduced-dimensional transformation technique and the weighted matrix in the l_1 norm constraint minimization, the sparsity of the solution is effective and enhanced. Thus, the proposed method provides superior angle estimation performance.

Fig. 5 shows the RMSE of the DOA estimation versus snapshots with different methods, where $M=N=8$ and SNR=0 dB are used. From Fig. 5, it can be observed that the proposed method has better angle estimation performance than RD-ESPRIT and RD-Capon in all the snapshots region. On the other hand, the angle estimation performance of the proposed method is closed to CRB and gradually improved with J increasing.

Fig. 6 illustrates the angle estimation performance of the proposed method with different transmit and receive antennas, respectively, where SNR=0 dB and $J=300$. From Fig. 6, it is clearly shown that the angle estimation performance of the proposed method is improved with the number of antennas increasing. Multiple antennas improve the angle estimation performance because of the diversity gain.

5. Conclusion

In this paper, we have proposed a sparse representation scheme for DOA estimation in monostatic MIMO radar. The proposed method firstly exploits the reduced-dimensional transformation and SVD-technique to reduce the complexity of sparsity signal reconstruction, and then a novel weighted matrix is formulated to enhance the sparsity of the solution in MMV problem. Finally, the DOA can be estimated by solving the reweighted l_1 norm constrained minimization problem. The proposed method works well for coherent targets without any decorrelation procedure, and has low sensitivity to the assumed number of targets, and also provides better angle estimation performance than RD-ESPRIT and RD-Capon algorithms.

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