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Nonuniform linear array DOA estimation using EM criterion ☆

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ABSTRACT

In this letter, we address the problem of Direction of Arrival (DOA) estimation with nonuniform linear array in the context of sparse Bayesian learning (SBL) framework. The nonuniform array output is deemed as an incomplete-data observation, and a hypothetical uniform linear array output is treated as an unavailable complete-data observation. Then the Expectation-Maximization (EM) criterion is directly utilized to iteratively maximize the expected value of the complete-data log likelihood under the posterior distribution of the latent variable. The novelties of the proposed method lie in its capability of interpolating the actual received data to a virtual uniform linear array, therefore extending the achievable array aperture. Simulation results manifests the superiority of the proposed method over off-the-shelf algorithms, specially on circumstances such as low SNR, insufficient snapshots, and spatially adjacent sources.

1. Introduction

The estimation of signal Direction of Arrival (DOA) from observed snapshots of a sensor array is a fundamental research topic in many fields [1,2], such as sonar, radar, wireless communications, radio seismology, etc. Most existing methodologies for the DOA estimation are devised for Uniform Linear Arrays (ULA). However, the problem of estimating DOA for Nonuniform Linear Arrays (NLA) is also of tremendous interest in various applications [3–5]. For example, when operating long uniform arrays, often some of the sensors do not function and their outputs must be ignored, leading to missing-data problems [6]. Recently, T.E. Tuncer et al. [7] have studied the estimation of DOA in the NLA case. This method can convert an NLA manifold to a ULA form, in order to fill the missing values of the NLA received data. In this respect, the interpolation is sector-dependent and requires previously determining the unknown source and noise variances. It turns out that the mapping error of the array interpolation matrix due to the violation of the aforementioned restriction is large compared to the errors due to finite length signals and noise which dominate the DOA estimation performance. Therefore high-resolution DOA estimates are hardly available by implementing the array interpolation technique proposed by T.E. Tuncer et al. In this paper, we study the DOA estimation method that can process multiple correlated signals for an NLA (i.e., a ULA where some sensors are omitted).

State-of-the-art DOA estimation techniques can be accomplished based on two main streams, that is, subspace-based estimators and sparsity-inducing estimators. Among the former category, representative example is MUSIC method [8,9]. However, this method can no longer be directly applied in the case of correlated sources, due to the rank deficiency of the data covariance matrix. To circumvent this issue, spatial smoothing technique [10] is reasonably implemented to restore the rank of the corresponding noise-free covariance matrix. Nevertheless, when applying this spatial smoothing-based MUSIC (SS-MUSIC) method to an NLA, only a part of the physical array (i.e., a subarray which has a contiguous number of elements without any holes) can be utilized, implying that this technique cannot achieve the entire available degrees-of-freedom (DOF) or array aperture for DOA estimation. As an alternative technique for direction finding of coherent sources, sparse signal reconstruction theory [11] attracts continued research interest in recent years. Methods falling into this framework include L1-SVD [12], L1-SRACV [13], SPICE [14], and so forth. Previous literature has verified the superiority of sparsity-inducing algorithms in super resolution and robustness, especially in much demanding scenarios with spatially adjacent signals, limited snapshots and low SNR. Unfortunately, those methods are primarily designed to work in the raw data domain, and for NLA (as we mentioned above, a ULA with some of the sensors deteriorated), reconstructing the missing-data is not under consideration.

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As it becomes clear from the above discussion, the bottlenecks faced by the aforementioned approaches motivate us to explore another new sparse DOA estimation algorithm for the NLA correlated source case. Concretely, this paper aims at developing an efficient sparsity-aware direction finding algorithm for NLA in the context of Sparse Bayesian Learning (SBL) framework [15], mainly due to the fact that the SBL technique surpasses its subspace-based and sparsity-driven counterparts in adaption to much rigorous scenarios [16]. In this line of reasoning, the principle of SBL is followed and the missing-data is iteratively interpolated based on the Expectation-Maximization (EM) criterion [17]. Specifically, each iteration of the proposed method is composed of two steps: in the E-step, the posterior distribution of the complete-data (i.e., the data received by a hypothetical ULA), given the incomplete-data (i.e., the data received by the actual NLA) and the previous estimate of DOA, is calculated; in the subsequent M-step, the expected value of the complete-data log likelihood, under the aforementioned posterior distribution, is maximized. The developed iterative approach will enable us to improve the interpolation using the previously estimated DOA. Analytical and numerical results show the superiority of our proposed high-resolution method over existing methods in DOA estimation precision.

2. Proposed DOA estimation approach

2.1. Signal model

Assume that K far-field independent narrowband sources incident on an M-element NLA from directions $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_K]^T$, and their discretized baseband waveforms are expressed as $s_k(t)$, for $k = 1, 2, \dots, K$. Then the array output at time t can be written as:

$$\mathbf{x}(t) = \sum_{k=1}^{K} \mathbf{b}(\theta_k) s_k(t) + \mathbf{n}(t) = \mathbf{B}(\boldsymbol{\theta})\mathbf{S}(t) + \mathbf{n}(t) \tag{1}$$

where the $K \times 1$ vector $\mathbf{S}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T$ contains the complex amplitude of the source signals, $\mathbf{n}(t) = [n_1(t), n_2(t), \dots, n_M(t)]^T$ denotes an $M \times 1$ i.i.d. white Gaussian noise vector following the distribution $\mathcal{N}(\mathbf{0}, \sigma_n^2 \mathbf{I}_M)$ and statistically independent of all the signals, $\mathbf{B}(\boldsymbol{\theta}) = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \dots, \mathbf{b}(\theta_K)]$ represents the $M \times K$ array manifold matrix and $\mathbf{b}(\theta_k)$ is the steering vector of the kth source:

$$\mathbf{b}(\theta_k) = \left[1, e^{j\frac{2\pi d_2}{\lambda}\sin\theta_k}, \dots, e^{j\frac{2\pi d_M}{\lambda}\sin\theta_k}\right]^T \tag{2}$$

here $d_i, i = 1, 2, \dots, M$, denotes the location of the ith array sensor, λ represents the signal wavelength, and the first sensor is assumed as the reference, i.e., $d_1 = 0$.

Grouping the sampled signal, then the received data $\mathbf{X} = [\mathbf{x}(t_1), \mathbf{x}(t_2), \dots, \mathbf{x}(t_N)]$ can be expressed as:

$$\mathbf{X} = \mathbf{B}(\boldsymbol{\theta})\mathbf{S} + \mathbf{N} \tag{3}$$

where $\mathbf{X} = [\mathbf{x}(t_1), \mathbf{x}(t_2), \dots, \mathbf{x}(t_N)]$, $\mathbf{S} = [\mathbf{s}(t_1), \mathbf{s}(t_2), \dots, \mathbf{s}(t_N)]$, $\mathbf{N} = [\mathbf{n}(t_1), \mathbf{n}(t_2), \dots, \mathbf{n}(t_N)]$, and N is the sample number.

2.2. EM based spatial interpolation approach

The NLA can be considered as a sublattice array of a ULA with M' ($M' > M$) sensors where some sensors are omitted. Under this assumption, the NLA output and the hypothetical ULA output can be seen as incomplete data and complete data, respectively. In the subsequent development, the EM criterion is implemented in order to reconstruct the complete data and the DOA set $\boldsymbol{\theta}$.

Without loss of generality, let $\mathbf{X}$ and $\mathbf{Y}$ denote the incomplete data and complete data, respectively. $\mathbf{Y}$ is composed of the actual observed data and the latent data corresponding to the missing sensors output. Notice that the NLA can be described by a binary vector $\mathbf{p}$ of length M'. If the mth sensor of the virtual ULA is contained in the NLA, the mth component of $\mathbf{p}$ is 1; otherwise it is zero. As a consequence, equation (3) can be described equivalently by left-multiplying the virtual ULA output with a transformation matrix $\mathbf{P}$

$$\mathbf{X} = \mathbf{P}\mathbf{Y} \tag{4}$$

where the $M \times M'$ matrix $\mathbf{P}$ is constructed by eliminating the zero rows of $\mathrm{diag}(\mathbf{p})$.

Basically, the EM criterion is a two-stage iterative optimization technique for finding maximum likelihood solutions for probabilistic models having latent variables. Now we use the EM criterion to maximize the posterior distribution $p(\mathbf{S}|\mathbf{X})$ for models in which we have introduced a prior $p(\mathbf{S})$ over the parameters. Notice that, for any distribution $q(\mathbf{Y})$, the following decomposition holds true [18]:

$$\ln p(\mathbf{X}|\mathbf{S}) = L(q, \mathbf{S}) + KL(q||p) \tag{5}$$

where

$$L(q, \mathbf{S}) = \int q(\mathbf{Y}) \ln\left\{\frac{p(\mathbf{X}, \mathbf{Y}|\mathbf{S})}{q(\mathbf{Y})}\right\} d\mathbf{Y} \tag{6}$$

$$KL(q||p) = -\int q(\mathbf{Y}) \ln\left\{\frac{p(\mathbf{Y}|\mathbf{X}, \mathbf{S})}{q(\mathbf{Y})}\right\} d\mathbf{Y} \tag{7}$$

and it is worth noting that, $KL(q||p)$ is the Kullback–Leibler divergence between $q(\mathbf{Y})$ and the posterior distribution $p(\mathbf{Y}|\mathbf{X}, \mathbf{S})$. Recall that. $KL(q||p) \geq 0$, with equality if, and only if, $q(\mathbf{Y}) = p(\mathbf{Y}|\mathbf{X}, \mathbf{S})$, it can be deduced that $L(q, \mathbf{S}) \leq \ln p(\mathbf{X}|\mathbf{S})$, in other words, $L(q, \mathbf{S})$ is a lower bound on $\ln p(\mathbf{X}|\mathbf{S})$.

Then we introduce a prior distribution $p(\mathbf{S})$ over the parameters, together with the product rule of probability $p(\mathbf{S}|\mathbf{X}) = p(\mathbf{S}, \mathbf{X})/p(\mathbf{X})$, to obtain:

$$\ln p(\mathbf{S}|\mathbf{X}) = \ln p(\mathbf{S}, \mathbf{X}) - \ln p(\mathbf{X}) \tag{8}$$

Plugging the decomposition (5) into (8) leads to a more explicit formulation of $\ln p(\mathbf{S}|\mathbf{X})$:

$$\begin{aligned}\ln p(\mathbf{S}|\mathbf{X}) &= \ln p(\mathbf{X}|\mathbf{S}) + \ln p(\mathbf{S}) - \ln p(\mathbf{X}) \\ &= L(q, \mathbf{S}) + KL(q||p) + \ln p(\mathbf{S}) - \ln p(\mathbf{X}) \\ &\geq L(q, \mathbf{S}) + \ln p(\mathbf{S}) - \ln p(\mathbf{X}) \end{aligned} \tag{9}$$

where $\ln p(\mathbf{X})$ is a constant. Now we can optimize the right-hand side of (9) alternately w.r.t. q and $\mathbf{S}$, which gives rise to the E-step equation and M-step equation as for the standard EM algorithm, respectively. In the E-step, the lower bound $L(q, \mathbf{S}^{(\mathrm{old})})$ is maximized w.r.t. $q(\mathbf{Y})$ while holding $\mathbf{S}^{(\mathrm{old})}$ fixed, and this largest value will occur when the Kullback–Leibler divergence $KL(q||p)$ vanishes, that is to say, when $q(\mathbf{Y}) = p(\mathbf{Y}|\mathbf{X}, \mathbf{S}^{(\mathrm{old})})$. By virtue of the fact that, $\mathbf{X}$ and $\mathbf{Y}$ are jointly Gaussian, we remark that the conditional distribution $p(\mathbf{Y}|\mathbf{X}, \mathbf{S}^{(\mathrm{old})})$ is again Gaussian. Making use of the conditional distribution $p(\mathbf{X}|\mathbf{Y}) = \mathcal{N}(\mathbf{X}|\mathbf{P}\mathbf{Y}, \mathbf{0})$ and the marginal distribution $p(\mathbf{Y}) = \mathcal{N}(\mathbf{Y}|\mathbf{A}(\boldsymbol{\theta})\mathbf{S}^{(\mathrm{old})}, \beta^{(\mathrm{old})-1}\mathbf{I}_{M'})$, where $\mathbf{A}(\boldsymbol{\theta})$ is the manifold matrix for the virtual ULA, $\beta^{(\mathrm{old})}$ is the inverse variance of the distribution, it is easy to obtain:

$$p\left(\mathbf{Y}|\mathbf{X}, \mathbf{S}^{(\mathrm{old})}\right) = \mathcal{N}\left(\mathbf{Y}|\hat{\mathbf{Y}}, \sum_{\mathbf{Y}}\right) \tag{10}$$

where

$$\hat{\mathbf{Y}} = \mathbf{A}(\boldsymbol{\theta})\mathbf{S}^{(\text{old})} + \mathbf{P}^H\left(\mathbf{X} - \mathbf{P}\mathbf{A}(\boldsymbol{\theta})\mathbf{S}^{(\text{old})}\right) \tag{11}$$

$$\sum_{\mathbf{Y}} = \beta^{(\text{old})-1}\left(\mathbf{I}_{M'} - \mathbf{P}^H\mathbf{P}\right) \tag{12}$$

Substituting $q(\mathbf{Y}) = p(\mathbf{Y}|\mathbf{X}, \mathbf{S}^{(\text{old})})$ into the right hand side of (9), we see that, in the subsequent M step, the quantity that is being maximized takes the form:

$$\begin{aligned}\int & p\left(\mathbf{Y}|\mathbf{X}, \mathbf{S}^{(\text{old})}\right) \ln p(\mathbf{X}, \mathbf{Y}|\mathbf{S})d\mathbf{Y} \\ &- \int p\left(\mathbf{Y}|\mathbf{X}, \mathbf{S}^{(\text{old})}\right) \ln q(\mathbf{Y})d\mathbf{Y} + \ln p(\mathbf{S}) \\ &= Q\left(\mathbf{S}, \mathbf{S}^{(\text{old})}\right) + \ln p(\mathbf{S}) + \text{const}\end{aligned} \tag{13}$$

where $Q(\mathbf{S}, \mathbf{S}^{(\text{old})}) = \int p(\mathbf{Y}|\mathbf{X}, \mathbf{S}^{(\text{old})}) \ln p(\mathbf{X}, \mathbf{Y}|\mathbf{S})d\mathbf{Y}$ denotes the expectation of the complete-data log likelihood.

So far we have obtained the outline of the EM algorithm as follows:

1). Initialize the parameters $\mathbf{S}^{(\text{old})}$;
2). E-step: Evaluate $p(\mathbf{Y}|\mathbf{X}, \mathbf{S}^{(\text{old})})$;
3). M-step: Evaluate $\mathbf{S}^{(\text{new})}$ given by

$$\mathbf{S}^{(\text{new})} = \arg\max_{\mathbf{S}} Q\left(\mathbf{S}, \mathbf{S}^{(\text{old})}\right)$$

where $Q(\mathbf{S}, \mathbf{S}^{(\text{old})}) = \int p(\mathbf{Y}|\mathbf{X}, \mathbf{S}^{(\text{old})}) \ln p(\mathbf{X}, \mathbf{Y}|\mathbf{S})d\mathbf{Y}$.

4). Check the convergence of $\mathbf{S}$. If no, let $\mathbf{S}^{(\text{old})} \leftarrow \mathbf{S}^{(\text{new})}$, go back to 2). If yes, stop.

2.3. Proposed DOA estimation algorithm

The application of the general EM criterion to the DOA estimation problem at hand requires only the determination of $Q(\mathbf{S}, \mathbf{S}^{(\text{old})})$. In what follows next, we consider a zero-mean isotropic Gaussian prior governed by a variance parameter vector $\boldsymbol{\gamma}$ so that

$$p(\mathbf{S}|\boldsymbol{\gamma}) = \prod_{i=1}^{N} \mathcal{N}(\mathbf{S}_{.i}|\mathbf{0}, \boldsymbol{\Gamma}) \tag{14}$$

where $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_g]^T$, g denotes the sampling grid number and satisfies $g >> K$, $\boldsymbol{\Gamma} = \text{diag}(\boldsymbol{\gamma})$.

Since we choose a conjugate Gaussian prior distribution (14), the corresponding posterior will also be Gaussian, and it can be written down directly in the following form:

$$p(\mathbf{S}|\mathbf{Y}) = \prod_{i=1}^{N} \mathcal{N}\left(\mathbf{S}_{.i}|\hat{\mathbf{S}}_{.i}, \sum_{\mathbf{S}}\right) \tag{15}$$

where

$$\begin{aligned}\sum_{\mathbf{S}} &= \left[\boldsymbol{\Gamma}^{-1} + \beta\mathbf{A}^H(\boldsymbol{\theta})\mathbf{A}(\boldsymbol{\theta})\right]^{-1} \\ &= \boldsymbol{\Gamma} - \boldsymbol{\Gamma}\boldsymbol{A}^H(\boldsymbol{\theta})\left[\beta^{-1}\mathbf{I}_{M'} + \mathbf{A}(\theta)\boldsymbol{\Gamma}\boldsymbol{A}^H(\boldsymbol{\theta})\right]^{-1}\mathbf{A}(\boldsymbol{\theta})\boldsymbol{\Gamma}\end{aligned} \tag{16}$$

$$\hat{\mathbf{S}}_{\bullet i} = \sum_{\mathbf{S}}\left\{\mathbf{A}^H(\boldsymbol{\theta})\beta\hat{\mathbf{Y}}_{.i}\right\} = \boldsymbol{\Gamma}\boldsymbol{A}^H(\boldsymbol{\theta})\sum^{-1}\hat{\mathbf{Y}}_{.i} \tag{17}$$

$$\sum = \beta^{(\text{old})-1}\mathbf{I}_{M'} + \mathbf{A}(\boldsymbol{\theta})\boldsymbol{\Gamma}\boldsymbol{A}^H(\boldsymbol{\theta}) \tag{18}$$

It is easy to see that, on the right hand side of (13), $\boldsymbol{\gamma}$ only appears in $\ln p(\mathbf{S})$, so each component of $\boldsymbol{\gamma}$ is evaluated by setting the derivatives of $\ln p(\mathbf{S}|\boldsymbol{\gamma})$ w.r.t. γ_i to zero:

$$\gamma_i = \|\hat{\mathbf{S}}_{i.}\|_2^2/N + \left(\sum_{\mathbf{S}}\right)_{ii}, (i = 1, 2, \ldots, g) \tag{19}$$

The optimization w.r.t. β gives rise to maximizing $Q(\mathbf{S}, \mathbf{S}^{(\text{old})})$ w.r.t. β, because β only appears in $Q(\mathbf{S}, \mathbf{S}^{(\text{old})})$:

$$\begin{aligned}Q\left(\mathbf{S}, \mathbf{S}^{(old)}\right) = \frac{M'N}{2}\ln\beta - \frac{\beta}{2}\sum_{i=1}^{N}\Big\{&E\left(\mathbf{Y}_{.i}^H\mathbf{Y}_{.i}\right) \\ &- 2E\left(\mathbf{Y}_{.i}^H\right)\mathbf{A}(\boldsymbol{\theta})\hat{\mathbf{S}}_{.i} + \hat{\mathbf{S}}_{.i}^H\mathbf{A}^H(\boldsymbol{\theta})\mathbf{A}(\boldsymbol{\theta})\mathbf{S}_{.i}\Big\}\end{aligned} \tag{20}$$

Setting the derivatives w.r.t. β to zero, we obtain the quantity β:

$$\beta = M'N/\sum_{i=1}^{N}\left\{\left\|\hat{\mathbf{Y}}_{.i} - \mathbf{A}(\boldsymbol{\theta})\hat{\mathbf{S}}_{.i}\right\|_2^2 + \text{Tr}\left[\mathbf{A}(\boldsymbol{\theta})\sum_{\mathbf{S}}\mathbf{A}^H(\boldsymbol{\theta})\right]\right\} \tag{21}$$

At the end of this section, the proposed DOA estimation algorithm is summarized below.

1). Select initial values $\mathbf{Y}^{(\text{old})}$, $\boldsymbol{\gamma}^{(\text{old})}$, $\mathbf{S}^{(\text{old})}$ and $\beta^{(\text{old})}$.
2). Apply the EM algorithm for received NLA signal.
   a) Construct the received virtual ULA signal $\mathbf{Y}^{(\text{new})}$ according to (11);
   b) Compute $\mathbf{S}^{(\text{new})}$ and $\sum_{\mathbf{S}}^{(\text{new})}$ according to (17) and (16), respectively, by using $\boldsymbol{\Gamma}^{(\text{old})}$ and $\mathbf{Y}^{(\text{new})}$;
   c) Evaluate $\boldsymbol{\gamma}^{(\text{new})}$ via (19), by using $\mathbf{S}^{(\text{new})}$ and $\sum_{\mathbf{S}}^{(\text{new})}$;
   d) Estimate the precise parameter $\beta^{(\text{new})}$ via (21), by using $\mathbf{Y}^{(\text{new})}$, $\mathbf{S}^{(\text{new})}$ and $\sum_{\mathbf{S}}^{(\text{new})}$.
3). Check for convergence of $\mathbf{S}$ or $\boldsymbol{\gamma}$. If the convergence criterion is not satisfied, then let $\mathbf{Y}^{(\text{old})} \leftarrow \mathbf{Y}^{(\text{new})}$, $\boldsymbol{\gamma}^{(\text{old})} \leftarrow \boldsymbol{\gamma}^{(\text{new})}$, $\mathbf{S}^{(\text{old})} \leftarrow \mathbf{S}^{(\text{new})}$, $\beta^{(\text{old})} \leftarrow \beta^{(\text{new})}$, and return to step 2. Otherwise stop.

Once the hyperparameter vector $\boldsymbol{\gamma}$ is estimated, the indexes of its nonzero elements indicate the signal DOAs.

2.4. Complexity

In this section, we compare computation costs of proposed method and other existing methods. The proposed method is an iterative algorithm whose computational complexity equals the complexity per iteration times the number of iterations. For each iteration, the complexity of the calculation of the complete data $\hat{\mathbf{Y}}$ is $(M' - M)gN + M'MN \approx (M' - M)gN$ since $M' \ll g$; estimating the hyperparameter vector $\boldsymbol{\gamma}$ requires approximately M'^2g operations. Thus, for one iteration, the number of operations required is $O[(M' - M)gN + M'^2g]$.

In contrast, each L1-SVD, L1-SRACV and SPICE iteration require $O(g^3K^3)$ [12], $O(M^3g^3)$ [13] and $O(M^3N)$ [14] operations, respectively. Besides, the number of iterations required by the proposed method in order to converge is small (lower than 100). As a result, the computational cost of the proposed method is smaller than those of L1-SVD and L1-SRACV, but a little higher than that for SPICE. MUSIC is decidedly less costly, that is, $O(M^3)$ [8,9] than all of these methods, but this makes no sense as it performs much worse than other methods in low SNR or insufficient snapshots adaptation.

3. Simulation results

In this section, several numerical simulations are carried out to show the merits of implementing our proposed method to NLA under various scenarios, and compared with those of SS-MUSIC, L1-SVD, L1-SRACV, and SPICE, as well as the stochastic Cramer–Rao lower bound (CRLB). The simulations are based on a 7-element NLA with its physical sensors described by $\mathbf{p} =$

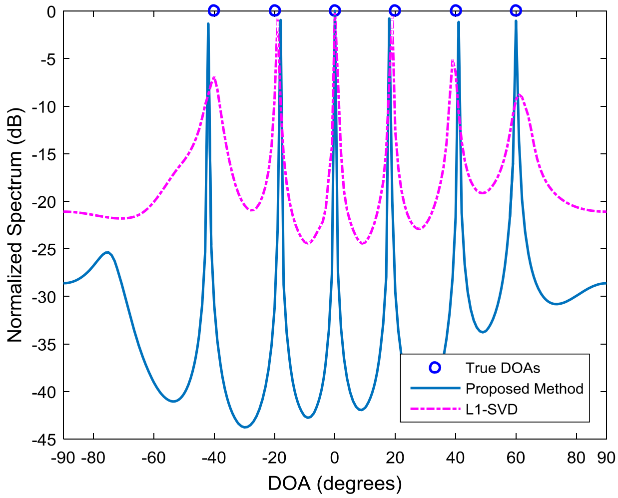

Fig. 1. Normalized spatial spectra for L1-SVD and the proposed method.

$[1\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0]^T$. Multiple spatially uncorrelated narrowband source signals, which are samples from an uncorrelated complex Gaussian process with zero mean, impinging on this sensor array. The measurements are corrupted by temporally and spatially uncorrelated white Gaussian noises. The direction grid is set to have 180 points sampled from $-90°$ to $90°$ with $1°$ intervals. All experiments are carried out in MATLAB v.2015a on a PC with a windows 7 system and a 3.40 GHz dual-core CPU.

Consider six equal-power signals that arrive from $[-40°, -20°, 0°, 20°, 40°, 60°]$. All signals are coherent, i.e., correlated with correlation coefficient of 1. The total number of snapshots is set to 100, and the SNR for the six signals is set to 0 dB. The normalized spectrum obtained using L1-SVD and proposed method is depicted in Fig. 1. We observe that the proposed method has higher resolution than that of L1-SVD. This can be attributed to the fact that, our proposed method exploits the interpolated complete data, which efficiently extends the aperture of the NLA, while L1-SVD only uses the raw array output data.

The root mean square error (RMSE) of our proposed algorithm and other existing methods is now tested in terms of the input SNR. Assume that three correlated sources incident on the NLA from directions of $-10°$, $20°$ and $28°$. The snapshot number is fixed at 100. The 1st and 2nd signals are correlated with a correlation factor 0.6. The 1st and 3rd signals are correlated with a correlation factor 0.8. Moreover, the forward/backward spatial smoothing is applied for MUSIC to decorrelate the correlated signals, using a 4-element smoothing subarray. For SS-MUSIC, the DOA estimates can be found by locating the peaks of the null spectrum, which is constructed via a one-dimensional search. Since the true DOAs of the signals belong to the continuous angle space and are unlikely to lie on the pre-specified grid precisely, SS-MUSIC suffers from basis mismatch. In addition, SS-MUSIC also requires the source number and does not have reliable performance in the moderate/low SNR or small snapshots regime, no matter how finely the discrete grid points are chosen. With regard to the simulation setups presented in this Section, numerous experiments have validated that the adopted discretization grid set is dense enough, because the DOA estimation precision of SS-MUSIC improves only slightly when the sampled interval decreases further, and too dense a grid is computationally prohibitive and might result in computational instability. The RMSE curves shown in Fig. 2 are obtained via 500 Monte Carlo trials, and indicate that among the tested methods, the proposed methods owns the best estimation performance w.r.t. variation of SNR. This is due to the fact that, our proposed method implements the EM algorithm to mimic a virtual ULA with wider aperture, which facilitates its significant

Table 1
Average running times of respective algorithms (sec).

Algorithms	Proposed	L1-SRACV	SPICE	L1-SVD	SS-MUSIC
Time	0.1161	0.6242	0.0372	0.2069	0.0038

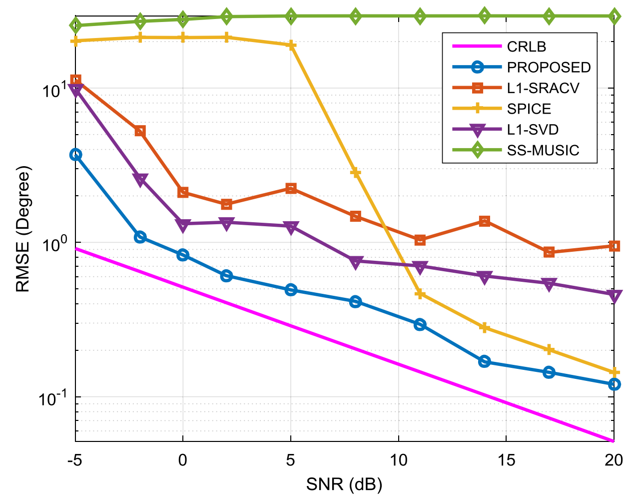

Fig. 2. RMSE versus SNR for three correlated sources.

predominance over other counterparts in SNR adaptation. Moreover, unlike other sparsity-promoting methods, which employ the ℓ_1-norm instead of ℓ_0-norm to enforce sparsity and have a poor performance on describing the true observation model, our proposed method involves maximizing a posterior distribution using a sparsity-inducing prior distribution. Several preliminary literatures have demonstrated theoretically that, by doing so, the global minimum is always left unaltered unlike standard ℓ_1-norm relaxations. This ensures that any approximate descent method is guaranteed to locate the maximally sparse solution. The average running times of respective algorithms are also provided in Table 1 where the SNR is fixed to 0 dB. We see that SS-MUSIC is the most computationally efficient algorithm, and excluding SPICE, the proposed method has a lower computational complexity than the other two sparsity-inducing methods. However, by considering the inferior performance provided by SPICE and SS-MUSIC method, as shown in Fig. 2, the proposed method is a preferred technology when handling NLA DOA estimation problem.

Next, we keep the SNR at 0 dB, and increase the number of snapshots from 20 to 400. The remaining parameters of the experiments are the same as before. The RMSE curves shown in Fig. 3 indicate that, the proposed approach is superior to the other compared methods all along.

Now, we fix the snapshot number at 100 and the SNR of the two incident signals at 10 dB, the source signals are correlated with a correlation factor 0.7. When the angle separation of the two signals varies from $5°$ to $15°$, the RMSE of the five DOA estimators are given in Fig. 4. This experiment again supports the predominance of our proposed method over the other state-of-the-art counterparts in DOA estimation precision.

We also investigate the RMSE of each algorithm in the case of coherent (i.e., the correlation coefficient is 1) sources. Assume three coherent signals impinging onto the NLA from directions of $-10°$, $20°$ and $28°$. The number of snapshots is 100. All signals have equal SNR increasing from -5 dB to 20 dB. The statistical results are shown in Fig. 5. It is clear from Fig. 5 that our proposed algorithm is able to achieve a much smaller RMSE than the other four methods. The relative performance of each DOA estimator versus snapshots number and angle separation are deducible

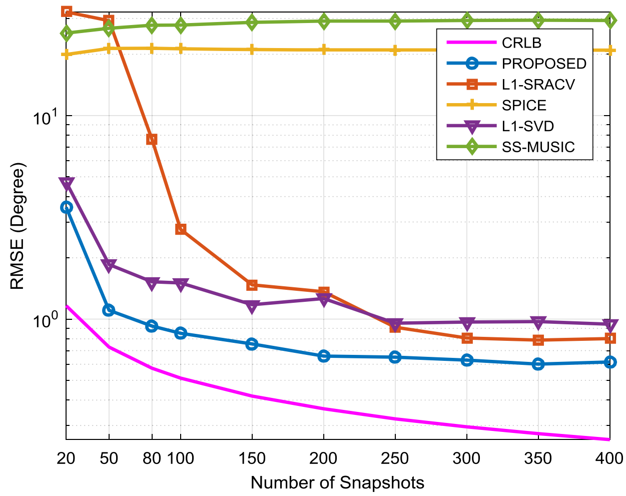

Fig. 3. RMSE versus number of snapshots for three correlated sources.

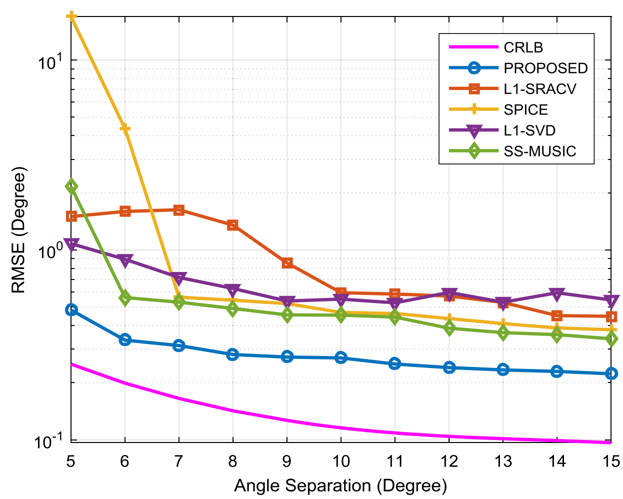

Fig. 4. RMSE versus angle separation of two correlated sources.

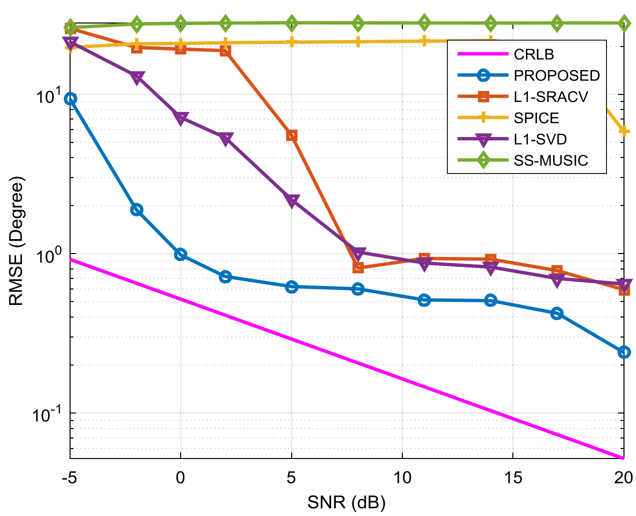

Fig. 5. RMSE versus SNR for three coherent sources.

from Fig. 3 and Fig. 4, respectively, and thus are not provided here due to space limitations.

Finally, we repeat the same simulation for a 10-element ULA. The three coherent sources are the same as before located at

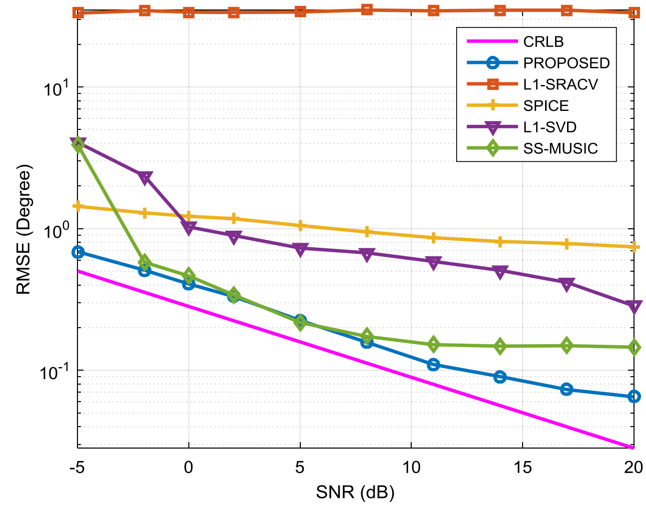

Fig. 6. RMSE versus SNR for three coherent sources (ULA case).

$[-10°, 20°, 28°]$. The RMSE performance of the tested methods are compared in Fig. 6 with respect to the input SNR. It is evident that the proposed method achieves significant improvement over the other methods for all the input SNR values being evaluated. The relative performance of each DOA estimator versus snapshots number and angle separation are deducible from Fig. 3 and Fig. 4, respectively, and thus are not provided here due to space limitations.

4. Conclusion

In this letter, we have extended the mathematical theory of SBL to detect sources observed by NLA. The proposed algorithm offers a significant enhancement in array aperture over the currently employed methods by interpolating the existing received data to emulate observations at a virtual long ULA, therefore an improved resolution capability can be obtained. Furthermore, the proposed method is tested using a specific NLA configuration and is shown to successfully estimate the DOAs of correlated sources.

For nonregular arrays, the intersensor separations are not a multiple of $\lambda/2$. The sensors are placed in an arbitrary way. The nonregular arrays are used when the construction of a ULA is not possible due to physical constraints. Frankly, our proposed DOA estimation approach cannot be generalized to the case of nonregular arrays. In fact, the nonregular array sensors cannot be deemed as a part of the virtual ULA sensors, thus we cannot simply complete the existing sensors in order to form the complete data. In other words, the transformation matrix $\mathbf{P}$ cannot be any noninvertible transformation, since the choice of complete data may critically affect the complexity and the rate of convergence of the algorithm, and the unfortunate choice of complete data may yield a completely useless algorithm. As this direction is out of the main scope of this work, it will be left as a future study.

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