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On improvement of joint estimation of DOA and PPS coefficients impinging on ULA

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ABSTRACT

This paper considers joint estimation of the direction-of-arrival (DOA) and coefficients of polynomial-phase signal (PPS) that impinge an uniform linear sensor array (ULA). Recently introduced method referred to as the polynomial beamformer (PB) being the maximum likelihood estimator for these parameters suffers from high computational complexity due to a multidimensional search. Hence, its applicability is limited only to the second-order PPSs while it is unfeasible for higher-order PPSs as it is heavily reliant on the PPS order. A low complexity estimator that utilizes similar strategy as the PB is proposed in this paper. Statistical study shows that the proposed approach outperforms the state-of-the-art methods in terms of signal-to-noise ratio (SNR) threshold and mean-squared error (MSE).

1. Introduction

Joint estimation of the direction-of-arrival (DOA) and coefficients of polynomial phase signals (PPSs) impinging an uniform linear array (ULA) of sensors has been considered in numerous research papers [2,3,6,8,10,13,14,23]. Its main application field is in the underwater acoustics where hydrophone arrays have several hundreds sensors [15]. The cornerstone technique for joint estimation of the PPS coefficients and DOA is proposed by Gershman et al. [13]. This technique is referred to as the polynomial beamformer (PB). The main drawback of the PB is high computational complexity. The special case of the PB referred to as the chirp beamformer (CB) is designed for the second-order PPSs. Namely, the CB requires solving a 3D optimization problem. For higher-order PPSs the PB becomes unfeasible since it requires the $(K + 1)$-dimensional search where K being the order of PPS.

Several sub-optimal strategies have been proposed addressing this drawback [7,10]. In these methods, multidimensional search is replaced with a number of 1D searches. However, achieved accuracy is limited, especially for the DOA estimate where mean-squared error (MSE) is significantly above the Cramer-Rao lower bound (CRLB). Furthermore, the error in estimation of the highest-order PPS coefficient propagates and affects the estimation of the DOA. Both errors in estimation propagate and influence the estimation accuracy for lower-order PPS coefficients.

In this paper, we propose a novel algorithm for joint estimation of DOA and PPS coefficients. To improve DOA estimation, the algorithm preforms a search over a set of assumed angles. For each angle coefficients of the PPS are estimated and a vector consisting of the angle and coefficients estimates is formed. Similarly to the PB-based approach, the optimal vector is determined by maximizing the objective function. Instead of multidimensional search in the PB-based approach we perform a 1D search over a set of angles. Three main goals are achieved by the proposed strategy: reduced calculation complexity, avoidance of error propagation from the estimation of highest-order PPS coefficient and simple generalization for all high-order PPSs. Recently, similar strategy has been exploited in case of parameter estimation of high-order PPS where 1D search is performed over a generic parameter (window width in the short-time Fourier transform) instead of the multidimensional search required for ML estimation [11,12,20] and is referred to as the quasi-ML (QML) estimator.

The paper is organized as follows. The considered ULA and signal model are described in Section 2.1, and related joint estimators of DOA and PPS are reviewed in Sections 2.2 and 2.3. The novel estimator is proposed in Section 3 with its performance evaluated and compared to existing methods on a numerical study in Section 4. Concluding remarks are given in Section 5.

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2. Theoretical background

2.1. Array signal model

Assume a constant amplitude PPS $x(n)$ impinging an ULA with M omnidirectional sensors. The output can be written as [10]

$$\mathbf{y}(n) = \mathbf{a}(\theta, n)x(n) + \mathbf{v}(n), \quad |n| \le (N-1)/2, \tag{1}$$

where $\mathbf{a}(\theta, n)$ is $M \times 1$ array steering vector, $\mathbf{v}(n)$ is $M \times 1$ vector of i.i.d. complex Gaussian zero-mean noise samples, and N is number of samples. The Kth order PPS $x(n)$ is defined as

$$x(n) = Ae^{j\phi(n)} = Ae^{j\sum_{k=0}^{K} a_k (n\Delta)^k}, \tag{2}$$

where A is amplitude, $\phi(n)$ is phase with coefficients a_k, $k = 0, \ldots, K$, and Δ is sampling interval. Steering vector $\mathbf{a}(\theta, n)$ is modeled as in [13]

$$\mathbf{a}(\theta, n) = [1, e^{j\omega(n)\psi}, \ldots, e^{j\omega(n)(M-1)\psi}]^T, \ \omega(n) = \frac{d\phi(n)}{dn}$$
$$= \sum_{k=0}^{K-1} (k+1)a_{k+1}(\Delta n)^k, \ \psi = d\frac{\sin(\theta)}{c}, \tag{3}$$

where $\omega(n)$ is instantaneous frequency (IF) of PPS, θ is DOA, d is inner-sensor spacing, and c is speed of propagation.

Our goal is to estimate vector $\mathbf{V} = [\theta, a_1, \ldots, a_K]$ from observations $\mathbf{y}(n)$.

Some of existing techniques for joint estimation of the DOA and PPS coefficients are reviewed below.

2.2. Polynomial beamformer

The PB estimator proposed in [13] can be summarized as

$$\hat{\mathbf{V}} = \arg\max_{\mathbf{B}} \mathrm{PB}(\mathbf{B}), \ \mathrm{PB}(\mathbf{B})$$
$$= \frac{1}{MN}\left|\sum_{n=-(N-1)/2}^{(N-1)/2} \mathbf{y}^H(n)\exp\left(j\sum_{k=1}^{K} b_k (n\Delta)^k\right)\mathbf{a}(\zeta, n)\right|^2, \tag{4}$$

where $\mathbf{B} = [\zeta, b_1, \ldots, b_K]$ is the $(K+1)$-dimensional search vector and ζ and b_i, $i \in [1, K]$ correspond to angle and PPS coefficients, respectively. Ideally, if there are no errors in estimation, $\mathbf{B} = [\zeta, b_1, \ldots, b_K] = [\theta, a_1, \ldots, a_K]$. $(\cdot)^H$ is the Hermitian operator and PB $(\cdot)$ is the PB objective function. The applicability of this technique is limited to PPS of low-order. This conclusion readily follows from the $(K+1)$-dimensional search required for the PB, hence, for $K > 2$ it is computationally unfeasible. The special case of the PB, referred as the CB, is derived for K=2. Furthermore, the genetic algorithm is used to reduce computational complexity of the search procedure [13].

2.3. Suboptimal estimators

Alternatives for reducing computational complexity of the PB are techniques based on the phase-differentiation (PD) proposed in [4,10,17,22]. The PD is performed by calculating the high-order instantaneous moments (HIM). The Kth order HIM of $x(n)$, is defined in recursive form as [21]:

$$\mathrm{HIM}_K[x(n), \tau] = \mathrm{HIM}_2[\mathrm{HIM}_{K-1}[x(n), \tau], \tau], \ \mathrm{HIM}_1[x(n), \tau] = x(n), \ \mathrm{HIM}_2[x(n), \tau] = x(n+\tau)x^*(n-\tau), \tag{5}$$

where τ is the lag parameter. Having in mind Kth order PPS impinging the mth sensor $x_m(n)$ is of the form (see (2) and (3))

$$x_m(n) = Ae^{j\left(a_o + \sum_{k=1}^{K} a_k ((n\Delta)^k + k\psi m(n\Delta)^{k-1})\right)}, \tag{6}$$

Kth order HIM of $x_m(n)$ equals [10]

$$\mathrm{HIM}_K[x_m(n), \tau] = De^{j2^{K-1}K!\tau^{K-1}\Delta^K a_K n}e^{j2^{K-1}K!\tau^{K-1}\Delta^{K-1}a_K\psi m}, \tag{7}$$

where $D = A^{2^{K-1}}e^{j2^{K-1}(K-1)!(\tau\Delta)^{K-1}a_{K-1}}$. From (7) it follows that $\mathrm{HIM}_K[x_m(n), \tau]$ is product of two complex sinusoids, e.q. one with time index n and the other with time index m. Maximizing the discrete Fourier transform (DFT) calculated along n and m, respectively, parameter estimates $\hat{a}_K$ and $\hat{\theta}$ can be derived from frequencies of the complex sinusoids [1,5,11,19]. The DFT of the corresponding $\mathrm{HIM}_K[x_m(n), \tau]$ is referred to as the high-order ambiguity function (HAF) [10]. The principal HAF-based joint estimator of DOA and PPS is proposed in [10] with efficient realization. Hereafter, it is referred to as the E-HAF (E stands for 'efficient'). The E-HAF results are significantly improved with the refined-HAF (R-HAF) [7]. It is summarized within the following algorithm.

Algorithm: R-HAF. Form a matrix $\mathbf{MHIM}_K[y(n), \tau]$ as

$$\mathbf{MHIM}_K[y(n), \tau] = \begin{bmatrix} \mathrm{HIM}_K[y_0(-(N-1)/2), \tau] & \mathrm{HIM}_K[y_0(-(N-1)/2+1), \tau] & \cdots & \mathrm{HIM}_K[y_0((N-1)/2), \tau] \\ \mathrm{HIM}_K[y_1(-(N-1)/2), \tau] & \mathrm{HIM}_K[y_1(-(N-1)/2+1), \tau] & \cdots & \mathrm{HIM}_K[y_1((N-1)/2), \tau] \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{HIM}_K[y_{M-1}(-(N-1)/2), \tau] & \mathrm{HIM}_K[y_{M-1}(-(N-1)/2+1), \tau] & \cdots & \mathrm{HIM}_K[y_{M-1}((N-1)/2), \tau] \end{bmatrix}. \tag{8}$$

1. **Set** k=K
2. Estimate coefficient a_k as

$$\hat{a}_k = \frac{\mathrm{argmax}_\omega\left\{\sum_m |Y_k^m(\omega, \tau)|\right\}}{2^{k-1}k!\tau^{k-1}\Delta^k},$$

where $Y_k^m(\omega, \tau)$ is the DFT of mth row of matrix $\mathbf{MHIM}_k[y(n), \tau]$. If $k \ne K$ **go to** Step 4.
3. Estimate the DOA as

$$\hat{\theta} = \arcsin\left(\frac{c}{d}\frac{\mathrm{argmax}\left\{\sum_n |Y_K^n(\omega, \tau)|\right\}}{2^{K-1}K!\tau^{K-1}\Delta^K\hat{a}_K}\right), \tag{9}$$

where $Y_K^n(\omega, \tau)$ is the DFT of nth column of matrix $\mathbf{MHIM}_k[y(n), \tau]$.
4. Dechirp estimated parameters from $y_m(n)$, $m = 0, \ldots, M-1$ as

$$y_m(n) = y_m(n)\exp(-j\hat{a}_k((n\Delta)^k + k\hat{\psi}m(n\Delta)^{k-1}))$$

where $\hat{\psi} = d\sin(\hat{\theta})/c$. Here we have used computer programming notation where the result from the right hand side of expression updates the left hand side.
5. **If** $k > 0$ set $k = k-1$ and **go to** Step 2.

Calculation complexity of the R-HAF with $\mathrm{O}(KMN\log_2 N)$ required operations is significantly lower than for the PB, $\mathrm{O}(MNN_D^{K+1})$, where N_D is the size of the search space in considered direction [9]. It should be noted that, calculation complexity of the R-HAF exhibits linear dependence of phase order, whereas in the case of the PB calculation complexity exhibits the exponential dependence.

The drawbacks of the R-HAF approach are mainly caused by error propagation in the estimation of the highest-order coefficient a_K toward lower-order ones and DOA estimation. In addition, DOA estimation is done using non-linear function $\arcsin(\cdot)$ (see (9)) that has a negative impact on accuracy. Furthermore, error in DOA estimation propagates and affects estimation of lower-order PPS coefficients a_k, $k < K$. Simulation results in [7] clearly demonstrate these effects since the MSEs in estimation of θ and a_k, $k < K$, are significantly above the CRLB.

In the next section a novel method is proposed for improving accuracy in the joint estimation of PPS coefficients and the DOA.

3. The proposed method

Instead of estimating the DOA as in the R-HAF, we make an assumption that the angle is known in advance. Then, the PPS coefficients are estimated for the assumed angle. Such procedure is performed over all angles within a considered set. A vector is formed of an angle and corresponding PPS coefficients. In this way, ensemble of vectors is obtained $\mathbf{R} = \{[\theta_i, \hat{a}_1^{(i)}, \ldots, \hat{a}_K^{(i)}], i = 1, \ldots, N_\theta\}$, where $\theta^{(i)}$ and N_θ are the assumed angle and number of angles in the search space, respectively. The optimal estimate is determined to be the vector from the ensemble that maximizes the PB objective function (4). In this way we overcome the negative impact of nonlinear function arcsin $(\cdot)$ on the accuracy of estimation of the DOA. Furthermore, angle and the highest-order coefficient are now independently estimated. Hence, errors in estimation of the DOA do not propagate toward the lower-order PPS coefficients a_k for $k < K$. By combining all these benefits the MSE of parameter estimates is significantly reduced with respect to other methods. The proposed method is referred to as the quasi-PB (QPB), due to similarity of the evaluation scheme for the PB, but with the significant difference in number of searches performed, e.g. only over a single (angle) parameter with respect to the $(K + 1)$-dimensional search as in the original PB.

3.1. Algorithm summary

The QPB algorithm can be summarized as follows.

Algorithm: QPB estimator. set i=0
for each θ_i **from considered set** θ
set k=K
while $k > 0$
for m=0–$M - 1$
Calculate HAF along n axis

$$Y_k^m(\omega, \tau) = \mathrm{DFT}_n[\mathrm{HIM}_k[y_m(n), \tau]]. \tag{10}$$

end for
Sum the obtained HAFs

$$\overline{Y}_k(\omega, \tau) = \sum_m |Y_k^m(\omega, \tau)|. \tag{11}$$

Estimate coefficient a_k as

$$\hat{a}_k^{(i)} = \frac{\mathrm{argmax}_\omega |\overline{Y}_k(\omega, \tau)|}{2^{k-1} k! \tau^{k-1} \Delta^k}. \tag{12}$$

Dechirp the estimated parameter from $y_m(n)$, $m = 0, \ldots, M - 1$ as

$$y_m(n) = y_m(n) \exp(-j \hat{a}_k^{(i)}((n\Delta)^k + k\psi_i m (n\Delta)^{k-1})). \tag{13}$$

where $\psi_i = d \sin(\theta_i)/c$.
set $k = k - 1$
end while
set

$$\mathbf{R}(i) = [\theta_i, \hat{a}_1^{(i)}, \ldots, \hat{a}_K^{(i)}]. \tag{14}$$

set $i = i + 1$
end for
Find the optimal estimate $\mathbf{R}_{\mathrm{opt}}$ from ensemble $\mathbf{R}$ using the PB objective function

$$\mathbf{R}_{\mathrm{opt}} = \arg \max_{\mathbf{R}(i)} \left| \sum_n \mathbf{y}^H(n) \exp\left(j \sum_{k=1}^{K} \hat{a}_k^{(i)} (n\Delta)^k \right) \mathbf{a}(\theta_i, n) \right|^2. \tag{15}$$

Refine $\mathbf{R}_{\mathrm{opt}}$ using Nelder-Mead simplex method [16] with optimization function (4) to obtain $\mathbf{R}_{\mathrm{opt}}^{\mathrm{ref}}$.
The estimated parameters are

$$[\hat{\theta}, \hat{a}_1, \ldots, \hat{a}_K] = \mathbf{R}_{\mathrm{opt}}^{\mathrm{ref}}. \tag{16}$$

3.2. Calculation complexity

Now we derive the calculation complexity of the QPB method for joint estimation of the PPS coefficients and DOA counting only the most demanding operations. In order to calculate the HAF, $Y_k^m(\omega, \tau)$, one has to start with calculation of the HIM in $N_{HIM} = N - 2(k - 1)\tau$ points. Calculation of the Kth order HIM in one point requires $(K - 1)$ -complex multiplications and $2(K - 1)$ -real additions. Hence, the overall calculation complexity of the HIM is $N_{HIM}(K - 1)$-complex multiplications and $2N_{HIM}(K - 1)$-real additions. Furthermore, as the calculation complexity of the DFT is $O(N \log_2 N)$ complex operations (additions and multiplications), the overall complexity of HAF, $Y_k^m(\omega, \tau)$, is $O(N \log_2 N)$. Finally, the overall complexity of the QPB is $O(KMN_\theta N \log_2 N)$, where N_θ is the number of angles in a search set. For the sake of comparison with respect to the other techniques we assume the following $K = 3$, $N_D = N = 200$, $M = 50$ and two different values for $N_\theta \in \{10, 200\}$. Calculation complexity for the PB, R-HAF, and QPB for these assumed values is summarized in Table 1.

3.3. Multicomponent case

The proposed method can be extended to the case of multi-component PPS impinging an ULA of sensors.

In this case, the signal captured by mth sensor is of the following form

$$y_m(n) = \sum_{l=1}^{L} x_m^{(l)}(n) + v_m(n) = \sum_{l=1}^{L} A^{(l)} e^{j(a_0^{(l)} + \sum_{k=1}^{K} a_k^{(l)}((n\Delta)^k + k\psi^{(l)} m (n\Delta)^{k-1})} + v_m(n),\ \psi = d\frac{\sin(\theta^{(l)})}{c},$$

where L is the number of signal components and $x_m^{(l)}(n)$ is the lth signal component with its corresponding vector of parameters $\{\theta^{(l)}, A^{(l)}, a_0^{(l)}, \ldots, a_K^{(l)}\}$. We assume that $A^{(1)} \geq A^{(2)} \geq \cdots \geq A^{(L)}$.

Vector $\{\theta^{(l)}, A^{(l)}, a_0^{(l)}, \ldots, a_K^{(l)}\}$, $l = 1, \ldots, L$ can be estimated in the component-by-component manner which is described as follows. Firstly, parameters of the strongest component, $x_m^{(1)}(n)$, are estimated using the QPB algorithm. Next, in the overall signal, $y_m(n)$, the influence of the estimated strongest component is mitigated by eliminating the DC component from its spectrum. Then, vector of parameters of the next component in terms of power, $x^{(2)}(n)$, are estimated. The procedure is repeated until all signal components are estimated.

The detailed estimation algorithm is given below.

Algorithm: Modification for multicomponent signals. for l=1–L
Estimate vector $\{\theta^{(l)}, A^{(l)}, a_0^{(l)}, \ldots, a_K^{(l)}\}$ of the signal component $x^{(l)}(n)$ using the QPB algorithm.
for m=0–$M - 1$
Demodulate signal $y_m(n)$ using $\{\theta^{(l)}, A^{(l)}, a_0^{(l)}, \ldots, a_K^{(l)}\}$ estimates

$$y_m(n) = y_m(n) e^{j(-\sum_{k=1}^{K} \hat{a}_k^{(l)}((n\Delta)^k + k\hat{\psi}^{(l)} m (n\Delta)^{k-1})}, \ \hat{\psi}^{(l)} = d\frac{\sin(\hat{\theta}^{(l)})}{c}.$$

Eliminate the DC component of $x_m^{(l)}(n)$

Table 1
Comparison of the calculation complexity of the PB, R-HAF and QPB for assumed values $K = 3$, $N_D = N = 200$, $M = 50$ and $N_\theta \in \{10, 200\}$.

PB	R-HAF	QPB
$O(MNN_D^{K+1})$	$O(KMN \log_2 N)$	$O(KMN_\theta N \log_2 N)$
$1.6 \cdot 10^{13}$	$2.3 \cdot 10^5$	$4.58 \cdot 10^7 (N_\theta = 200)$, $2.29 \cdot 10^6 (N_\theta = 10)$

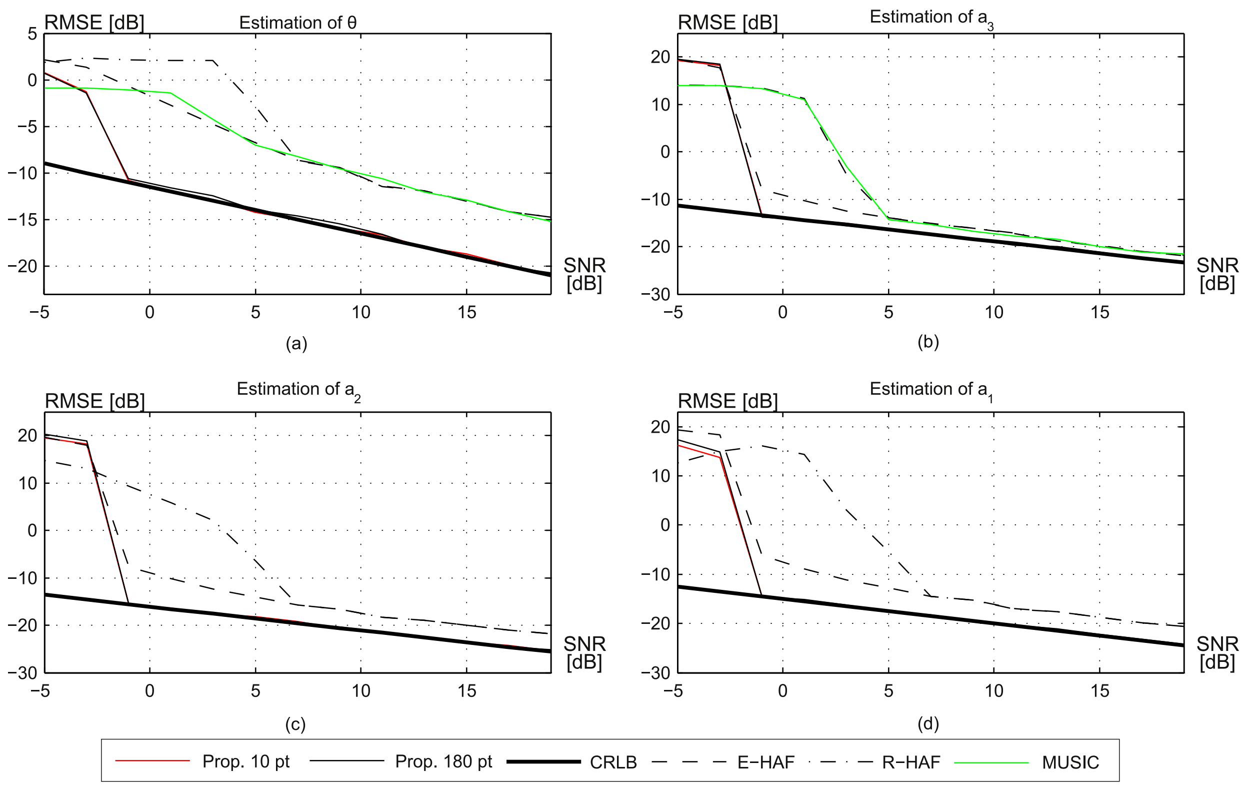

Fig. 1. RMSEs for θ, a_3, a_2 and a_1 as a function of SNR: (a) RMSE for θ, (b) RMSE for a_3, (c) RMSE for a_2 and (d) RMSE for a_1.

$Y_m(\omega) = \mathrm{DFT}\{y_m(n)\}$, $Y_m(0) = 0$, $y_m(n) = \mathrm{IDFT}\{Y_m(\omega)\}$.

Modulate signal $y_m(n)$ using $\{\theta^{(l)}, A^{(l)}, a_0^{(l)}, \ldots, a_K^{(l)}\}$ parameter estimates

$$y_m(n) = y_m(n) e^{j\left(\sum_{k=1}^{K} \hat{a}_k^{(l)}((n\Delta)^k + k\hat{\psi}^{(l)} m (n\Delta)^{k-1})\right)}, \quad \hat{\psi}^{(l)} = d\frac{\sin(\hat{\theta}^{(l)})}{c}.$$

end for
end for

It should be noted that when signal consists of several components, the HAF suffers from cross-terms that degrade performance estimation. To reduce cross-terms influence in the QPB algorithm one should calculate product HAF (PHAF) instead of the HAF [4]. As for the monocomponent case, the parameters' estimates can be further refined using an approach similar to [18].

4. Simulation results

Example 1. The proposed method is compared to the E-HAF, R-HAF and MUSIC-based method from [6]. The third-order PPS is considered as $x(n) = A\exp(j(\pi + 3\pi(n\Delta) + 13\pi(n\Delta)^2 - 7\pi(n\Delta)^3))$, with $|n\Delta| \le 1$, A=1, M=50, N=257 and $\theta = 0.5242$. Search in the proposed approach for DOA is performed using two sets with 10 and 180 equally separated angles in the search space $[0, \pi]$. In both sets of experiments angle $\theta = 0.5242$ is off the search grid. For example, the closest point on the search grid for the case of 10 angles in the DOA search space is 0.6981, i.e. it is offset by 0.1739 rad (about 10°) from the considered θ. The simulation study shows that such a relatively large displacement of the grid does not cause a drop in the performance of the proposed algorithm due to application of the Nelder-Mead simplex method [16]. ULA parameters are $d = 0.2$ m and $c = 1500$ m/s. Performance is evaluated by means of the root MSE (RMSE):

$$RMSE(a) = 10\log_{10}\sqrt{\frac{1}{N_{sim}}\sum_{q=1}^{N_{sim}}(a - \hat{a}_q)^2},$$

where a is the true coefficient value, $\hat{a}_q$ is the estimate in the qth trial and N_{sim} is the number of Monte Carlo simulations. We set $N_{sim} = 200$. Results are shown in Fig. 1, where RMSEs in estimation of θ, a_3, a_2, and a_1 are plotted. RMSE is depicted versus SNR. The considered techniques are also compared with the CRLB.

As it can be seen from Fig. 1 the QPB has a similar performance for both numbers of elements in the DOA set (10 and 180 elements in search sets). Reason for this lies in the fact that the Nelder-Mead simplex method [16] coupled with the optimization function (4) is able to improve results of the estimation procedure. The QPB method outperforms the alternatives from [7,10] (referred to as the E-HAF and R-HAF respectively) and [6] (here referred to as the MUSIC). The SNR threshold for both the QPB and R-HAF is at −1 dB while for the E-HAF it is at about 7 dB. The RMSE for both, E-HAF and R-HAF, is significantly higher than for the QPB. For θ this difference is about 7 dB, for a_3 it is about 2 dB, while for a_2 and a_1 it is about 4 dB above the SNR threshold. The MUSIC-based algorithm has a similar performance as the R-HAF for parameters θ and a_3. RMSEs of lower order phase parameters obtained by MUSIC approach are significantly above the CRLB due to the error propagation effect (they are not shown on Fig. 1c) and d)). It is important to note that the RMSE of the QPB for the entire operational SNR range is very close to the CRLB as it is evident from Fig. 1. Now, if we put our focus on the RMSE of the DOA estimation it is obvious that the QPB significantly improves estimation

Table 2
Parameters of multi-component third-order PPS considered in Example 2.

I Component		II Component	
$\psi^{(1)}$	$4.5603\cdot10^{-5}$	$\psi^{(2)}$	$8.5705\cdot10^{-5}$
$a_3^{(1)}$	−21.991	$a_3^{(2)}$	6.2832
$a_2^{(1)}$	40.841	$a_2^{(2)}$	18.85
$a_1^{(1)}$	9.4248	$a_1^{(2)}$	−2.5133
$a_0^{(1)}$	0.7854	$a_0^{(2)}$	−0.7854

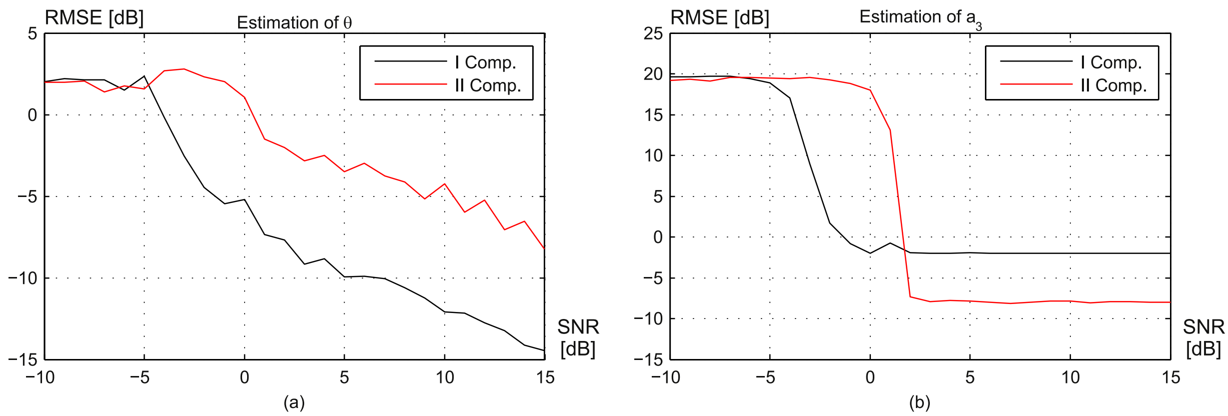

Fig. 2. RMSE of θ and a_3 estimates obtained with 300 Monte Carlo simulations.

accuracy of the DOA, this being our initial goal. Improvement in the estimation accuracy of θ benefits the lower-order PPS coefficients in terms of the RMSE as observable in Fig. 1. Similar conclusions can be drawn for phase coefficient a_0 and amplitude A.

Example 2. The proposed method is evaluated on two-component third-order PPS with amplitudes $A^{(1)} = 1$ and $A^{(2)} = 0.65$ and phase parameters given in Table 2. Number of signal samples and ULA parameters are the same as in Example 1. DOAs for the first and second components are $\theta^{(1)} = 0.3491$ and $\theta^{(2)} = 0.6981$, respectively. Search space over DOA consists of 10 equally separated values from interval $[0, \pi]$. DOA parameters have been chosen is such a way that they are not in the search grid. RMSEs of θ and a_3 obtained using 300 Monte Carlo simulations are plotted in Fig. 2. The x-axis at Fig. 2 corresponds to the SNR for the strongest component $A^{(1)}$, i.e., $\mathrm{SNR} = 10\log_{10}((A^{(1)})^2/\sigma^2)$.

The SNR thresholds of first and second components are at 0 dB and 2 dB, respectively. For parameter θ, RMSE of component I is by 6 dB lower with respect to RMSE of component II. A similar difference applies to a_3 parameter. However, estimation of component II has lower RMSE since it is estimated after elimination of the strongest component, while accuracy of the strongest component estimation is affected by a presence of a weaker component. Accuracy above the SNR threshold can probably be refined using a technique similar to the Pham-Zoubir approach [18].

5. Conclusion

Novel and improved method for joint estimation of DOA and PPS coefficients of signals impinging the ULA is proposed in the paper. PPS coefficients are estimated for each angle from the assumed angle set. Optimal estimates are selected by maximizing the objective function. The accuracy achieved with this method is significantly better for estimates of the DOA and lower-order PPS coefficients than for suboptimal procedures. Calculation complexity analysis for proposed method is also derived. It is shown that small increase in complexity with respect to alternative suboptimal strategies is paid-off by a significant accuracy improvement.

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