



Short communication

Mixed far-field and near-field source localization based on subarray cross-cumulant[☆]Zhi Zheng^{a,*}, Mingcheng Fu^a, Wen-Qin Wang^a, Hing Cheung So^b^a School of Information and Communication Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China^b Department of Electronic Engineering, City University of Hong Kong, Hong Kong, China

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ABSTRACT

This paper presents a new algorithm for mixed far-field and near-field source localization using a uniform linear array (ULA). Firstly, the ULA is divided into two overlapping subarrays to construct two special cross-cumulant matrices of the subarray outputs, which are only characterized by directions-of-arrival (DOAs) of the sources. Then, the shift invariance structure in the cumulant domain is derived, and the DOAs of all sources are estimated by the TLS-ESPRIT method. Finally, with the estimated DOAs, the range estimates of near-field sources are obtained via one-dimensional search, and the types of sources are also distinguished. The developed algorithm involves neither DOA search nor parameter pairing. Furthermore, it exhibits a higher localization accuracy than the traditional methods. Simulation results are presented to demonstrate the performance of the proposed algorithm.

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1. Introduction

Source localization is a fundamental problem in many fields such as radar, sonar, wireless communications, electronic surveillance and seismic exploration [1–4]. In the past decades, lots of high-resolution methods like MUSIC [5] and ESPRIT [6] have been developed to locate far-field (FF) sources, i.e., direction-of-arrival (DOA) estimation. In near-field (NF) source scenarios, both DOA and range parameters need to be estimated, and various algorithms [7–20] were also presented for near-field source localization.

However, in some practical applications, both FF and NF sources may coexist. In the mixed source scenarios, the pure FF or NF localization methods may be invalid. To tackle this issue, some new approaches have been recently developed, including high-order MUSIC [21], second-order MUSIC [22], mixed-order MUSIC [23], sparse reconstruction methods [24,25], spatial differencing methods [26,27] and others [28–30]. The aforementioned approaches [21–25,29] estimate the DOAs of sources using MUSIC spectrums or sparse recovery techniques. Therefore, they are computationally intensive. Although other methods [26–28,30] implement DOA estimation by the generalized ESPRIT method [31] based on two shift subarrays, the generalized ESPRIT method [31] still need to perform

spectral search different from the traditional ESPRIT method [6]. The high-order ESPRIT method [11] can achieve search-free DOA estimates, but it is limited to the near-field source scenarios. Moreover, the high-order ESPRIT method [11] only exploits half of the array degree of freedom and involves parameter pairing processing.

To overcome the above drawbacks, in this paper, we devise a new algorithm for mixed source localization using a uniform linear array (ULA). Firstly, we design two special cross-cumulant matrices based on two overlapping subarrays of the ULA, and derive the shift invariance structure in the cumulant domain. Then, the DOAs of all sources are estimated by the total least squares (TLS)-ESPRIT method. Finally, with the DOA estimates, the ranges of near-field sources are estimated by one-dimensional (1-D) search and the types of sources are also distinguished. Our approach avoids DOA search and parameter pairing process, and it provides an improved localization accuracy. Its superiorities over the traditional methods are verified by simulation results.

Notation: The superscripts $*$, T , H and \dagger denote the conjugate, transpose, conjugate transpose and pseudo-inverse, respectively; $E\{\cdot\}$ represents the statistical expectation, and $\arg(\cdot)$ stands for the argument of a complex number.

2. Signal model

Consider K narrowband sources impinging on a symmetric ULA of $2M + 1$ sensors with inter-element spacing d , as shown in Fig. 1. Suppose the center of array be the phase reference point, the sig-

[☆] Fully documented templates are available in the elsarticle package on CTAN.

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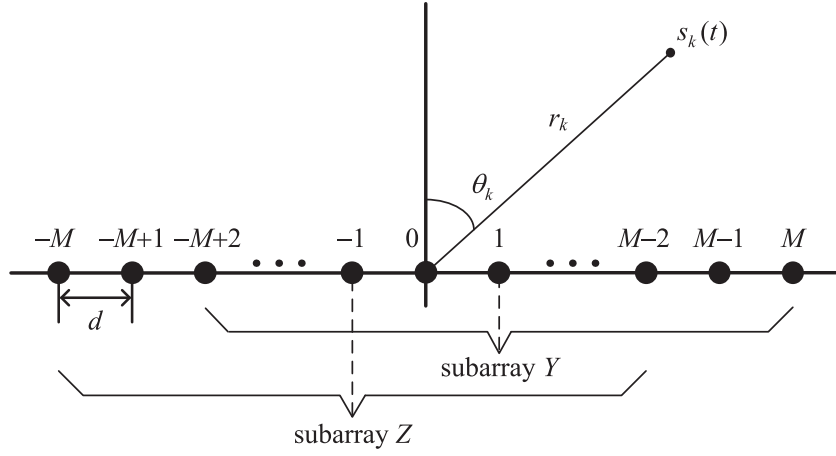


Fig. 1. Uniform linear array configuration.

nal received by the m th sensor can be expressed as [21,22]

$$x_m(t) = \sum_{k=1}^K s_k(t) e^{j\tau_{mk}} + n_m(t) \quad (1)$$

where $s_k(t)$ is the k th source waveform, $n_m(t)$ denotes the m th sensor noise, and τ_{mk} represents the propagation time of the k th source between the 0th and m th sensor. When the k th source is a near-field one, τ_{mk} has the following form:

$$\tau_{mk} = m\omega_k + m^2\phi_k \quad (2)$$

where ω_k and ϕ_k are given by

$$\omega_k = -2\pi \frac{d}{\lambda} \sin \theta_k \quad (3)$$

$$\phi_k = \pi \frac{d^2}{\lambda r_k} \cos^2 \theta_k \quad (4)$$

with λ being the signal wavelength. θ_k and r_k are the DOA and range of the k th source, respectively. According to the definition of the Fresnel region [14], r_k should belong to the Fresnel region, i.e., $r_k \in [0.62(D^3/\lambda)^{1/2}, 2D^2/\lambda]$, where $D = 2Md$ denotes the array aperture. Otherwise, if the k th source is a far-field one, τ_{mk} has the form of:

$$\tau_{mk} = m\omega_k. \quad (5)$$

In matrix form, (1) can be expressed as

$$\mathbf{x}(t) = \mathbf{A}_F \mathbf{s}_F(t) + \mathbf{A}_N \mathbf{s}_N(t) + \mathbf{n}(t) \quad (6)$$

where $\mathbf{y}(t)$ and $\mathbf{n}(t)$ are $(2M+1) \times 1$ complex vectors, and

$$\mathbf{x}(t) = [x_{-M}(t), \dots, x_0(t), \dots, x_M(t)]^T \quad (7)$$

$$\mathbf{A}_F = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_{K_1})] \quad (8)$$

$$\mathbf{A}_N = [\mathbf{a}(\theta_{K_1+1}, r_{K_1+1}), \dots, \mathbf{a}(\theta_K, r_K)] \quad (9)$$

$$\mathbf{s}_F(t) = [s_1(t), \dots, s_{K_1}(t)]^T \quad (10)$$

$$\mathbf{s}_N(t) = [s_{K_1+1}(t), \dots, s_K(t)]^T \quad (11)$$

$$\mathbf{n}(t) = [n_{-M}(t), \dots, n_0(t), \dots, n_M(t)]^T \quad (12)$$

where

$$\mathbf{a}(\theta_k, r_k) = [e^{j(-M\omega_k + M^2\phi_k)}, \dots, 1, \dots, e^{j(M\omega_k + M^2\phi_k)}]^T \quad (13)$$

denotes the $(2M+1) \times 1$ steering vector. Note that in the received signal model (6), the first K_1 sources are assumed to be FF sources and the remaining $(K - K_1)$ are NF sources.

Throughout this paper, the following assumptions are required to hold:

- (1) The source signals are statistically independent, zero-mean random processes with nonzero kurtosis.
- (2) The sensor noise is additive spatially white Gaussian process with zero-mean, and independent of the source signals.
- (3) The source number K is known or accurately estimated by the information theoretic criteria [32].

3. Proposed approach

3.1. DOA estimation of near-field and far-field sources

Based on the above assumptions, the fourth-order cumulant of the array outputs is defined as

$$\begin{aligned} & \text{cum}\{x_m(t), x_p^*(t), x_n^*(t), x_q(t)\} \\ &= \text{cum}\left\{\sum_{k=1}^K s_k(t) e^{j(m\omega_k + m^2\phi_k)}, \left(\sum_{k=1}^K s_k(t) e^{j(p\omega_k + p^2\phi_k)}\right)^*, \right. \\ & \quad \left. \left(\sum_{k=1}^K s_k(t) e^{j(n\omega_k + n^2\phi_k)}\right)^*, \sum_{k=1}^K s_k(t) e^{j(q\omega_k + q^2\phi_k)}\right\} \\ &= \sum_{k=1}^K e^{j\{[(m-p)-(n-q)]\omega_k + [(m^2-p^2)-(n^2-q^2)]\phi_k\}} \\ & \quad \times \text{cum}\{s_k(t), s_k^*(t), s_k^*(t), s_k(t)\} \\ &= \sum_{k=1}^K c_{4,s_k} e^{j\{[(m-p)-(n-q)]\omega_k + [(m^2-p^2)-(n^2-q^2)]\phi_k\}} \end{aligned} \quad (14)$$

where $c_{4,s_k} = \text{cum}\{s_k(t), s_k^*(t), s_k^*(t), s_k(t)\}$ is the kurtosis of the k th source signal, and $m, p, n, q \in [-M, M]$.

Let $p = -m$ and $q = -n$, (14) can be further written as

$$\begin{aligned} & \text{cum}\{x_m(t), x_{-m}^*(t), x_n^*(t), x_{-n}(t)\} \\ &= \sum_{k=1}^K c_{4,s_k} e^{j2m\omega_k} (e^{j2n\omega_k})^*, \quad m, n \in [-M, M]. \end{aligned} \quad (15)$$

As shown in Fig. 1, we divide the ULA into two overlapping subarrays Y and Z to construct two cross-cumulant matrices of the subarray outputs, from which we can derive the shift invariance structure in the cumulant domain. The received vectors of subarrays Y and Z are given by

$$\begin{aligned} \mathbf{y}(t) &= [y_{-M+1}(t), \dots, y_0(t), \dots, y_{M-1}(t)]^T \\ &= [x_{-M+2}(t), \dots, x_1(t), \dots, x_M(t)]^T, \end{aligned} \quad (16)$$

$$\begin{aligned} \mathbf{z}(t) &= [z_{-M+1}(t), \dots, z_0(t), \dots, z_{M-1}(t)]^T \\ &= [x_{-M}(t), \dots, x_{-1}(t), \dots, x_{M-2}(t)]^T. \end{aligned} \quad (17)$$

It is obvious that the m th element of $\mathbf{y}(t)$ is $x_{m+1}(t)$, while the m th element of $\mathbf{z}(t)$ is $x_{m-1}(t)$.

Using the subarray outputs and (15), we construct two cross-cumulant matrices \mathbf{C}_1 and \mathbf{C}_2 . The (\tilde{m}, \tilde{n}) th element of \mathbf{C}_1 and \mathbf{C}_2 are:

$$\begin{aligned} \mathbf{C}_1(\tilde{m}, \tilde{n}) &= \text{cum}\{y_{\tilde{m}-M}, z_{-(\tilde{m}-M)}^*, y_{\tilde{n}-M}^*, z_{-(\tilde{n}-M)}\} \\ &= \text{cum}\{x_{\tilde{m}-M+1}, x_{-(\tilde{m}-M+1)}^*, x_{\tilde{n}-M+1}^*, x_{-(\tilde{n}-M+1)}\} \\ &= \sum_{k=1}^K c_{4,s_k} e^{j2(\tilde{m}-M+1)\omega_k} (e^{j2(\tilde{n}-M+1)\omega_k})^* \\ &= \sum_{k=1}^K c_{4,s_k} e^{j2(\tilde{m}-M)\omega_k} (e^{j2(\tilde{n}-M)\omega_k})^* \end{aligned} \quad (18)$$

$$\begin{aligned} \mathbf{C}_2(\tilde{m}, \tilde{n}) &= \text{cum}\{y_{\tilde{m}-M}, z_{-(\tilde{m}-M)}^*, y_{\tilde{n}-M}^*, y_{-(\tilde{n}-M)}\} \\ &= \text{cum}\{x_{\tilde{m}-M+1}, x_{-(\tilde{m}-M+1)}^*, x_{\tilde{n}-M-1}^*, x_{-(\tilde{n}-M-1)}\} \\ &= \sum_{k=1}^K c_{4,s_k} e^{j2(\tilde{m}-M+1)\omega_k} (e^{j2(\tilde{n}-M-1)\omega_k})^* \\ &= \sum_{k=1}^K c_{4,s_k} e^{j2(\tilde{m}-M)\omega_k} (e^{j2(\tilde{n}-M)\omega_k})^* e^{j4\omega_k} \end{aligned} \quad (19)$$

where $\tilde{m}, \tilde{n} \in [1, 2M-1]$. Note that the cross-cumulant matrices \mathbf{C}_1 and \mathbf{C}_2 are only characterized by the DOAs.

In a compact matrix form, \mathbf{C}_1 is expressed as

$$\mathbf{C}_1 = \mathbf{B}\mathbf{C}_{4s}\mathbf{B}^H \quad (20)$$

where $\mathbf{C}_{4s} = \text{diag}[c_{4,s_1}, c_{4,s_2}, \dots, c_{4,s_K}]$, virtual “array manifold matrix” $\mathbf{B} = [\mathbf{b}(\theta_1), \dots, \mathbf{b}(\theta_K)] \in \mathbb{C}^{(2M-1) \times K}$, and

$$\mathbf{b}(\theta_k) = [e^{-j2(M-1)\omega_k}, e^{-j2(M-2)\omega_k}, \dots, 1, \dots, e^{j2(M-2)\omega_k}, e^{j2(M-1)\omega_k}]^T. \quad (21)$$

Similarly, \mathbf{C}_2 can also be represented as

$$\mathbf{C}_2 = \mathbf{B}\Phi\mathbf{C}_{4s}\mathbf{B}^H \quad (22)$$

where $\Phi = \text{diag}[e^{j4\omega_1}, e^{j4\omega_2}, \dots, e^{j4\omega_K}]$.

Combining \mathbf{C}_1 and \mathbf{C}_2 , we form the $(4M-2) \times (4M-2)$ matrix:

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2^H \\ \mathbf{C}_2 & \mathbf{C}_1 \end{bmatrix} = \tilde{\mathbf{B}}\mathbf{C}_{4s}\tilde{\mathbf{B}}^H \quad (23)$$

where

$$\tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{B}\Phi \end{bmatrix} \in \mathbb{C}^{(4M-2) \times K}. \quad (24)$$

Performing eigenvalue decomposition (EVD) of \mathbf{C} yields

$$\mathbf{C} = \mathbf{E}_s \mathbf{\Lambda}_s \mathbf{E}_s^H + \mathbf{E}_n \mathbf{\Lambda}_n \mathbf{E}_n^H \quad (25)$$

where $\mathbf{\Lambda}_s \in \mathbb{C}^{K \times K}$ and $\mathbf{\Lambda}_n \in \mathbb{C}^{(4M-2-K) \times (4M-2-K)}$ are the diagonal matrices containing the K largest and $(4M-2-K)$ smallest eigenvalues of \mathbf{C} , respectively, $\mathbf{E}_s \in \mathbb{C}^{(4M-2) \times K}$ and $\mathbf{E}_n \in \mathbb{C}^{(4M-2) \times (4M-2-K)}$ are composed of the eigenvectors of \mathbf{C} corresponding to the K largest and $(4M-2-K)$ smallest eigenvalues, respectively.

Based on the subspace theory, \mathbf{E}_s spans the column space of $\tilde{\mathbf{B}}$. This means that there is an invertible $K \times K$ matrix \mathbf{T} such that

$$\mathbf{E}_s = \tilde{\mathbf{B}}\mathbf{T}. \quad (26)$$

Let \mathbf{E}_1 and \mathbf{E}_2 be the upper and the lower $(2M-1) \times K$ half matrices of \mathbf{E}_s , respectively. From (26), we have

$$\mathbf{E}_1 = \mathbf{B}\mathbf{T}, \quad \mathbf{E}_2 = \mathbf{B}\Phi\mathbf{T} \quad (27)$$

The relations in (27) is combined to result in

$$\mathbf{E}_2 = \mathbf{E}_1\Psi \quad (28)$$

where $\Psi = \mathbf{T}^{-1}\Phi\mathbf{T}$. Eq. (28) can be solved by the TLS criterion [6] to find Ψ whose eigenvalues are related to the DOAs of sources. Let \mathbf{V} be the $2K \times 2K$ matrix of right singular vectors of the matrix $[\hat{\mathbf{E}}_1 \hat{\mathbf{E}}_2]$. If the matrix is partitioned into four $K \times K$ submatrices as

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_{11} & \mathbf{V}_{12} \\ \mathbf{V}_{21} & \mathbf{V}_{22} \end{bmatrix}, \quad (29)$$

then the solution of (28) is given by

$$\hat{\Psi} = -\mathbf{V}_{12}\mathbf{V}_{22}^{-1}. \quad (30)$$

Assuming that γ_k is the k th eigenvalue of $\hat{\Psi}$, the DOA estimate of the k th source is

$$\hat{\theta}_k = \sin^{-1} \left(-\frac{\arg(\gamma_k)}{8\pi d/\lambda} \right), \quad k = 1, \dots, K \quad (31)$$

3.2. Source classification and range estimation

Now we perform EVD on the covariance matrix $\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\}$:

$$\mathbf{R} = \mathbf{U}_s \mathbf{\Sigma}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{\Sigma}_n \mathbf{U}_n^H \quad (32)$$

where $\mathbf{\Sigma}_s \in \mathbb{C}^{K \times K}$ and $\mathbf{\Sigma}_n \in \mathbb{C}^{(2M+1-K) \times (2M+1-K)}$ are the diagonal matrices containing the K largest and $(2M+1-K)$ smallest eigenvalues of \mathbf{R} , respectively, $\mathbf{U}_s \in \mathbb{C}^{(2M+1) \times K}$ and $\mathbf{U}_n \in \mathbb{C}^{(2M+1) \times (2M+1-K)}$ are composed of the eigenvectors of \mathbf{R} corresponding to the K largest and $(2M+1-K)$ smallest eigenvalues, respectively.

With the DOA estimates $\{\hat{\theta}_k, k = 1, \dots, K\}$, the range estimates $\{\hat{r}_k, k = 1, \dots, K\}$ are obtained by substituting each $\hat{\theta}_k$ into the following spectral function

$$f(\theta, r) = [\mathbf{a}^H(\theta, r) \mathbf{U}_n \mathbf{U}_n^H \mathbf{a}(\theta, r)]^{-1}. \quad (33)$$

Note that $\hat{\theta}_k$ and \hat{r}_k achieve automatic pairing without any extra processing. In fact, we can easily distinguish the types of sources. When $\hat{r}_k \in [0.62(D^3/\lambda)^{1/2}, 2D^2/\lambda]$, the k th source is a near-field one. On the contrary, when $\hat{r}_k \in [2D^2/\lambda, +\infty)$, the k th source is a far-field one and let \hat{r}_k be ∞ .

3.3. Discussion

1) *Element Spacing Requirement*: It is worth noting that the proposed algorithm requires $d \leq \lambda/8$ to avoid the ambiguity of the phase for the elements of Φ , $e^{j4\omega_k} = e^{-j8\pi d/\lambda \sin \theta_k}$, $k = 1, \dots, K$. 2) *Source Number Limitation*: Since the dimension of fourth-order cumulant matrix \mathbf{C}_1 and \mathbf{C}_2 is $2M-1$, the proposed algorithm can locate $2M-2$ sources at most using a ULA of $2M+1$ sensors. In contrast, second-order MUSIC and high-order MUSIC

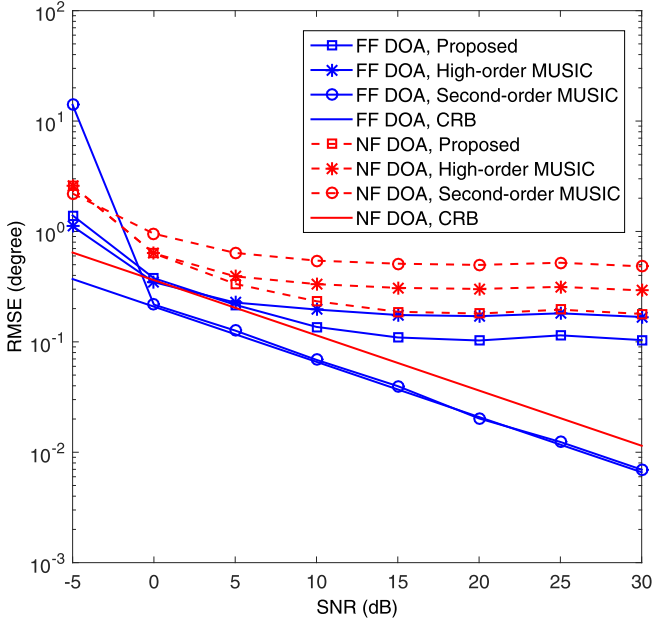


Fig. 2. RMSEs of DOA estimates for one FF and one NF source versus SNR.

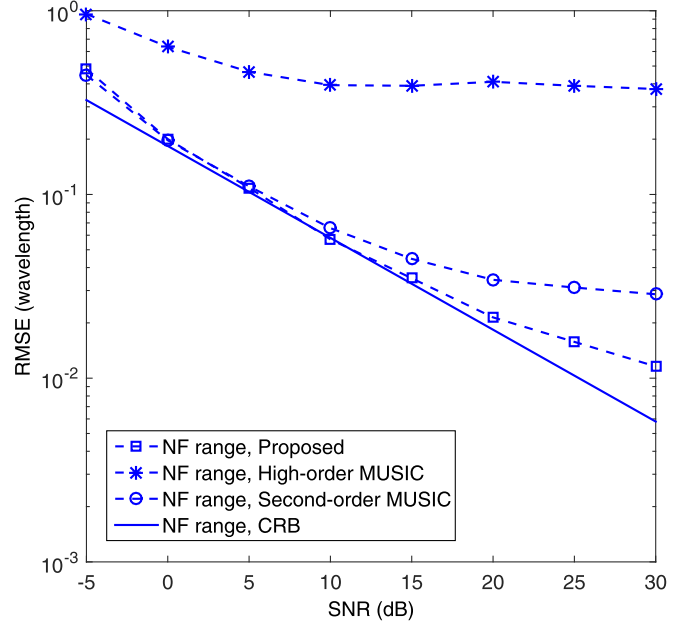


Fig. 3. RMSEs of range estimates for one FF and one NF source versus SNR.

can deal with up to M and $2M$ sources, respectively [21,22]. 3) *Computational Complexity*: For the proposed algorithm, the major computations involve cumulant matrix construction, covariance matrix construction, their EVDs and range search, the resulting multiplications required are $O(18(2M-1)^2N + \frac{4}{3}(4M-2)^3 + (2M+1)^2N + \frac{4}{3}(2M+1)^3 + K_S(2M+1)^2)$, where S_r is the number of search points in the Fresnel region. While high-order MUSIC and second-order MUSIC need to perform DOA search besides range search, and their multiplications are respectively $O(9(2M+1)^2N + 9(4M+1)^2N + \frac{4}{3}(2M+1)^3 + \frac{4}{3}(4M+1)^3 + \frac{4}{3}(2M+1)^2K + S_\theta(2M+1)^2)$ and $O((2M+1)^2N + (M+2)^2N + \frac{4}{3}(2M+1)^3 + \frac{4}{3}(M+2)^3 + 2S_\theta(2M+1)^2 + (K-K_1)S_r(2M+1)^2)$, where S_θ is the number of search points in the angular domain. Note that the proposed approach does not include $O(S_\theta(2M+1)^2)$, while both second-order MUSIC and high-order MUSIC contain a complexity of $O(S_\theta(2M+1)^2)$. Obviously, the proposed approach has lower computational burden than high-order MUSIC.

4. Simulation results

In this section, simulation examples are presented to assess the proposed algorithm, which is compared to the high-order MUSIC [21], second-order MUSIC [22] and Cramer–Rao bound (CRB) [22]. A 11-element ULA with inter-element spacing $d = \lambda/8$ is considered, and its Fresnel region is $0.8665\lambda < r < 3.125\lambda$. The additive noise is assumed to be spatial white complex Gaussian, and the signal-to-noise ratio (SNR) is defined relative to each signal. The results are evaluated by the root mean square error (RMSE) based on 500 Monte-Carlo trials.

In the first example, we consider the mixed source scenario with one far-field source and one near-field source, which are located at $(-5^\circ, +\infty)$ and $(55^\circ, 1.5\lambda)$, respectively. The SNR varies from -5 dB to 30 dB, and the number of snapshots is set as $N = 600$. Figs. 2 and 3 respectively display the RMSEs of DOA and range estimates using the proposed algorithm. For comparison, the RMSEs of high-order MUSIC, second-order MUSIC and the CRB are also presented. As it can be seen, second-order MUSIC has the best performance for FF DOA estimation, which is close to the CRB across a wide range of SNR, but it suffers severe performance degradation when the SNR is less than 0 dB. In contrast, the proposed

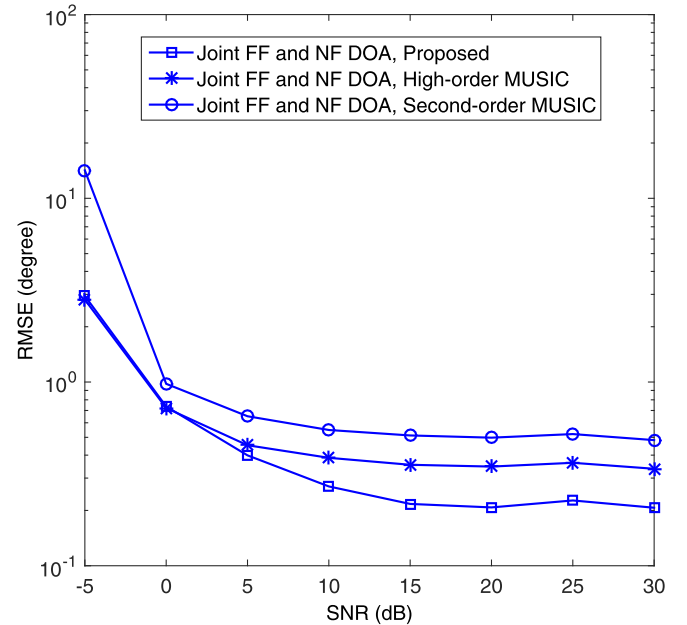


Fig. 4. RMSEs of joint FF and NF DOA estimates for one FF and one NF source versus SNR.

method and high-order MUSIC achieve better performance at low SNRs because the cumulants can restrain the noise effect. For NF DOA estimation, the proposed method outperforms both second-order MUSIC and high-order MUSIC. Jointly considering FF and NF DOA estimation, the proposed method is also better than the latter two, as shown in Fig. 4. For NF range estimation, the proposed method has obvious advantages over high-order MUSIC, and it also outperforms second-order MUSIC at high SNRs.

In the second example, we consider the situation with only two near-field sources. The other simulation parameters are the same with those in the first example, except that two near-field sources are located at $(-5^\circ, 1.45\lambda)$ and $(50^\circ, 1.6\lambda)$. The RMSEs of DOA and range estimates using the proposed algorithm are respectively shown in Figs. 5 and 6, where the RMSEs of high-order MUSIC,

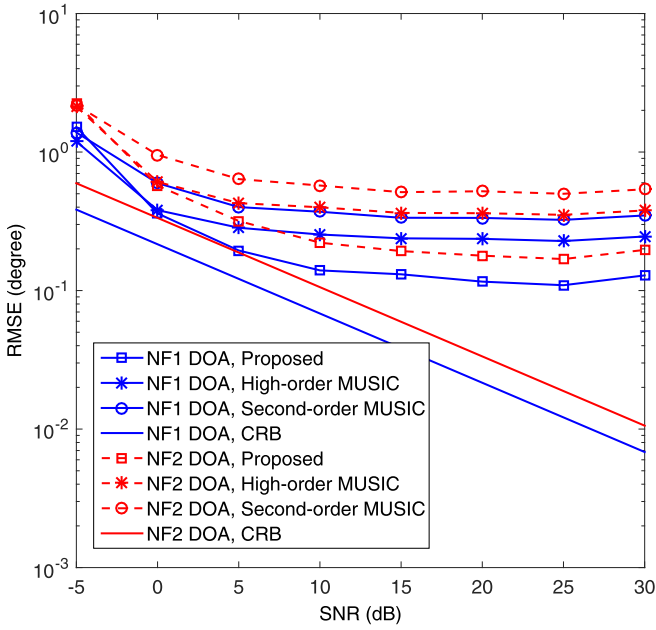


Fig. 5. RMSEs of DOA estimates for two NF sources versus SNR.

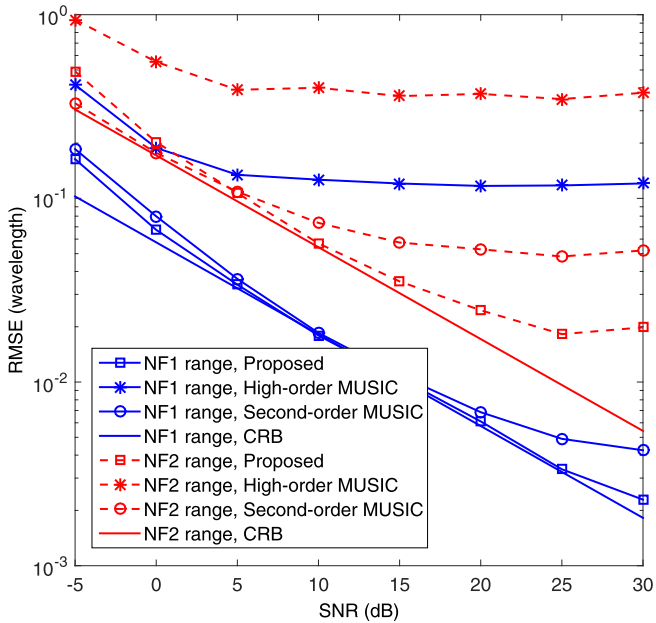


Fig. 6. RMSEs of range estimates for two NF sources versus SNR.

second-order MUSIC and the CRB are also plotted for comparison. For two NF sources, the proposed method exhibits better DOA estimates than both high-order MUSIC and second-order MUSIC across a wide range of SNR. For NF range estimation, both the proposed method and second-order MUSIC are obviously better than high-order MUSIC, but the proposed method outperforms second-order MUSIC at high SNRs. Therefore, our approach has higher localization accuracy than Liang and Liu [21] and He et al. [22] in near-field source scenarios.

5. Conclusion

In this paper, a new approach for mixed source localization has been developed. The presented approach performs DOA estimation based on the shift invariance structure in the cumulant do-

main, and estimates the ranges of sources by 1-D search. Compared to the existing methods, our approach does not require DOA search and parameter pairing, and thus it is computationally more efficient. Simulation results indicate that the presented approach achieves better performance than traditional methods including the second-order MUSIC and high-order MUSIC in near-field and mixed source scenarios.

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