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Single snapshot DOA estimation by compressive sampling

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ABSTRACT

Direction-of-arrival (DOA) estimation has attracted a lot of attention on variety of research areas. A lot of high-resolution DOA estimation algorithms has been proposed, including MUSIC and ESPRIT. Most of these methods are subspace-based and won not work when the snapshot number is limited. In this paper, a high-resolution method with single snapshot is proposed basing on compressive sampling theory. The echoes are sparsely recovered in spatial domain, and high-resolution DOA estimation can be described with a underdetermined equation solving problem. Three algorithms, including diagonal loading least squares, ℓ_1 Regularization, and Orthogonal Matching Pursuit (OMP), are adopt to solve the problem. In comparison with MUSIC, the novel method have several advantages: Firstly, it can be applied in the single snapshot scenario. Secondly, it does not estimate source number. Thirdly, it works well when the sources is coherent. Fourthly, it also works when the sensor number is less than source number. Simulation results show that compressive sampling methods can estimate the DOA accurately, especially for large array scenario.

1. Introduction

In order to locate the emitting sources, DOA estimation is a basic and important technique in array signal processing, and has broad applications in sonar, radar, etc. There are a lot of high-resolution DOA estimation algorithms, such as MUSIC, ESPRIT. Most of these methods are subspace-based, requiring a correlation matrix which depends on the statistics of array measurement. When the snapshot number is limited or even only one as the statistics is not quite accurate, and subspace methods and spatial spectrum estimation by beamforming degrade dramatically.

The keypoint of most former algorithms for single snapshot DOA estimation is to reconstruct the correlation matrix and its inverse. Literature [1] utilizes the rank-1 correlation matrix $\boldsymbol{R}$, which is calculated from one snapshot of array measurement, and the inverse matrix is replaced by the pseudo-inverse $\boldsymbol{R}^{\dagger} = \boldsymbol{R}/\mathrm{tr}\,(\boldsymbol{R}\boldsymbol{R}^{\mathrm{H}})$. However, its resolving capability is not so good.

Compressive sampling (CS), or compressed sensing, makes a lot of progress in the last decade and impressive performance has been reported in several applications such as fusion of images, ultra wideband communications [2], and underwater acoustic communications [3].

In this paper, CS is introduce to single snapshot DOA estimation. First, a novel signal model is proposed by sampling sparsely in the spatial domain. And then, the CS techniques are adopted to solve the underdetermined system. The method escapes from estimating source number and does not need to restrict the sources' correlativity. Furthermore, it is still effective when the sensor number is less than the source number.

This paper is organized as follows. Section 2 presents the sparse sampling based novel model. In Section 3, diagonal loading least squares, ℓ_1 Regularization, and Greedy pursuit are introduced to estimate DOAs. Simulation results are provided to illustrate the performances comparing with MUSIC algorithm. Finally, the conclusions are summarized in Section 5.

2. Signal model by sparse representation

Without loss of generality, we assume a uniform-linear-array (ULA) of M elements, spaced half of the center wavelength apart. Consider K far field narrowband waves impinging the array from different directions. The received array data can be seemed as a sum of K echoes from multiple paths, with different directions θ_k:

$$\boldsymbol{x}(t) = \sum_{k=1}^{K} \boldsymbol{a}_k s_k(t) + \boldsymbol{n}(t) \tag{1}$$

where $s_k(t)$ is the source signal and $\boldsymbol{a}_k$ is the steering vector. $n(t)$ is additive Gaussian noise and uncorrelated with the source signals. $\boldsymbol{x}(t) \in \mathbb{C}^M$.

Aim to estimate the DOA, a direction response $\boldsymbol{h}$ is considered as:

$$h(t,\vartheta) = \sum_{k=1}^{K} \sigma_k(t)\delta(\vartheta - \theta_k) = \begin{cases} \sigma_k(t), & \vartheta = \theta_k \\ 0, & else \end{cases} \tag{2}$$

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where $\sigma_k(t)$ denotes the kth signal's amplitude at t. A parameter set of ϑ is defined: $\vartheta \in \Theta = \{\vartheta_1, \vartheta_2, \ldots, \vartheta_N\}$. For ULA, Θ can be set from $-90°$ to $90°$ with a step $\Delta\vartheta$, or a pre-estimated direction range. N is the spatial sampling number. $\boldsymbol{h} = [h(\vartheta_1), \ \ldots, \ h(\vartheta_N)]^{\mathrm{H}}$. $\boldsymbol{A} = [\boldsymbol{a}(\vartheta_1), \ \ldots, \ \boldsymbol{a}(\vartheta_N)]$. $\boldsymbol{h} \in \mathbb{C}^N$, $\boldsymbol{A} \in \mathbb{C}^{M\times N}$. As a result, Eq. (1) can be represented:

$$\boldsymbol{x}_i = \boldsymbol{A}\boldsymbol{h}_i + \boldsymbol{n}_i, \quad i = 1, \ldots, N_t \tag{3}$$

$\boldsymbol{x}_i$ is the ith snapshot and $\boldsymbol{x}_i = \boldsymbol{x}(i\Delta t)$. Δt is the sampling period and N_t snapshots are observed. $\boldsymbol{X} = [\boldsymbol{x}_1, \ \ldots, \ \boldsymbol{x}_{N_t}]$. $\boldsymbol{X} \in \mathbb{C}^{M\times N_t}$. A direction response can be estimated from one snapshot, and it should be a sparse vector containing only few nonzero entries. The Least Squares solution is gotten from the optimization problem:

$$\min_{\boldsymbol{h}} \|\boldsymbol{x} - \boldsymbol{A}\boldsymbol{h}\|^2 \tag{4}$$

In practical case, $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A}$ is not invertible, and in hence, the performance of LS by $\boldsymbol{A}$'s Pseudo-inverse is not satisfactory. Several procedures are introduced to solve the problem in next section. Furthermore, the procedure is not only constrained in ULA. If the array is extended to other shapes, such as planar array, 2-D direction can be estimated with a 2-D grid.

3. Super resolution by compressive sampling

Take into account $\boldsymbol{h}$'s sparsity, the problem is formulated by the ℓ_0-norm:

$$\min_{\boldsymbol{h}} \|\boldsymbol{x} - \boldsymbol{A}\boldsymbol{h}\|^2 + \lambda\|\boldsymbol{h}\|_0 \tag{5}$$

$\lambda > 0$, and $\|\boldsymbol{h}\|_0$ counts the number of $\boldsymbol{h}$'s nonzero components. The other form of ℓ_0-norm minimization is K-sparse approximation

$$\min_{\boldsymbol{h}} \|\boldsymbol{x} - \boldsymbol{A}\boldsymbol{h}\|^2, \ s.t. \ \|\boldsymbol{h}\|_0 \leqslant K \tag{6}$$

It's NP-hard combinatory optimization problem and the nonzero components are constrained below a certain value K. Two categories of sub-optimal strategies have been adopted to approximately solve the problem, relaxation of the cost function, and greedy pursuit strategies or non-convex local optimization.

3.1. Relaxation of the cost function

The regularization techniques are operated to relax the cost function by replacing ℓ_0-norm with convex norm. Some extra terms are added to the cost function. A generalized form for weighted norm approximation problem is

$$\min_{\boldsymbol{h}} \|\boldsymbol{x} - \boldsymbol{A}\boldsymbol{h}\|^2 + \|\boldsymbol{W}\boldsymbol{h}\|_p \tag{7}$$

or

$$\min_{\boldsymbol{h}} \|\boldsymbol{W}\boldsymbol{h}\|_p, \ s.t. \ \|\boldsymbol{x} - \boldsymbol{A}\boldsymbol{h}\|^2 < \epsilon \tag{8}$$

When $\boldsymbol{W}$ is $\lambda\boldsymbol{I}$ ($\boldsymbol{I}$ is an identity matrix) and $p = 0$, it degrades to Eq. (5). However, Eq. (5) is not a convex problem. If $p \geqslant 1$, it would transform to a convex optimization problem and many techniques, such as interior-method [4], can be applied. Some well-established software can solve the problem efficiently, e.g. SeDuMi toolbox [5].

3.1.1. Diagonal loading least squares (ℓ_2 Regularization)

Diagonal Loading Least Squares (DL-LSs) is suggested [6] to estimate time delay. It can be used here to estimate DOAs and means an answer to an optimization problem:

$$\min_{\boldsymbol{h}} \|\boldsymbol{x} - \boldsymbol{A}\boldsymbol{h}\|^2 + \|\boldsymbol{W}\boldsymbol{h}\|_2^2 \tag{9}$$

$$\hat{\boldsymbol{h}} = (\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A} + \boldsymbol{W}^{\mathrm{H}}\boldsymbol{W})^{-1}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{x} \tag{10}$$

For simplicity, $\boldsymbol{W} = \lambda\boldsymbol{I}$. Introducing a regularization item makes the result to satisfy $(\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A} + \boldsymbol{W}^{\mathrm{H}}\boldsymbol{W})\boldsymbol{h} = \boldsymbol{A}^{\mathrm{H}}\boldsymbol{x}$instead of $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A}\boldsymbol{h} = \boldsymbol{A}^{\mathrm{H}}\boldsymbol{x}$. If $\boldsymbol{W}$ is orthogonal to $\boldsymbol{h}$, i.e. $\boldsymbol{W}\boldsymbol{h} = \boldsymbol{0}$, the veracity would not be harmed. However, $\boldsymbol{W}$ is difficult to determine. In order to simplify the function, a sub-optimal solution is proposed: The primary estimated response, $\bar{\boldsymbol{h}}$'s reciprocal is introduced as diagonal loading item. It should be noted that the absolute value should be calculated to keep the loading items' positive. $\boldsymbol{W} = \mathrm{diag}(\boldsymbol{1}./\,|\,\bar{\boldsymbol{h}}|)$. $\bar{\boldsymbol{h}} = (\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A} + \lambda\boldsymbol{I})^{-1}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{x}$, where λ is set as the noise's power. λ is of small importance, because re-loading weakens improper λ's impact.

DLLS is a case of Tikhonov regression, of which the Tikhonov matrix is $\boldsymbol{W}$.

3.1.2. ℓ_1 Regularization

Unfortunately, ℓ_2 minimization is difficult to find a K-sparse solution, instead returning a non-sparse result with many nonzero elements. It also shows in simulation A that DLLS has the poorest resolution capability. Hence, ℓ_1 minimization is dominate for sparse recovery. LASSO is short for Least Absolute Selection and Shrinkage Operator, estimating Least Squares parameters with a penalty on the ℓ_1 norm, and several different methods are presented in [7]. In our single snapshot DOA application, when the signals are complex signals, $\boldsymbol{A}$ and $\boldsymbol{x}$ in Eq. (5) for LASSO should be replaced by [real ($\boldsymbol{A}$);imag ($\boldsymbol{A}$)] and [real ($\boldsymbol{x}$);imag ($\boldsymbol{x}$)].

Most of the relaxation algorithms for Eq. (5) are some special cases of Eq. (8), while LASSO with $\boldsymbol{I} = \boldsymbol{W}$ and $p = 1$.

3.2. Greedy pursuit

Matching Pursuit is a classical greedy algorithm [8]. By projecting $\boldsymbol{h}$ automatically to all of the selected elements orthogonally, The vector $\boldsymbol{h}$ is iteratively calculated. In the first iteration, the approximation error is the signal itself, and the nonzero element of $\boldsymbol{h}$ is supposed as one. After an implementation, the index is determined corresponding to the largest item of inner product between the approximation error and $\boldsymbol{A}$'s columns. Some researchers proposed the improved methods. Orthogonal Matching Pursuit [9] achieves better performance by projecting $\boldsymbol{h}$ to the dictionaries. OMP requires more computation and storage cost than MP. [10] proposed fast approximations, in form of Gradient Pursuit, Conjugate Gradient Pursuit and Approximate Conjugate Gradient Pursuit. These methods have similar computation complexities and memory requirements.

4. Simulation

In the numerical simulations, single-snapshot DOA is performed by several CS tools that have been introduced in the last section.

4.1. Algorithms compare

A ULA with 30 sensors is assumed. The received data is composed of five narrowband signals shown in Table 1. The center frequency $f_c = 2$ KHz and the sampling frequency $f_s = 10$ KHz. 100 snapshots are observed.

All of the three CS methods can estimate DOA accurately in Fig. 1. OMP is computed by QR decomposition, and LASSO by Grafting [10]. Single-snapshot estimation results are fluctuant with time. The average of single-snapshot estimation results are also shown in Figs. 1 and 2. The CS methods are robust for both coherent and incoherent signals, and can be applied in scenario of single snapshot, while MUSIC requires snapshot number larger than sensor number. Furthermore, the CS methods are still effective when sources number is greater than sensor number in Fig. 3.

Table 1
Directions and SNR of the signals.

	Signal 1	Signal 2	Signal 3	Signal 4	Signal 5
Direction/°	−35	−15	0	5	40
SNR/dB	0	5	10	5	0

4.2. Detection probability of single snapshot vs. SNR

The random noise disturbance will cause numerically instability and some echoes may be failed to be estimated. The phenomena is especially severe in low SNR or little sensor-number case. Only signal 3 is being in the received data and other simulation conditions are same as in simulation A. Fig. 4 shows the detection probability becomes larger with SNR in the single snapshot scenario.

4.3. Resolving probability vs. sensor-number

Keep only the signal 3 and 4 with SNRs both 5 dB. Change the sensor number to observe resolving probability. θ_1 and θ_2 are the real directions, when $\hat{\theta}_1$ and $\hat{\theta}_2$ are the estimated ones. For an experiment, if $|\hat{\theta}_i - \theta_i| \leqslant \zeta$, and $|\hat{\theta}_1 - \theta_1| + |\hat{\theta}_2 - \theta_2| < |\hat{\theta}_1 - \hat{\theta}_2|$, the

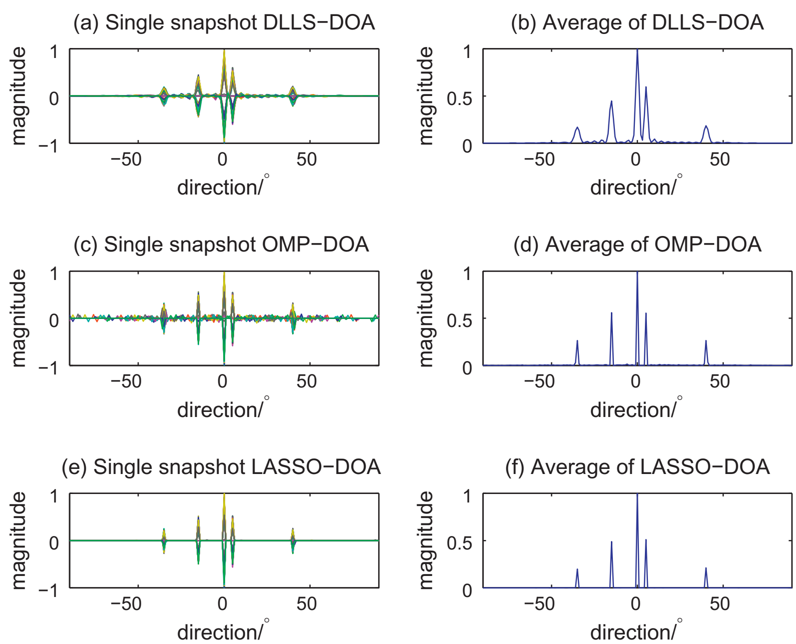

Fig. 1. DOA estimation when $M = 30$, $K = 5$.

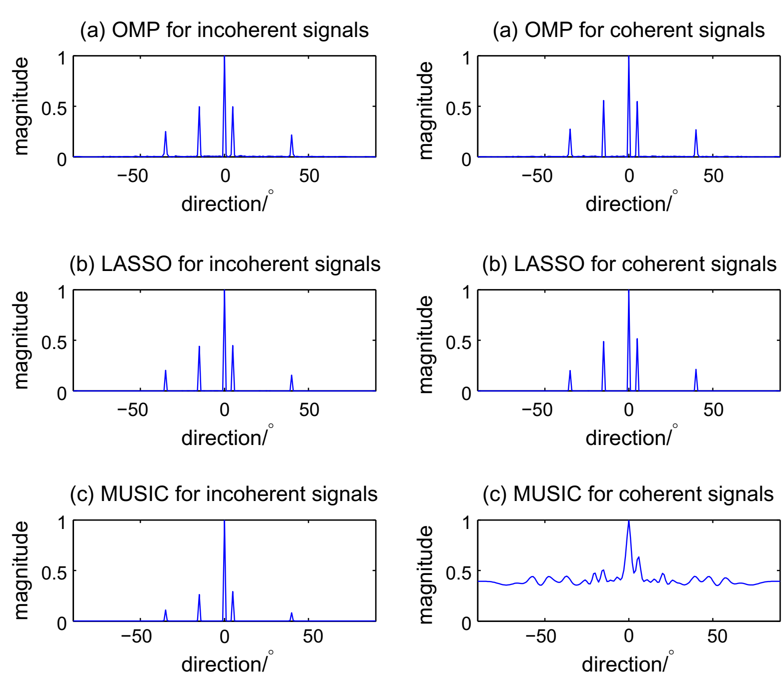

Fig. 2. Uncorrelated and correlated signals DOA estimation by MUSIC and CS.

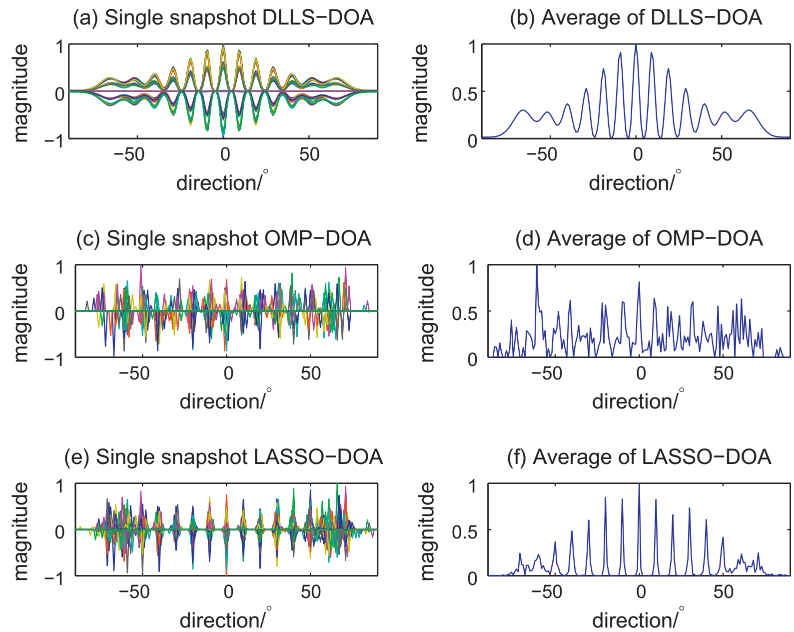

Fig. 3. DOA estimation when $M = 14$, $K = 15$.

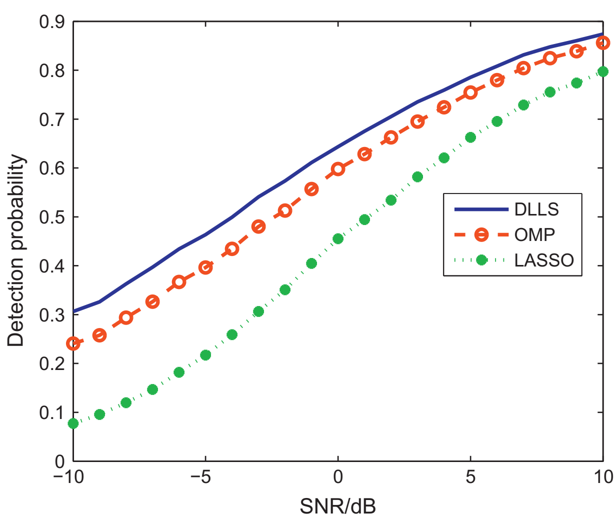

Fig. 4. Detection probability of single snapshot vs. SNR.

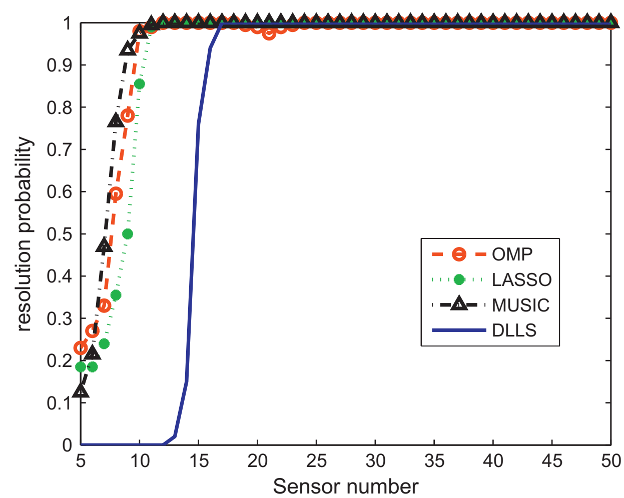

Fig. 5. Resolving probability vs. M.

two echoes are distinguished successfully; otherwise, they are distinguished unsuccessfully. Some little positive ζ denotes error threshold to determine weather the echo estimated exactly and is set as 1° herein. N_{est}experiments are done and N_{sucess} ones are successful. N_{sucess}/N_{est} is resolving probability. Resolving probability is gained by 200 Monte Carlo simulations of uncorrelated sources as Fig. 5. The average results of single snapshot by CS's has similar performance as MUSIC for uncorrelated signals.

5. Conclusion

Basing on compressive sampling theory, single snapshot DOA estimation is transformed to underdetermined equation solving problem, which can be further worked out with CS favorite methods including convex optimization and greedy pursuit. Simulations show the proposed method's excellent performance. More details, including the computing quantities and the differences between the above CS methods for this underdeterminated system, will be studied in the future.

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