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Transmit beamforming for DOA estimation based on Cramer–Rao bound optimization in subarray MIMO radar

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ABSTRACT

Compared to conventional phased-array radar, MIMO radar benefiting from its extra degrees of freedom brought by waveform diversity allows to optimize the Cramer–Rao Bound (CRB) for Direction-of-arrival (DOA) estimation more freely. In this paper, under the premise that the general angular directions of targets are known as priori, a new transmit beamforming method for subarray MIMO radar is proposed with the application to improve the performance of DOA estimator for multiple targets. The CRB expression for DOA estimation of subarray MIMO radar is derived firstly. Then, the correlation matrix of the transmitted waveforms is optimized to minimize the CRB for DOA estimation. Once the optimized correlation matrix is determined, eigendecomposition method is applied to calculate the subarray beamforming weights. Meanwhile, fewer orthogonal waveforms are transmitted in the proposed method compared to conventional MIMO radar, which means that less number of subarrays will be used. The reduction in the number of transmitted orthogonal waveforms can effectively reduce the computational complexity. The proposed method obtains the optimized tradeoff between the effective aperture of virtual array and coherent gain, and consequently improves the performance of DOA estimator. Simulation results show that the proposed method has a superior performance compared with the existing methods.

1. Introduction

In recent years, MIMO radar as an emerging field of radar research has attracted more and more attentions of scientific researchers [1,2]. Depending on the array element configuration, MIMO radar can be classified into two categories: separated distributed MIMO radar [3] and co-located distributed MIMO radar [4,5]. Compared with conventional phased-array radar, MIMO radar may have a similar array structure, whereas the transmitted waveforms may be quite different from each other. Due to the extra degrees of freedom offered by waveform diversity, MIMO radar obtains superior capabilities compared with conventional phased-array radar, such as higher resolution and better parameter identifiability[[6–8].

Estimating DOAs of multiple targets is one of the most important applications of radar system in practice. Some classic DOA estimation algorithms have been applied to MIMO radar, such as ESPRIT and MUSIC [9–13]. The methods proposed in [9,10] both take full advantage of the rotational invariance property of the uniform linear array to estimate the DOA of target. The ESPRIT-like algorithm for non-uniform array MIMO radar is also proposed in [12]. As we all know, spatial angular resolution of radar is inversely proportional to the effective aperture of array. For the reason that larger virtual effective aperture can be obtained in MIMO radar, in some scenarios, these

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algorithms applied by MIMO radar will have better performance than that applied by conventional phased-array radar. However, the performance of DOA estimation algorithms is still affected by other factors. For example, ESPRIT algorithm is also subject to SNR [14]. MIMO radar transmitting orthogonal waveforms is faced with the problem of SNR gain loss, which is unfavorable for DOA estimation.

In order to mitigate the effects of the SNR gain loss, there are many energy focus methods proposed for MIMO radar recently. These methods can be generally classified into two categories.

One is utilizing partially correlated signals as the transmitted waveforms [15–18], which we refer to as Partially Correlated Waveforms (PCW) MIMO radar for convenience. The basic idea of PCW MIMO radar is to optimize the correlation matrix of transmitted waveforms firstly, and then to design the partially correlated signals according to the optimized correlation matrix. By focusing the transmit energy within the interested regions, this method will achieve better parameter identifiability. Meanwhile, some partially correlated signals design methods are developed for PCW MIMO radar [19–21]. The BPSK and polyphase coded signals which have the approximately correlation matrix to a desired one can be obtained respectively by the methods proposed in [19,20] with high computational cost.

The other category is subarray MIMO radar. The transmit array of the subarray MIMO radar is divided into several subarrays, and the transmit waveforms are coherent within each subarray while orthogonal among the subarrays. Actually, subarray MIMO radar can be regarded as a tradeoff between phased-array and MIMO radar, and thus it is also called as phased-MIMO radar in [22]. Overlapped and non-overlapped subarray configurations are discussed in [22,23] and [24] respectively. However, one of the disadvantages in subarray MIMO radar is that the transmit beam-pattern optimization with respect to the beamforming weights of the subarrays is often hard to be resolved by convex optimization algorithms directly. Besides, there is no adaptive principle for the division of subarrays so far. In essence, PCW MIMO radar and subarray MIMO radar both sacrifice partial effective aperture of virtual array for coherent transmit gain. The signal models of them are equivalent mathematically.

In this paper, a new transmit beamforming algorithm based on the optimization of CRB for DOA estimation is proposed for subarray MIMO radar, under the premise that the general angular directions of targets are known as priori. Firstly, the correlation matrix of transmitted waveforms is optimized to minimize the CRB for DOA estimation. It is worth noting that the optimized correlation matrix of transmit waveforms is obtained without considering the configurations of the subarrays. Secondly, eigendecomposition method is applied in the optimized correlation matrix to determine the configurations of the subarrays. Generally, the correlation matrix of transmitted waveforms is not always full rank, which makes it possible to transmit less number of orthogonal waveforms in the proposed method. Although less number of orthogonal waveforms will result in the reductions of the effective aperture of virtual array and the number of DOAs that can be estimated, the SNR gain of each virtual array will be increased and the huge amount of computational complexity of MIMO radar at the receiver will be reduced. In fact, part of the extra degrees of freedom are converted into transmit coherent gain, which will improve the DOA estimation performance. Therefore, the proposed method develops a way to achieve the optimized tradeoff between the effective aperture of virtual array and coherent gain, which obtains the optimized DOA estimation performance.

Actually, the optimization method based on CRB is proposed in [18], but it cannot be applied in subarray MIMO radar directly because the algorithm is based on PCW MIMO radar. The concept of subarray MIMO radar is first proposed in [22] and the further researches can be found in [26]. Nevertheless, the methods of optimization for subarray beamforming weights or the subarry division are not investigated by them. The particular contributions of this paper are as follows. Firstly, the CRB for DOA estimation of subarray MIMO radar are derived, and the CRB based optimization method for PCW MIMO radar is extended to subarray MIMO radar. Then, the method we proposed provides an effective principle for the division of subarrays and determining the minimum number of transmitted orthogonal waveforms. Meanwhile, the proposed method also enables us to design the correlation matrix more freely without consideration of the partially correlated signals design. Due to the optimized tradeoff between the effective aperture of virtual array and coherent gain, the proposed method can achieve a better DOA estimation performance with less computational complexity compared to the existing algorithms.

The paper is organized as follows. In Section 2, we briefly introduce the signal models of conventional MIMO radar, PCW MIMO radar and subarray MIMO radar respectively. In Section 3, a new transmit beamforming algorithm based on CRB optimization is proposed for subarray MIMO radar. The performance of the proposed algorithm is analyzed in Section 4. The simulations results that show the advantages of the proposed method are presented in Section 5 which is followed by the conclusions in Section 6.

2. Signal models of MIMO radar

2.1. Conventional MIMO radar

Consider a MIMO radar equipped with a transmit array of M_t antennas and a receive array of M_r antennas. The transmit and receive arrays are both uniform linearly array (ULA), and they are assumed to be closely located so that both of them can see a target located in the far-field at the same spatial angle. In conventional MIMO radar, the M_t antennas of transmit array are used to transmit M_t orthogonal waveforms. Assume that there are K targets existing in the far-field, then the complex envelop of the signal received at the kth target is modeled as

$$\mathbf{r}_k(\mathbf{t}) = \sqrt{\frac{\mathbf{E}}{\mathbf{M_t}}}\boldsymbol{a}(\boldsymbol{\theta}_\mathbf{k})^T \boldsymbol{\Phi}(\mathbf{t}) \tag{1}$$

where $\boldsymbol{a}(\theta)$ represents the transmit steering vector, θ_k represents the spatial angle of the kth target to the transmit array, $\mathbf{\Phi}(t)$ is $M_t \times 1$ normalized transmitted orthogonal waveforms, E denotes the total transmit energy, and $(\cdot)^T$ stands for the transpose. Then, the output of the receive array can be written as

$$\mathbf{X}(t) = \sum_{k=1}^{K} \alpha_k \boldsymbol{b}(\theta_k) r_k(t) + \boldsymbol{Z}(t) = \sqrt{\frac{E}{M_t}} \sum_{k=1}^{K} \alpha_k \boldsymbol{b}(\theta_k) \boldsymbol{a}(\theta_k)^T \mathbf{\Phi}(t) + \boldsymbol{Z}(t) \quad (2)$$

where α_k represents the reflection coefficient of the kth target, $\boldsymbol{b}(\theta)$ represents the receive steering vector, and $\mathbf{Z}(t)$ is $M_r \times 1$ zero-mean white Gaussian noise term with the variance of σ^2. Eq. (2) can also be rewritten as

$$\boldsymbol{X}(t) = \sqrt{\frac{E}{M_t}} \mathbf{BSA}^T \boldsymbol{\Phi}(t) + \boldsymbol{Z}(t) \quad (3)$$

where

$$\mathbf{B} = [\boldsymbol{b}(\theta_1), \ldots, \boldsymbol{b}(\theta_K)] \quad (4)$$

$$\mathbf{A} = [\boldsymbol{a}(\theta_1), \ldots, \boldsymbol{a}(\theta_K)] \quad (5)$$

$$\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_K]^T \quad (6)$$

$$\boldsymbol{S} = diag(\boldsymbol{\alpha}) \quad (7)$$

2.2. PCW MIMO radar

Consider a MIMO radar with the same array structure in Section 2.1 transmitting partially correlated signals $\mathbf{\Psi}(t)$ via its transmit antennas. Therefore, the energy received at the spatial angle θ_k can be described as

$$P(\theta_k) = \boldsymbol{a}^H(\theta_k) E\{\mathbf{\Psi}(t)\mathbf{\Psi}^H(t)\} \boldsymbol{a}(\theta_k) = \boldsymbol{a}^H(\theta_k) \boldsymbol{R}_{\Psi} \boldsymbol{a}(\theta_k) \quad (8)$$

where

$$\boldsymbol{R}_{\Psi} = E\{\mathbf{\Psi}(t)\mathbf{\Psi}^H(t)\} = \frac{1}{L} \boldsymbol{\Psi}\boldsymbol{\Psi}^H \quad (9)$$

with L denoting the number of snapshots, and $(\cdot)^H$ representing the Hermitian transpose.

Note that we can freely design the transmit beampattern to focus the transmit energy within the spatial region of interest by optimizing the correlation matrix $\boldsymbol{R}_{\psi}$ in (8). Usually, the correlation matrix of transmitted waveforms should satisfy one of the constraints below:

$$trace(\boldsymbol{R}_{\Psi}) = E \quad (10)$$

or

$$(\boldsymbol{R}_{\Psi})_{ii} = E/M_t, \quad i = 1, \ldots, M_t \quad (11)$$

The constraint (10) guarantees the total energy of transmitted waveforms to be E while the constraint (11) even requires the transmit energy of each antenna to be equal with each other. Similarly to the conventional MIMO radar, the output of the receive array of PCW MIMO radar can be written as

$$\boldsymbol{X}(t) = \mathbf{BSA}^T \boldsymbol{\Psi}(t) + \boldsymbol{Z}(t) \quad (12)$$

2.3. Subarray MIMO radar

The coherent waveform is transmitted by each subarray in subarray MIMO radar, so that transmit coherent gain can be obtained by each of them. Assume that the transmit array in Section 2.1 is partitioned to N subarrays, the beamforming weights of which are defined as $\boldsymbol{c}_n (n=1,\ldots,N)$ with a dimension of $M_t \times 1$. Note that the antennas corresponding to the nonzero elements in $\boldsymbol{c}_n$ constitute the nth subarray. Therefore, the transmit energy of the nth subarray received at the angle θ can be written as

$$s_n(t,\theta) = \boldsymbol{c}_n^H \boldsymbol{a}(\theta) \mathbf{\Phi}_n(t), \quad n = 1, \ldots, N \quad (13)$$

where $\Phi_n(t)$ is the nth waveform in orthogonal waveforms set $\mathbf{\Phi}(t)$. Thus, the sum of all the beams of subarrays pointing at the spatial angle θ can be written as

$$s(t,\theta) = \sum_{n=1}^{N} \boldsymbol{c}_n^H \boldsymbol{a}(\theta) \Phi_n(t) = (\mathbf{C}^H \boldsymbol{a}(\theta))^T \boldsymbol{\Phi}_N(t) \quad (14)$$

where $\mathbf{C} = [\boldsymbol{c}_1,\ldots,\boldsymbol{c}_N]$ is the transmit beamforming weight matrix with a dimension of $M_t \times N$, and $\mathbf{\Phi}_N(t) = [\Phi_1(t),\ldots,\Phi_N(t)]^T$ is the $N \times 1$ transmitted orthogonal waveforms. In order to ensure the total transmit energy to be E or the other even stricter constraint in (11), $\boldsymbol{C}$ should satisfy

$$\sum_{n=1}^{N} ||c_n||_2^2 = E \quad (15)$$

or

$$||\tilde{\boldsymbol{c}}_1||_2^2 = \cdots = ||\tilde{\boldsymbol{c}}_{M_t}||_2^2 = E/M_t \quad (16)$$

where $\tilde{c}_m (m=1,\ldots,M_t)$ represents the mth row of $\mathbf{C}$, and $||\cdot||_2$ stands for Euclidean norm.

Thus, the signal model of the subarray MIMO radar can be described as

$$\boldsymbol{X}(t) = \mathbf{BS}(\mathbf{C}^H \mathbf{A})^T \mathbf{\Phi}_N(t) + \boldsymbol{Z}(t) \quad (17)$$

Eq. (17) can also be rewritten as

$$\boldsymbol{X}(t) = \mathbf{BSA}^T (\mathbf{C}^* \mathbf{\Phi}_N(t)) + \boldsymbol{Z}(t) \quad (18)$$

where $(\cdot)^*$ represents conjugation operator. Compared with the signal model in Eq. (12), the subarray MIMO radar can be regarded as another form of PCW MIMO radar. In other words, the partially correlated signals can be realized by the linear combinations of a group of orthogonal waveforms [25], i.e.

$$\tilde{\boldsymbol{\Psi}}(t) = \boldsymbol{C}^* \mathbf{\Phi}_N(t) \quad (19)$$

As a result, the correlation matrix of the waveforms transmitted by the subarray MIMO radar can be defined as

$$\boldsymbol{R_C} = \frac{1}{L} \tilde{\boldsymbol{\Psi}} \tilde{\boldsymbol{\Psi}}^H = \frac{1}{L} \boldsymbol{C}^* \mathbf{\Phi}_N \mathbf{\Phi}_N^H \boldsymbol{C}^T = \boldsymbol{C}\boldsymbol{C}^H \quad (20)$$

where $\mathbf{\Phi}_N \mathbf{\Phi}_N^H / L = \mathbf{I}_{N \times N}$. Indeed, Eq. (20) reveals the equivalent relationship between the signal models of PCW MIMO radar and subarray MIMO radar. However, there is no effective principle to determine the minimum number N of the orthogonal waveforms yet.

3. Transmit beamforming based on CRB optimization for DOA estimation

The CRB for DOA of the unknown target represents the best performance of any unbiased estimator. The stochastic and deterministic CRB matrixes for DOA estimation have been given in [26]. However, according to Stoica and Moses [28], the deterministic CRB matrix for DOA estimation of the subarray MIMO radar assumed in Section 2 can be calculated as follows (The derivation can be found in Appendix A.):

$$\boldsymbol{CRB}_{DOA}(\boldsymbol{\theta}) = \frac{\sigma^2}{2L}\{Re(\boldsymbol{D}^H \boldsymbol{\Pi}_{\boldsymbol{V}}^{\perp} \boldsymbol{D} \odot \hat{\mathbf{P}})\}^{-1} \tag{21}$$

where

$$\boldsymbol{D} \triangleq [\boldsymbol{d}(\theta_1), \ldots, \boldsymbol{d}(\theta_K)] \tag{22}$$

$$\boldsymbol{d}(\theta) \triangleq \frac{\partial \mathbf{v}(\theta)}{\partial \theta} = \left(\boldsymbol{C}^H \frac{\partial \boldsymbol{a}(\theta)}{\partial \theta}\right) \otimes \boldsymbol{b}(\theta) + (\boldsymbol{C}^H \boldsymbol{a}(\theta)) \otimes \frac{\partial \boldsymbol{b}(\theta)}{\partial \theta} \tag{23}$$

$$\boldsymbol{V} \triangleq [\mathbf{v}(\theta_1), \ldots, \mathbf{v}(\theta_K)] \tag{24}$$

$$\mathbf{v}(\theta) \triangleq (\boldsymbol{C}^H \boldsymbol{a}(\theta)) \otimes \boldsymbol{b}(\theta) \tag{25}$$

$$\boldsymbol{\Pi}_{\boldsymbol{V}}^{\perp} = \boldsymbol{I} - \boldsymbol{V}(\boldsymbol{V}^H \boldsymbol{V})^{-1} \mathbf{V}^H \tag{26}$$

$$\hat{\mathbf{P}} = \boldsymbol{S}\boldsymbol{S}^H \tag{27}$$

where $\otimes$ represents Kronecker product, and $\odot$ represents Hadamard product. Note that Eq. (21) is similar to the deterministic CRB matrix in [26]. However, in Eq. (21) the energy of each subarray may be quite different, whereas they are equal in [26].

From (21), it can be noticed that the CRB matrix for DOA estimation is related to the subarray beamforming weights $\boldsymbol{C}$. In order to achieve the best performance of DOA estimation, the CRB matrix is minimized by optimizing the weight matrix $\boldsymbol{C}$ under either the constraint in (15) or (16). Here the constraint in (15) is focused, and the optimization problems can be formulated as

$$\min_{\boldsymbol{C}}\ trace(\boldsymbol{CRB}_{DOA}(\boldsymbol{\theta})) \text{s.t.} \sum_{n=1}^{N} ||c_n||_2^2 = E \tag{28}$$

Note that the similar optimization problem for PCW MIMO radar has been proposed in [18] and here is extended to subarray MIMO radar. Obviously, the optimization problem for subarray MIMO radar in Eq. (28) is a quadratic programming problem, which is hard to be solved by convex optimization algorithms directly. However, since the equivalent relationship between PCW MIMO radar and subarray MIMO radar, the CRB matrix in (21) can also be rewritten as a function of the waveform correlation matrix $\boldsymbol{R_C}$. Considering the relationship in (20), it enables us to optimize the correlation matrix of transmitted waveforms firstly, and then to calculate the subarray beamforming weights $\boldsymbol{C}$ indirectly. Similar to the method used in [18], the FIM with respect to all the unknown parameters in (17) can be written as

$$\mathbf{F} = \frac{2L}{\sigma^2}\begin{bmatrix} Re(\mathbf{F}_{11}) & Re(\mathbf{F}_{12}) & -Im(\mathbf{F}_{12}) \\ Re^T(\mathbf{F}_{12}) & Re(\mathbf{F}_{22}) & -Im(\mathbf{F}_{22}) \\ -Im^T(\mathbf{F}_{12}) & -Im^T(\mathbf{F}_{22}) & Re(\mathbf{F}_{22}) \end{bmatrix} \tag{29}$$

where

$$\begin{aligned}\mathbf{F}_{11} = {} & (\dot{\mathbf{B}}^H \dot{\mathbf{B}}) \odot (\boldsymbol{S}^H \mathbf{A}^H \boldsymbol{R_C} \mathbf{A} \boldsymbol{S}) + (\dot{\mathbf{B}}^H \mathbf{B}) \odot (\boldsymbol{S}^H \mathbf{A}^H \boldsymbol{R_C} \dot{\mathbf{A}} \boldsymbol{S}) \\ & + (\mathbf{B}^H \dot{\mathbf{B}}) \odot (\boldsymbol{S}^H \dot{\mathbf{A}}^H \boldsymbol{R_C} \mathbf{A} \boldsymbol{S}) + (\mathbf{B}^H \mathbf{B}) \odot (\boldsymbol{S}^H \dot{\mathbf{A}}^H \boldsymbol{R_C} \dot{\mathbf{A}} \boldsymbol{S})\end{aligned} \tag{30}$$

$$\mathbf{F}_{12} = (\dot{\mathbf{B}}^H \mathbf{B}) \odot (\boldsymbol{S}^H \mathbf{A}^H \boldsymbol{R_C} \boldsymbol{A}) + (\mathbf{B}^H \mathbf{B}) \odot (\boldsymbol{S}^H \dot{\mathbf{A}}^H \boldsymbol{R_C} \mathbf{A}) \tag{31}$$

$$\mathbf{F}_{22} = (\mathbf{B}^H \mathbf{B}) \odot (\mathbf{A}^H \boldsymbol{R_C} \mathbf{A}) \tag{32}$$

$$\dot{\mathbf{B}} = \left[\frac{\partial \boldsymbol{b}(\theta_1)}{\partial \theta_1}, \ldots, \frac{\partial \boldsymbol{b}(\theta_K)}{\partial \theta_K}\right] \tag{33}$$

$$\dot{\mathbf{A}} = \left[\frac{\partial \boldsymbol{a}(\theta_1)}{\partial \theta_1}, \ldots, \frac{\partial \boldsymbol{a}(\theta_K)}{\partial \theta_K}\right] \tag{34}$$

with $Re(\cdot)$ and $Im(\cdot)$ denoting the real and imaginary part of a complex-valued matrix, respectively. Then, the corresponding CRB matrix is

$$\boldsymbol{CRB} = \boldsymbol{F}^{-1} \tag{35}$$

Since we only care about the DOA performance of the subarray MIMO radar, the optimization problem in (28) can be cast as a semidefinite program (SDP) based on the Trace-Opt criterion algorithm in [18] and the references therein, i.e.,

$$\begin{aligned} & \min_{\{t_k\}_{k=1}^{K}, \boldsymbol{R_C}} \quad \sum_{k=1}^{K} t_k \\ & s.t. \quad \begin{bmatrix} \boldsymbol{F} & \boldsymbol{e}_k \\ \boldsymbol{e}_k^T & t_k \end{bmatrix} \\ & trace(\boldsymbol{R_C}) = E \geq 0, \quad k = 1, \ldots, K \end{aligned} \tag{36}$$

where $\{t_k\}$ are auxiliary variables, and $\boldsymbol{e}_k$ denotes the kth column of the unit matrix with a dimension of $3K$. Since $\mathbf{F}$ is a linear function of the correlation matrix $\boldsymbol{R_C}$, the constraints in the above SDP are either linear matrix inequalities or linear equalities in the elements of the waveform correlation matrix $\boldsymbol{R_C}$. The SDP in (36) can be resolved by optimization software easily, such as SeDuMi and SDPT3.

Once the optimized correlation matrix $\boldsymbol{R_C}$ obtained, the next step is to calculate the subarray beamforming weights $\boldsymbol{C}$ according to the relationship in (20). In this paper, eigendecomposition method is applied in $\boldsymbol{R_C}$, i.e.,

$$\boldsymbol{R_C} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H \tag{37}$$

where $\boldsymbol{\Lambda}$ is a diagonal matrix with the eigenvalues of $\boldsymbol{R_C}$ on its diagonal, and $\mathbf{U}$ are the eigenvectors of $\boldsymbol{R_C}$. Therefore, the subarray beamforming weights $\boldsymbol{C}$ can be calculated as follows:

$$\boldsymbol{C} = \boldsymbol{U} sqrt(\boldsymbol{\Lambda}) \tag{38}$$

where $sqrt(\cdot)$ denotes the square roots of each element in a matrix.

In fact, the optimized waveform correlation matrix is not always full rank, and some of the eigenvalues are very small (often less than 1% of the largest eigenvalue). In other words, these zero or small eigenvalues and their corresponding eigenvectors make few, if any, contribution to the overall transmit beam-pattern. Therefore, in the proposed method only the N largest eigenvalues (whose sum exceeds 99% of the total energy) and their corresponding eigenvectors are selected to constitute the subarray beamforming weights, i.e.

$$\boldsymbol{C} = \tilde{\boldsymbol{U}}[sqrt(\tilde{\Lambda}) \cdot \Delta] \tag{39}$$

where $\tilde{\Lambda}$ is the $N \times N$ diagonal matrix whose diagonal elements are the N principal eigenvalues of $\boldsymbol{R_C}$, $\tilde{\boldsymbol{U}}$contains corresponding eigenvectors, and $\Delta$is the correction factor for transmit energy ensuring that

$$trace(\boldsymbol{CC}^H) = E \tag{40}$$

4. Performance analysis

In this section, the performance of the proposed transmit beamforming method for subarray MIMO radar is analyzed in terms of the power transmitted by each subarray, subarray transmit beamforming gain, maximum effective aperture of the virtual array, and the computational complexity associated with eigendecomposition based DOA estimation techniques.

In the proposed method, the number and configurations of the subarrays can be adjusted adaptively to the priori information of targets. However, unlike other subarray MIMO radar, the energy transmitted in our method is not uniformly distributed in each subarray. According to (39), we could calculate the transmit energy of each subarray by

$$E_n = \frac{\Lambda_n}{\sum_{n=1}^{N} \Lambda_n} E, \quad n = 1, \ldots, N \tag{41}$$

where Λ_n is the nth principal eigenvalue of $\boldsymbol{R_C}$. The energy non-uniform distribution makes it possible to achieve an approximate correlation matrix of transmitted waveforms to the optimized one. Therefore, the transmit beamforming gain obtained by each subarray can be written as $G_n^2 \cdot E_n$, where

$$G_n = |\boldsymbol{c}_n \boldsymbol{a}(\theta)|, \quad n = 1, \ldots, N \tag{42}$$

At the receiver, the antennas belonging to one subarray which transmit the same waveform are equivalent to one virtual transmit antenna. Thanks to the fact that the configuration of sub-arrays can be adjusted adaptively according to the need of beamforming in our method, the maximum effective aperture of the virtual array is not fixed all the time actually.

The computational complexity of eigendecomposition based DOA estimation techniques will increase with the numbers of the transmitted orthogonal waveforms and the receive array antennas. The proposed method can transmit fewer orthogonal waveforms with the computational complexity of $O(N^3 M_r^3)$. Noting that N is smaller than M_t, it results in the decrease of computational complexity.

For the reason of comparison, we also consider the conventional MIMO radar, PCW MIMO radar and other subarray MIMO radar in the following simulation section. The performance analysis of them can be found in [22,26]. Actually, all other MIMO radar signal models can be considered as the special cases of the model in (16) by choosing $\boldsymbol{C}$ appropriately. The proposed method can design $\boldsymbol{C}$ much more freely, so that it has a superior DOA estimation performance.

5. Simulation results

In all of the simulation experiments in this section, a MIMO radar system is assumed to have a transmit array of $M_t = 10$ omni-directional antennas spaced half a wavelength apart, which is also used as the receive array. The additive noise is Gaussian zero-mean σ^2-variance spatially and temporally white. To compare the performance of the proposed method with other existing methods, five beamforming methods are considered: (a) the proposed transmit beamforming method, (b) the beamforming method proposed in [26], (c) the beamforming method for conventional MIMO radar, (d) the beamforming method in [22] for overlapped subarray MIMO radar, and (e) the PCW MIMO radar. In all the five signal models, the total transmit energy is set as $E = M_t$. In addition, except the model in (e) transmitting the partially correlated signals, the other four models transmit orthogonal signals. Throughout our experiments, the number of snapshots is set as $L = 256$ and each method has completed 400 independent runs in the Monte-Carlo analysis and the probability of source resolution test respectively.

Case 1. In this case, two uncorrelated targets are assumed to be located at spatial angle 1° and −1° respectively. The transmit energy in model (a) is concentrated at both 1° and −1°. According to our method, the optimized correlation matrix of transmitted waveforms can be obtained by the optimization in (36). Applying eigendecomposition method, the largest two eigenvalues and the corresponding eigenvectors are used to calculate the two subarray transmit beamforming weights by (39), i.e., $\boldsymbol{C} = [\boldsymbol{c}_1, \boldsymbol{c}_2]$.

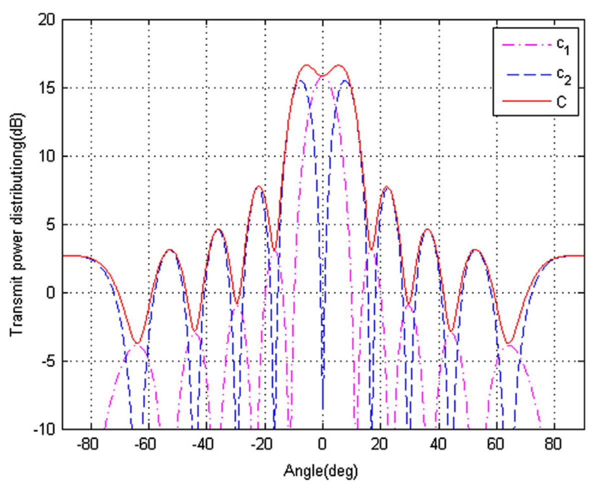

Fig. 1. The sub-array beam-patterns and overall beam-pattern of model (1) in Case 1.

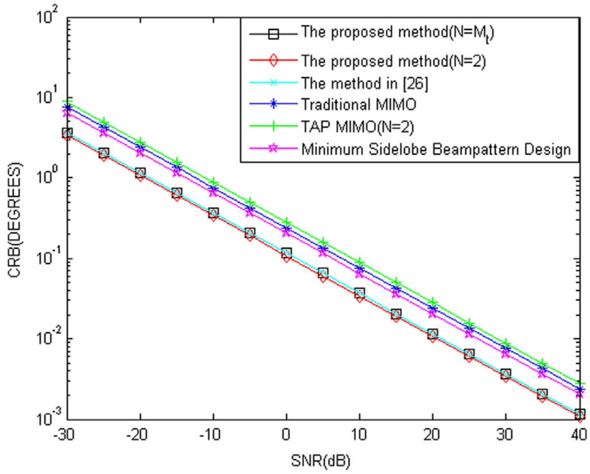

Fig. 2. The CRBs for DOA estimation of all the methods in Case 1.

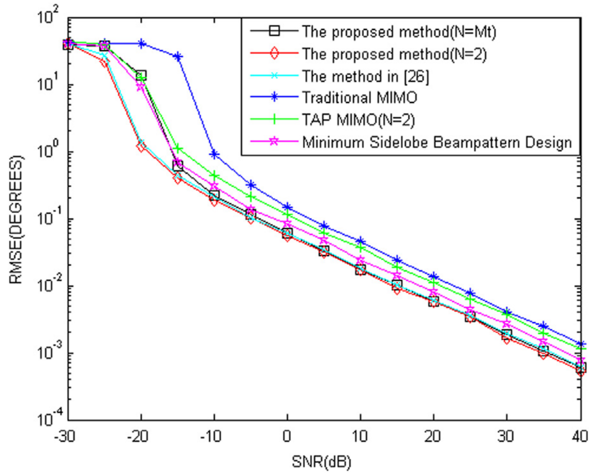

Fig. 3. The RMSEs versus SNR for all the methods in Case 1.

Fig. 1 shows the beam-patterns of the two subarrays and the overall beam-pattern synthesized by our method in model (a). In model (b), the transmit energy is focused within the spatial region $[-5^\circ, 5^\circ]$, and the transmit beamforming weights are calculated based on the method in [26]. The orthogonal waveforms are transmitted by each antenna in model (c) with no beamforming weights applied. In model (d), the transmit array is divided to two overlapped subarrays of 9 antennas (which is also called TAP MIMO radar), and the transmit beamforming weights in [14] are used to focus the energy within $[-5^\circ, 5^\circ]$. The model (e) employees the Minimum Sidelobe Beam-pattern Design method in [4], and the partially correlated signals designed according to the optimized correlation matrix are transmitted, whose energy is also focused within $[-5^\circ, 5^\circ]$. The CRBs for all methods are plotted in Fig. 2. Note that the case of applying Eq. (38) to calculate $\mathbf{C}$ in model (a) is also given in Fig. 2. It can be seen that the model (b) has the worst CRB for DOA estimation, for the reason of its smallest effective aperture of virtual array $M_r\lambda/2$. Although model (c) has no transmit beamforming gain, it has a better CRB due to its larger effective aperture of $(M_t+M_r-1)\lambda/2$ compared to model (d). However, the proposed method in model (a) can achieve the optimized tradeoff between the effective aperture of virtual array and the transmit beamforming gain, thus it has the best CRB.

In the Monte-Carlo analysis, the eigendecomposition based DOA estimation technique (ESPRIT algorithm) in [10] is applied to estimate the DOAs. Fig. 3 shows the RMSEs of results versus SNR for all models tested. It is easy to see that the method in (a) has a superior performance of DOA estimation than the methods in (c)–(e) under the SNR range shown in Fig. 3, and is slightly better than the method in (b). It is worth noting that the two cases in our proposed method that respectively employ less subarrays ($N=2$) and full subarrays ($N=M_t$) have the approximate performance under the high SNR, but the former has better performance under the low SNR. It is because of that both cases can obtain the estimated correlation matrix accurately under high SNR, whereas

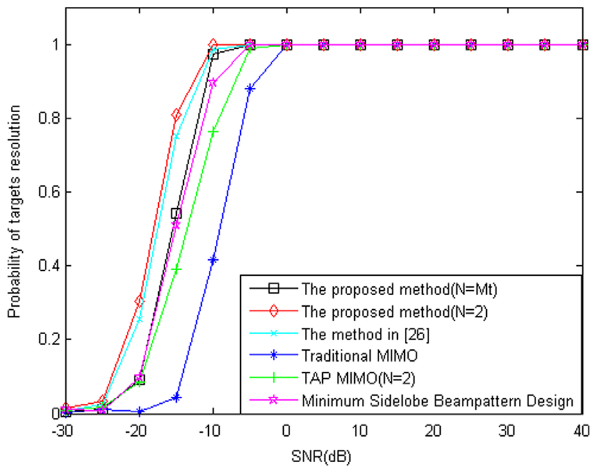

Fig. 4. The probability of resolution versus SNR for all the methods in Case 1.

higher transmit beamforming gain will be more conducive to the accuracy of correlation matrix estimation when SNR is poor. In addition, the computational complexity of ESPRIT algorithm is related to the number of subarrays according to [10]. The methods in (a), (b) and (d) have the least computational complexity of $O(2^3M_r^3)$, whereas the others have it with $O(M_t{}^3M_r^3)$.

It is defined in [27] that targets can be considered to be resolved if the following is satisfied:

$$|\tilde{\theta}_n-\theta_n| \le \frac{\Delta\theta}{2}, \quad n=1,2 \tag{43}$$

where $\Delta\theta=|\theta_1-\theta_2|$. Fig. 4 shows the probability of resolution versus SNR for all the methods. From this figure, it is easy to find that the conventional MIMO radar has the worst probability of resolution under low SNR for the reason of no transmit beamforming applied. The proposed method ($N=2$) in (a) has the best probability of resolution performance obviously.

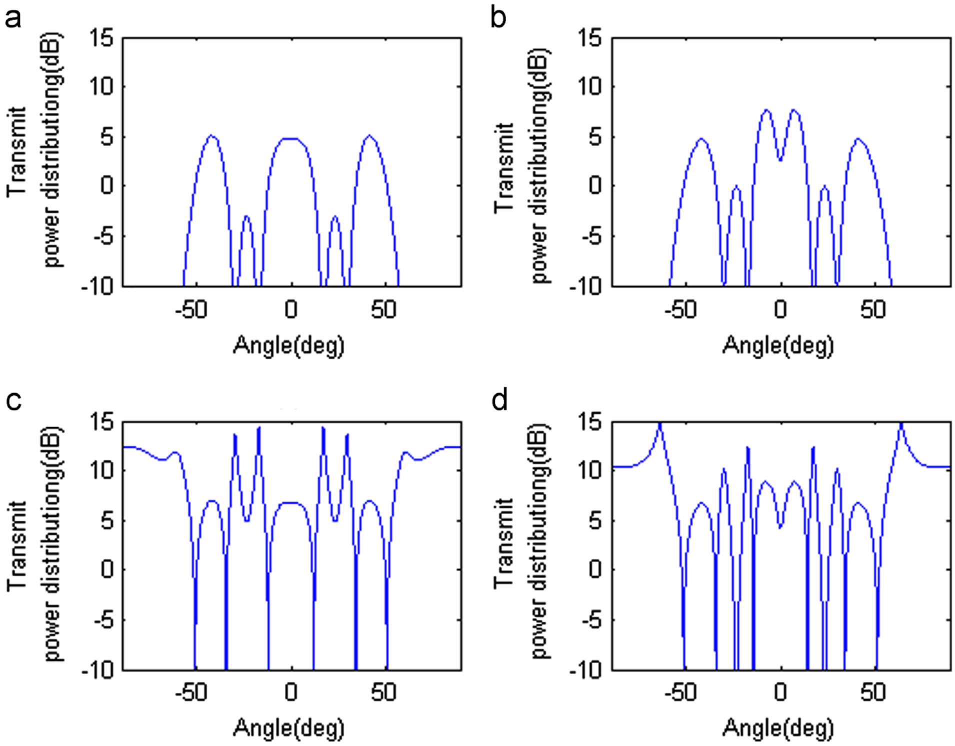

Fig. 5. The 4 subarray transmit beam-patterns of proposed method in Case 2.

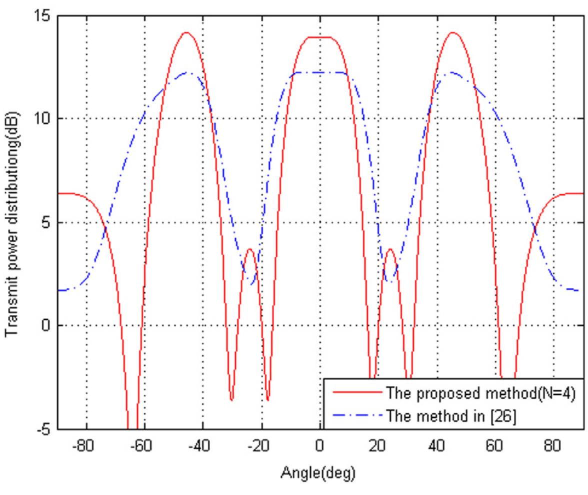

Fig. 6. The overall transmit beam-patterns of the two methods in Case 2.

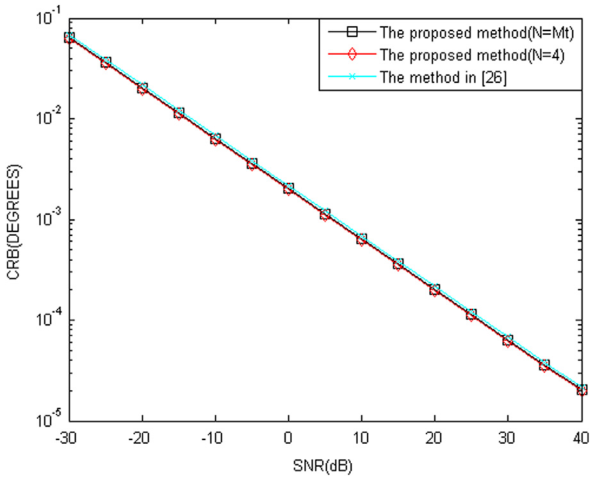

Fig. 7. The CRBs versus SNR for the methods in Case 2.

Case 2. In this case, four uncorrelated targets are assumed to be located at -45°, -5°, 5° and 45° respectively. In Case 1, the performance of model (a) and (b) are much better than others, we only consider the two methods in this case consequently.

In model (a), the transmit energy is concentrated at -45°, -5°, 5° and 45° respectively by solving the SDP in (35). Four largest eigenvalues whose sum exceeds 99% of the total energy are obtained, and the subarray beamforming weights $\boldsymbol{C}=[\boldsymbol{c}_1,\boldsymbol{c}_2,\boldsymbol{c}_3,\boldsymbol{c}_4]$ are calculated by Eq.(39). Fig. 5 shows the beam-patterns of the four subarrays, and the overall beam-pattern can be seen in Fig. 6. According to the method in [26], the transmit energy of model (b) is focused within $[-50^\circ,-40^\circ]\cup[-10^\circ,10^\circ]\cup[40^\circ,50^\circ]$, and six principal eigenvalues whose sum exceeds 99% of the total energy are obtained. The transmit beam-pattern of model (b) is also plotted in Fig. 6.

Fig. 7 shows the CRBs versus SNR for the methods in model (a) and (b) (including the $N=M_t$ case in (a)). It can be noticed from this figure, the proposed method is still

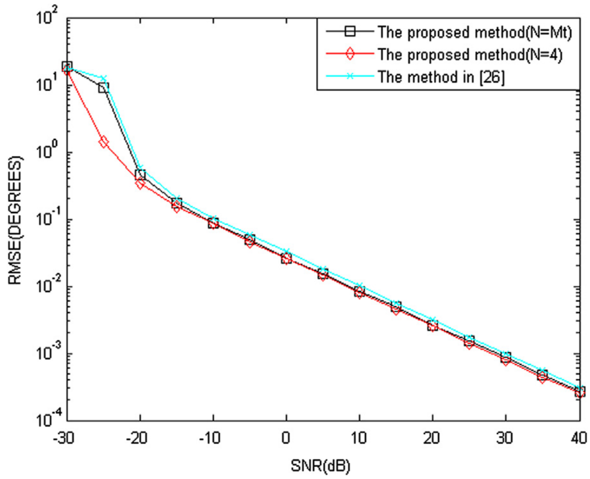

Fig. 8. The RMSEs versus SNR for the methods in Case 2.

superior to the method of [26] in the case of dispersed targets, for the reason that higher transmit beamforming gain is obtained by our method.

The Monte-Carlo analysis based on ESPRIT algorithm for DOA estimation is tested for the methods above. The result given in Fig. 8 shows that under the SNR range, the proposed method has better performance for DOA estimation. Especially under the low SNR, the proposed method which transmits fewer orthogonal waveforms can significantly enhance the performance of DOA estimation. Meanwhile, the computational complexity of the ESPRIT algorithm based DOA estimator in our method has been greatly reduced due to the reduction in the number of subarrays. The computational complexity of our method is $O(4^3M_r^3)$, while the $N=M_t$ case and model (b) have it with $O(10^3M_r^3)$ and $O(6^3M_r^3)$ respectively. Therefore, in this case the method we proposed also has the best performance for DOA estimation with the least computational complexity.

6. Conclusion

In this paper, the proposed transmit beamforming for subarray MIMO radar can achieve a superior performance of DOA estimation with the minimum number of subarrays owing to the following two reasons. First, the proposed method can achieve the theoretical minimum CRB for DOA estimation by optimizing the correlation matrix of transmitted waveforms. Then, fewer orthogonal waveforms transmitted in our method makes the transmit energy further concentrated at the region of interest, which also reduces the computational complexity of the eigendecomposition based DOA estimation techniques. Meanwhile, the proposed method optimizes the subarray beamforming weights indirectly by optimizing the correlation matrix of transmitted waveforms as the first step, which avoids the difficulty in optimizing the subarray beamforming weights directly and saves the computational cost of designing the partially correlated signals according to the optimized correlation matrix.

Acknowledgments

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Appendix A

Considering the signal model in (17), the FIM with respect to the unknown parameters $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$ can be written as [27]

$$\boldsymbol{F}(\theta_i,\theta_j)=2Re\,tr\left[\frac{\partial(\mathbf{B}\boldsymbol{S}(\mathbf{C}^H\mathbf{A})^T\boldsymbol{\Phi}_N)^H}{\partial\theta_i}\mathbf{Q}^{-1}\frac{\partial(\mathbf{B}\boldsymbol{S}(\mathbf{C}^H\mathbf{A})^T\boldsymbol{\Phi}_N)}{\partial\theta_j}\right] \tag{A1}$$

where $\mathbf{Q}=\sigma^2\mathbf{I}$, for the reason that noise is Gaussian with zero-mean and variance of σ^2. Eq. (A1) can also be rewritten as

$$\begin{aligned}\boldsymbol{F}(\theta_i,\theta_j)&=\frac{2}{\sigma^2}Re\,tr\left[\frac{\partial(\mathbf{B}\boldsymbol{S}(\mathbf{C}^H\mathbf{A})^T)}{\partial\theta_j}\boldsymbol{\Phi}_N\boldsymbol{\Phi}_N{}^H\frac{\partial(\mathbf{B}\boldsymbol{S}(\mathbf{C}^H\mathbf{A})^T)^H}{\partial\theta_i}\right]\\&=\frac{2L}{\sigma^2}Re\,tr\left[\frac{\partial(\mathbf{B}\boldsymbol{S}(\mathbf{C}^H\mathbf{A})^T)^H}{\partial\theta_i}\ \frac{\partial(\mathbf{B}\boldsymbol{S}(\mathbf{C}^H\mathbf{A})^T)}{\partial\theta_j}\right]\end{aligned} \tag{A2}$$

where the facts $tr(\mathbf{AB})=tr(\mathbf{BA})$ and $\boldsymbol{\Phi}_N\boldsymbol{\Phi}_N^H/L=\mathbf{I}_{N\times N}$ are used. Then, using the fact that $tr(\mathbf{AB})=vec(\mathbf{A}^T)^T\,vec(\mathbf{B})^T$, we can obtain

$$\boldsymbol{F}(\theta_i,\theta_j)=\frac{2L}{\sigma^2}Re\left\{vec\left[\frac{\partial(\mathbf{B}\boldsymbol{S}(\mathbf{C}^H\mathbf{A})^T)^H}{\partial\theta_i}\right]^H vec\left[\frac{\partial(\mathbf{B}\boldsymbol{S}(\mathbf{C}^H\mathbf{A})^T)}{\partial\theta_j}\right]\right\} \tag{A3}$$

It is easy to be verified that

$$\begin{aligned}\frac{\partial(\mathbf{B}\boldsymbol{S}(\mathbf{C}^H\mathbf{A})^T)}{\partial\theta_i}&=\dot{\mathbf{B}}e_ie_i{}^T\boldsymbol{S}(\mathbf{C}^H\mathbf{A})^T+\mathbf{B}\boldsymbol{S}e_ie_i{}^T(\mathbf{C}^H\dot{\mathbf{A}})^T\\&=\frac{\partial\boldsymbol{b}(\theta_i)}{\partial\theta_i}\cdot s_i(\mathbf{C}^H\boldsymbol{a}(\theta_i))^T+\boldsymbol{b}(\theta_i)\cdot s_i\left(\mathbf{C}^H\frac{\partial\boldsymbol{a}(\theta_i)}{\partial\theta_i}\right)^T\end{aligned} \tag{A4}$$

where $\boldsymbol{e}_i$ represents the ith column of unit matrix. Therefore, we have

$$\begin{aligned}vec\left(\frac{\partial(\mathbf{B}\boldsymbol{S}(\mathbf{C}^H\mathbf{A})^T)}{\partial\theta_i}\right)&=\left[(\mathbf{C}\boldsymbol{a}(\theta_i))\otimes\frac{\partial\boldsymbol{b}(\theta_i)}{\partial\theta_i}+(\mathbf{C}\frac{\partial\boldsymbol{a}(\theta_i)}{\partial\theta_i})\otimes\boldsymbol{b}(\theta_i)\right]\cdot s_i\\&=\boldsymbol{d}(\theta_i)\cdot s_i\end{aligned} \tag{A5}$$

where the fact $vec(\mathbf{AVB}^T)=(\mathbf{B}\otimes\mathbf{A})vec(\mathbf{V})$ is applied. Define

$$\boldsymbol{\Delta}\triangleq[\boldsymbol{d}(\theta_1)s_1,\ldots,\boldsymbol{d}(\theta_K)s_K] \tag{A6}$$

Thus, the FIM with respect to $\boldsymbol{\theta}$ part can be written as

$$\boldsymbol{F}(\boldsymbol{\theta},\boldsymbol{\theta})=\frac{2L}{\sigma^2}Re\{\boldsymbol{\Delta}^H\cdot\boldsymbol{\Delta}\} \tag{A7}$$

Similarly, we have

$$\begin{aligned}vec\left[\frac{\partial(\mathbf{B}\boldsymbol{S}\mathbf{A}^T)}{\partial\overline{\alpha}_i}\right]&=vec(\mathbf{B}\boldsymbol{e}_i\boldsymbol{e}_i^T\mathbf{A}^T)=vec\left[\boldsymbol{b}(\theta_i)\cdot(\mathbf{C}^H\boldsymbol{a}(\theta_i))^T\right]\\&=(\mathbf{C}^H\boldsymbol{a}(\theta_i))\otimes\boldsymbol{b}(\theta_i)\end{aligned} \tag{A8}$$

and

$$vec\left[\frac{\partial(\mathbf{BSA}^T)}{\partial\tilde{\alpha}_i}\right]=vec(j\mathbf{B}\boldsymbol{e}_i\boldsymbol{e}_i^T\mathbf{A}^T)=vec\left[j\boldsymbol{b}(\theta_i)\cdot(\boldsymbol{Ca}(\theta_i))^T\right]$$
$$=j(\boldsymbol{Ca}(\theta_i))\otimes\boldsymbol{b}(\theta_i) \tag{A9}$$

where $\overline{\alpha}$ and $\tilde{\alpha}$ denotes the real and imaginary part of reflection coefficient. Hence,

$$\boldsymbol{F}(\boldsymbol{\theta},\overline{\boldsymbol{\alpha}})=\frac{2L}{\sigma^2}Re\{\boldsymbol{\Delta}^H\cdot\boldsymbol{V}\} \tag{A10}$$

and

$$\boldsymbol{F}(\boldsymbol{\theta},\tilde{\boldsymbol{\alpha}})=-\frac{2L}{\sigma^2}Im\{\boldsymbol{\Delta}^H\cdot\boldsymbol{V}\} \tag{A11}$$

We also have

$$\boldsymbol{F}(\overline{\boldsymbol{\alpha}},\overline{\boldsymbol{\alpha}})=\boldsymbol{F}(\tilde{\boldsymbol{\alpha}},\tilde{\boldsymbol{\alpha}})=\frac{2L}{\sigma^2}Re\{\boldsymbol{V}^H\cdot\boldsymbol{V}\} \tag{A12}$$

As a result, the FIM of subarray MIMO radar can be written as

$$\boldsymbol{F}=\frac{2L}{\sigma^2}Re\left\{\begin{bmatrix}\boldsymbol{\Delta}^H\\ \boldsymbol{V}^H\\ -j\boldsymbol{V}^H\end{bmatrix}\begin{bmatrix}\boldsymbol{\Delta} & \boldsymbol{V} & j\boldsymbol{V}\end{bmatrix}\right\} \tag{A13}$$

It is worth noting that (A13) can be transformed into the form as FIM of (29) by applying the relationship in (20). They are equivalent to each other in the mathematic expression. According to the method in [28], define that

$$\boldsymbol{G}=(\boldsymbol{V}^H\boldsymbol{V})^{-1}\boldsymbol{V}^H\boldsymbol{\Delta} \tag{A14}$$

and

$$\boldsymbol{T}=\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ -\overline{\boldsymbol{G}} & \boldsymbol{I} & \boldsymbol{0}\\ -\tilde{\boldsymbol{G}} & \boldsymbol{0} & \boldsymbol{I}\end{bmatrix} \tag{A15}$$

which satisfies the following

$$\begin{bmatrix}\boldsymbol{\Delta} & \boldsymbol{V} & j\boldsymbol{V}\end{bmatrix}\boldsymbol{T}=\begin{bmatrix}(\boldsymbol{\Delta}-\boldsymbol{VG}) & \boldsymbol{V} & j\boldsymbol{V}\end{bmatrix}=\begin{bmatrix}\boldsymbol{\Pi}_{\boldsymbol{V}}^{\perp} & \boldsymbol{V} & j\boldsymbol{V}\end{bmatrix} \tag{A16}$$

hence, we could have

$$\boldsymbol{T}^H\mathbf{F}\boldsymbol{T}=\frac{2L}{\sigma^2}Re\left\{\boldsymbol{T}^H\begin{bmatrix}\boldsymbol{\Delta}^H\\ \boldsymbol{V}^H\\ -j\boldsymbol{V}^H\end{bmatrix}\begin{bmatrix}\boldsymbol{\Delta} & \boldsymbol{V} & j\boldsymbol{V}\end{bmatrix}\boldsymbol{T}\right\}$$
$$=\frac{2L}{\sigma^2}Re\begin{bmatrix}\boldsymbol{\Delta}^H\boldsymbol{\Pi}_{\boldsymbol{V}}^{\perp}\boldsymbol{\Delta} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{V}^H\boldsymbol{V} & j\boldsymbol{V}^H\boldsymbol{V}\\ \boldsymbol{0} & -j\boldsymbol{V}^H\boldsymbol{V} & \boldsymbol{V}^H\boldsymbol{V}\end{bmatrix} \tag{A17}$$

then we have

$$\boldsymbol{CRB}=\boldsymbol{F}^{-1}=(\boldsymbol{T}(\boldsymbol{T}^H\mathbf{F}\boldsymbol{T})\boldsymbol{T}^H)^{-1}$$
$$=\begin{bmatrix}\frac{2L}{\sigma^2}Re(\boldsymbol{\Delta}^H\boldsymbol{\Pi}_{\boldsymbol{V}}^{\perp}\boldsymbol{\Delta}) & x & x\\ x & x & x\\ x & x & x\end{bmatrix}^{-1} \tag{A18}$$

Since we only care about the DOA estimation, the symbol x represents the other elements that we do not care, and the first element in (A18) is the CRB matrix being concerned. For

$$\left[\boldsymbol{\Delta}^H\boldsymbol{\Pi}_{\boldsymbol{V}}^{\perp}\boldsymbol{\Delta}\right]_{kp}=(\mathbf{d}(\theta_k)\cdot s_k)^H\boldsymbol{\Pi}_{\boldsymbol{V}}^{\perp}(\mathbf{d}(\theta_p)\cdot s_p)$$
$$=[\mathbf{d}^H(\theta_k)\boldsymbol{\Pi}_{\boldsymbol{V}}^{\perp}\mathbf{d}(\theta_p)]\cdot(s_k s_p) \tag{A19}$$

we can obtain the CRB matrix for DOA estimation that

$$\boldsymbol{CRB}_{DOA}(\boldsymbol{\theta})=\frac{\sigma^2}{2L}\{Re(\mathbf{D}^H\boldsymbol{\Pi}_{\boldsymbol{V}}^{\perp}\mathbf{D}\odot\hat{\mathbf{P}})\}^{-1} \tag{A20}$$

References

[1] E. Fishler, A. Haimovich, R. Blum, D. Chizhik, L. Cimini, R. Valenzuela, MIMO radar: an idea whose time has come, in: Proceeding of IEEE Radar Conference, 2004, pp. 71–78.
[2] F. Daum, J. Huang, MIMO radar: snake oil or good idea? IEEE Aerosp. Electron. Syst. Mag. 24 (2009) 8–12.
[3] M. Haimovich, R.S. Blum, L.J. Cimini, MIMO radar with widely separated antennas, IEEE Signal Process. Mag. 25 (2008) 116–129.
[4] J. Li, P. Stoica, MIMO radar with co-located antenna: review of some recent work, IEEE Signal Process. Mag. 24 (2007) 106–114.
[5] R. Boyer, Co-located MIMO radar with orthogonal waveform coding: Cramér–Rao Lower bound, in: Proceedings of 3rd IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP), 2009, pp. 149–152.
[6] D.W. Bliss, K.W. Forsythe, Multiple-input multiple-output (MIMO) radar and imaging: degrees of freedom and resolution, in: Proceedings of the 37th Asilomar Conference on Signals, Systems & Computers, Pacific Grove, CA, 2003, pp. 54–59.
[7] J. Li, P. Stoica, MIMO radar-Diversity means superiority, MIMO Radar Signal Processing, Wiley-IEEE Press, New York, 2009, 1–64.
[8] Z. Shenghua, L. Hongwei, L.i. Baochang, Y. Kuiying, Adaptive MIMO radar target parameter estimation with Kronecker-product structured interference covariance matrix, Signal Process. 92 (2012) 1177–1188.
[9] I. Bekkerman, J. Tabrikian, Target detection and localization using MIMO radars and sonars, IEEE Trans. Signal Process. 54 (2006) 3873–3883.
[10] C. Duofang, C. Baixiao, Q. Guodong, Angle estimation using ESPRIT in MIMO radar, Electron. Lett. 44 (2008) 770–771.
[11] C. Jinli, G. Hong, S. Weimin, Angle estimation using ESPRIT without pairing in MIMO radar, Electron. Lett. 44 (2008) 1422–1423.
[12] L. Caicai, L. Guisheng, Z. Shengqi, W. Sunyong, An ESPRIT-like algorithm for coherent DOA estimation based on data matrix decomposition in MIMO radar, Signal Process. 91 (2011) 1803–1811.
[13] J.L. Chen, H. Gu, W.M. Su, A new method for joint DOD and DOA estimation in bistatic MIMO radar, Signal Process. 90 (2010) 714–718.
[14] A..Hassanien, S..Vorobyov, Direction finding for MIMO radar with colocated antennas using transmit beamspace preprocessing, in: Proceedings of 3rd IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP), 2009, pp. 181–184.
[15] J. Li, P. Stoica, Y. Xie, On probing signal design for MIMO radar, presented at the 40th Asilomar Conference on Signals, Systems & Computers, Pacific Grove, CA, October 2006, pp. 31–35.
[16] D.R. Fuhrmann, G.S. Antonio, Transmit beamforming for MIMO radar systems using signal cross-correlation, IEEE Trans. Aerosp. Electron. Syst. 44 (2008) 171–186.
[17] S. Ahmed, J.S. Thompson, B. Mulgrew, Y. Petillot, Unconstrained synthesis of covariance matrix for MIMO radar transmit beampattern, IEEE Trans. Signal Process. 59 (2011) 3837–3849.
[18] J. Li, P. Stoica, K. Forsythe, D. Bliss, Range compression and waveform optimization for MIMO radar: a Cramér–Rao bound based study, IEEE Trans. Signal Process. 56 (2008) 218–232.
[19] P. Stoica, J. Li, X. Zhu, Waveform synthesis for diversity-based transmit beampattern design, IEEE Trans. Signal Process. 56 (2008) 2593–2598.
[20] S. Ahmed, J.S. Thompson, B. Mulgrew, Y. Petillot, Finite alphabet constant-envelope waveform design for MIMO radar, IEEE Trans. Signal Process. 59 (2011) 5326–5337.
[21] E. Grossi, M. Lops, L. Venturino, Min-max waveform design for MIMO radars under unknown correlation of the target scattering, Signal Process. 92 (2012) 1550–1558.

[22] A. Hassanien, S. Vorobyov, Phased-MIMO radar: a tradeoff between phased-array and MIMO radars, IEEE Trans. Signal Process. 58 (2010) 3137–3151.
[23] H. Li, B. Himed, Transmit subaperturing for MIMO radars with collocated antennas, IEEE J. Sel. Top. Signal Process. 4 (2010) 1–4.
[24] D. Wilcox, M. Sellathurai, On MIMO radar subarrayed transmit beamforming, IEEE Trans. Signal Process. 60 (2012) 2076–2081.
[25] P.E. Berry, D. Yau, Optimal fast-time beamforming with linearly independent waveforms, Signal Process. 89 (2009) 492–501.
[26] A. Hassanien, S. Vorobyov, Transmit energy focusing for DOA estimation in MIMO radar with colocated antennas, IEEE Trans. Signal Process. 59 (2011) 2669–2682.
[27] H.L. Van Trees, Optimum Array Processing, Wiley, New York, 2002.
[28] P. Stoica, R.L. Moses, Spectral Analysis of Signals, Prentice-Hall, Upper Saddle River, NJ, 2005.