ELSEVIER Chinese Astronomy and Astrophysics 36 (2012) 229–237

CHINESE
ASTRONOMY
AND ASTROPHYSICS

An Analysis of Light Periods of BL Lac Object S5 0716+714 with the MUSIC Algorithm† ⋆

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Abstract The multiple signal classification (MUSIC) algorithm is introduced to the estimation of light periods of BL Lac objects. The principle of the MUSIC algorithm is given, together with a testing on its spectral resolution by using a simulative signal. From a lot of literature, we have collected a large number of effective observational data of the BL Lac object S5 0716+714 in the three optical wavebands V, R, and I from 1994 to 2008. The light periods of S5 0716+714 are obtained by means of the MUSIC algorithm and average periodogram algorithm, respectively. It is found that there exist two major periodic components, one is the period of (3.33 ± 0.08) yr, another is the period of (1.24 ± 0.01) yr. The comparison of the performances of periodicity analysis of two algorithms indicates that the MUSIC algorithm has a smaller requirement on the sample length, as well as a good spectral resolution and anti-noise ability, to improve the accuracy of periodicity analysis in the case of short sample length.

Key words: BL Lacertae Objects: individual—galaxies: fundamental parameters—methods: analytical

1. INTRODUCTION

The light variability means the variation of radiation flux of an object with the time, to study the light variability of AGNs is an extremely meaningful task. As an important subclass of AGNs, BL Lac objects exhibit violent light variations, observable in different

† Supported by Scientific Research Foundation of Shaanxi Province
Received 2010–11–08; revised version 2011–08–15
⋆ A translation of *Acta Astron. Sin.* Vol. 53, No. 1, pp. 1–8, 2012
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doi:10.1016/j.chinastron.2012.07.001

wavebands, with the timescales from several days to several years. Along with the progress of observational techniques and the use of advanced high-accuracy observational instruments, even the variation timescales in the order of hours or seconds have been observed. The variation timescale, the correlation of light variations in different wavebands, and the time delay etc. provide us with important information for understanding the central structure of BL Lac objects and the physical process in which, for example, from the timescale of light variations, we can estimate the size and brightness temperature of radiation region, and so on[1].

There are many methods available for analyzing the light periods, to select an effective method of periodicity analysis seems to be especially important. The Jurkevich method, discrete correlation analysis, CLEAN algorithm, power spectrum analysis, wavelet analysis, high-order spectrum analysis, structure function method etc are commonly used for the periodicity analysis of light curves. We have made certain comparative studies on the periodogram method of classical spectrum estimation, the autoregressive modeling method of modern spectrum estimation and the double-spectrum analysis of high-order spectrum analysis, the result indicates that these methods are effective for the long-periodic light variations[2−6]. However, the application of these methods has some prerequisites and certain limitations, if several different statistical methods are adopted simultaneously to make a comparative analysis on the obtained data, a rather believable conclusion may be obtained. In this paper, the MUSIC algorithm realized by performing an eigen decomposition is introduced, which can be used for the measurement of spatial direction and the estimation of power spectrum. In recent years, the MUSIC algorithm has been widely and successfully used for estimating the direction of an arrived wave beam, but its another application in power spectrum estimation is rarely seen in literatures. This algorithm needs little a priori information, and possesses the high-resolution characteristic of modern spectrum estimation algorithms, with a less requirement on the sample length and a simple principle, this method can give a high-resolution and high-stability spectrum estimation under the condition of low signal-noise ratio.

S5 0716+714 is a famous BL Lac object, and it has received consistently attentions for a long time, for example, Impey et al.[7] made the optical observations on S5 0716+714 in a period as long as 3 years, Qian et al.[8] made the optical observations at the Xing-long station of National Astronomical Observatories of Chinese Academy of Sciences from 1994 to 2000, and Nesci et al.[9] observed the light variations of S5 0716+714 at the optical waveband from February 1995 to March 2003. With the rather rich data, many authors have obtained the relatively consistent long periods by different methods.

This paper has made a briefly introduction to the basic principle of the MUSIC spectrum estimation method, and made a test on the MUSIC algorithm by using a simulative signal with a low signal-noise ratio. In order to verify the good performance in the analysis of long-periodic variations, we have made a comparison with the average periodogram method. We have had both methods apply to the data of historic light variations of S5 0716+714 at the three optical wavebands V, R and I, to examine the possibly existed long-periods of light variations, and made a brief discussion on the results obtained with both methods.

2. BASIC PRINCIPLE OF THE MUSIC ALGORITHM FOR SPECTRUM ESTIMATION

MUSIC is the abbreviation for "Multiple Signal Classification", the MUSIC spectrum estimation algorithm is a typical representative of spatial spectrum estimation techniques, based on the theorem of matrix eigen-decomposition, it divides the information space into two orthogonal subspaces, namely the signal subspace and the noise subspace, by the orthogonality of vectors in respectively the noise subspace and the signal subspace, the frequency components contained in the signal can be estimated, and therefore the signal periods[10].

A signal contains always the signal of interest and the noise interference, generally, we have to separate them from each other and suppress the noise interference. The MUSIC algorithm is actually an algorithm which can extract the useful signal from the noise interference by through an eigen decomposition, resulting in a power spectrum with a peak value at the place of signal frequency and small values at other places. Only if the sample length N is greater than or equal to $P+1$ (P is the number of signal sources), can all the frequency components in the sample be estimated, and therefore can the periods be obtained as the reciprocals of frequencies.

From the signal to be analyzed, a data vector $\boldsymbol{x} = [x_0, x_1, \cdots, x_N]^T$ is constructed, here T means the operator of transpose, the correlation matrix of the data vector is

$$\boldsymbol{R}_x = E[\boldsymbol{x}^* \boldsymbol{x}^{\mathrm{H}}]\,, \tag{1}$$

in which $*$ expresses the operator of complex conjugation, H means the operator of conjugate transpose. The eigen decomposition of the matrix $\boldsymbol{R}_x$ can be expressed as

$$\hat{\boldsymbol{R}}_x = \sum_{i=1}^{N} \lambda_i \boldsymbol{\nu}_i^* \boldsymbol{\nu}_i^{\mathrm{H}}\,, \tag{2}$$

in which λ_i is the i-th eigen value, $\boldsymbol{\nu}_i$ is the eigen vector corresponding to λ_i. The signal is composed of the signal sources of number p and the internal noises with the variance of σ_n^2, and

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p > \sigma_n^2, \lambda_{p+1} = \lambda_{p+2} = \cdots = \lambda_N = \sigma_n^2\,. \tag{3}$$

The eigen vectors corresponding to the eigen values $\lambda_1, \lambda_2, \cdots, \lambda_p$ compose the signal space, its correlation matrix is $\boldsymbol{R}_s$, and the eigen vectors corresponding to the eigen values $\lambda_{p+1}, \lambda_{p+2}, \cdots, \lambda_N$ compose the noise space, its correlation matrix is $\boldsymbol{R}_n$, therefore we have

$$\boldsymbol{R}_x = \boldsymbol{R}_{\mathrm{s}} + \boldsymbol{R}_{\mathrm{n}} = \sum_{k=1}^{p} \lambda_k \boldsymbol{\nu}_k \boldsymbol{\nu}_k^{\mathrm{H}} + \sigma_n^2 \boldsymbol{I}\,. \tag{4}$$

Because of the orthogonality of the signal subspace and the noise subspace, we can obtain the spectrum estimation of the MUSIC algorithm[10]:

$$\hat{\boldsymbol{P}}_{\mathrm{MUSIC}}(f) = \frac{1}{\sum\limits_{i=p+1}^{N} |\boldsymbol{e}^{\mathrm{H}} \boldsymbol{\nu}_i|^2} = \frac{1}{|\boldsymbol{e}^{\mathrm{H}} \boldsymbol{V}_n|^2}\,, \tag{5}$$

$\boldsymbol{e} = [1, e^{j2\pi f}, \cdots, e^{j(N-1)2\pi f}]$ expresses the signal vector, $\boldsymbol{V}_n = [\nu_{p+1}, \nu_{p+2}, \cdots, \nu_N]$ expresses the eigen vector of $\boldsymbol{R}_x$ corresponding to the zero eigen values of number $N-P$, $\boldsymbol{e}^H\boldsymbol{\nu}_i$ is the projection of $\boldsymbol{e}$ onto the noise subspace. Corresponding to each frequency f_i contained in the signal, P_{MUSIC} will appear a peak value

$$\hat{\boldsymbol{P}}_{\mathrm{MUSIC}}(f=f_i) = \max_{f=f_i} \frac{1}{|\boldsymbol{e}^{\mathrm{H}}\boldsymbol{V}_n|^2} = \hat{\mathrm{P}}_{\max} . \tag{6}$$

In practical applications, we generally divide f into hundreds of equally spaced units, and calculate $\hat{\boldsymbol{P}}_{MUSIC}(f_i)$ by substituting each f_i into the above formula to find out f_i corresponding to all peak values, as soon as the peak values of the spectrum are derived, the signal frequencies are determined.

3. TESTING BY A SIMULATIVE PERIODIC SIGNAL

For testing the reliability and performance of the MUSIC spectrum estimation, we adopt a simulative signal, and make a comparison with the average peridogram method. Since the actual astronomical signal contains always a noise component, the simulative signal contains three frequency components and a noise component, namely

$$x(t) = \sum_{i=1}^{3} A_i \sin(2\pi f_i t) + \xi_i , \tag{7}$$

in which $A_1 = A_2 = A_3 = 6$, $f_1 = 0.1\,\mathrm{Hz}$, $f_2 = 0.15\,\mathrm{Hz}$, $f_3 = 0.3\,\mathrm{Hz}$. For being more representative, the frequency f_1 has a small difference from f_2 but a rather large difference from f_3. The observation noise ξ_i is a white noise. In order to compare the anti-noise ability between two spectral analysis methods, the signal-noise ratio is gradually reduced. In this experiment, the sampling frequency is 1000 Hz, the initial number of sampling points is 256, the sampled sequence is used to make respectively the average periodogram and MUSIC spectrum estimations, the adopted signal and the results obtained from the spectrum estimations are given in Fig.1, in which the horizontal axis represents frequency, the vertical axis represents the amplitude of the estimated power spectrum, and for the contrast of peak values, the diagrams of power spectrum are normalized.

From the above simulation results, we can find that even for a small-size sample, the MUSIC algorithm can improve the frequency resolution by reducing the realtime requirement. The corresponding frequencies and therefore the periods can be found from their frequency-domain peaks, and under the condition of low signal-noise ratio, useful signals can be accurately detected by noise suppression. In the process of astronomical observations, affected by many factors, the observational data unavoidably suffer from much noise interferences, the periodogram spectral analysis may result in false periods, and can not detect accurately the signal frequencies, in contrast, the MUSIC spectrum estimation can detect accurately the signal frequencies even under the condition of low signal-noise ratio. The above simulation results have proved the feasibility and superiority of the MUSIC algorithm in spectrum estimation.

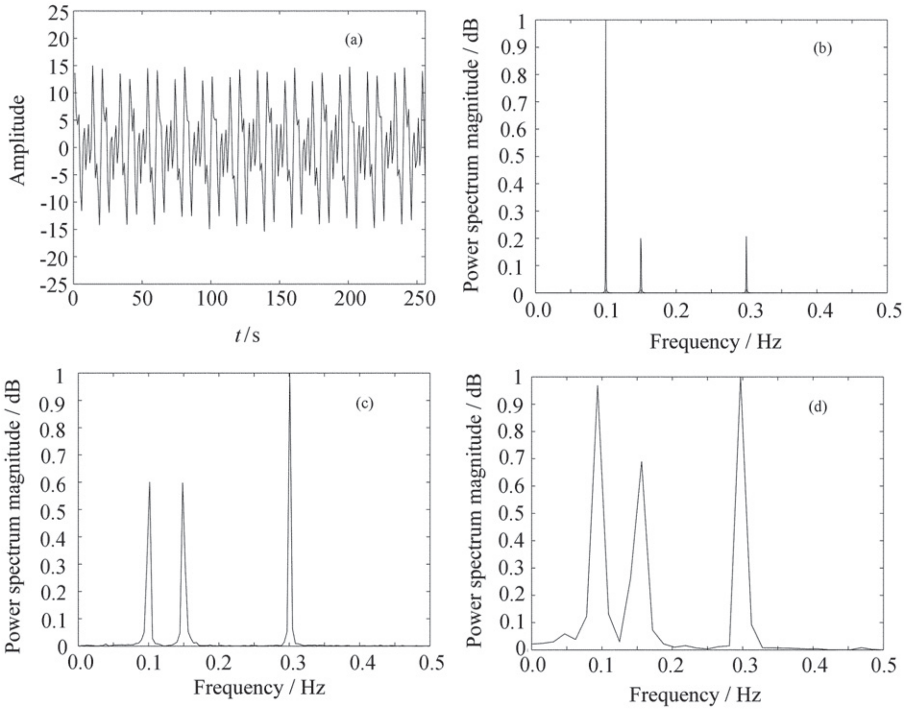

Fig. 1 Simulative signal. (a) The original oscillogram; (b) The MUSIC power spectrum; (c) The average periodogram power spectrum (N=256); (d) The average periodogram power spectrum (N=64)

4. CALCULATION AND ANALYSIS OF LIGHT PERIODS OF THE BL LAC OBJECT S5 0716+714

The BL Lac object S5 0716+714 has a high variability at different wavebands, since it was discovered, a considerable number of observed data have been accumulated, the published literature revealed that it exhibits the light variations at short timescale, medium timescale, and long timescale[7−9,11−23]. In order to determine the actual light periods, the sufficient data observed in a long duration are necessary. We have collected the observational data of S5 0716+714 at the three optical wavebands (V, R and I) in the duration from 1994 to 2008, including the observed data from 1994 to 2001 given by Qian et al.[8] and Raiteri et al.[11], the data from 1996 to 2003 given by Montagni et al.[12], the data from 2003 to 2006 given by Wu et al.[13−14], and the data in 2008 provided by Poon et al.[15]. The collected data are richer than those used for analyzing the light periods of S5 0716+714 by Zhang H.J. et al.[16−18], they have given the light curves of S5 0716+714 at the three optical wavebands (V, R, and I).

In actual astronomical observations, generally several times of observations are made on the every object in one month, therefore there are multiple data points for each month, this is one of the reasons to cause noises, unfavorable to the periodicity analysis, in order to eliminate this effect, we have made the monthly average on the collected data of the historic light variations of S5 0716+714 at the optical I, R and V wavebands. For the points without data, they are replaced by interpolated values, then a new time series is constructed according to the order of time. After the data preprocessing, the equally-spaced observational data are used to make the analysis of long-periodic light variations by using respectively the MUSIC algorithm and the average periodogram method.

The power spectra of S5 0716+714 at the optical I, R, and I wavebands obtained respectively by the average periodogram algorithm and the MUSIC algorithm are given in Fig.2, in which the horizontal axis expresses frequency, the vertical axis expresses the amplitude of the estimated spectrum. Fig.2(a), Fig.2(c) and Fig.2(e) are the results obtained by using the MUSIC algorithm, Fig.2(b), Fig.2(d) and Fig.2(f) are the results obtained by using the average periodogram algorithm, Fig.2(a), Fig.2(b) are the results for the I band, Fig.2(c), Fig.2(d) are the results for the R band, and Fig.2(e), Fig.2(f) are the results for the V band. If at a frequency appears a distinct peak, then it is concluded that in the original observation series this frequency component has a larger spectral intensity, being a major periodic component of the series. From the six spectra in the figure, we can find that at the frequency of about $(0.025\pm0.0006)\,\mathrm{month}^{-1}$, the power spectrum has a distinct peak, indicating that the observational data contain a periodic component of about (3.33 ± 0.08) yr, this main period is consistent with the conclusions made by other authors[6,11,17−18].

The purpose of the spectral analysis on S5 0716+714 is to find out the major periodic components that probably hidden in the data of historic light variations, because these major periodic components differ in intensity, in the spectrum diagram the major periodic component with strongest intensity, namely that corresponds to the peak, is easy to be discriminated, but those with smaller intensities will be easy neglected.

In Fig.2(a) the I-band main peak has a very large amplitude, and the peak is quite sharp, in other places the spectrum is rather flat, with no significant secondary peaks, the data exhibit an obvious periodicity. In Fig.2(b) the spectrum has a secondary peak at the frequency $(0.032\pm0.0008)\,\mathrm{month}^{-1}$, corresponding to the period of (2.60 ± 0.07) yr, this has never been mentioned by other people before, but the corresponding intensity is very small, whether it is a true light period should be further verified by even more data. At the I band, the MUSIC algorithm and average periodogram algorithm have almost the same accuracy of frequency estimation.

In Fig.2(c), the main-peak frequency is not exactly $(0.025\pm0.0006)\,\mathrm{month}^{-1}$, there exists a small line shift, the spectral line is rather smooth, and the line width is rather broad, easy to cause an estimation error. Relative to the main peak, the secondary peak has a not very small amplitude, its frequency is $(0.067\pm0.001)\,\mathrm{month}^{-1}$, corresponding to the period of (1.24 ± 0.01) yr, this result is rather consistent with the 1.10 yr period obtained by Zhang H. J. et al.[16]. Zhang H. J. et al. believed that the 1.10 yr period may have a harmonic relation with the period of 3.30 yr that obtained by Raiteri et al.[11], so the 1.10 yr period seems to be the true period. Even so, this period would not the true period of the object, but that caused by the earth rotation. Whether the 1.10 yr period is a true period or not, it needs to be confirmed by future observations.

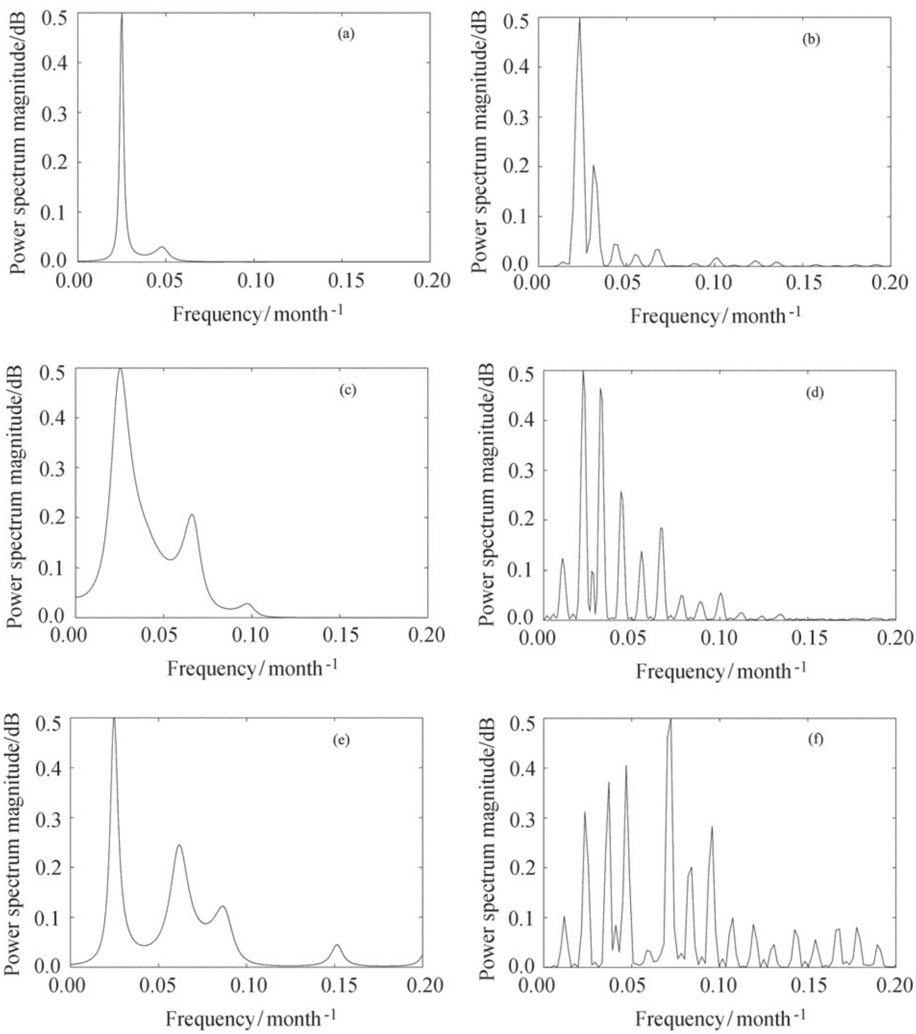

Fig. 2 The power spectrum estimation of S5 0716+714. (a) The MUSIC power spectrum (I-band); (b) The average periodogram power spectrum (I-band); (c) The MUSIC power spectrum (R-band); (d) The average periodogram power spectrum (R-band); (e) The MUSIC power spectrum (V-band); (f) The average periodogram power spectrum (V-band)

From Fig.2(d), we can find also a secondary period of (1.24$\pm$0.01) yr, and that in the spectrum obtained by the average periodogram algorithm exist many false peaks, it is because that the quality of the data used by this method depends on the signal-noise ratio, systematic error, and data distribution. Fig.2(e) has a secondary peak with the frequency of (0.062$\pm$0.007) month^{-1} corresponding to the period of (1.34$\pm$0.01) yr, which differs from the period of (1.24$\pm$0.01) yr that obtained from Fig.2(c) and Fig.2(d), the reason is that the observational data are always finitely long, in the power spectrum estimation, the correlation matrix in the signal space is estimated from the finitely long data, in the V-band data a strong noise component exists, under the influence of the strong noise component, when the sample of observational data is rather small, a certain error will be unavoidable[34]. In Fig.2(f) we can find many peaks, the strong secondary peaks influence the judgement of the main peak, the serious grating effect and line splitting appear, and therefore false peaks, because of the increased appearance probability of false peaks and the randomness of their positions, these false peaks greatly interfere the judgement of signal periods, thus the quality of spectrum estimation declines significantly.

5. DISCUSSIONS

Now, the periodicity analysis and study on the light variations of the BL Lac object 05 0716+714 have obtained many results. The study on the short-timescale light variations shows the existence of short-timescale periodicity, and the study on the long-timescale light variations indicates also the existence of long-timescale periodicity. Heidt et al.[19] obtained a possible 4 d period at the optical waveband, Wu et al.[13−14] found that it has the 0.17 d, 0.11 d and 0.23 d periods, Qian et al.[20] suggested a period of 10 d, Quirrenbach et al.[21] inferred that its period varies from 1 d to 7 d, Ma et al.[22] obtained a period of 14 d, Sagar et al.[23] summarized a period of about 30 d, Katajainen et al.[1] observed the time interval between two maximum fluxes being 60±70 d. By adopting the discrete Fourier transform, discrete correlation analysis, and structure function method, Raiteri et al.[11] found that its light period is 3.3 yr at the optical waveband, and that the light period is 5.5∼6 yr at the radio waveband. Zhang H. J. et al.[16] obtained the 1.1 yr period at the optical B, V, R, and I wavebands, and obtained the period of 1 160 d with the wavelet analysis method[17−18], Qian et al.[8] found the light period of 5.3 yr. Based on the updated data, we have successively made the periodicity analysis on the optical data of S5 0716+714 by using the autoregression method and the double-spectrum method, and found that S5 0716+714 has the periods of ∼3.33 yr and 3.47 yr[6]. As far as the above description concerned, the question, whether the variability of this source has one or multiple periodic components, seems to be still in arguments, in addition, even for the same data sample, the periodicity derived from the light curve may differ at some extent.

In this paper we have applied the MUSIC algorithm and the average periodogram algorithm to the data of historic light variations of the BL Lac object S5 0716+714 at the V, R, and I wavebands, and obtained the periods of (3.33±0.08) yr and (1.24±0.01) yr. This result is basically consistent with the conclusions made by Raiteri et al.[11] and Zhang H. J. et al.[16−17] with the other methods. Both the simulation experiment and the periodicity analysis of S5 0716+714 indicate that with a clear physical concept, simple methodology and high computing efficiency, the peridogram method has become a basic method for the power spectrum estimation, but when the observational data are not very much or under the condition of low signal-noise ratio, false peaks may occur, leading to false periods and affecting its practical application. Generally speaking, by using an eigen decomposition the MUSIC algorithm has improved significantly the performance of spectrum estimation under the condition of low signal-noise ratio, in addition to a high resolution, it can theoretically approach to an infinitely high accuracy, and it is a representative of modern spectrum estimation. When the sample length is rather short, it can estimate the periodic components in a signal with a rather high accuracy, this is of important significance in the application of periodicity analysis of astronomical data. The MUSIC algorithm is superior to the periodogram algorithm in both estimation accuracy and frequency resolution.

ACKNOWLEDGEMENT The author wishes to thank the Referee for valuable opinion, Prof. Wu Xue-bing of Astronomical Department of Beijing University for teaching and guidance, Profs. Zhang Yi-ming, He Ning, Fu Ming-xing, Pang Qiao, Ren Ya-Jie, He Junfeng of Shaanxi University of Technology for warm-hearted helps.

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