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# A Two-Dimensional DOA Tracking Algorithm Using PAST with L-Shape Array

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**Abstract**

In this paper, we proposed a 2D-DOA tracking algorithm using projection approximation and subspace tracking (PAST) with L-shape Array. The proposed algorithm obtains signal subspace by PAST, without any eigen-value decomposition (EVD) of the covariance matrix, Compared with off-line algorithms, PAST algorithm has a much lower computational complexity. Finally we use ESPRIT algorithm to achieve automatically paired 2D-DOA estimation.

*Keywords:* 2D-DOA tracking; PAST;L-shape Array

## 1. Introduction

Direction-of-arrival (DOA) estimation is a very important part of signal processing theory and has been widely used in many fields[1-2]. And many algorithms have been proposed for DOA estimation，including multiple signal classification (MUSIC) algorithm[3] and estimation of signal parameters via rotational invariance technique (ESPRIT) algorithm[4]. However, these algorithms consider the signal targets are stationary targets, therefore the condition of moving signal targets is not considered. The off-line algorithms with high computational complexity of obtaining signal subspace, which not suitable for DOA tracking.

Recently, in order to achieve DOA tracking, subspace tracking algorithms have been proposed. Among them, Projection Approximation Subspace Tracking (PAST) algorithm[6] and Projection Approximation Subspace Tracking with deflation (PASTd) algorithm[7-8] are widely used. Both of these two algorithms avoid eigen-value decomposition or singular value decomposition and reduce the computational complexity[9-11]. In this paper ,we use PAST algorithm to achieve DOA tracking With L-Shape Array.


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The remainder of this paper is structured as follows: Section 2 introduce L-shape array, which is the frame of PAST algorithm, while Section 3 proposes the PAST algorithm for 2D-DOA tracking. In Section 4, we use simulation to verify the effectiveness of the proposed algorithm, and give the conclusion of proposed algorithm in Section 5.

## 2. Data Model

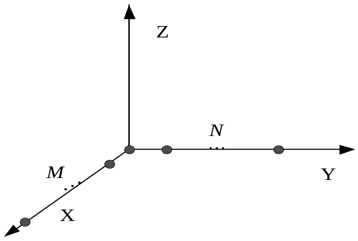


Fig.1. L-shape Array

Consider an L-shaped array in fig.1, which is constituded by *M* array elements in *x* axis and *N* array elements in *y* axis, has *M*+*N*-1 array elements. Element spacing is *d*. Suppose there are *K* sources, whose DOA is $(\theta_k, \phi_k)$ $k = 1, \cdots, K$, $\theta_k$ and $\phi_k$ is the elevation angle and azimuth angle of *k*-th source. Sources irradiated onto the L-shaped array, then the direction vectors of *k*-th source in *x* axis and *y* axis are[12]:

$$\mathbf{a}_x = [1 \ \cdots \ \exp(-j2\pi d(M-1)\sin\theta_k \cos\phi_k / \lambda)]^T \tag{1}$$

$$\mathbf{a}_y = [\exp(-j2\pi d \sin\theta_k \sin\phi_k / \lambda) \ \cdots \ \exp(-j2\pi d(N-1)\sin\theta_k \sin\phi_k / \lambda)]^T \tag{2}$$

Then the direction vector in *x* axis and *y* axis are:

$$\mathbf{A}_x = \begin{bmatrix} 1 & \cdots & 1 \\ e^{-j\frac{2\pi}{\lambda} d \sin\theta_1 \cos\phi_1} & \cdots & e^{-j\frac{2\pi}{\lambda} d \sin\theta_K \cos\phi_K} \\ \vdots & \ddots & \vdots \\ e^{-j\frac{2\pi}{\lambda}(M-1) d \sin\theta_1 \cos\phi_1} & \cdots & e^{-j\frac{2\pi}{\lambda}(M-1) d \sin\theta_K \cos\phi_K} \end{bmatrix} \mathbf{A}_y = \begin{bmatrix} & \cdots & \\ e^{-j\frac{2\pi}{\lambda} d \sin\theta_1 \sin\phi_1} & \cdots & e^{-j\frac{2\pi}{\lambda} d \sin\theta_K \sin\phi_K} \\ \vdots & \ddots & \vdots \\ e^{-j\frac{2\pi}{\lambda}(N-1) d \sin\theta_1 \sin\phi_1} & \cdots & e^{-j\frac{2\pi}{\lambda}(N-1) d \sin\theta_K \sin\phi_K} \end{bmatrix} \tag{3}$$

We can see that $\mathbf{A}_x$ and $\mathbf{A}_y$ are both Vandermonde matrixs. The received signal on the L-shaped array is

$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} \mathbf{A}_x \\ \mathbf{A}_y \end{bmatrix} \mathbf{s}(t) + \mathbf{n}(t) \\ &= \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \end{aligned} \tag{4}$$

Wherein, $\mathbf{s}(t) \in \mathbb{C}^{M+N-1 \times N_s}$ denotes the random signal received by the array elements. $\mathbf{n}(t) \in \mathbb{C}^{M+N-1 \times F}$ denotes Gaussian white noise generated by the array elements.

## 3. Tracking with PAST Algorithm

Define an unconstrained cost function:

$$\mathbf{J}(\mathbf{W}) = \mathbf{E}\{\left\| \mathbf{x} - \mathbf{W}\mathbf{W}^{\mathrm{H}}\mathbf{x} \right\|^2\} \tag{5}$$

After simplification:
We can find:

$$\mathbf{E}\{\mathbf{x}^{\mathrm{H}}\mathbf{x}\} = \sum_{i=1}^{M} \mathbf{E}\{\left|\mathbf{x}_i\right|^2\} = \mathrm{tr}(\mathbf{E}\{\mathbf{x}\mathbf{x}^{\mathrm{H}}\}) = \mathrm{tr}(\mathbf{C}) \tag{6}$$

$$\mathbf{E}\{\mathbf{x}^{\mathrm{H}}\mathbf{W}\mathbf{W}^{\mathrm{H}}\mathbf{x}\} = \mathrm{tr}(\mathbf{E}\{\mathbf{W}^{\mathrm{H}}\mathbf{x}\mathbf{x}^{\mathrm{H}}\mathbf{W}\}) = \mathrm{tr}(\mathbf{W}^{\mathrm{H}}C\mathbf{W}) \tag{7}$$

$$\mathbf{E}\{\mathbf{x}^{\mathrm{H}}\mathbf{W}\mathbf{W}^{\mathrm{H}}\mathbf{W}\mathbf{W}^{\mathrm{H}}\mathbf{x}\} = \mathrm{tr}(\mathbf{E}\{\mathbf{W}^{\mathrm{H}}\mathbf{W}\mathbf{x}\mathbf{x}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{W}\}) = \mathrm{tr}(\mathbf{W}^{\mathrm{H}}C\mathbf{W}\mathbf{W}^{\mathrm{H}}\mathbf{W}) \tag{8}$$

We can use trace function to represent the objective function:

$$\mathbf{J}(\mathbf{W}) = \mathrm{tr}(C) - 2\mathrm{tr}(\mathbf{W}^{\mathrm{H}}C\mathbf{W}) + \mathrm{tr}(\mathbf{W}^{\mathrm{H}}C\mathbf{W}\mathbf{W}^{\mathrm{H}}\mathbf{W}) \tag{9}$$

Among them, $\mathbf{W}$ is $M \times N$ matrix, suppose the rank is $N$ .

Actually, subspace changes with time, we use the subspace $W(t\text{-}1)$ which is got at time $t$-1 and metadata array at time $t$ to get the subspace W($t$). We choose gradient descent, we can get :

$$\nabla\mathbf{J} = [-2\mathbf{C} + \mathbf{C}\mathbf{W}\mathbf{W}^{\mathrm{H}} + \mathbf{W}\mathbf{W}^{\mathrm{H}}\mathbf{C}]\mathbf{W} \tag{10}$$

So：

$$\begin{aligned}\mathbf{W}(t)&=\mathbf{W}(t\text{-}1)\text{-}\mu\nabla J(\mathbf{W}(t\text{-}1))\\ &=\mathbf{W}(t\text{-}1)\text{-}\mu[\text{-}2\hat{\mathbf{C}}(t)+\hat{\mathbf{C}}(t)\mathbf{W}(t\text{-}1)\mathbf{W}^{H}(t\text{-}1)\\ &\quad+\mathbf{W}(t\text{-}1)\mathbf{W}^{H}(t\text{-}1)\hat{\mathbf{C}}(t)]\mathbf{W}(t\text{-}1)\end{aligned} \tag{11}$$

We take $\hat{\mathbf{C}}(t)=\mathbf{x}(t)\mathbf{x}^{H}(t)$ and $\mathbf{y}(t)=\mathbf{x}(t)\mathbf{x}^{H}(t)$ into the last formula:

$$\begin{aligned}\mathbf{W}(t)=\mathbf{W}(t\text{-}1)\text{-}\mu[&2\mathbf{x}(t)\mathbf{y}^{H}(t) - \mathbf{x}(t)\mathbf{y}^{H}(t)\\ &\times\mathbf{W}^{H}(t\text{-}1)\mathbf{W}(t\text{-}1)\text{-}\mathbf{W}(t\text{-}1)\mathbf{W}^{H}(t\text{-}1)\mathbf{y}(t)\mathbf{y}^{H}(t)]\end{aligned} \tag{12}$$

Because $\mathbf{W}^{\mathrm{H}}(t\text{-}1)\mathbf{W}(t\text{-}1)=\mathbf{I}$ , we can get:

$$\mathbf{W}(t)=\mathbf{W}(t\text{-}1)+\mu[\mathbf{x}(t) - \mathbf{W}(t\text{-}1)\mathbf{y}(t)]\mathbf{y}^{H}(t) \tag{13}$$

As the ability of $\mathbf{W}(\mathrm{t})$ to track time-varying subspace is bad and has slow convergence. In order to solve the problem, we can define a new exponential weighting function:

$$\begin{aligned}J(\mathbf{W}(t)) &= \sum_{i=1}^{t}\beta^{t-i}\left\|\mathbf{x}(i) - \mathbf{W}(t)\mathbf{W}^{H}(t)\mathbf{x}(i)\right\|^{2}\\ &=\mathrm{tr}[\mathbf{C}(t)]\text{-}2\mathrm{tr}[\mathbf{W}^{H}(t)\mathbf{C}(t)\mathbf{W}(t)]+\mathrm{tr}[\mathbf{W}^{H}(t)\mathbf{C}(t)\mathbf{W}(t)\mathbf{W}^{H}(t)\mathbf{W}(t)]\end{aligned} \tag{14}$$

where $0 < \beta \le 1$, $\beta$ represents forgetting factor, $\mathbf{C}(t) = \sum_{i=1}^{t}\beta^{t-i}\mathbf{x}(i)\mathbf{x}^{\mathrm{H}}(i)$ . Further, we can use:

$$\mathbf{y}(i)=\mathbf{W}^{H}(i\text{-}1)\mathbf{x}(i) \approx \mathbf{W}^{H}(t)\mathbf{x}(i) \tag{15}$$

we can get the fixed cost function:

$$J(\mathbf{W}(t)) = \sum_{i=1}^{t}\beta^{t-i}\left\|\mathbf{x}(i) - \mathbf{W}(t)\mathbf{y}(i)\right\|^{2} \tag{16}$$

At this time, the optimal solution of $\min \mathrm{J}(\mathbf{W}(\mathrm{t}))$ is wiener filter:

$$\mathbf{W}(t)=\mathbf{C}_{xy}(t)\mathbf{C}_{yy}^{-1}(t) \tag{17}$$

The relationship of autocorrelation matrix $\mathbf{C}_{\mathrm{yy}}(\mathrm{t})$ and cross-correlation matrix $\mathbf{C}_{\mathrm{xy}}(\mathrm{t})$ are:

$$\mathbf{C}_{xy}(t) = \sum_{i=1}^{t}\beta^{t-i}\mathbf{x}(i)\mathbf{y}^{\mathrm{H}}(i) = \beta\mathbf{C}_{xy}(t-1) + \mathbf{x}(t)\mathbf{y}^{\mathrm{H}}(t) \tag{18}$$

$$\mathbf{C}_{yy}(t) = \sum_{i=1}^{t}\beta^{t-i}\mathbf{y}(i)\mathbf{y}^{\mathrm{H}}(i) = \beta\mathbf{C}_{yy}(t-1) + \mathbf{y}(t)\mathbf{y}^{\mathrm{H}}(t) \tag{19}$$

Take (18), (19) into (17), we can get proposed (PAST) algorithm with matrix inverse theorem. The main steps of PAST algorithm are:

Table.1. Algorithmic process of PAST

select $\mathbf{P}(0)$ and $\mathbf{W}(0)$ appropriately。
For $k$=1,2,$\cdots$
$\mathbf{y}(k)=\mathbf{W}^H(k\text{-}1)\mathbf{x}(k)$
$f(k)=\mathbf{P}(k\text{-}1)\mathbf{y}(k)$
$\mathbf{g}(k)=f(k)/[\beta+\mathbf{y}^H(k)f(k)]$
$\mathbf{P}(k)=1/\beta * Tri\{\mathbf{P}(k\text{-}1)-\mathbf{g}(k)f^H(k)\}$
$\mathbf{e}(k)=\mathbf{x}(k)-\mathbf{W}(k\text{-}1)\mathbf{y}(k)$
$\mathbf{W}(k)=\mathbf{W}(k\text{-}1)+\mathbf{e}(k)\mathbf{g}^H(k)$
End

For PAST algorithm, we can set the initial value of $\mathbf{P}(0)$ and $\mathbf{W}(0)$ a matrix or its subarray. In this section, we use PAST algorithm to update signal subspace continuously, then use ESPRIT algorithm to estimate two angles of the sources.

The angles estimated by ESPRIT algorithm will be matched automatically, but can't be related with the sources. Now the classic association algorithms include track splitting method and nearest neighbor method and so on. Ref. [13] use uniform array to estimate twice at target subspace, which can achieve the association of the angles and sources by reconstruction of the covariance matrix method. This section use the method which is put forward in Ref. [13] to associate the sources and angles.

What's given below is the algorithm steps of DOA tracking in L-shaped array based on PAST:

(1) Get the signal output at time *t* based on the matrix direction of the L-shaped array.
(2) Select appropriate $\mathbf{P}(0)$ and $\mathbf{W}(0)$, get the signal subspace $\mathrm{V}_\mathrm{S}(t)$ based on PAST algorithm.
(3) ESPRIT algorithm use the signal subspace which is solved out by PAST to estimate the azimuth $\theta(t)$ and $\phi(t)$ of signals.
(4) Use the association algorithm in this section to associate the angles and sources.
(5) Repeat steps (1)-(5) to compute the angles at time *t*+1.

## 4. Simulations

We used 1000 Monte Carlo experiments to verify the performance of the PAST algorithm. Define root mean square error (RMSE):

$$RMSE=\frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{1000}\sum_{l=1}^{1000}\left[\left(\hat{\theta}_{k,l}-\theta_k\right)^2+\left(\hat{\phi}_{k,l}-\phi_k\right)^2\right]} \tag{20}$$

where $\hat{\theta}_{k,l},\hat{\phi}_{k,l}$ are the estimates of $\theta_k,\phi_k$ in the *l*-th Monte Carlo trial, respectively. We set $d=\lambda/2$.

*M*, *N* and *K* are the number of the array elements in *x* axis, *y* axis and the number of sources. Suppose that sources move in line or curve, it means azimuth and elevation changes linearly with time and the number of sources keep stable in the moving. We set $\beta$=0.97 .The time of tracking is 60 seconds, tracking once every second. Snapshots $N_s$ is 60, then use PAST algorithm to estimated signal subspace.

Fig.2 present azimuth angles and elevation angles of two moving sources. We set SNR= 25dB and number of array elements *M*=16, *N*=16. Fig.2 show the trajectories of DOA with time elapses. We can see that the DOA of sources can be estimated clearly during the tracking time.

First picture of Fig.3 illustrates relative change of two - dimensional angles of moving sources. From the picture, What can be concluded is also that proposed algorithm can effectively achieve DOA tracking of moving sources. Second picture of Fig.3 shows the angle tracking performance varies with SNR. The picture presents proposed algorithm has better tracking accuracy at higher SNR. Figs.4 show angle tracking performance of proposed algorithm varies with *M* or *N*. We can see that the angle tracking performance of proposed algorithm is improved with the increasing of array elements. Multiple antennas improve angle estimation performance because of diversity gain.

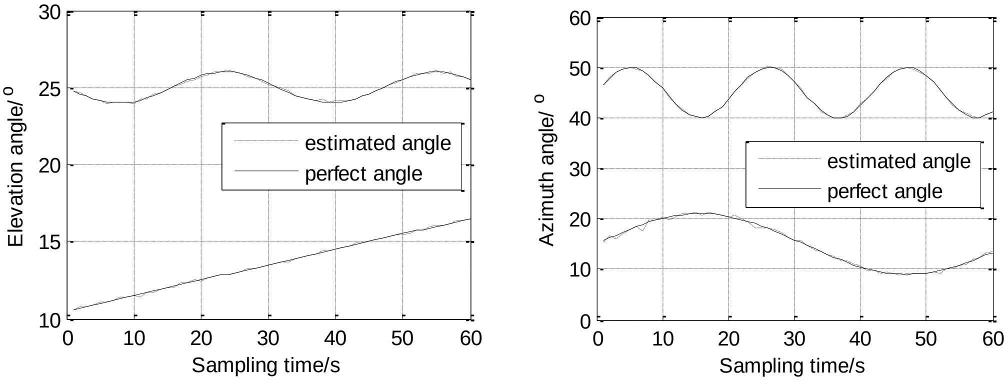


Fig. 2. (1) The estimated and perfect elevation angles of two sources ;(2) The estimated and perfect azimuth angles of two sources

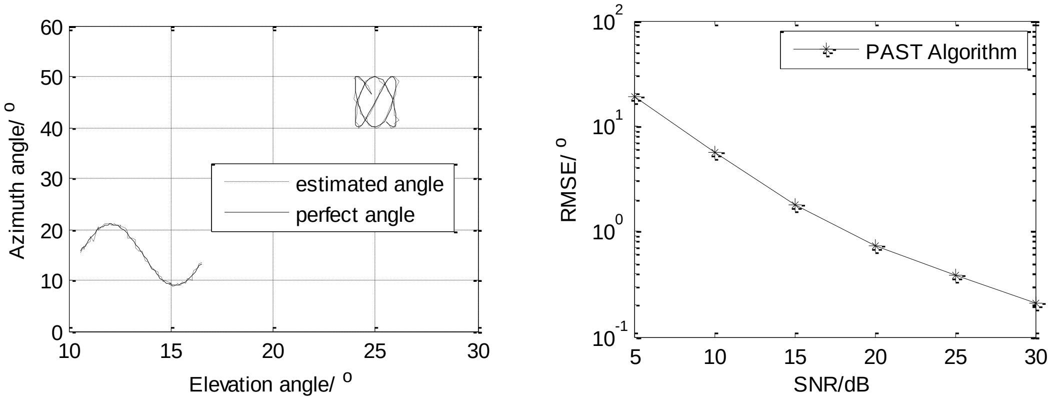


Fig. 3. (1)The azimuth angles and elevation angles of two sources ;(2) Angle tracking performance varies with SNR,

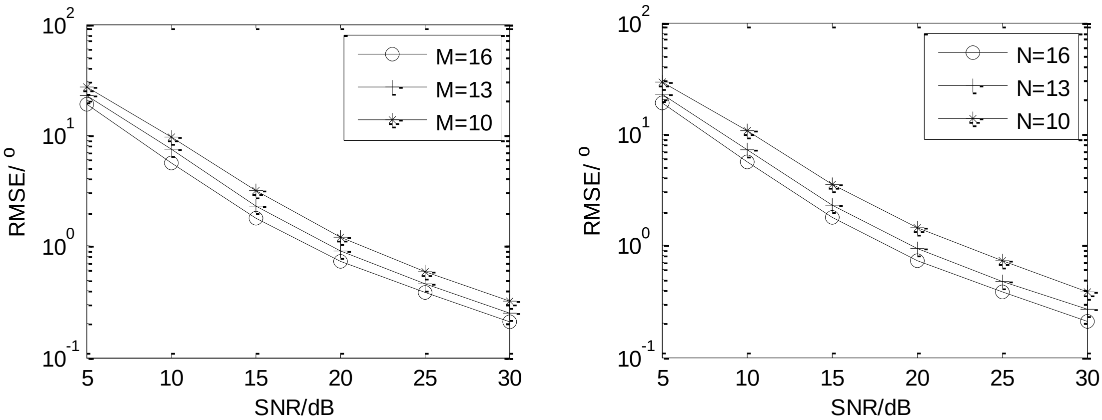


Fig. 4. (1)DOA tracking performance varies with *M* ;(2)DOA tracking performance varies with *N*

## 5. Conclusion

In this paper, we proposed a 2D-DOA tracking algorithm using PAST with L-shape Array. The proposed algorithm obtains signal subspace by PAST, without any EVD of covariance matrix, which has a much lower computational complexity than off-line algorithms. Then we use ESPRIT algorithm to achieve 2D-DOA estimation, finally we associate the estimated angles with the sources. Through simulation experiments, the effectiveness of proposed algorithm has been verified.

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