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An eigenvector based method for estimating DOA and sensor gain-phase errors

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ABSTRACT

This paper addresses the problem of direction of arrival (DOA) estimation in the presence of sensor gain-phase errors. We derive an interesting conclusion that the Cramer–Rao Bound (CRB) on DOA and gain-phase error estimation accuracy is independent of sensor phase errors. Then, we propose a new method by using the Hadamard product of the signal eigenvectors and its conjugate. In addition, we derive a sufficient condition for unambiguous DOA estimation of the proposed method.
We observe that the proposed method performs independently of phase errors, which is same as the CRB. Furthermore, it is robust to the correlated signals. Simulation results verify the above-mentioned observations and the effectiveness of the proposed method.

1. Introduction

The direction of arrival (DOA) estimation in array signal processing has been a strong interest because of its applications such as jammer localization, radio emitter tracking, and mobile communications [1], [2]. DOA estimation methods involve beamforming methods such as the capon's beamformer [3], maximum likelihood (ML) methods [4], [5], and eigenstructure-based methods [6–10]. In these methods, it is assumed that the array manifold is known. However, this assumption is rarely satisfied in practice. The array errors such as gain-phase errors, mutual coupling, and sensor position errors will degrade substantially the performance of most of DOA estimation algorithms [2], [11].

With the assumption that array model errors are deterministic unknown quantities, self-calibration techniques [12–15] estimate the array errors and DOAs simultaneously. The methods in [12], [13] jointly estimate the DOAs of signals and array errors. However, these methods suffer from suboptimal convergence due to the involved joint iteration, and they fail in large gain-phase errors. The methods in [14,15] avoid the above-mentioned iteration. However, these methods are limited to the arrays of special structures (for instance, a uniformly spaced linear array).

In this paper, the problem of the DOA estimation in the presence of constant but unknown gain-phase errors is addressed, which is of significance because most of DOA estimation methods are sensitive to the unavoidable gain-phase errors caused by thermal effects, aging of components, the environment changes around the sensor array, and so on [11], [12]. Intuitively, one may speculate that the CRB on DOA and gain-phase error estimation accuracy increases as phase errors become larger. Unlike the speculation, we derive an interesting conclusion that the CRB is independent of phase errors. To the best of our knowledge, the performance of most of the existing methods [12], [13] for estimating DOA and array errors degrade significantly with increasing phase errors and deviate from the CRB in large phase errors.

In our previous work [16], we proposed an eigenstructure method for estimating DOA and sensor gain-phase errors, which performs independently of phase errors. Due to the phase error independence, during the operation life of an array, the phase error calibration can be avoided if the method in [16] is employed. Following [16], another DOA estimation method was developed in [17], which also attains the phase-error independence. The methods in [16] and [17] both suffer from correlated signals, because the involved gain error estimation requires the assumption of independent signals. However, correlated signals are commonly encountered in practice. In addition, the method in [17] involves the estimation of a parameter related to the signal power, which is non-convex optimization. Therefore, the feasibility of the two methods in [16] and [17] is limited.

In this paper, we propose a method for estimating DOA and gain-phase errors, in which the DOA and gain error estimation is developed based on the Hadamard product of the signal eigenvec-

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tors and their conjugates. Like the methods in [16] and [17], the proposed method performs independently of phase errors as the CRB does. In contrast to the methods in [16] and [17], the proposed method works well in the case of correlated signals and it does not require the information of signal power. In addition, we derive a sufficient condition which guarantees the unambiguous DOA estimates of the proposed method and give a combined strategy which utilizes the proposed method and the WF method in [12] together. Simulation results are conducted to verify the performance of the proposed method and the combined strategy.

This paper is organized as follows. Section 2 describes the preliminaries. Section 3 derives the phase error independence of the CRB on DOA. The proposed method is given in Section 4. Some discussions are contained in Section 5. Simulation results are presented in Section 6. Finally, a conclusion is given in Section 7.

2. Preliminaries

In this paper, the superscripts *, T, and H represent the conjugate, transpose, and conjugate transpose operations, respectively. Consider an array with M omni-directional sensors located on the same plane and numbered 1 through M, where Sensor 1 is taken as reference. Suppose that there are K narrow-band far-field signals $\{s_k(t)\}_{k=1}^{K}$ with the center wavelength λ and the DOAs $\{\theta_k\}_{k=1}^{K}$. Moreover, the signals and the array sensors are assumed in the same plane. The vector of M sensor outputs can then be written as

$$\mathbf{r}_0(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{n}(t)$ is a zero-mean complex Gaussian random process, $\mathbf{s}(t) = [s_1(t), \cdots, s_K(t)]^{\mathrm{T}}$, $\mathbf{A} = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_K)]$, and $\mathbf{a}(\theta_k)$ is the ideal steering vector for the kth signal with DOA θ_k, given by

$$\mathbf{a}(\theta_k) = \left[1, e^{-j2\pi d_{2,k}/\lambda}, \cdots, e^{-j2\pi d_{M,K}/\lambda}\right]^{\mathrm{T}} \tag{2}$$

where $d_{m,k}$ is the distance from Sensor 1 to Sensor m along θ_k

$$d_{m,k} = x_m \cos\theta_k + y_m \sin\theta_k. \tag{3}$$

On the model (1), two general assumptions are introduced below, which are considered to hold throughout this paper.

Assumption 1. $\mathbf{n}(t)$ *is spatially white, that is,* $\mathrm{E}[\mathbf{n}(t)\mathbf{n}^{\mathrm{H}}(t)] = \sigma_n^2\mathbf{I}$ *where* σ_n^2, $\mathrm{E}[\bullet]$, *and* $\mathbf{I}$ *denote the noise power, the expectation operator, and the identity matrix, respectively.*

Assumption 2. $s_k(t)$ *and* $\mathbf{n}(t)$ *are independent of each other.*

In the following, for simplicity we omit the time variable. With Assumptions 1 and 2, the covariance matrix of the array output vector can be written as

$$\mathbf{R}_0 = \mathrm{E}\left[\mathbf{r}_0\mathbf{r}_0^{\mathrm{H}}\right] = \mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}} + \sigma_{\mathrm{n}}^2\mathbf{I} \tag{4}$$

where $\mathbf{R}_s = \mathrm{E}[\mathbf{s}\mathbf{s}^{\mathrm{H}}]$.

Denote the deterministic gain and phase errors of Sensor m with g_m and φ_m, respectively, where $g_1 = 1$ and $\varphi_1 = 0$. In the presence of gain and phase errors, (1) and (4) can be rewritten as

$$\mathbf{r} = \mathbf{\Phi}\mathbf{G}\mathbf{A}\mathbf{s} + \mathbf{n} \tag{5}$$

$$\mathbf{R} = \mathbf{\Phi}\mathbf{G}\mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{\Phi}^{\mathrm{H}} + \sigma_n^2\mathbf{I} \tag{6}$$

where $\mathbf{\Phi}$ and $\mathbf{G}$ are diagonal matrices and their m-th diagonal elements Φ_{mm} and G_{mm} are $e^{j\varphi_m}$ and g_m, respectively.

In practice, the number of samples is finite. Thus, $\mathbf{R}$ must be estimated by

$$\widehat{\mathbf{R}} = \frac{1}{N_s}\sum_{t=1}^{N_s}\mathbf{r}(t)\mathbf{r}^{\mathrm{H}}(t), \tag{7}$$

where N_s defines the number of samples.

Through this paper, "ˆ" will denote the estimate of the quantity over which it appears. This estimate is a result of replacing $\mathbf{R}$ with $\widehat{\mathbf{R}}$.

Thus, the problem addressed here is to estimate DOA and gain-phase errors simultaneously.

3. Independence of CRB on phase errors

Let $\odot$ denote dot product, i.e., element-wise multiplication. Based on [18], we have that the prior matrix, involved in the fisher information matrix, is zero, and the CRB on the covariance matrix of the estimation errors of DOAs and gain-phase errors can be expressed as

$$\left[C_{\mathrm{CR}}(\boldsymbol{\theta}, \mathbf{g}, \boldsymbol{\varphi})\right]^{-1} = \frac{2N_s}{\sigma_n^2}\mathrm{Re}\begin{bmatrix}\mathbf{J}_{\theta\theta} & \mathbf{J}_{\theta g} & \mathbf{J}_{\theta\varphi}\\ \mathbf{J}_{g\theta} & \mathbf{J}_{gg} & \mathbf{J}_{g\varphi}\\ \mathbf{J}_{\varphi\theta} & \mathbf{J}_{\varphi g} & \mathbf{J}_{\varphi\varphi}\end{bmatrix} \tag{8}$$

where $\mathbf{J}_{g\theta} = \mathbf{J}_{\theta g}^{\mathrm{T}}$, $\mathbf{J}_{\varphi\theta} = \mathbf{J}_{\theta\varphi}^{\mathrm{T}}$, $\mathbf{J}_{\varphi g} = \mathbf{J}_{g\varphi}^{\mathrm{T}}$,

$$\mathbf{J}_{\theta\theta} = \left(\mathbf{D}_\theta^{\mathrm{H}}\mathbf{P}_{\mathrm{B}}^{\perp}\mathbf{D}_\theta\right)\odot\mathbf{Y}^{\mathrm{T}} \tag{9}$$

$$\mathbf{J}_{\theta g} = \left(\mathbf{D}_\theta^{\mathrm{H}}\mathbf{P}_{\mathrm{B}}^{\perp}\right)\odot\left(\mathbf{D}_g\mathbf{Y}\right)^{\mathrm{T}} \tag{10}$$

$$\mathbf{J}_{\theta\varphi} = \left(\mathbf{D}_\theta^{\mathrm{H}}\mathbf{P}_{\mathrm{B}}^{\perp}\right)\odot\left(\mathbf{D}_\varphi\mathbf{Y}\right)^{\mathrm{T}} \tag{11}$$

$$\mathbf{J}_{gg} = \left(\mathbf{D}_g\mathbf{Y}\mathbf{D}_g^{\mathrm{H}}\right)\odot\left(\mathbf{P}_{\mathrm{B}}^{\perp}\right)^{\mathrm{T}} \tag{12}$$

$$\mathbf{J}_{g\varphi} = \left(\mathbf{D}_g\mathbf{Y}\mathbf{D}_\varphi^{\mathrm{H}}\right)\odot\left(\mathbf{P}_{\mathrm{B}}^{\perp}\right)^{\mathrm{T}} \tag{13}$$

$$\mathbf{J}_{\varphi\varphi} = \left(\mathbf{D}_\varphi\mathbf{Y}\mathbf{D}_\varphi^{\mathrm{H}}\right)\odot\left(\mathbf{P}_{\mathrm{B}}^{\perp}\right)^{\mathrm{T}}. \tag{14}$$

$\mathbf{B} = \mathbf{\Phi}\mathbf{G}\mathbf{A} = [\mathbf{b}_1, \cdots, \mathbf{b}_K] = [\mathbf{d}_1^{\mathrm{T}}, \cdots, \mathbf{d}_M^{\mathrm{T}}]^{\mathrm{T}}$ (i.e., $\mathbf{b}_i$ denotes the i-th column of $\mathbf{B}$, and $\mathbf{d}_j$ denotes the j-th row of $\mathbf{B}$),

$$\mathbf{D}_\theta = \left[\frac{\partial\mathbf{b}_1}{\partial\theta_1}\frac{\partial\mathbf{b}_2}{\partial\theta_2}\cdots\frac{\partial\mathbf{b}_K}{\partial\theta_K}\right], \qquad \mathbf{D}_g = \left[\frac{\partial\mathbf{d}_1^{\mathrm{T}}}{\partial g_1}\frac{\partial\mathbf{d}_2^{\mathrm{T}}}{\partial g_2}\cdots\frac{\partial\mathbf{d}_M^{\mathrm{T}}}{\partial g_M}\right]^{\mathrm{T}},$$

$$\mathbf{D}_\varphi = \left[\frac{\partial\mathbf{d}_1^{\mathrm{T}}}{\partial\varphi_1}\frac{\partial\mathbf{d}_2^{\mathrm{T}}}{\partial\varphi_2}\cdots\frac{\partial\mathbf{d}_M^{\mathrm{T}}}{\partial\varphi_M}\right]^{\mathrm{T}}, \qquad \mathbf{P}_{\mathrm{B}}^{\perp} = \mathbf{I} - \mathbf{B}(\mathbf{B}^{\mathrm{H}}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{H}},$$

and $\mathbf{Y} = \mathbf{R}_s\mathbf{B}^{\mathrm{H}}\mathbf{R}^{-1}\mathbf{B}\mathbf{R}_s$.

Since $\mathbf{\Phi}\mathbf{\Phi}^{\mathbf{H}} = \mathbf{I}$, we have

$$\mathbf{D}_\theta = \mathbf{\Phi}\mathbf{G}\left[\frac{\partial\mathbf{a}(\theta_1)}{\partial\theta_1}\frac{\partial\mathbf{a}(\theta_2)}{\partial\theta_2}\cdots\frac{\partial\mathbf{a}(\theta_K)}{\partial\theta_K}\right] = \mathbf{\Phi}\mathbf{G}\breve{\mathbf{A}}. \tag{15}$$

We rewrite $\mathbf{A} = [\mathbf{h}_1^{\mathrm{T}}, \cdots, \mathbf{h}_M^{\mathrm{T}}]^{\mathrm{T}}$, then we have $\mathbf{d}_m = g_m e^{j\varphi_m}\mathbf{h}_m$. Following that, we obtain $\frac{\partial\mathbf{d}_m}{\partial g_m} = e^{j\varphi_m}\mathbf{h}_m$ and $\frac{\partial\mathbf{d}_m}{\partial\varphi_m} = je^{j\varphi_m}\mathbf{h}_m$. Thus, we get

$$\mathbf{D}_g = \mathbf{\Phi}\mathbf{A} \tag{16}$$

$$\mathbf{D}_\varphi = j\mathbf{\Phi}\mathbf{G}\mathbf{A} \tag{17}$$

$$\mathbf{P}_{\mathrm{B}}^{\perp} = \mathbf{\Phi}\left(\mathbf{I} - \mathbf{G}\mathbf{A}\left(\mathbf{A}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\right)\mathbf{\Phi}^{H} = \mathbf{\Phi}\mathbf{T}_1\mathbf{\Phi}^{H} \tag{18}$$

$$\mathbf{R}^{-1} = \left(\mathbf{\Phi}^{\mathrm{H}}\right)^{-1}\left(\mathbf{G}\mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}} + \sigma_n^2\mathbf{I}\right)^{-1}\mathbf{\Phi}^{-1} = \left(\mathbf{\Phi}^{\mathrm{H}}\right)^{-1}\mathbf{T}_2\mathbf{\Phi}^{-1} \tag{19}$$

$$\mathbf{Y} = \mathbf{R}_s\mathbf{A}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{T}_2\mathbf{G}\mathbf{A}\mathbf{R}_s. \tag{20}$$

According to (15) and (18), and based on $\mathbf{\Phi}^H\mathbf{\Phi} = \mathbf{I}$, we have $\mathbf{D}_\theta^{\mathrm{H}}\mathbf{P}_{\mathrm{B}}^{\perp}\mathbf{D}_\theta = \breve{\mathbf{A}}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{T}_1\mathbf{G}\breve{\mathbf{A}}$. Thus

$$\mathbf{J}_{\theta\theta} = \left(\breve{\mathbf{A}}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{T}_1\mathbf{G}\breve{\mathbf{A}}\right)\odot\mathbf{Y}^{\mathrm{T}}. \tag{21}$$

Substituting (15) and (16) into (10), we obtain

$$\mathbf{J}_{\theta g} = \left(\breve{\mathbf{A}}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{T}_1\mathbf{\Phi}^{\mathrm{H}}\right) \odot \left((\mathbf{GAY})^{\mathrm{T}}\mathbf{\Phi}\right). \tag{22}$$

Since the matrix $\mathbf{\Phi}$ is diagonal and $\mathbf{\Phi}^H\mathbf{\Phi} = \mathbf{I}$, $\mathbf{J}_{\theta g}$ can be simplified as

$$\begin{aligned}\mathbf{J}_{\theta g} &= \left(\breve{\mathbf{A}}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{T}_1\right) \odot (\mathbf{GAY})^{\mathrm{T}}\mathbf{\Phi}^H\mathbf{\Phi} \\ &= \left(\breve{\mathbf{A}}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{T}_1\right) \odot (\mathbf{GAY})^{\mathrm{T}}.\end{aligned} \tag{23}$$

Similarly, we have

$$\mathbf{J}_{\theta\varphi} = j\left(\breve{\mathbf{A}}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{T}_1\right) \odot (\mathbf{GAY})^{\mathrm{T}} \tag{24}$$

$$\begin{aligned}\mathbf{J}_{gg} &= \left(\mathbf{\Phi}\mathbf{\Phi}^{\mathrm{H}}\right)\left(\left(\mathbf{GAYA}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\right) \odot \mathbf{T}_1^{\mathrm{T}}\right)\left(\mathbf{\Phi}^{\mathrm{H}}\mathbf{\Phi}\right) \\ &= \left(\mathbf{GAYA}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\right) \odot \mathbf{T}_1^{\mathrm{T}}\end{aligned} \tag{25}$$

$$\mathbf{J}_{g\varphi} = -j\left(\mathbf{GAYA}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\right) \odot \mathbf{T}_1^{\mathrm{T}} \tag{26}$$

$$\mathbf{J}_{\varphi\varphi} = \left(\mathbf{GAYA}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\right) \odot \mathbf{T}_1^{\mathrm{T}}. \tag{27}$$

According to (21)–(27), we conclude that $\mathbf{J}_{\theta\theta}$, $\mathbf{J}_{\theta g}$, $\mathbf{J}_{\theta\varphi}$, $\mathbf{J}_{gg}$, $\mathbf{J}_{g\varphi}$, and $\mathbf{J}_{\varphi\varphi}$ are independent of phase errors, so that the matrix $C_{\mathrm{CR}}(\boldsymbol{\theta}, \mathbf{g}, \mathbf{\Phi})$ is independent of phase errors.

In consequence, the CRB on DOA and gain-phase error estimation accuracy is independent of phase errors. In the following, we develop a method which performs independently of phase errors as the CRB does.

4. The proposed method

4.1. Joint DOA and gain-error estimation

In this paper, we assume that the number of signals is known. The number of signals can be estimated by the Minimum Description Length (MDL) and the Akaike Information Criterion (AIC) methods [19].

Decomposing $\mathbf{R}$ yields

$$\mathbf{R} = \sum_{m=1}^{M} \gamma_m \mathbf{v}_m (\mathbf{v}_m)^{\mathrm{H}} \tag{28}$$

where the eigenvalues $\{\gamma_m\}_{m=1}^{M}$ are listed in descending order, and $\{\mathbf{v}_m\}_{m=1}^{M}$ are the corresponding eigenvectors. Then, the signal eigenvectors of the matrix $\mathbf{R}$ are $\{\mathbf{v}_m\}_{m=1}^{K}$ and the remaining ones are the noise eigenvectors.

As shown in [6], the space spanned by the practical steering vectors is equal to the space spanned by the signal eigenvectors of the matrix $\mathbf{R}$, that is

$$span\{\mathbf{\Phi}\mathbf{G}\mathbf{a}_1, \ldots, \mathbf{\Phi}\mathbf{G}\mathbf{a}_K\} = span\{\mathbf{v}_1, \ldots, \mathbf{v}_K\}. \tag{29}$$

Based on (29), it is known that

$$\mathbf{v}_k = \xi_{1k}\mathbf{\Phi}\mathbf{G}\mathbf{a}(\theta_1) + \xi_{2k}\mathbf{\Phi}\mathbf{G}\mathbf{a}(\theta_2) + \cdots + \xi_{Kk}\mathbf{\Phi}\mathbf{G}\mathbf{a}(\theta_K), \tag{30}$$

where $\xi_{1k}, \xi_{2k}, \cdots, \xi_{Kk}$ are constant and not all of them are equal to zero.

Then

$$\mathbf{v}_k \odot \mathbf{v}_l^* = \sum_{i=1}^{K}\sum_{n=1}^{K} \xi_{ik}\xi_{nl}^* \mathbf{G}_1 \mathbf{a}(\theta_i) \odot \mathbf{a}^*(\theta_n) \tag{31}$$

where $\mathbf{G}_1 = \mathbf{G}\mathbf{G}$.

Denote

$$\mathbf{b}(\theta_i, \theta_n) = \mathbf{a}(\theta_i) \odot \mathbf{a}^*(\theta_n) \tag{32}$$

From (31), we have

$$\begin{aligned}&span\left\{\mathbf{v}_k \odot \mathbf{v}_l^* \text{ where } k, l = 1, \cdots, K\right\} \\ &\quad = span\left\{\mathbf{G}_1\mathbf{b}(\theta_i, \theta_n) \text{ where } i, n = 1, \cdots, K\right\}.\end{aligned} \tag{33}$$

Define $\mathbf{V}'_s$ as a matrix whose column are composed of $\mathbf{v}_k \odot \mathbf{v}_l^*$, $i, j = 1, \cdots, K$, and

$$\mathbf{P}_{\mathbf{V}'_s}^{\perp} = \mathbf{I} - \mathbf{V}'_s\left((\mathbf{V}'_s)^{\mathrm{H}}\mathbf{V}'_s\right)^{-1}(\mathbf{V}'_s)^{\mathrm{H}}. \tag{34}$$

Denote $\mathbf{A}'$ as a matrix whose columns are composed of $\mathbf{G}_1\mathbf{b}(\theta_i, \theta_j)$, $i, j = 1, \cdots, K$, and

$$\mathbf{P}_{\mathbf{A}'}^{\perp} = \mathbf{I} - \mathbf{A}'\left((\mathbf{A}')^{\mathrm{H}}\mathbf{A}'\right)^{-1}(\mathbf{A}')^{\mathrm{H}}. \tag{35}$$

According to the uniqueness of the projection matrix onto a subspace and from (32), it follows that

$$\mathbf{P}_{\mathbf{V}'_s}^{\perp} = \mathbf{P}_{\mathbf{A}'}^{\perp}. \tag{36}$$

Since $\mathbf{P}_{\mathbf{A}'}^{\perp}$ is independent of phase errors, it follows from (36) that $\mathbf{P}_{\mathbf{V}'_s}^{\perp}$ is independent of phase errors.

Based on (32), we have

$$\mathbf{P}_{\mathbf{V}'_s}^{\perp}\mathbf{G}_1\mathbf{b}(\theta_i, \theta_n) = \mathbf{0}. \tag{37}$$

Due to $(\mathbf{P}_{\mathbf{V}'_s}^{\perp})^{\mathrm{H}} = \mathbf{P}_{\mathbf{V}'_s}^{\perp}$ and $\mathbf{P}_{\mathbf{V}'_s}^{\perp}\mathbf{P}_{\mathbf{V}'_s}^{\perp} = \mathbf{P}_{\mathbf{V}'_s}^{\perp}$, (37) leads to

$$\left(\mathbf{G}_1\mathbf{b}(\theta_i, \theta_n)\right)^{\mathrm{H}}\mathbf{P}_{\mathbf{V}'_s}^{\perp}\left(\mathbf{G}_1\mathbf{b}(\theta_i, \theta_n)\right) = 0. \tag{38}$$

The gain errors and DOAs are then estimated by minimizing the cost function below.

$$J = \sum_{i=1}^{K}\sum_{n=1}^{K}\left(\mathbf{G}_1\mathbf{b}(\theta_i, \theta_n)\right)^{\mathrm{H}}\mathbf{P}_{\mathbf{V}'_s}^{\perp}\left(\mathbf{G}_1\mathbf{b}(\theta_i, \theta_n)\right). \tag{39}$$

Note that

$$\mathbf{G}_1\mathbf{b}(\theta_i, \theta_n) = \widetilde{\mathbf{B}}(\theta_i, \theta_n)\boldsymbol{\kappa} \tag{40}$$

where $\widetilde{\mathbf{B}}(\theta_i, \theta_n) = \mathrm{diag}\{\mathbf{b}(\theta_i, \theta_n)\}$ (diag$\{\bullet\}$ defines a diagonal matrix) and $\boldsymbol{\kappa} = [g_1^2, \cdots, g_M^2]^T$.

An iterative procedure for estimating the DOAs and gain errors is summarized as follows.

Step (1). Initialization: set $l = 0$; and set the gain error of each sensor to be 1, that is, $\widehat{\boldsymbol{\kappa}}^0 = [1, 1, \cdots, 1]^{\mathrm{T}}$.

Step (2). DOA estimation: the DOAs of signals can be estimated as

$$\begin{aligned}&\left(\widehat{\theta}_i^{(l)}, \widehat{\theta}_n^{(l)}\right) = \arg\max_{\theta' > \theta + \Delta\theta} P^{(l)}(\theta, \theta'), \\ &\quad \text{for } i, n = 1, \cdots, K;\ i \neq n\end{aligned} \tag{41}$$

where $P^{(l)}(\theta, \theta')$ defines a two-dimensional spatial spectrum

$$P^{(l)}(\theta, \theta') = \left(\left(\widehat{\mathbf{G}}_1^{(l)}\mathbf{b}(\theta, \theta')\right)^{\mathrm{H}}\widehat{\mathbf{P}}_{\mathbf{V}'_s}^{\perp}\left(\widehat{\mathbf{G}}_1^{(l)}\mathbf{b}(\theta, \theta')\right)\right)^{-1}. \tag{42}$$

In (41), $\theta' > \theta + \Delta\theta$ is involved because both $\mathbf{b}(\theta_i, \theta_n)$ and $\mathbf{b}(\theta_n, \theta_i)$ satisfy (37), and $\Delta\theta$ is a small amount, implying that we do not search the region where θ and θ' are close to each other. This is because $\mathbf{b}(\theta, \theta)$ always satisfies (37) and the peaks of $P(\theta, \theta')$ in (41) always occur on the points located on the line $\theta = \theta'$ whether there are signals or not.

Step (3). Gain-error estimation: Define $\mathbf{w} = [1, 0, 0, \cdots, 0]^{\mathrm{T}}$. Like the derivation in [12], we obtained that $\boldsymbol{\kappa}$ can be estimated by using the DOA estimates from Step 2, that is

$$\widehat{\boldsymbol{\kappa}}^{(l+1)} = \mathbf{Q}_{(l)}^{-1}\mathbf{w}/\left(\mathbf{w}^{\mathrm{H}}\mathbf{Q}_{(l)}^{-1}\mathbf{w}\right) \tag{43}$$

$$\mathbf{Q}_{(l)} = \sum_{i=1}^{K}\sum_{n=1}^{K}\widetilde{\mathbf{B}}^{\mathrm{H}}\left(\widehat{\theta}_i^{(l)}, \widehat{\theta}_n^{(l)}\right)\widehat{\mathbf{P}}_{\mathrm{V}_s'}^{\perp}\widetilde{\mathbf{B}}\left(\widehat{\theta}_i^{(l)}, \widehat{\theta}_n^{(l)}\right). \tag{44}$$

Then, we have

$$\widehat{g}_m^{(l+1)} = sqrt\left(\widehat{\kappa}^{(l+1)}(m)\right) \tag{45}$$

$$\widehat{\mathbf{G}}^{(l+1)} = diag\left\{\widehat{g}_1^{(l+1)}, \widehat{g}_2^{(l+1)}, \cdots, \widehat{g}_M^{(l+1)}\right\} \tag{46}$$

$$\widehat{\mathbf{G}}_1^{(l+1)} = \widehat{\mathbf{G}}^{(l+1)}\widehat{\mathbf{G}}^{(l+1)} \tag{47}$$

where $\widehat{\kappa}^{(l+1)}(m)$ is the m-th element of the vector $\widehat{\boldsymbol{\kappa}}^{(l+1)}$.

Step (4). Termination condition:

Compute

$$\widehat{J}^{(l)} = \sum_{i=1}^{K}\sum_{n=1}^{K}\left(\widehat{\mathbf{G}}_1^{(l+1)}\mathbf{b}\left(\widehat{\theta}_i^{(l)}, \widehat{\theta}_n^{(l)}\right)\right)^{\mathrm{H}}\widehat{\mathbf{P}}_{\mathrm{V}_s'}^{\perp}\left(\widehat{\mathbf{G}}_1^{(l+1)}\mathbf{b}\left(\widehat{\theta}_i^{(l)}, \widehat{\theta}_n^{(l)}\right)\right). \tag{48}$$

If $\widehat{J}^{(l-1)} - \widehat{J}^{(l)} > \varepsilon$ (a present threshold), then $l = l+1$ and go to Step 2. If $\widehat{J}^{(l-1)} - \widehat{J}^{(l)} \le \varepsilon$, done.

Remark 1. Since $\widehat{\mathbf{P}}_{\mathrm{V}_s'}^{\perp}$ is independent of phase errors, the new DOA estimation by (41) is independent of phase errors.

Remark 2. From (32), we can see that the new DOA estimation requires that the number of signals is larger than one.

4.2. Phase error estimation

The phase error estimation is the same as that used in [16], for the integrality of this paper, they are briefly included. Readers are referred to [16] for more details.

Let $\boldsymbol{\Phi} = [\varphi_1, \varphi_2, \cdots, \varphi_M]^{\mathrm{T}}$. Based on the DOA estimates by the new DOA estimation method, phase errors can be estimated by the WF method in [12]. Denote $\widetilde{\mathbf{D}}(\theta_k) = \mathrm{diag}\{\mathbf{a}(\theta_k)\}$ (diag{•} defines a diagonal matrix), and $\angle(\bullet)$ as taking the phase of a complex number. Phase errors can be estimated by

$$\widehat{\boldsymbol{\Phi}} = \angle(\boldsymbol{\beta}) \tag{49}$$

where

$$\boldsymbol{\beta} = \boldsymbol{\psi}^{-1}\mathbf{w}/\left(\mathbf{w}^{\mathrm{H}}\boldsymbol{\psi}^{-1}\mathbf{w}\right) \tag{50}$$

$$\boldsymbol{\psi} = \sum_{k=1}^{K}\widetilde{\mathbf{D}}^{\mathrm{H}}(\theta_k)\widehat{\mathbf{V}}_{\mathrm{n}}\widehat{\mathbf{V}}_{\mathrm{n}}^{\mathrm{H}}\widetilde{\mathbf{D}}(\theta_k) \tag{51}$$

$$\widehat{\mathbf{V}}_{\mathrm{n}} = [\widehat{\mathbf{v}}_{K+1}, \cdots, \widehat{\mathbf{v}}_M]. \tag{52}$$

In [16], it was proved that the phase error estimation by (49) performs independently of phase errors. Therefore, the proposed method for estimating DOA and sensor gain-phase errors performs independently of phase errors.

Like the combined strategy in [16], a combined strategy is proposed by combining the proposed method in this paper and WF method, which is briefly described as follows.

Step (1). Estimate the DOAs and gain-phase errors by the proposed method.

Step (2). The gain-phase error estimates from step 1 are taken as the initial values of the gain-phase errors in the joint iteration process of the WF method. Consequently, the DOA and gain-phase error estimates by the WF method are taken as the final estimates.

Since the gain-phase error estimates from Step 1 are approximated to their true values in the case of moderate gain errors, the optimal convergence of the WF method is attained. Thus, the proposed combined strategy performs independently of phase errors.

5. Discussions

5.1. Array configuration free of DOA estimation ambiguity

For two pairs of directions $(\theta_1, \theta_2) \ne (\theta_3, \theta_4)$ and $(\theta_1, \theta_2) \ne (\theta_4, \theta_3)$, the DOA estimates by the proposed method are ambiguous if $\mathbf{a}(\theta_1) \odot \mathbf{a}^*(\theta_2) = \mathbf{a}(\theta_3) \odot \mathbf{a}^*(\theta_4)$. Note that the ambiguity of DOA estimation mentioned in this paper is equivalent to the pair-wise linearly dependence of the steering vectors corresponding to two different directions defined in [20], [21]. We provide a sufficient condition to guarantee the unambiguous DOA estimates by the proposed DOA estimation method in Theorem 1 below.

Theorem 1. *If an array contains at least four sensors with the coordinates (x_i, y_i), where $(x_1, y_1) = (0, 0)$, $y_2 = 0$, $x_2 \le \lambda/4$, and $0 < |y_4 - y_3| \le \lambda/4$, and the DOAs of signals $\theta_k \in [-90°, 90°]$, then there is no ambiguity of the DOA estimates by the proposed method.*

The proof of Theorem 1 is given in Appendix A.

Remark 3. Theorem 1 implies that the proposed method is not suitable for a linear array.

5.2. Differences from the method in [16]

The DOA estimation in [16] is based on the Hadamard product of the array output and its conjugate, and the gain-error estimation utilizes the diagonal elements of the covariance matrix which degrades in the presence of correlated signals. In contrast, the DOA and gain-error estimations in this paper are based on the Hadamard product of the signal eigenvectors and their conjugates, which remains a good performance in the case of correlated signals.

In addition, the method in [16] has different spatial spectrum for the real and complex signals. In contrast, the proposed method gives the same spectrum no matter the signals are real or complex and thus it is more concise. Furthermore, the array configuration for the method in [16] requires that four sensors are arranged in a fixed manner. However, the array configuration for the proposed method requires that two pairs of sensors are, respectively, arranged in two different fixed manners. Thus, the array configuration for the proposed method is relaxed than that for the method in [16]. The cost of the proposed method is its increased computation complexity due to the iteration between the DOA estimation and the gain error estimation.

6. Simulation results

In the following, the range of the DOAs of signals is set to be $[-90°, 90°]$. The gain errors $\{g_m\}_{m=1}^{M}$ and phase errors $\{\varphi_m\}_{m=1}^{M}$ are generated by $g_m = 1 + \sqrt{12}\sigma_g\beta_m$, and $\varphi_m = \sqrt{12}\sigma_\varphi\eta_m$, respectively, where β_m and η_m are independent and identically distributed random variables distributed uniformly over $[-0.5, 0.5]$; σ_g and σ_φ are the standard deviations of g_m and φ_m, respectively. Note that by simulation, we observe that the proposed method might not converge when σ_g is larger than 0.3. In most of case, σ_g is less than 0.3. In the simulations below, we assume $\sigma_g = 0.2$, and the powers of signals with different directions are assumed to be equal. The sensitivity of the WF method to the phase errors was shown in [16] and herein we omit the results of WF method.

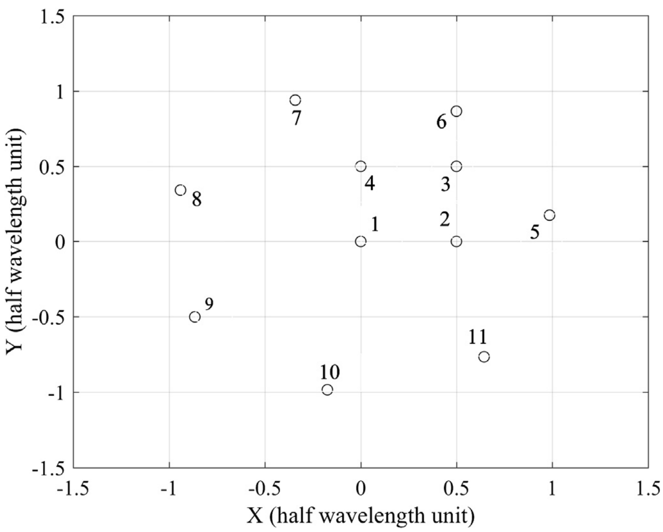

Fig. 1. Array configuration.

6.1. Effect of correlated signals

The sensor configuration is shown in Fig. 1, where Sensors 1–4 are located on the four vertices of a square with its side equal to $\lambda/4$ and Sensors 5–11 lie on a uniform circular array with radius $\lambda/2$. It is noted that the arrangement of Sensors 1–4 satisfies the sufficient condition for unambiguous DOA estimates by the method in [16], while Sensors 1, 2, 4, and 6 satisfies the sufficient condition for unambiguous DOA estimates by the proposed method in this paper. Suppose that there are two signals with directions $\theta_1 = -30°$ and $\theta_2 = 20°$, respectively. The SNR and the number of samples are 30 dB and 200, respectively. $\sigma_\varphi = 50°$.

We first investigate the case of uncorrelated signals. Fig. 2 shows the spatial spectrum of $P(\theta, \theta')$ for the proposed method and the method in [16]. From Fig. 2, we can see that both of the method in [16] and the proposed method produce a peak near the DOA pair $(-30°, 20°)$. In addition, the proposed method has narrower mainlobe and lower sidelobe.

Next, we investigate the case when the two signals are correlated with a correlation coefficient equal to 0.5, and the spatial spectrum is shown in Fig. 3. From Fig. 3, we observe that the method in [16] cannot correctly estimate the DOAs. This is because that the gain error estimation of the method in [16] requires the assumption of the independence of the two signals. On contrast, the proposed method does not require the above-mentioned assumption and thus it remains a good performance.

Fig. 4 shows the average root mean square error (ARMSE) of gain error, phase error, and DOA estimates versus the correlation coefficient of the two signals, based on 500 Monte Carlo trials, when SNR is equal to 10 dB. From Fig. 4, we observe that the proposed method is robust to the correlated signals. However, the method in [16] deteriorates when the correlation coefficient is larger than 0.1. Correspondingly, the combined strategy is robust to the correlated signal and the one in [16] gets worse when the correlation coefficient is larger than 0.6. Moreover, the combined method approaches to the CRB.

6.2. Effect of phase errors

We set the correction coefficient of the two signals and SNR are equal to 0 and 10 dB, respectively. Other simulation parameters are the same as those in Section 6.1. The ARMSE curves of gain error, phase error, and DOA estimates against the standard deviation of the phase error σ_φ are shown in Fig. 5(a), Fig. 5(b),

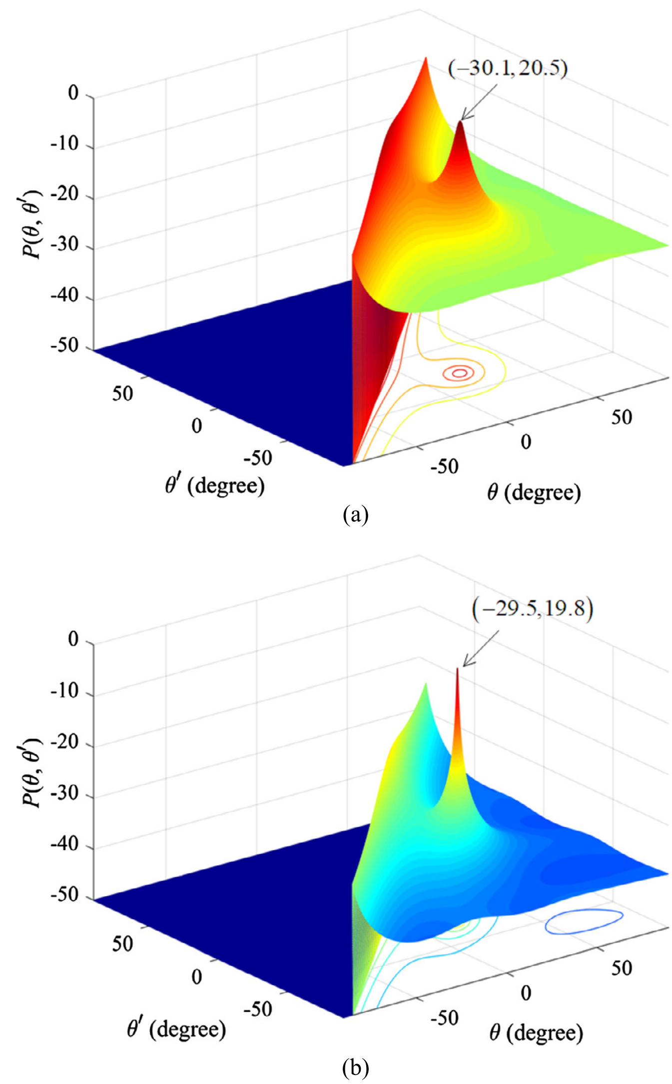

Fig. 2. The two-dimensional spatial spectrum given by (a) the method in [16]; (b) the proposed method, when the two signals are uncorrelated.

and Fig. 5(c), respectively. From Fig. 5, we observe that the proposed method and combined strategy, and the ones in [16] behave independently of phase errors. On the other hand, the proposed method has higher accuracy compared to the method in [16]. Because the gain and phase error estimation have a similar trend as the DOA estimation, we omit their results in the following.

6.3. Effect of SNR

We set the correction coefficient of the two signals is equal to 0 and $\sigma_\varphi = 50°$. Other simulation parameters are the same as those in Section 6.1. The ARMSE of DOA estimates against the SNR is shown in Fig. 6.

From Fig. 6, it is illustrated that all the aforementioned methods behave better with increasing SNR. The proposed method has higher estimation accuracy than the method in [16].

6.4. Effect of sample number

Consider that the correction coefficient of the two signals is equal to 0, $\sigma_\varphi = 50°$, and the SNR is equal to 20 dB. Other simulation parameters are the same as those in Section 6.1. The ARMSE of the DOA estimates versus the number of samples is shown in

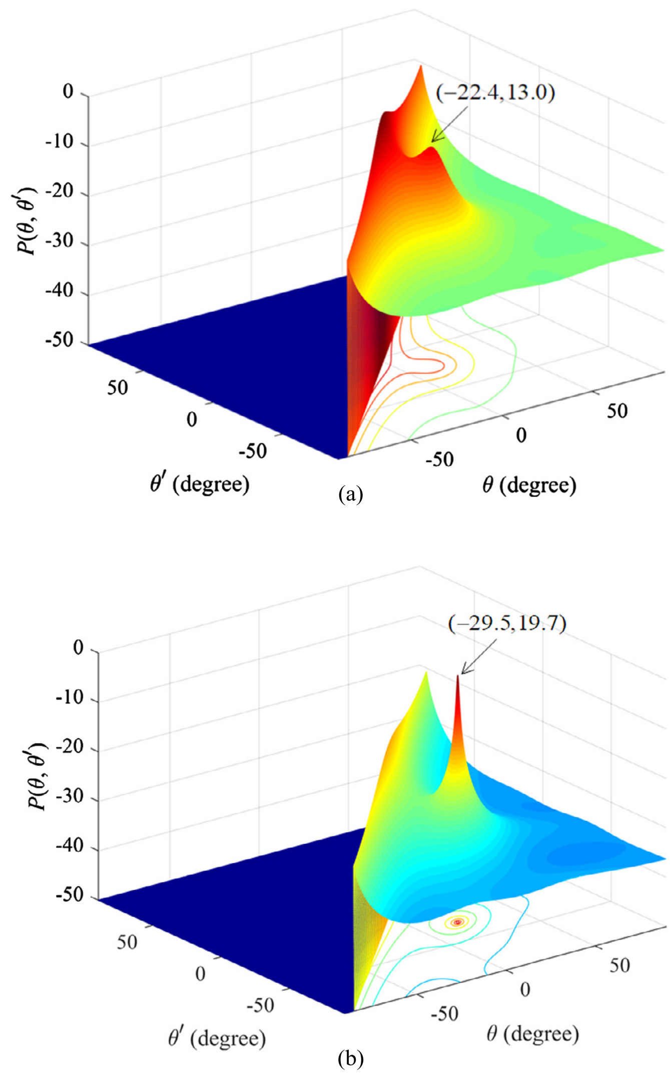

Fig. 3. The two-dimensional spatial spectrum given by (a) the method in [16]; (b) the proposed method, when the correlation coefficient of the two signals is equal to 0.5.

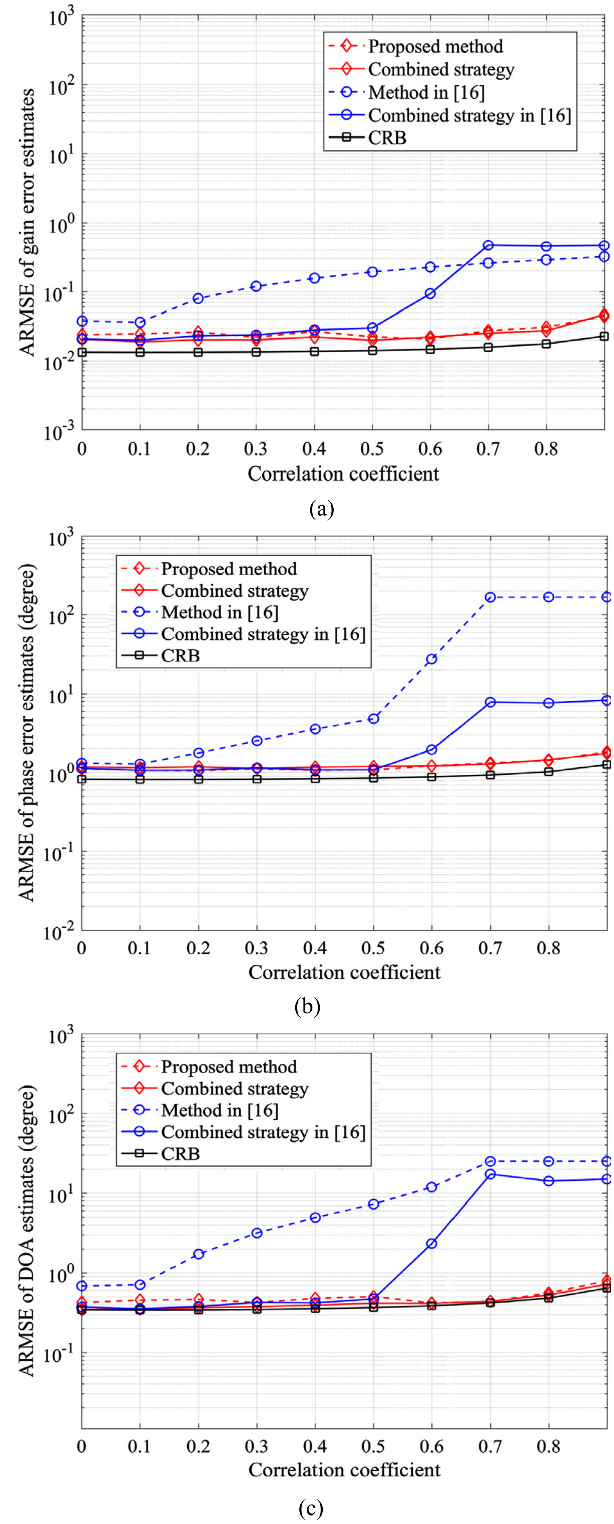

Fig. 4. ARMSE versus correlation coefficient; (a) gain error estimates; (b) phase error estimates; (c) DOA estimates.

Fig. 7. From Fig. 7, it is shown that all the aforementioned methods behave better as the sample number increases, which is due to the fact that the sampled covariance matrix is closer to its true value as the number of samples increases.

Overall, we observe that the proposed method has higher estimation accuracy than the method in [16]. On the other hand, both the combined strategy and the one in [16] have the DOA estimation accuracy approximately to the CRB, while the performance of the proposed method and the method in [16] deviates from the CRB.

7. Conclusion

In this paper, the DOA estimation problem in the presence of senor gain-phase errors is addressed. We first derive that the Cramer–Rao Bound (CRB) on DOA and gain-phase error estimation accuracy is independent of phase errors. Following that, we propose a method for estimating DOA and gain-phase errors by using the Hadamard product of the eigenvectors and their conjugates. In addition, we derive a sufficient condition for the unambiguous DOA estimation of the proposed method.

The proposed method performs independently of phase errors as the CRB does. Moreover, it is robust to the correlated signals and does not require the information of the signal power, which makes it more feasible to the practical usage. The main drawback is its increased computational complexity. Furthermore, a combined strategy is developed by combining the proposed and WF methods. The combined strategy inherits the advantages of the proposed and WF methods, such as phase-error independence, robustness to the correlated signals, and high estimation accuracy. Simulation results demonstrated the above-mentioned performance of the proposed

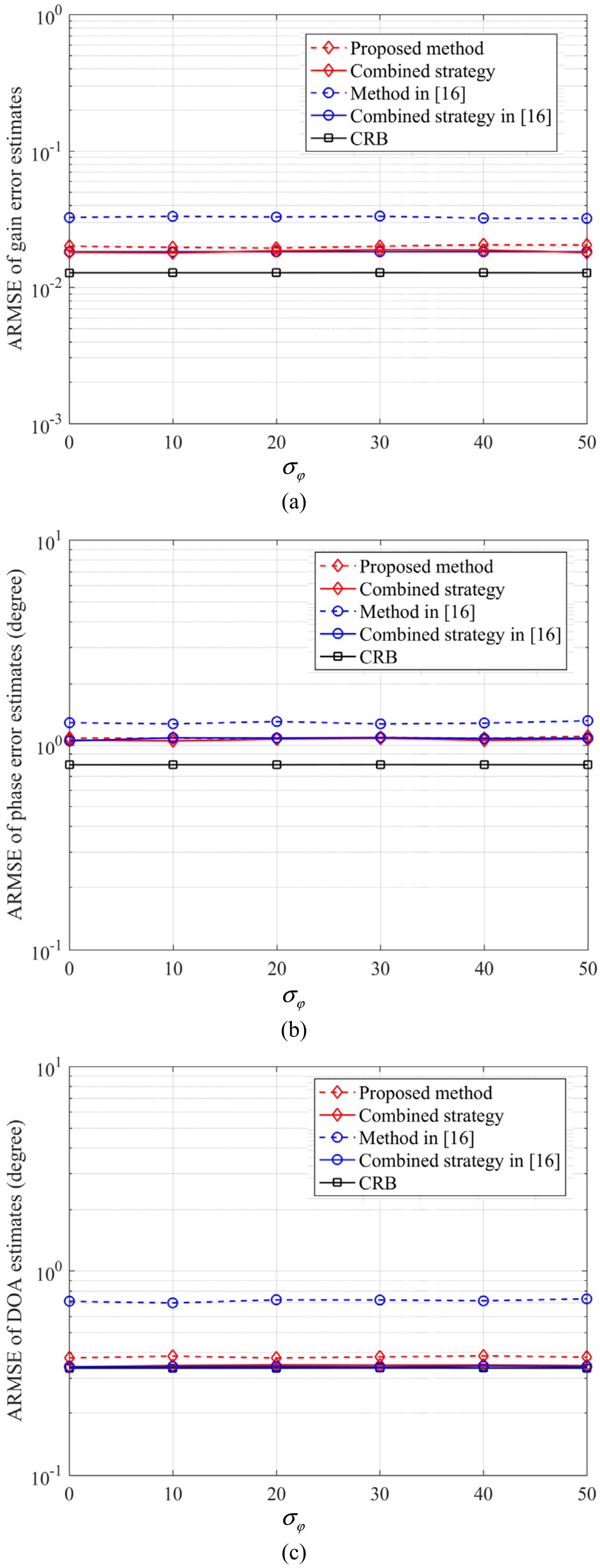

Fig. 5. ARMSE versus σ_φ; (a) gain error estimates; (b) phase error estimates; (c) DOA estimates.

method and the combined strategy. In addition, it is illustrated that the estimation accuracy of the proposed method is higher than that of the method in [16], however, it deviates from the CRB. On the other hand, the combined strategy has an estimation accuracy approximately to the CRB.

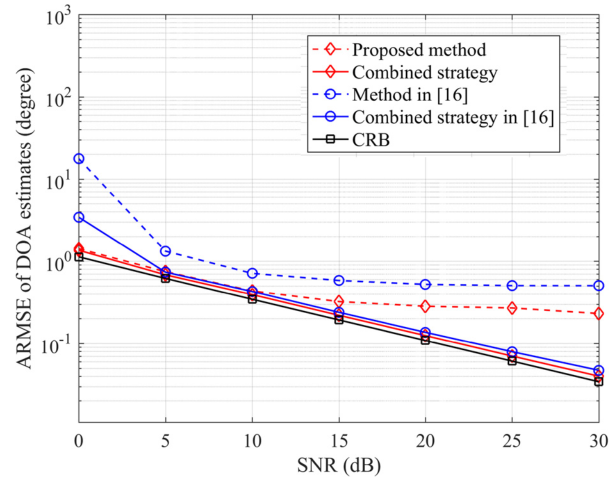

Fig. 6. ARMSE of DOA estimates against SNR.

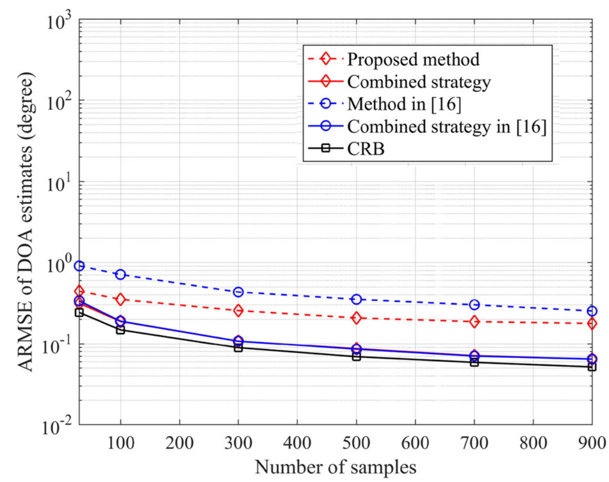

Fig. 7. ARMSE of DOA estimates against number of samples.

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Appendix A. Proof of Theorem 1

Consider an array composed of four sensors with coordinates (x_i, y_i) for $i = 1, \cdots, 4$, where $(x_1, y_1) = (0, 0)$, $y_2 = 0$, $r_{2,1} = x_2 \leq \lambda/4$, and $0 < |y_4 - y_3| \leq \lambda/4$. Additionally, the DOAs of signals are confined in $[-90°, 90°]$. Assume that there are four directions such that $(\theta_1, \theta_2) \neq (\theta_3, \theta_4)$ and $(\theta_1, \theta_2) \neq (\theta_4, \theta_3)$. The assumption that (θ_3, θ_4) is ambiguous means

$$\mathbf{a}(\theta_1) \odot \mathbf{a}^*(\theta_2) = \mathbf{a}(\theta_3) \odot \mathbf{a}^*(\theta_4). \tag{53}$$

For Sensors 2, 3, and 4, (53) leads to

$$\begin{aligned} &\exp(-j2\pi d_{2,1}/\lambda) \odot \exp(j2\pi d_{2,2}/\lambda) \\ &\quad = \exp(-j2\pi d_{2,3}/\lambda) \odot \exp(j2\pi d_{2,4}/\lambda) \end{aligned} \tag{54}$$

$$\begin{aligned} &\exp\big(-j2\pi (d_{4,1} - d_{3,1})/\lambda\big) \odot \exp\big(j2\pi (d_{4,2} - d_{3,2})/\lambda\big) \\ &\quad = \exp\big(-j2\pi (d_{4,3} - d_{3,3})/\lambda\big) \odot \exp\big(j2\pi (d_{4,4} - d_{3,4})/\lambda\big). \end{aligned} \tag{55}$$

Based on (3), (54) and (55), we have

$$\begin{aligned} &\exp\big(-j2\pi x_2(\cos\theta_1 - \cos\theta_2)/\lambda\big) \\ &\quad = \exp\big(-j2\pi x_2(\cos\theta_3 - \cos\theta_4)/\lambda\big) \end{aligned} \tag{56}$$

$$\exp\left(\frac{-j2\pi}{\lambda}(x_4-x_3)(\cos\theta_1-\cos\theta_2) + (y_4-y_3)(\sin\theta_1-\sin\theta_2)\right) = \exp\left(\frac{-j2\pi}{\lambda}(x_4-x_3)(\cos\theta_3-\cos\theta_4) + (y_4-y_3)(\sin\theta_3-\sin\theta_4)\right) \tag{57}$$

As a result, we obtain

$$\cos\theta_1-\cos\theta_2=\cos\theta_3-\cos\theta_4+\lambda n_1/x_2 \tag{58}$$

$$(x_4-x_3)(\cos\theta_1-\cos\theta_2)+(y_4-y_3)(\sin\theta_1-\sin\theta_2) = (x_4-x_3)(\cos\theta_3-\cos\theta_4)+(y_4-y_3)(\sin\theta_3-\sin\theta_4) + n_2\lambda \tag{59}$$

where n_1 and n_2 are integers.

Since $x_2 \le \lambda/4$, $(\theta_1,\theta_2) \ne (\theta_3,\theta_4)$ and $(\theta_1,\theta_2) \ne (\theta_4,\theta_3)$, n_1 is zero. Thus, (58) can be rewritten as

$$\cos\theta_1-\cos\theta_2=\cos\theta_3-\cos\theta_4. \tag{60}$$

From (59) and (60), it follows that

$$(y_4-y_3)(\sin\theta_1-\sin\theta_2) = (y_4-y_3)(\sin\theta_3-\sin\theta_4)+n_2\lambda \tag{61}$$

Since $0<|y_4-y_3|\le\lambda/4$, $(\theta_1,\theta_2) \ne (\theta_3,\theta_4)$ and $(\theta_1,\theta_2) \ne (\theta_4,\theta_3)$, from (61) we have $n_2=0$, and

$$\sin\theta_1-\sin\theta_2=\sin\theta_3-\sin\theta_4. \tag{62}$$

Simplifying (60) and (62), we obtain

$$\sin\frac{\theta_1+\theta_2}{2}\sin\frac{\theta_1-\theta_2}{2}=\sin\frac{\theta_3+\theta_4}{2}\sin\frac{\theta_3-\theta_4}{2} \tag{63}$$

$$\cos\frac{\theta_1+\theta_2}{2}\sin\frac{\theta_1-\theta_2}{2}=\cos\frac{\theta_3+\theta_4}{2}\sin\frac{\theta_3-\theta_4}{2}. \tag{64}$$

Dividing (63) by (64) and summarization of square of (63) and square of (64) yield (65) and (66), respectively

$$\frac{\sin\frac{\theta_1+\theta_2}{2}}{\cos\frac{\theta_1+\theta_2}{2}}=\frac{\sin\frac{\theta_3+\theta_4}{2}}{\cos\frac{\theta_3+\theta_4}{2}} \tag{65}$$

$$\left(\sin\frac{\theta_1-\theta_2}{2}\right)^2=\left(\sin\frac{\theta_3-\theta_4}{2}\right)^2. \tag{66}$$

It follows from (65) and (66) that

$$\frac{\theta_1+\theta_2}{2}=\frac{\theta_3+\theta_4}{2}+q_1\pi \tag{67}$$

$$\frac{\theta_1-\theta_2}{2}=\frac{\theta_3-\theta_4}{2}+q_2\pi \tag{68}$$

where q_1 and q_2 are integers.

Simplifying (67) and (68) yields

$$\theta_1=\theta_3+(q_1+q_2)\pi \tag{69}$$

$$\theta_2=\theta_4+(q_1-q_2)\pi. \tag{70}$$

Since $|\theta_k|\le 90°$, $(\theta_1,\theta_2) \ne (\theta_3,\theta_4)$ and $(\theta_1,\theta_2) \ne (\theta_4,\theta_3)$, we have that (69) and (70) cannot be simultaneously true. Thus, (53) does not hold. As a consequence, there is no ambiguity of the DOA estimates.

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