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Short communication

Sequential DOA estimation method for multi-group coherent signals ☆

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ABSTRACT

In order to cope with multi-group coherent signals induced by multipath propagation, this paper presents a novel approach to estimate the direction of arrivals (DOAs) of each group sequentially. For each time, we just estimate the DOAs of one group which has the minimum coherent signals based on forward spatial smoothing (FSS), and the information of this group is eliminated from the array covariance matrix with oblique projection technique. Then we estimate the next group similarly until all groups are estimated. Afterwards, we use the forward/backward spatial smoothing (FBSS) to further improve the estimation accuracy of the DOAs in each group separately. The new method not only utilizes less array sensors to resolve multi-group coherent signals, but also estimates the same DOAs from different groups. Simulation results are presented to demonstrate the effectiveness of the proposed method especially in low signal-to-noise ratio (SNR) and small number of snapshots.

1. Introduction

Due to multipath propagation, there are several groups of coherent signals impinging on the array in real environment, which is a common scenario in wireless communication and smart jammers. For narrowband sources, a preprocessing technique should be used to decorrelate the coherent signals, such that the conventional subspace-based methods like MUSIC [1] can be utilized to estimate the direction of arrivals (DOAs) of all paths. The most representative one is the technique of spatial smoothing [2,3], which divides an array into multiple overlapping subarrays and uses the sum of covariance matrices of these subarrays to realize decorrelation. But these methods sacrifice too much of array aperture. They are not applicable to the situation where some signals from different groups have the same DOA, since all DOAs are estimated simultaneously. The maximum likelihood [4] is proposed for DOA estimation of coherent signals without losing array aperture, but it brings a serious computation load in practical application. The MODE (root-WSF) algorithm is developed in [5], which can estimate the DOAs without a time-consuming parameter searching. The matrix pencil [6,7] is another important method, which can estimate the DOAs of coherent signals easily without additional processing of spatial smoothing. However, the matrix pencil method obtains good estimation performance only in high signal-to-noise ratio (SNR). The approach based on the fourth-order cumulants in [8,9] estimates each group of coherent signals separately. Although the number of signals it resolved can exceed the number of the sensors, it needs a large number of snapshots and a high computational burden because of the high-order cumulants. In recent years, many methods, such as [10–13], are developed for a scenario where the uncorrelated and coherent signals coexist. These methods estimate the DOAs of uncorrelated and coherent signals in two separate stages. However, the multi-group coherent signals are also decorrelated as a whole in [10–12]. In [13], an effective method utilizes the steering vector singular value decomposition (SVSVD) to estimate the DOAs of the coherent signals in each group. Unfortunately, this method achieves bad performance to estimate multi-group coherent signals and has a "saturation behavior" regardless of the SNR. Recently, the techniques of sparse representation and sparse recovery, such as [14–17], have provided a new perspective of DOA estimation by using the spatial sparsity in the array signal model. These methods are prevalent, since they can estimate coherent signals without any decorrelation operation for an arbitrary array. Especially, a SRWSF method in [17] based on optimal weighting matrix improves the performance in low SNR. However, these methods require a large number of array sensors to achieve good performance for multi-group coherent signals.

In this paper, an novel method is proposed to estimate each group of coherent signals sequentially under multi-group environment. It can work in the case where some signals have the

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same DOA as long as they are from different groups. This method is based on forward spatial smoothing (FSS) in conjunction with subspace-based method, and it does not need high-order statistics. The DOAs of one group which has the minimum coherent signals are estimated first, whose contributions are eliminated from the array covariance matrix by exploiting the property of oblique projection. Subsequently, the DOAs of one group which has the second-minimum coherent signals are estimated, and so on. Moreover, after all DOAs are estimated, we use the forward/backward spatial smoothing (FBSS) to improve the performance of DOA estimation. The new method not only resolves multi-group coherent signals with less array sensors, but also has a better DOA estimation accuracy in low SNR and small number of snapshots.

2. Signal model

Consider a signal scenario that several narrowband far-field sources undergo multipath propagation producing several groups of delayed and scaled replicas. Let K groups of coherent signals impinge on a uniform linear array composed of M sensors, where the interspacing between adjacent sensors is equal to half the wavelength. The multipath signals are caused by the kth source $s_k(t)$ with power σ_k^2, and the collection of these signals is referred to as the kth group, $k = 1, \ldots, K$. Assume that there are $L_k \geq 2$ paths associated with the kth source $s_k(t)$ and the signal corresponding to the lth multipath is from direction θ_{kl} for $l = 1, \ldots, L_k$. Assume that the signals in different groups are uncorrelated with each other. The $M \times 1$ array output can be expressed as

$$\mathbf{x}(t) = \sum_{k=1}^{K}\sum_{l=1}^{L_k}\mathbf{a}(\theta_{kl})\rho_{kl}s_k(t) + \mathbf{n}(t) = \sum_{k=1}^{K}\mathbf{B}_k\boldsymbol{\rho}_k s_k(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{a}(\theta_{kl}) = \left[1, \ldots, e^{-j(M-1)\pi \sin\theta_{kl}}\right]^{\mathrm{T}}$ is the steering vector, ρ_{kl} is the complex fading coefficient of the lth multipath in the kth group and $|\rho_{kl}| \leq 1$, $\boldsymbol{\rho}_k = \left[\rho_{k1}, \ldots, \rho_{kL_k}\right]^{\mathrm{T}}$, $\rho_{k1} = 1$, $\mathbf{B}_k = \left[\mathbf{a}(\theta_{k1}), \ldots, \mathbf{a}(\theta_{kL_k})\right]$, $\mathbf{a}_k = \mathbf{B}_k\boldsymbol{\rho}_k$, $\mathbf{A} = \left[\mathbf{a}_1, \ldots, \mathbf{a}_K\right]$, $\mathbf{s}(t) = \left[s_1, \ldots, s_K\right]^{\mathrm{T}}$, and $\mathbf{n}(t)$ is the additive noise vector. Assume that the entries of $\mathbf{s}(t)$ and $\mathbf{n}(t)$ are zero mean wide-sense stationary random processes. It is assumed that the entries of $\mathbf{n}(t)$ have identical power σ_n^2 and are uncorrelated with each other and the signals. The operators $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$, $(\cdot)^{\#}$ and $E\{\cdot\}$ denote conjugate, transpose, conjugate transpose, inverse, Moore-Penrose inverse and expectation, respectively. The symbol diag $\{z_1, z_2\}$ stands for a diagonal matrix with elements z_1, z_2 and blkdiag $\{\mathbf{Z}_1, \mathbf{Z}_2\}$ stands for a block diagonal matrix with entries $\mathbf{Z}_1, \mathbf{Z}_2$. The array covariance matrix is

$$\mathbf{R}_x = E\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma_n^2\mathbf{I}_M \tag{2}$$

where $\mathbf{R}_s = \mathrm{diag}\left\{\sigma_1^2, \ldots, \sigma_K^2\right\}$ represents the covariance matrix of $\mathbf{s}(t)$ and $\mathbf{I}_M$ indicates an $M \times M$ identity matrix. K groups of coherent signals are equivalent to K virtual sources. When $K < M$, by performing eigen-decomposition of the $\mathbf{R}_x$, we have

$$\mathbf{R}_x = \mathbf{U}_s\boldsymbol{\Lambda}_s\mathbf{U}_s^H + \mathbf{U}_n\boldsymbol{\Lambda}_n\mathbf{U}_n^H \tag{3}$$

where $\mathbf{U}_s$ consists of the eigenvectors related to the K largest eigenvalues and the columns of $\mathbf{U}_s$ span the signal subspace. $\mathbf{U}_n$ consists of the eigenvectors corresponding to the $(M-K)$ smallest eigenvalues spanning the noise subspace. Here $\boldsymbol{\Lambda}_s = \mathrm{diag}\{\lambda_1, \ldots, \lambda_K\}$, $\boldsymbol{\Lambda}_n = \mathrm{diag}\{\lambda_{K+1}, \ldots, \lambda_M\}$, and λ_i, $i = 1, \ldots, M$ represent the eigenvalues of the $\mathbf{R}_x$ in descending order. From (2), we can obtain

$$\mathbf{R}_0 = \mathbf{A}\mathbf{R}_s\mathbf{A}^H = \mathbf{R}_x - \sigma_n^2\mathbf{I}_M = \sum_{k=1}^{K}\sigma_k^2\mathbf{B}_k\boldsymbol{\rho}_k\boldsymbol{\rho}_k^H\mathbf{B}_k^H = \mathbf{U}_s\left(\boldsymbol{\Lambda}_s - \sigma_n^2\mathbf{I}_K\right)\mathbf{U}_s^H \tag{4}$$

3. Proposed algorithms

3.1. DOA estimation

The DOA estimation method is proposed to deal with multi-group coherent signals sequentially, which is based on FSS. Firstly, we define the Nth FSS, which means dividing a uniform linear array with M sensors into overlapping subarrays of size $(M-N)$. Therefore, the number of subarrays is $(N+1)$. The covariance matrix of the Nth FSS can be formulated by

$$\mathbf{R}_N^f = \sum_{m=0}^{N}\mathbf{W}_{M-N,m}\mathbf{R}_0\mathbf{W}_{M-N,m}^H = \sum_{k=1}^{K}\sum_{m=0}^{N}\mathbf{W}_{M-N,m}\mathbf{B}_k\mathbf{Z}_{0k}\mathbf{B}_k^H\mathbf{W}_{M-N,m}^H \tag{5}$$

where $\mathbf{W}_{a,b} = \left[\mathbf{O}_{a,b}, \mathbf{I}_a, \mathbf{O}_{a,(M-a-b)}\right]$, $\mathbf{O}_{a,b}$ is an $a \times b$ zero matrix, and $\mathbf{Z}_{0k} = \sigma_k^2\boldsymbol{\rho}_k\boldsymbol{\rho}_k^H$. Define $\mathbf{Z}_{Nk}$, $k = 1, \ldots, K$ as

$$\mathbf{Z}_{Nk} = \sigma_k^2\sum_{m=0}^{N}\boldsymbol{\Phi}_k^m\boldsymbol{\rho}_k\boldsymbol{\rho}_k^H\boldsymbol{\Phi}_k^{-m} = \sigma_k^2\mathbf{Q}_{Nk}\mathbf{Q}_{Nk}^H \tag{6}$$

where $\boldsymbol{\Phi}_k = \mathrm{diag}\left\{e^{-j\pi\sin\theta_{k1}}, \ldots, e^{-j\pi\sin\theta_{kL_k}}\right\}$, $\mathbf{Q}_{Nk} = \boldsymbol{\Gamma}_k\left(\mathbf{W}_{N+1,0}\mathbf{B}_k\right)^{\mathrm{T}}$, here $\boldsymbol{\Gamma}_k = \mathrm{diag}\left\{\rho_{k1}, \ldots, \rho_{kL_k}\right\}$. we can rewrite (6) as

$$\mathbf{R}_N^f = \sum_{k=1}^{K}\mathbf{W}_{M-N,0}\mathbf{B}_k\mathbf{Z}_{Nk}\mathbf{B}_k^H\mathbf{W}_{M-N,0}^H = \sum_{k=1}^{K}\sigma_k^2\tilde{\mathbf{B}}_{Nk}\mathbf{Q}_{Nk}\mathbf{Q}_{Nk}^H\tilde{\mathbf{B}}_{Nk}^H = \tilde{\mathbf{A}}_N\tilde{\mathbf{Q}}_N\tilde{\mathbf{A}}_N^H \tag{7}$$

where $\tilde{\mathbf{B}}_{Nk} = \mathbf{W}_{M-N,0}\mathbf{B}_k$, $\tilde{\mathbf{A}}_N = \left[\tilde{\mathbf{B}}_{N1}, \ldots, \tilde{\mathbf{B}}_{NK}\right]$, and $\tilde{\mathbf{Q}}_N = \mathrm{blkdiag}\left\{\sigma_1^2\mathbf{Q}_{N1}\mathbf{Q}_{N1}^H, \ldots, \sigma_K^2\mathbf{Q}_{NK}\mathbf{Q}_{NK}^H\right\}$. We can see that $\boldsymbol{\Gamma}_k$ is an $L_k \times L_k$ diagonal matrix and $\left(\mathbf{W}_{N+1,0}\mathbf{B}_k\right)^{\mathrm{T}}$ is an $L_k \times (N+1)$ Vandermonde matrix. So, when $L_k \leq (N+1)$ $\mathbf{Q}_{Nk}$ will be full row rank and $\mathbf{Z}_{Nk}$ will be non-singular, which has been proved strictly in [2]. The groups whose coherent signals are less than or equal to $(N+1)$ are decorrelated. The rank of the kth block diagonal entry in $\tilde{\mathbf{Q}}_N$ is $\min\{L_k, N+1\}$. When $(M-N)$ is greater than the rank of $\tilde{\mathbf{Q}}_N$, the rank of $\mathbf{R}_N^f$ is the same as that of $\tilde{\mathbf{Q}}_N$. We can directly apply MUSIC algorithm on $\mathbf{R}_N^f$ to estimate the DOAs of these groups whose coherent signals are less than or equal to $(N+1)$. Let $N+1 = L_{\min}$, where $L_{\min} = \min\{L_k, k = 1, \ldots, K\}$, so the group which has the minimum number of coherent signals will be estimated firstly by the Nth FSS. After estimating the DOAs of this group, the related signal components are eliminated from the array covariance matrix by the technique of oblique projection. Then, the second-minimum group is estimated with corresponding subarrays in FSS. More details are given as follows.

For the convenience of description, assume that the number of coherent signals in each group is satisfied with $L_k = k+1$, $k = 1, \ldots, K$. When $N = 1$, from (5)–(7) we have

$$\mathbf{R}_1^f = \sum_{m=0}^{1}\mathbf{W}_{M-1,m}\mathbf{R}_0\mathbf{W}_{M-1,m}^H = \sum_{k=1}^{K}\tilde{\mathbf{B}}_{1k}\mathbf{Z}_{1k}\tilde{\mathbf{B}}_{1k}^H \tag{8}$$

where $\mathbf{Z}_{1k} = \sigma_k^2\mathbf{Q}_{1k}\mathbf{Q}_{1k}^H$. We can see that $\mathbf{Q}_{1k}$ is an $L_k \times 2$ matrix, and the rank of $\mathbf{Q}_{1k}$, $k = 1, \ldots, K$ equals 2. So the rank of the block diagonal matrix $\tilde{\mathbf{Q}}_1$ is $2K$. When $2K < M-1$, the rank of $\mathbf{R}_1^f$ will also be $2K$, but only the first group $L_1 = 2$ will be decorrelated. We implement the MUSIC algorithm on $\mathbf{R}_1^f$, and L_1 peaks are formed. Then the DOAs of L_1 coherent signals in the first group are estimated. Before estimating the DOAs in the second group, the signal components in the first group are eliminated from the array covariance matrix by the oblique projection. According to the definition [18], we define a new matrix as follows.

$$\mathbf{E}_1 = \mathbf{D}_1\left(\mathbf{D}_1^H\mathbf{R}_0^{\#}\mathbf{D}_1\right)^{-1}\mathbf{D}_1^H\mathbf{R}_0^{\#} \tag{9}$$

where $\mathbf{R}_0$ is given in (4) and $\mathbf{D}_1 = \mathbf{a}_1 = \mathbf{B}_1\boldsymbol{\rho}_1$. Since $\mathbf{B}_1$ is known by using the estimated DOAs of the first group, we only need to estimate $\boldsymbol{\rho}_1$. Referring to [19]

$$\boldsymbol{\rho}_k = \frac{\left(\mathbf{B}_k^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{B}_k\right)^{-1}\boldsymbol{\eta}_k}{\boldsymbol{\eta}_k^H\left(\mathbf{B}_k^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{B}_k\right)^{-1}\boldsymbol{\eta}_k} \tag{10}$$

where $\boldsymbol{\eta}_k = \left[1, 0, \dots, 0\right]^T_{1\times L_k}$.We define a new matrix

$$\mathbf{H}_1 = \left(\mathbf{I}_M - \mathbf{E}_1\right)\mathbf{R}_0\left(\mathbf{I}_M - \mathbf{E}_1\right)^H = \sum_{k=2}^{K}\mathbf{B}_k\mathbf{Z}_{0k}\mathbf{B}_k^H \tag{11}$$

From (11), one can see that the components in the first group have been eliminated. Then, we continue to apply FSS on $\mathbf{H}_1$ for $N = 2$,

$$\mathbf{R}_2^f = \sum_{m=0}^{2}\mathbf{W}_{M-2,m}\mathbf{H}_1\mathbf{W}_{M-2,m}^H = \sum_{k=2}^{K}\tilde{\mathbf{B}}_{2k}\mathbf{Z}_{2k}\tilde{\mathbf{B}}_{2k}^H \tag{12}$$

By performing eigen-decomposition on $\mathbf{R}_2^f$, the DOAs of coherent signals in the second-minimum group are estimated. We continue to define the oblique projection matrix $\mathbf{E}_2 = \mathbf{D}_2\left(\mathbf{D}_2^H\mathbf{R}_0^\#\mathbf{D}_2\right)^{-1}\mathbf{D}_2^H\mathbf{R}_0^\#$, and $\mathbf{D}_2 = \left[\mathbf{a}_1, \mathbf{a}_2\right]$ then

$$\mathbf{H}_2 = \left(\mathbf{I}_M - \mathbf{E}_2\right)\mathbf{R}_0\left(\mathbf{I}_M - \mathbf{E}_2\right)^H = \sum_{k=3}^{K}\mathbf{B}_k\mathbf{Z}_{0k}\mathbf{B}_k^H \tag{13}$$

For $N = 3$, $\mathbf{R}_3^f = \sum_{m=0}^{3}\mathbf{W}_{M-3,m}\mathbf{H}_2\mathbf{W}_{M-3,m}^H = \sum_{k=3}^{K}\tilde{\mathbf{B}}_{3k}\mathbf{Z}_{3k}\tilde{\mathbf{B}}_{3k}^H$. We estimate the DOAs of L_3 coherent signals from the third-minimum group. Repeating the above procedure until the coherent signals in the K group are estimated.

3.2. Improved method

The proposed method can achieve satisfactory DOA estimation accuracy. After all DOAs have been estimated, we can further improve the performance when $K > 1$. Since the DOAs and fading coefficients have been estimated, K matrices can be constructed as [19]

$$\mathbf{F}_k = \left(\mathbf{I} - \mathbf{G}_k\right)\mathbf{U}_s\mathbf{U}_s^H\left(\mathbf{I} - \mathbf{G}_k\right)^H, \; k = 1, \dots, K \tag{14}$$

where $\mathbf{U}_s$ is given in (3), $\mathbf{G}_k = \boldsymbol{\psi}_k\left(\boldsymbol{\psi}_k^H\mathbf{V}_k\boldsymbol{\psi}_k\right)^{-1}\boldsymbol{\psi}_k^H\mathbf{V}_k$, $\boldsymbol{\psi}_k = \left[\mathbf{a}_1, \dots, \mathbf{a}_{k-1}, \mathbf{a}_{k+1}, \dots, \mathbf{a}_K\right]$, and $\mathbf{V}_k = \mathbf{I} - \mathbf{a}_k\left(\mathbf{a}_k^H\mathbf{a}_k\right)^{-1}\mathbf{a}_k^H$. The columns of $\mathbf{A}$ also span the signal subspace, which is $\mathbf{U}_s = \mathbf{AC}$, and $\mathbf{C}$ is a $K \times K$ full rank matrix. Then

$$\left(\mathbf{I} - \mathbf{G}_k\right)\mathbf{U}_s = \left[\mathbf{0}, \dots, \mathbf{0}, \mathbf{a}_k, \mathbf{0}, \dots, \mathbf{0}\right]\mathbf{C} = \mathbf{a}_k\mathbf{c}_k^T \tag{15}$$

where $\mathbf{c}_k^T$ is the kth row of $\mathbf{C}$. From (15), $\mathbf{F}_k$ only contains the components of the kth group, and the interference caused by other groups has been eliminated. Then we can apply spatial smoothing algorithm on $\mathbf{F}_k$, $k = 1, \dots, K$ to improve DOA estimation accuracy in each group separately. After getting the estimates of DOAs in the kth group, $\mathbf{a}_k$ is updated in $\boldsymbol{\psi}_{k+1}$ to estimate the DOAs in the $\left(k + 1\right)$ group.

Different from FSS used in Section 3.1, here we use FBSS on $\mathbf{F}_k$. In [3], it has been proved that FBSS can decorrelate the coherent signals efficiently under the mild restrictions that the equal elements of the set $\left\{\varepsilon_{kl} = \frac{\rho_{kl}^* e^{j(M-1)\pi\,\sin\,\theta_{kl}}}{\rho_{kl}}, l = 1, \dots, L_k\right\}$ must be at most of the number of forward subarrays. However, when some of the members of the set $\left\{\varepsilon_{kl}, l = 1, \dots, L_k\right\}$ are close in low SNR, the rank of forward/backward smoothed covariance matrix will be deficient with limited subarrays. Since the coherent signals in each group are estimated sequentially in Section 3.1, firstly the minimum group, then the second-minimum group, and so on, the number of subarrays in each spatial smoothing is very limited. To ensure that the coherent signals are decorrelated correctly, we use FSS in Section 3.1, which has been proved strictly in [2]. However, $\mathbf{F}_k$ in (15) only contains one group of coherent signals and the others have been eliminated. There are enough subarrays, so we can use FBSS on $\mathbf{F}_k$ to further improve DOA estimation accuracy.

3.3. Discussion

Theoretical analysis shows that only the group which has $L_k = k + 1$ coherent signals are able to form peaks in the MUSIC spectrum for the kth FSS. However, the latter groups of coherent signals may still form some peaks in the MUSIC spectrum in the case of low SNR and finite samples, so we need to choose correct angles. Since the forward spatially smoothed covariance matrix $\mathbf{R}_k^f$ at most forms r (the rank of $\mathbf{R}_k^f$) peaks in the MUSIC spectrum, we choose $k + 1$ angles from all peaks (less than or equal to r) as a group and select the group as the correct one which makes the following formula minimum

$$g = \mathbf{a}^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{a} \tag{16}$$

There will be a possible scenario that different groups have the same number of coherent signals. For simplicity, we take three groups, $L_1 = 2, L_2 = 2, L_3 = 3$, as an example. At a time, only one group of DOAs is estimated. One group of two DOAs making (16) minimum is first chosen from all peaks, and their contributions are eliminated by oblique projection. Then other groups of two DOAs and three DOAs are estimated sequentially.

Since the proposed method estimates the DOAs of each group sequentially, we can use less array elements to resolve multi-group coherent signals. Assume that the number of coherent signals in each group is satisfied with $L_k = k + 1$, $k = 1, \dots, K$, where $K = 2p$ represents even number and $K = 2p - 1$ clarifies odd number. The number of array elements is at least $\left[p\left(K + 1 - p\right) + K + 2\right]$ for the proposed method, while $\left(\left\lceil\frac{(K+1)^2}{2}\right\rceil + K\right)$ for FBSS, where $\lceil\cdot\rceil$ is a ceiling operator. when $K = 3$, the number of array sensors is at least 9 for the proposed method, while 11 for FBSS.

4. Simulation results

In this section, simulation results are presented to illustrate the performance of the proposed method. We select FBSS [3], MODE [5], SVSVD [13] and SRWSF [17] to be the comparative methods, and name the proposed methods in Sections 3.1 and 3.2 as method 1 and method 2, respectively. Assume that all sources are of equal power and the input SNR is defined as $10\log_{10}\left(\sigma_k^2/\sigma_n^2\right)$. The MUSIC spectrum search range is from -90° to 90° with 0.1° spacing. The results are measured by two performance indices, called the detection probability (DP) and root-mean-square error (RMSE). DP is defined as DP=$\frac{t_s}{t_m}$, where t_s represents the times of success, and t_m denotes the number of Monte-Carlo trial, here $t_m = 500$. A detection of the I signals is considered successful if and only if $\max\left(\left|\hat{\theta}_{kl} - \theta_{kl}\right|\right) \le \xi$, $k = 1, \dots, K$, $l = 1, \dots, L_k$, where $I = \sum_{k=1}^{K}L_k$, $\hat{\theta}_{kl}$ is the estimate of θ_{kl}, and ξ is set 2.5°. RMSE is calculated by 500 successful Monte-Carlo trials and is defined as RMSE=$\sqrt{\frac{1}{500I}\sum_{n=1}^{500}\sum_{k=1}^{K}\sum_{l=1}^{L_k}\left(\hat{\theta}_{kl}(n) - \theta_{kl}\right)^2}$, where $\hat{\theta}_{kl}(n)$ is the estimate of θ_{kl} for the nth successful Monte-Carlo trial.

The first simulation considers three groups of coherent signals from $\left[-50^\circ, 35^\circ\right]$, $\left[52^\circ, -24^\circ, 25^\circ\right]$ and $\left[-14^\circ, 15^\circ, -37^\circ, 0^\circ, 43^\circ\right]$. The fading amplitudes are $\left[1, 0.9\right]$, $\left[1, 0.9, 0.8\right]$ and $\left[1, 0.9, 0.9, 0.8, 0.8\right]$,

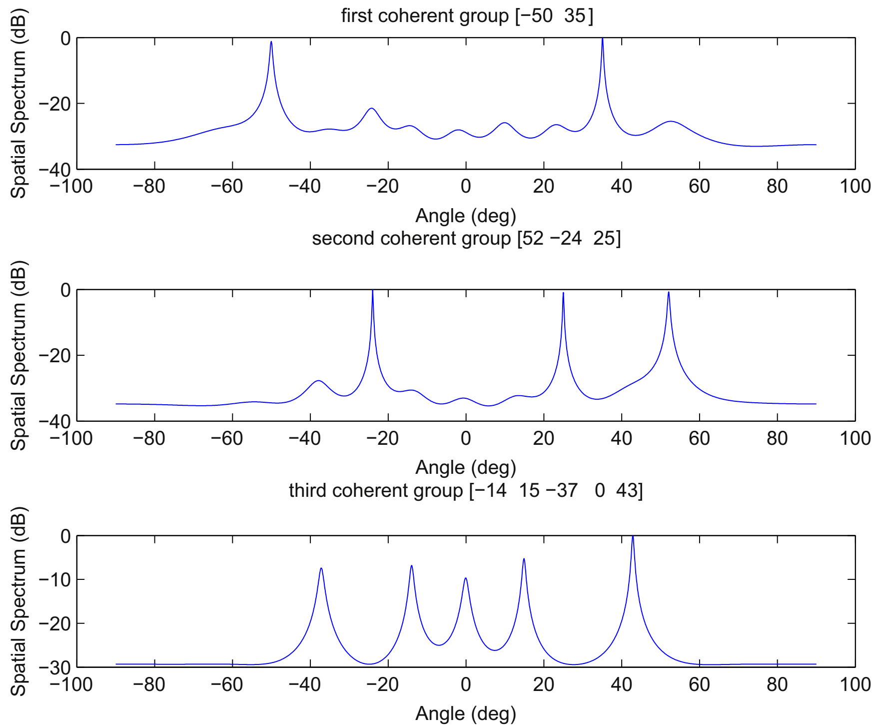

Fig. 1. The pseudo spectrum for each coherent group with SNR 0 dB and 500 snapshots.

and the fading phases are $\left[56.72^{\circ}, 270.05^{\circ}\right]$, $\left[243.03^{\circ}, 33.5^{\circ}, 140.76^{\circ}\right]$ and $\left[112.23^{\circ}, 78.56^{\circ}, 32.09^{\circ}, 160.07^{\circ}, 95.3^{\circ}\right]$, respectively.

The pseudo spectrum of each coherent group is shown in Fig. 1 for the proposed method 1 with 500 snapshots and the input SNR being 0 dB. The number of array elements is 14. We can see that the proposed method can form sharp peaks at correct DOAs in each group.

Figs. 2 and 3 show the performance curves of the DOAs versus input SNR for the proposed method, FBSS, MODE, SVSVD, and SRWSF. The number of sensors and snapshots is set 14 and 200. The number of subarrays in method 2 is 4, 6 and 8 for three groups, respectively. FBSS method has 8 subarrays for fair comparison. Fig. 2 illustrates that the detection probability of the proposed method is superior to FBSS, MODE, SVSVD and SRWSF. Furthermore, no matter how low SNR is, the successful detection probability of the proposed algorithm is always 100%. From Fig. 3, we can see that both the proposed methods have better DOA estimation accuracy than others in low SNR. The estimation performance of MODE is better than method 1 in high SNR, but still worse than method 2.

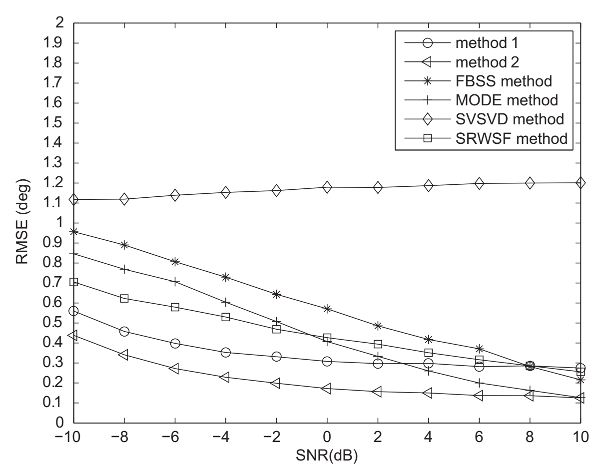

Fig. 3. RMSEs of the DOA estimates versus SNR with 200 snapshots.

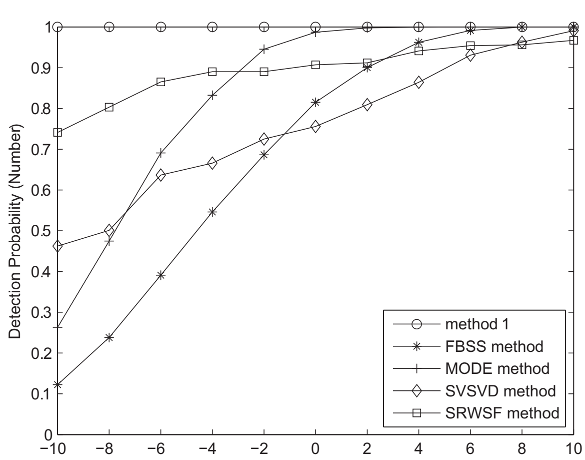

Fig. 2. Detection Probability of the DOA estimates versus SNR with 200 snapshots.

Figs. 4 and 5 display the performance curves of the DOAs against the number of snapshots. The number of sensors and SNR is set 14 and −5dB. The number of subarrays in method 2 is 4, 6 and 8 for three groups, respectively. FBSS has 8 subarrays for fair comparison. It can be seen from Fig. 4 that the detection probability of the proposed method is 100%, which is higher than FBSS, MODE, SVSVD, and SRWSF. The superiority of the proposed algorithm is more pronounced in small number of snapshots. Fig. 5 indicates that both the proposed methods always have better DOA estimation accuracy than others, and the performance is significant.

Figs. 6 and 7 show the effect of array sensors on the performance of those methods. FBSS method estimates all DOAs at one time, so it fails when $M = 11$ and $M = 12$. Let FBSS have 8 subarrays when $M \geq 14$, but it only has 6 subarrays when $M = 13$. SVSVD also fails when $M = 11$ and $M = 12$. The number of

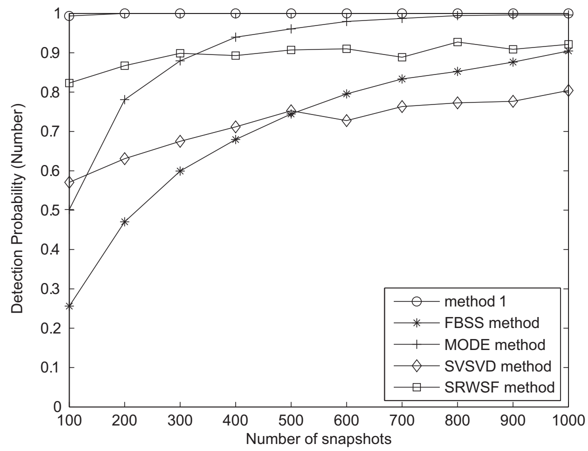

Fig. 4. Detection Probability of the DOA estimates versus the number of snapshots with SNR -5 dB.

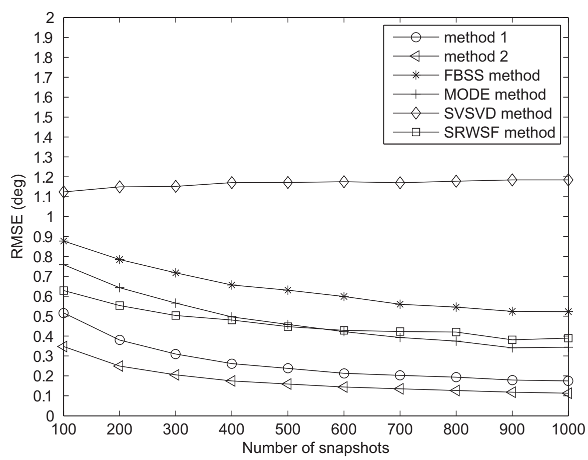

Fig. 5. RMSEs of the DOA estimates versus the number of snapshots with SNR -5 dB.

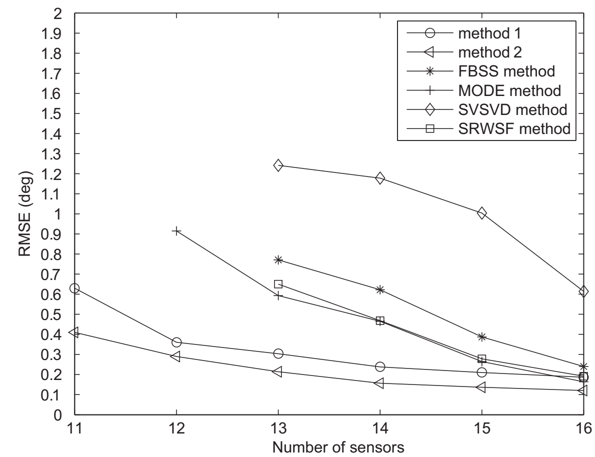

Fig. 7. RMSEs of the DOA estimates versus the number of sensors with SNR -5 dB and 500 snapshots.

subarrays in method 2 is 4, 6 and 8 for three groups, respectively. The input SNR is set -5dB, and the number of snapshots is 500. The detection probability and RMSEs versus the number of sensors are shown in Figs. 6 and 7, respectively. In Fig. 6, it is shown that MODE method obtains bad detection probability when $M = 11$ and $M = 12$. Also, the detection probability of SRWSF is zero when $M = 11$ and $M = 12$. From Fig. 7, it is observed that the performance of both the proposed methods outperforms others. Both figures indicate that the proposed method not only uses less array sensors to estimate multi-group coherent signals, but also has better estimation performance.

In the second simulation, we consider the scenario in which different groups have the same DOA. There are three groups of coherent signals coming from $[-50^\circ, 35^\circ]$, $[52^\circ, -24^\circ, 35^\circ]$ and $[-14^\circ, 15^\circ, -37^\circ, 0^\circ, 35^\circ]$, respectively. The fading amplitudes and phases are set with the same as those given in the first simulation. The number of sensors and snapshots is set 13 and 500, respectively. FBSS, MODE and SRWSF cannot deal with this scenario, since all DOAs are estimated simultaneously. Method 2 is based on FBSS and the number of subarrays is 4, 6 and 8 for three groups, respectively. Fig. 8 shows the detection probability versus SNR. It can be seen that the detection probability of the proposed method is close to 100% even the input SNR being -10dB, while the detection probability of SVSVD is zero even the input SNR being 10dB. Although SVSVD estimates the DOAs of coherent signals in each group, some DOAs are always missing in this scenario. The RMSEs of the DOA estimates versus SNR is shown in Fig. 9. From this figure, we can see that both the proposed methods have satisfactory performance and method 2 achieves better DOA estimation accuracy.

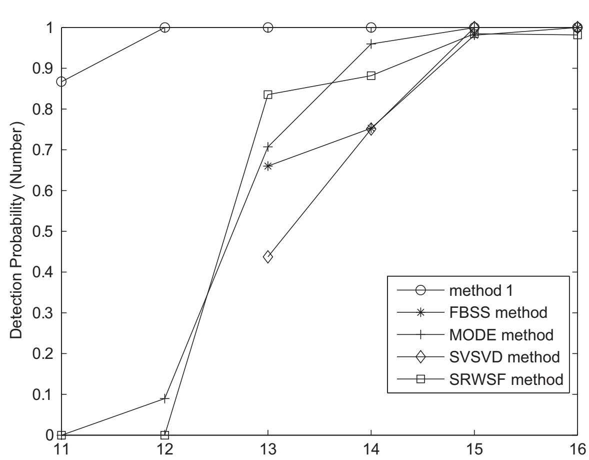

Fig. 6. Detection Probability of the DOA estimates versus the number of sensors with SNR -5 dB and 500 snapshots.

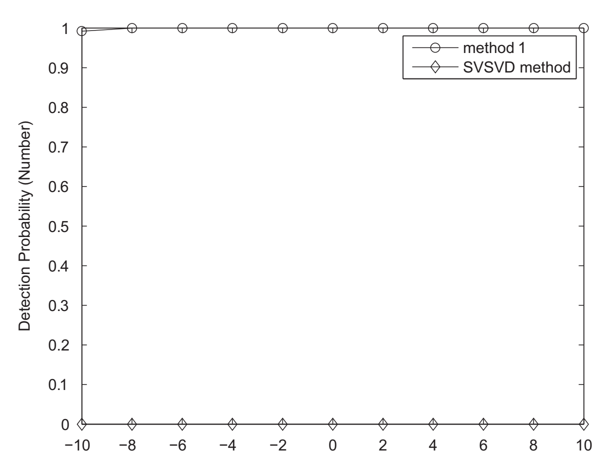

Fig. 8. Detection Probability of the DOA estimates versus SNR with 500 snapshots.

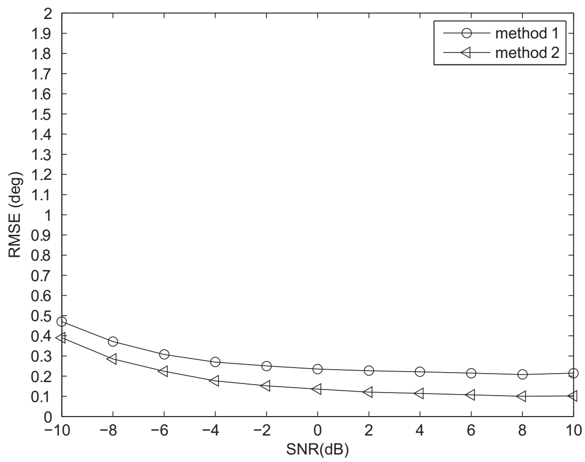

Fig. 9. RMSEs of the DOA estimates versus SNR with 500 snapshots.

5. Conclusion

In this paper, we have presented a novel DOA estimation method for multi-group coherent signals. The new method based on FSS estimates the DOAs of each group sequentially. After all groups of DOAs are estimated, the accuracy of DOAs is further improved based on FBSS. The new method can estimate the same DOA as long as they are from different groups and utilize less array sensors to estimate multi-group coherent signals. Simulation results have illustrated that the new method and the improved method both have good performance especially in low SNR and small number of snapshots.

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