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Fast communication

Wideband cyclic MUSIC algorithms ☆

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Abstract

In this paper, we propose two cyclic-MUSIC-like DOA estimation algorithms for wideband cyclostationary signals, the averaged cyclic MUSIC (ACM) algorithm and the extended wideband cyclic MUSIC (EWCM) algorithm. Compared with the existing spectral correlation signal subspace fitting (SC-SSF) algorithm, which was also developed for wideband cyclostationary signals, our new methods have more relaxed requirements on the signals. Simulation results show that under those circumstances, both our methods are superior to the existing SC-SSF algorithm.

Keywords: Direction-of-arrival (DOA); Uniform linear array (ULA); Cyclic MUSIC

1. Introduction

Many man-made signals encountered in communications such as BPSK, FSK, AM signals exhibit cyclostationarity [3]. By exploiting this cyclostationarity property, cyclic MUSIC [2] has been shown to be able to separate signals with different cycle frequencies, thus performing signal selective direction of arrival (DOA) estimation, which is superior to the conventional MUSIC [5]. However, both the conventional MUSIC and cyclic MUSIC were originally developed for narrowband signals. Although many papers can be found on improving the performance of cyclic MUSIC, most of them [1,4,7] are still designed for narrowband signals. Little work can be found on DOA estimation for wideband cyclostationary signals, except the spectral correlation signal subspace fitting (SC-SSF) algorithm [6]. This algorithm is asymptotically consistent for either narrowband or wideband sources. But as we will show later in this paper, that SC-SSF does not work if the second order cyclic statistics of the sources are the same.

In this paper, we propose two cyclic-MUSIC-like DOA estimation algorithms for wideband cyclostationary signals. First, we propose an

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averaged (conjugate) cyclic MUSIC (ACM) algorithm by averaging the cyclic or cyclic conjugate correlation matrix over the time delay τ. Then, we propose an extended wideband cyclic MUSIC (EWCM) algorithm by exploiting both averaged cyclic correlation and averaged cyclic conjugate correlation. Both our methods work well when the second order cyclic statistics of the sources are the same, while SC-SSF fails. Furthermore, when the cyclic correlaion of a signal is related to its cyclic conjugate correlation, as will be described in Section 4.3, our EWCM algorithm can detect DOAs of more sources than the number of antennas.

2. Background

2.1. Cyclostationarity

For a given signal $s(t)$, its cyclic correlation and cyclic conjugate correlation are defined as [3]

$$r_{ss}^{\alpha}(\tau) = \left\langle s\left(t+\frac{\tau}{2}\right)s^{*}\left(t-\frac{\tau}{2}\right)\mathrm{e}^{-\mathrm{j}2\pi\alpha t}\right\rangle, \tag{1}$$

$$r_{ss^{*}}^{\alpha}(\tau) = \left\langle s\left(t+\frac{\tau}{2}\right)s\left(t-\frac{\tau}{2}\right)\mathrm{e}^{-\mathrm{j}2\pi\alpha t}\right\rangle, \tag{2}$$

where $[\cdot]^{*}$ denotes complex conjugate and $\langle\cdot\rangle$ denotes time average. α is referred to as cycle frequency. Many man-made communication signals exhibit cyclostationarity due to modulation, periodic gating, etc. They usually have cycle frequencies at twice the carrier frequency or multiples of the baud rate or combinations of these. Moreover, some signals may have both nonzero cyclic correlation and nonzero cyclic conjugate correlation.

For a given vector $\mathbf{x}(t)$, its cyclic correlation matrix and cyclic conjugate correlation matrix are defined as [2]

$$\mathbf{R}_{\mathbf{xx}}^{\alpha}(\tau) = \left\langle \mathbf{x}\left(t+\frac{\tau}{2}\right)\mathbf{x}^{H}\left(t-\frac{\tau}{2}\right)\mathrm{e}^{-\mathrm{j}2\pi\alpha t}\right\rangle, \tag{3}$$

$$\mathbf{R}_{\mathbf{xx}^{*}}^{\alpha}(\tau) = \left\langle \mathbf{x}\left(t+\frac{\tau}{2}\right)\mathbf{x}^{\mathrm{T}}\left(t-\frac{\tau}{2}\right)\mathrm{e}^{-\mathrm{j}2\pi\alpha t}\right\rangle, \tag{4}$$

where $[\cdot]^{H}$ denotes conjugate transpose and $[\cdot]^{\mathrm{T}}$ denotes transpose.

2.2. Existing spectral correlation signal subspace fitting (SC-SSF) algorithm

To look into the SC-SSF algorithm, consider a uniform linear array (ULA) of size N with intersensor spacing d, that receives I signals of interest (SOI) with cycle frequency α from directions θ_i, $i=1,\ldots,I$. The incident waves are assumed to be $s_i(t)=a_i(t)\mathrm{e}^{\mathrm{j}2\pi f_0 t}$, where $a_i(t)$ is a baseband signal with unspecified bandwidth, that is, the signals can be wideband, and f_0 is the carrier frequency. Other signals that either have different cycle frequencies or do not exhibit cyclostationarity are considered as interference. Then the signal induced at the nth antenna can be written, in its baseband form as

$$x_n(t) = \sum_{i=1}^{I} a_i(t+(n-1)\Delta_i)\mathrm{e}^{\mathrm{j}2\pi f_0(n-1)\Delta_i} + n_n(t), \tag{5}$$

where $\Delta_i = d\sin\theta_i/c$ with c as the propagation speed, and $n_n(t)$ includes noise as well as interference induced at the nth antenna.

SC-SSF calculates the cyclic autocorrelation of the received signal at each antenna (Eq. (11) in [6]), which can be rewritten as

$$r_{x_n x_n}^{\alpha}(\tau) = \sum_{i=1}^{I} r_{a_i a_i}^{\alpha}(\tau)\mathrm{e}^{\mathrm{j}2\pi\alpha(n-1)\Delta_i}. \tag{6}$$

Here $n_n(t)$ disappears since it does not share the same cycle frequency as the SOI, and the cross terms also disappear as the signals are assumed to be mutually cyclically uncorrelated [6].

Now define $\mathbf{r}_a(\tau) = [r_{a_1 a_1}^{\alpha}(\tau),\ldots,r_{a_I a_I}^{\alpha}(\tau)]^{\mathrm{T}}$ and $\mathbf{r}_x(\tau) = [r_{x_1 x_1}^{\alpha}(\tau),\ldots,r_{x_N x_N}^{\alpha}(\tau)]^{\mathrm{T}}$. From (6), $\mathbf{r}_x(\tau)$ could be written as

$$\mathbf{r}_x(\tau) = \mathbf{A}(\alpha)\mathbf{r}_a(\tau), \tag{7}$$

where

$$\mathbf{A}(\alpha) = [\mathbf{a}(\alpha,\theta_1),\ldots,\mathbf{a}(\alpha,\theta_I)] \tag{8}$$

is the steering matrix composed of steering vectors $\mathbf{a}(f,\theta)|_{f=\alpha,\theta=\theta_i}$ for $i=1,\ldots,I$. Here $\mathbf{a}(f,\theta)$ is defined as

$$\mathbf{a}(f,\theta) = [1,\mathrm{e}^{\mathrm{j}2\pi f d\sin\theta/c},\ldots,\mathrm{e}^{\mathrm{j}2\pi f(N-1)d\sin\theta/c}]^{\mathrm{T}} \tag{9}$$

On the other hand, the following signal model is considered in MUSIC for the narrowband case

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{10}$$

where $\mathbf{x}(t) = [x_1(t), \ldots, x_N(t)]^{\mathrm{T}}$ is the received signal vector, $\mathbf{s}(t) = [s_1(t), \ldots, s_I(t)]^{\mathrm{T}}$ is the source signal vector, $\mathbf{n}(t) = [n_1(t), \ldots, n_N(t)]^{\mathrm{T}}$ is the noise vector, and $\mathbf{A}$ is actually $\mathbf{A}(f_0)$ defined in (8). Clearly (7) is in the same form as (10). Thus MUSIC can be applied, and the remaining steps of SC-SSF based on (7) are the same as those of MUSIC. Since (7) can be obtained regardless of the signal bandwidth, SC-SSF is considered as asymptotically consistent for either narrowband or wideband sources.

But similar to MUSIC, for SC-SSF to work, $\langle \mathbf{r}_a(\tau)\mathbf{r}_a^H(\tau)\rangle$ should be full rank, which means $r^{\alpha}_{a_i a_i}(\tau)$ and $r^{\alpha}_{a_j a_j}(\tau)$ as functions of τ should be incoherent for $i \neq j$, or their second order cyclic statistics should be different. However, the assumption that two signals are mutually cyclically uncorrelated does not ensure that their second order cyclic statistics are different. For example, two randomly generated BPSK signals with the same baud rate and the same pulse shaping are mutually cyclically uncorrelated, but their second order cyclic statistics are the same. Thus SC-SSF would not work in this example.

3. Averaged (conjugate) cyclic MUSIC (ACM)

Instead of calculating the cyclic autocorrelation as in SC-SSF, we calculate the cross cyclic correlation of signals between different antennas, which is expressed as in (5), and obtain

$$\begin{aligned}
r^{\alpha}_{x_p x_n}(\tau) &= \left\langle x_p\left(t+\frac{\tau}{2}\right) x_n^*\left(t-\frac{\tau}{2}\right) \mathrm{e}^{-\mathrm{j}2\pi\alpha t}\right\rangle \\
&= \sum_{i=1}^{I} \left\langle a_i\left(t+\frac{\tau}{2}+(p-1)\Delta_i\right) \right. \\
&\quad \left. \times a_i^*\left(t-\frac{\tau}{2}+(n-1)\Delta_i\right)\mathrm{e}^{-\mathrm{j}2\pi\alpha t}\right\rangle \\
&\quad \times \mathrm{e}^{\mathrm{j}2\pi f_0 (p-n)\Delta_i}, \\
&= \sum_{i=1}^{I} r^{\alpha}_{a_i a_i}(\tau+(p-n)\Delta_i)\,\mathrm{e}^{\mathrm{j}2\pi\frac{\alpha}{2}(p+n-2)\Delta_i} \\
&\quad \times \mathrm{e}^{\mathrm{j}2\pi f_0 (p-n)\Delta_i}, \\
&= \sum_{i=1}^{I} \mathrm{e}^{\mathrm{j}2\pi(f_0+\frac{\alpha}{2})(p-1)\Delta_i} r^{\alpha}_{a_i a_i}(\tau+(p-n)\Delta_i) \\
&\quad \times \mathrm{e}^{-\mathrm{j}2\pi(f_0-\frac{\alpha}{2})(n-1)\Delta_i}.
\end{aligned} \tag{11}$$

The assumptions made here are the same as SC-SSF mentioned in [6], i.e., interference and noise do not share the same cycle frequency as the SOI, and signals are mutually cyclically uncorrelated. Note that $r^{\alpha}_{x_p x_n}(\tau)$ is actually the (p, n)-th element of $\mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau)$ defined in (3). Our goal here is to obtain a matrix which is in a form similar to that for Cyclic MUSIC, i.e.,

$$\mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau) = \mathbf{A}\mathbf{R}^{\alpha}_{\mathbf{ss}}(\tau)\mathbf{A}^H. \tag{12}$$

To achieve this goal, we propose to average $r^{\alpha}_{x_p x_n}(\tau)$ over the time delay $\tau \in [\tau_1, \tau_2]$ and obtain

$$\begin{aligned}
\sum_{\tau=\tau_1}^{\tau_2} r^{\alpha}_{x_p x_n}(\tau) &= \sum_{i=1}^{I} \mathrm{e}^{\mathrm{j}2\pi(f_0+\frac{\alpha}{2})(p-1)\Delta_i} \\
&\quad \times \left[\sum_{\tau=\tau_1}^{\tau_2} r^{\alpha}_{a_i a_i}(\tau+(p-n)\Delta_i)\right] \\
&\quad \times \mathrm{e}^{-\mathrm{j}2\pi(f_0-\frac{\alpha}{2})(n-1)\Delta_i}.
\end{aligned} \tag{13}$$

By appropriately choosing τ_1 and τ_2, we can make the middle factor of (13) independent of p and n. Using $\langle\cdot\rangle_\tau$ to denote this averaging, we can write

$$\begin{aligned}
\langle r^{\alpha}_{x_p x_n}\rangle_\tau &= \sum_{i=1}^{I} \mathrm{e}^{\mathrm{j}2\pi(f_0+\frac{\alpha}{2})(p-1)\Delta_i} \langle r^{\alpha}_{a_i a_i}\rangle_\tau \\
&\quad \times \mathrm{e}^{-\mathrm{j}2\pi(f_0-\frac{\alpha}{2})(n-1)\Delta_i}.
\end{aligned} \tag{14}$$

Thus by stacking up $\langle r^{\alpha}_{x_p x_n}\rangle_\tau$, the obtained averaged cyclic correlation matrix, denoted by $\langle \mathbf{R}^{\alpha}_{\mathbf{xx}}\rangle_\tau$ can be written as

$$\langle \mathbf{R}^{\alpha}_{\mathbf{xx}}\rangle_\tau = \mathbf{A}(f_0+\alpha/2)\mathbf{M}_1\mathbf{A}^H(f_0-\alpha/2), \tag{15}$$

where $\mathbf{A}(f_0+\alpha/2)$ is defined in (8) with the parameter α replaced by $f_0+\alpha/2$ and

$$\mathbf{M}_1 = \mathrm{diag}[\langle r^{\alpha}_{a_1 a_1}\rangle_\tau, \ldots, \langle r^{\alpha}_{a_I a_I}\rangle_\tau]. \tag{16}$$

The range of the lags, τ_1 and τ_2 in (13), are designer-chosen parameters. The cyclic correlation of a signal usually have large values only in a certain interval of τ, say $[\tau_{\min}, \tau_{\max}]$. The length of this interval is approximately the symbol duration or twice the symbol duration [1]. Therefore, not all possible lags need to be included. By examining the middle factor of (13), a general guidance for choosing τ_1 and τ_2 would be to make $\tau_1+(N-1)\Delta_i<\tau_{\min}$ and $\tau_2-(N-1)\Delta_i>\tau_{\max}$, where N is the number of antennas. This choice ensures that for all possible p and n, all significant values of cyclic correlation will be included in the averaging, thus the averaged cyclic correlation will be independent of p and n. Choosing too few lags will not achieve (15). On the other hand, choosing too many lags not only makes the algorithm inefficient, but also causes performance degradation since at some time lags, the cyclic correlation of the signal is as small as the cyclic correlation of interference or noise. In our experience, $\tau_2-\tau_1$ should be a couple of symbol durations.

Now, if applying cyclic conjugate correlation, by the deduction similar to the above, we obtain

$$\langle \mathbf{R}^{\alpha}_{\mathbf{xx}^*}\rangle_\tau = \mathbf{A}(f_0+\alpha/2)\mathbf{M}_2\mathbf{A}^{\mathrm{T}}(f_0+\alpha/2), \tag{17}$$

where

$$\mathbf{M}_2 = \mathrm{diag}[\langle r^{\alpha}_{a_1a_1^*}\rangle_\tau,\ldots,\langle r^{\alpha}_{a_Ia_I^*}\rangle_\tau]. \tag{18}$$

Both (15) and (17) are in a form similar to (12), thus the remaining steps to estimate the DOA are the same as those for cyclic MUSIC. The difference is only that the frequency for evaluating the steering vectors which are lying in the left signal subspace of (15) and (17) is $f_0+\alpha/2$, while it is f_0 for cyclic MUSIC in (12).

Since $\mathbf{M}_1$ and $\mathbf{M}_2$ are of full rank by their structures, we know that for ACM to work, we only need to make the assumption that the sources are mutually cyclically uncorrelated, while SC-SSF further requires that the second order statistics of the sources are different as discussed above.

4. Extended wideband cyclic MUSIC (EWCM)

In this section, we propose an extended wideband cyclic MUSIC (EWCM) algorithm by modifying the extended cyclic MUSIC algorithm in [1] which was developed for narrowband signals, and then combining with the ACM algorithm discussed above. This EWCM algorithm has all the merits of extended cyclic MUSIC, but now applicable to wideband cyclostationary signals.

4.1. Existing extended cyclic MUSIC

The extended cyclic MUSIC algorithm is based on the data model (10) as cyclic MUSIC, thus it only works for the narrowband case. This algorithm calculates an extended cyclic correlation matrix by applying both cyclic correlation and cyclic conjugate correlation as

$$\mathbf{R}^{\alpha}_{\mathrm{CE}}(\tau) = \begin{bmatrix} \mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau) & \mathbf{R}^{\alpha}_{\mathbf{xx}^*}(\tau) \\ [\mathbf{R}^{\alpha}_{\mathbf{xx}^*}(\tau)]^* & [\mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau)]^* \end{bmatrix}, \tag{19}$$

where $\mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau)$ and $\mathbf{R}^{\alpha}_{\mathbf{xx}^*}(\tau)$ are defined as in (3) and (4). Using (12) and a similar expression for cyclic conjugate correlation, (19) becomes

$$\mathbf{R}^{\alpha}_{\mathrm{CE}}(\tau) = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^* \end{bmatrix} \begin{bmatrix} \mathbf{R}^{\alpha}_{\mathbf{ss}}(\tau) & \mathbf{R}^{\alpha}_{\mathbf{ss}^*}(\tau) \\ [\mathbf{R}^{\alpha}_{\mathbf{ss}^*}(\tau)]^* & [\mathbf{R}^{\alpha}_{\mathbf{ss}}(\tau)]^* \end{bmatrix} \times \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^* \end{bmatrix}^H \tag{20}$$

where $\mathbf{R}^{\alpha}_{\mathbf{ss}}(\tau)$ is the cyclic correlation matrix, and $\mathbf{R}^{\alpha}_{\mathbf{ss}^*}(\tau)$ is the cyclic conjugate correlation matrix, of the source vector $\mathbf{s}(t)$. Since the dimension of $\mathbf{R}^{\alpha}_{\mathrm{CE}}(\tau)$ is $2N\times 2N$, but its rank is equal to I' with $I\leqslant I'\leqslant 2I$ depending on the property of the cyclostationary sources, the number of sources that can be estimated by this method is I_s with $(N-1)\leqslant I_s\leqslant 2(N-1)$. The performance is also improved. (See [1] for more details.)

4.2. Construction of the extended wideband cyclic correlation matrix

To develop our EWCM algorithm, our goal is to construct an extended wideband cyclic correlation matrix similar to (20). If we apply the idea of averaging in ACM discussed in Section 3 directly to (19), we obtain

$$\langle \mathbf{R}^{\alpha}_{\mathrm{CE}}\rangle_\tau = \begin{bmatrix} \langle \mathbf{R}^{\alpha}_{\mathbf{xx}}\rangle_\tau & \langle \mathbf{R}^{\alpha}_{\mathbf{xx}^*}\rangle_\tau \\ [\langle \mathbf{R}^{\alpha}_{\mathbf{xx}^*}\rangle_\tau]^* & [\langle \mathbf{R}^{\alpha}_{\mathbf{xx}}\rangle_\tau]^* \end{bmatrix}, \tag{21}$$

where $\langle \mathbf{R}^{\alpha}_{\mathbf{xx}}\rangle_\tau$ and $\langle \mathbf{R}^{\alpha}_{\mathbf{xx}^*}\rangle_\tau$ are expressed as in (15) and (17), and the lower two sub-matrices are their complex conjugate, respectively, which can be written as

$$[\langle \mathbf{R}^{\alpha}_{\mathbf{xx}^*}\rangle_\tau]^* = \mathbf{A}^*(f_0+\alpha/2)\mathbf{M}_2^*\mathbf{A}^H(f_0+\alpha/2), \tag{22}$$

$$[\langle \mathbf{R}^{\alpha}_{\mathbf{xx}}\rangle_\tau]^* = \mathbf{A}^*(f_0+\alpha/2)\mathbf{M}_1^*\mathbf{A}^{\mathrm{T}}(f_0-\alpha/2). \tag{23}$$

Unfortunately, $\langle \mathbf{R}^{\alpha}_{\mathrm{CE}}\rangle_\tau$ cannot be written in a form similar to (20).

To overcome this problem, we replace the lower two sub-matrices of $\langle \mathbf{R}^{\alpha}_{\mathrm{CE}}\rangle_\tau$ with $[\langle \mathbf{R}^{-\alpha}_{\mathbf{xx}^*}\rangle_\tau]^*$ and $[\langle \mathbf{R}^{-\alpha}_{\mathbf{xx}}\rangle_\tau]^*$, and obtain an extended wideband cyclic correlation matrix denoted by $\mathbf{R}_{\mathrm{EW}}$, i.e.,

$$\mathbf{R}_{\mathrm{EW}} = \begin{bmatrix} \langle \mathbf{R}^{\alpha}_{\mathbf{xx}}\rangle_\tau & \langle \mathbf{R}^{\alpha}_{\mathbf{xx}^*}\rangle_\tau \\ [\langle \mathbf{R}^{-\alpha}_{\mathbf{xx}^*}\rangle_\tau]^* & [\langle \mathbf{R}^{-\alpha}_{\mathbf{xx}}\rangle_\tau]^* \end{bmatrix} \tag{24}$$

Note that the only difference is that the cycle frequency for evaluating the lower two matrices is $-\alpha$ now. Thus replacing α in (22) and (23) with $-\alpha$, we obtain

$$[\langle \mathbf{R}^{-\alpha}_{\mathbf{xx}^*}\rangle_\tau]^* = \mathbf{A}^*(f_0-\alpha/2)\mathbf{M}_3\mathbf{A}^H(f_0-\alpha/2), \tag{25}$$

$$[\langle \mathbf{R}^{-\alpha}_{\mathbf{xx}}\rangle_\tau]^* = \mathbf{A}^*(f_0-\alpha/2)\mathbf{M}_4\mathbf{A}^{\mathrm{T}}(f_0+\alpha/2), \tag{26}$$

where

$$\mathbf{M}_3 = \mathrm{diag}[\langle r^{-\alpha}_{a_1a_1^*}\rangle^*_\tau, \ldots, \langle r^{-\alpha}_{a_Ia_I^*}\rangle^*_\tau], \tag{27}$$

$$\mathbf{M}_4 = \mathrm{diag}[\langle r^{-\alpha}_{a_1a_1}\rangle^*_\tau, \ldots, \langle r^{-\alpha}_{a_Ia_I}\rangle^*_\tau]. \tag{28}$$

Now the extended wideband cyclic correlation matrix $\mathbf{R}_{\mathrm{EW}}$ can be written in a compact form as

$$\mathbf{R}_{\mathrm{EW}} = \begin{bmatrix} \mathbf{A}(f_0+\alpha/2) & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^*(f_0-\alpha/2) \end{bmatrix} \begin{bmatrix} \mathbf{M}_1 & \mathbf{M}_2 \\ \mathbf{M}_3 & \mathbf{M}_4 \end{bmatrix} \times \begin{bmatrix} \mathbf{A}^H(f_0-\alpha/2) & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{\mathrm{T}}(f_0+\alpha/2) \end{bmatrix}. \tag{29}$$

It is easy to show that an equivalent way to construct $\mathbf{R}_{\mathrm{EW}}$ is

1. Construct $\mathbf{x}_e = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}^* \end{bmatrix}$.
2. Calculate the cyclic correlation matrix of $\mathbf{x}_e$, $\mathbf{R}^{\alpha}_{\mathbf{x}_e\mathbf{x}_e}(\tau)$.
3. Average $\mathbf{R}^{\alpha}_{\mathbf{x}_e\mathbf{x}_e}(\tau)$ over the time delay τ to obtain $\langle \mathbf{R}^{\alpha}_{\mathbf{x}_e\mathbf{x}_e}\rangle_\tau$, which is equivalent to $\mathbf{R}_{\mathrm{EW}}$.

4.3. DOA estimation for EWCM

Comparing (29) with (20), we notice that they are in the same form. Thus to solve the DOA estimation problem for EWCM, we can refer to [1]. First define an extended normalized steering vector

$$\mathbf{b}(\theta,\mathbf{c}) = \frac{\mathbf{B}(\theta)\mathbf{c}}{\|\mathbf{B}(\theta)\mathbf{c}\|}, \tag{30}$$

where $\mathbf{c} = [c_1\ c_2]^{\mathrm{T}}$ is a constant vector to be determined, and $\mathbf{B}(\theta)$ is defined as

$$\mathbf{B}(\theta) = \begin{bmatrix} \mathbf{a}(f_0+\alpha/2,\theta) & \mathbf{0} \\ \mathbf{0} & \mathbf{a}^*(f_0-\alpha/2,\theta) \end{bmatrix}. \tag{31}$$

Then find the maxima of the following spatial spectrum in terms of θ as the DOA estimate

$$P(\theta) = \left[\min_{\mathbf{c}} \frac{\mathbf{c}^H\mathbf{B}^H(\theta)\mathbf{E}_n\mathbf{E}_n^H\mathbf{B}(\theta)\mathbf{c}}{\mathbf{c}^H\mathbf{c}}\right]^{-1} = [\lambda_{\min}[\mathbf{B}^H(\theta)\mathbf{E}_n\mathbf{E}_n^H\mathbf{B}(\theta)]]^{-1}, \tag{32}$$

where $\lambda_{\min}[\cdot]$ denotes the minimum eigenvalue, and $\mathbf{E}_n$ is the left noise subspace of $\mathbf{R}_{\mathrm{EW}}$ estimated by applying the Singular Value Decomposition (SVD).

Now let us investigate the condition on which our new EWCM could detect more sources than the number of antennas. Consider the following matrix which corresponds to the ith source in the center matrix on the right-hand side of Eq. (29),

$$\mathbf{r}_i = \begin{bmatrix} \langle r^{\alpha}_{a_ia_i}\rangle_\tau & \langle r^{\alpha}_{a_ia_i^*}\rangle_\tau \\ \langle r^{-\alpha}_{a_ia_i^*}\rangle^*_\tau & \langle r^{-\alpha}_{a_ia_i}\rangle^*_\tau \end{bmatrix}. \tag{33}$$

If the determinant of $\mathbf{r}_i$ is 0, then the ith source will only contribute an additional 1 to the rank of $\mathbf{R}_{\mathrm{EW}}$ in (29). Otherwise, the contribution of the ith source to the rank of $\mathbf{R}_{\mathrm{EW}}$ will be 2, supposing the rank has not reached $2N-1$. Thus the maximum number of SOI that can be detected simultaneously is between $N-1$ and $2(N-1)$, depending on the cyclic correlation properties of the sources, i.e., whether (33) has rank 1 or 2.

5. Simulation results

Three simulations have been carried out to illustrate the effectiveness of our algorithms. In all of these simulations, the SOI is assumed to be wideband BPSK signals with raised cosine shaping of a roll-off factor 0.7. The carrier frequency, f_0, is 20 MHz and the symbol rate, f_b, is 10 MHz. The interference has a different symbol rate at 3 MHz. 3200 snapshots are used. Averaging of the cyclic correlation is performed from $\tau = -30$ to $\tau = 30$, i.e., a total of 60 lags are used.

Simulation 1: In this simulation, an antenna array with 4 antennas are assumed to receive two SOI from DOAs of 20° and 50°. The signal to noise ratio (SNR) is 15 dB. Simulation result is shown in Fig. 1. We notice that since the second order cyclic statistics of the two BPSK signals are the same, SC-SSF failed to detect their DOAs, while our ACM algorithm detected these two DOAs successfully.

Simulation 2: This simulation is tested to show that more sources can be detected than the number of array antennas by our EWCM algorithm. Five SOI are impinging on the array with 4 antennas from −50°, −20°, 5°, 30°, and 60° respectively. SNR is assumed to be 15 dB. Simulation result is shown in Fig. 2. Clearly all DOAs of these 5 wideband signals are detected simultaneously with only 4 antennas.

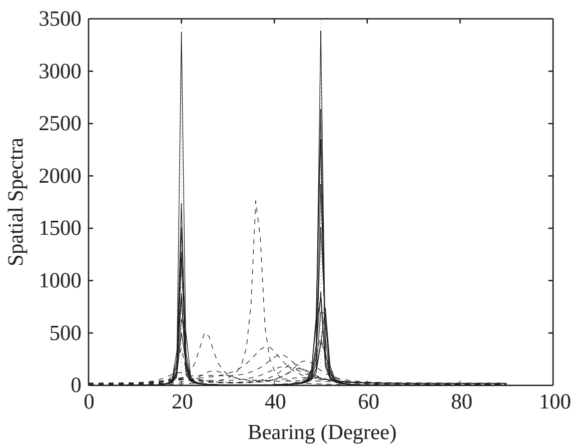

Fig. 1. DOA estimation for two wideband BPSK signals with same second order cyclic statistics: — Proposed ACM method; - - SC-SSF.

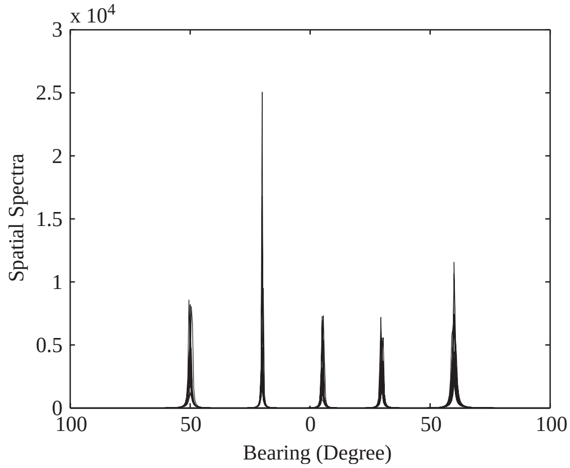

Fig. 2. 5 wideband BPSK signals are detected with 4 antennas by the proposed EWCM algorithm.

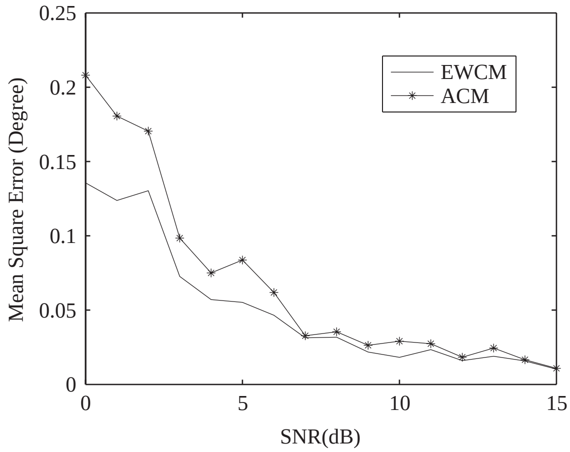

Fig. 3. Mean square error of the detected DOA over SNR by ACM and EWCM algorithms.

Simulation 3: This simulation is tested to show the effect of SNR on the mean square error of the DOA estimates. One SOI is impinging on the array with 4 antennas from 20°. One interference with a different cycle frequency is coming from a DOA of 22°. SNR is assumed to be varied from 0 dB to 15 dB. From Fig. 3, we can see both our methods successfully suppress the interference and detect the DOA with good accuracy.

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