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DOA estimation based on reduced-rank multistage Wiener filter

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Abstract

A new DOA estimator based on the reduced-rank multistage Wiener filter (MWF) is proposed this paper. This method estimates the DOAs of the impinging signals in the framework of beamforming and the number of sources (NOS) is hence not required. Besides improving performance over the classical subspace-based methods under small sample size condition, the new algorithm offers a significant reduction in computational complexity. In addition, a computation-free rank selection criterion is developed for the reduced-rank MWF, which plays the role of determining the NOS in the subspace-based methods. This criterion has better performance than the conventional information theoretic criteria when the sample size is small.

Keywords: DOA estimation; low computational complexity; *multistage Wiener filter; reduced-rank processing*

1. Introduction

The direction-of-arrival (DOA) estimation has received considerable attentions in the past several decades and numerous DOA estimators have been proposed by researchers. Among the most popular techniques are the MUSIC[1] and ESPRIT[2] algorithms, which offer a good compromise between estimation accuracy and computational complexity. However, these eigen-structure based algorithms require precise knowledge of the number of sources (NOS). If the NOS is incorrectly estimated, e.g., when the available snapshots is limited, the estimation accuracy and resolution of these algorithms will decrease. Akaike information criterion (AIC) and minimum description length (MDL) are the most widely used criteria to determine the NOS[3] of the impinging signals. However, they tend to estimate wrong NOS in the situations of low signal-to-noise ratio (SNR) and small sample size.

Although the conventional beamformer and Capon estimator[4] do not require the knowledge of NOS, their resolution is low. The high resolution DOA estimators without knowing the NOS have been presented recently[5,6]. However, like the conventional MUSIC algorithm, these techniques suffer considerable performance degradation when there is no enough snapshots to estimate the covariance matrix of the input signals. In addition, the method described in [5] has a high computational complexity. Although the minimum norm-like (MNL) algorithm presented in [6] reduces computational cost, it will produce the pseudo peaks in the spatial spectrum.

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In this paper we present a new technique for the problem of DOA estimation based on the reduced-rank multistage Wiener filter[7] (MWF). This method does not require the NOS and needs smaller snapshots than other methods because it works in a low dimensional subspace. In addition, it is much more efficient than all of the aforementioned algorithms. Determining the dimension of the reduced-rank subspace is a key problem in reduced-rank processing (RRP). We use the power of the signal components in the Krylov subspace as the rank selection criterion, which has the same performance of the AIC and MDL criteria when the number of snapshots is large and better performance when the number of snapshots is small.

2. DOA estimation using adaptive interference canceller

Supposing that J narrowband signals with DOAs of (θ_1,ϕ_1), (θ_2,ϕ_2), $\cdots$, (θ_J,ϕ_J) impinge on an array with M omnidirectional sensors. The received signal vector $\mathbf{z}(k)$ of the k th snapshot can be modeled as

$$\mathbf{z}(k)=\sum_{j=1}^{J}s_j(k)\mathbf{v}(\theta_j,\phi_j)+\mathbf{n}(k) \tag{1}$$

where $s_j(k)$ is the waveform of the j th signal, $\mathbf{v}(\theta_j,\phi_j)\in\mathbb{C}^M$ is the array manifold vector corresponding to direction (θ_j,ϕ_j) and $\mathbf{n}(k)\in\mathbb{C}^M$ is the additive noise vector. Although the DOA estimation algorithm presented in this paper can be applied to arbitrary array geometry, we consider a uniform linear array (ULA) for simplicity, thus the parameters to be estimated are the elevation angles θ_j.

The adaptive interference canceller that contains a main channel and an array of auxiliary channels is one of the earliest applications of the adaptive antenna array, as shown in Fig. 1. The incident signals are present in both of the main channel and auxiliary channels. The auxiliary channels are used to estimate the signals in the main channel by the adaptive weight vector $\mathbf{w}_a$. Then the estimated signals are subtracted from the main channel, leading to nulls in the directions of the incident signals. Hence the adaptive interference canceller can be used as a DOA estimator. The adaptive weight vector $\mathbf{w}_a$ and the weight vector of the canceller $\mathbf{w}$ are given by[3]

$$\mathbf{w}_a=\mathbf{R}_{\mathbf{x}}^{-1}\mathbf{r}_{\mathbf{x}d},\quad \mathbf{w}=[1;-\mathbf{w}_a]=\left[1,-\mathbf{w}_a^T\right]^T \tag{2}$$

where $\mathbf{R}_{\mathbf{x}}=E[\mathbf{x}(k)\mathbf{x}^H(k)]$ is the covariance matrix of $\mathbf{x}(k)$ and $\mathbf{r}_{\mathbf{x}d}=E[\mathbf{x}(k)d^*(k)]$ is the cross-correlation vector between $\mathbf{x}(k)$ and $d(k)$. The spatial spectrum can be expressed as

$$P(\theta)=-20\log_{10}\left(\left|\mathbf{w}^H\mathbf{v}(\theta)\right|\right) \tag{3}$$

3. DOA estimation based on reduced-rank MWF

The RRP is a commonly used technique in array processing as the NOS is usually smaller than the number of sensors. Compared with full-rank processing, RRP not only has a faster convergence rate and a lower computational complexity, but also has robustness against the array model errors[8]. The fast convergence rate implies that the RRP needs a small amount of training data to achieve a desired performance. This feature is very important for the application in which the sample size is limited.

Two widely used RRP techniques are the projections onto the eigen-subspace and Krylov subspace, respectively. The Krylov subspace method not only has a fewer computations than the eigen-subspace method, but also obtains a

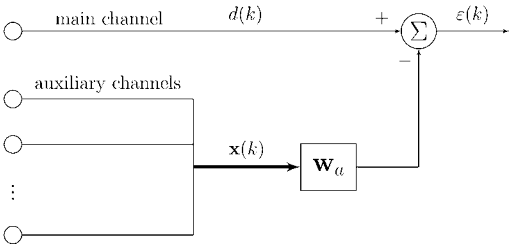

Fig. 1. Adaptive interference canceller.

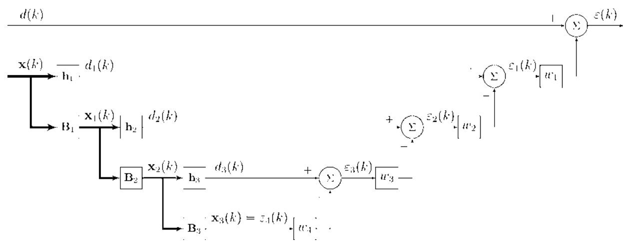

Fig. 2. Multistage Wiener Filter for M=4.

lower rank when applied to Wiener filter[7]. Besides multistage Wiener filter, there exists other approaches to construct the Krylov subspaces, e.g., Lanczos iteration and conjugate gradient algorithm.

MWF solves the Wiener equation (2) by a cascaded chain of scalar Wiener filters, as shown in Fig. 2. The input signal $\mathbf{x}_{i-1}(k)$ at the i th stage is decomposed into two part: $d_i(k)$ and $\mathbf{x}_i(k)$. $d_i(k)$ is the projection of $\mathbf{x}_{i-1}(k)$ onto the direction of $\mathbf{h}_i$ and $\mathbf{x}_i(k)$ is the projection of $\mathbf{x}_{i-1}(k)$ onto the range space of $\mathbf{B}_i$, where $\mathbf{h}_i$ is the normalized version of the cross-correlation vectors $\mathrm{E}[\mathbf{x}_{i-1}(k)d^*_{i-1}(k)]$ and $\mathbf{B}_i$ is the blocking matrix whose columns are orthogonal to $\mathbf{h}_i$. It is known that MWF works in the Krylov subspace. In other words, if we stop the forward decomposition process at the r th stage, then we get a reduced-rank solution of (2) in an r-dimensional Krylov subspace[9]. Because MWF solves the Wiener equation in an iterative manner, it has fewer computations than the matrix inversion based methods if the iteration process is stopped at an appropriate stage.

From Fig. 2 we know that the reduced-rank adaptive weight vector $\mathbf{w}_a^{(r)}$ can be expressed as

$$\mathbf{w}_a^{(r)} = w_1\mathbf{h}_1 - w_1 w_2 \mathbf{B}_1\mathbf{h}_2 + \cdots + (-1)^{r+1}\prod_{i=1}^{r} w_i \prod_{i=1}^{r-1}\mathbf{B}_i\mathbf{h}_r \tag{4}$$

It is easy to show that if the blocking matrix is chosen as $\mathbf{B}_i = \mathbf{I} - \mathbf{h}_i\mathbf{h}_i^H$, the corresponding MWF is known as the correlation subtractive structure MWF, then $\mathbf{w}_a^{(r)}$ can be simplified into

$$\mathbf{w}_a^{(r)} = w_1\mathbf{h}_1 - w_1 w_2 \mathbf{h}_2 + \cdots + (-1)^{r+1}\prod_{i=1}^{r} w_i \mathbf{h}_r \tag{5}$$

As in all reduced-rank processing, determining the dimension of the reduced-rank subspace is a key problem in MWF. Some fast and accurate rank selection methods for MWF have been proposed[10]. Here we use $\sigma_{d_i}^2 = \mathrm{E}[|d_i(k)|^2]$ as the rank selection criterion since $\sigma_{d_i}^2$ has already been calculated by MWF. The idea behind this criterion is that $d_i(k)$ is the projection of the input signal onto the orthonormal basis vector $\mathbf{h}_i$. If the signals have been completely cancelled by the previous $r-1$ stages, $d_r(k)$ will only contain the noise; therefore, $\sigma_{d_r}^2$ is approximately equal to the noise power σ_n^2.

The MNL algorithm minimizes the array output power with a quadratic constraint on the weight vector[6], i.e.,

$$\min \mathbf{w}^H \mathbf{R}_\mathbf{z} \mathbf{w} \quad \text{s.t.} \quad |w_1|^2 + \beta \|\mathbf{w}\|^2 = c \tag{6}$$

where β and c are positive constants and the solution is independent of c. The proposed algorithm can also be formulated as an constrained optimization problem as follows,

$$\min \mathbf{w}^H \mathbf{R}_\mathbf{z} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}_a \in K_r(\mathbf{R}_\mathbf{x}, \mathbf{r}_{\mathbf{x}d}), \quad w_1 = 1 \tag{7}$$

where $K_r(\mathbf{R}_\mathbf{x}, \mathbf{r}_{\mathbf{x}d}) = \mathrm{span}\left\{\mathbf{r}_{\mathbf{x}d}, \mathbf{R}_\mathbf{x}\mathbf{r}_{\mathbf{x}d}, \cdots, \mathbf{R}_\mathbf{x}^{r-1}\mathbf{r}_{\mathbf{x}d}\right\}$ is the r -dimensional Krylov subspace. Since the number of nulls in beam pattern is proportional to the degrees of freedom of the adaptive weight vector $\mathbf{w}_a$, constraining $\mathbf{w}_a$ in a low-dimensional subspace can effectively reduce the false peaks in the spatial spectrum. Moreover, the norm of $\mathbf{w}_a^{(r)}$ increases with the increase of rank r , the constraints in (7) imply that

$$|w_1|^2 + \|\mathbf{w}\|^2 < T_r = 2 + \|\mathbf{w}_a^{(r+1)}\|^2 \tag{8}$$

which has a similar form of the constraint in (6). The spatial spectrums of Capon, MNL and MWF estimators are plotted in Fig. 3. The number of snapshots is $K = 2M = 20$ and the input SNR at sensor level is 10 dB. We see from the figure that the proposed method achieves compromise between high-resolution and false peaks.

The MNL algorithm is the most efficient high resolution DOA estimator at present with a computational complexity of $O(M^2 + NM)$, where N is the total number of search directions. However, the costs of computing the covariance matrix $\mathbf{R}_\mathbf{z}$ and the parameter β are $O(MK^2)$ and $O(M^3)$, respectively[6]. Hence, the true complexity of the MNL estimator is $\max\left\{O(M^3), O(MK^2), O(NM)\right\}$. The cost of computing $\mathbf{w}_a$ by MWF is $O(rM^2)$ and the cost of computing the spatial spectrum is $\mathrm{O}(\mathrm{NM})$. Therefore, the total computational complexity of our algorithm is $\max\left\{O(rM^2), O(NM)\right\}$. Considering r is usually much smaller than M , our algorithm is more efficient than MNL and other algorithms.

4. Simulation results

A 10-element ULA is used in our simulations and 1000 random trials are run for each experiment. First let we examine the probabilities of resolution of different algorithms versus the SNR when $K = 10$ snapshots are available. Two uncorrelated narrowband signals impinge on the array from 72$^\mathrm{o}$ and78$^\mathrm{o}$, respectively. They are regarded to be resolved if $P(\theta)$ provides obvious peak at each direction θ_i ($i = 1, 2$) within the angular interval $[\theta_i - \Delta\theta, \theta_i + \Delta\theta]$ with $\Delta\theta = |\theta_1 - \theta_2|/2$. The results are shown in Fig. 4. It is seen from the figure that our algorithm has a higher probability of resolution than other algorithms except the ESTRIT with the NOS known. However, if AIC or MDL criterion is used to estimate the NOS, the performance of ESPRIT degrades substantially. This can be further explained in Fig. 5, which plots the estimation accuracy of the NOS by the criteria of AIC, MDL and MWF. Since the rank determined by RRP is equal to the number of incident waves, it can be used as an estimate of the NOS. The result indicates that the reduced-rank MWF achieves a reasonable compromise between AIC and MDL criteria when the snapshots number is large, while offering a significant improvement in estimation accuracy when the snapshots number is small.

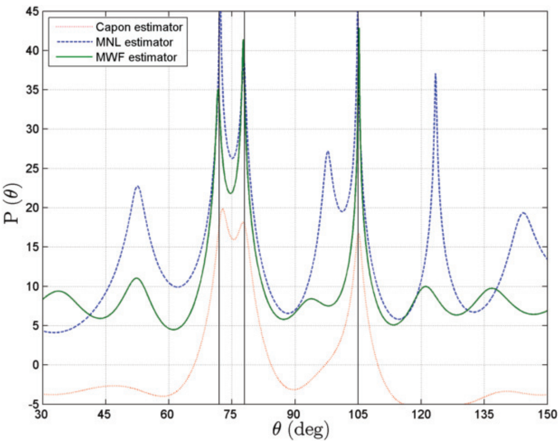

Fig. 3. Spatial spectrum of Capon, MNL and MWF.

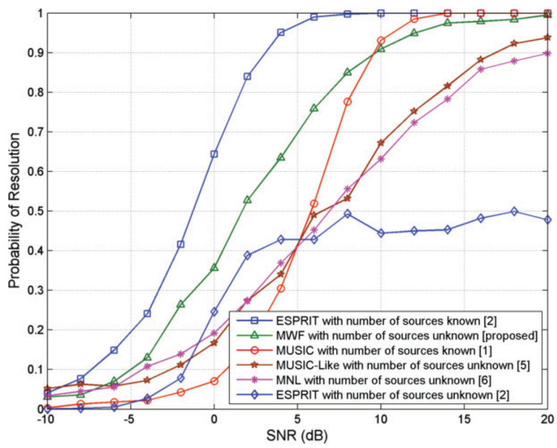

Fig. 4. Probability of resolution versus SNR for 10 snapshots.

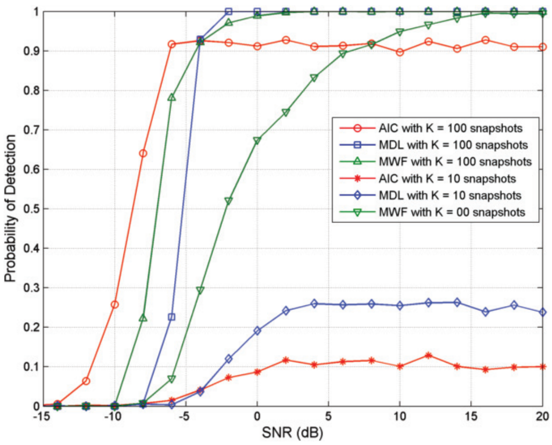

Fig. 5. Probability of detection of AIC, MDL and MWF.

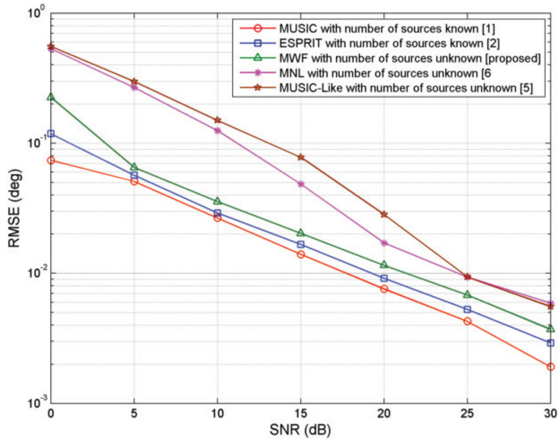

Fig. 6. Average RMSE versus SNR for 20 snapshots.

Next we consider the root-mean-squared errors (RMSE) of different algorithms versus the SNR when $K = 20$ snapshots are available. Three uncorrelated narrowband signals impinge on the array from 72^o, 78^o and 105^o, respectively. The results are shown in Fig. 6, where RMSE is normalized by the half-power beamwidth and averaged over the runs that the sources are successfully resolved. We know from the figure that the new algorithm has higher estimation accuracy than the MUSIC-Like and MNL algorithms with the NOS unknown, and a slightly worse accuracy than the MUSIC and ESPRIT algorithms with the NOS exactly known.

5. Conclusions

A new DOA estimator based on the reduced-rank multistage Wiener filter was proposed in this paper. This method does not require the information of the number of sources. It outperforms other algorithms when the sample size is small. In addition, it is much more efficient than the subspace based methods. Although we use the MWF to accomplish the reduced-rank processing in Krylov subspace, other techniques such as Lanczos iteration and conjugate gradient algorithm can also be used.

References

1. R. O. Schmidt. Multiple emitter location and signal parameter estimation. *IEEE Trans. Antennas Propag.*; 1986, vol. 34, p. 276-280.
2. R. Roy, T. Kailath. ESPRIT–estimation of signal parameters via rotational invariance techniques. *IEEE Trans. Acoust., Speech, Signal Process.*; 1989, vol. 37, p. 984-995.
3. H. L. Van Trees. *Optimum Array Processing*. New York: John Wiley & Sons, Inc., 2002.
4. J. Capon. High-resolution frequency-wavenumber spectrum analysis. *Proc. IEEE*; 1969. vol. 57, p. 1408–1418.
5. Y. Zhang and B. P. Ng. Music-like doa estimation without estimating the number of sources. *IEEE Trans. Signal Process.*; 2010, vol. 58, p. 1668-1676.
6. V. V. Reddy, M. Mubeen and B. P. Ng. Reduced-complexity super-resolution doa estimation with unknown number of sources. *IEEE Signal Process. Lett.*; 2015, vol. 22, p. 772-776.
7. J. S. Goldstein, I. S. Reed and L. L. Scharf. A multistage representation of the wiener filter based on orthogonal projections. *IEEE Trans. Inf. Theory*; 1998, vol. 44, p. 2943-2959.
8. M. Zhang, A. Zhang and Q. Yang. Robust adaptive beamforming based conjugate gradient algorithms. *IEEE Trans. Signal Process.*; 2016, vol. 64, p. 6046-6057.
9. L. L. Scharf et al. Subspace expansion and the equivalence of conjugate direction and multistage wiener filters. *IEEE Trans. Signal Process.*; 2008, vol. 56, p. 5013-5019.
10. M. Zhang, A. Zhang and J. Li. Fast and accurate rank selection methods for multistage wiener filter. *IEEE Trans. Signal Process.*; 2016, vol. 64, p. 973-984.