Signal Processing 157 (2019) 261–265

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Short communication

Non-coherent direction of arrival estimation utilizing linear model approximation

Ye Tian [a,b,*], Jiaxin Shi [a], Yanru Wang [a], Qiusheng Lian [a,b]

[a] School of Information Science and Engineering, Yanshan University, Qinhuangdao, Hebei 066004, China
[b] Hebei Key Laboratory of Information Transmission and Signal Processing, Qinhuangdao, Hebei 066004, China

ARTICLE INFO

Article history:
Received 28 March 2018
Revised 28 November 2018
Accepted 14 December 2018
Available online 15 December 2018

Keywords:
Direction of arrival
Non-coherent
Linear model approximation
Nested CADiS

ABSTRACT

This fast communication presents a non-coherent direction of arrival (DOA) estimation algorithm utilizing the elements squared of the array covariance. Very different from previous non-coherent DOA estimation algorithms, the nonlinear model produced by the squared operation is approximately cast as a linear model, under a high-power reference source. And then, the DOA estimation is efficiently obtained via the mature ℓ_1-norm minimization. The proposed algorithm can be applied to arbitrary arrays. Focused on the estimation accuracy, the nested coprime array with displaced subarrays (CADiS) is exploited. To resolve the ambiguity problems in the scenario that the sources impinging on the array from the range $[-90^\circ, 90^\circ]$, a simple but effective strategy is introduced. The proposed algorithm performs independent of the phase errors and can provide improved DOA estimation accuracy with reduced computational complexity in large snapshot case. Simulation results demonstrate the effectiveness and superiority of the proposed algorithm.

1. Introduction

In the last few decades, research on direction of arrival (DOA) estimation has received a widespread amount of attention, and many high-resolution algorithms (such as MUSIC and ESPRIT [1]) have been investigated for dealing with this issue. By assuming that ideal phase synchronization is available at the elements of the array, these kind of methods can provide excellent performance. However, the ideal condition is difficult to achieve in practical applications since phase uncertainties always exist. As a result, the estimator's performance could degrade substantially.

To cope with array phase errors, several DOA estimation methods that suitable for compensating phase errors have been proposed in recent years. By constructing a special second-order or fourth-order statistics matrix, the Hadamard Product based algorithms [2,3] are proposed, such methods belongs to subspace framework, and all involve two-dimensional spatial spectrum searches. Unlike subspace techniques, the emerging sparse reconstruction (SR) technique is also introduced for DOA estimation in the presence of phase errors. In [4,5], the two-step iterative SR based algorithm is addressed. By transforming a non-convex optimization problem into two convex optimization subproblems, the DOA and phase errors are jointly estimated. However, it requires enormous computation time, and is only limited to the case that the array phase errors are small. In 2015, a very different approach, termed as non-coherent DOA estimation [6], is proposed by Haley et al., where the DOA estimation problem is formulated as a nonlinear optimization in a SR framework, and is solved via a modified version of a phase retrieval algorithm (namely GESPAR [7]). A key advantage of non-coherent DOA estimation is that the performance is independent of sensor phase errors, whereas the cost is in that multiple reference sources are required to avoid ambiguities in multiple sources scenario and the global minima cannot be guaranteed.

In this paper, we present another non-coherent direction of arrival (DOA) estimation algorithm. Instead of using the magnitude squared of the measurements suggested in [6], the elements squared of the array covariance are adopted, which is more favorable for suppressing noise as well as reducing computational complexity. In order to simplify the solving process and also to improve the estimation accuracy, the nonlinear model produced by the squared operation is approximated by a linear model under a high-power reference source. Consequently, the DOAs are easily obtained via the ℓ_1-norm minimization. To further improve the DOA estimation performance, the recently proposed nested CADiS [8] is utilized. Meanwhile, we also develop an alternate work

* Corresponding author School of Information Science and Engineering, Yanshan University, Qinhuangdao, Hebei 066004, China.
E-mail address: tianye@ysu.edu.cn (Y. Tian).

https://doi.org/10.1016/j.sigpro.2018.12.009
0165-1684/

strategy for high-power reference sources to resolve the ambiguity problems. By conducting numerical simulations, we show the superiority of the proposed algorithm.

Notations: Capital (small case) bold letters are used to denote matrices (column vectors), vec(·), diag(·), $\mathrm{E}\{\cdot\}$, $|\cdot|^2$, $\|\cdot\|_2$ and $\|\cdot\|_1$ denote the vectorization, diagonalization, expectation, squared operation, ℓ_2-norm and ℓ_1-norm, respectively. $\otimes$ denotes the Kronecker product, the superscripts T, * and H imply the transpose, complex conjugate and conjugate transpose, respectively.

2. Problem formulation

We consider K sources from distinct angles $\theta_1, \ldots, \theta_K$ impinging on an array with L omni-directional sensors. These sensors are labeled by $1, 2, \ldots, L$, and the coordinate of sensor l is denoted by $(\bar{x}_l, \bar{y}_l)$. Take the array phase errors into account and let the first sensor as the phase reference point, the array output can be expressed as

$$\mathbf{x}(t) = \mathbf{\Phi}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{x}(t) = [x_1(t), \ldots, x_L(t)]^T$, $\mathbf{s}(t) = [s_1(t), \ldots, s_K(t)]^T$, $\mathbf{n}(t) = [n_1(t), \ldots, n_L(t)]^T$, $\mathbf{A} = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_K)]$, whose kth column is expressed as $\mathbf{a}(\theta_k) = [e^{-j2\pi d_{1,k}/\lambda}, \ldots, e^{-j2\pi d_{L,k}/\lambda}]^T$ with $d_{l,k} = \bar{x}_l \sin\theta_k + \bar{y}_l \cos\theta_k$ and λ representing the carrier wavelength. $\mathbf{\Phi} = \mathrm{diag}([e^{j\phi_1}, \ldots, e^{j\phi_L}]^T)$, where ϕ_l is the phase error of sensor l. Without loss of generality, we set that $\phi_1 = 0$.

Throughout the paper, the following four assumptions are required to hold.

- The source signals $\mathbf{s}(t)$ are zero-mean, stationary and uncorrelated;
- The sensor noises $\mathbf{n}(t)$ are zero-mean, white Gaussian, and statistically independent of $\mathbf{s}(t)$;
- The reference source has known DOA and is placed to one end of the visible region;
- The power of the reference source is far higher than the other sources and has been pre-measured without other sources impinging.

3. The proposed algorithm

3.1. Basic idea for DOA estimation

Based on the above assumptions, the array covariance matrix can be written as

$$\mathbf{R_0} = \mathrm{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{\Phi}\mathbf{A}\mathbf{R}_s\mathbf{A}^H\mathbf{\Phi}^H + \sigma_n^2\mathbf{I} \tag{2}$$

where $\mathbf{R}_s = \mathrm{E}\{\mathbf{s}(t)\mathbf{s}^H(t)\} = \mathrm{diag}([P_1, \ldots, P_K]^T)$, and P_k represents the power of kth signal, σ_n^2 is the noise power and $\mathbf{I}$ is an $L \times L$ identity matrix. In practice, we only obtain the estimated result $\hat{\mathbf{R}}_\mathbf{0}$ of $\mathbf{R_0}$ according to $\bar{T}$ snapshots, i.e., $\hat{\mathbf{R}}_\mathbf{0} = \bar{T}^{-1}\sum_{t=1}^{\bar{T}} \mathbf{x}(t)\mathbf{x}^H(t)$, and they may be approximately equal if $\bar{T}$ is adequate, otherwise (2) does not hold and the performance of the corresponding algorithm will deteriorate, which is a recognized fact in DOA estimation and has been adopted by a large number of literatures. In the following for simplicity we assume that $\bar{T}$ is adequate and omit the time variable.

Decomposing $\mathbf{R}_0$, we can get the estimate of σ_n^2 by averaging $L-K$ small eigenvalues. Consequently, by subtracting the noise term, we have

$$\mathbf{R}_1 = \mathbf{R_0} - \sigma_n^2\mathbf{I} = \mathbf{\Phi}\mathbf{A}\mathbf{R}_s\mathbf{A}^H\mathbf{\Phi}^H. \tag{3}$$

Taking the elements squared of the array covariance in (3), we obtain

$$|\mathbf{R}_1|^2 = |\mathbf{\Phi}\mathbf{A}\mathbf{R}_s\mathbf{A}^H\mathbf{\Phi}^H|^2 = |\mathbf{A}\mathbf{R}_s\mathbf{A}^H|^2. \tag{4}$$

Vectorizing $|\mathbf{R}_1|^2$ in (4) yields

$$\mathbf{r} = \mathrm{vec}(|\mathbf{R}_1|^2) = |\mathrm{vec}(\mathbf{A}\mathbf{R}_s\mathbf{A}^H)|^2 = \left|\sum_{k=1}^{K} \mathbf{b}(\theta_k)P_k\right|^2 = |\mathbf{B}\mathbf{p}|^2 \tag{5}$$

where $\mathbf{B} = [\mathbf{b}(\theta_1), \ldots, \mathbf{b}(\theta_K)]$, $\mathbf{b}(\theta_k) = \mathbf{a}^*(\theta_k) \otimes \mathbf{a}(\theta_k)$, and $\mathbf{p} = [P_1, \ldots, P_K]^T$.

Notice that $\mathbf{\Phi}$ is a diagonal matrix with elements $\{e^{j\phi_l}\}_{l=1}^{L}$, thus Eq. (4) holds. Based on the nonlinear model (5), the GESPAR algorithm in [7] can be directly applied for DOA estimation. However, the ambiguities associated with GESPAR are difficult to resolve in multiple sources scenario, and the global minima cannot be guaranteed. In view of this, a high-power reference source is utilized in this paper.

The mth element of $\mathbf{r}$ is given by

$$\begin{aligned} r_m &= \left|P_1 e^{-j2\pi z_{m,1}/\lambda} + \cdots + P_K e^{-j2\pi z_{m,K}/\lambda}\right|^2 \\ &= \left|\sum_{k=1}^{K} P_k e^{-j2\pi z_{m,k}/\lambda}\right|^2 \qquad m = 1, \ldots, L^2 \end{aligned} \tag{6}$$

where $z_{m,k} = -d_{l,k} + d_{q,k}$, $m = (l-1)L + q$, $1 \le l, q \le L$. Without loss of generality, we let the first source as the reference source, then r_m can be expanded as

$$\begin{aligned} r_m &= P_{ref}^2 + 2P_{ref}\sum_{k=2}^{K} P_k \cos[2\pi(z_{m,k} - z_{m,ref})/\lambda] \\ &\quad + \left|\sum_{k=2}^{K} P_k e^{-j2\pi z_{m,k}/\lambda}\right|^2. \end{aligned} \tag{7}$$

Since we assume that the power of the reference source is far higher than the other sources, or more precisely $P_{ref} \gg \sum_{k=2}^{K} P_k$. Define $\Delta_m = |\sum_{k=2}^{K} P_k e^{-j2\pi z_{m,k}/\lambda}|^2/(2P_{ref})$, r_m can be expressed as

$$r_m = P_{ref}^2 + 2P_{ref}\sum_{k=2}^{K} P_k \cos[2\pi(z_{m,k} - z_{m,ref})/\lambda] + 2P_{ref}\Delta_m \tag{8}$$

which directly leads to

$$\begin{aligned} \bar{r}_m &= (r_m - P_{ref}^2)/(2P_{ref}) = \sum_{k=2}^{K} P_k \cos[2\pi(z_{m,k} - z_{m,ref})/\lambda] + \Delta_m \\ &\approx \sum_{k=2}^{K} P_k \cos[2\pi(z_{m,k} - z_{m,ref})/\lambda]. \end{aligned} \tag{9}$$

The vector form is then given by

$$\bar{\mathbf{r}} = [\bar{r}_1, \ldots, \bar{r}_{L^2}]^T = \mathbf{C}\bar{\mathbf{p}} + \mathbf{\Delta} \approx \mathbf{C}\bar{\mathbf{p}} \tag{10}$$

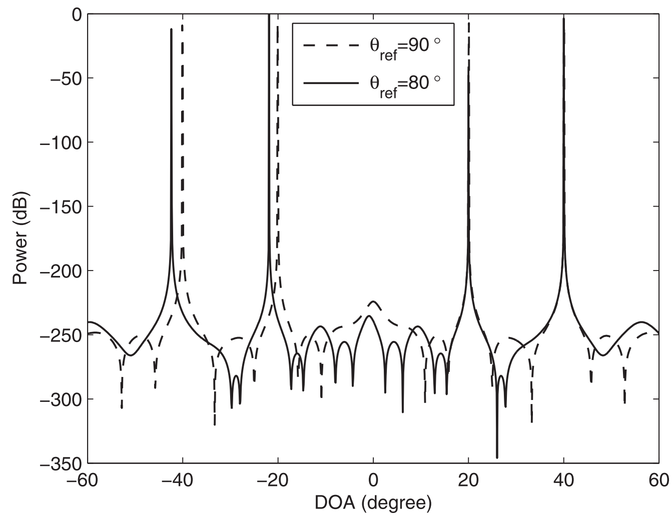
where $\mathbf{C} = [\mathbf{c}(\theta_1), \ldots, \mathbf{c}(\theta_K)]$, $\mathbf{c}(\theta_k) = [\cos 2\pi(z_{1,k} - z_{1,ref})/\lambda, \ldots, \cos 2\pi(z_{L^2,k} - z_{L^2,ref})/\lambda]^T$, $\bar{\mathbf{p}} = [P_2, \ldots, P_K]^T$, and $\mathbf{\Delta} = [\Delta_1, \ldots, \Delta_{L^2}]^T \to \mathbf{0}$. It can be easily observed that $\bar{\mathbf{r}}$ is approximately expressed as a linear model.

Based on the linear model $\bar{\mathbf{r}}$, the DOA estimation problem can be efficiently solved via the mature ℓ_1-norm minimization. Let $\mathbf{D}$ and $\mathbf{p}^\circ$ respectively denote the sensing matrix and sparse vector, which are defined over a finite grid $\bar{\theta}_1, \ldots, \bar{\theta}_Q$, $Q \gg K-1$. Consequently, the DOAs can be estimated by the following optimization problem

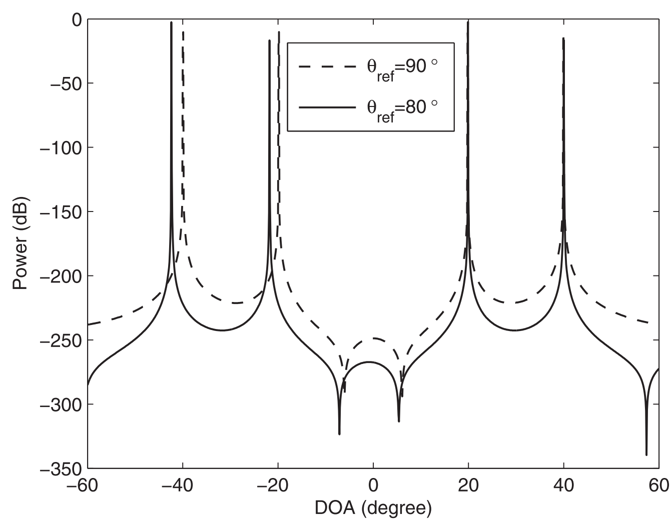
$$\hat{\mathbf{p}} = \arg\min_{\bar{\mathbf{p}}} \left[\frac{1}{2}\|\bar{\mathbf{r}} - \mathbf{D}\bar{\mathbf{p}}\|_2 + \beta\|\bar{\mathbf{p}}\|_1\right] \tag{11}$$

where β is a penalty parameter and can be selected properly via L-curve method [9]. By finding the indexes of nonzero coefficients in $\hat{\mathbf{p}}$, the DOAs are finally obtained.

According to the theorem proven by Donoho et al. [10], the estimation error of ℓ_1-norm minimization related to (11) can be



(a)



(b)

Fig. 1. The spectrum obtained with different reference sources. (a) Nested CADiS; (b) ULA.

bounded by

$$\|\hat{\mathbf{p}} - \bar{\mathbf{p}}\|_2^2 \le \frac{(\varepsilon + \eta)^2}{1 - \Gamma(4K - 5)} \tag{12}$$

where Γ denotes the mutual coherence of the sensing matrix $\mathbf{D}$, $\|\boldsymbol{\Delta}\|_2 \le \varepsilon$, $\|\bar{\mathbf{r}} - \mathbf{D}\bar{\mathbf{p}}\|_2 \le \eta$ and $\eta \ge \varepsilon$. Note that $\bar{\mathbf{r}}$ will be an approximately noise-free model, and η, ε will be small parameters, provided that the number of snapshots is adequate and the power of reference target is high enough. Therefore, one can conclude from the theorem in [10] that a good and stable DOA estimation can be effectively guaranteed via (11). In detailed implementations, $P_{ref} \ge 10K$ and $\bar{T} \ge 200$ are needed.

The linear model approximation strategy exploited in this paper is a generalized one, and can be applied to arbitrary arrays. Here, only the nested CADiS introduced by Qin et al. [8] is used since it is easy to construct and it is possible to improve the direction finding performance. According to the characteristics of the nested CADiS, $\bar{r}_m$ can be rewritten as

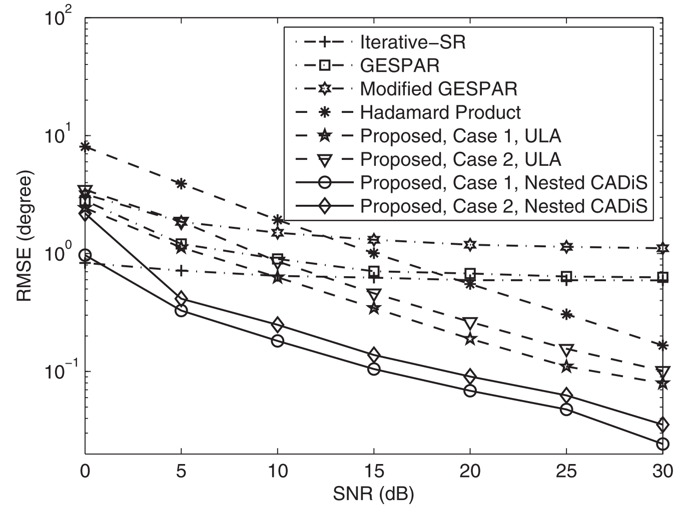


Fig. 2. RMSE of DOA estimates versus SNR.

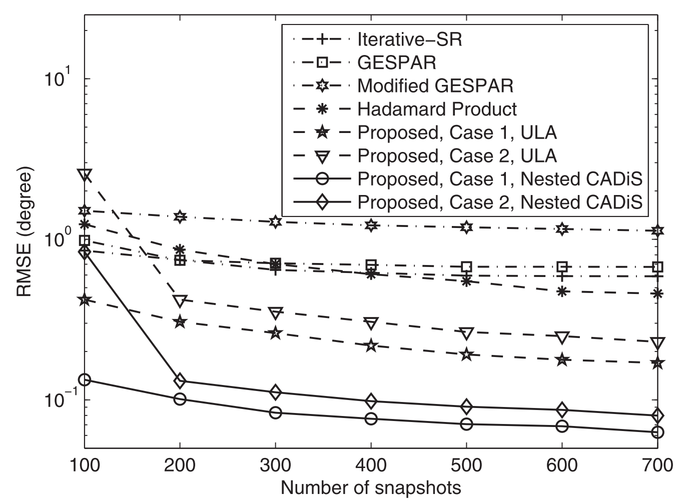


Fig. 3. RMSE of DOA estimates versus the number of snapshots.

$$\bar{r}_m \approx \sum_{k=2}^{K} P_k \cos z_m(\omega_k - \omega_{ref}), \tag{13}$$

where $z_m = -p_l + p_q,\ m = (l-1)L + q,\ 1 \le l,\ q \le L,\ \{p_1, p_2, \ldots, p_L\}$ are the multiple of fundamental inter-sensor spacing d of the nested CADiS, $\omega_k = 2\pi d \sin(\theta_k)/\lambda$, and $d \le \lambda/2$. As discussed in [8], the nested CADiS with $L = M + N - 1$ sensors can yield $2MN + 1$ consecutive lags with the range $[-MN, MN]$, which means that the number of degrees of freedom is greatly increased.

Remark 1. The robustness of the proposed strategy relies on the received reference power P_{ref}, which may embed a bias in practical applications, i.e., $\tilde{P}_{ref} = P_{ref} + \Delta_p$, and yields $\tilde{r}_m \approx \tilde{P}_{ref}^2 + 2\tilde{P}_{ref}\sum_{k=2}^{K} P_k \cos z_m(\omega_k - \omega_{ref})$. However, if Δ_p is not large or is comparable to Δ_m in size, we can easily obtain that $\bar{r}_m = (\tilde{r}_m - P_{ref}^2)/(2P_{ref}) \approx \sum_{k=2}^{K} P_k \cos z_m(\omega_k - \omega_{ref})$. This indicates that a good linear approximation model can also be obtained if Δ_p is restricted to a small range. A large number of simulations validate that $|\Delta_p| \le 2$ is required for $P_{ref} \ge 10K$ with $P_2 = \cdots = P_K = 1$.

Remark 2. The method in [6] and our algorithm all belong to non-coherent processing framework. The computational complexity of our algorithm mainly in covariance matrix construction and

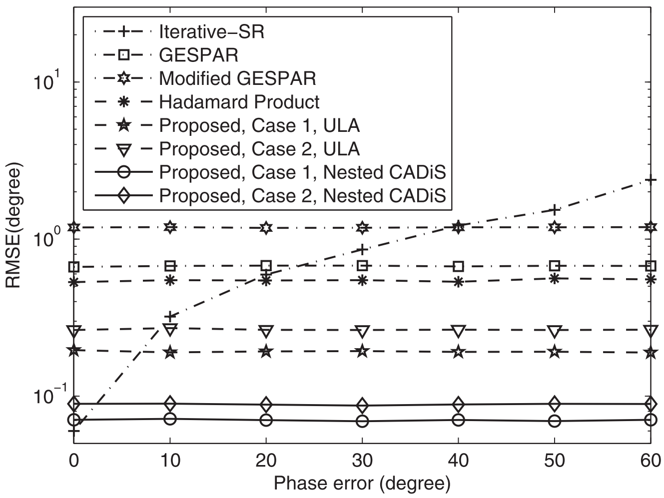


Fig. 4. RMSE of DOA estimates versus phase errors.

twice sparse recovery processes, which require $O(L^2\bar{T})+O(2G^3)$. As a comparison, the method in [6] requires $O(I_t LG\bar{T})$ for $\bar{T}$ snapshots and I_t iterations. Typically assuming $L \ll G$, $I_t \geq 100$, thus our algorithm is more suitable for large number of snapshot scenario. By conducting numerical simulations, we find that the average running time of the method in [6] is about $0.025\bar{T}$ seconds, with $G=180$ and $I_t=100$. Whereas the average running time of our algorithm takes about 0.5 seconds when $\bar{T} \leq 5000$, which indicates that our algorithm is computationally more efficient than the method in [6], provided that $\bar{T} > 20$.

3.2. Ability to avoid ambiguities

We assume that the reference source with high power is placed to one end of the visible region. To simplify the discussion on ambiguities, we ignore the influence of $\mathbf{\Delta}$, and let $\omega_{ref}=\pi$ with $\theta_{ref}=90^\circ$, $d=\lambda/2$. In this case, (13) becomes

$$\bar{r}_m = \sum_{k=2}^{K} -P_k \cos z_m(\omega_k). \tag{14}$$

which indicates that there will be no ambiguity problem if the DOAs of multiple sources are restrict to the range [−90°, 0] or [0, 90°]. However, this limited condition only satisfies a few applications. If the sources impinging on the array from [−90°, 90°], one can easily find that a phase mirroring ambiguity appears, since if $\omega_k = 2\pi d \sin(\theta_k)/\lambda$ is a solution to the DOA estimation problem, then so is $-\omega_k = 2\pi d \sin(-\theta_k)/\lambda$. The following proposition shows that if we let two high-power reference sources work in an alternate manner, the ambiguity problem can be well resolved.

Proposition 1. *Given an approximate linear model* $\bar{\mathbf{r}}$, *the DOAs of multiple sources in the presence of phase errors can be estimated unambiguously if two high-power reference sources work in an alternate manner and its known DOAs are set properly.*

Proof. Let a high-power reference source with known DOA ω_{ref1} work on a period of time of length $\bar{T}/2$. Influenced by the mirroring ambiguity problem, except for $\Delta\omega_1 = \omega_k - \omega_{ref1}$, $-\Delta\omega_1 = \omega_{ref1} - \omega_k \in [-2\pi, 2\pi]$ is also a solution. Although we assume that the reference source is placed to one end of the visible region, a false source related to $-\Delta\omega_1$ also appears, which satisfies

$$\omega_{ref1} - \omega_k = \omega'_k - \omega_{ref1} + 2k_1\pi, \quad \text{for } \forall z_m, \quad k_1 = -1, 0, 1 \tag{15}$$

where $\omega'_k = 2\pi d \sin(\theta'_k)/\lambda$, and θ'_k denotes the false DOA, $k_1 = 1$ with $\theta_{ref1} \to 90^\circ$, $k_1 = 0$ with $\theta_{ref1} \to 0^\circ$ and $k_1 = -1$ with $\theta_{ref1} \to -90^\circ$.

On another period of time of same length $\bar{T}/2$, we let the second high-power reference source with $\omega_{ref2}(\neq \omega_{ref1})$ work, whereas the first one stops, which directly results in

$$\omega_{ref2} - \omega_k = \omega''_k - \omega_{ref2} + 2k_2\pi, \quad \text{for } \forall z_m, \quad k_2 = -1, 0, 1. \tag{16}$$

Since $\omega_{ref2} \neq \omega_{ref1}$, we have

$$\omega'_k + 2k_1\pi \neq \omega''_k + 2k_2\pi. \tag{17}$$

If the DOAs of two reference sources are set at the same end of the visible region, inequality $\omega'_k \neq \omega''_k$ holds. That is, the DOAs correspond to the false sources are different by solving the optimization problem (11) twice with different reference sources, whereas the DOAs correspond to the true sources are same. As a final result, the ambiguity problem is resolved effectively. □

Remark 3. The number of required reference sources in [6] is increased linearly as the number of sources with unknown DOAs. As a comparison, at most two reference sources are required for our algorithm.

4. Simulations

In this section, the performance of the proposed algorithm is investigated, and compared with those of Hadamard Product based algorithm [2], iterative SR based algorithm [5] and modified GESPAR algorithm [6] with ULA, as well as GESPAR algorithm [7] with our model $\mathbf{r}$. For better demonstrating the effectiveness, both ULA and nested CADiS are utilized for the proposed algorithm. The number of sensors is set to be 6 with $M=4, N=3$ for nested CADiS, and $d=\lambda/2$. The power of reference source is 100, whereas the power of the other sources is 1. The signal-to-noise ratio (SNR) is defined as $\text{SNR} = -10\log_{10}(\sigma_n^2)$. In the following experiments, two sources located at $\theta_1 = 20^\circ, \theta_2 = 40^\circ$ are considered. For case 1, the DOAs are restrict to [0, 90°], a reference source with $\theta_{ref} = 90^\circ$ is used, whereas for case 2, the DOAs belong to $[-90^\circ, 90^\circ]$, two reference sources with $\theta_{ref1} = 90^\circ, \theta_{ref2} = 70^\circ$ work in alternate manner, and the effective number of snapshots for each reference source is reduced by half. The root mean square error (RMSE) of DOA estimation is obtained by 500 independent Monte-Carlo trials.

In the first experiment, we examine the ability of the proposed algorithm to avoid ambiguities. The SNR and snapshot number are set to be 10dB and 500, respectively. The phase errors are drawn from a uniform distribution in [0, 20°]. The spectrum obtained by the proposed algorithm is shown in Fig. 1, where we can easily distinguish that $\theta_1 = 20^\circ, \theta_2 = 40^\circ$ are the true sources.

In the second experiment, we evaluate the RMSE of DOA estimations in different SNRs and different number of snapshots. In Fig. 2, the total number of snapshots is fixed at 500, whereas the SNR varies from 0dB to 30dB. In Fig. 3, the SNR is set to be 20dB, whereas the number of snapshots varies from 100 to 700. The phase errors are same with the first experiment. We can see that our algorithm using nested CADiS performs much better than the compared algorithms. Even if the number of snapshots is reduced by half (i.e., case 2) and ULA is utilized, our algorithm also outperforms the other methods when SNR ≥ 10dB and snapshot number > 100.

In the last experiment, we show the effect of phase errors on the performance of DOA estimation. The simulation condition is same with the first experiment, except that the maximum value of phase errors varies from 10° to 70°. As can be seen in Fig. 4, the proposed algorithm performs independent of the phase errors and can provide improved DOA estimation accuracy.

5. Conclusion

In this paper, we present a new algorithm to solve non-coherent DOA estimation problem. Instead of using magnitude-only measurements, the elements squared of the array covariance are exploited. By transforming the nonlinear model to an approximate linear model under a high-power reference source, the DOAs of multiple sources are efficiently obtained. Both simulation results and theoretical analysis validate that it can performs better than the other-state-of-the-art algorithms without ambiguities.

Acknowledgement

This work was supported in part by the National Natural Science Foundation of China under Grants 61601398, 61772453, and in part by the National Natural Science Foundation of Hebei Province under Grant F2016203100.

References

[1] S. Pillai, Array Signal Processing, New York, NY, USA:, Springer, 1989.
[2] S. Cao, Z. Ye, D. Xu, X. Xu, A hadamard product based method for DOA estimation and gain-phase error calibration, IEEE Trans. Aero. Elec. Syst. 49 (2) (2013) 1224–1232.
[3] S. Cao, Z. Ye, N. Hu, X. Xu, DOA estimation based on fourth-order cumulants in the presence of sensor gain-phase errors, Signal Process. 93 (2013) 2581–2585.
[4] L. Zhao, H. Liu, Y. Li, Y. Zhou, DOA estimation under sensor gain and phase uncertainties, in: Proceedings of the International Conference on Information Fusion, 2015, pp. 209–213.
[5] H. Liu, L. Zhao, Y. Li, X. Jing, T.K. Truong, A sparse-based approach for DOA estimation and array calibration in uniform linear array, IEEE Sens. J. 16 (15) (2016) 6018–6027.
[6] H. Kim, A.M. Haimovich, Y.C. Eldar, Non-coherent direction of arrival estimation from magnitude-only measurements, IEEE Signal Process. Lett. 22 (7) (2015) 925–929.
[7] Y. Shechtman, A. Beck, Y.C. Eldar, GESPAR: efficient phase retrieval of sparse signals, IEEE Trans. Signal Process. 62 (4) (2014) 928–938.
[8] S. Qin, Y.D. Zhang, M.G. Amin, Generalized coprime array configurations for direction of arrival estimation, IEEE Trans. Signal Process. 63 (6) (2015) 1377–1390.
[9] P.C. Hansen, D.P. O'Leary, The use of the l-curve in the regularization of discrete ill-posed problems, SIAM J. Sci. Comput. 14 (6) (1993) 1487–1503.
[10] D.L. Donoho, M. Elad, V.N. Temlyakov, Stable recovery of sparse overcomplete representations in the presence of noise, IEEE Trans. Inf. Theory 52 (1) (2005) 6–18.