

Fast communication

An efficient DOA estimation method in multipath environment

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ABSTRACT

An efficient direction of arrival (DOA) estimation method is proposed with uniform linear array (ULA) in multipath environment. By applying a transformation, we can convert a constructed complex matrix to a real one, and then utilize the real matrix combined with our proposed criterion to estimate the DOAs of uncorrelated signals. Afterwards, the contributions of uncorrelated signals are eliminated, and then several new matrices without the information of uncorrelated signals are constructed to resolve the remaining coherent signals. The proposed estimation method overcomes the shortcomings of the existing methods and has satisfactory performance. Simulation results confirm the theoretical analysis and show the effectiveness of the proposed method.

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1. Introduction

Direction of arrival (DOA) estimation of multiple narrowband signals is an important problem in array signal processing including radar, sonar, radio astronomy, and mobile communications. Many prominent algorithms, such as multiple signal classification (MUSIC), estimation of signal parameters via rotation invariance techniques (ESPRIT), have been developed over the years [1,2]. However, those high-resolution methods will fail to work in real environments when signals are highly correlated or coherent due to multipath propagation.

In order to decorrelate the coherent signals, a technique referred to as spatial smoothing [3,4] has been proposed and studied. Their solutions are based on a preprocessing scheme that divides the total array into several overlapped subarrays. Then the average of the subarray covariance matrices is used to resolve the coherent signals combined with the high-resolution methods. However, the number of signals they resolved

cannot exceed the number of array sensors. A deflation approach is proposed in [5], but it cannot utilize all the constructed Toeplitz matrices, otherwise its performance will be highly degraded. In [6], a non-Toeplitz matrix is constructed to resolve more signals by exploiting the symmetric configuration of uniform linear array (ULA). However, the computational complexity is high when using all the constructed matrices, otherwise the performance may not be very well by using only one constructed matrix. Since [5,6] both use MUSIC to resolve the uncorrelated signals, their computational complexity of spatial spectrum calculation and peak searching is very high. Besides, in the case of noise and finite samples, they may get false DOA estimations caused by coherent signals [7].

Many other methods have also been developed for DOA estimation of coherent signals, such as the maximum likelihood (ML) method [8] and the subspace-based methods [9]. In [10], the method of root weighted subspace fitting (root-WSF) is proposed, and it can obtain the DOA estimates without a time-consuming parameter searching. In [11], the method based on higher-order cumulants can resolve more signals since it resolves each group of coherent signals separately. Unfortunately, the method requires large number of snapshots and suffers from burdensome computation. The idea of matrix pencil (MP) is utilized in [12]

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based on the spatial samples of the data. It can find DOA easily in the presence of coherent signals without spatial smoothing. However, the required SNR is too high to put the method into application.

In this letter, an efficient method for DOA estimation is proposed with ULA in multipath environment. Firstly, we convert a constructed complex matrix to a real matrix along with their eigenvectors by using a transformation. Then using the eigenvectors of the real matrix that correspond to signal subspace, we can resolve the uncorrelated signals with our proposed criterion. Afterwards, the uncorrelated signals are eliminated by utilizing the symmetric configuration of the ULA, and then several new matrices that without the information of uncorrelated signals are constructed to resolve the coherent signals. The proposed method can resolve signals with low computational complexity but high performance.

The notations that will be used in this letter are listed as follows. The notations $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^\dagger$, $E\{\cdot\}$, $\|\cdot\|$ and $|\cdot|$ denote conjugate, transpose, conjugate transpose, Moore–Penrose inverse, expectation, Frobenius norm and take absolute value, respectively. The notation $\text{diag}\{z_1, z_2\}$ represents a diagonal matrix with diagonal entries z_1, z_2 and $\text{blkdiag}\{\mathbf{Z}_1, \mathbf{Z}_2\}$ represents a block diagonal matrix with diagonal entries $\mathbf{Z}_1, \mathbf{Z}_2$. The notation $\mathbf{Z}(a:b, c:d)$ denotes the matrix constructed by the elements from a to b rows and c to d columns of \mathbf{Z} .

2. Problem formulation

Consider a ULA with $N = 2M - 1$ sensors and take the middle one as the reference. Let θ_k represent the DOA of an impinging signal, and then the steering vector is

$$\mathbf{a}(\theta_k) = [v_k^{1-M}, \dots, v_k^{-1}, 1, v_k^1, \dots, v_k^{M-1}]^T, \quad v_k = e^{j2\pi d \sin \theta_k / \lambda}, \quad (1)$$

where λ is the carrier wavelength of the signal, and d is the interspacing.

Suppose K narrowband signals with distinct DOAs impinging on the array. Without loss of generality, assume the first K_u signals are uncorrelated, and the signal that comes from direction θ_k corresponds to the propagation of the far-field source $s_k(t)$ with power σ_k^2 , $k = 1, \dots, K_u$. The rest are D groups of K_c coherent signals, which come from D statistically independent far-field sources $s_k(t)$ with power σ_k^2 , $k = K_u + 1, \dots, K_u + D$, and with P_k multipath signals for each far-field source ($K_c = K - K_u = \sum_{k=K_u+1}^{K_u+D} P_k$). In the k th coherent group, the signal that corresponds to the p th multipath propagation of the far-field source $s_k(t)$ comes from the direction θ_{kp} , $p = 1, \dots, P_k$, and the complex attenuation coefficient is α_{kp} . Assume the $K_u + D$ far-field sources are uncorrelated with each other, and then the uncorrelated signals and coherent signals in different groups will also be uncorrelated with each other. The $N \times 1$ array output vector $\mathbf{x}(t)$ is then given by

$$\begin{aligned} \mathbf{x}(t) &= \sum_{k=1}^{K_u} \mathbf{a}(\theta_k) s_k(t) + \sum_{k=K_u+1}^{K_u+D} \sum_{p=1}^{P_k} \mathbf{a}(\theta_{kp}) \alpha_{kp} s_k(t) + \mathbf{n}(t) \\ &= \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \end{aligned} \quad (2)$$

where $\mathbf{A} = [\mathbf{A}_u, \mathbf{A}_c \mathbf{\Gamma}]$, $\mathbf{A}_u = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_{K_u})]$, $\mathbf{A}_c = [\mathbf{A}_{c,K_u+1}, \dots, \mathbf{A}_{c,K_u+D}]$ with $\mathbf{A}_{c,k} = [\mathbf{a}(\theta_{k1}), \dots, \mathbf{a}(\theta_{kP_k})]$, $\mathbf{\Gamma} = \text{blkdiag}\{\alpha_{K_u+1}, \dots, \alpha_{K_u+D}\}$ with $\alpha_k = [\alpha_{k1}, \dots, \alpha_{kP_k}]^T$ containing attenuation information of the k th coherent group, $\mathbf{s}(t) = [\mathbf{s}_u^T(t), \mathbf{s}_c^T(t)]^T$, $\mathbf{s}_u(t) = [s_1(t), \dots, s_{K_u}(t)]^T$, $\mathbf{s}_c(t) = [s_{K_u+1}(t), \dots, s_{K_u+D}(t)]^T$, and $\mathbf{n}(t)$ is the $N \times 1$ noise vector with the power of each entry equals to σ_n^2 . Assume the entries of $\mathbf{s}(t)$ and $\mathbf{n}(t)$ are zero mean wide-sense stationary random processes, and the entries of $\mathbf{n}(t)$ are uncorrelated with each other and the signals, such as the white Gaussian noise. With $\mathbf{x}(t)$, we can obtain the array covariance matrix

$$\mathbf{R}_x = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma_n^2\mathbf{I}_N, \quad (3)$$

where $\mathbf{R}_s = \text{blkdiag}\{\mathbf{R}_u, \mathbf{R}_c\}$ is the far-field source covariance matrix with $\mathbf{R}_u = E\{\mathbf{s}_u(t)\mathbf{s}_u^H(t)\} = \text{diag}\{\sigma_1^2, \dots, \sigma_{K_u}^2\}$, $\mathbf{R}_c = E\{\mathbf{s}_c(t)\mathbf{s}_c^H(t)\} = \text{diag}\{\sigma_{K_u+1}^2, \dots, \sigma_{K_u+D}^2\}$, and \mathbf{I}_N represents an $N \times N$ identity matrix.

3. DOA estimation of the proposed method

3.1. DOA estimation of the uncorrelated signals

The methods for DOA estimation of uncorrelated signals in [5–7] are all MUSIC, which has some inherent shortcomings. In MUSIC, the spatial spectrum calculation needs about $[(N+1) \times (N - K_u - D) + 1] \times (180/\varepsilon + 1)$ complex multiplications, where ε denotes the scanning interval in $[-90^\circ, 90^\circ]$. Obviously, when ε is small, such as 0.1° , the computational complexity is very high. The method of MUSIC also needs peak searching, which further increases the algorithm complexity. Moreover, the contributions from a group of coherent signals may be sufficiently close to that of a single point signal. Then the coherent signals may cause false peaks in the MUSIC spectrum in the case of noise and finite samples [7], which result in the false DOA estimates of uncorrelated signals. Here we will propose an efficient and low complexity estimation method for uncorrelated signals. Define

$$\begin{aligned} \mathbf{B}_u &= \text{diag}\{v_1, v_2, \dots, v_{K_u}\}, \\ \mathbf{B}_c &= \text{blkdiag}\{\mathbf{B}_{c,K_u+1}, \mathbf{B}_{c,K_u+2}, \dots, \mathbf{B}_{c,K_u+D}\}, \end{aligned} \quad (4)$$

with $\mathbf{B}_{c,k} = \text{diag}\{v_{k1}, v_{k2}, \dots, v_{kP_k}\}$. Under the assumption in Section 2, we can estimate the noise term by averaging the $N - K_u - D$ smallest eigenvalues of \mathbf{R}_x . Then we can eliminate the noise term from the covariance matrix of \mathbf{R}_x and obtain the noise-free covariance matrix \mathbf{R}_x' . Afterwards, we construct a new matrix as [13]

$$\begin{aligned} \mathbf{R}_z &= \mathbf{R}_x' + \Pi_N \mathbf{R}_x'^* \Pi_N \\ &= [\mathbf{A}_u, \mathbf{A}_c \mathbf{\Gamma}, \Pi_N (\mathbf{A}_c \mathbf{\Gamma})^*] \text{blkdiag}\{2\mathbf{R}_u, \mathbf{R}_c, \mathbf{R}_c^*\} \\ &\quad \times [\mathbf{A}_u, \mathbf{A}_c \mathbf{\Gamma}, \Pi_N (\mathbf{A}_c \mathbf{\Gamma})^*]^H \\ &= [\mathbf{A}_u, \mathbf{A}_c \mathbf{C}] \text{blkdiag}\{2\mathbf{R}_u, \mathbf{R}_c, \mathbf{R}_c^*\} [\mathbf{A}_u, \mathbf{A}_c \mathbf{C}]^H, \end{aligned} \quad (5)$$

where Π_N is an $N \times N$ exchange matrix with ones on its anti-diagonal and zeros elsewhere, $\mathbf{C} \triangleq [\mathbf{\Gamma}, \mathbf{B}_c^{1-N} \mathbf{\Gamma}^*]$.

By performing eigen-decomposition, \mathbf{R}_z can be expressed as

$$\mathbf{R}_z = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H, \quad (6)$$

where $\mathbf{\Lambda}_s = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_{K_u+2D}\}$ with $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{K_u+2D}$ being the nonzero eigenvalues, and $\mathbf{U}_s = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{K_u+2D}]$ with the columns being the eigenvectors correspond to the eigenvalues. Because the column space of \mathbf{U}_s is equal to that of $[\mathbf{A}_u, \mathbf{A}_c \mathbf{C}]$, there is a $(K_u + 2D) \times (K_u + 2D)$ full rank matrix \mathbf{T} that makes $\mathbf{U}_s = [\mathbf{A}_u, \mathbf{A}_c \mathbf{C}] \mathbf{T}$.

Note that the above eigen-decomposition is not needed for our method, and it is only for a convenience description. Furthermore, we define a new matrix \mathbf{R}_T as

$$\mathbf{R}_T = \mathbf{Q}_N^H \mathbf{R}_z \mathbf{Q}_N, \quad (7)$$

where \mathbf{Q}_N is a sparse matrix as

$$\mathbf{Q}_{2n} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_n & j\mathbf{I}_n \\ \mathbf{I}_n & -j\mathbf{I}_n \end{bmatrix}$$

when $N = 2n$ is even, and

$$\mathbf{Q}_{2n+1} = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{I}_n & \mathbf{0} & j\mathbf{I}_n \\ \mathbf{0} & \sqrt{2} & \mathbf{0} \\ \mathbf{I}_n & \mathbf{0} & -j\mathbf{I}_n \end{bmatrix}$$

when $N = 2n + 1$ is odd.

Referring to Theorem 3 in [12], we can see that the matrix \mathbf{R}_T is a real matrix. With (6), the eigen-decomposition of \mathbf{R}_T can be expressed as

$$\mathbf{R}_T = \mathbf{E}_s \mathbf{\Lambda}_s \mathbf{E}_s^T, \quad (8)$$

and the matrix of eigenvectors $\mathbf{E}_s = \mathbf{Q}_N^H \mathbf{U}_s \mathbf{J}$ is also real, where \mathbf{J} is a full rank matrix and satisfies $\mathbf{J} \mathbf{\Lambda}_s \mathbf{J}^H = \mathbf{\Lambda}_s$. Afterwards, we define two real matrices as

$$\mathbf{G}_1 = \mathbf{Q}_{N-1}^H ([\mathbf{I}_{N-1}, \mathbf{0}_{(N-1) \times 1}] + [\mathbf{0}_{(N-1) \times 1}, \mathbf{I}_{N-1}]) \mathbf{Q}_N, \quad (9)$$

$$\mathbf{G}_2 = j\mathbf{Q}_{N-1}^H ([\mathbf{I}_{N-1}, \mathbf{0}_{(N-1) \times 1}] - [\mathbf{0}_{(N-1) \times 1}, \mathbf{I}_{N-1}]) \mathbf{Q}_N. \quad (10)$$

Using \mathbf{G}_1 and \mathbf{G}_2 , we further construct a matrix \mathbf{F} as

$$\mathbf{F} = (\mathbf{G}_1 \mathbf{E}_s)^* (\mathbf{G}_2 \mathbf{E}_s). \quad (11)$$

To make the following derivation clearer, we define \mathbf{U}_{s1} and \mathbf{U}_{s2} as the first $N-1$ rows and last $N-1$ rows of \mathbf{U}_s , \mathbf{A}_{u1} and \mathbf{A}_{c1} as the first $N-1$ rows of \mathbf{A}_u and \mathbf{A}_c , respectively. Obviously, $\mathbf{U}_{s1} = [\mathbf{A}_{u1}, \mathbf{A}_{c1} \mathbf{C}] \mathbf{T}$ and $\mathbf{U}_{s2} = [\mathbf{A}_{u1} \mathbf{B}_u, \mathbf{A}_{c1} \mathbf{B}_c \mathbf{C}] \mathbf{T}$.

Then we can rewrite \mathbf{F} as

$$\begin{aligned} \mathbf{F} &= (\mathbf{Q}_{N-1}^H ([\mathbf{I}_{N-1}, \mathbf{0}_{(N-1) \times 1}] + [\mathbf{0}_{(N-1) \times 1}, \mathbf{I}_{N-1}]) \mathbf{Q}_N \mathbf{Q}_N^H \mathbf{U}_s)^* \\ &\quad \times (j\mathbf{Q}_{N-1}^H ([\mathbf{I}_{N-1}, \mathbf{0}_{(N-1) \times 1}] - [\mathbf{0}_{(N-1) \times 1}, \mathbf{I}_{N-1}]) \mathbf{Q}_N \mathbf{Q}_N^H \mathbf{U}_s) \\ &= (\mathbf{Q}_{N-1}^H (\mathbf{U}_{s1} + \mathbf{U}_{s2}) \mathbf{J})^* (j\mathbf{Q}_{N-1}^H (\mathbf{U}_{s1} - \mathbf{U}_{s2}) \mathbf{J}) \\ &= (\mathbf{T})^{-1} (\mathbf{Q}_{N-1}^H ([\mathbf{A}_{u1}, \mathbf{A}_{c1} \mathbf{C}] + [\mathbf{A}_{u1} \mathbf{B}_u, \mathbf{A}_{c1} \mathbf{B}_c \mathbf{C}]))^* \\ &\quad \times (j\mathbf{Q}_{N-1}^H ([\mathbf{A}_{u1}, \mathbf{A}_{c1} \mathbf{C}] - [\mathbf{A}_{u1} \mathbf{B}_u, \mathbf{A}_{c1} \mathbf{B}_c \mathbf{C}])) (\mathbf{T}) \\ &= j(\mathbf{T})^{-1} (\mathbf{Q}_{N-1}^H [\mathbf{A}_{u1} \mathbf{\Psi}_u, \mathbf{A}_{c1} \mathbf{\Psi}_c \mathbf{C}])^* \\ &\quad \times (\mathbf{Q}_{N-1}^H [\mathbf{A}_{u1} \mathbf{\Psi}_u', \mathbf{A}_{c1} \mathbf{\Psi}_c' \mathbf{C}]) (\mathbf{T}), \end{aligned} \quad (12)$$

where $\mathbf{\Psi}_u = \mathbf{I}_{K_u} + \mathbf{B}_u$, $\mathbf{\Psi}_c = \mathbf{I}_{K_c} + \mathbf{B}_c$, $\mathbf{\Psi}_u' = \mathbf{I}_{K_u} - \mathbf{B}_u$, and $\mathbf{\Psi}_c' = \mathbf{I}_{K_c} - \mathbf{B}_c$. Moreover, \mathbf{F} can be further written as

$$\begin{aligned} \mathbf{F} &= j(\mathbf{T})^{-1} ([\mathbf{A}_{u1} \mathbf{\Psi}_u, \mathbf{A}_{c1} \mathbf{\Psi}_c \mathbf{C}]^H \mathbf{Q}_{N-1} \mathbf{Q}_{N-1}^H [\mathbf{A}_{u1} \mathbf{\Psi}_u, \mathbf{A}_{c1} \mathbf{\Psi}_c \mathbf{C}])^{-1} \\ &\quad \times [\mathbf{A}_{u1} \mathbf{\Psi}_u, \mathbf{A}_{c1} \mathbf{\Psi}_c \mathbf{C}]^H \mathbf{Q}_{N-1} \mathbf{Q}_{N-1}^H [\mathbf{A}_{u1} \mathbf{\Psi}_u', \mathbf{A}_{c1} \mathbf{\Psi}_c' \mathbf{C}] (\mathbf{T}) \\ &= j(\mathbf{T})^{-1} \begin{bmatrix} \mathbf{\Psi}_u^H \mathbf{A}_{u1}^H \mathbf{A}_{u1} \mathbf{\Psi}_u & \mathbf{\Psi}_u^H \mathbf{A}_{u1}^H \mathbf{A}_{c1} \mathbf{\Psi}_c \mathbf{C} \\ \mathbf{C}^H \mathbf{\Psi}_c^H \mathbf{A}_{c1}^H \mathbf{A}_{u1} \mathbf{\Psi}_u & \mathbf{C}^H \mathbf{\Psi}_c^H \mathbf{A}_{c1}^H \mathbf{A}_{c1} \mathbf{\Psi}_c \mathbf{C} \end{bmatrix}^{-1} \\ &\quad \times \begin{bmatrix} \mathbf{\Psi}_u^H \mathbf{A}_{u1}^H \mathbf{A}_{u1} \mathbf{\Psi}_u' & \mathbf{\Psi}_u^H \mathbf{A}_{u1}^H \mathbf{A}_{c1} \mathbf{\Psi}_c' \mathbf{C} \\ \mathbf{C}^H \mathbf{\Psi}_c^H \mathbf{A}_{c1}^H \mathbf{A}_{u1} \mathbf{\Psi}_u' & \mathbf{C}^H \mathbf{\Psi}_c^H \mathbf{A}_{c1}^H \mathbf{A}_{c1} \mathbf{\Psi}_c' \mathbf{C} \end{bmatrix} (\mathbf{T}) \\ &= (\mathbf{T})^{-1} \begin{bmatrix} j\mathbf{\Psi}_u' \mathbf{\Psi}_u^{-1} & \mathbf{V}_1 \\ \mathbf{0}_{2D \times K_u} & \mathbf{V}_2 \end{bmatrix} (\mathbf{T}), \end{aligned} \quad (13)$$

where

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = j \begin{bmatrix} \mathbf{\Psi}_u^H \mathbf{A}_{u1}^H \mathbf{A}_{u1} \mathbf{\Psi}_u & \mathbf{\Psi}_u^H \mathbf{A}_{u1}^H \mathbf{A}_{c1} \mathbf{\Psi}_c \mathbf{C} \\ \mathbf{C}^H \mathbf{\Psi}_c^H \mathbf{A}_{c1}^H \mathbf{A}_{u1} \mathbf{\Psi}_u & \mathbf{C}^H \mathbf{\Psi}_c^H \mathbf{A}_{c1}^H \mathbf{A}_{c1} \mathbf{\Psi}_c \mathbf{C} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{\Psi}_u^H \mathbf{A}_{u1}^H \mathbf{A}_{c1} \mathbf{\Psi}_c' \mathbf{C} \\ \mathbf{C}^H \mathbf{\Psi}_c^H \mathbf{A}_{c1}^H \mathbf{A}_{c1} \mathbf{\Psi}_c' \mathbf{C} \end{bmatrix}.$$

As can be seen that $j\mathbf{\Psi}_u' \mathbf{\Psi}_u^{-1}$ is a diagonal matrix, the k th element of which is $\tan(\pi d \sin \theta_k / \lambda)$. Then there are K_u eigenvalues of \mathbf{F} equal to $\tan(\pi d \sin \theta_1 / \lambda), \dots, \tan(\pi d \sin \theta_{K_u} / \lambda)$, respectively. By performing eigen-decomposition of \mathbf{F} , we can get the nonzero eigenvalues $\gamma_1, \gamma_2, \dots, \gamma_{K_u+2D}$. Thus, the estimation of θ_k can be obtained as follows:

$$\hat{\theta}_k = \arcsin \left\{ \frac{\lambda}{\pi d} \arctan \{\gamma_k\} \right\}, \quad k = 1, 2, \dots, K_u + 2D. \quad (14)$$

However, in addition to the DOAs of K_u uncorrelated signals, we can also obtain the DOAs of $2D$ false estimations which associated with the D groups of coherent signals. Here, we will propose a criterion to choose the correct DOAs of uncorrelated signals from the $K_u + 2D$ estimates.

By performing eigen-decomposition of \mathbf{R}_x , we can obtain a matrix \mathbf{U}_n , the columns of which are the eigenvectors correspond to the $N - K_u - D$ smallest eigenvalues and span the noise subspace. When θ is the DOA of uncorrelated signals, the steering vector $\mathbf{a}(\theta)$ is orthogonal to the column space of \mathbf{U}_n . Then we calculate the value of $\eta_k = \|\mathbf{a}^H(\hat{\theta}_k) \mathbf{U}_n\|$. When K_u and D are known, we can select the DOAs that with the K_u smallest values of η_k as the DOA estimates of uncorrelated signals.

From above, we can see that our method does not need spectrum calculation and peak searching, and only needs several matrix operations. Besides, the data used in our algorithm have been transformed into real number, and then the computational complexity of our method is significantly reduced. Compared with MUSIC, our method overcomes all its aforementioned shortcomings.

3.2. DOA estimation of the coherent signals

The decorrelation method for coherent signals in our letter also uses the symmetric configuration of the ULA as [5,6]. For \mathbf{R}_x , its (n, m) th entry $n, m = 1, 2, \dots, N$ can be

expressed as

$$\begin{aligned} \mathbf{R}_x(n, m) = & \sum_{k=1}^{K_u} (\sigma_k^2 v_k^{n-M}) v_k^{M-m} \\ & + \sum_{k=K_u+1}^{K_u+D} \sum_{p=1}^{P_k} \left(\sigma_k^2 \rho_{kp}^* \sum_{l=1}^{P_k} \rho_{kl} v_{kl}^{n-M} \right) v_{kp}^{M-m} \\ & + \sigma_n^2 \delta(n, m), \end{aligned} \quad (15)$$

where $\delta(\cdot)$ is the kronecker delta function. Afterwards, we define a matrix \mathbf{Z} with \mathbf{R}_x as

$$\mathbf{Z} = \mathbf{R}_x - \Pi_N \mathbf{R}_x^T \Pi_N. \quad (16)$$

Obviously, the information of uncorrelated signals and noise are eliminated with (16), and \mathbf{Z} only contains the information of coherent signals. Furthermore, the (n, m) th entry of \mathbf{Z} can be expressed as

$$Z(n, m) = \sum_{k=K_u+1}^{K_u+D} \sum_{p=1}^{P_k} \left(\sigma_k^2 \sum_{l=1}^{P_k} (\rho_{kp}^* \rho_{kl} - \rho_{kp} \rho_{kl}^*) v_{kl}^{n-M} \right) v_{kp}^{M-m}. \quad (17)$$

Afterwards, we use the n th row $n = 1, 2, \dots, N$ of \mathbf{Z} to construct the following Toeplitz matrix as

$$\begin{aligned} \mathbf{H}(n) = & [\mathbf{Z}^T(n, M : N), \mathbf{Z}^T(n, M-1 : N-1), \dots, \mathbf{Z}^T(n, 1 : M)]^T \\ = & \mathbf{A}_1 \text{diag}\{z_{K_u+1,1}^n, \dots, z_{K_u+1,P_{K_u+1}}^n, \dots, z_{K_u+D,P_{K_u+D}}^n\} \mathbf{A}_1^H, \end{aligned} \quad (18)$$

where $\mathbf{A}_1 = \mathbf{A}_c(M : N, 1 : K_c)$, $z_{kp}^n = \sigma_k^2 \sum_{l=1}^{P_k} (\rho_{kp}^* \rho_{kl} - \rho_{kp} \rho_{kl}^*) v_{kl}^{n-M}$, $k = K_u + 1, \dots, K_u + D$, $p = 1, \dots, P_k$. Then we can use the matrices constructed by all the rows of \mathbf{Z} to get the DOA estimation of coherent signals.

Unfortunately, the simple average of these matrices as that did in [5] may get poor performance, and some numerical examples will be given in Section 4. This is because that z_{kp}^n , $k = K_u + 1, \dots, K_u + D$, $p = 1, \dots, P_k$ have different phases, and then the addition of z_{kp}^n will cause the cancellation of themselves. In some cases, the cancellation will even cause the rank deficient of the final averaged matrix, which will result in one or more poor DOA estimations. Since the value of z_{kp}^n is related with the attenuation coefficients and we have no knowledge of them in advance, it is very difficult to choose some of the Toeplitz matrices to avoid the above degradation. Besides, it will inevitably lose some information of the received data by using part of these Toeplitz matrices. To solve this problem, we can take the following operations.

With eigen-decomposition of these Toeplitz matrices, we have

$$\mathbf{H}(n) \mathbf{U}_H(n) = \mathbf{U}_H(n) \mathbf{\Lambda}_H(n), \quad n = 1, 2, \dots, N, \quad (19)$$

where $\mathbf{U}_H(n)$ and $\mathbf{\Lambda}_H(n)$ are the corresponding matrices of eigenvectors and eigenvalues, respectively. By taking the absolute value of these diagonal matrices $\mathbf{\Lambda}_H(n)$, we can construct N new matrices as

$$\mathbf{H}'(n) = \mathbf{U}_H(n) |\mathbf{\Lambda}_H(n)| \mathbf{U}_H^H(n), \quad n = 1, 2, \dots, N. \quad (20)$$

Afterwards, we can use the average of $\mathbf{H}'(n)$, $n = 1, 2, \dots, N$ that $\bar{\mathbf{H}}(n) = \sum_{n=1}^N \mathbf{H}'(n)/N$ with MUSIC or ESPRIT to resolve the coherent signals.

3.3. Discussion of the number of the resolvable signals

In this section, the number of sensors required for DOA estimation will be analyzed. Consider K_u uncorrelated signals and D groups of K_c coherent signals impinging on the array as in Section 2, where $K_u + K_c = K$. To estimate and eliminate the noise term in Section 3.1, we need $N - 1 \geq K_u + D$. According to the construction of the matrix \mathbf{F} in (11), we need $N - 1 \geq K_u + 2D$. Then we need $N \geq K_u + 2D + 1$ sensors to estimate the DOAs of uncorrelated signals. Moreover, referring to the construction of the Toeplitz matrix $\mathbf{H}(n)$ in (18) of Section 3.2, we need $M - 1 \geq K_c$ i.e. $N \geq 2K_c + 1$ sensors to estimate the DOAs of coherent signals. Then the method we proposed requires at least $N = \max(K_u + 2D + 1, 2K_c + 1)$ sensors to resolve all the signals. From the above analysis, we can see that our method can resolve more signals even than the number of sensors.

4. Simulations

In this section, two simulations are provided to illustrate the effectiveness of our method. The number of snapshots is denoted by N_p , and \mathbf{R}_x is estimated through these snapshots as $\hat{\mathbf{R}}_x = \sum_{n=1}^{N_p} \mathbf{x}(n) \mathbf{x}^H(n)/N_p$. Assume all sources are of equal power σ_s^2 , and the SNR is defined as $10 \log(\sigma_s^2/\sigma_n^2)$. The estimation method for uncorrelated signals and method 1 for coherent signals in [5] are selected to be the comparative methods. For simplicity, we assume that the numbers of sources and signals are known, otherwise they can be estimated by the existing robust detection methods. When using MUSIC to resolve the uncorrelated signals in [5], the scanning is performed over $[-90^\circ, 90^\circ]$ with an interval of 0.1° . To reduce the computational complexity, we use ESPRIT to resolve the coherent signals in our method and method 1 in [5]. Define the root mean square error (RMSE) of the DOA estimates from 200 Monte Carlo trials as the performance index

$$\text{RMSE} = \sqrt{\sum_{n=1}^{200} \sum_{k=1}^{K_s} (\hat{\theta}_k(n) - \theta_k)^2 / (200K_s)}, \quad (21)$$

where $\hat{\theta}_k(n)$ is the estimate of θ_k for the n th Monte Carlo trial, and K_s is the number of all the uncorrelated or all the coherent signals.

In the first simulation, we consider four uncorrelated signals coming from $[-59^\circ, -31^\circ, 15^\circ, 28^\circ]$ and two groups of six coherent signals coming from $[-12^\circ, 13^\circ, 55^\circ]$ and $[-45^\circ, 5^\circ, 40^\circ]$. The complex attenuation coefficients of coherent signals are $[-0.3957 + 0.8083j, -0.5380 + 0.2656j, 0.9335 - 0.3585j]$ and $[0.6907 - 0.1138j, -0.5280 + 0.6010j, -0.5909 - 0.8068j]$, respectively. In this simulation, we use a 13-element ULA with half wavelength interspacing, and the matrices used for DOA estimation of coherent signals are averaged over $n = 7, 8$

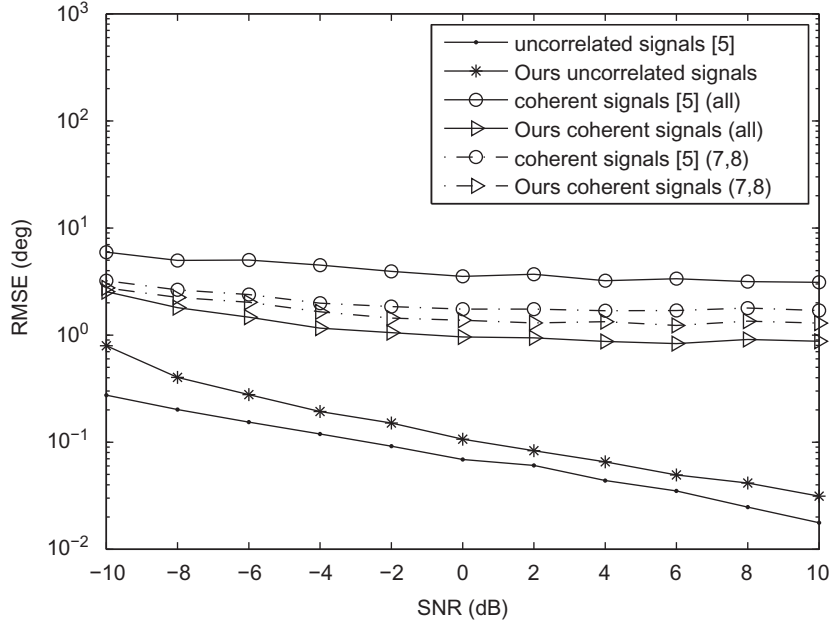


Fig. 1. RMSE of the DOA estimates versus SNR for uncorrelated and coherent signals, the number of snapshots is 500.

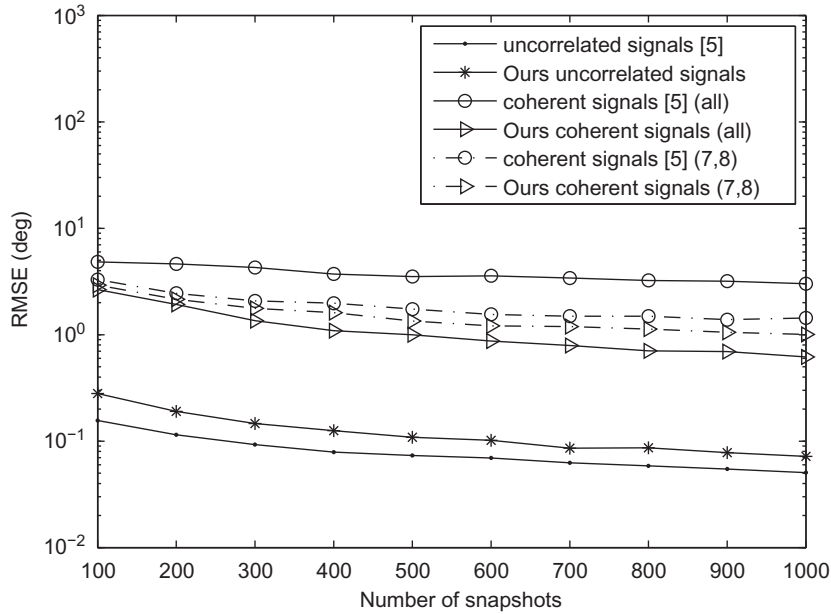


Fig. 2. RMSE of the DOA estimates versus number of snapshots for uncorrelated and coherent signals, the SNR is 0 dB.

and $n = 1, 2, \dots, 13$, respectively. The RMSE of the DOA estimates versus SNR is shown in Fig. 1 with $N_p = 500$, and the RMSE of the DOA estimates versus number of snapshots is shown in Fig. 2 with SNR = 0 dB. As shown in the figures, our method outperforms the method in [5] when average over the matrices $n = 7, 8$. Besides, we obtain the best performance by averaging over all the matrices, while the method in [5] gets the worst estimation. This is because that our method can effectively avoid the cancellation of different complex

values of z_{kp}^n , and then the addition of the new constructed matrices in our method will get more information without any other influence. Although our method has a little performance degradation for uncorrelated signals than MUSIC, the computational complexity of our method is largely reduced compared with MUSIC and our overall performance is satisfactory. Moreover, the two-step process of our method can be parallelized easily to further speed up the processing in practical application.

The second simulation considers six uncorrelated signals coming from $[-43^\circ, -21^\circ, -14^\circ, 5^\circ, 24^\circ, 47^\circ]$ and a group of four coherent signals coming from $[-49^\circ, -17^\circ, 13^\circ, 58^\circ]$. The complex attenuation coefficients of coherent signals are $[-0.5218 + 0.8531j, -0.0153 + 0.8999j, 0.4956 + 0.4944j, -0.3358 - 0.7261j]$. We use a nine-element ULA with half wavelength interspacing, and the matrices used for DOA estimation of coherent signals are averaged over $n = 5, 6$ and $n = 1, 2, \dots, 9$, respectively.

Note that the number of signals exceeds the number of sensors in this simulation. Referring to the analysis in Section 3.3, the number of signals in this simulation is the maximum number we can resolve for a nine-element ULA. Besides, the method in [5] also needs at least nine sensors to resolve these signals. The RMSE of the DOA estimates versus SNR is shown in Fig. 3 with $N_p = 500$, and the RMSE of the DOA estimates versus number of snapshots is shown in Fig. 4 with SNR = 0 dB. From the two figures, we

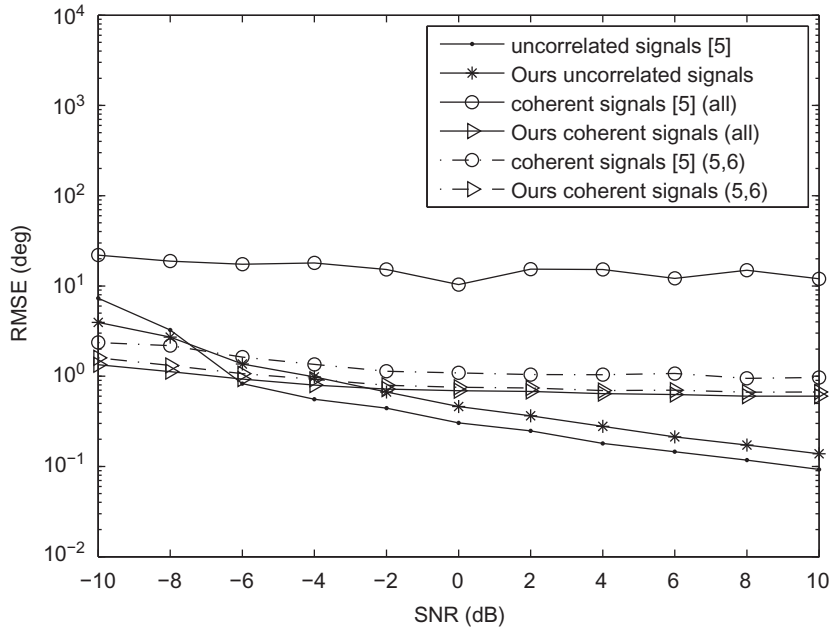


Fig. 3. RMSE of the DOA estimates versus SNR for uncorrelated and coherent signals, the number of snapshots is 500.

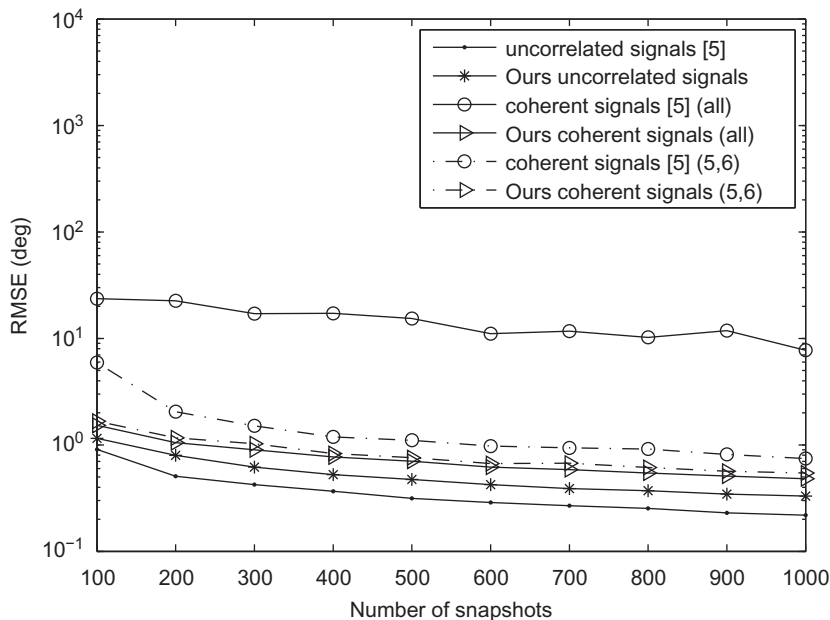


Fig. 4. RMSE of the DOA estimates versus number of snapshots for uncorrelated and coherent signals, the SNR is 0 dB.

can see that our method outperforms the method in [5] when average over the matrices $n = 5, 6$. Especially, the DOA estimation method for coherent signals in [5] fail to work when average over all the matrices (RMSE is greater than 10° in most cases) because the addition of z_{kp}^n causes the cancellation to a large extent. On the contrary, we can obtain a very accurate estimation by averaging over all the matrices. As can be seen from the simulation results, the performance of our proposed method is satisfactory when it processes the maximum number of resolvable signals with a given ULA, which further demonstrates the efficiency of our method.

5. Conclusion

In this letter, we present an efficient DOA estimation method in multipath environment. Since our DOA estimation of uncorrelated signals only needs several matrix operations and does not need spatial spectrum calculation and peak searching, the computational complexity is significantly reduced. Besides, a transformation is applied to convert the complex matrix to real matrix, and hence further reduce the complexity. When resolving the coherent signals, we overcome the shortcomings of [5], and can utilize more information of the received data. Simulation results validate the effectiveness of our method and show a satisfactory performance.

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