Signal Processing 93 (2013) 3423–3429

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Removing the outliers in root-MUSIC via pseudo-noise resampling and conventional beamformer

Volodymyr Vasylyshyn *

Kharkov Air Force University, 61023 Kharkov, Ukraine

ARTICLE INFO

Article history:
Received 3 August 2012
Received in revised form
22 May 2013
Accepted 24 May 2013
Available online 13 June 2013

Keywords:
DOA estimation
Pseudo-noise resampling
Beamformer
Root-MUSIC

ABSTRACT

A joint use of pseudo-noise resampling technique and a conventional beamformer is proposed to mitigate the effect of outliers in Root-MUSIC estimator. After resampling of Root-MUSIC via pseudo-randomly generated noise we combine it with a conventional beamformer (in the sense that identification of the signal roots is based on the power of the conventional beamformer response). The resulting estimator can be referred to as modified resampled Root-MUSIC. The estimator bank is formed from a number of modified resampled Root-MUSIC. Censored selection of the results of modified pseudo-randomly resampled Root-MUSIC estimators is exploited based on an appropriate local performance test.

1. Introduction

Antenna array signal processing is used in many fields to extract the desired information from the data received at an array of sensors. In many applications (radar, synthetic aperture radar, seismology, wireless communications), the estimation of the directions of arrival (DOAs) of signals is one of the most important tasks.

A large number of DOA estimators having different characteristics have been proposed. In many cases of practical interest with multiple sources, the eigenstructure estimators (such as Pisarenko estimator, MUSIC, Min-Norm, Root-MUSIC, Root-Min-Norm, ESPRIT) have demonstrated improved performance as compared with techniques such as conventional beamforming. Furthermore, the eigenstructure estimators offer significantly lower computational load as compared with ML-like estimators and constitute a very important class of DOA estimation estimators.

However, eigenstructure estimators have a strong performance breakdown as the number of snapshots or the signal-to-noise ratio (SNR) go down below a certain threshold. This phenomenon is known as the threshold effect and occurs as a result of outliers in DOA estimation problem [1–5].

Numerous authors have attempted to lower the signal-to-noise ratio (SNR) threshold of eigenstructure methods. We consider approaches that motivated by the success of modern resampling schemes (e.g., bootstrap and jackknife). These schemes based on initial data generate the artificial data that are useful for the cases of small signal-to-noise ratios or small number of the data samples.

One of recent approaches employs additional information obtained when several DOA estimation techniques forming the so-called estimator bank are computed and exploited simultaneously for the single batch of data (e.g., for a given sample covariance matrix). In [2], an efficient estimator bank was formulated, referred to as pseudorandom joint estimation strategy (PR-JES). Pseudorandomly weighted MUSIC estimators have been used as appropriate underlying techniques for PR-JES. In this case the resampling of the eigenvectors of a sampled covariance matrix is performed.

In [3,4] an approach has been proposed which exploits a resampling of initial data for a chosen eigenstructure estimator (MUSIC in [3] and Unitary ESPRIT in [4]) via pseudo-random noise to generate the underlying

* Tel.: +3809 8239 6165.
E-mail addresses: volodvas@rambler.ru, vladvas@ukr.net

0165-1684/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.sigpro.2013.05.026

estimators for the estimator bank. In a sense, a similar idea of bootstrapping with artificial noise was proposed in [5]. It was shown that the noise injection to the inputs of neural networks during training permits to produce a more independent set of estimators.

It is noted in [3,4], that the resampling itself cannot improve the DOA estimation performance due to adding pseudo-random noise to original data. However, the use of a local performance test, which tests the estimator behavior for some fixed data batch in the single trial (i.e. before statistical averaging) after each resampling to select the only outlier – free estimators from the whole number of resampled estimators, can bring performance improvement [3,4].

A likelihood ratio-based quality assessment method for detecting the outlying estimates in the estimator bank based direction finding methods has been introduced in [6]. The useful approach for the identifying and curing of the outlying estimates has been introduced in [7].

In this paper, which is based on the results of [8], a generalization of pseudo-noise approach [3] when applying it to Root-MUSIC is presented. We combine the two approaches for removing the outliers—the pseudo-noise approach and approach of [1]. In other words, we combine pseudo-randomly resampled Root-MUSIC and a conventional beamformer. The result of such combination is a modified resampled Root-MUSIC. The use of conventional beamformer makes the resampled Root-MUSIC robust against outliers associated with the false roots [1]. The proposed modification increases the accuracy of source DOA estimation in threshold region that is verified by computer simulations.

2. Signal model

Let a uniform linear array (ULA) be composed of M sensors and let it receive the signals from V far-field narrowband sources. It is assumed that signals and noise are stationary zero mean uncorrelated random processes and besides, the noises are spatially and temporally white. The $M \times 1$ vector of observation can be described by

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{x}(t)$ is the noisy observation vector, $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_V)]$ is the $M \times V$ matrix of source direction vectors

$$\mathbf{a}(\theta) = [1, \exp(j\omega), \ldots, \exp(j(M-1)\omega)]^T. \tag{2}$$

Here $\mathbf{a}(\theta)$ is the $M \times 1$ steering vector corresponding to the direction θ, $j = \sqrt{-1}$, $\Theta = [\theta_1, \ldots, \theta_V]^T$ is the $V \times 1$ vector of signal DOA's, $\mathbf{s}(t)$ is the $V \times 1$ signal vector, $\mathbf{n}(t)$ is the $M \times 1$ additive noise vector, $\omega = 2\pi d \sin\theta/\lambda$ is the phase shift (which specifies the source angle of arrival) between elements, d is the interelement spacing, λ is the wave length, $(\cdot)^T$ stands for transpose. The angles $[\theta_1, \ldots, \theta_V]$ are measured with respect to the normal of array axis. The first sensor of array is selected as the reference point.

The $M \times M$ data covariance matrix is given by

$$\mathbf{R} = E[\mathbf{x}(t)\,\mathbf{x}^H(t)] = \mathbf{A}(\boldsymbol{\theta})\,\mathbf{S}\,\mathbf{A}^H(\boldsymbol{\theta}) + \sigma^2\mathbf{I}, \tag{3}$$

where $\mathbf{S} = E[\mathbf{s}(t)\,\mathbf{s}^H(t)]$ is the $V \times V$ signal covariance matrix, σ^2 is the noise variance, $\mathbf{I}$ is the identity matrix, $E[\cdot]$ and $(\cdot)^H$ stand for expectation operator and Hermitian transpose, respectively. It is assumed that sources are uncorrelated and $\mathbf{S}$ has a full rank.

In practice, the sample covariance matrix, obtained from N snapshots

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{x}(t)\mathbf{x}^H(t) = \frac{1}{N}\mathbf{X}\mathbf{X}^H, \tag{4}$$

is used as an estimate of $\mathbf{R}$, and $\mathbf{X} = [\mathbf{x}(t_1), \ldots, \mathbf{x}(t_N)]$ is the $M \times$ N data matrix.

In order to estimate the DOAs by eigenstructure estimators it is necessary to compute the eigendecomposition (ED) of $\hat{\mathbf{R}}$

$$\hat{\mathbf{R}} = \hat{\mathbf{E}}_s\hat{\boldsymbol{\Lambda}}_s\hat{\mathbf{E}}_s^H + \hat{\mathbf{E}}_n\hat{\boldsymbol{\Lambda}}_n\hat{\mathbf{E}}_n^H, \tag{5}$$

where $\hat{V} \times \hat{V}$ and $(M-\hat{V}) \times (M-\hat{V})$ diagonal matrices $\hat{\boldsymbol{\Lambda}}_s$ and $\hat{\boldsymbol{\Lambda}}_n$ contain $\hat{V}$ and $M-\hat{V}$ signal- and noise- subspace eigenvalues. In their turn, the columns of $M \times \hat{V}$ and $M \times (M-\hat{V})$ matrices $\hat{\mathbf{E}}_s$ and $\hat{\mathbf{E}}_n$ contain the signal and noise eigenvectors. $\hat{V}$ is any consistent estimate of the number of sources [9].

3. Joint application of pseudo-noise resampling and the conventional beamformer

The idea of the pseudo-noise resampling technique is motivated by the success of modern resampling schemes (e.g., bootstrap and jackknife [10]) in signal processing [11]. When this technique is applied to Root-MUSIC, its aim is to mitigate the effect of outliers and to recover the Root-MUSIC performance using artificial data obtained by pseudo-noise resampling. The same data snapshots that have already been used in Root-MUSIC are used for pseudo-noise resampling.

Root-MUSIC is a variation of MUSIC estimator based on a polynomial formulation facilitated by the ULA structure and estimates the DOAs via rooting the polynomial

$$P_{rm}(z) = \mathbf{a}^T(z^{-1})\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^H\mathbf{a}(z), \tag{6}$$

where $\mathbf{a}(z) = [1, z, \ldots, z^{M-1}]^T$, $z = \exp(j\omega)$. Note, that $P_{rm}(z)$ is a polynomial of degree $2(M-1)$, whose roots occur in mirrored pairs with respect to unit circle. After selecting $M-1$ roots lying inside the unit circle we can obtain the DOA estimates from the phases of V roots, which have the largest magnitude $(z_v, v = 1, \ldots, V)$

$$\hat{\theta}_v = \arcsin((\lambda/2\pi d)\arg(z_v)). \tag{7}$$

In the case of bootstrap resampling scheme the bootstrap samples are generated from the original data set. Each bootstrap sample has ζ elements, generated by sampling with replacement ζ times from the original data set [10,11]. Let us consider the details of the pseudo-noise resampling and assume that K resampling runs are performed for the obtained data batch (sequence of snapshots).

The idea of pseudo-noise resampling scheme is to perturb the original measured data matrix $\mathbf{X} = [\mathbf{x}(t_1), \ldots, \mathbf{x}(t_N)]$ by means of artificially generated pseudo-random noise [3,4]

$$\mathbf{Y}_i = \mathbf{X} + \tilde{\mathbf{Z}}_i, \tag{8}$$

where $\mathbf{Y}_i = [\mathbf{y}_i(t_1), \ldots, \mathbf{y}_i(t_N)]$ is the $M \times N$ resampled data matrix, $i = 1, \ldots, K$. $\tilde{\mathbf{Z}}_i$ is the matrix of independent zero-mean circular pseudo-noise obtained by a Gaussian random generator such that [3]

$$E(\tilde{\mathbf{Z}}_i) = 0,\ E(\tilde{\mathbf{Z}}_i \tilde{\mathbf{Z}}_i^H) = \sigma^2_{\tilde{Z}_i} N\mathbf{I},\ E(\tilde{\mathbf{Z}}_i \tilde{\mathbf{Z}}_i^T) = 0 \tag{9}$$

Each new matrix $\tilde{\mathbf{Z}}_i$ (and resampled data matrix $\mathbf{Y}_i$, respectively) allows to realize the corresponding resampled DOA estimator. In some resampling runs, the pseudo-noise $\tilde{\mathbf{Z}}_i$ can permute the original noise $\mathbf{N} = [\mathbf{n}(t_1), \ldots, \mathbf{n}(t_N)]$ in a favorable way for the exploited DOA estimator. Application of some local performance test [3] allows to select these successful resampling runs (or successfully resampled estimators in such runs) and improve the estimation performance.

The variance of the pseudo-noise $\sigma^2_{\tilde{Z}_i}$ should be approximately the same as the variance of the original noise σ^2 [12]. Such choice enables maintaining an acceptable signal-to-noise ratio (SNR) in the synthetic (resampled) data. According to [3] the variance of pseudo noise can be determined as

$$\sigma^2_{\tilde{Z}_i} = p\,\sigma^2, \tag{10}$$

where $p \sim 1$ is the parameter chosen by user. It was shown in [3] that $p = 0.3$ is the nearly optimal value of the pseudo-noise parameter p. The consistent estimate of σ^2 can be obtained as in [13].

In each resampling run after perturbation (8) of the measured data matrix by means of artificially generated pseudo-random noise, it is necessary to perform steps similar to those of non-resampled modified Root-MUSIC (form the covariance matrix of $\mathbf{Y}_i$, perform the eigendecomposition of the covariance matrix). Let

$$\mathbf{R}_{Y_i} = E[\mathbf{y}_i(t)\,\mathbf{y}_i^H(t)] = \mathbf{R} + \sigma^2_{\tilde{Z}_i}\mathbf{I} \tag{11}$$

be the covariance matrix and

$$\hat{\mathbf{R}}_{Y_i} = \frac{1}{N}\mathbf{Y}_i \mathbf{Y}_i^H \tag{12}$$

be the sample estimate of a $\mathbf{R}_{Yi}$. Eigendecomposition of (12) can be presented in a form similar to (5)

$$\hat{\mathbf{R}}_{Y_i} = \hat{\mathbf{U}}_{s,i}\hat{\mathbf{\Phi}}_{s,i}\hat{\mathbf{U}}_{s,i}^H + \hat{\mathbf{U}}_{n,i}\hat{\mathbf{\Phi}}_{n,i}\hat{\mathbf{U}}_{n,i}^H. \tag{13}$$

After K resampling runs the K different eigenvalue and eigenvector sets are available. These sets can be used for the DOA estimation and detection. The ith eigenvalue set consists of diagonal elements of the matrices $\hat{\mathbf{\Phi}}_{s,i}$ and $\hat{\mathbf{\Phi}}_{n,i}$.

Resampled Root-MUSIC corresponding to ith resampling is given by

$$P_{rrm,i}(z) = \mathbf{a}^T(z^{-1})\hat{\mathbf{U}}_{n,i}\hat{\mathbf{U}}_{n,i}^H\mathbf{a}(z). \tag{14}$$

Here the term "resampled" in the name of the method means that it is based on the eigenstructure of the covariance matrix formed from the data resampled by pseudonoise.

The modified resampled Root-MUSIC estimators $(i = 1, \ldots, K)$ are the combinations of resampled Root-MUSIC estimators $(i = 1, \ldots, K)$ with the robust conventional beamformer.

In the modified Root-MUSIC [1] the identification of the signal roots is based on the power of conventional beamformer response toward the candidate directions ($M-1$ directions, associated with $M-1$ roots of Root-MUSIC lying inside the unit circle). The conventional beamformer can be expressed as

$$P_{BF}(\theta) = \mathbf{a}^H(\theta)\mathbf{Q}\hat{\mathbf{R}}\mathbf{Q}\mathbf{a}(\theta), \tag{15}$$

where $\mathbf{Q}$ is an arbitrary diagonal weighting matrix ($\mathbf{Q} = \mathbf{I}$ corresponds to the Bartlett beamformer, and appropriately designed $\mathbf{Q}$ corresponds to the low-sidelobe Dolph–Chebyshev beamformer [1]). Similar to the modified Root-MUSIC [1] the modified resampled Root-MUSIC includes the following steps:

Step 1. Calculate the ith resampled Root-MUSIC polynomial (14). Find $M-1$ candidate roots and $M-1$ candidate angles $\theta_{cand,\ 1}, \ldots, \theta_{cand,\ M-1}$ associated with these roots, $\hat{\theta}_{\mathrm{cand,\ m}} = \arcsin((\lambda/2\pi d)\arg(z_{\mathrm{m}}))$, $m = 1, \ldots, M-1$.
Step 2. Use the angles $\theta_{\mathrm{cand,\ m}}$, $m = 1, \ldots, M-1$ and (15) to obtain the set of beamformer outputs $P_{BF}(\theta_{\mathrm{cand,\ m}})$, $m = 1, \ldots, M-1$. Find the V maximal elements from this set and obtain the signal DOA estimates $\hat{\theta}_1, \ldots, \hat{\theta}_V$ as the angles associated with these elements.

Fig. 1 shows the conventional beamformer output and roots radii of Root-MUSIC and resampled Root-MUSIC for the case when $M = 8$ and there are two equal-power sources with angular coordinates $\theta_1 = 20^\circ$ and $\theta_2 = 26^\circ$, $SNR = -8dB$. It can be seen from Fig. 1 that in the case of successfully resampled Root-MUSIC, the use of the beamformer function identifies $V = 2$ signal roots and obtains the signal DOA estimates $\hat{\theta}_1 = 22.55^\circ$, $\hat{\theta}_2 = 27.2^\circ$. The estimates of Root-MUSIC and modified Root-MUSIC in this case are $\hat{\theta}_1 = 22.54^\circ$, $\hat{\theta}_2 = 43.2^\circ$. Therefore, the joint utilization of pseudo-noise resampling and the conventional beamformer improves the performance of Root-MUSIC estimator.

Furthermore, the obtained improvement in performance can be explained in the following way. It is known that the

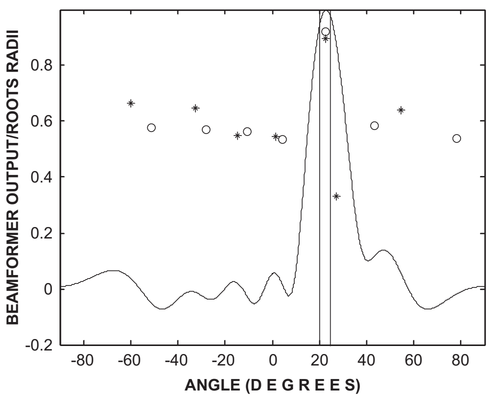

Fig. 1. The output of the conventional beamformer ($\mathbf{Q} = \mathbf{I}$), the roots radii of the Root-MUSIC (°) and those of resampled Root-MUSIC (*). Scenario with $M = 8$ and two equal-power sources impinging from $\theta_1 = 20^\circ$ and $\theta_2 = 26^\circ$, SNR $= -8$ dB.

non-outlying DOA estimates tend to be close to each other and to form a cluster [7]. The outlying DOA estimates are usually located far away from such a cluster. This fact, for example, is used in [7] to compute the final DOA estimate of the estimator bank. In this paper we find a cluster with signal DOA estimates (i.e. a cluster with non-outlying DOA estimates) with the help of the conventional beamformer. In the case of Fig. 1 the estimates $\hat{\theta}_1 = 22.55^\circ$ and $\hat{\theta}_2 = 27.2^\circ$ form such cluster.

Let us denote the modified resampled Root-MUSIC estimators as $P_{mrrm,i}$. Given the matrix $\mathbf{X}$ we form the estimator bank from the $i = 1, \ldots, K$ resampled estimators

$$B = \{P_{mrrm,i}, i = 1, \ldots, K\}_{\mathbf{X}}, \tag{16}$$

where K is the dimension of estimator bank. The PR-JES [2] technique combines the results of multiple "parallel" underlying estimators from the estimator bank via the so-called censored averaging. It is based on the local performance test which detects the failures in underlying estimators, remove the outliers and group the "successful" estimators. For the estimators based on the polynomial rooting the local performance test can be formulated in the following way:

The estimator obtains not less than $\hat{V}$ DOA estimates associated with the corresponding signal roots and localized in the sectors of the source localization $\hat{\theta}_c$.

The proposed generalized pseudo-noise resampling technique includes the following steps:

Step 1. Estimate the number of sources V using one of known methods [9].

Step 2. Specify the sectors of source localization as F nonoverlapping intervals (for example, using (15))

$$\hat{\theta}_c = [\theta_{1L},\ \theta_{1R}] \cup [\theta_{2L},\ \theta_{2R}] \cup \ldots \cup\ [\theta_{FL},\ \theta_{FR}], \tag{17}$$

where θ_{fL}, θ_{fR}, $f = 1, \ldots, F$ are the left and right boundaries of each subinterval, respectively. Here F is the number of most significant peaks of the conventional beamformer output [2].

Step 3. Calculate the Root-MUSIC estimator and test it using the local performance test. If Root-MUSIC satisfies the local performance test, then the DOA estimates of this estimator are taken as the resultant DOA estimates. Terminate the algorithm (i.e. go to step 7). If Root-MUSIC does not satisfy the local performance test, then go to the next step.

Step 4. Estimate the variance of noise σ^2 according to [13] and define the variance of pseudo-noise $\sigma^2_{\tilde{z}_i}$ from Eq.(10).

Step 5. Generate K resampled estimators (14) by means of repeating K resampling runs with $\sigma^2_{\tilde{z}_i}$ (by means of adding the new matrix $\tilde{\mathbf{Z}}_i$ to the fixed $\mathbf{X}$ on the ith resampling run). Combine each of K resampled estimators with the conventional beamformer (generate K modified resampled Root-MUSIC estimators $P_{mrrm,i}$). The estimator bank (16) is completed.

Step 6. Test each of $P_{mrrm,i}$, $i = 1, \ldots, K$, using the local performance test. If any Q_s $(0 < Q_s \le K)$ estimators $\tilde{P}_{mrrm,i}$, $i = 1, \ldots, Q_s$, from the total number of K estimators satisfy the local performance test, then estimate the v-th DOA as

$$\hat{\theta}_v = med\{\tilde{\hat{\theta}}_v^{(1)}, \tilde{\hat{\theta}}_v^{(2)}, \cdots, \tilde{\hat{\theta}}_v^{(Q_s)}\}, v = 1, \ldots, \hat{V}, \tag{18}$$

where $\tilde{\hat{\theta}}_v^{(1)} < \tilde{\hat{\theta}}_v^{(2)} < \cdots < \tilde{\hat{\theta}}_v^{(Q_s)}$ is the ordered set of "preliminary" DOA estimates of v-th source ($\tilde{\hat{\theta}}_v^{(i)}$ is the estimate of v-th source obtained at step 2 of the corresponding modified resampled Root-MUSIC estimator), med$\{\mathbf{c}\}$ is the median value of the elements in vector $\mathbf{c}$,

$$med\{c_1, \ldots, c_h\} = \begin{cases} (\xi_{h/2} + \xi_{h/2+1})/2 & \text{if } h \text{ is even} \\ \xi_{(h+1)/2} & \text{if } h \text{ is odd} \end{cases} \tag{19}$$

where {ξ1,…,ξh}=sort {c1,…,ch}, and sort {•} denotes the operator of sorting in ascending (descending) order. If all members of estimator bank (16) do not satisfy the local performance test then estimate the v-th DOA as

$$\hat{\theta}_v = med\{\hat{\theta}_v^{(1)}, \hat{\theta}_v^{(2)}, \ \ldots, \hat{\theta}_v^{(K)}\}, v = 1, \ldots, \hat{V}, \tag{20}$$

where $\hat{\theta}_v^{(1)} < \hat{\theta}_v^{(2)} < \cdots < \hat{\theta}_v^{(K)}$ is the ordered set of "preliminary" DOA estimates and each of them is associated with the v-th closest to the unit circle root selected from the $M-1$ roots of corresponding polynomial (angles associated with these $M-1$ roots are localized in the whole array field of view $[-90^\circ, 90^\circ]$).

Step 7. Stop.

It should be noted that usual averaging of the DOA estimates instead of median value can be used in (18) and (20). However, it is known that the median is a robust estimate of the mean. Therefore, the median filtering is expected to additionally reduce the undesirable effect of outliers [2].

Step 3 guarantees that the asymptotic behavior of proposed approach will not be worse than that of Root-MUSIC. If local performance test is accepted for Root-MUSIC, then no further processing is necessary.

Eq. (18) corresponds to the so-called censored averaging [2], whereas (20) corresponds to the case when all estimators are "unsuccessful", and, therefore, there is no reason to prefer one estimator to another.

The sector estimation problem is discussed in detail in [2] and it is shown that the moderate errors when estimating the θ_c do not result in significant changes of PR-JES performance compared with the case with known localization sectors. This fact is used in simulation (the sectors are assumed to be known). The sources can be unresolved within each sector at the moment of sectors estimation.

It is obvious that the computational load of the proposed estimator bank is higher than that of the conventional Root-MUSIC. However, if the dimension of the estimator bank is not very high ($K = 10$–100) this cost is significantly less than that of the stochastic or deterministic ML techniques. When increasing K we can expect the monotonous improvement of the SNR threshold. Furthermore, in order to reduce the computational cost of the proposed approach, the singular value decomposition of the data matrix $\mathbf{Y}_i$ can be performed instead of eigenvalue decomposition of $\hat{\mathbf{R}}_{Y_i}$ and multiphase systolic algorithms can be used to solve spectral decomposition problem [14].

The available K eigenvalue sets obtained with the help of pseudo-noise resampling can be used to realize a detection procedure similar to the procedure of [15], which is expected to be better than the usual MDL method used in [2]. It is shown in [15] that the bootstrap-based detection method outperforms the MDL for small sample sizes and for low SNR. Besides, it is also comparable with the more powerful sphericity test. So, the ideas of [15] can be used together with the proposed approach in order to improve the performance of source detection and, consequently, to lower the DOA estimation SNR threshold [2].

The proposed approach can also be used for the frequency estimation. The features of the data model for the case of estimation of complex sine wave frequencies for the single-experiment ($N=1$) data are considered in [16]. For the application of the eigenstructure methods, it is necessary to perform the procedure similar to the procedure of spatial smoothing in antenna array signal processing. We divide the initial data with the $M \times 1$ length into the overlapping segments with the length M_s and receive the $M_s \times (M - M_s + 1)$ data matrix and $M_s \times M_s$ covariance matrix. The $M \times 1$ pseudo-noise vector should be added to original data before segmentation. The rest of the steps are similar to the steps presented before.

4. Simulation results

The experimental investigations of the obtained approach were conducted by computer simulation. The simulation results are presented in Figs. 2–5. For Figs. 2–4 we assumed the ULA to have $M=8$ sensors spaced at $d=\lambda/2$. The approximate array beamwidth is $\Delta_R = 2/(M-1)\ rad \approx 16.6^\circ$. The number of data samples was taken $N=100$, and $G=1000$ independent simulation runs were performed to obtain each simulation point. The two uncorrelated equally powered sources with angular coordinates $\theta_1 = 20^\circ$, $\theta_2 = 26^\circ$ were assumed. Therefore, the source localization sector [2–4] can be determined as $\theta_c = [\theta_1 - \Delta_R/2,\ \theta_2 + \Delta_R/2] = [11.7,\ 34.3]$.

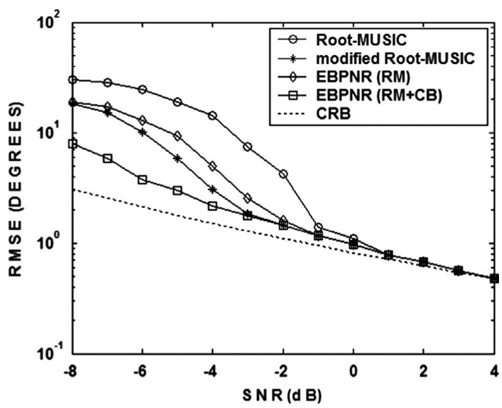

Fig. 2. RMSEs of the Root-MUSIC, the modified Root-MUSIC, the estimator bank composed of pseudo-noise resampled Root-MUSIC estimators (EBPNR (RM)), the proposed estimator bank composed of pseudo-noise resampled Root-MUSIC estimators combined with conventional beamformer (EBPNR RM+CB)) versus the SNR.

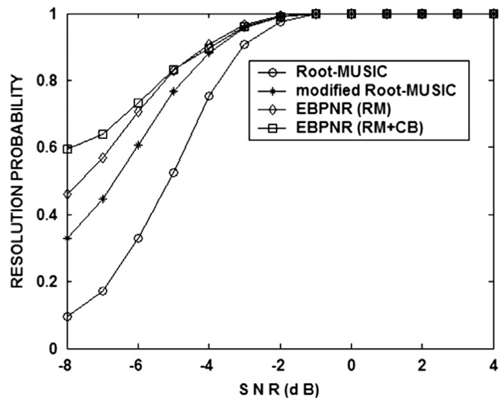

Fig. 3. Resolution probabilities of the estimator bank composed of the pseudo-noise resampled Root-MUSIC estimators (EBPNR (RM)), the proposed estimator bank (EBPNR (RM+CB)) (with K=30 and p=0.3), the modified Root-MUSIC and the Root-MUSIC versus the SNR.

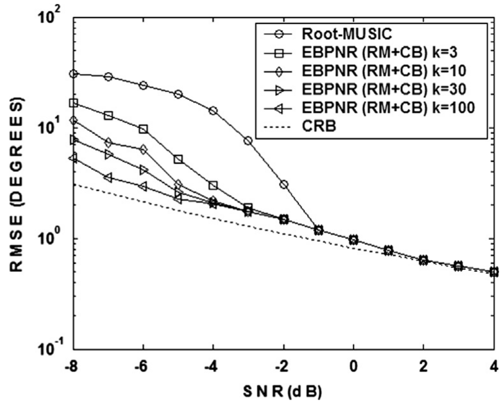

Fig. 4. RMSEs of the proposed estimator bank (EBPNR (RM+CB)) for different values of bank dimension k versus SNR.

The sample RMSE (root mean square error) was additionally averaged over sources [1–4]. Therefore, the sample RMSE was computed as

$$RMSE = \sqrt{\frac{1}{GV}\sum_{g=1}^{G}\sum_{v=1}^{V}[(\hat{\theta}_v(g) - \theta_v)^2]}, \tag{21}$$

where $\hat{\theta}_v(g)$ denotes the DOA estimate of the vth source obtained from a particular estimator at the gth run, whereas θ_v is the corresponding true DOA.

In all experiments (except the fifth one presented in Fig. 5) we assumed that the source localization sectors and the number of sources are known. This was due to the results in [2] about the influence of the accuracy of sector estimation on the accuracy of DOA estimates and due to the results in [15] about better performance of bootstrap-based detection procedure compared with MDL.

Figs. 2 and 3 show the DOA estimation RMSE and the resolution probability of the Root-MUSIC, modified Root-MUSIC, the estimator bank composed of pseudo-noise resampled Root-MUSIC estimators (EBPNR (RM)), the proposed estimator bank composed of pseudo-noise resampled Root-MUSIC estimators combined with the conventional beamformer (EBPNR (RM+CB)) versus SNR.

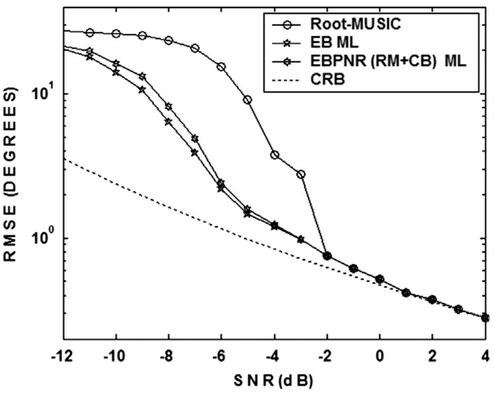

Fig. 5. RMSEs of the Root-MUSIC, the estimator bank composed of pseudorandomly weighted Root-MUSIC estimators with ML-based choice of the final set of estimates (EB ML), the proposed estimator bank with ML-based choice of the final set of estimates (EBPNR (RM+CB) ML) versus the SNR.

In other words, the resampled Root-MUSIC and modified resampled Root-MUSIC are used as underlying estimators for EBPNR (RM) and EBPNR (RM+CB), respectively. Here the SNR is defined by $SNR = 10\log_{10}(\sigma_s^2/\sigma^2)$, where σ_s^2 and σ^2 are the powers of signal and noise, respectively. Stochastic CRB is also shown in Fig. 2. The pseudo-noise parameter $p = 0.3$ was taken and the dimension of the estimator bank was $K = 30$.

The sources were said to be resolved in the gth run if [2]

$$\sum_{v=1}^{2} |\hat{\theta}_v(g) - \theta_i| < |\theta_1 - \theta_2|. \tag{22}$$

The figures show that the proposed estimator bank (EBPNR RM+CB)) has better threshold performance and resolution probability than the original one (EBPNR (RM)) and both (conventional and modified) Root-MUSIC estimators. On the other hand, Fig. 2 also shows that the asymptotic performances of above-mentioned estimators are identical.

In Fig. 4 the RMSEs of the DOA estimates of the proposed estimator bank (EBPNR (RM+CB)) for different values of the estimator bank dimension versus SNR are displayed. We can see that performance of the proposed technique becomes better in the threshold region as the dimension of the estimator bank is increases.

In order to compare the proposed approach and approach of [2] instead of the source angular sectors another well-known technique to identify the outliers in the estimates obtained from the estimator bank was used. This is explained by the fact that in [6] the latter technique is discussed. Each set of the DOA estimates from each estimator is substituted into the likelihood function and the set of estimates that yields the highest value of this function is picked [17]. So, no preliminary estimates of the source angular sectors are required in this case (however, computational load of this final step is higher).

The DOA estimation RMSE of the estimator bank composed of pseudo-randomly weighted Root-MUSIC estimators with ML-based choice of the final set of estimates (EB ML) and RMSE of the proposed estimator bank with ML-based choice of the final set of estimates (EBPNR (RM+CB) ML) versus the SNR are displayed in Fig. 5. In this case $M = 10$, $\theta_1 = 10^\circ$, $\theta_2 = 15^\circ$, $K = 10$ and $p = 0.3$. It can be seen that the performance of proposed approach is slightly worse than performance of the EB ML (i.e. PR-JES with ML choice of the final set of estimates) which is based on eigenvector resampling. However, the use of available K eigenvalue sets as input data for the detection procedure of [15] can change this relationship due to improved performance of [15] as compared to MDL that used together with [2].

The results of the simulation performed for the case of frequency estimation ($M = 64$, $N = 1$, $M_s = 45$, $V = 2$ harmonic components at normalized frequencies $f_1 = 0.2$, $f_2 = 0.215$ $(\Delta f < 1/M)$ were corrupted by a zero mean white Gaussian noise) also confirm the improved performance of proposed approach.

5. Conclusion

The proposed approach has been shown to enable the mitigation of outliers in Root-MUSIC. Our technique improves the statistical performance of Root-MUSIC in the threshold region, where the outliers have a strong influence on the performance of Root-MUSIC. The conventional beamformer performs the search for the signal DOA estimate cluster in order to reduce the effect of outlying estimates that may potentially be involved in computing the DOA estimates after pseudo-noise resampling.

The performance of the proposed approach in the case of a known number of sources is slightly worse than performance of PR-JES. However, in the case of an unknown number of sources, the situation can be changed if we will use available K eigenvalue sets and realize a detection procedure of [15], which is expected to be better than the usual MDL method used in [2]. This possibility is absent in the case of [2,6,7].

The computational burden of the proposed approach is higher than that of PR-JES [2] because eigenvalue decomposition is performed K times. However, the adding of the pseudo-noise to covariance matrix (data matrix) can be considered as updating of the covariance matrix (data matrix). Therefore, the methods of updating the eigenvalue decomposition of the original covariance matrix (or singular value decomposition of the data matrix) such as those considered in [18] and multiphase systolic algorithms for spectral decomposition [14] can be used for reducing computational burden.

The proposed approach can be also applied to Root-Min-Norm estimator and moreover, can be used for the frequency estimation. As the directions of future studies it is possible to indicate the application of results of [6,7] to cure the outlying DOA estimates, the realization of a new detection procedure and a comparison of the resulted approach with [6,7]. Furthermore, it can be expected that the use of the improved estimate of the noise power obtained in [19] could additionally improve the performance of the proposed approach. It is also of interest to consider the case when the length of pseudo-noise data will be larger than the length of original data [5].

References

[1] A.B. Gershman, J. Ringelstein, J.F. Bohme, Removing the outliers in Root-MUSIC via conventional beamformer, Signal Processing 60 (1997) 251–254.
[2] A.B. Gershman, Pseudo-randomly generated estimator banks: a new tool for improving the threshold performance of direction finding, IEEE Transactions on Signal Processing 46 (May) (1998) 1351–1364.
[3] A.B. Gershman, J.F. Bohme, A pseudo-noise approach to direction finding, Signal Processing 71 (May) (1998) 1–13.
[4] A.B. Gershman, M. Haardt, Improving the performance of Unitary ESPRIT via pseudo-noise resampling, IEEE Transactions on Signal Processing 47 (August) (1999) 2305–2308.
[5] Y. Raviv, N. Intrator, Bootstrapping with Noise: an effective regularization technique, Connection Science 8 (3-4) (1996) 355–372.
[6] P. Parvazi, A.B. Gershman, Y.I. Abramovich, Detecting outliers in the estimator bank-based direction finding techniques using the likelihood ratio quality assessment, Proceedings of ICASSP (2007) 1065–1068.
[7] P. Parvazi, A.B. Gershman, Y.I. Abramovich, Improving the threshold performance of the estimator bank direction finding technique using outliers identification and cure, Proceedings of ISWPC (2008) 270–273.
[8] V.I. Vasylyshyn, Improving the performance of Root-MUSIC via pseudo-noise resampling and conventional beamformer, In: Proceedings of the Third Microwaves, Radar and Remote Sensing Symposium, Kyiv, 2011, pp.246–249.
[9] M Wax, T. Kailath, Detection of signals by information theoretic criteria, IEEE Transactions on Acoustics, Speech, & Signal Processing ASSP-33 (1985) 387–392.
[10] B. Efron, R Tibshirani, An Introduction to the Bootstrap, NewYork, Chapman & Hall, 1993.
[11] A.M. Zoubir, D.R. Iskander, Bootstrap Techniques for Signal Processing, Cambridge University press, 2004.
[12] B.D. Carlson, Covariance matrix estimation errors and diagonal loading in adaptive arrays, IEEE Transactions on Aerospace and Electronic Systems 24 (July) (1988) 397–401.
[13] P. Stoica, K.C. Sharman, Maximum likelihood methods for direction-of-arrival estimation, IEEE Transactions on ASSP- 38 (1990) 1132–1143.
[14] K.R. Liu, K. Yao, Multiphase systolic algorithms for spectral decomposition, IEEE Transactions on Signal Processing 40 (1992) 190–201.
[15] R.F. Brcich, A.M. Zoubir, P. Pelin, Detection of sources using bootstrap techniques, IEEE Transactions on Signal Processing 50 (2) (2002) 206–215.
[16] P. Stoica, R.L Moses, Introduction to Spectral Analysis, Prentice hall, 1997.
[17] A.B. Gershman, P. Stoica, New MODE-based techniques for direction finding with an improved threshold performance, Signal Processing 76 (3 Aug.) (1999) 221–235.
[18] R.D. DeGroat, E.M. Dowling, D.A. Linebarger, Subspace tracking, in: V.K. Madisetti, D.B. Williams (Eds.), The Digital Signal Processing Handbook, CRC Press, Boca Raton, FL, 1998.
[19] R.E. Bethel, K.L. Bell, Maximum likelihood approach to joint array detection/estimation, IEEE Transactions on Aerospace and Electronic Systems 40 (July) (2004) 1060–1072.