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# Analysis of Estimation of Direction of Arrival by Comparative Study

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## Abstract

New smart antenna systems use spatial diversity techniques to avoid the noise and interference from unwanted directions. That means, they ‘adapt’ or ‘modify’ the antenna beam in an intended direction from where the transmission is coming from. Since, most antenna systems today are the arrays with multiple radiators, such beam forming technique requires fairly accurate estimate of direction of arrival. The performance of smart antenna systems in that case gets greatly affected by how accurate the directional of arrival (DOA) algorithm in the array signal processing works. The work done here investigates the DOA algorithms implemented with MUSIC, ESPRIT and another technique that uses mechanical rotation of the antenna array. In simulation using MATLAB, mechanical array rotation can be implemented by creating nulls in the direction of interferes. This paper produces comparative study from results obtained in these three approaches considering three dimensional scenarios.

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**Keywords:** *smart antenna, array signal processing, DOA, MUSIC, ESPRIT, array rotation*

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## Introduction

With new technologies and new generations of the protocols and wireless devices coming to market, there is huge burst of wireless communication systems. This needs more system capacity for transmission and reception.

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Typically, it can be achieved by allocating new portion of spectrum or increasing system bandwidth. But both these approaches need global level coordination since the frequency spectrum being limited resource [1]. The possible approach then becomes to use the available spectrum with more efficiency. With increasing complexity of source and channel coding techniques, remaining choice to achieve spectrum efficiency is multiple access techniques [2]. Space Division Multiple Access (SDMA) poses a promising approach by proper signal processing.

In smart antenna systems, direction of arrival (DOA) estimation is done by signal processing system by creating nulls in the direction of interferes and by creating peaks in the directions of intended transmitters [3-4]. This signal processing becomes major task in smart antenna array processing because receiver will receive full signal along with interference. Its next task is to extract useful information like number of signals arrived from multipath, signal strengths, directions, and noise level available into channel.

Many approaches had been used and experimented with around the globe including multiple signal classification (MUSIC) [5-7] and estimation of signal parameters using rotational in-variance techniques (ESPRIT) [8-10] and some novel techniques [11-12]. Most of these techniques provide excellent performance under assumption of white Gaussian noise. That means, performance of these methods is subjected to noise model. If this noise model is changed or noise is colored noise [13], these systems don't offer good performance [14].

Ability of smart antenna arrays to achieve high performance is majorly affected by antenna array structure. Fundamentally array geometries considered are: one dimensional (1D), two dimensional (2D) and three dimensional (3D) [15-16]. In this paper three dimensional geometry is assumed while developing the simulation codes in MATLAB. These codes deal with azimuth as well as elevation angle. One and two dimensional arrays create problems while locating the transmitting sources with accuracy. Organization of this paper is as follows; section 2 gives general smart array architecture and its relation with direction of arrival. Section 3 discusses about DOA algorithms MUSIC, ESPRIT and mechanical rotation of antenna array, in general. Performance comparison is discussed in section 4. Final conclusions are presented in section 5.

### Smart Antenna Array structure

A smart antenna array system is basically an antenna array interfaced with digital signal processing (DSP) system. High frequency communication signals received by an array are forwarded to receiver front end and then DSP system. Analog to digital converted signal samples are fed to DOA estimation algorithm. Beamforming weights are calculated by antenna array and fed to all antenna elements individually. These weights are calculated so that resulting antenna array radiation pattern (beam) will adapt itself to the direction of arrival of signal of interest. Fig. 1 shows the general block diagram of such a system.

### DOA Algorithms

DOA signal processing algorithms are basically classified into quadratic (non-subspace) type and subspace type [4]. These methods generally rely on physical size of antenna array and result in high error probability. Example of such method is Bartlett and Capon method [17-22]. Examples of subspace DOA methods are MUSIC and ESPRIT. These methods do not rely on physical aperture of antenna array and provide high accuracy and resolution. Let us consider array of  $N$  antenna elements receives  $P$  signals such that each antenna element has zero mean Gaussian noise. Array output can be represented as,

$$y(t) = \sum_{i=1}^N w_i x_i(t) \quad (1)$$

We have,

$$x(t) = a(\theta)s(t) + n(t) \quad (2)$$

Here terms can be given and defined as  $x(t) = [x_1(t), x_2(t), \dots, x_N(t)]^T$  and  $a(\theta) = [a(\theta_1), \dots, a(\theta_N)]$  are received array data and array manifold matrix, respectively. Also,  $s(t)$  is source waveform vector while  $n(t)$  is noise vector.

Array manifold vector relates to array beam steering vector as,

$$a(\theta_i) = [1, e^{\frac{j2\pi d}{\lambda} \sin \theta_i}, \dots, e^{\frac{j2\pi(N-1)d}{\lambda} \sin \theta_i}]^T \quad (3)$$

In the above equation,  $d$  stands for antenna separation in the array and  $\lambda$  is wavelength of operation of array.

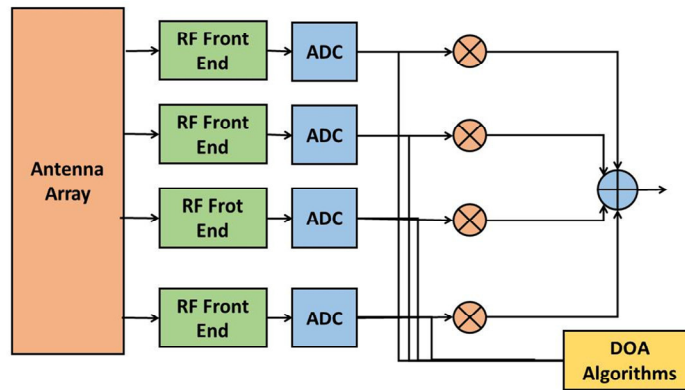


Fig. 1. General smart antenna array system

Correlation matrix of order  $N \times N$  can be obtained assuming that received signal as stationary random process,

$$R = E[(x(t) - m_x(t)) \cdot (x(t) - m_x(t))^H] \quad (4)$$

Here,  $(\cdot)^H$  indicates conjugate matrix. While calculating these equations some important assumptions have been made, these are, first, signals and Gaussian noise are stationary processes and ergodic zero mean complex valued. Second, transmitting signal sources are uncorrelated. Third, steering vectors are linearly independent and signal sources are statistically independent. And final, number of sources  $P$  and number of antennas  $N$  satisfy the equation,

$$N \geq 2P + 2 \quad (5)$$

Calculation of co-variance matrix then done as,

$$\hat{R}_{xx} = \frac{1}{K} \sum_{t=1}^K X X^H \quad (6)$$

At this stage, we can apply DOA algorithms for accurate estimation.

## MUSIC

Multiple signal classification (MUSIC) is one of most popular algorithm for high accuracy, high resolution direction of arrival estimation. MUSIC can estimate DOA from number of signals arrived by using eigen value decomposition structure of input co-variance matrix. MUSIC obtains correlation between various input signals by locating peaks in MUSIC spatial spectrum. Co-variance matrix for the source signals  $S$  can be written as,

$$S = E[s(t)s^H(t)] \quad (7)$$

MUSIC requires  $S$  to be non-singular matrix. However, this assumption exempts linear uniform array. Array steering vector  $a(\theta)$  depends on the antenna array geometry. From the given array structure, Eigen values can be decomposed into matrix and we plot MUSIC pseudo-spectrum using,

$$P_{MUSIC} = \frac{1}{a(\theta)^H E_N E_N^H a(\theta)} \quad (8)$$

The spatial spectrum obtained is shown in Fig. 2. Two sources were assumed and equal number of interferes assumed. The algorithm is efficiently able to estimate the direction of arrival when SNR 50dB. In three dimensions, it can clearly identify DOA for both the sources, in terms of azimuth angle and elevation angle.

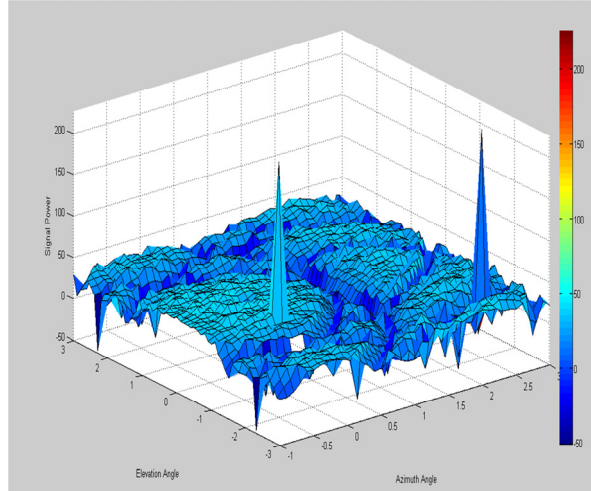


Fig. 2. Spatial spectrum using MUSIC algorithm

## ESPRIT

Estimation of signal parameter via rotational in-variance technique (ESPRIT) is similar to MUSIC in mathematical foundation. However, it uses underlying data model and generates estimates taking into account the imperfections in array geometry. This way, ESPRIT is more robust than MUSIC [10-11]. A rotational in-variance characteristic of the signal subspace is the main feature used in ESPRIT. When array elements placed equidistant and form matched pairs with  $M$  array elements. Total numbers of doublets are,  $m = M/2$ . Signal subspace is computed for two sub-arrays, which in turn result into two vectors. It also assumes existence of non-singular matrix which will help in eigenvalue decomposition. ESPRIT is based on the fact that between any two adjacent antennas in antenna array, phase shift is constant; whenever steering vector is taken into consideration.

General steps followed in ESPRIT are [9]:

1. Estimate correlation matrix  $R$  and find its eigenvalue decomposition  $R = QAQ^H$ .

2. Partition matrix  $Q$ , so as to obtain matrix corresponding to  $M$  largest eigen values which spans the signal subspace.
3. Using least squares method, obtain estimate for matrix of order  $M \times M$ .
4. Find eigen values of this  $M \times M$  matrix. Diagonal elements of this matrix are the values of estimates, we are looking for.
5. Calculate DOA from ESPRIT equations.

Authors have developed ESPRIT code, assuming similar data as in MUSIC implementation. The result obtained for three dimensional simulation using ESPRIT are shown in Fig. 3. It shows histograms of azimuth and elevation angles for four sources. It is clear that the algorithm can identify their azimuth and elevation angles distinctly.

### Mechanical Rotation of Antenna Array

Mechanical rotation of antenna array method is proposed in [11] which discusses DOA estimation by rotating the antenna array plane for azimuth direction. At any antenna element from the array, available signal is given by,

$$x_n = Ae^{j\frac{\omega n c \cos \varphi}{\lambda}} \quad (9)$$

Equation (9) gives any time instance,  $A$  being the amplitude of signal &  $\varphi$  in the above equation gives DOA.

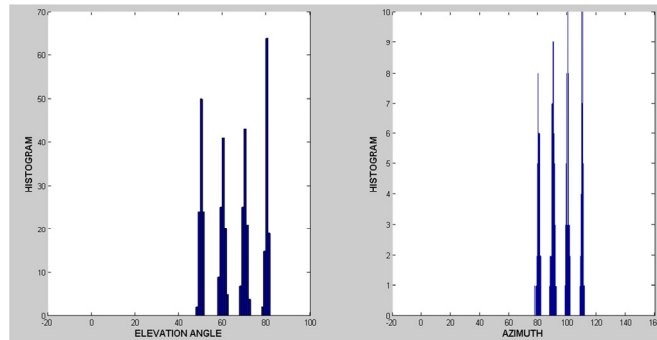


Fig. 3. Histogram plots obtained using ESPRIT algorithm

Now, we want to mechanically rotate the antenna array plane, then  $\varphi$  becomes  $(\varphi + \delta\varphi)$ . Equation (9) then modified in terms of  $(\varphi + \delta\varphi)$ .

$$x'_n = Ae^{j\frac{\omega n c \cos(\varphi + \delta\varphi)}{\lambda}} \quad (10)$$

Taking the frequency ratios from equations (9) and (10), we can obtain the constant value  $K$ , which is known from spectral analysis of the signals. Rearranging the equation in terms of  $\varphi$ ,

$$\varphi = \tan^{-1} \left[ \frac{\cos(\delta\varphi) - K}{\sin(\delta\varphi)} \right] \quad (11)$$

However, in this paper, authors have implemented the code considering azimuth as well as elevation angle. For the assumed scenario of 2014 snapshots and SNR = 50dB, authors have compared mechanical rotation method with MUSIC and ESPRIT. Mechanical rotation of antenna array focuses on fundamental of DOA estimation that the DOA algorithm should be able to create nulls in the direction of unwanted signals (interferes) and should be able to create peaks in the direction of sources. In the implementation, it is found that three dimensional DOA estimation using mechanical rotation of antenna array gives closer estimates. It can be observed from Fig. 5.

Classical beam-former uses Fourier transform to estimate the direction of arrivals for the transmitter from the spectral analysis. But as can be observed from Fig. 4 here, it does not provide accuracy for the DOA. There can be high probability of noise and interference. Authors have compared classical beam-former DOA estimate with the method of rotation of antenna array plane mechanically.

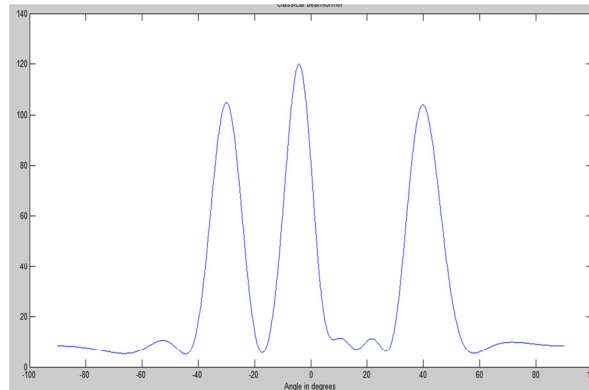


Fig. 4. DOA from classical beamformer

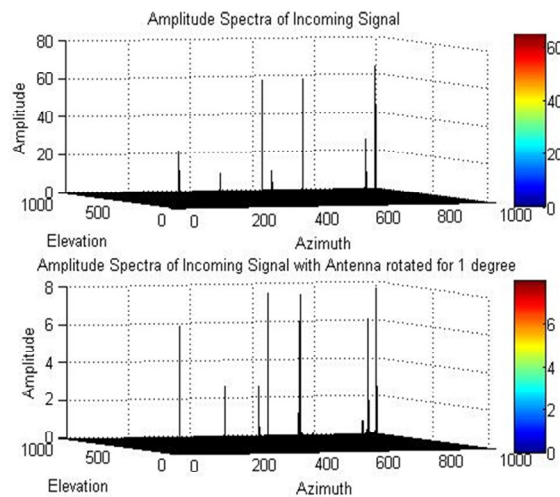


Fig. 5. DOA using mechanical rotation of antenna array in 3D

### Performance Comparison

Location of the two sources put at  $-40^\circ$  and  $+40^\circ$ , was accurately estimated by mechanical rotation of antenna array method. The MATLAB code is implemented in three dimensions. MUSIC algorithm is popularly used for DOA estimation. But ESPRIT can provide more robustness and accuracy. But when physical parameters of array are considered including antenna spacing, phase shift, number of elements in the antenna array; in such case, mechanical rotation of antenna array method gives better accuracy for three dimensions. It can also be noted that when  $P > N$ , this method can properly detect all the direction of arrivals. Authors have also tried to increase the accuracy by increasing number of snapshots and result found to be agreement.

## Conclusion

DOA estimation methods MUSIC, ESPRIT and mechanical rotation of antenna array are compared for three dimensional approach. By comparison it was concluded that mechanical rotation of antenna array elements gives combined features of MUSIC and ESPRIT. It provides accuracy and robustness.

## Future Work

Authors are working on building the prototype for DOA estimation using mechanical rotation of antenna array elements in real life application. This prototype will be proof-of-concept model for the simulated results.

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