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DOA and Doppler Frequency Estimation Based on Sub-aperture MUSIC

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Abstract: Most available algorithms for DOA estimation in monostatic MIMO radar involve the computation of covariance matrix and its inversion or eigendecomposition, of which the computational complexity is expensive, especially in the case that both the number of sensors and snapshots are large. To solve this problem, a high-resolution algorithm for joint direction-of-arrival (DOA) and Doppler Frequency estimation based on sub-aperture MUSIC in monostatic MIMO radar is presented. Firstly, the received data matrix form each transmitter is evenly divided into four sub-blocks. Then two-dimensional fast Fourier transforms (2D-FFTs) is applied to each sub-block in order to achieve coherent integration. By utilizing the data corresponding to the peaks of coherent integration in each sub-block, a reduced-dimensional data vector is constructed for joint DOA and Doppler frequency estimation via root-MUSIC. Since the full-dimensional covariance matrix estimation and eigendecomposition are avoided, the computational complexity is relatively small. Simulation results demonstrate the effectiveness of the proposed method with slightly better performance than that of ESPRIT-based method.

Keywords: DOA, Doppler frequency, two-dimensional fast Fourier transforms (2D-FFTs), sub-aperture MUSIC, monostatic MIMO radar.

1. INTRODUCTION

There is a growing awareness that multiple-input multiple-output (MIMO) radar is of value since it can provide better spatial resolution, interference rejection and extended degrees of freedom (DOFs) as well(Bliss *et al.* 2003; Fishler *et al.* 2006). So far MIMO radar can be classified as two broad categories. The first kind is small-scaled coherent MIMO radar, of which the antennas are closed spaced and therefore both the DOA and direction of departure (DOD) are identical for all the receive sensors and all the transmit ones, respectively. The other kind is large-scaled nocoherent MIMO radar with widely separated antennas. Several classical algorithms like Capon, MUSIC and ESPRIT have been applied to MIMO radar. In a recent study, Yan *et al.*(2008) have proposed a Capon-based algorithm for multitarget identification and localization. And the polynomial rooting technique for DOA estimation in monostatic MIMO radar is introduced in order to avoid angle searching(Xie *et al.* 2010). However, as we all know, the resolution of Capon-based algorithm is poor, especially at low SNR. Different from Capon technique, MUSIC spectrum indicate the orthogonality of signal-subspace and noise-subspace. Therefore, it can provide much better resolution. Xie *et al.*(2012) have presented direction finding with automatic pairing for bistatic MIMO radar by utilizing the property of the kronecker product. However, it requires general search in support region, which is computationally expensive. ESPRIT method is popular for its high resolution and unnecessariness of angle search. Chen *et al.*(2008) found that DOAs and DODs of targets can be obtained via the one-dimensional ESPRIT transmit array and receive array, respectively. By exploiting ESPRIT and SVD of cross-correlation matrix of the received data from two transmit subarray, Chen *et al.* (2010) showed that the influence of spatial colored noise can be eliminated. Cao (2010) developed a method of joint estimation of angle and Doppler frequency based on ESPRIT by utilizing the rotational factor produced by time delay sampling. Nevertheless, the above algorithms involve the computation of covariance matrix and its inversion or eigendecomposition. Hence, its computational cost is relatively high, especially in the case of MIMO radar system mainly owing to the increased DOFs.

In this paper, high-resolution DOA and Doppler frequency estimation based on sub-aperture MUSIC in monostatic MIMO radar (named as sub-aperture MUSIC for brevity) is presented. Without loss of generality, we assume that there are two targets in the mainlobe. And, firstly, the received data matrix of each transmitter is evenly portioned into four sub-blocks. Then two-dimensional fast Fourier transforms (Boroujeny and Lim, 1992) (2D-FFTs) is applied to each sub-block in order to achieve coherent integration. DOA and Doppler frequency can be obtained by a reduced-dimensional data vector, which is produced in terms of the data corresponding to the peaks of coherent integration in each sub-block, via root-MUSIC technique(Bencheikh *et al.* 2010). Note that both the spatial steering vector and temporal steering vector of the sub-aperture MUSIC are ambiguous. Furthermore, the normalized spatial and temporal frequencies are introduced to eliminate the ambiguities. As the full-dimensional covariance matrix estimation and eigendecomposition are avoided, the computational burden of the sub-aperture method can be alleviated.

The remainder of the paper is organized as follows. In Section 2, we present the monostatic MIMO radar signal model, followed by the sub-aperture MUSIC method in Section 3. The computational complexity of the proposed method and numerical examples are provided in Section 4 and Section 5, respectively. Finally, Section 6 concludes the paper.

2. MONOSTATIC MIMO RADAR SIGNAL MODEL

Consider a uniform linear array (ULA) with M-elements, of which the interelement spacing is half-wavelength. Assume that each antenna emits different waveform, and then the waveform of the mth antenna can be expressed as

$$g_m(\tau) = \sum_{l=0}^{L} p_m(\tau - lT_r)e^{j2\pi f_c \tau} \tag{1}$$

where

$$p_m(\tau) = \begin{cases} \tilde{p}_m(\tau) & 0 \leqslant \tau \leqslant \tau_s \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$\int g_n(\tau) g_m^*(\tau) d\tau = \delta_{nm} \tag{3}$$

L is the number of pulses in coherent processing interval(CPI). $\tilde{p}_m(\tau)$ denotes the complex envelope of transmit signal. τ represents the fast time delay. T_r denotes the pulse repetition period(PRP). f_c and τ_s denote the carrier frequency and pulse width, respectively. $(\cdot)^*$ denotes the conjugate operator. δ_{nm}denotes the Kronecker function.

At each receive antenna, a matched filterbank is utilized to extract the transmitted waveforms from the reflected signals. Suppose that there exist P far-field point targets located at the same range bin. The DOA of the pth moving target, of which the radial velocity with respect to radar is v_p, is denoted by θ_p. Then the receiving data of the lth pulse can be rewritten in matrix form as

$$\boldsymbol{Y}(l) = \boldsymbol{A}_r(\theta)\boldsymbol{\Lambda}(l)\boldsymbol{A}_t^{\mathrm{T}}(\theta) + \boldsymbol{N}(l) \tag{4}$$

where

$$\boldsymbol{A}_r(\theta) = [\boldsymbol{a}_r(\theta_1), \boldsymbol{a}_r(\theta_2), \cdots, \boldsymbol{a}_r(\theta_P)]^{\mathrm{T}} \tag{5}$$

$$\boldsymbol{a}_r(\theta_p) = \left[1, \cdots, e^{j\pi(M-1)\sin\theta_p}\right]^{\mathrm{T}} \tag{6}$$

$$\boldsymbol{\Lambda}(l) = \mathrm{diag}\left(\sqrt{K}\beta_1 e^{j2\pi f_{d1} T_r l}, \cdots, \sqrt{K}\beta_P e^{j2\pi f_{dP} T_r l}\right) \tag{7}$$

$$\boldsymbol{A}_t(\theta) = [\boldsymbol{a}_t(\theta_1), \boldsymbol{a}_t(\theta_2), \cdots, \boldsymbol{a}_t(\theta_P)]^{\mathrm{T}} \tag{8}$$

$$\boldsymbol{a}_t(\theta_p) = \left[1, \cdots, e^{j\pi(M-1)\sin\theta_p}\right]^{\mathrm{T}} \tag{9}$$

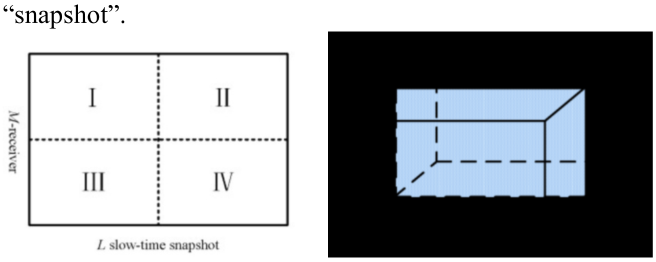
$l = 0, 1, \cdots, L-1$. $(\cdot)^{\mathrm{T}}$ denotes the transpose operator. $\boldsymbol{a}_r(\theta_p)$ and $\boldsymbol{a}_t(\theta_p)$ denote the receive steering vector and transmit steering vector of the p th target, respectively. $\mathrm{diag}(\cdot)$ denotes the diagonalization operator. $f_{dp} = 2v_p/\lambda$. K represents the number of fast-time snapshots. $\boldsymbol{N}(l)$ denotes the noise matrix after matched filtering.

In the most available algorithms for DOA estimation, the DOFs of the transmitter is exploited in the receive site. However, the computational complexity of covariance matrix estimation and its inversion is also increased, especially in the case that the number of sensors and snapshots are relatively large.

3. SUB-APERTURE MUSIC

In what follows, an algorithm with less computational complexity for joint DOA and Doppler frequency estimation based on sub-aperture MUSIC is developed.

The data cube of Eq.4 can be described as Fig.1(a). Without loss of generality, we put the transmit-dimension as "snapshot".



(a) (b)

Fig. 1. Receiving Data for(a) Data cube of the receiving output and (b) Cross section of the receiving data

Note that the output of the mth transmit antenna can be expressed as the cross-section in Fig.1(a), viz. the shaded section. Then we partition this data matrix uniformly into four blocks non-overlapped, as shown in Fig.1(b). From Fig. 1(b), we can see that the corresponding spatial steering vector and temporal steering vector can be expressed respectively as

$$\boldsymbol{s}_s(\theta) = [1, \exp(j\pi M \sin\theta/2)]^{\mathrm{T}} \tag{10}$$

$$\boldsymbol{s}_t(f) = [1, \exp(j\pi L f T_r 2)]^{\mathrm{T}} \tag{11}$$

Let

$$\begin{aligned} \boldsymbol{Z}_m &= [\boldsymbol{Y}_m(1), \boldsymbol{Y}_m(2), \cdots, \boldsymbol{Y}_m(L)] \\ &\triangleq \begin{bmatrix} \boldsymbol{Z}_{11} & \boldsymbol{Z}_{12} \\ \boldsymbol{Z}_{21} & \boldsymbol{Z}_{22} \end{bmatrix} \end{aligned} \tag{12}$$

where $\boldsymbol{Y}_m(l)$, which corresponds to the receive data of the mth column of the matrix $\boldsymbol{Y}(l)$, denotes the receive data of the m th transmit antenna and the l th pulse. $\boldsymbol{Z}_{ij} \in \mathbb{C}^{(M/2)\times(L/2)}$, $(i, j = 1, 2)$. (Here the received data matrix is uniformly divided into four sub-blocks non-overlapped on the assumption that both M and L are evens. Obviously, $\boldsymbol{Z}_m$ can be partitioned like overlapped smoothing with loss of sub-aperture).

For simplicity, assume that there are two targets (In practical applications, the number of targets can be obtained by AIC or MDL, and so on) located in the mainlobe width (known as Rayleigh limit). In this case, it is impossible to obtain the real parameters for the targets by directly 2D-FFTs of $\boldsymbol{Z}_m$. In what follows, we will develop a novel algorithm for joint estimation of DOA and Doppler frequency by exploiting 2D-FFTs technique.

Define

$$\boldsymbol{\Theta}_f \triangleq \mathcal{FT}2[\boldsymbol{Z}_m] = \boldsymbol{F}_{11} + \boldsymbol{F}_{12} + \boldsymbol{F}_{21} + \boldsymbol{F}_{22} \tag{13}$$

where

$$\boldsymbol{F}_{11}=\mathcal{FT}2\begin{bmatrix}\boldsymbol{Z}_{11} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\quad \boldsymbol{F}_{12}=\mathcal{FT}2\begin{bmatrix}\mathbf{0} & \boldsymbol{Z}_{12}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}$$
$$\boldsymbol{F}_{21}=\mathcal{FT}2\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \boldsymbol{Z}_{21} & \mathbf{0}\end{bmatrix}\quad \boldsymbol{F}_{22}=\mathcal{FT}2\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{Z}_{22}\end{bmatrix} \tag{14}$$

$\mathcal{FT}2[\cdot]$denotes the 2D-FFTs operator. $\mathbf{0}$ denotes the zero matrix of size $(M/2)\times(L/2)$.

The two-dimensional subscripts corresponding to the largest peak (denoted as$(\hat{\theta}_{\text{est}},\hat{f}_{\text{est}})$) of $\boldsymbol{\Theta}_f$ are written symbolically as i_x and i_y, respectively. If there are more than one peak exceeding the detection threshold, source number estimation algorithms should be applied to each peak to determine which one requires further processing. In this paper, the size of the reduced-dimensional vector is defined as 4, of which the DOFs is sufficient, on the preceding assumption that there exist two targets in the mainlobe. If more than two targets are within the mainlobe, the length of the reduced-dimensional vector should be increased. Then a reduced-dimensional data vector can be constructed as

$$\boldsymbol{b}(m)=\left[\boldsymbol{F}_{11}(i_x,i_y,m),\tilde{\boldsymbol{F}}_{12}(i_x,i_y,m),\tilde{\boldsymbol{F}}_{21}(i_x,i_y,m),\tilde{\boldsymbol{F}}_{22}(i_x,i_y,m)\right]^{\text{T}} \tag{15}$$

where

$$\tilde{\boldsymbol{F}}_{12}(i_x,i_y,m)=e^{j\pi i_y}\boldsymbol{F}_{12}(i_x,i_y,m) \tag{16}$$

$$\tilde{\boldsymbol{F}}_{21}(i_x,i_y,m)=e^{j\pi i_x}\boldsymbol{F}_{21}(i_x,i_y,m) \tag{17}$$

$$\tilde{\boldsymbol{F}}_{22}(i_x,i_y,m)=e^{j\pi i_x}e^{j\pi i_y}\boldsymbol{F}_{22}(i_x,i_y,m) \tag{18}$$

$\boldsymbol{F}_{11}(i_x,i_y,m)$ denotes the (i_x,i_y)th element of the matric $\boldsymbol{F}_{11}$ of the mth snapshot.

Covariance matrix of $\boldsymbol{b}(m)$ can be estimated by

$$\boldsymbol{R}_b=\frac{1}{M}\sum_{m=1}^{M}\boldsymbol{b}(m)\boldsymbol{b}^{\text{H}}(m) \tag{19}$$

Then the DOA and Doppler frequency of the targets can be calculated by two-dimensional searches

$$\left(\hat{\theta},\hat{f}\right)=\arg\max_{\theta,f}\frac{1}{(\boldsymbol{s}_s(\theta)\otimes\boldsymbol{s}_t(f))^{\text{H}}\boldsymbol{u}_n\boldsymbol{u}_n^{\text{H}}(\boldsymbol{s}_s(\theta)\otimes\boldsymbol{s}_t(f))} \tag{20}$$

where $\boldsymbol{u}_n$ consists of the eigenvectors corresponding to the last two eigenvalues via eigendecomposition of $\boldsymbol{R}_b$ in terms of the previous assumption.

It should be noted that there exist $M/2$ and $L/2$ ambiguities of both the spatial steering vector and temporal steering vector as given in (10) and (11) since $|M\sin\theta/2|>1$ and $|LfT_r|>1$ for large M and$L$, respectively. Note that the $(\hat{\theta}_{\text{est}},\hat{f}_{\text{est}})$ obtained by previous 2D-FFTs can be utilized to narrow the range of search so as to ensure the uniqueness of the estimate. However, it also requires two-dimensional searches with parameter estimation accuracy depending on the search step. Fortunately, both the normalized spatial frequency ϕ_s and temporal frequency ϕ_t have a range of$[-1,1)$. Thus, by (10) and (11), we have

$$\boldsymbol{s}_s(\phi_s)=[1,\exp(j\pi\phi_s)]^{\text{T}} \tag{21}$$

$$\boldsymbol{s}_t(\phi_t)=[1,\exp(j\pi\phi_t)]^{\text{T}} \tag{22}$$

In order to reduce the computational complexity of two-dimensional searches, root-MUSIC method is adopted. Define the noise-subspace as follows

$$\boldsymbol{p}_n=\boldsymbol{u}_n\boldsymbol{u}_n^{\text{H}}=\begin{bmatrix}\boldsymbol{p}_{11} & \boldsymbol{p}_{12}\\ \boldsymbol{p}_{21} & \boldsymbol{p}_{22}\end{bmatrix} \tag{23}$$

where $\boldsymbol{p}_{ij},(i,j=1,2)$. According to the orthogonality of signal- and noise-subspace, we have the following cost function

$$(\boldsymbol{s}_s(\phi_s)\otimes\boldsymbol{s}_t(\phi_t))^{\text{H}}\boldsymbol{p}_n(\boldsymbol{s}_s(\phi_s)\otimes\boldsymbol{s}_t(\phi_t))=0 \tag{24}$$

Based on the property of the Kronecker product and (23), we obtain

$$\boldsymbol{s}_t^{\text{T}}\left(z_t^{-1}\right)\left[\sum_{i,j=1}^{2}z_s^{j-i}\boldsymbol{p}_{ij}\right]\boldsymbol{s}_t\left(z_t^{-1}\right)=0 \tag{25}$$

where $z_t=\exp(j\pi\phi_t)$, $z_s=\exp(j\pi\phi_s)$.

For $P\leqslant M(N-1)$, we have

$$\det\left[\sum_{i,j=1}^{2}z_s^{j-i}\boldsymbol{p}_{ij}\right]=0 \tag{26}$$

Then the normalized spatial frequency can be estimated by polynomial rooting

$$\hat{\phi}_{sp}=\text{angle}(\hat{z}_{sp})\Big/\pi,\qquad p=1,2 \tag{27}$$

where $\text{angle}(\kappa)$ denotes the phase of κ.

The DOA can be yielded in terms of $\hat{\theta}_{\text{est}}$ obtained and the principle of the interferometer as follows

$$\hat{\theta}_p=\arcsin\left(2\left(2N_s+\hat{\phi}_{sp}\right)\Big/M\right) \tag{28}$$

where $N_s=\text{round}\left(M\sin\hat{\theta}_{\text{est}}/4\right)$. $\arcsin(\cdot)$ denotes the arcsine operation. $\text{round}(\cdot)$ represents the function of round.

Upon substituting the $\hat{\theta}_p$ obtained in (28), we get

$$\boldsymbol{s}_t^{\text{T}}\left(z_t^{-1}\right)\left[\sum_{i,j=1}^{2}\hat{z}_{sp}^{j-i}\boldsymbol{p}_{ij}\right]\boldsymbol{s}_t\left(z_t^{-1}\right)=0 \tag{29}$$

Similarly, the root $\hat{z}_{tp}$ which closest to the unit circle can be calculated by polynomial rooting technique, then the Doppler frequency for the pth target can be computed as

$$\hat{f}_{dp}=\left(N_t+\hat{\phi}_{tp}\right)\Big/LT_s \tag{30}$$

where $N_t=\text{round}\left(LT_s\hat{f}_{\text{est}}\right)$, $\hat{\phi}_{tp}=\text{angle}(\hat{z}_{tp})/\pi$.

Clearly, the DOA and Doppler frequency of the targets can be paired automatically. The proposed method can be mainly applied to the condition of large number of array elements and snapshots. Since the full-dimensional covariance matrix estimation and eigendecomposition are avoided, the computational cost of the presented method is relatively slow. Secondly, the reduced-dimensional vector $\boldsymbol{b}(m)$ is of size 4×1, it requires less number of snapshots for subsequent parameters estimation in relative with the direct received data. Moreover, the normalized frequencies and the real polynomial rooting technique are introduced to further reduce the computational complexity somewhat.

In this paper, we primarily discuss the situation of uniformly partition of the receive matrix non-overlapped. As forementioned, the algorithm can be extended to the overlapped case but with much more complexity. A further study will be made in the subsequent work.

4. COMPUTATIONAL COMPLEXITY

In monostatic MIMO radar of transceiver system with an-M element array, assume that there are P targets in the range bin of interest. The number of snapshots for slow time is L. For simplicity, here we only consider the main computational complexity, such as the covariance matrix estimation and its eigendecomposition, the covariance matrix inversion as well as the polynomial rooting technique. For ESPRIT-based method (Cao, 2010), the complexity of estimation and eigendecomposition of the covariance matrix (Gloub and Loan, 1996) is $2M^4(L-1)+2M^6$, and that of the proposed method is $16M+64$. The complexity for calculating $\boldsymbol{\Theta}$ is $2M^6$, including the matrix pseudoinversion of $\mathbf{C}_{11}$. Hence, the total computational load of ESPRIT-based method is $2M^4(L-1)+4M^6$. In our algorithm, for time-decimal FFT, the computational cost of computing 2D-FFTs is $3M^2L/2\log_2(ML)$, while that of the polynomial rooting is 216. Thus, the gross calculation amount of our algorithm is about $3M^2L/2\log_2(ML)+(16M+280)P$.
We introduce a simply example to demonstrate the effectiveness of our algorithm. For a monostaitc MIMO radar with, $P=2$ and $L=512$, the computational load of the ESPRIT-based method and our algorithm is 5.37×10^9 and 1.10×10^7, respectively. Obviously, it is suggested that our algorithm has much smaller computational burden.

5. SIMULATION EXAMPLES

In this section, simulation examples are given to verify the effectiveness and superiority of the proposed method. A monostatic MIMO radar system, of which the array is a 32-antenna ULA with half-wavelength sensor spacing, is considered. Assume that each transmitter emits orthogonal waveform with the pulse width $10\mu s$ and pulse repetition frequency $10\mathrm{kHz}$. The number of snapshots of the slow time dimension is $L=512$. Without loss of generality, suppose that there are two targets coming from $\theta_1=16.5^\circ$ and $\theta_1=17^\circ$ relative to the broadside of the array in the range bin of interest. Obviously, the two targets are located in the mainlobe width. For comparison, the performance curves of the ESPRIT-based method (Cao, 2010) (abbreviated as ESPRIT-based method) are given.
The estimation results of the proposed method in the case of $\mathrm{SNR}=0\mathrm{dB}$ are provided in Fig.2. Fig.2(a) shows the output of the directly 2D-FFTs of the full-dimensional receive data. It is suggested that the two targets cannot be separated owing to the Rayleigh limit, which is consistent to the theoretical analysis. Hence, further processing is required. The paired results of the proposed method are plotted in Fig.2(b), in which the number of the independent trials is 20. The notations "o", "+" and "." denote the result of the directly 2D-FFTs, true value and the estimate of our algorithm, respectively. From Fig.2(b), it can be seen that the proposed method can separate the two targets effectively and accurately.

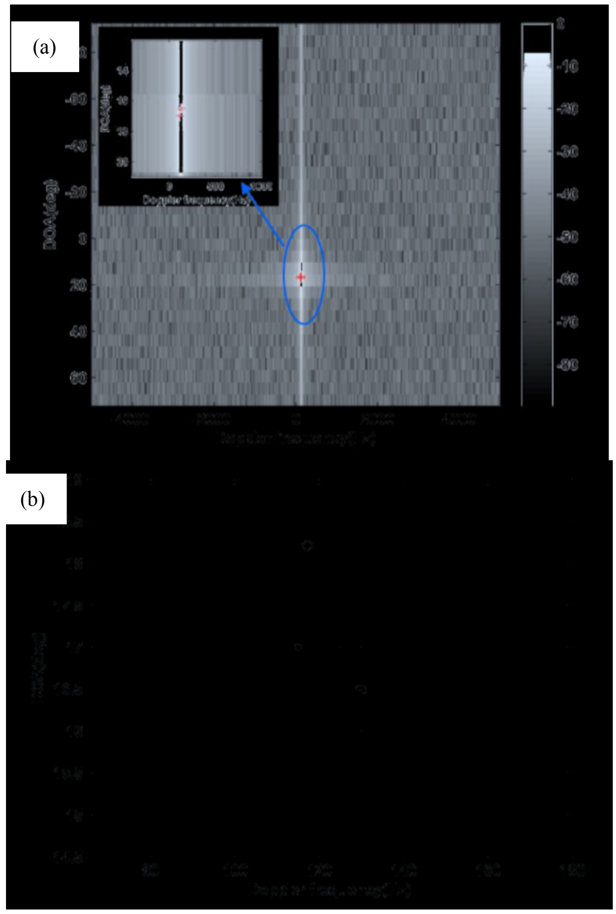
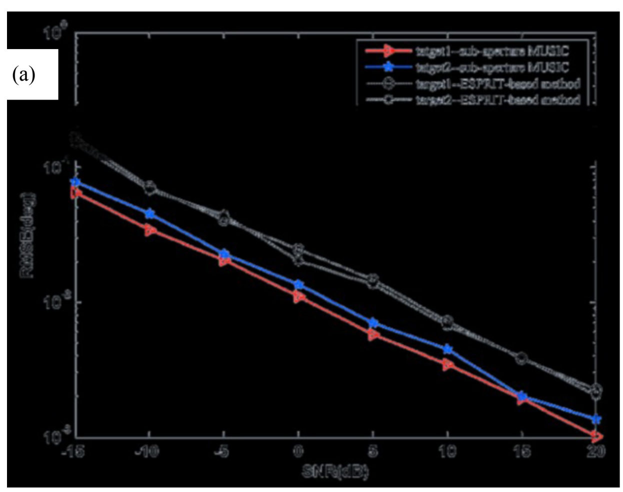


Fig. 2. Estimation results for (a) Directly 2D-FFT of full-dimensional data and (b) Sub-aperture MUSIC.

The performance of the parameter estimation versus the SNR is shown in Fig.3 with the number of Monte Carlo trials being100. From Fig.3, we can observe that the performance of the proposed method is slightly better than that of the ESPRIT-based method. Moreover, since the estimation and eigendecomposition of the directly received data are avoided, the computational complexity of the proposed method is relatively low. The superiority is much more notable for either large array or large number of snapshots.



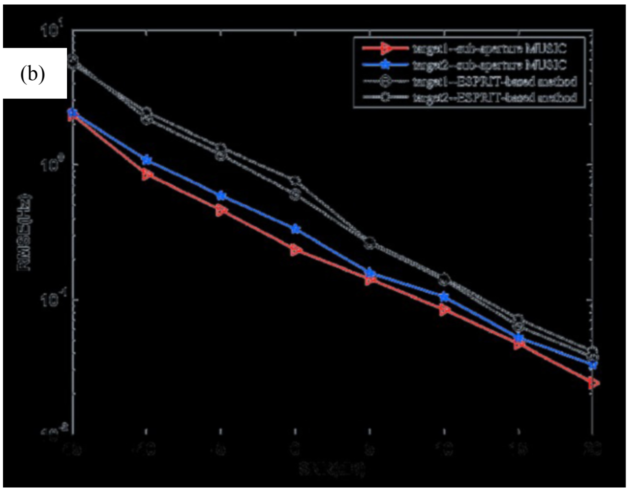


Fig. 3. RMSE of the targets for (a) DOA and (b) Doppler frequency.

6. CONCLUSIONS

In this paper, we developed an algorithm for joint DOA and Doppler frequency estimation based on sub-aperture MUSIC by exploiting 2D-FFTs technique, which avoids the full-dimensional covariance estimation and eigendecomposition. Compared with the ESPRIT-based method, the proposed method can reduce the computation burden significantly and provide slightly better performance of parameters estimation. In addition, the growth rate of the computational burden of the ESPRIT-based method is much faster than that of our algorithm when the number of either sensors or snapshots increases. Numerical simulations confirm the performance of the method presented.

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