



Performance improvement of direction finding algorithms in non-homogeneous environment through data fusion



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ABSTRACT

This paper proposes a new effective approach to improve the performance of DOA (Direction Of Arrival) algorithms when bursts affect the array data. The proposed approach is based on the combining of data fusion techniques and the results of theoretical performance analysis of conventional DOA algorithms. For this purpose, the received array data is first split in M time-segments. Then, the DOAs are estimated from each data segment using a conventional DOA algorithm. The obtained estimates are fused using the federated fusion algorithm according to their statistical accuracy obtained from the well-documented performance analysis of the considered algorithm. As proof of concept of the proposed approach, numerical experiments have been conducted by considering the MUSIC algorithm. The obtained results show that the new algorithm outperforms the conventional one in terms of accuracy in a non-homogeneous environment. Therefore, it exhibits enhanced robustness capability. Moreover, it reduces the memory cost and computational complexity which makes it suitable for real time applications. To our knowledge, it is the first time that theoretical performance analysis results are exploited for the derivation of new subspace-DOA methods.

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1. Introduction

Direction finding and spectral estimation are challenging issues that involve subspace processing techniques. Typical methods are the Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) [1,2] and Multiple Signal Classification (MUSIC) algorithm [3,4]. So far, many variants of MUSIC algorithm have been studied under the assumption of stationary additive Gaussian or second order model observation noise [5]. Many approaches are considered in the literature to improve the DOA estimation such as data fusion techniques [6–8]. When the array output signals are affected by impulsive noise, sudden bursts or sharp spikes [9], the above techniques become useless. Note that the duration and occurrence of the aforementioned phenomenon are arbitrary, and may originate from natural changes or jamming.

Estimation problem in presence of these types of noise has been considered in several references such as [10–19] and robust algorithms are presented. In particular, the well-known high res-

olution algorithms such as MUSIC and ESPRIT are based on the estimation of the desired subspace computed from the covariance matrix. To improve the robustness of these algorithms in impulsive environments, several authors propose to use a robust estimate of the covariance matrix [18,20]. These algorithms show better performance but suffer from high computational cost.

In this paper, we propose a new effective approach to overcome the problem of DOA estimation in a non-homogeneous environment. Our approach is based on data fusion technique and the well-documented performance analysis results of the considered DOA algorithm. In fact, the received array output data can be divided into segments such that one can expect these segments to not exhibit the same information quality on the DOA sources. This information can then be exploited by using data fusion concepts [6] to improve the performance of the DOA estimation.

Note that data fusion concepts are a well-known and are applied in many technological areas to improve parameter estimation. The novelty introduced in this paper can be summarized into two points: (i) a data fusion technique is implemented locally for the same data array (time data segmentation) in contrast to existing methods that use the spatial diversity (delocalized sensors) as in [8,7], and (ii) to our knowledge, it is the first time that the the-

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oretical performance analysis of a DOA algorithm is used to derive a new version of the DOA algorithm under consideration.

To show the effectiveness of the proposed approach, one can consider the conventional MUSIC algorithm which is applied to each segment to provide DOA estimates. The latter are then fused on the basis of their accuracy to obtain a fused DOA using the federated fusion algorithm [21]. The DOA accuracy expression is obtained from the well-established performance analysis of DOA MUSIC-based algorithms [22–25].

The aim of this paper is to demonstrate that the proposed approach based on data fusion concepts and the well-known performance analysis can improve significantly subspace based algorithms. Herein, we limit ourselves to the general case of Gaussian environments with arbitrary bursts. Moreover, the bursts are assumed to be randomly distributed over the observed array data with unknown variances and duration.

To illustrate our claims, the performance of the proposed DOA estimation technique, referred to as FU-MUSIC, is compared with respect to the conventional MUSIC algorithm and its robust version [20] in terms of Root Mean Square Error (RMSE) and angle resolution. The obtained results are promising and show that data fusion techniques can improve efficiently the performance of the conventional MUSIC algorithm in non-homogeneous environments. Moreover, if the conventional MUSIC is substituted by a robust version of MUSIC, our approach is still effective.

This paper is organized as follows: In Section 2, the problem formulation is provided. Section 3 details our proposed approach. Some discussions on the effectiveness of our approach are made in Section 4 and Section 5 presents the results of numerical experiments supporting our claims. Finally, concluding remarks are given in Section 6.

2. Data model

Consider K narrow-band far-field signals received by a uniform linear array of L antennas ($L > K$). The array output vector $\mathbf{x}(t) = [x_1(t) \dots x_L(t)]^T$ is given by,

$$\mathbf{x}(t) = A(\theta)\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where $A(\theta) = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_K)]$ is the steering matrix and its i th column ($1 \leq i \leq K$) is given by $\mathbf{a}(\theta_i) = [1 \ e^{-j\varphi_i} \ e^{-j2\varphi_i} \dots \ e^{-j(L-1)\varphi_i}]^T$, with $\varphi_i = 2\pi \frac{d}{\lambda} \sin\theta_i$, λ is the propagation wavelength, d is the inter-element spacing and θ_i is the i th DOA. $\mathbf{n}(t)$ is an additive white stationary noise with zero mean and covariance matrix, $\mathbf{R}_{nn} = E[\mathbf{n}(t)\mathbf{n}^H(t)] = \sigma_n^2 \mathbf{I}$ where $(\cdot)^H$ denotes the transpose conjugate operator. $\mathbf{s}(t) = [s_1(t) \dots s_K(t)]^T$ is the source signal vector where the sources are assumed to be uncorrelated. The current paper aims to present a new approach to improve the accuracy of DOA estimation when the observed data is affected by bursts.

3. The proposed approach

Herein, data fusion is used to improve the DOA algorithms by considering their theoretical accuracy performance analysis. In order to explain our proposed approach, we have chosen to consider the MUSIC algorithm and its well-documented accuracy performance to derive a new version.

Note that, this approach can be applied to other subspace-based DOA algorithms as long as the theoretical accuracy performance of the latter are available.

3.1. MUSIC algorithm and its performance analysis

One can compute the subspace decomposition by applying the Singular Value Decomposition (SVD) method on the observed data

matrix $\mathbf{X} = [\mathbf{x}(1) \ \mathbf{x}(2) \ \dots \ \mathbf{x}(T)]$ where T is the observation sample size. In this case, the subspace decomposition is given by

$$\mathbf{X} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H = [\mathbf{U}_{ss} \ \mathbf{U}_{nn}] \begin{bmatrix} \mathbf{\Lambda}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_{nn} \end{bmatrix} [\mathbf{U}_{ss} \ \mathbf{U}_{nn}]^H \quad (2)$$

where $\mathbf{\Lambda}_{ss}$ is a diagonal matrix whose entries are the K largest singular values that are associated to the columns of \mathbf{U}_{ss} and $\mathbf{\Lambda}_{nn}$ is a diagonal matrix whose entries are the least $L - K$ singular values that are associated to the columns of \mathbf{U}_{nn} . \mathbf{U}_{ss} and \mathbf{U}_{nn} span the signal subspace and the noise subspace, respectively.

Let \mathbf{u} be any vector of the noise subspace. Thus, for any vector \mathbf{v} of the signal subspace, we have $\mathbf{v}^H \mathbf{u} = 0$. In particular, this property holds for the steering vectors that span the signal subspace (i.e. $\mathbf{a}^H(\theta_k) \mathbf{u} = 0$, $k = 1, \dots, K$).

Based on this property, the MUSIC algorithm [26] provides the estimated directions of arrival that correspond to the K local minima of the spectrum function given by

$$\mathbf{F}_{MUSIC} = \mathbf{a}^H(\theta) \mathbf{U}_{nn} \mathbf{U}_{nn}^H \mathbf{a}(\theta) \quad (3)$$

Let $\Delta\theta_k$ be the error due to the MUSIC algorithm for the estimation of θ_k such

$$\hat{\theta}_k = \theta_k + \Delta\theta_k \quad (4)$$

where $\hat{\theta}_k$ is the estimate of θ_k for $k = 1, \dots, K$. From [22–24], and assuming that the elements of the perturbation matrix of \mathbf{X} denoted $\Delta\mathbf{X}$ are uncorrelated circular random variables with equal variances σ_n^2 , the performance analysis of the MUSIC algorithm provides the general expression for perturbation of DOA estimate as:

$$\Delta\theta_k = \frac{\text{Im}[\alpha_k^H \Delta\mathbf{X} \beta_k]}{\gamma_k} \quad (5)$$

where the parameters α_k , β_k , and γ_k are given by:

$$\alpha_k = \mathbf{J} \mathbf{U}_{nn} \mathbf{U}_{nn}^H \mathbf{a}^{(1)}(\theta_k) \quad (6)$$

$$\beta_k = \mathbf{V}_{ss} \mathbf{\Lambda}_{ss}^{-1} \mathbf{U}_{ss}^H \mathbf{a}(\theta_k) \quad (6)$$

$$\gamma_k = \mathbf{a}^{(1)}(\theta_k)^H \mathbf{U}_{nn} \mathbf{U}_{nn}^H \mathbf{a}^{(1)}(\theta_k) \quad (7)$$

and where $\mathbf{a}^{(1)}(\theta_k)$ stands for the first derivative of the steering vector $\mathbf{a}(\theta_k)$ (i.e. $\frac{\partial \mathbf{a}(\theta)}{\partial \theta}$).

From Eq. (5) and using the above notations, a direct computation of the mean squared error is given by [23]:

$$E_{\Delta\mathbf{X}}(\Delta\theta_k)^2 = \frac{E_{\Delta\mathbf{X}}[\text{Im}(\beta_k^H \Delta\mathbf{X} \alpha_k)]^2}{2\gamma_k^2} \quad (8)$$

$$= \frac{\|\alpha_k\|^2 \|\beta_k\|^2 \sigma_n^2}{2\gamma_k^2} \quad (9)$$

where the notation $\|\cdot\|$ stands for the Frobenius norm. However, the computation of the DOA estimation error (by using Eq. (9)) requires the priori knowledge of the noise power σ_n^2 . The latter can be estimated as the mean of the least eigenvalues of the data covariance matrix of the observed data associated to the noise subspace.

3.2. New algorithms for DOA estimation

DOA algorithms can be improved by using data fusion technique and by considering their theoretical accuracy performance as shown below.

In the proposed methodology, the data matrix \mathbf{X} is, first, divided into data segments. Then, for each data segment, the DOA

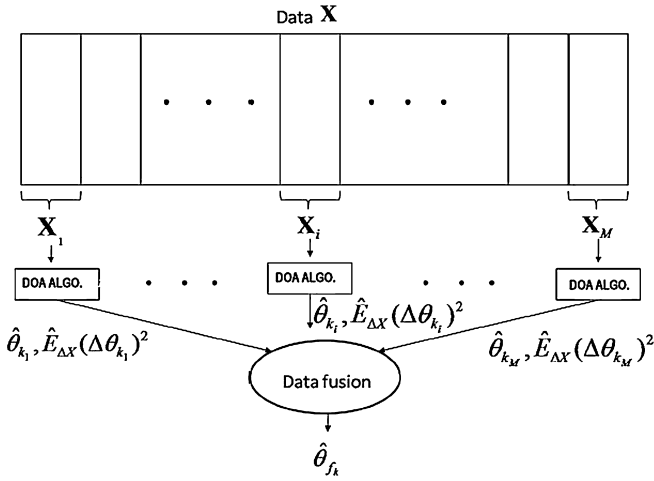


Fig. 1. The proposed approach: New DOA Algorithm.

Table 1

Summary of the proposed algorithm.

New DOA Algorithm	
Step 1:	Decompose the data matrix of dimension $(L \times T)$ into M data segments of $(L \times \frac{T}{M})$;
Step 2:	Estimate $\hat{\theta}_k, k = 1, \dots, K$ by using the DOA algorithm for each data segment;
Step 3:	Compute the estimation error $\hat{P}_{k_m} = \hat{E}_{\Delta X_m}(\Delta\theta_{k_m})^2$;
Step 4:	According to the federate fusion algorithm [21], compute $\hat{\theta}_{f_k}$.

estimates are provided by the DOA algorithm under consideration and its associated accuracies are estimated. In the case of the MUSIC algorithm, one uses Eq. (9) where θ_k is substituted by $\hat{\theta}_k$. Finally, a weighted combination of the DOA estimates according to their estimated accuracies provides a fused DOA for the different sources. For this purpose, we consider the federated fusion technique presented in [21]. The proposed algorithm is illustrated in Fig. 1.

Recall that the data matrix \mathbf{X} which contains T samples is of dimension $(L \times T)$. \mathbf{X} is now divided into M data segments $\mathbf{X}_m, m = 1, \dots, M$, (without loss of generality, we assume that T is a multiple of M) and each segment is of dimension $(L \times \frac{T}{M})$. Then, one applies DOA algorithm independently on each data segment (as shown in Fig. 1) in order to obtain their associated DOA and corresponding estimated accuracy $\hat{E}_{\Delta X_m}(\Delta\theta_{k_m})^2, k = 1, \dots, K$ and $m = 1, \dots, M$. The notation θ_{k_m} stands for the k th DOA of the m th data segment.

Denote by $\hat{P}_{k_m} = \hat{E}_{\Delta X_m}(\Delta\theta_{k_m})^2$ the estimated accuracy which corresponds to the m th data segment and k th DOA estimate $\hat{\theta}_{k_m}$. One can express the fused DOA estimates as follows

$$\hat{\theta}_{f_k} = \frac{1}{P_k^*} \sum_{m=1}^M \hat{P}_{k_m}^{-1} \hat{\theta}_{k_m} \quad (10)$$

where

$$P_k^* = \sum_{m=1}^M \hat{P}_{k_m}^{-1} \quad (11)$$

which corresponds to a normalization term and $\hat{\theta}_{f_k}$ stands for the k th DOA provided by the new DOA algorithm. The latter is summarized in Table 1. To illustrate the effectiveness of the proposed algorithm, numerical experiments are provided and discussed in Section 5.

4. Discussion

Herein, some comments are provided in order to highlight the advantages of the proposed approach:

To our knowledge, it is the first time that the theoretical performance analysis of a DOA algorithm are exploited to develop new algorithms for the DOA estimation. For example, when MUSIC algorithm is considered, the new algorithm referred to as FU-MUSIC uses Eq. (9) to estimate the accuracy of MUSIC algorithm for each segment where the parameters $\theta_k, \sigma^2, \mathbf{U}_n, \mathbf{U}_s$ and \mathbf{V}_s are substituted by their estimates provided by the MUSIC algorithm in order to estimate its accuracy \hat{P}_{k_m} .

Enhanced robustness: From a data fusion point of view, the proposed algorithm is based on a decentralized fusion scheme [27] where the erroneous DOA estimates are mitigated through their corresponding estimated accuracies. Therefore, the new DOA algorithm is efficient when bursts affect the observed data in comparison to the original algorithm. To exhibit the robustness of the proposed approach, simulation results are presented in Section 5 where a robust DOA algorithm is considered based on the normalized sample covariance matrix.

Numerical complexity: For a given matrix of dimension $(L \times T)$ where $T > L$, the computational cost of the SVD algorithm [28] is of order $O(T^3)$ flops. Based on this result, one can see that the numerical cost of FU-MUSIC is much lower as compared to the MUSIC algorithm. Indeed, the complexity of the MUSIC algorithm, when computing the SVD of an $(L \times T)$ matrix, is of order $O(T^3)$ flops. Whereas, FU-MUSIC needs the SVD computation of M matrix of dimension $(L \times \frac{T}{M})$ which is in order of $O(M(\frac{T}{M})^3)$ flops. Moreover, if M processors are available, the order of the numerical complexity is reduced to $O((\frac{T}{M})^3)$ flops per processor.

Memory space: By using conventional DOA algorithm, one needs to allocate sufficient space to store the whole observation data of dimension $(L \times T)$. However, the proposed approach requires only a small memory space to store matrices of dimension $(L \times \frac{T}{M})$. In fact, only one segment is stored in the buffer at once to compute the DOA and its associated estimation accuracy using the conventional DOA algorithm. The same buffer is used by the next data segment and so on until processing the overall observed data.

According to these advantages, it is suggested that the proposed algorithm is more suitable for real time implementation.

Note that our proposed approach can be generalized to other DOA's estimation algorithms (e.g. ESPRIT and Root-MUSIC) when the analytical expressions of their estimation accuracy are available.

5. Numerical experiments

This section provides a performance evaluation of our approach. For this purpose, we consider the MUSIC algorithm to estimate DOAs for each segment and the new algorithm in this case is referred to as FU-MUSIC as explained earlier. Let K source signals impinging on a Uniform Linear Antenna (ULA) with $L = 8$ sensors. The signals are observed for $T = 100$ samples (unless stated otherwise). For FU-MUSIC, we consider $M = 10$ segments and we assume that bursts are generated in such a way they affect the whole corrupted segment.

The source signal is assumed to be a zero mean circular complex Gaussian process with variance σ_s^2 while bursts are generated from a two term complex Gaussian mixture as in reference [29], given by

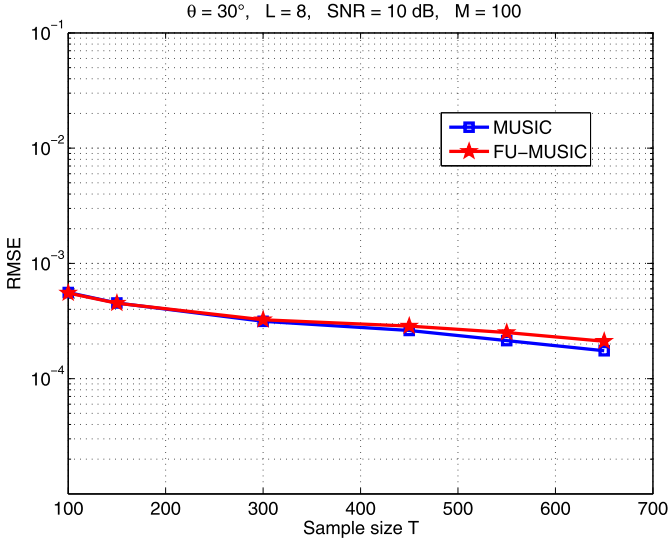


Fig. 2. DOA estimation vs sample size: without bursts.

$$b_i(t) = (1 - \nu)N_c(0, \sigma^2) + \nu N_c(0, \kappa\sigma^2) \quad (12)$$

where $b_i(t)$ denotes the noise at the i th sensor, ν is the percentage of contamination with $0 < \nu < 1$ and $N_c(0, \sigma^2)$ is a circular complex Gaussian distribution with zero mean and variance σ^2 . The parameter κ denotes the contamination factor and usually $\kappa \gg 1$. In our simulations, $\nu = 0.9$ and $\kappa = 100$.

The affected data rate is defined as $\tau = \frac{T_s}{T} \times 100\%$ where T_s is the size of the affected data. In the following numerical experiments, the affected data rate is $\tau = 20\%$ (unless stated otherwise). The burst variance σ^2 is chosen to be 40 dB higher than the variance of the signal of interest.

Let's first investigate the performance of MUSIC and FU-MUSIC algorithms when data is not affected by bursts. For this purpose, their performance are given with respect to sample size T for $\text{SNR} = 10$ dB as shown in Fig. 2. In the latter, the aforementioned algorithms show a similar performance. This result shows that the fusion process over all segments has the same effect as the use of the whole samples in the computation of the MUSIC algorithm.

Now, we are interesting on the behavior of the algorithm under consideration in the presence of bursts. For that, four scenarios are considered: (i) FU-MUSIC is compared with respect to MUSIC for the estimation of DOAs (one then two DOAs) when the affected data rate is $\tau = 20\%$. Note that MUSIC algorithm is applied to the whole observed data. (ii) Then, we compare the robust versions of MUSIC and FU-MUSIC for one source located at $\theta = 30^\circ$. (iii) After that, we consider only FU-MUSIC algorithm and we investigate its robustness with respect to the rate of the affected data. (iv) Finally, we study the ability of FU-MUSIC to distinguish between two closely spaced sources. All results are obtained after 100 Monte Carlo runs.

Scenario 1:

Figs. 3 to 4 are obtained for one source located at $\theta = 30^\circ$. One can observe that FU-MUSIC provides a good DOA estimate in contrast to MUSIC algorithm that does not perform at all. This is related to the fact that MUSIC algorithm performs well in homogeneous environment which is not the case herein because of the bursts that affect the observed data (For more details see [26]). Moreover, using FU-MUSIC, the erroneous DOA estimates in the corrupted segments are mitigated through their estimated accuracies while accurate ones are expanded to improve the fused DOA accuracy. To understand this phenomenon, Fig. 5 plots the analytical expression of the estimation accuracy \hat{P}_{k_m} with respect to segments. In

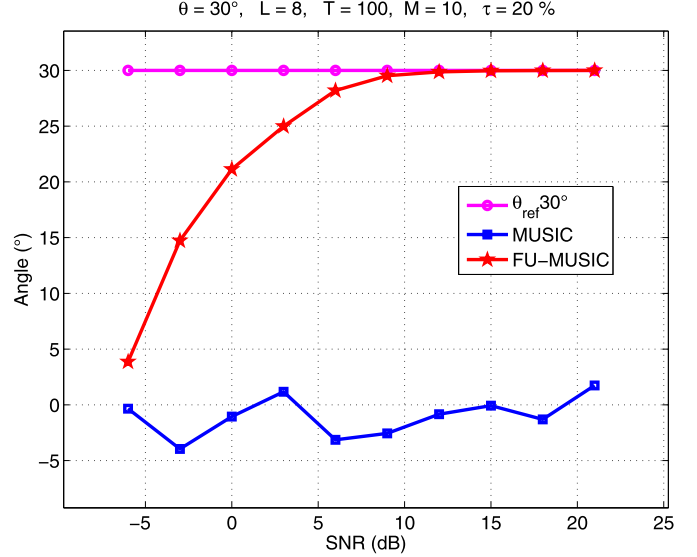


Fig. 3. DOA estimation vs SNR.

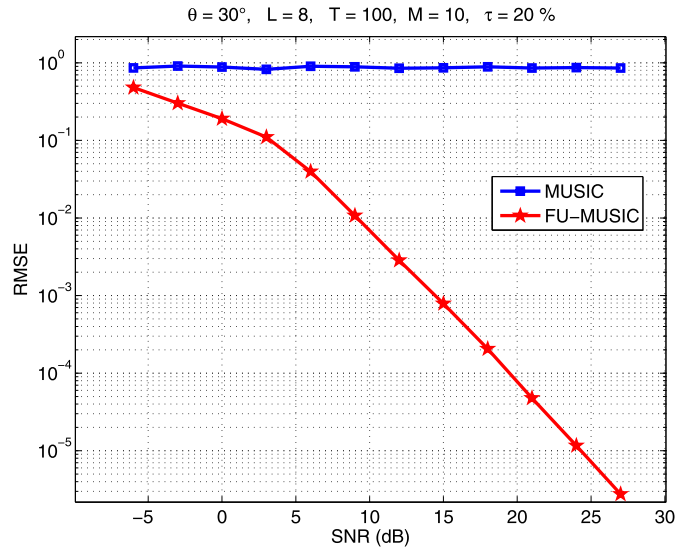


Fig. 4. DOA estimation vs SNR.

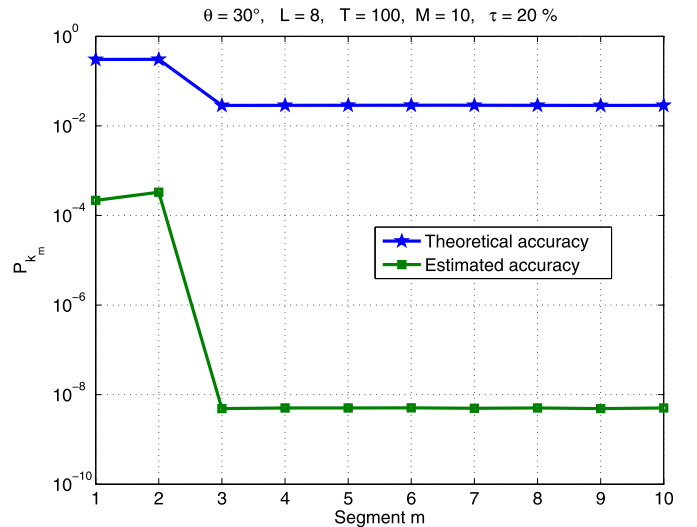


Fig. 5. Estimation accuracy vs segments: only segments 1 & 2 are corrupted by bursts.

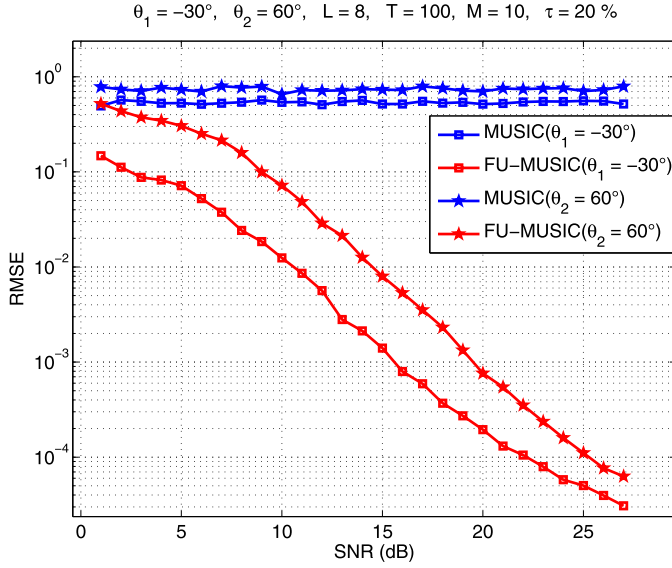


Fig. 6. DOA estimation vs SNR.

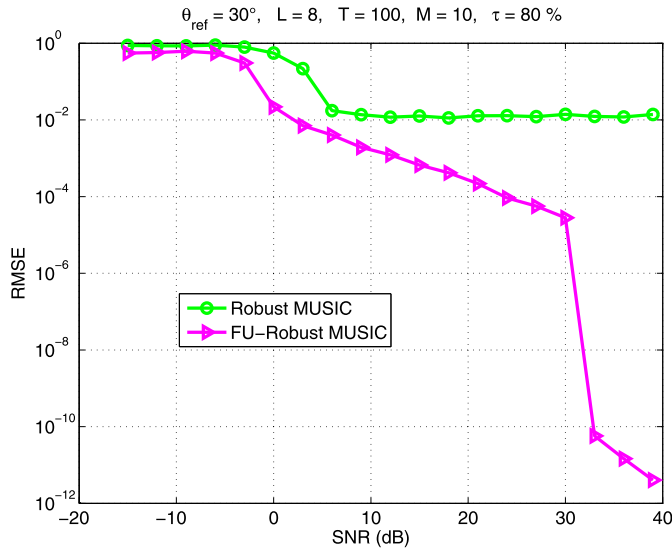


Fig. 7. DOA estimation vs SNR.

the legend of Fig. 5, Theoretical accuracy refers to the accuracy P_{k_m} obtained with the actual values of θ_k , σ_n^2 , \mathbf{U}_n , \mathbf{U}_s and \mathbf{V}_s while Estimated accuracy refers to the estimated accuracy \hat{P}_{k_m} (which is used in FU-MUSIC algorithm) obtained with the estimated values of $\hat{\theta}_k$, $\hat{\sigma}_n^2$, $\hat{\mathbf{U}}_n$, $\hat{\mathbf{U}}_s$ and $\hat{\mathbf{V}}_s$. Since we are not seeking a truthful estimate quality (i.e. P_{k_m}), \hat{P}_{k_m} is used herein just like an indicator of the estimation trend in a given segment. Without loss of generality, this figure is obtained by corrupting only the first and second segments through the numerical runs. One can observe that the two curves have the same trend which means that the estimated accuracy \hat{P}_{k_m} keeps the information on the erroneous DOA estimates and can be considered as a good indicator of the performance accuracies.

Fig. 6 shows the Root Mean Squared Error (RMSE) of the estimated DOAs of two sources located at -30° and 60° with respect to the normal to the sensor array. From this figure, one can observe that FU-MUSIC exhibits a good estimation of the two DOAs.

Scenario 2:

The Robust MUSIC is obtained by considering a robust estimate of the covariance matrix called NSCM (Normalized Sample Covariance

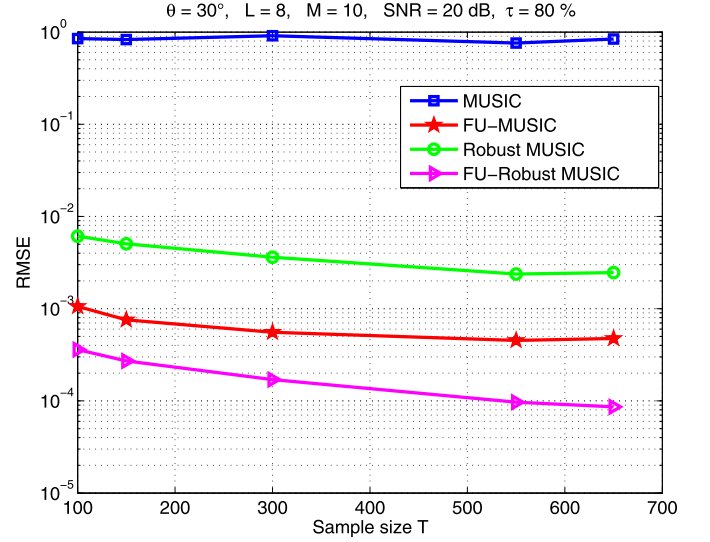


Fig. 8. DOA estimation vs sample size.

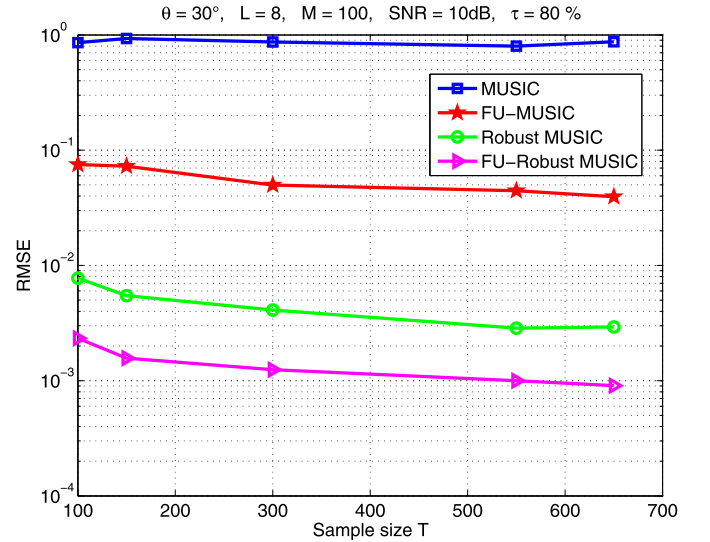


Fig. 9. DOA estimation vs sample size.

Matrix) given in [11,20]. In that case, the analytical expression of the NSCM is given by

$$\mathbf{y}(t) = \begin{cases} \frac{\mathbf{x}(t)}{\|\mathbf{x}(t)\|} & \text{if } \mathbf{x}(t) \neq \mathbf{0} \\ \mathbf{0} & \text{otherwise.} \end{cases} \quad (13)$$

$$\mathbf{R}_{NSCM} = \frac{L}{T} \sum_{t=1}^T \mathbf{y}(t) \mathbf{y}^H(t) \quad (14)$$

To derive the Robust MUSIC algorithm, one extract the minor subspace from the eigen-decomposition of the NSCM ' \mathbf{R}_{NSCM} '. Fig. 7 is given by considering Robust MUSIC algorithm and its improved version obtained using data fusion concept, the resulting algorithm is referred to as FU-Robust MUSIC. From this figure, one can see that if a robust DOA algorithm such as Robust MUSIC is combined with our approach, the latter improves significantly its performance. More precisely, we investigate the performance with respect to the sample size T . As shown in Fig. 8, both FU-MUSIC and FU-Robust MUSIC outperform MUSIC and Robust MUSIC for SNR = 20 dB. Also, for a lower SNR (10 dB), Robust MUSIC performs better than FU-MUSIC and FU-Robust MUSIC outperforms Robust MUSIC as shown in Fig. 9.

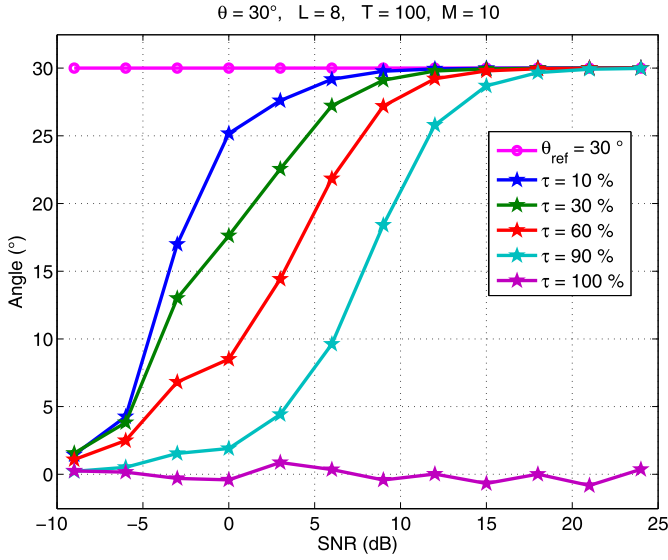


Fig. 10. DOA estimation vs rate of affected data.

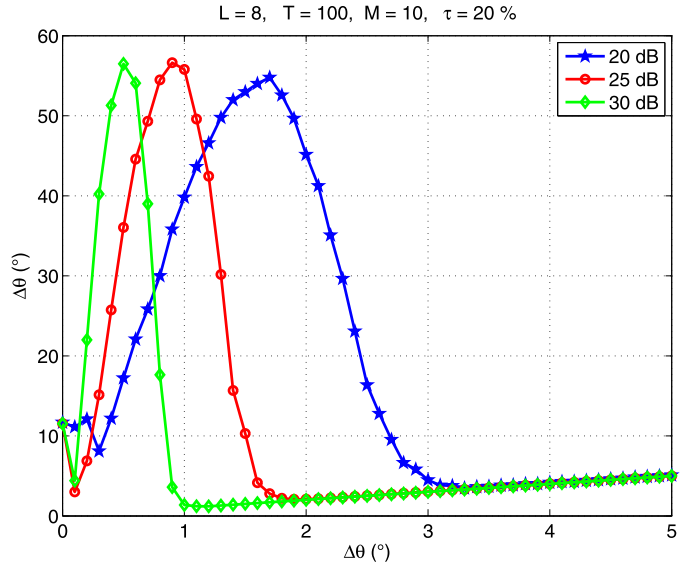


Fig. 12. DOA estimation of two closely spaced sources.

SNR = 30 dB. One can observe that the Angular Resolution Limit (ARL) [30] of FU-MUSIC for SNR = 20 dB is $\Delta\theta = 3^\circ$. For higher SNR values, one gets a lower ARL. According to Fig. 12, the ARL of the proposed algorithm for SNR = 25 dB and SNR = 30 dB are $\Delta\theta = 1.5^\circ$ and $\Delta\theta = 1^\circ$, respectively.

6. Conclusion

This paper introduces a new effective approach based on data fusion concepts and on the results of the theoretical performance analysis of DOA algorithms. To our knowledge, it is the first time that theoretical performance analysis results are used to develop new algorithms for the DOA estimation. The handled numerical experiments reveal that the proposed approach is robust with respect to non-homogeneous environment. Also, the new algorithm has much lower computational complexity with respect to the original algorithm complexity, thanks to the partitioning of the observed data into small size segments. This complexity reduction allows a real time implementation of the new proposed algorithm. Moreover, the considered approach, herein, can be generalized to other DOA estimation algorithms (e.g. ESPRIT) or even a robust form of them when the analytical expression of their estimated accuracy is available.

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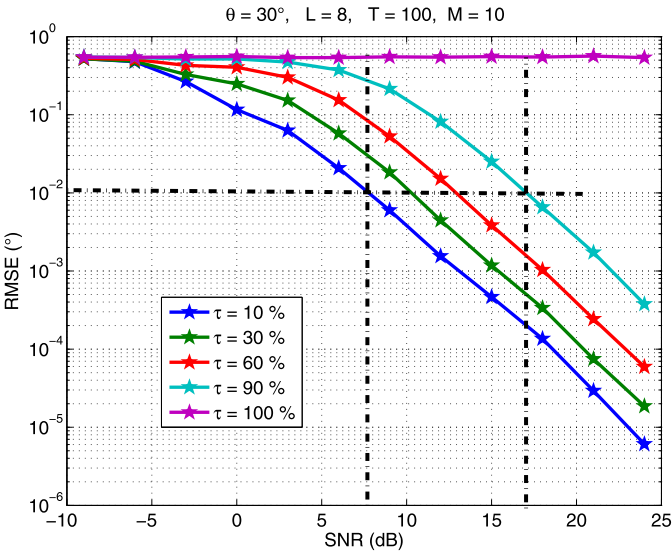


Fig. 11. DOA estimation vs rate of affected data.

Scenario 3:

Now, we consider only the proposed FU-MUSIC algorithm and we evaluate its performance with respect to the affected data rate (τ). Figs. 10 and 11 show the DOA estimate and the corresponding RMSE obtained through FU-MUSIC when bursts affect the observation data at different rates $\tau = 10\%$, $\tau = 30\%$, $\tau = 60\%$, $\tau = 90\%$ and $\tau = 100\%$. The obtained results reveal that FU-MUSIC is efficient even when 90% of the observation data are affected by bursts. Also, one can observe that when more data are affected by bursts, higher SNR values are needed to compensate the effect of erroneous DOA estimates. For example, one needs SNR = 7.5 dB to get $\text{RMSE} = 10^{-2}$ rad when the affected data rate is $\tau = 10\%$ while SNR = 17 dB is required to obtain the same estimation accuracy when $\tau = 90\%$.

Scenario 4:

Herein, we are interested in the ability of FU-MUSIC to distinguish between two closely spaced sources. Two sources are considered, the first one is located at $\theta_1 = 30^\circ$ and the second source is located at $\theta_2 = \theta_1 + \Delta\theta$ where $\Delta\theta = 0.01^\circ, \dots, 5^\circ$. Fig. 12 presents the separation angle estimate $\hat{\Delta\theta}$ for SNR = 20 dB, SNR = 25 dB and

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