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Multiple snapshot direct data domain approach and ESPRIT method for direction of arrival estimation

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Abstract

In this study, the performance of the matrix pencil (MP) method and the estimation of signal parameters using rotational invariance technique (ESPRIT) method have been compared under varying number of snapshots taken. The matrix pencil method applies the technique directly to the data to estimate the poles, and works well under the correlated signal case, as opposed to ESPRIT, a statistical subspace based estimation technique, which will work after applying some additional spatial smoothing techniques to tackle the ill conditioning problem of the covariance matrix. Simulation results are provided to show the performance of these two methods.

Keywords: Matrix pencil; Multiple snapshots; ESPRIT

1. Introduction

In the literature, there are many algorithms that exist to find the direction of arrival (DOA) of the sources coming to the antenna arrays. Each method has its advantages and disadvantages. Super resolution techniques that are based on the eigen-structure of the input covariance matrix including multiple signal classification (MUSIC), root-MUSIC, and estimation of signal parameters via rotational invariance techniques (ESPRIT) generate the high resolution DOA estimation. The MUSIC algorithm proposed by Schmidt [1] returns the pseudo-spectrum at all frequency samples. Conventional signal processing algorithms using the covariance matrix work on the premise that the signals impinging on the array are not fully correlated or coherent. Under uncorrelated conditions, the source covariance matrix satisfies the full rank condition, which is the basis of the eigen-decomposition. Many techniques involve modification of the covariance matrix through a preprocessing scheme called spatial smoothing. Sarkar [2] utilized the matrix pencil to get the DOA of the signals in a coherent multi-path environment. In the matrix pencil method, based on the spatial samples of the data, the analysis is done on a snapshot-by-snapshot basis, and therefore nonstationary environments can be handled easily. Unlike the conventional covariance matrix techniques, the matrix pencil method can find DOA easily in the presence of multi-path coherent signal without performing additional processing of spatial soothing.

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This paper focuses on comparison of MP and ESPRIT methods. These two methods differ in the fact that MP method is directly applied on the data itself, where in ESPRIT; the covariance matrix is used to estimate the poles, and corresponding DOA. The rest of the paper is organized as follows. In Section 2, MP method is explained in detail and formulation is given for the multiple snapshot case. A short overview is given about ESPRIT in Section 3. Simulations are given in Section 4, followed by the conclusions.

2. Matrix pencil method

The narrowband sources located in the far field of a uniformly spaced array consisting of isotropic omni directional point sensors radiating in free space is considered. This results in a uniform linear array (ULA). The focus here is to use the unitary transform to convert the complex matrices used in the MP formulation to real matrices and use these matrices to estimate the DOA of multiple signals simultaneously impinging on the ULA. The vector $x(n)$ is the set of voltages measured at the feed point of the antenna elements of the ULA. Therefore, $x(t)$ can be modeled by a sum of complex exponentials, i.e.,

$$y(t) = x(t) + n(t) = \sum_{i=1}^{M} R_i e^{s_i t} + n(t), \tag{1}$$

where $y(t) =$ observed voltages at a specific instance t. $n(t) =$ noise associated with the observation $x(t) =$ actual noise free signal. Therefore, one can write the sampled signal as

$$y(p) = \sum_{i=1}^{M} R_i z_i^p + n(p), \quad p = 0, 1, \ldots, N-1, \tag{2}$$

where

$$z_i = e^{j\frac{2\pi}{\lambda} d \sin(\theta)}, \quad i = 1, 2, \ldots, M. \tag{3}$$

In this study, it has been assumed that the damping factor α_i is equal to zero. The objective is to find the best estimates for θ. Let us consider the matrix Y, which is obtained directly from $x(p)$. Y is a Hankel matrix, and each column of Y is a windowed part of the original data vector, $\{x(0) \quad x(1) \quad x(2) \quad \ldots \quad x(N-1)\}$. It is assumed that there are N data samples.

$$Y = \begin{bmatrix} x(0) & x(1) & \ldots & x(L-1) \\ x(1) & x(2) & \ldots & x(L) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L) & x(N-L+1) & \ldots & x(N-1) \end{bmatrix}_{(N-L+1)\times(L)}. \tag{4}$$

The parameter L is called the pencil parameter. L is chosen between $N/3$ and $N/2$ for efficient noise filtering [3]. The variance of the estimated values of R_i and z_i will be minimal, if the values of L are chosen in this range. From the matrix Y, we can define two sub-matrices, say

$$Y_a = \begin{bmatrix} x(0) & x(1) & \ldots & x(L-1) \\ x(1) & x(2) & \ldots & x(L) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L-1) & x(N-L) & \ldots & x(N-2) \end{bmatrix}_{(N-L)\times(L)}, \tag{5}$$

$$Y_b = \begin{bmatrix} x(1) & x(2) & \ldots & x(L) \\ x(2) & x(3) & \ldots & x(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L) & x(N-L+1) & \ldots & x(N-1) \end{bmatrix}_{(N-L)\times(L)}. \tag{6}$$

One can also write

$$Y_a = Z_a R Z_b, \tag{7}$$

$$Y_b = Z_a R_0 Z_0 Z_b, \tag{8}$$

where

$$Z_a = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{(N-L-1)} & z_2^{(N-L-1)} & \cdots & z_M^{(N-L-1)} \end{bmatrix}_{(N-L)\times(M)}, \tag{9}$$

$$Z_b = \begin{bmatrix} 1 & z_1 & \cdots & z_1^{(L-1)} \\ 1 & z_2 & \cdots & z_2^{(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_M & \cdots & z_M^{(L-1)} \end{bmatrix}_{M\times L}, \tag{10}$$

$$Z_0 = \operatorname{diag}[z_1, z_2, \ldots, z_M], \tag{11}$$

$$R_0 = \operatorname{diag}[R_1, R_2, \ldots, R_M]. \tag{12}$$

Now, let us consider the matrix pencil

$$Y_b - \lambda Y_a = Z_a R_0 [Z_0 - \lambda I] Z_b. \tag{13}$$

Here I is the $M \times M$ identity matrix. One can show that the rank of $Y_b - \lambda Y_a$ will be M, provided that $M \leqslant L \leqslant N - M$ [1]. However, if $\lambda = z_i$, $i = 1, 2, \ldots, M$ the ith row of $[Z_0 - \lambda I]$ is zero, then the rank of this matrix is $M - 1$. Therefore, the parameters z_i can be found as the generalized eigenvalues of the matrix pair $\{Y_a, Y_b\}$. So, the solution to the problem can be reduced to an ordinary eigenvalue problem, and z_i will be the eigenvalues of

$$\left\{Y_a^+ Y_b - \lambda I\right\}, \tag{14}$$

where Y_a^+ is the Moore–Penrose pseudo inverse of Y_a, which is defined as

$$Y_a^+ = \left\{Y_a^H Y_a\right\}^{-1} Y_a^H. \tag{15}$$

The DOA is obtained by $\theta_i = \sin^{-1}(\operatorname{Im}(\log z_i)/\pi d)$, where $z_i = e^{j\frac{2\pi}{\lambda} d \sin(\theta)}$.

For the noisy data, the singular value decomposition is useful to reduce some of the noise effect. The matrix Y can be written as

$$Y = U \Sigma V^H. \tag{16}$$

Here, U and V are unitary matrices whose columns are the eigenvectors of YY^H and $Y^H Y$, respectively. Σ is the singular values of Y, which are located on the main diagonals in the descending order $\sigma_1 \geqslant \sigma_2 \geqslant \cdots \geqslant \sigma_{\min}$. If the data is noiseless, the first M singular values are nonzero, the rest is zero, where

$$\sigma_i > 0, \quad i = 1, \ldots, M,$$

$$\sigma_i = 0, \quad i = M+1, \ldots, \min\bigl(L,\ (N-L+1)\bigr).$$

If the data is noisy, M needs to be estimated. The ratio of each of the singular value to the largest one determines the value of M. The criteria $\sigma_i/\sigma_{\max} \approx 10^{-r}$ is used to find M, where r is the number of accurate significant decimal digits of the data vector $y(n)$. For data contaminated with noise, the noise effect is reduced by discarding the eigenvectors corresponds to the noise. After computing SVD of data matrix Y, the matrix space is divided into two subspaces. They are signal subspace and noise subspace. The matrices Y_a and Y_b are constructed from the signal subspace matrix.

2.1. Multiple snapshot case

The formulation is given for the one snapshot case before. In the case of multiple snapshots, the Hankel matrix structure is constructed for each snapshot. Let say, there are K snapshots, for each snapshot the Hankel matrix is constructed from (17). These Hankel matrices are appended side by side as in (18),

$$Y_k = \begin{bmatrix} x_k(0) & x_k(1) & \dots & x_k(L-1) \\ x_k(1) & x_k(2) & \dots & x_k(L) \\ \vdots & \vdots & \ddots & \vdots \\ x_k(N-L) & x_k(N-L+1) & \dots & x_k(N-1) \end{bmatrix}_{(N-L+1)\times(L)}, \quad k = 1, 2, \dots, K, \tag{17}$$

$$Y_E = [Y_1 \quad Y_2 \quad \dots \quad Y_K]_{(N-L+1)\times(KL)}. \tag{18}$$

The size of the extended matrix Y_E is $(N-L+1) \times (KL)$. The matrices Y_{E1} and Y_{E2} can be written as below by deleting the first and last row as done before in one single snapshot MP.

$$Y_{E1} = \begin{bmatrix} x_1(0) & x_1(1) & \dots & x_1(L-1) & x_2(0) & x_2(1) & \dots & x_2(L-1) \\ x_1(1) & x_1(2) & \dots & x_1(L) & x_2(1) & x_2(2) & \dots & x_2(L) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1(N-L-1) & x_1(N-L) & \dots & x_1(N-2) & x_2(N-L-1) & x_2(N-L) & \dots & x_2(N-2) \end{matrix}$$

$$\begin{matrix} \dots \end{matrix} \begin{matrix} x_K(0) & x_K(1) & \dots & x_K(L-1) \\ x_K(1) & x_K(2) & \dots & x_K(L) \\ \vdots & \vdots & \ddots & \vdots \\ x_K(N-L-1) & x_K(N-L) & \dots & x_K(N-2) \end{bmatrix}, \tag{18a}$$

and

$$Y_{E2} = \begin{bmatrix} x_1(1) & x_1(2) & \dots & x_1(L) & x_2(1) & x_2(2) & \dots & x_2(L) \\ x_1(2) & x_1(3) & \dots & x_1(L+1) & x_2(2) & x_2(3) & \dots & x_2(L+1) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1(N-L) & x_1(N-L+1) & \dots & x_1(N-1) & x_2(N-L) & x_2(N-L+1) & \dots & x_2(N-1) \end{matrix}$$

$$\begin{matrix} \dots \end{matrix} \begin{matrix} x_K(1) & x_K(2) & \dots & x_K(L) \\ x_K(2) & x_K(3) & \dots & x_K(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ x_K(N-L) & x_K(N-L+1) & \dots & x_K(N-1) \end{bmatrix}. \tag{18b}$$

The matrix pencil is written by using these two equations. The same procedure is applied to (18a) and (18b). This extended data matrix will increase the computational complexity in singular value decomposition calculation.

3. Esprit

ESPRIT is another parameter estimation technique, based on the fact that in the steering vector, the signal at one element is a constant phase shift from the previous element. ESPRIT is a subspace based estimation technique where the poles are computed using Covariance matrix [4]. The data model consists of M signals. By using the data model (2).

The correlation matrix of y can be written as

$$R = E[yy^H] = E\big[(x+n)(x+n)^H\big], \tag{19}$$

$$R = AR_sA^H + R_n, \tag{20}$$

where A is defined as $A = [a(\theta_1) \quad a(\theta_2) \quad \dots \quad a(\theta_M)]$, and $a(\theta) = [1 \quad e^{j\theta} \quad \dots \quad e^{j(M-1)\theta}]$. Assume that the signal and noise are uncorrelated. R_s is the signal covariance matrix and the rank is M, equal to the number of the signals coming to the array, and computed as $R_s = \frac{1}{N}\sum_{n=1}^{N} x(n)x^H(n)$. R_n is the noise covariance matrix, and $R_n = \sigma^2 I$. In real application, we only have a limited number of time samples, so covariance matrix can be computed from these limited numbers of samples as

$$R \approx \frac{1}{N}\sum_{n=1}^{N} y(n)y^H(n). \tag{21}$$

Let E_s contain the M largest eigenvectors corresponds signal subspace of the matrix R. E_s and A have the same range, so we can write $E_s = AT$, where T is a nonsingular matrix. Let us define two matrices, E_1 and E_2, by deleting first and last row of the covariance matrix, respectively,

$$E_2 = E_1\Phi, \tag{22}$$

where $\Phi = T^{-1}\Omega T$. The eigenvalues of Φ will give the poles that we are looking for. Matrix pencil and ESPRIT have some similarities. Both algorithms estimate a diagonal matrix whose entries are the poles of the system. The major difference is that ESPRIT works with the signal subspace as defined by the correlation matrix, while matrix pencil works with the data directly. MP has a significant saving in terms of computation load and very suitable for real time applications. Increasing the size of the Hankel matrix in MP method will increase the computational load [5].

4. Simulations

In this section, the computer simulation results are given to illustrate the performance of the methods. The noisy signal model is formulated from (2). $n(k)$ is treated as a zero mean Gaussian white noise with variance σ^2. Uniformly spaced arrays (ULA) of omni directional isotropic point sensors are considered in this study. The distance between any two elements of the ULA is half a wavelength. $y(k)$ is the voltage induced at each of the antenna elements, for $k = 0, 1, \ldots, N-1$. It is assumed that there are 8 antenna elements and two signals are impinging on the array with amplitudes $R_1 = R_2 = 1$. The signals are coming from 30° and 45°.

The performance of these two estimation methods is compared under different number of snapshots. The RMSE of the estimators are plotted in Fig. 1 for single snapshot, 8 snapshots and 16 snapshots cases. 500 Monte Carlo simulations are conducted. As can be seen from Fig. 1, MP method has a lower variance than the ESPRIT. Noise for each run is independent of each other. The optimum value for the pencil length, L, is chosen to be 4 for efficient noise filtering.

The histogram plot is given for 1000 trials for two signals for single snapshot and 8 snapshot cases in Figs. 2–9.

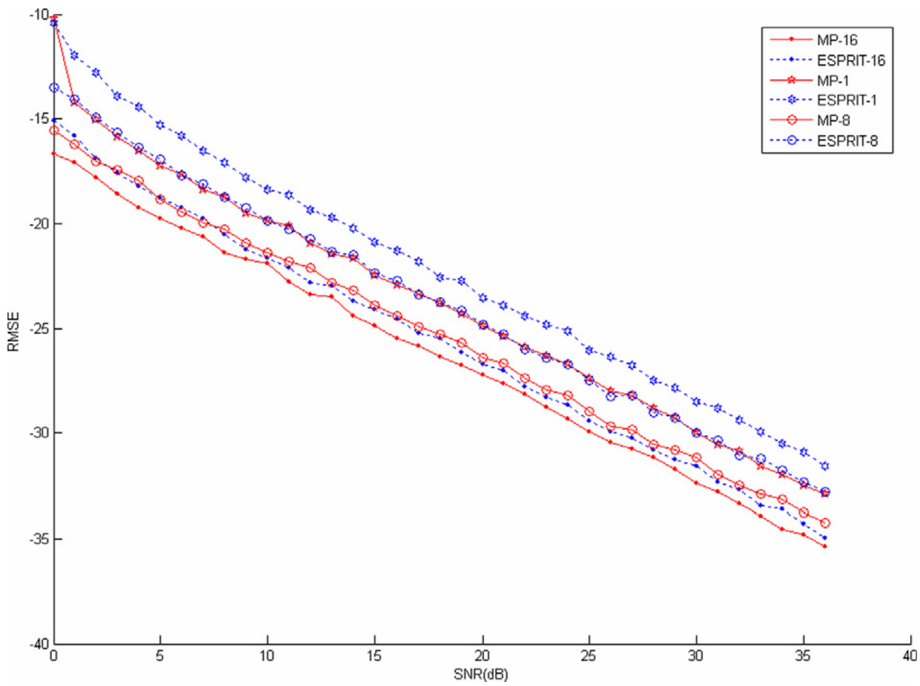

Fig. 1. Comparing of the performance of MP and ESPRIT for different number of snapshots.

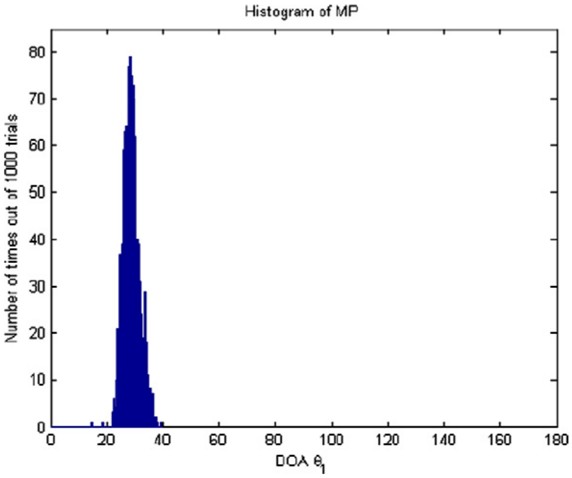

Fig. 2. Accuracy of MP, for θ_1, one snapshot.

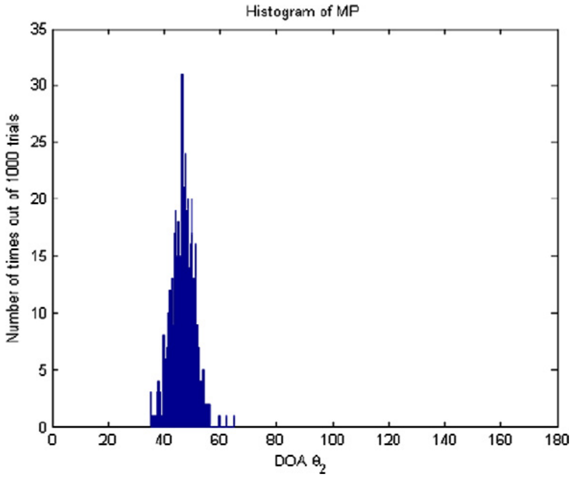

Fig. 3. Accuracy of MP, for θ_2, one snapshot.

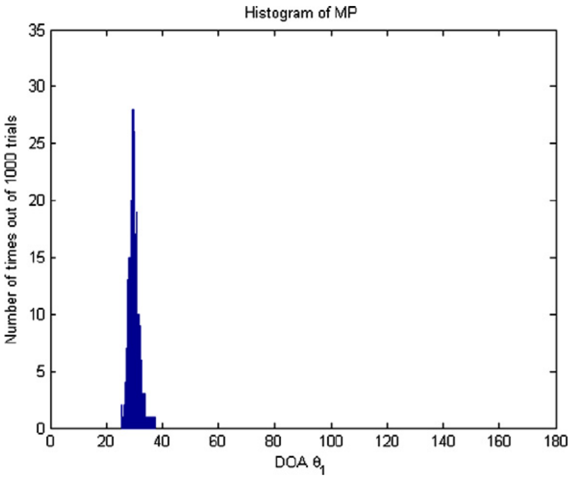

Fig. 4. Accuracy of MP, for θ_1, 8 snapshots.

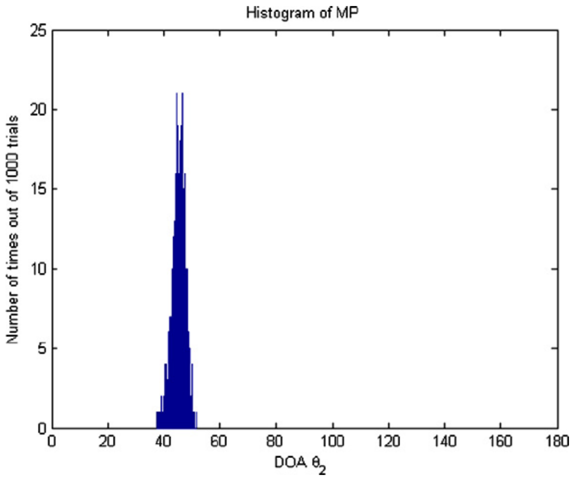

Fig. 5. Accuracy of MP, for θ_2, 8 snapshots.

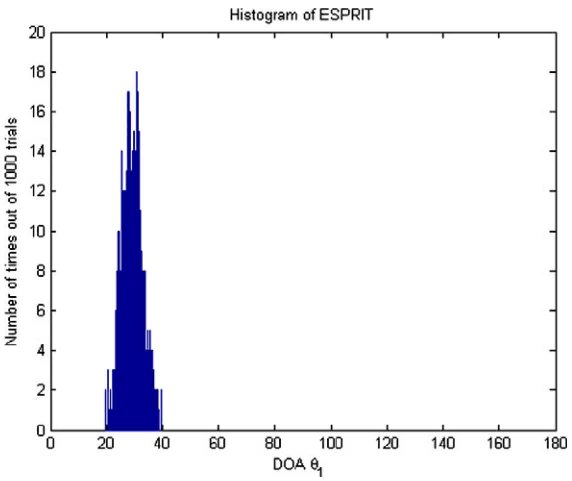

Fig. 6. Accuracy of ESPRIT, for θ_1, one snapshot.

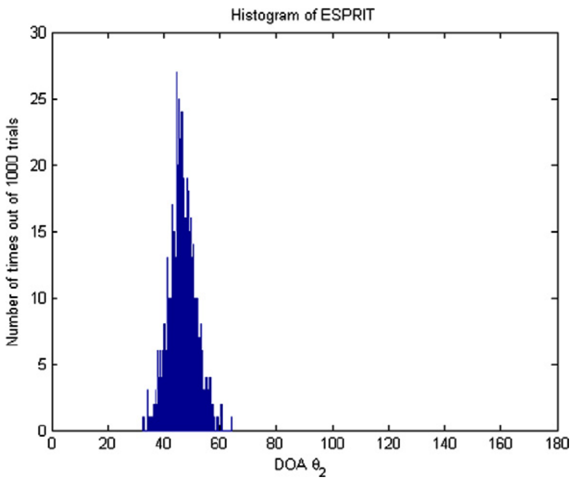

Fig. 7. Accuracy of ESPRIT, for θ_2, one snapshot.

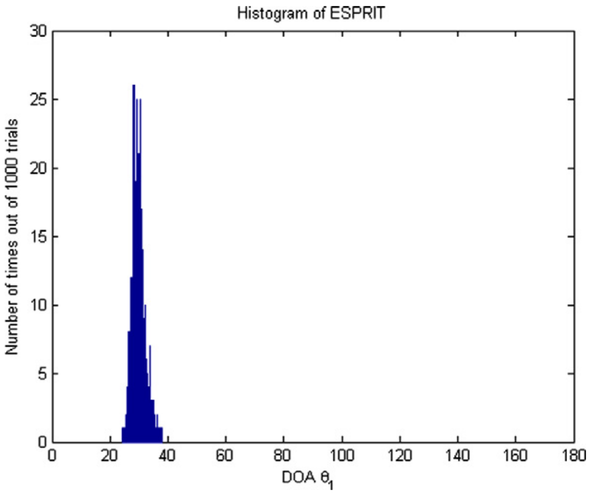

Fig. 8. Accuracy of ESPRIT, for θ_1, 8 snapshots.

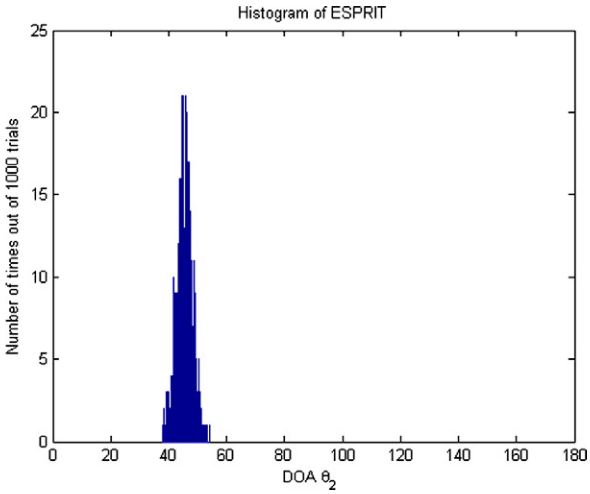

Fig. 9. Accuracy of ESPRIT, for θ_2, 8 snapshots.

5. Conclusion

In this paper, it has been shown that the as we take more samples for the direct data domain approach the variance of the estimator gets smaller. The matrix pencil method results are compared with statistical ESPRIT method under the small number of samples. Matrix pencil method outperforms the ESPRIT. The computational load increases for the MP method since the matrix size increased by appending the Hankel matrices side by side. MP works even under the correlated signal case, as opposed the ESPRIT it loses the full rank requirement and needs some additional spatial smoothing techniques so that it will work for the correlated signal case.

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