Int. J. Electron. Commun. (AEÜ) 71 (2017) 131–138

Contents lists available at ScienceDirect

International Journal of Electronics and Communications (AEÜ)

journal homepage: www.elsevier.com/locate/aeue

Regular paper

Improved 2D direction of arrival estimation with a small number of elements in UCA in the presence of mutual coupling ☆

Guo-jun Jiang [a], Ying-ning Dong [a,b], Xing-peng Mao [a,b,*], Yong-tan Liu [a,b]

[a] School of Electronics and Information Engineering, Harbin Institute of Technology, Harbin, China
[b] Collaborative Innovation Center of Information Sensing and Understanding at Harbin Institute of Technology, Harbin, China

ARTICLE INFO

Article history:
Received 13 May 2016
Accepted 13 October 2016

Keywords:
2D DOA estimation
Mutual coupling
Beamspace transformation
Uniform circular array
Rank reduction theory (RARE)

ABSTRACT

With a small number of elements in uniform circular arrays (UCAs), the performance of beamspace transformation (BT) based methods for 2D direction-of-arrival (DOA) estimation in the presence of mutual coupling suffers serious degradation. The performance breakdown is associated with the residual term which is introduced by the BT. To solve this problem, a novel algorithm which modifies the beamspace data is proposed in this paper. Based on the estimated azimuth angles obtained by a UCA-RARE-like method and the estimated elevation angles obtained by a new searching method, the beamspace data is iteratively revised by removing its residual component. Thus, unbiased 2D DOA estimates can be obtained depending on the revised beamspace data. Simulation results are given to verify the effectiveness of the proposed algorithm and a comparison is made with results from existed methods.

1. Introduction

Uniform circular array (UCA) has attracted great attention for direction-of-arrival (DOA) estimation as it can simultaneously estimate both azimuth and elevation angles. Estimating 2D DOAs with UCA [1–5] is of practical importance in a variety of applications, such as mobile communications, sonar, and radar. However, the well-known subspace-based algorithm, i.e., the 2D searching method over the MUSIC-spectrum, has high computational cost. To come up with computationally efficient DOA estimation for UCAs, the beamspace transformation (BT) [1,6], which is based on the phase mode excitation, is applied to transform the element space manifold without Vandermode structure into the desired beamspace manifold with Vandermode structure. Then various algorithms based on the BT are introduced, such as UCA RB-MUSIC [1], UCA-ESPRIT [1], and UCA-RAnk REduction (RARE) [2,3], among which the UCA-RARE is more attractive since it estimates the azimuth angle decoupled from the elevation angle.

The mutual coupling effect between elements in the array can destroy the underlying model assumptions for DOA estimation [7–12]. In practice, the mutual coupling is rather significant in UCAs. In [13], a hybrid algorithm, i.e., UCA-RARE/Root-MUSIC, is presented to yield bias-free 2D DOA estimation in the presence of mutual coupling. Nevertheless, the inherent ambiguous estimates by the UCA-RARE have not been resolved. A decoupled method considering elevation-dependent mutual coupling is devised by using compact UCAs in [14]. Depending on the derived formulation of the beamspace array manifold, the azimuth estimation can be decoupled from the elevation estimation without the exact knowledge of the mutual coupling [14]. And a 1D search algorithm is performed for elevation estimates. In [15], a modified UCA-RARE algorithm for azimuth angle estimates and a rooting algorithm for elevation angle estimates are applied for sparse UCAs in the presence of mutual coupling.

However, the aforementioned algorithms do not remove the error introduced by the BT. The BT performs appropriately only when the number of elements in a UCA is sufficiently large enough to avoid aliasing in a steering vector of the mode space. Otherwise, the residual term introduced by the BT brings unignorable error to the DOA estimation. In [6], a technique which eliminates the residual term is proposed to improve the performance for DOA estimation. Nevertheless, only the azimuth estimates and the first-order approximation of the residual term are considered in absence of mutual coupling. In [15], an appropriate truncated degree is provided to reduce the truncation errors introduced by the manifold decomposition technique. However, it is only used to enhance the estimate accuracy for elevation angles.

☆ This work was supported by the National Nature Science Foundation of China (61171180); and the Fundamental Research Funds for the Central Universities (Grant No. HIT. MKSTISP. 2016 13 and HIT. MKSTISP. 2016 26).

* Corresponding author at: School of Electronics and Information Engineering, Harbin Institute of Technology, Harbin 150001, China.

E-mail addresses: hitjiangguojun@163.com (G.-j. Jiang), dongyn@hit.edu.cn (Y.-n. Dong), mxp@hit.edu.cn (X.-p. Mao), liuyt@hit.edu.cn (Y.-t. Liu).

http://dx.doi.org/10.1016/j.aeue.2016.10.012
1434-8411/

In this paper, we propose an improved algorithm for 2D DOA estimation with a small number of elements in the presence of mutual coupling. Firstly, initial azimuth angles are obtained by the UCA-RARE-like method proposed in [14] and initial elevation angles are estimated by our proposed elevation–estimation method. Then, the beamspace data is modified based on the initial azimuth and elevation estimates, and DOAs can be obtained depending on the modified data. Finally, an iterative technique is performed to enhance the accuracy of DOA estimation.

The paper is organized as follows. Section 2 reviews the fundamental theory of both azimuth angle and elevation angle estimation. Section 3 presents the problem formulation. In Section 4, a new searching method for elevation angle estimation is proposed. Then a novel algorithm for 2D DOA estimation is theoretically derived. In Section 5, error analyses for both azimuth and elevation angles are presented. Simulation results and the comparison with existed methods are given in Section 6. Finally, Section 7 concludes this paper.

2. Fundamental theory

2.1. Array signal model

Assume a UCA composed of P identical elements. Consider K far-field narrowband signals impinging on the UCA from directions $\{(\theta_k,\phi_k),\quad k=1,2,\ldots,K\}$, with $\theta_k\in[0,\pi/2]$ indicating the elevation angle of the kth signal measured from the Z-axis and $\phi_k\in[0,2\pi)$ denoting the corresponding azimuth angle measured from the X-axis counter-clockwise. The received data vector at time t in absence of mutual coupling is given by

$$\mathbf{x}(t)=\mathbf{A}(\theta,\phi)\mathbf{s}(t)+\mathbf{n}(t), \tag{1}$$

where $\mathbf{s}(t)=[s_1(t),s_2(t),\ldots,s_K(t)]^{\mathrm{T}}\in\mathbb{C}^{K\times1}$is the signal vector with $(\cdot)^{\mathrm{T}}$ denoting the transposition operator, $\mathbf{n}(t)\in\mathbb{C}^{P\times1}$ is the additive white Gaussian noise vector, and the $P\times K$ element space manifold matrix is

$$\mathbf{A}(\theta,\phi)=[\mathbf{a}(\theta_1,\phi_1),\mathbf{a}(\theta_2,\phi_2),\ldots,\mathbf{a}(\theta_K,\phi_K)], \tag{2}$$

with $\mathbf{a}(\theta_i,\phi_i)=[e^{j2\pi R\sin(\theta_i)\cos(\phi_i-r_1)/\lambda},\ldots,e^{j2\pi R\sin(\theta_i)\cos(\phi_i-r_P)/\lambda}]^{\mathrm{T}}$. Here, R is the array radius, $r_p=2\pi(p-1)/P$ is the element location with $p=1,\ldots,P$, and λ is the wavelength.

In the presence of mutual coupling, the steering vector is expressed as [14]

$$\tilde{\mathbf{a}}(\theta,\phi)=\mathbf{C}(\theta)\mathbf{a}(\theta,\phi), \tag{3}$$

where $\mathbf{C}(\theta)$ is the elevation-dependent mutual coupling matrix (MCM) of a UCA. Because of the circulant MCM and the symmetry structure of $\mathbf{c}(\theta)$, i.e., the first row of the MCM, $\tilde{\mathbf{a}}(\theta,\phi)$ can be rewritten as [14]

$$\tilde{\mathbf{a}}(\theta,\phi)=\mathbf{a}(\theta,\phi)\otimes\mathbf{c}(\theta), \tag{4}$$

where $\otimes$ is the periodic convolution of discrete sequences.

The BT is utilized to get a beamspace manifold with Vandermonde structure and computationally efficient rooting algorithms can be introduced based on it. In absence of mutual coupling, the beamspace steering vector is

$$\mathbf{b}(\theta,\phi)=\mathbf{W}\mathbf{a}(\theta,\phi)=\mathbf{T}(z)\mathbf{g}(\theta)+\mathbf{d}(\theta,\phi), \tag{5}$$

$$[\mathbf{g}(\theta)]_k=j^kJ_k(kr\sin\theta),\quad k=1,2,\ldots,M+1, \tag{6}$$

$$\mathbf{T}(z)=\begin{bmatrix}\mathbf{Q}(z) & 0\\ 0 & 1\\ \prod\mathbf{Q}(1/z) & 0\end{bmatrix}, \tag{7}$$

$$\mathbf{Q}(z)=diag\{z^{-M},z^{-M+1},\ldots,z^{-2},z^{-1}\}. \tag{8}$$

Here $z=e^{j\phi}$, $[\mathbf{W}]_{s,t}=1/\sqrt{P}e^{j2\pi st/P}$ with $s=-M,\ldots,M$ and $t=0,\ldots,P-1$, $M=\lceil kR\rceil$ is the highest order mode with $\lceil\cdot\rceil$ denoting the ceiling operation, J_k is the first kind Bessel function of order k, $\prod$ is an anti-diagonal matrix, and $\mathbf{d}(\theta,\phi)$ is a distortion term which can be neglected when the antenna distance is less than $\lambda/2$ [14].

2.2. Azimuth angle estimation

The UCA-RARE only relies on the symmetry properties of the UCA configuration, and can decouple azimuth estimation from elevation estimation in a search-free way. It is numerically efficient as it avoids the 2D exhaustive search over the MUSIC-spectrum. In [14], a UCA-RARE-like method for azimuth estimation in the presence of mutual coupling has been exploited. Recalling (5), the steering vector with elevation-dependence mutual coupling is

$$\begin{aligned}\tilde{\mathbf{b}}_1(\theta,\phi)&=\mathbf{W}\tilde{\mathbf{a}}(\theta,\phi)=\mathbf{W}(\mathbf{a}(\theta,\phi)\otimes\mathbf{z}(\theta))\\&=\begin{bmatrix}\mathbf{DFT}_{(M+1:-1:1)}[\mathbf{a}(\theta,\phi)]\\ \mathbf{IDFT}_{(2:M+1)}[\mathbf{a}(\theta,\phi)]\end{bmatrix}\odot\mathbf{m}(\theta)=\mathbf{W}\mathbf{a}(\theta,\phi)\odot\mathbf{m}(\theta)\\&\approx\mathbf{T}(z)\mathbf{g}'(\theta),\end{aligned} \tag{9}$$

where $\mathbf{m}(\theta)=\begin{bmatrix}\mathbf{DFT}_{(M+1:-1:1)}[\mathbf{z}(\theta)]\\ \mathbf{N}\cdot\mathbf{IDFT}_{(2:M+1)}[\mathbf{z}(\theta)]\end{bmatrix}=\begin{bmatrix}\mathbf{m}_1(\theta)\\ \mathbf{m}_2(\theta)\end{bmatrix}$ and $\mathbf{g}'(\theta)=\mathbf{m}_1(\theta)\odot\mathbf{g}(\theta)$ with $\odot$ indicating the Hadamard product of vectors. Then the beamspace data is

$$\begin{aligned}\mathbf{y}_1(t)&=\mathbf{W}\Big(\tilde{\mathbf{A}}(\theta,\phi)\mathbf{s}(t)+\mathbf{n}(t)\Big)\\&\approx[\mathbf{T}(z_1)\mathbf{g}'(\theta_1),\ldots,\mathbf{T}(z_K)\mathbf{g}'(\theta_K)]\mathbf{s}(t)+\mathbf{n}_1(t)\\&\approx\mathbf{B}_1(\theta,\phi)\mathbf{s}(t)+\mathbf{n}_1(t)\end{aligned} \tag{10}$$

where $\tilde{\mathbf{A}}(\theta,\phi)=[\tilde{\mathbf{a}}(\theta_1,\phi_1),\ldots,\tilde{\mathbf{a}}(\theta_K,\phi_K)]$ and $\mathbf{B}_1(\theta,\phi)=\left[\tilde{\mathbf{b}}_1(\theta_1,\phi_1),\ldots,\tilde{\mathbf{b}}_1(\theta_K,\phi_K)\right]$ are the element space and beamspace manifolds in the presence of mutual coupling, respectively, $\mathbf{n}_1(t)=\mathbf{W}\mathbf{n}(t)$ is white Gaussion noise with distribution $(0,\sigma^2\mathbf{I}_{2M+1})$, and $\mathbf{I}_{2M+1}$ is an identity matrix. The eigenvalue decomposition (EVD) of the beamspace data covariance matrix is

$$\begin{aligned}\mathbf{R_y}&=E\{\mathbf{y}_1(t)\mathbf{y}_1^{\mathrm{H}}(t)\}=\mathbf{B}_1(\theta,\phi)\mathbf{S}\mathbf{B}_1^{\mathrm{H}}(\theta,\phi)+\sigma^2\mathbf{I}_{2M+1}\\&=\mathbf{E_s}\Lambda_\mathbf{s}\mathbf{E_s}^{\mathrm{H}}+\mathbf{E_n}\Lambda_\mathbf{n}\mathbf{E_n}^{\mathrm{H}},\end{aligned} \tag{11}$$

where $\mathbf{S}=E\{\mathbf{s}(t)\mathbf{s}^{\mathrm{H}}(t)\}$ is the signal covariance matrix with $E\{\cdot\}$ standing for the expectation operator, $(\cdot)^{\mathrm{H}}$ indicates the Hermitian Transpose, and $\mathbf{E_s}$ and $\mathbf{E_n}$ denote the signal and noise subspaces, respectively. Then we have

$$\mathbf{g}'^{\mathrm{H}}(\theta)\mathbf{T}^{\mathrm{H}}(z)\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}\mathbf{T}(z)\mathbf{g}'(\theta)=0. \tag{12}$$

If the elevation angle is known, (12) can be expressed as a polynomial in $z=e^{j\phi}$, and a root-MUSIC method can be used to estimate the azimuth angles [16–18]. Nevertheless, there is no prior knowledge of the elevation angles. Then a function for azimuth estimation, which is similar to the UCA-RARE, is

$$f(\phi)=\arg\min_{\phi}\det\left[\mathbf{T}^{\mathrm{H}}(z)\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}\mathbf{T}(z)\right], \tag{13}$$

where $\det[\cdot]$ is the determinant of a matrix. In this case, the azimuth estimates can be achieved by the polynomial rooting of (13) without the exact knowledge of elevation angles and mutual coupling. However, spurious azimuth angles, e.g., $\phi_i+\pi$ for $\phi_i<\pi,\phi_j-\pi$ for $\phi_j>\pi$, and $(\phi_i+\phi_j)/2$ for $\theta_i=\theta_j$, are also estimated. They will be canceled in the elevation angle estimation.

2.3. Elevation angle estimation

To obtain the elevation angle estimates, a closed-form algorithm similar to UCA-ESPRIT is utilized in the original UCA-RARE algorithm [2]. However, there are some shortcomings that make it unsuitable in the presence of elevation-dependence mutual coupling [14]. A searching method depending on the null space of $\Psi(\phi) = \mathbf{T}^{H}(\phi)\mathbf{E_n}\mathbf{E_n^{H}}\mathbf{T}(\phi)$ is used in [14]. For each azimuth angle estimate, a null-space matrix $\Gamma(\phi)$ of $\Psi(\phi)$ is constructed, and then a 1D search for elevation parameter in the range $0^\circ \leqslant \theta \leqslant 90^\circ$ is utilized to find the elevation angles with their corresponding $\mathbf{g}'(\theta)$ belonging to $\Gamma(\phi)$. Note that for every spurious azimuth estimate, there is no $\mathbf{g}'(\theta)$ belonging to $\Gamma(\phi)$. In this way, all the spurious azimuth estimates can be detected and removed. Thus, the desired 2D DOAs can be achieved.

3. Problem formulation

Recall (5) and (9), the beamspace steering vector in the presence of the elevation-dependence mutual coupling can be rewritten as

$$\tilde{\mathbf{b}}_2(\theta,\phi) = \mathbf{Wa}(\theta,\phi)\odot\mathbf{m}(\theta) = \mathbf{T}(z)\mathbf{g}'(\theta) + \mathbf{d}(\theta,\phi)\odot\mathbf{m}(\theta). \tag{14}$$

Let $\varepsilon = \mathbf{d}(\theta,\phi)\odot\mathbf{m}(\theta)$ be the residual term of the beamspace steering vector in the presence of mutual coupling. According to (14), the beamspace steering vector contains two terms: $\mathbf{T}(z)\mathbf{g}'(\theta)$ and ε. With a large number of elements, the first term is dominant and is suitable for rooting methods, whereas the second term is small and is generally ignored. However, when the number of elements is not large enough, the residual term may have significant values and introduce errors for 2D DOA estimation, especially in the presence of mutual coupling [15].

Referring to (14), ε is not a constant and depends on the actual angles. As a result, the error in DOA estimation introduced by ε is related to the actual angles. The behavior of ε as a function of both elevation and azimuth angles is analyzed here. Consider a UCA consisted of nine monopoles elements with a radius $R = 0.7\lambda$. Here and in the following, the mutual coupling coefficients are given in Table 1. Fig. 1 depicts the residual term versus elevation and azimuth angles of the Mth excitation mode. It can be seen that the residual term keeps unchanged when the azimuth angle changes, whereas it increases as the elevation angle increases.

4. Proposed algorithm

4.1. Proposed elevation–estimation algorithm

The accuracy of elevation estimates obtained by the searching method in Section 2 can still be improved. In this section, we propose a new 1D search-based algorithm to enhance the accuracy for elevation estimation. In our new method for elevation estimation, the search function is

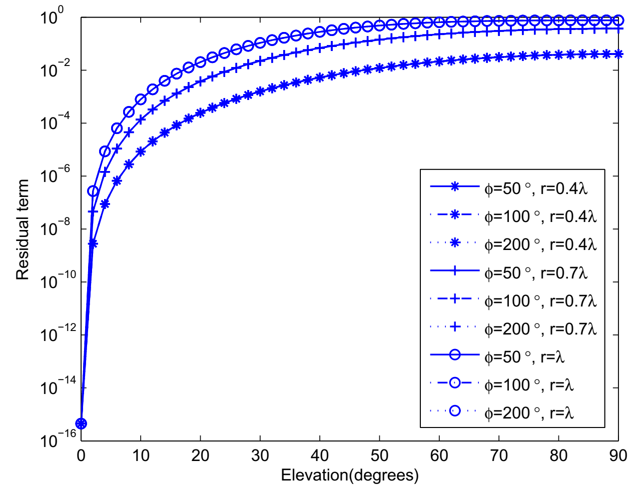

Fig. 1. Residual term versus elevation and azimuth angles.

$$f(\theta) = \mathbf{g}'^{H}(\theta)\mathbf{T}^{H}(\dot{z})\mathbf{E_n}\mathbf{E_n^{H}}\mathbf{T}(\dot{z})\mathbf{g}'(\theta), \tag{15}$$

where $\dot{z}$ denotes the azimuth angles that have been estimated, including the spurious estimates. For every estimated azimuth angle, we calculate $f(\theta)$ over a discrete number of θ-values in the range $\theta \in [0, \pi/2]$ and search for a corresponding minimum spectrum value. Then the K smallest spectrum values are chosen among all the minimum spectrum values, and their corresponding K azimuth-elevation angles are the estimated DOAs. Though a greater computational load is needed for the search-based method compared with the original UCA-RARE, the inherent shortcomings of the original UCA-RARE can be overcome.

Consider two uncorrelated signals with directions $[\theta_1 = 30^\circ, \phi_1 = 100^\circ]$ and $[\theta_2 = 48^\circ, \phi_2 = 118^\circ]$ impinging on the UCA with nine monopoles elements. The number of snapshots is set as 1000. For each simulation, 500 independent Monte Carlo trials are used. Fig. 2 shows the mean square error (MSE) performance of elevation estimates in the presence of mutual coupling. It can be seen from Fig. 2 that the proposed elevation–estimation algorithm outperforms the algorithm in [14] for both the two elevation angles. This is because the proposed elevation–estimation algorithm does not need the computation for the null space of $\Psi(\phi)$, which may introduce small errors for the following searching process. Moreover, the MSE obtained by the well-known 2D MUSIC algorithm is also depicted in Fig. 2. Here and in the following, the search step for the MUSIC in both azimuth and elevation angles is 0.01°. The 2D MUSIC algorithm is the most straightforward technique for DOA estimation while it has high computational cost. The average MATLAB-runtime of the 2D MUSIC algorithm is 729.02s (simulated on a 3.3 GHz Core i5 Processor), whereas the average MATLAB-runtime by the UCA-RARE-like/proposed elevation–estimation algorithm is 0.87s. Fig. 2 shows that the

Table 1
The mutual coupling coefficients at different elevation angles.

Elevation	C_2	C_3	C_4	C_5
10°	−0.6305 + 0.4215i	0.0284 + 0.5312i	0.3110 + 0.3089i	0.3583 + 0.2039i
20°	−0.6332 + 0.4200i	0.0280 + 0.5317i	0.3108 + 0.3090i	0.3580 + 0.2041i
30°	−0.6359 + 0.4164i	0.0264 + 0.5327i	0.3104 + 0.3100i	0.3579 + 0.2049i
40°	−0.6377 + 0.4118i	0.0236 + 0.5336i	0.3096 + 0.3118i	0.3576 + 0.2066i
50°	−0.6386 + 0.4073i	0.0206 + 0.5338i	0.3083 + 0.3136i	0.3569 + 0.2086i
60°	−0.6389 + 0.4035i	0.0180 + 0.5336i	0.3068 + 0.3150i	0.3559 + 0.2102i
70°	−0.6390 + 0.4008i	0.0162 + 0.5332i	0.3057 + 0.3159i	0.3552 + 0.2113i
80°	−0.6390 + 0.3992i	0.0151 + 0.5330i	0.3050 + 0.3165i	0.3547 + 0.2120i
90°	−0.6390 + 0.3987i	0.0147 + 0.5330i	0.3048 + 0.3166i	0.3545 + 0.2122i

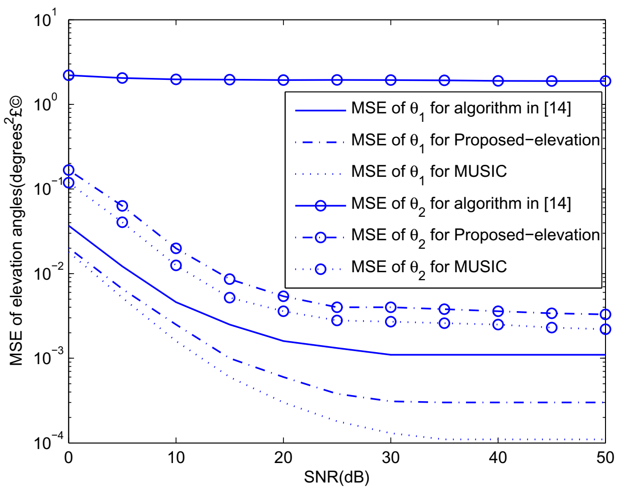

Fig. 2. MSE of elevation estimates versus SNR.

MSE of the proposed elevation–estimation method is close to the MSE of the 2D MUSIC.

4.2. Proposed algorithm for azimuth and elevation estimation

Working in beamspace reconstructs the Vandermande structure of the manifold and allows computationally efficient algorithms for DOA estimation. However, it has already been proved that the performance of element-space algorithms is superior to that of beamspace algorithms [19]. This is because the residual term exists when the BT is utilized, and the residual term is significant with a small number of elements (from six to ten elements). To reduce the residual term and improve performance for 2D DOA estimation, a new algorithm for both azimuth and elevation estimation is proposed in this section. The proposed method can improve the performance of both azimuth and elevation estimation when the number of elements is small with low computational complexity.

Recalling (10), the beamspace data can be rewritten as

$$\mathbf{y}_2(t) = \mathbf{W}\left(\tilde{\mathbf{A}}(\theta,\phi)\mathbf{s}(t) + \mathbf{n}(t)\right) = \mathbf{B}_2(\theta,\phi)\mathbf{s}(t) + \mathbf{n}_1(t) \\ = \mathbf{B}_1(\theta,\phi)\mathbf{s}(t) + \Delta\mathbf{y}(t) + \mathbf{n}_1(t), \tag{16}$$

with

$$\mathbf{B}_2(\theta,\phi) = \mathbf{W}\tilde{\mathbf{A}}(\theta,\phi) = \mathbf{B}_1(\theta,\phi) + [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_K], \tag{17}$$

$$\varepsilon_i = \mathbf{d}(\theta_i, \phi_i) \odot \mathbf{m}(\theta_i), \quad i = 1, \ldots, K, \tag{18}$$

and

$$\Delta\mathbf{y}(t) = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_K]\mathbf{s}(t) = \sum_{i=1}^{K}\varepsilon_i \mathbf{s}_i(t). \tag{19}$$

Comparing (16) with (10), it can be seen that $\mathbf{y}_2(t)$ comprises three summands while $\mathbf{y}_1(t)$ consists of two terms. The second term of $\mathbf{y}_2(t)$ is introduced by the residual term as shown in (19). Usually, $\Delta\mathbf{y}(t)$ can be ignored with a large number of elements and then the azimuth angles can be estimated by (13). However, with a small number of elements, the value of $\Delta\mathbf{y}(t)$ may be significant and (10) is no longer suitable for the calculation of the beamspace data. In this case, the performance for 2D DOA estimation degrades seriously.

The key idea of the proposed algorithm is to revise the beamspace data by eliminating the residual component $\Delta\mathbf{y}(t)$, which is introduced by the residual term of the beamspace steering vector. First, the initial azimuth and elevation angles $\left[\left(\hat{\theta}_1, \hat{\phi}_1\right), \left(\hat{\theta}_2, \hat{\phi}_2\right), \ldots, \left(\hat{\theta}_K, \hat{\phi}_K\right)\right]$ are estimated by using the UCA-RARE-like/proposed elevation–estimation method. Note that the proposed searching method for elevation estimation is utilized here. Then based on the initial DOAs, the amplitudes of the signals can be calculated by minimizing the squared norm of the difference between the observations and estimates as

$$\hat{\mathbf{s}}(t) = \arg\min_{\mathbf{s}} \left\| \mathbf{y}(t) - \hat{\mathbf{B}}_2\left(\hat{\theta}, \hat{\phi}\right)\mathbf{s} \right\|_2^2. \tag{20}$$

The corresponding solution is performed by the least square technique as

$$\hat{\mathbf{s}}(t) = (\hat{\mathbf{B}}_2^H \hat{\mathbf{B}}_2)^{-1} \hat{\mathbf{B}}_2^H \mathbf{y}(t). \tag{21}$$

Let $\Delta\hat{\mathbf{y}}_i(t) = \hat{\varepsilon}_i \hat{\mathbf{s}}_i(t)$ and it is given by

$$\Delta\hat{\mathbf{y}}_i(t) = \hat{\varepsilon}_i \hat{\mathbf{s}}_i(t) = \mathbf{d}\left(\hat{\theta}_i, \hat{\phi}_i\right) \odot \mathbf{m}\left(\hat{\theta}_i\right)\hat{\mathbf{s}}_i(t), \tag{22}$$

Then we have

$$\Delta\hat{\mathbf{y}}(t) = \Delta\hat{\mathbf{y}}_1(t) + \Delta\hat{\mathbf{y}}_2(t) + \cdots + \Delta\hat{\mathbf{y}}_K(t). \tag{23}$$

Thus the second term $\Delta\hat{\mathbf{y}}(t)$ in (16) can be calculated by (22) and (23).

However, it is not easy to calculate $\hat{\varepsilon}_i$ since it is a sum of an infinite number of terms [6]. For convenience, $\Delta\hat{\mathbf{y}}(t)$ can be obtained in another way. Considering (17), we have

$$[\hat{\varepsilon}_1, \hat{\varepsilon}_2, \ldots, \hat{\varepsilon}_K] = \hat{\mathbf{B}}_2\left(\hat{\theta}, \hat{\phi}\right) - \hat{\mathbf{B}}_1\left(\hat{\theta}, \hat{\phi}\right). \tag{24}$$

In this case, $\Delta\hat{\mathbf{y}}(t)$ can be calculated as

$$\begin{aligned}\Delta\hat{\mathbf{y}}(t) &= [\hat{\varepsilon}_1, \hat{\varepsilon}_2, \ldots, \hat{\varepsilon}_K]\hat{\mathbf{s}}(t) = \left(\hat{\mathbf{B}}_2 - \hat{\mathbf{B}}_1\right)\hat{\mathbf{s}}(t) \\ &= \left(\hat{\mathbf{B}}_2 - \hat{\mathbf{B}}_1\right)(\hat{\mathbf{B}}_2^H \hat{\mathbf{B}}_2)^{-1} \hat{\mathbf{B}}_2^H \mathbf{y}(t) \\ &= (\hat{\mathbf{B}}_2 - \hat{\mathbf{B}}_1)\mathbf{P_B}\mathbf{y}(t),\end{aligned} \tag{25}$$

where $\mathbf{P_B}$ is given as

$$\mathbf{P_B} = (\hat{\mathbf{B}}_2^H \hat{\mathbf{B}}_2)^{-1} \hat{\mathbf{B}}_2^H. \tag{26}$$

The modified beamspace data is given by

$$\begin{aligned}\hat{\mathbf{y}}(t) &= \mathbf{y}(t) - \Delta\hat{\mathbf{y}}(t) = \mathbf{y}(t) - \left(\hat{\mathbf{B}}_2 - \hat{\mathbf{B}}_1\right)\mathbf{P_B}\mathbf{y}(t) \\ &= \left[\mathbf{I} - \left(\hat{\mathbf{B}}_2 - \hat{\mathbf{B}}_1\right)\mathbf{P_B}\right]\mathbf{y}(t).\end{aligned} \tag{27}$$

With the modified data $\hat{\mathbf{y}}(t)$, we use the UCA-RARE-like/proposed elevation–estimation method again to estimate the 2D DOAs, which are more accurate compared with the initial DOAs. Furthermore, by repeating (21), (25), and (27), we can obtain a new revised beamspace data and then estimate DOAs with the new beamspace data. This process can be repeated many times. To get the convergence, the element-space covariance matrix $\mathbf{R_x}$ is constructed and the EVD of $\mathbf{R_x}$ is

$$\mathbf{R_x} = E\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\} = \mathbf{E_{xs}}\Lambda_{\mathbf{xs}}\mathbf{E_{xs}}^H + \mathbf{E_{xn}}\Lambda_{\mathbf{xn}}\mathbf{E_{xn}}^H. \tag{28}$$

Compute

$$J_{\mathbf{x}}^n = \left\| \mathbf{E_{xn}}^H \mathbf{a}\left(\hat{\theta}, \hat{\phi}\right) \right\|^2 \tag{29}$$

using the estimated DOAs. If $J_{\mathbf{x}}^n - J_{\mathbf{x}}^{n+1} > \zeta$, then update the iteration counter $n = n + 1$ and repeat (21), (25), and (27) until

$$J_{\mathbf{x}}^n - J_{\mathbf{x}}^{n+1} \le \zeta, \tag{30}$$

with ζ being a threshold. In most cases, two or three iterations are enough for the proposed algorithm to get reliable performance, and further iterations only bring a little bit of performance improvement. Therefore in our simulations, we stop iterating after one or two iterations. With the above considerations, the steps of the proposed algorithm for azimuth and elevation estimation are listed in Fig. 3. It is interesting to notice that the proposed algorithm is suitable for the BT based methods in absence of the mutual coupling. Actually, these methods can be considered as special examples of the algorithms in the presence of the mutual coupling when the MCM is an identity matrix.

4.3. Computational complexity

Assume N number of snapshots are available. The computational complexity to calculate $\mathbf{R_x}$ is $O\left(P^2N\right)$, while its EVD is $O\left(P^3\right)$. The complexity for $\mathbf{R_y}$ is $O\left((2M+1)^2N\right)$, while its EVD is $O\left((2M+1)^3\right)$. The computations of polynomial rooting and 1D searching are $O((2M+1)I_1)$ and $O\left(\frac{90}{\Delta\theta_1}(M+1)^2I_1\right)$, respectively, with I_1 and $\Delta\theta_1$ being the number of estimated azimuth angles and the scanning internal, respectively. The complexity of calculating $\hat{\mathbf{y}}$ or $\mathbf{R}_{\hat{\mathbf{y}}}$ is about $O\left((2M+1)^2N\right)$. When $N \gg P$, the complexity of the proposed algorithm is $O\left(P^2N+(1+2I_2)(2M+1)^2N+\frac{90}{\Delta\theta_1}(M+1)^2I_1(1+I_2)\right)$ where I_2 is the number of iterations. The computational complexity of the algorithm in [14] is about $O\left((2M+1)^2N+\frac{360}{\Delta\theta_1}(M+1)I_1\right)$. And the complexity of the MUSIC is about $O\left((2M+1)^2N+\frac{90}{\Delta\theta_2}\cdot\frac{360}{\Delta\phi_2}(2M+1)^2\right)$ with $\Delta\theta_2$ and $\Delta\phi_2$ being the scanning internals of the MUSIC in the elevation and azimuth angles, respectively.

Assume $P=9$, $N=500$, $\Delta\theta_1=0.06^\circ$, $\Delta\theta_2=0.01^\circ$, and $\Delta\phi_2=0.01^\circ$. The computational complexity is shown in Table 2. As a matter of fact that $I_2=1$ or 2 is enough for the proposed algorithm to get good performance, we show the computational complexity of the proposed algorithm with one or two iterations. It can be seen

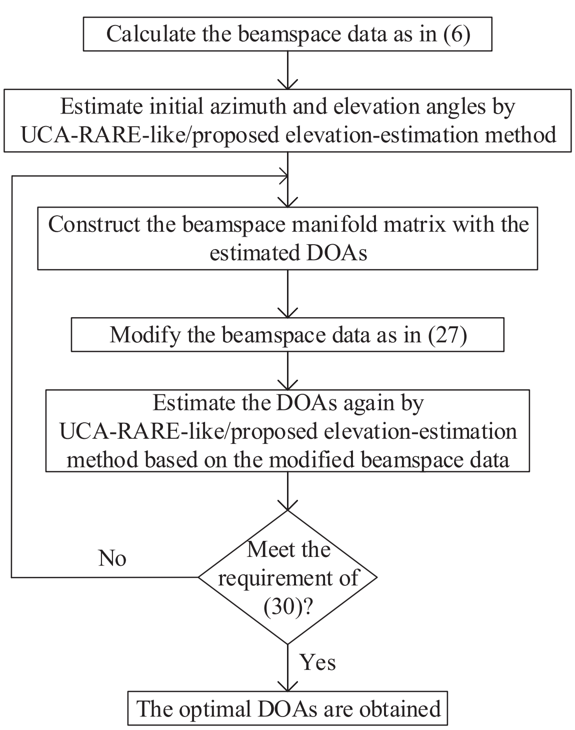

Fig. 3. Proposed algorithm for azimuth and elevation estimation.

Table 2
Computational complexity.

Proposed with $I_2=1$	4.62×10^5
Proposed with $I_2=2$	6.93×10^5
the algorithm in [14]	2.21×10^5
MUSIC	2.62×10^{10}

from Table 2 that the proposed algorithm with one or two iterations has much lower computational complexity than the MUSIC. And the computational complexity of the proposed algorithm with one iteration is close to that of the algorithm in [14].

5. Error analysis

5.1. Error analysis in azimuth estimation

According to (13), for a true azimuth angle ϕ, there is

$$\mathbf{P_1}(\phi)=\det\left\{\mathbf{T}^{\mathrm{H}}(\phi)\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}\mathbf{T}(\phi)\right\}=0. \tag{31}$$

The EVD of $\Psi(\phi)$ is

$$\Psi(\phi)=\mathbf{U_s}\Sigma_\mathbf{s}\mathbf{U_s}+\mathbf{U_n}\Sigma_\mathbf{n}\mathbf{U_n}. \tag{32}$$

If there are m signals with the azimuth angle being ϕ, $\phi+\pi$, or $\phi-\pi$, then $\Sigma_\mathbf{n}$ contains m zero eigenvalues and $\mathbf{U_n}$ contains the corresponding eigenvectors. Similarly as in [15], we define

$$\rho(\phi)=\sum_{l=1}^{m}\mathbf{u}_l^{\mathrm{H}}\Psi(\phi)\mathbf{u}_l=\sum_{l=1}^{m}\mathbf{u}_l^{\mathrm{H}}\mathbf{T}^{\mathrm{H}}(\phi)\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}\mathbf{T}(\phi)\mathbf{u}_l, \tag{33}$$

where $\mathbf{u}_l$ denotes the lth column of $\mathbf{U_n}$. For small enough errors, we can write

$$0=\rho'\left(\hat{\phi}\right)\approx\rho'(\phi)+\rho''(\phi)\left(\hat{\phi}-\phi\right), \tag{34}$$

where ϕ and $\hat{\phi}$ indicate the true and estimated azimuth angles, respectively, and $\rho'\left(\hat{\phi}\right)=(\partial\rho(\phi)/\partial\phi)|_{\phi=\hat{\phi}}$. To solve the error $(\hat{\phi}-\phi)$ in (34), we need to calculate the first and second derivatives of (33). Defining $\mathbf{T}'(\phi)=\partial\mathbf{T}(\phi)/\partial\phi$ and $\mathbf{T}''(\phi)=\partial\mathbf{T}'(\phi)/\partial\phi$, then the first derivative of $\rho(\phi)$ is

$$\rho'(\phi)=\sum_{l=1}^{m}2\mathrm{Re}\left\{\mathbf{u}_l^{\mathrm{H}}\mathbf{T}^{\mathrm{H}}(\phi)\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}\mathbf{T}'(\phi)\mathbf{u}_l\right\}, \tag{35}$$

where $\mathrm{Re}\{\cdot\}$ is the real part of the argument within the brackets. The second derivative of $\rho(\phi)$ is

$$\rho''(\phi)=\sum_{l=1}^{m}2\mathrm{Re}\left\{\mathbf{u}_l^{\mathrm{H}}\mathbf{T}^{\mathrm{H}}(\phi)\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}\mathbf{T}''(\phi)\mathbf{u}_l\right\} + \sum_{l=1}^{m}2\mathbf{u}_l^{\mathrm{H}}\mathbf{T}'^{\mathrm{H}}(\phi)\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}\mathbf{T}'(\phi)\mathbf{u}_l. \tag{36}$$

Submitting (35) and (36) into (34), the error in azimuth estimation can be calculated as

$$\hat{\phi}-\phi=-\frac{\rho'(\phi)}{\rho''(\phi)}. \tag{37}$$

Moreover, when $m=1$, the error can also be given as

$$\hat{\phi}-\phi=\frac{\mathrm{Re}\left\{\mathbf{g}'^{\mathrm{H}}(\theta)\mathbf{T}'^{\mathrm{H}}(\phi)\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}\mathbf{T}(\phi)\mathbf{g}'(\theta)\right\}}{\mathbf{g}'^{\mathrm{H}}(\theta)\mathbf{T}'^{\mathrm{H}}(\phi)\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}\mathbf{T}'(\phi)\mathbf{g}'(\theta)}. \tag{38}$$

As $\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}\tilde{\mathbf{b}}_2(\theta,\phi)=\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}(\mathbf{T}(\phi)\mathbf{g}'(\theta)+\varepsilon)=0$, then we have $\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}\mathbf{T}(\phi)\mathbf{g}'(\theta)=-\mathbf{E_n}\mathbf{E_n}^{\mathrm{H}}\varepsilon$. In this case, (38) can be rewritten as

$$\hat{\phi} - \phi = -\frac{\mathrm{Re}\left\{\mathbf{g}'^{\mathrm{H}}(\theta)\mathbf{T}'^{\mathrm{H}}(\phi)\mathbf{E}_{\mathbf{n}}\mathbf{E}_{\mathbf{n}}^{\mathrm{H}}\varepsilon\right\}}{\mathbf{g}'^{\mathrm{H}}(\theta)\mathbf{T}'^{\mathrm{H}}(\phi)\mathbf{E}_{\mathbf{n}}\mathbf{E}_{\mathbf{n}}^{\mathrm{H}}\mathbf{T}'(\phi)\mathbf{g}'(\theta)}. \tag{39}$$

5.2. Error analysis in elevation estimation

An expression for the error in elevation estimation can be found by expanding the first derivative of (15) with respect to θ and evaluating at $\hat{\theta}$. For small enough errors [6,15], we have

$$0 = f'\left(\hat{\theta}\right) \approx f'(\theta) + f''(\theta)\left(\hat{\theta} - \theta\right), \tag{40}$$

where θ and $\hat{\theta}$ are the true and estimated elevation angles, respectively, and $f'\left(\hat{\theta}\right) = (\partial f(\theta)/\partial\theta)|_{\theta=\hat{\theta}}$. In order to solve (40), let $\tilde{g}(\theta) = \partial g'(\theta)/\partial\theta$ and $\tilde{\tilde{g}}(\theta) = \partial\tilde{g}(\theta)/\partial\theta$. Then the first derivative of $f(\theta)$ is given by

$$f'(\theta) = 2\mathrm{Re}\left\{\tilde{\mathbf{g}}^{\mathrm{H}}(\theta)\mathbf{T}^{\mathrm{H}}\mathbf{E}_{\mathbf{n}}\mathbf{E}_{\mathbf{n}}^{\mathrm{H}}\mathbf{T}\mathbf{g}'(\theta)\right\}. \tag{41}$$

The second derivative of $f(\theta)$ is expressed as

$$f'(\theta) = 2\mathrm{Re}\left\{\mathbf{g}'^{\mathrm{H}}(\theta)\mathbf{T}^{\mathrm{H}}\mathbf{E}_{\mathbf{n}}\mathbf{E}_{\mathbf{n}}^{\mathrm{H}}\mathbf{T}\tilde{\tilde{g}}(\theta)\right\} + 2\tilde{\mathbf{g}}^{\mathrm{H}}(\theta)\mathbf{T}^{\mathrm{H}}\mathbf{E}_{\mathbf{n}}\mathbf{E}_{\mathbf{n}}^{\mathrm{H}}\mathbf{T}\tilde{\mathbf{g}}(\theta). \tag{42}$$

Combining (40)–(42), the error for elevation angle can be calculated as

$$\hat{\theta} - \theta = -\frac{f'(\theta)}{f''(\theta)}. \tag{43}$$

6. Simulation results

We first investigate the error versus azimuth angles with a fixed elevation angle $\theta = 50^{\circ}$, and the error versus elevation angles with an unchanged azimuth angle $\phi = 60^{\circ}$. The number of snapshots is 1000 and SNR = 10 dB. Fig. 4 shows the estimated error and the approximated error, which is calculated by (39) or (43), versus azimuth and elevation angles in the case of eight or ten elements. The noise is a stationary and zero mean Gaussian random process. It can be seen that the approximated error describes the corresponding estimated error in azimuth or elevation estimates with high fidelity. The error behaves like a sinusoid as a function of azimuth angles whereas it increases as the elevation angle changes from 0° to 90°. When the elevation angle is small, the error is small. With high elevation angles, the error increases quickly and has serious influence on DOA estimates.

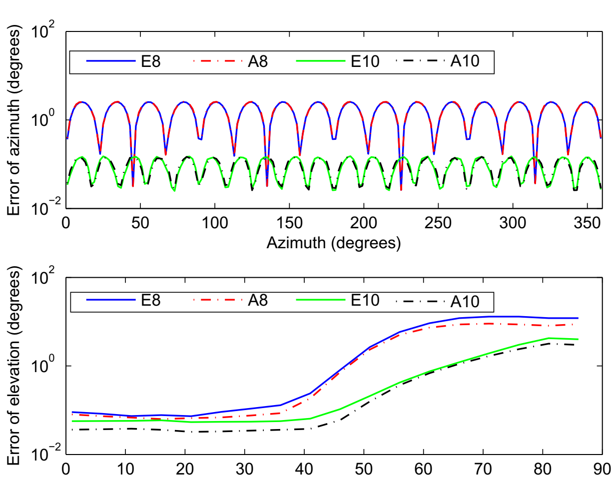

Fig. 4. The estimated error and the approximated error versus azimuth angles and elevation angles (Estimated error with $P = 8$ (E8), Approximated error with $P = 8$ (A8), Estimated error with $P = 10$ (E10), and Approximated error with $P = 10$ (A10)).

To verify the efficiency of the proposed algorithm for azimuth and elevation estimates, a UCA composed of nine monopoles elements is assumed. Two uncorrelated signals at $[\theta_1, \phi_1] = [40^{\circ}, 50^{\circ}]$

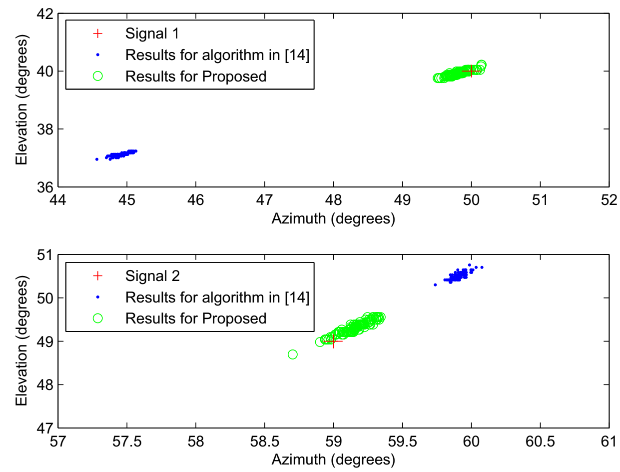

Fig. 5. Results of the algorithm in [14] and the proposed method in the presence of mutual coupling for two signals $[\theta_1, \phi_1] = [40^{\circ}, 50^{\circ}]$ and $[\theta_2, \phi_2] = [49^{\circ}, 59^{\circ}]$.

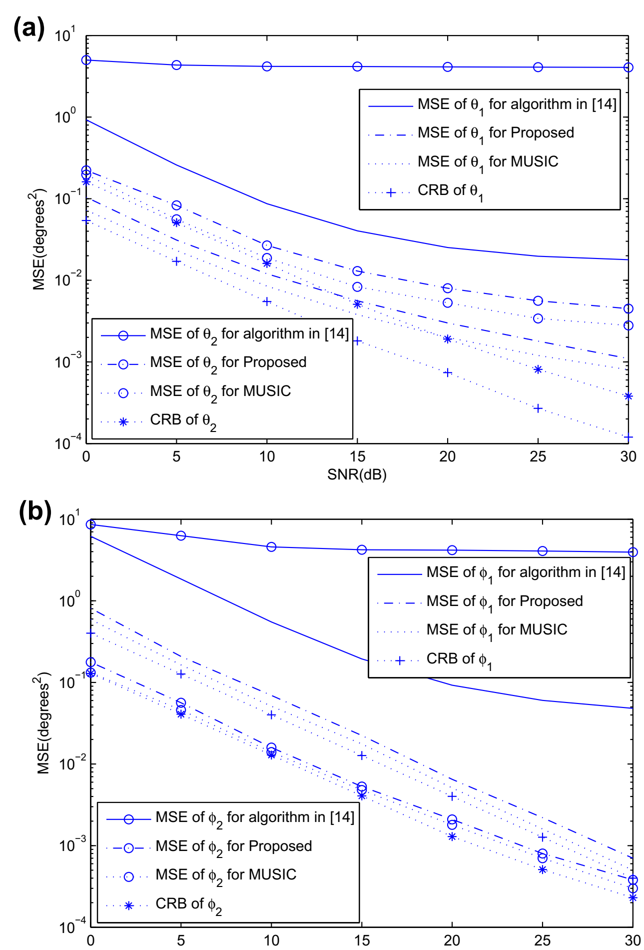

Fig. 6. MSEs of elevation estimates (a) and MSEs of azimuth estimates (b) versus SNR.

and $[\theta_2, \phi_2] = [49^\circ, 59^\circ]$ impinge on the UCA with 1000 snapshots and 100 independent Monte-Carlo runs. We consider the case of one repeat of the proposed algorithm. Fig. 5 shows the results of the two signals by the algorithm in [14] and the proposed algorithm, respectively, in the condition of SNR = 15 dB. It can be seen that both the azimuth and elevation estimates of the two signals by the algorithm in [14] deviate from the true angles, whereas the elevation and azimuth estimates obtained by the proposed algorithm distribute around the true ones.

Fig. 6 illustrates the performance of the proposed algorithm on MSE versus SNR in comparison with the traditional algorithm. Two uncorrelated signals at $[\theta_1, \phi_1] = [32^\circ, 30^\circ]$ and $[\theta_2, \phi_2] = [42^\circ, 46^\circ]$ are considered. The numbers of snapshots and Monte-Carlo simulations are 1000 and 500, respectively. In Fig. 6(a), the proposed algorithm outperforms the algorithm in [14] for both the two elevation angles. Moreover, the MSE performance of the MUSIC [15] and the CRBs are also shown in Fig. 6. The MSEs obtained by the proposed algorithm are close to both the MSEs obtained by the MUSIC and the CRBs. A same conclusion can be drawn for the azimuth angles in Fig. 6(b). Fig. 7 depicts the MSE performance of the elevation and azimuth angles with snapshots varied from 200 to 1000 at the SNR of 15 dB. From Fig. 7, the MSE obtained by the proposed algorithm is smaller than that obtained by the algorithm in [14]. And the proposed algorithm has good performance as well as the MUSIC. The results in Figs. 6 and 7 show that the proposed algorithms can achieve better performance for 2D DOA estimation versus SNR and snapshots than the algorithm in [14]. These results

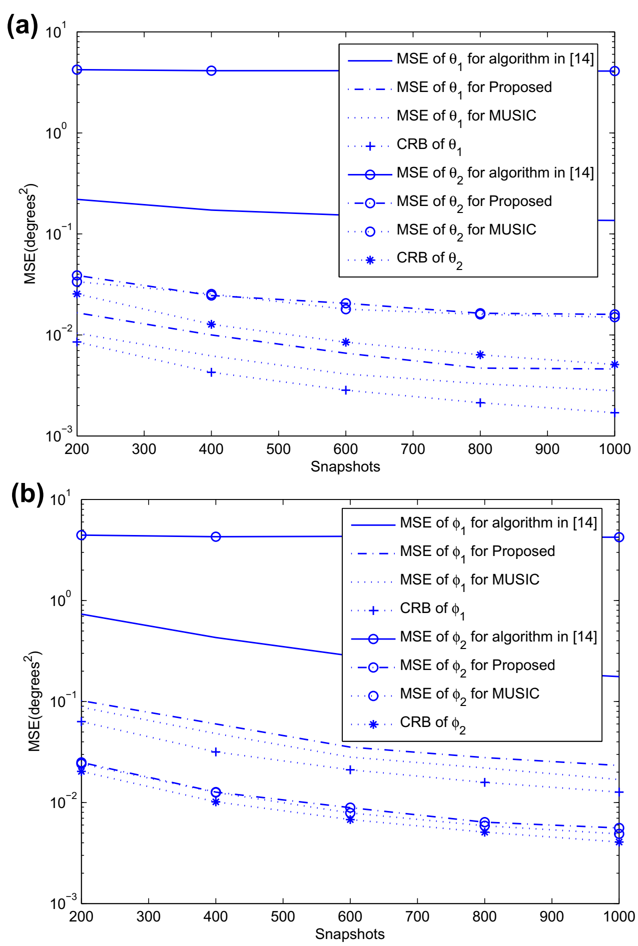

Fig. 7. MSEs of elevation estimates (a) and MSEs of azimuth estimates (b) versus snapshots.

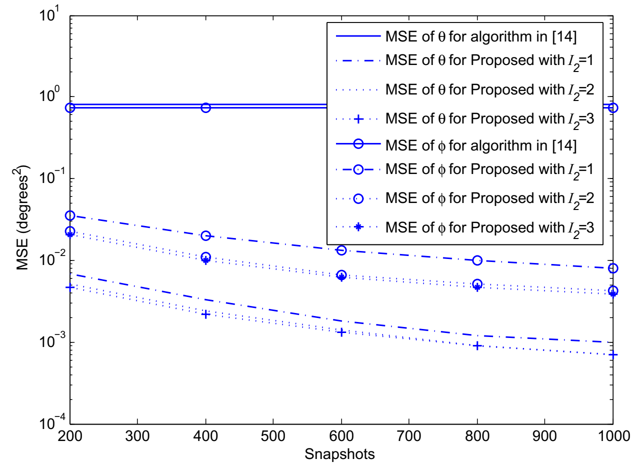

Fig. 8. MSEs obtained by the proposed algorithm with three iterations ($I_2 = 3$), proposed algorithm with two iterations ($I_2 = 2$), proposed algorithm with one iteration ($I_2 = 1$), and the algorithm in [14] for three signals.

are achieved by modifying the beamspace data through eliminating its residual component.

Fig. 8 illustrates the MSE performance of the proposed algorithm with one, two, and three iterations for three signals at $[\theta_1, \phi_1] = [20^\circ, 20^\circ], [\theta_2, \phi_2] = [40^\circ, 58^\circ]$, and $[\theta_3, \phi_3] = [60^\circ, 80^\circ]$ in the condition of SNR = 25 dB. Referring to Fig. 8, the MSE performance of the proposed algorithm with one, two or three iterations is much better than the algorithm in [14]. Moreover, the MSE performance of the proposed algorithm with two or three iterations is improved compared to that with one iteration, however, the performance improvement is not significant. In addition, the proposed algorithm with two or three iterations requires higher computational complexity.

7. Conclusion

A novel algorithm for 2D DOA estimation is proposed for UCA with a small number of elements in the presence of mutual coupling. When the number of elements is small, the performance of traditional BT based methods suffers significant degradation. In this paper, the analysis of the residual term is presented. Moreover, for the proposed method, we first use a UCA-RARE-like method for azimuth estimates and apply a new searching method for elevation estimates. Then, the beamspace data is modified based on the initial DOAs, and an iterative technique is utilized to improve the accuracy for DOA estimation. Simulations are presented to show the effectiveness of the proposed method on SNR, snapshots and number of iterations. It can be concluded that the proposed algorithm enormously improves the performance for azimuth estimates as well as for elevation estimates.

References

[1] Mathews CP, Zoltowski MV. Eigenstructure techniques for 2-D angle estimation with uniform circular arrays. IEEE Trans Signal Process 1994;42:2395–407.
[2] Pesavento M, Bohme JF. Direction of arrival estimation in uniform circular arrays composed of directional elements. In: Proceedings of Sensor Array and Multichannel Signal Process. p. 503–7.
[3] Goossens R, Rogier H. 2-D direction-of-arrival estimation in the presence of mutual coupling by exploiting the symmetry in a uniform circular array. In: IEEE Antennas Propag Society Int Symposium. p. 5283–6.
[4] Lee J, Song I, Kwon H, Lee SR. Low-complexity estimation of 2D DOA for coherently distributed sources. Signal Process 2003;83:1789–802.
[5] You G, Qiu T, Yang J. A novel DOA estimation algorithm of cyclostationary signal based on UCA in impulsive noise. Int J of Electron Commun 2013;67:491–9.

[6] Belloni F, Koivunen V. Beamspace transform for UCA: error analysis and bias reduction. IEEE Trans Signal Process 2006;54:3078–89.
[7] Hui HT. Improved compensation for the mutual coupling effect in a dipole array for direction finding. IEEE Trans Antennas Propag 2003;51:2498C2503.
[8] Rogier H, Bonek E. Analytical spherical-mode-based compensation of mutual coupling in uniform circular arrays for direction-of-arrival estimation. Int J Electron Commun 2006;60:179C89.
[9] Dai J, Xu W, Zhao D. Real-valued DOA estimation for uniform linear array with unknown mutual coupling. Signal Process 2012;92:2056–65.
[10] Goossens R, Rogier H. Direction-of-arrival and polarization estimation with uniform circular arrays in the presence of mutual coupling. Int J Electron Commun 2008;62:199–206.
[11] Wang D. Improved active calibration algorithms in the presence of channel gain/phase uncertainties and sensor mutual coupling effects. Circuit Syst Signal Process 2015;34:1825–68.
[12] Yang B, Viberg M, McKelvey T, et al. Array response interpolation and DOA estimation with array response decomposition. Signal Process 2016;125:97–109.
[13] Goossens R, Rogier H. A hybrid UCA-RARE/Root-MUSIC approach for 2-D direction of arrival estimation in uniform circular arrays in the presence of mutual coupling. IEEE Trans Antennas Propag 2007;55:841–9.
[14] Wang BH, Hui HT, Leong MS. Decoupled 2D direction of arrival estimation using compact uniform circular arrays in the presence of elevation-dependent mutual coupling. IEEE Trans Antennas Propag 2010;58:747–55.
[15] Xie J, He Z, Li H, Li J. 2D DOA estimation with sparse uniform circular arrays in the presence of mutual coupling. EURASIP J Adv Signal Process 2011;2011:1–18.
[16] Goossens R, Rogier H, Werbrouck S. UCA Root-MUSIC with sparse uniform circular arrays. IEEE Trans Signal Process 2008;56:4095C4099.
[17] Vasylyshyn V. Removing the outliers in root-MUSIC via pseudo-noise resampling and conventional beamformer. Signal Process 2013;93:3423–9.
[18] Qian C, Huang L, So HC. Improved unitary root-MUSIC for DOA estimation based on pseudo-noise resampling. IEEE Signal Process Lett 2014;21:140–4.
[19] Stoica P, Nehorai A. Comparative performance study of element-space and beam-space MUSIC estimators. Circuit Syst Signal Process 1991;10:285–92.