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DOA estimation of rectilinear signals with a partly calibrated uniform linear array

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ABSTRACT

This paper investigates the problem of direction-of-arrival (DOA) estimation of rectilinear or strictly second-order noncircular signals with a partly calibrated uniform linear array (ULA). Consider that the uncalibrated portion of the array suffers from unknown gains and phases, an extended data model corresponding to a virtual (extended) array is presented by taking the noncircularity of the signals into account. On this basis and given the signal subspace matrix associated with the virtual array, a linear equation is derived to determine the unknown gains and phases. Then, the DOAs are found through the eigenvalue decomposition of a matrix related to the signal subspace matrix and array gains and phases. Since spatial spectrum search is not required, the proposed method is computationally efficient. Moreover, it is able to handle at most $2M-3$ rectilinear signals with a partly calibrated ULA of M elements, even though only two neighboring elements of the array are calibrated. Numerical results also demonstrate that the proposed method has remarkable performance superiority brought by the rectilinearity.

1. Introduction

Measuring the directions-of-arrival (DOAs) of signals received by a sensor array plays an important role in a number of areas including radar, navigation, wireless communication and radio astronomy. The problem of DOA estimation has been an active research area for decades and stimulated a tremendous number of methods such as the classical multiple signal classification (MUSIC) [1] and estimation of signal parameters via rotational invariance techniques (ESPRIT) [2]. Among various array configurations for this purpose, uniform linear arrays (ULAs) are the most common and widely deployed arrays.

Although a well calibrated array system without imperfections is desirable, in practice it usually suffers from various uncertainties such as the unknown gains and phases caused by, say, the differences among the receivers utilized to demodulate and digitize the signals from the array elements [3–8]. In particular, the array might be partly calibrated, i.e., only a portion of the array is well calibrated. For instance, when we augment a well-constructed array mounted on a vehicle by placing a number of additional elements on the ground, the response of the off-vehicle elements may be poorly known [9]. The problem of DOA estimation with partly calibrated arrays has been paid great attention [9–12]. To estimate the DOAs and the gains and phases of the uncalibrated elements, an iterative algorithm was developed in [9] and a maximum likelihood (ML) based methodology was derived in [10]. Compared with these two methods, a computationally more efficient ESPRIT-like method was proposed in [11] and further investigated in [12] by examining the conditions ensuring the uniqueness of DOA estimates and identifiability. Moreover, a refining scheme for the ESPRIT-like method was presented in [12].

It is worth noting that in general the methods for DOA estimation with partly calibrated arrays implicitly assume that the received signals are complex circular. Nevertheless, in modern mobile communication systems and satellite systems, the signals are (strictly) noncircular when, for example, they are binary phase shift keying (BPSK) or offset quadrature phase shift keying (OQPSK) modulated. In other words, their second-order statistical characteristics are contained not only in the Hermitian covariance matrix, but also in the complex symmetric covariance matrix [13–18]. Therefore, it is of great interest to study how the rectilinearity of the signals can be exploited for DOA estimation with a partly calibrated ULA. To this end, the rectilinearity or strictly second-order noncircularity of signals is investigated and a rectilinear ESPRIT-like (Rec-ESPRIT-like) method is devised in this paper. With the rectilinearity property, an extended data model is introduced and a linear equation for gain and phase determination is derived. Once the unknown gains and phases have been estimated from the sig-

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nal subspace of the extended array covariance matrix, the DOA estimates of rectilinear signals can thus be obtained via eigenvalue decomposition (EVD). By taking advantage of the rectilinearity, the number of signals that can be handled by the Rec-ESPRIT-like method is considerably larger than the number of array elements. More specifically, at most $2M-3$ signals can be identified by a partly calibrated ULA of M elements even if only two neighboring elements are calibrated.

2. Data model

Consider a ULA of M elements and, without loss generality, assume that the first M_c sensors are calibrated. A total of L narrow-band signals impinge on the array from far-field with DOAs $\theta_1, \dots, \theta_L$. The array observation vector $\mathbf{x}(k)$ at time instant k can be written as

$$\mathbf{x}(k) = \mathbf{\Gamma}(\gamma)\mathbf{A}\mathbf{s}(k) + \mathbf{n}(k) \in \mathbb{C}^{M\times 1}, \tag{1}$$

where $\mathbf{\Gamma}(\cdot)$ is an operator returning a diagonal matrix whose main diagonal is composed of vector $\boldsymbol{\gamma}$, given by [12]

$$\gamma = [\mathbf{1}_{M_c}^T, \gamma_1, \gamma_2, \dots, \gamma_{M-M_c}]^T \in \mathbb{C}^{M\times 1}, \tag{2}$$

where $(\cdot)^T$ denotes the transpose operator, $\mathbf{1}_{M_c}$ represents an $M_c \times 1$ vector with ones for all entries and $\gamma_1, \gamma_2, \dots, \gamma_{M-M_c}$ contain the element gains $\{|\gamma_i|\}_{i=1}^{M_c}$ and phases $\{\arg(\gamma_i)\}_{i=1}^{M_c}$. It should be noted that the array unknown gains and phases are assumed to be angularly independent. $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_L)]$ is the steering matrix with $\mathbf{a}(\theta_l) = [1, \beta_l, \dots, \beta_l^{M-1}]^T$, $l = 1, \dots, L$, being the steering vectors. Here, we have $\beta_l = e^{j2\pi\lambda^{-1}d\sin\theta_l}$ with d and λ being the inter-element spacing and carrier wavelength, respectively. $\mathbf{s}(k)$ and $\mathbf{n}(k) \sim \mathcal{CN}(\mathbf{0}_{M\times 1}, \sigma_n^2\mathbf{I}_M)$ are the signal vector and additive Gaussian noise vector with zero mean and covariance matrix $\sigma_n^2\mathbf{I}_M$, respectively. σ_n^2, $\mathbf{0}_{M\times 1}$ and $\mathbf{I}_M$ are the noise variance, $M\times 1$ zero vector and $M\times M$ identity matrix, respectively.

In this work, we focus on strictly second-order noncircular or rectilinear signals which can be expressed as [14–18]

$$\mathbf{s}(k) = \mathbf{\Psi}\mathbf{s}_0(k) \in \mathbb{C}^{L\times 1}, \tag{3}$$

where $\mathbf{s}_0(k) \in \mathbb{R}^{L\times 1}$ is a real-valued signal vector, and $\mathbf{\Psi} = \mathrm{diag}\{e^{j\psi_1}, e^{j\psi_2}, \dots, e^{j\psi_L}\}$ is a diagonal matrix with diagonal entries $e^{j\psi_l}, l = 1, 2, \dots, L$, containing phase shifts that can be different for each signal. In the sequel, we shall show how the DOAs can be estimated with this property.

3. Proposed DOA estimation method

3.1. Extended data model

By taking advantage of the signal rectilinearity, we denote by $\mathbf{y}(k)$ the flipped and conjugated observation vector as

$$\mathbf{y}(k) = \mathbf{J}\mathbf{x}^*(k) \in \mathbb{C}^{M\times 1}, \tag{4}$$

where $\mathbf{J} \in \mathbb{R}^{M\times M}$ represents an exchange matrix with ones on its anti-diagonal and zeros elsewhere, and $(\cdot)^*$ denotes the complex conjugate. Therefore, an extended observation vector $\mathbf{z}(k) \in \mathbb{C}^{2M\times 1}$ can be constructed as

$$\mathbf{z}(k) = \begin{bmatrix}\mathbf{y}(\mathbf{k}) \\ \mathbf{x}(k)\end{bmatrix} = \mathbf{B}\mathbf{s}_0(k) + \mathbf{v}(k), \tag{5}$$

where $\mathbf{B} \in \mathbb{C}^{2M\times L}$ and $\mathbf{v}(k) \in \mathbb{C}^{2M\times 1}$ denote the extended steering matrix and extended noise vector, respectively, as

$$\mathbf{B} = \begin{bmatrix}\mathbf{J}\mathbf{\Gamma}(\gamma^*)\mathbf{A}^*\mathbf{\Psi}^* \\ \mathbf{\Gamma}(\gamma)\mathbf{A}\mathbf{\Psi}\end{bmatrix}, \quad \mathbf{v}(k) = \begin{bmatrix}\mathbf{J}\mathbf{n}^*(\mathbf{k}) \\ \mathbf{n}(k)\end{bmatrix} \sim \mathcal{CN}(\mathbf{0}_{2M\times 1}, \sigma_n^2\mathbf{I}_{2M}). \tag{6}$$

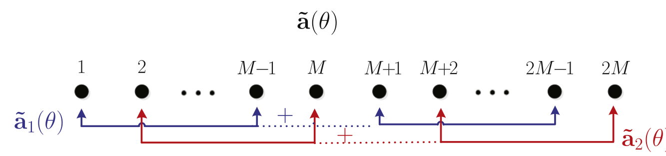

Fig. 1. Illustration of dividing $\tilde{\mathbf{a}}(\theta)$ into overlapping $\tilde{\mathbf{a}}_1(\theta)$ and $\tilde{\mathbf{a}}_2(\theta)$.

Specifically, we have $\mathbf{B} = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \dots, \mathbf{b}(\theta_L)]$ with $\mathbf{b}(\theta_l)$ being the extended steering vector as

$$\mathbf{b}(\theta_l) = \begin{bmatrix}\mathbf{J}\mathbf{\Gamma}(\gamma^*)\mathbf{a}^*(\theta_l)\mathbf{e}^{-\mathbf{j}\psi_l} \\ \mathbf{\Gamma}(\gamma)\mathbf{a}(\theta_l)e^{j\psi_l}\end{bmatrix} = \mathbf{\Gamma}(\tilde{\gamma})\tilde{\mathbf{a}}(\theta_l), \tag{7}$$

where $\tilde{\gamma} \in \mathbb{C}^{2M\times 1}$ and $\tilde{\mathbf{a}}(\theta_l) \in \mathbb{C}^{2M\times 1}$ are defined as

$$\tilde{\gamma} \triangleq [\gamma_{M-M_c}^*, \dots, \gamma_2^*, \gamma_1^*, \mathbf{1}_{2M_c}^T, \gamma_1, \gamma_2, \dots, \gamma_{M-M_c}]^T \tag{8a}$$

$$\tilde{\mathbf{a}}(\theta_l) \triangleq [\beta_l^{1-M}e^{-j\psi_l}, \dots, \beta_l^{-1}e^{-j\psi_l}, e^{-j\psi_l}, e^{j\psi_l}, \beta_l e^{j\psi_l}, \dots, \beta_l^{M-1}e^{j\psi_l}]^T. \tag{8b}$$

As a matter of fact, $\tilde{\mathbf{a}}(\theta_l)$ can be regarded as the steering vector of the virtual (extended) array in the absence of imperfections and $\tilde{\gamma}$ is the gain/phase vector of the virtual array. Furthermore, let $\tilde{\mathbf{A}} \triangleq [\tilde{\mathbf{a}}(\theta_1), \dots, \tilde{\mathbf{a}}(\theta_L)] \in \mathbb{C}^{2M\times L}$, one gets $\mathbf{B} = \mathbf{\Gamma}(\tilde{\gamma})\tilde{\mathbf{A}}$.

3.2. Gain/phase and DOA determination from $\mathbf{R}_z$

A careful examination of (8b) indicates that the virtual array possesses the property of shift-invariant. More specifically, as illustrated in Fig. 1, let $\tilde{\mathbf{a}}_1(\theta_l)$ be composed of the 1st to the $(M-1)$th entries plus the $(M+1)$th to the $(2M-1)$th entries of $\tilde{\mathbf{a}}(\theta_l)$, while $\tilde{\mathbf{a}}_2(\theta_l)$ be composed of the 2nd to the Mth entries plus the $(M+2)$th to the $2M$th entries of $\tilde{\mathbf{a}}(\theta_l)$, then mathematically we have

$$\tilde{\mathbf{a}}_2(\theta_l) = \mathbf{\Pi}_2\tilde{\mathbf{a}}(\theta_l) = \mathbf{\Pi}_1\tilde{\mathbf{a}}(\theta_l)\beta_l = \tilde{\mathbf{a}}_1(\theta_l)\beta_l, \tag{9}$$

where $\tilde{\mathbf{a}}_1(\theta_l) = \mathbf{\Pi}_1\tilde{\mathbf{a}}(\theta_l) \in \mathbb{C}^{(2M-2)\times 1}$, and $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ are given by $\mathbf{\Pi}_1 = \mathbf{I}_2 \otimes [\mathbf{I}_{M-1}, \mathbf{0}_{(M-1)\times 1}] \in \mathbb{C}^{(2M-2)\times 2M}$ and $\mathbf{\Pi}_2 = \mathbf{I}_2 \otimes [\mathbf{0}_{(M-1)\times 1}, \mathbf{I}_{M-1}] \in \mathbb{C}^{(2M-2)\times 2M}$. Here, $\otimes$ denotes the Kronecker product. According to (9), let us similarly define $\tilde{\mathbf{A}}_1 = \mathbf{\Pi}_1\tilde{\mathbf{A}} \in \mathbb{C}^{(2M-2)\times L}$ and $\tilde{\mathbf{A}}_2 = \mathbf{\Pi}_2\tilde{\mathbf{A}} \in \mathbb{C}^{(2M-2)\times L}$, then we have

$$\tilde{\mathbf{A}}_2 = \tilde{\mathbf{A}}_1\mathbf{\Phi}, \tag{10}$$

where $\mathbf{\Phi} \in \mathbb{C}^{L\times L}$ is a diagonal matrix given by

$$\mathbf{\Phi} = \mathrm{diag}\{\beta_1, \beta_2, \dots, \beta_L\}. \tag{11}$$

Since $\beta_l = e^{j2\pi\lambda^{-1}d\sin\theta_l}$, it is known that, if $\mathbf{\Phi}$ is available, then the DOAs can be readily extracted.

Now, let us proceed to the determination of $\mathbf{\Phi}$ based on the matrix $\mathbf{U}$ which spans the column space of $\mathbf{B}$, i.e., $\mathbf{U} = \mathbf{B}\mathbf{T}$, where $\mathbf{T} \in \mathbb{C}^{L\times L}$ is a nonsingular matrix. According to (5), it is known that $\mathbf{U}$ corresponds to the L principal eigenvectors of the extended array covariance matrix $\mathbf{R}_z = E\{\mathbf{z}(k)\mathbf{z}^H(k)\}$, where $E\{\cdot\}$ and $(\cdot)^H$ denote the statistical expectation and Hermitian transpose, respectively. On this basis, we obtain two matrices $\mathbf{U}_1 \in \mathbb{C}^{(2M-2)\times L}$ and $\mathbf{U}_2 \in \mathbb{C}^{(2M-2)\times L}$ as

$$\mathbf{U}_1 \triangleq \mathbf{\Pi}_1\mathbf{U} = \mathbf{\Pi}_1\mathbf{B}\mathbf{T} = \mathbf{\Pi}_1\mathbf{\Gamma}(\tilde{\gamma})\tilde{\mathbf{A}}\mathbf{T} = \mathbf{\Gamma}(\tilde{\gamma}_1)\tilde{\mathbf{A}}_1\mathbf{T} \tag{12a}$$

$$\mathbf{U}_2 \triangleq \mathbf{\Pi}_2\mathbf{U} = \mathbf{\Pi}_2\mathbf{B}\mathbf{T} = \mathbf{\Pi}_2\mathbf{\Gamma}(\tilde{\gamma})\tilde{\mathbf{A}}\mathbf{T} = \mathbf{\Gamma}(\tilde{\gamma}_2)\tilde{\mathbf{A}}_2\mathbf{T}, \tag{12b}$$

where $\tilde{\gamma}_1 = \mathbf{\Pi}_1\tilde{\gamma}$ and $\tilde{\gamma}_2 = \mathbf{\Pi}_2\tilde{\gamma}$. Recalling (10), it can be readily derived from (12) that $\mathbf{U}_2 = \mathbf{\Gamma}(\tilde{\gamma}_2)\tilde{\mathbf{A}}_1\mathbf{\Phi}\mathbf{T}$ and $\tilde{\mathbf{A}}_1 = \mathbf{\Gamma}^{-1}(\tilde{\gamma}_1)\mathbf{U}_1\mathbf{T}^{-1}$. Accordingly, we have

$$\mathbf{\Gamma}(\bar{\gamma})\mathbf{U}_2 = \mathbf{U}_1\mathbf{\Omega}, \tag{13}$$

where $\bar{\gamma} = \tilde{\gamma}_1 \oslash \tilde{\gamma}_2$ with $\oslash$ denoting element-wise division. More precisely, it can be expressed as

$$\bar{\gamma} = \left[\frac{\gamma^*_{M-M_c}}{\gamma^*_{M-M_c-1}}, \ldots, \frac{\gamma^*_2}{\gamma^*_1}, \frac{\gamma^*_1}{1}, \mathbf{1}^T_{2M_c-2}, \frac{1}{\gamma_1}, \frac{\gamma_1}{\gamma_2}, \ldots, \frac{\gamma_{M-M_c-1}}{\gamma_{M-M_c}}\right]^T. \quad (14)$$

In (13), $\mathbf{\Omega} \in \mathbb{C}^{L\times L}$ is given by $\mathbf{\Omega} = \mathbf{T}^{-1}\mathbf{\Phi}\mathbf{T}$. In other words, $\mathbf{\Omega}$ and $\mathbf{\Phi}$ are similar matrices, they have the same eigenvalues $\beta_1, \ldots, \beta_L$. This implies that the DOAs can be determined once $\mathbf{\Omega}$ is available. To this end, $\mathbf{\Omega}$ is written as

$$\mathbf{\Omega} = (\mathbf{U}_1^H\mathbf{U}_1)^{-1}\mathbf{U}_1^H\mathbf{\Gamma}(\bar{\gamma})\mathbf{U}_2. \quad (15)$$

In order to compute $\bar{\gamma}$ and thus $\mathbf{\Omega}$, substituting (15) into (13) yields

$$\mathbf{P}_1\mathbf{\Gamma}(\bar{\gamma})\mathbf{U}_2 = \mathbf{0}, \quad (16)$$

where $\mathbf{P}_1 = \mathbf{I}_{2M-2} - \mathbf{U}_1(\mathbf{U}_1^H\mathbf{U}_1)^{-1}\mathbf{U}_1^H$. Obviously, (16) is a linear equation with respect to $\bar{\gamma}$. Furthermore, denote by $\mathbf{u}_{2,l}$ the lth column of $\mathbf{U}_2$, i.e., $\mathbf{U}_2 = [\mathbf{u}_{2,1}, \mathbf{u}_{2,2}, \ldots, \mathbf{u}_{2,L}]$, and utilize the fact that $\mathbf{\Gamma}(\bar{\gamma})\mathbf{u}_{2,l} = \mathbf{\Gamma}(\mathbf{u}_{2,l})\bar{\gamma}$, then we have

$$\mathbf{P}_1\mathbf{\Gamma}(\mathbf{u}_{2,l})\bar{\gamma} = \mathbf{0}, \quad l = 1, 2, \ldots, L. \quad (17)$$

Let us define $\mathbf{Q} \triangleq [(\mathbf{P}_1\mathbf{\Gamma}(\mathbf{u}_{2,l}))^T, \ldots, (\mathbf{P}_1\mathbf{\Gamma}(\mathbf{u}_{2,L}))^T]^T \in \mathbb{C}^{(2M-2)L\times(2M-2)}$ such that (17) can be rewritten in a more compact form as

$$\mathbf{Q}\bar{\gamma} = \mathbf{0}. \quad (18)$$

As shown in (14), the middle $2M_c - 2$ entries of $\bar{\gamma}$ are equal to one, hence, we partition $\bar{\gamma}$ and $\mathbf{Q}$ as

$$\bar{\gamma} \triangleq [\bar{\gamma}_1^T, \mathbf{1}^T_{2M_c-2}, \bar{\gamma}_3^T]^T, \quad \mathbf{Q} \triangleq [\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3], \quad (19)$$

where $\mathbf{Q}_1$ and $\mathbf{Q}_3$ are, respectively, composed of the left and right $M - M_c$ columns, and $\mathbf{Q}_2$ contains the middle $2M_c - 2$ columns of $\mathbf{Q}$. As a result, we have

$$[\mathbf{Q}_1, \mathbf{Q}_3]\begin{bmatrix}\bar{\gamma}_1 \\ \bar{\gamma}_3\end{bmatrix} = -\mathbf{Q}_2\mathbf{1}_{2M_c-2}. \quad (20)$$

To proceed, defining $\mathbf{Q}_{1+3} \triangleq [\mathbf{Q}_1, \mathbf{Q}_3]$ and $\bar{\gamma}_{1+3} = [\bar{\gamma}_1^T, \bar{\gamma}_3^T]^T$ for notational simplicity, it can be readily derived that

$$\bar{\gamma}_{1+3} = -(\mathbf{Q}_{1+3}^H\mathbf{Q}_{1+3})^{-1}\mathbf{Q}_{1+3}^H\mathbf{Q}_2\mathbf{1}_{2M_c-2}. \quad (21)$$

As a consequence, given $\bar{\gamma}_{1+3}$, we can obtain $\bar{\gamma}$, $\mathbf{\Omega}$ and thus $\mathbf{\Phi}$. In addition, the array gain/phase parameters, $\gamma_1, \ldots, \gamma_{M-M_c}$, can be calculated from $\bar{\gamma}_1$ or $\bar{\gamma}_3$ as

$$\gamma_m = \prod_{i=M-M_c-m+1}^{M-M_c} \bar{\gamma}^*_{1,i}, \quad \text{or} \quad \gamma_m = \prod_{i=1}^{m} \bar{\gamma}^{-1}_{3,i}, \quad (22)$$

where $m = 1, \ldots, M - M_c$, $\bar{\gamma}_{1,i}$ and $\bar{\gamma}_{3,i}$ are the ith entry of $\bar{\gamma}_1$ and $\bar{\gamma}_3$, respectively.

3.3. Gain/phase and DOA determination from $\hat{\mathbf{R}}_z$

With the above method for gain/phase parameter estimation, the problem of DOA estimation can be readily tackled. More exactly, given K samples, the DOAs can be estimated with the proposed Rec-ESPRIT-like method as follows:

Step 1: Estimate the extended array covariance matrix as

$$\hat{\mathbf{R}}_z = \frac{1}{K}\sum_{k=1}^{K}\mathbf{z}(k)\mathbf{z}^H(k). \quad (23)$$

Step 2: Perform the EVD of $\hat{\mathbf{R}}_z$, obtain $\hat{\mathbf{U}}$ as the L principal eigenvectors, $\hat{\mathbf{U}}_1 = \mathbf{\Pi}_1\hat{\mathbf{U}}$ and $\hat{\mathbf{U}}_2 = \mathbf{\Pi}_2\hat{\mathbf{U}}$ as (12).

Step 3: Calculate $\hat{\mathbf{P}}_1$, $\hat{\mathbf{Q}}$, $\hat{\mathbf{Q}}_{1+3}$, $\hat{\mathbf{Q}}_2$ as (16)–(20), and

$$\hat{\bar{\gamma}}_{1+3} = -(\hat{\mathbf{Q}}_{1+3}^H\hat{\mathbf{Q}}_{1+3})^{-1}\hat{\mathbf{Q}}_{1+3}^H\hat{\mathbf{Q}}_2\mathbf{1}_{2M_c-2}. \quad (24)$$

Step 4: Construct $\hat{\bar{\gamma}} = [\hat{\bar{\gamma}}_1^T, \mathbf{1}^T_{2M_c-2}, \hat{\bar{\gamma}}_3^T]^T$, and obtain $\hat{\gamma}_m$ as (22), and estimate $\mathbf{\Omega}$ as

$$\hat{\mathbf{\Omega}} = (\hat{\mathbf{U}}_1^H\hat{\mathbf{U}}_1)^{-1}\hat{\mathbf{U}}_1^H\mathbf{\Gamma}(\hat{\bar{\gamma}})\hat{\mathbf{U}}_2. \quad (25)$$

Step 5: Carry out the EVD of $\hat{\mathbf{\Omega}}$ and estimate $\beta_1, \ldots, \beta_L$ as its eigenvalues.

Step 6: Estimate the DOAs according to the definition of β_l as follows:

$$\hat{\theta}_l = \arcsin\left\{\frac{\lambda}{2\pi d}\arg(\hat{\beta}_l)\right\}. \quad (26)$$

It is worth noting that once the estimates of the gain/phase parameters have been obtained as stated in Steps 3 and 4, we can also employ the MUSIC-like algorithm [16] for DOA estimation. A better performance in terms of DOA estimation accuracy is achievable at the cost of much more computational complexities. In this work, we shall not pursue this direction.

From the above steps, it can be seen that the proposed Rec-ESPRIT-like method involves the EVDs of $\hat{\mathbf{R}}_z \in \mathbb{C}^{2M\times 2M}$ and $\hat{\mathbf{\Omega}} \in \mathbb{C}^{L\times L}$, and the inversions of $\hat{\mathbf{Q}}_{1+3}^H\hat{\mathbf{Q}}_{1+3} \in \mathbb{C}^{(2M-2M_c)\times(2M-2M_c)}$ and $\hat{\mathbf{U}}_1^H\hat{\mathbf{U}}_1 \in \mathbb{C}^{L\times L}$. This indicates that the overall complexity of the proposed method is $\mathcal{O}(8M^3)$. Since the complexity of the ESPRIT-like method is $\mathcal{O}(M^3)$, the Rec-ESPRIT-like method has a relatively higher complexity.

To define the projector matrix $\mathbf{P}_1$ in (16), we need the condition $2M - 2 > L$. Consequently, we can only have $L \le 2M - 3$. In other words, the proposed method is able to handle at most $2M - 3$ rectilinear signals. This will also be verified by numerical results in the next section. Finally, it is worth mentioning that the proposed method makes use of a similar concept of the ESPRIT algorithm. Thus, besides ULAs, it is applicable to other arrays with shift-invariant structures.

4. Numerical results

In this section, four numerical examples are provided to illustrate the performance of the proposed Rec-ESPRIT-like method. In the first three examples, we assume $M = 8$ and $L = 3$, while in the fourth one we set $M = 5$ and $L = 7$. The signals are assumed to be uncorrelated, unless otherwise specified. The array elements are spaced by half-wavelength. The gain/phase vector is chosen as $\gamma = [\mathbf{1}^T_{M_c}, \ \mathbf{c}(M_c + 1 : M)]^T$ with $\mathbf{c} = [0.88e^{j\pi/6}, 1.45e^{-j\pi/8}, 0.94e^{-j\pi/4}, 0.8e^{j\pi/5}, 1.25e^{-j\pi/3}, 1.53e^{-j\pi/5}, 0.75e^{j\pi/4}, 1.36e^{-j\pi/10}]$. The phase shifts of the rectilinear signals are generated from the uniform distribution $\mathcal{U}[0, 2\pi]$ and thus $\mathbf{\Psi}$ can be obtained accordingly. In the first three example, we randomly choose $\mathbf{\Psi}$ as $\mathbf{\Psi} = \text{diag}\{0.5166 + j0.8562, -0.9022 - j0.4313, 0.8607 - j0.5090\}$. The root mean squared error (RMSE) of DOA estimation, calculated from 500 Monte Carlo experiments, is adopted as the performance measure. More precisely, the RMSE is computed as $\text{RMSE} = \sqrt{\frac{1}{LQ}\sum_{l=1}^{L}\sum_{q=1}^{Q}(\hat{\theta}_{l,q} - \theta_l)^2}$, where $\hat{\theta}_{l,q}$ represents the DOA estimate of the lth signal in the qth Monte Carlo trial and Q is the total number of Monte Carlo trials. The proposed Rec-ESPRIT-like method is compared to the ESPRIT-like method [11], ESPRIT-like method with known gains and phases, and Rec-ESPRIT-like method with known gains and phases. The noncircular Cramer–Rao bound (NC-CRB) and rectilinear Cramer–Rao bound (Rec-CRB) given in [14] are also provided.

Example 1. Assume that $M_c = 4$, $K = 500$ and the signal DOAs are $\{-8^\circ, 0^\circ, 10^\circ\}$. Fig. 2 shows the RMSEs of DOA estimation versus signal-to-noise ratio (SNR). It is seen that the Rec-ESPRIT-like method performs significantly better than the traditional ESPRIT-like method and can even outperform the ESPRIT-like method with known gains/phases. Moreover, the NC-CRB is much higher than

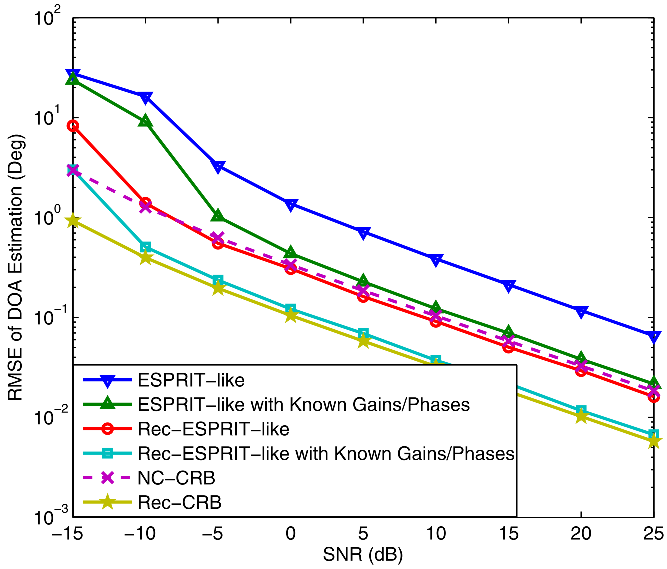

Fig. 2. RMSE of DOA estimation versus SNR, $M_c = 4$ and $K = 500$.

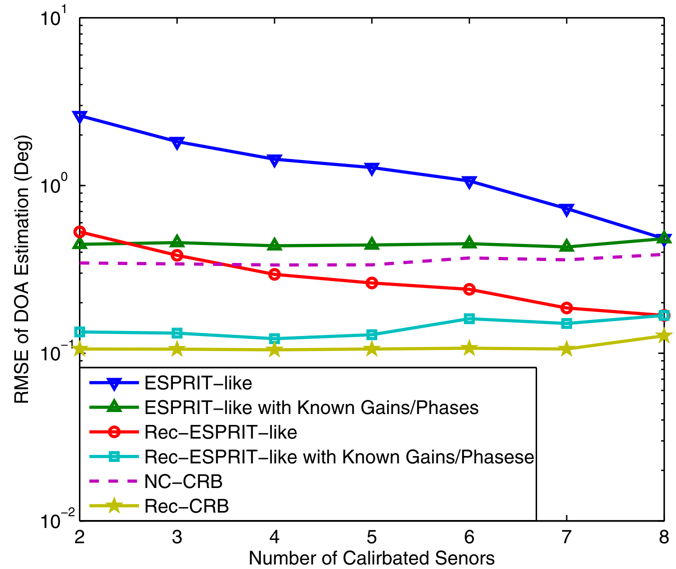

Fig. 4. RMSE of DOA estimation versus number of calibrated elements, SNR = 0 dB and $K = 500$.

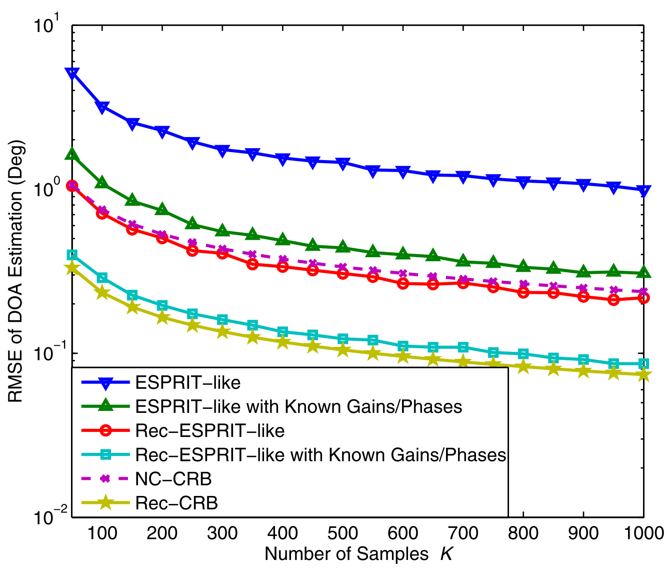

Fig. 3. RMSE of DOA estimation versus number of samples, $M_c = 4$ and SNR = 0 dB.

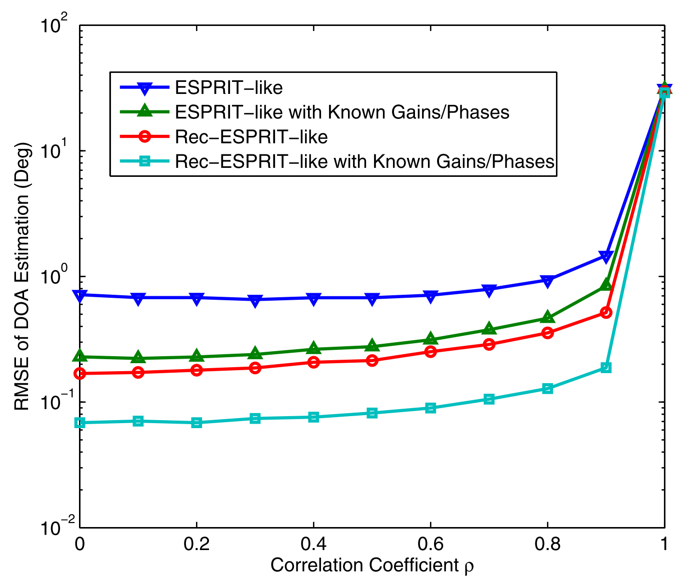

Fig. 5. DOA estimates versus correlation coefficient, $M_c = 4$, SNR = 5 dB and $K = 500$.

the Rec-CRB since it does not take the signal rectilinearity into account [14]. Additionally, it is noticed that there is a gap between the Rec-ESPRIT-like method and Rec-CRB. This is because the ESPRIT-like methods have not made full use of the array aperture. It is possible to narrow down the gap by employing MUSIC-like method and iterative refinement. However, it will not be considered in this work due to higher computation complexities and the interested reader is referred to the literature, e.g., [12] and [16].

Example 2. We now assume $M_c = 4$ and SNR = 0 dB, the RMSEs of DOA estimation versus the number of samples K are demonstrated in Fig. 3. Again, we can see the performance superiority of the proposed Rec-ESPRIT-like method by taking advantage of the signal rectilinearity.

Example 3. We now set SNR = 0 dB and $K = 500$, and vary the number of calibrated elements M_c from 2 to 8. The resulting RMSEs of DOA estimation versus M_c are drawn in Fig. 4. As expected, the DOA estimation accuracy becomes better along with the increase of the number of calibrated sensors. In particular, when $M_c = M = 8$, i.e., the whole array is well calibrated, the Rec-ESPRIT-like is reduced to the Rec-ESPRIT-like with known gains and phases, and therefore, they have the same behavior.

Example 4. In this example, we examine the performance of the proposed method when the signals are correlated. To do so, we fix $M_c = 4$, SNR = 5 dB and $K = 500$, and vary the correlation coefficient ρ between $s_1(t)$ and $s_2(t)$ (and between $s_3(t)$ and $s_2(t)$) from 0 (i.e, uncorrelated) to 1 (i.e., coherent). The RMSEs of DOA estimation versus correlation coefficient ρ are depicted in Fig. 5. It is seen that the proposed Rec-ESPRIT-like method is able to perform quite well provided that the signals are not strongly correlated (coherent).

Example 5. In the last example, the maximum number of signals that can be handled by the proposed Rec-ESPRIT-like method

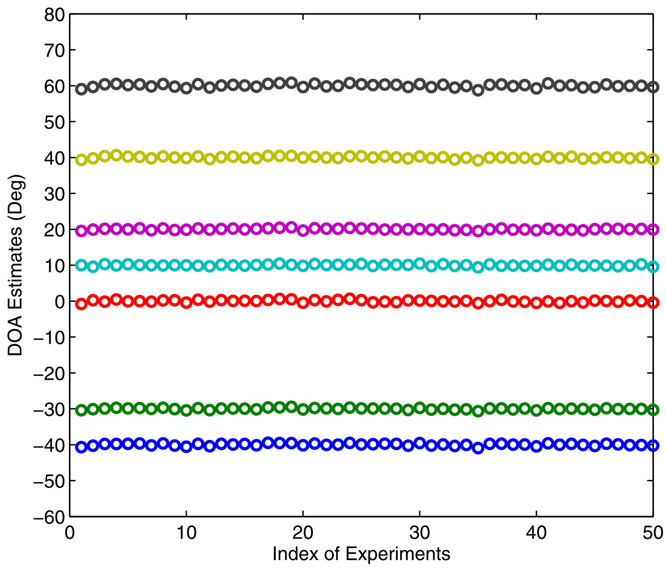

Fig. 6. DOA estimates versus index of experiments, $M_c = 2$, $L = 7$ and SNR = 20 dB.

in the case of $M_c = 2$ is tested. To this end, it is assumed that SNR = 20 dB, $M = 5$, $L = 2M - 3 = 7$, and the DOAs are $\{-40^\circ, -30^\circ, 0^\circ, 10^\circ, 20^\circ, 40^\circ, 60^\circ\}$. The resulting DOA estimates of 50 independent experiments are shown in Fig. 6. It is observed that the proposed Rec-ESPRIT-like method is capable of accurately estimating all of the seven DOAs. However, the traditional ESPRIT-like method which does not consider the signal rectilinearity can handle at most 3 signals in this scenario.

5. Conclusion

A new method, named as Rec-ESPRIT-like, is proposed for DOA estimation of rectilinear signals with a partly calibrated ULA. By making use of the signal rectilinearity, an extended data model is introduced and a linear equation is developed for gain/phase determination. It is shown that the DOA estimates can be achieved via EVD and the number of signals can be handled is much larger than the number of array elements. The effectiveness and superiority of the proposed method are verified by representative numerical examples.

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