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Improved two-dimensional DOA estimation algorithm for two-parallel uniform linear arrays using propagator method

Jianfeng Li *, Xiaofei Zhang, Han Chen

College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

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ABSTRACTS

In this paper, we study the problem of two-dimensional direction-of-arrival (DOA) estimation with two parallel uniform arrays (ULA). The computational loads of the propagator method (PM) can be significantly smaller since the PM does not require any eigen-value decomposition of the cross correlation matrix and singular value decomposition of the received data. We propose an improved two-dimensional DOA estimation algorithm for two-parallel uniform linear arrays using PM. Compared with Wu's algorithm, the proposed algorithm has better angle estimation performance, and lower complexity. Furthermore, the proposed algorithm can achieve automatically paired two-dimensional angle estimation. The simulation results verify the effectiveness of the proposed algorithm.

1. Introduction

Two-dimensional direction-of-arrival (DOA) estimation has been applied in many fields, such as radar, sonar, wireless communication [1–3]. The subspace-based methods, such as multiple signal classification (MUSIC) [4] and estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm [5,6], has been widely used in DOA estimation. However, they require eigen-value decomposition (EVD) of the cross correlation matrix and singular value decomposition (SVD) of the received data to obtain signal subspace or noise subspace, which are highly computationally intensive. It is well known that computational complexity of the propagator method (PM) [7] is significantly smaller because the PM does not require any EVD of the cross correlation matrix and SVD of the received data. In [8], Wu proposed a propagator method (PM)-based DOA estimation algorithm for two parallel uniform linear arrays, which required extra pairing match. Ref. [9] presented a computationally efficient 2-D DOA estimation for two parallel uniform linear array using PM, which has worse angle estimation performance than Wu's algorithm in [8]. In this paper, we propose an improved two-dimensional DOA estimation algorithm for two-parallel uniform linear arrays (ULAs) using PM. The proposed algorithm has the following advantages: (1) it can achieve automatically paired two-dimensional estimation of angles, while Wu's algorithm needs additional pair matching; (2) it has better angle estimation performance than Wu's algorithm; and (3) it has lower complexity than Wu's method.

The remainder of this paper is structured as follows. Section 2 develops the data model with two parallel uniform arrays, and Section 3 presents the improved propagator method (PM) algorithm for DOA estimation, the complexity analysis and Cramer–Rao bound (CRB). In Section 4, simulation results are presented to verify the improvement of the proposed algorithm, while the conclusions are shown in Section 5.

Notation: $(.)^T$, $(.)^H$, $(.)^{-1}$ and $(.)^+$ denote transpose, conjugate-transpose, inverse, pseudo-inverse operations, respectively; diag($\mathbf{v}$) stands for diagonal matrix whose diagonal element is a vector $\mathbf{v}$; $\mathbf{I}_K$ is a $K \times K$ identity matrix; $\odot$ is the Hadamard product; Re(.) is to get real

* Corresponding author.
E-mail address: lijianfengtin@126.com (J. Li).

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part of the complex; min(.) is to get minimum elements of an array; $E[.]$is expectation operator; $angle(\cdot)$ is to get the phase; $amp(\cdot)$ is to get the modulus.

2. Data model

In order to obtain the joint estimation of the elevation and azimuth angles, we consider the two parallel ULAs as Fig. 1, which is same with the array structure in [8]. The array is located in the x–y plane with interelement spacing d, and the coordinates of the two ULAs are $(d\cdot i,0),0\le i\le N$ and $(d\cdot l,0),0\le l\le N-1$, respectively. qSuppose that there are K far-field narrowband sources impinging on the arrays. Let θ_k and ϕ_k be the elevation and azimuth angles of the kth source, then the output of the two ULAs can be expressed as

$$\mathbf{x}_1(t)=\mathbf{A}_x\mathbf{s}(t)+\mathbf{w}_1(t) \tag{1}$$

$$\mathbf{x}_2(t)=\mathbf{A}_y\mathbf{\Phi}_y s(t)+\mathbf{w}_2(t) \tag{2}$$

where $\mathbf{A}_x=[\mathbf{a}_x(\theta_1,\phi_1),\mathbf{a}_x(\theta_2,\phi_2),\ldots,\mathbf{a}_x(\theta_K,\phi_K)]$, $\mathbf{s}(t)=[s_1(t),s_2(t),\ldots,s_K(t)]^T$, with $s_k(t)$ be the kth source, we can estimate the source number K by applying the method in [10] or assume K is known a priori. $\mathbf{a}_x(\theta_k,\phi_k)=[1,e^{-j2\pi d\cos\phi_k\sin\theta_k/\lambda},\ldots,e^{-j2\pi Nd\cos\phi_k\sin\theta_k/\lambda}]^T$ with λ being wavelength, $\mathbf{A}_y$ is the first N rows of $\mathbf{A}_x$, $\mathbf{\Phi}_y=diag[e^{-j2\pi d\sin\phi_1\sin\theta_1/\lambda},e^{-j2\pi d\sin\phi_2\sin\theta_2/\lambda},\ldots,e^{-j2\pi d\sin\phi_K\sin\theta_K/\lambda}]$. $\mathbf{w}_1(t)$ and $\mathbf{w}_2(t)$ are Gaussian white noise vector of zeros mean and variance σ^2 and are uncorrelated with $\mathbf{s}(t)$.

Define the array output as

$$\mathbf{x}(t)=\begin{bmatrix}\mathbf{x}_1(t)\\ \mathbf{x}_2(t)\end{bmatrix}=\mathbf{A}\mathbf{s}(t)+\mathbf{w}(t) \tag{3}$$

where

$$\mathbf{A}=\begin{bmatrix}\mathbf{A}_x\\ \mathbf{A}_y\mathbf{\Phi}_y\end{bmatrix},\quad \mathbf{w}(t)=\begin{bmatrix}\mathbf{w}_1(t)\\ \mathbf{w}_2(t)\end{bmatrix}.$$

We assume that $\mathbf{A}_x$ and $\mathbf{A}_y$ are constant for J samples, and then the received data is $\mathbf{X}=[\mathbf{x}(1),\mathbf{x}(2),\ldots,\mathbf{x}(J)]$, which can be expressed by

$$\mathbf{X}=\mathbf{A}\mathbf{S}+\mathbf{W} \tag{4}$$

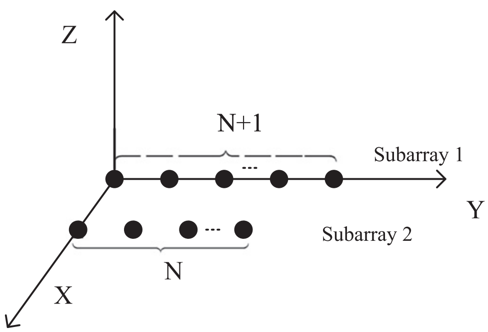

Fig. 1. Illustration of the array geometry.

where $\mathbf{W}\in\mathbb{C}^{(2N+1)\times J}$ is the noise matrix, $\mathbf{S}=[\mathbf{s}(1),\mathbf{s}(2),\ldots,\mathbf{s}(J)]\in\mathbb{C}^{K\times J}$.

3. Angle estimation algorithm

3.1. Improved DOA estimation using PM

Partition the matrix $\mathbf{A}$ as

$$\mathbf{A}=\begin{bmatrix}\mathbf{A}_1\\ \mathbf{A}_2\end{bmatrix} \tag{5}$$

where $\mathbf{A}_1$is a $K\times K$ nonsingular matrix, $\mathbf{A}_2$ is a $(2N+1-K)\times K$ matrix. Then $\mathbf{A}_2$ is a linear transformation of $\mathbf{A}_1$

$$\mathbf{P}^H\mathbf{A}_1=\mathbf{A}_2 \tag{6}$$

where $\mathbf{P}\in\mathbb{C}^{K\times(2N+1-K)}$ is the propagator matrix.

The covariance matrix of the received data is $\mathbf{R}=(1/J)\sum_{t=1}^{J}\mathbf{x}(t)\mathbf{x}^H(t)$, which is partitioned as

$$\mathbf{R}=\begin{bmatrix}\mathbf{R}_1 & \mathbf{R}_2\end{bmatrix} \tag{7}$$

where $\mathbf{R}_1\in\mathbb{C}^{(2N+1)\times K}$, $\mathbf{R}_2\in\mathbb{C}^{(2N+1)\times(2N+1-K)}$. In the noiseless case

$$\mathbf{R}_2=\mathbf{R}_1\mathbf{P}$$

Actually, there is always noise, and the propagator matrix can be estimated by the following minimization problem:

$$J_{csm}(\mathbf{P})=\|\mathbf{R}_2-\mathbf{R}_1\mathbf{P}\|_F^2$$

where $\|\cdot\|$ denotes Frobenius norm. The estimate of $\mathbf{P}$ is via

$$\hat{\mathbf{P}}=(\mathbf{R}_1^H\mathbf{R}_1)^{-1}\mathbf{R}_1^H\mathbf{R}_2 \tag{8}$$

We define $\mathbf{P}_c=\begin{bmatrix}\mathbf{I}_{K\times K}\\ \hat{\mathbf{P}}^H\end{bmatrix}$. In noiseless case, $\mathbf{P}_c\mathbf{A}_1=\mathbf{A}$, and we partition $\mathbf{P}_c$ as

$$\mathbf{P}_c=\begin{bmatrix}\mathbf{P}_x\\ \mathbf{P}_y\end{bmatrix},\quad \mathbf{P}_x\in\mathbb{C}^{(N+1)\times K},\quad \mathbf{P}_y\in\mathbb{C}^{N\times K}. \tag{9}$$

and use $\mathbf{P}_1$ to denote the first N rows of $\mathbf{P}_x$. According to (5) and (9),

$$\mathbf{P}_1\mathbf{A}_1=\mathbf{A}_y \tag{10a}$$

$$\mathbf{P}_y\mathbf{A}_1=\mathbf{A}_y\mathbf{\Phi}_y \tag{10b}$$

Then

$$\mathbf{\Psi}_y=\mathbf{P}_1^+\mathbf{P}_y=\mathbf{A}_1\mathbf{\Phi}_y\mathbf{A}_1^{-1} \tag{11}$$

We perform the eigen-value decomposition of $\mathbf{\Psi}_y$. The eigen-values are corresponding to the diagonal elements of $\mathbf{\Phi}_y$, and the eigenvectors are the estimate of $\mathbf{A}_1$

$$\mathbf{A}_1'=\mathbf{A}_1\mathbf{\Pi}$$

where $\mathbf{\Pi}$ is a permutation matrix, and $\mathbf{\Pi}^{-1}=\mathbf{\Pi}$, and then the estimate of $\mathbf{\Phi}_y$ is $\hat{\mathbf{\Phi}}_y=\mathbf{\Pi}\mathbf{\Phi}_y\mathbf{\Pi}$. Assume $\hat{\beta}_k$ is the kth diagonal element of $\hat{\mathbf{\Phi}}_y$.

Use $\mathbf{P}_2$ to denote the last N rows of $\mathbf{P}_x$, $\mathbf{P}_3$ to denote the first $(N-1)$ rows of $\mathbf{P}_y$, and $\mathbf{P}_4$ to denote the last $(N-1)$ rows of $\mathbf{P}_y$. We define $\mathbf{B}_1\triangleq\begin{bmatrix}\mathbf{P}_1\\ \mathbf{P}_3\end{bmatrix}\mathbf{A}_1'$ and $\mathbf{B}_2\triangleq\begin{bmatrix}\mathbf{P}_2\\ \mathbf{P}_4\end{bmatrix}\mathbf{A}_1'$. Assume $\mathbf{A}'=\mathbf{A}\mathbf{\Pi}$, In noiseless case, $\mathbf{P}_1\mathbf{A}_1'$ is the first N rows of $\mathbf{A}'$, $\mathbf{P}_2\mathbf{A}_1'$ is the second to $(N+1)$th row of $\mathbf{A}'$, $\mathbf{P}_3\mathbf{A}_1'$ is the $(N+2)$th to the last row of $\mathbf{A}'$, and $\mathbf{P}_4\mathbf{A}_1'$ is the last $(N-1)$

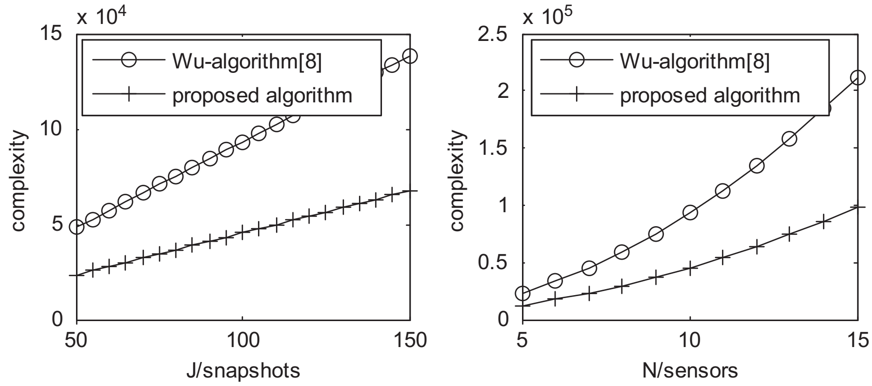

Fig. 2. Complexity comparison versus J and N.

rows of $\mathbf{A}'$, so

$$\mathbf{P}_1\mathbf{A}_1' = \mathbf{P}_2\mathbf{A}_1'\hat{\boldsymbol{\Phi}}_x \tag{12a}$$

$$\mathbf{P}_3\mathbf{A}_1' = \mathbf{P}_4\mathbf{A}_1'\hat{\boldsymbol{\Phi}}_x \tag{12b}$$

which leads to

$$\mathbf{B}_2 = \mathbf{B}_1\hat{\boldsymbol{\Phi}}_x \tag{12c}$$

where $\hat{\boldsymbol{\Phi}}_x = \boldsymbol{\Pi}\boldsymbol{\Phi}_x\boldsymbol{\Pi}$ with $\boldsymbol{\Phi}_x = diag[e^{-j2\pi d\cos\phi_1 \sin\theta_1/\lambda}, e^{-j2\pi d\cos\phi_2 \sin\theta_2/\lambda}, \ldots, e^{-j2\pi d\cos\phi_K \sin\theta_K/\lambda}]$.

We assume the kth diagonal element of $\mathbf{B}_1{}^{+}\mathbf{B}_2$ as $\hat{\alpha}_k$, and define $\hat{\eta}_k = -angle(\hat{\alpha}_k) - jangle(\hat{\beta}_k)$. Then the elevation and azimuth angles are estimated by

$$\hat{\phi}_k = angle(\hat{\eta}_k) \tag{13a}$$

$$\hat{\theta}_k = \sin^{-1}(amp(\hat{\eta}_k)\lambda/2\pi d) \tag{13b}$$

Till now, we have achieved the proposal for the improved PM algorithm for DOA estimation with two parallel ULAs. We show the major steps and complexity of the proposed algorithm as follows:

(1) Construct the covariance matrix…………………………………………………………………………$O\{(2N+1)^2 J\}$
(2) Partition the covariance matrix to estimate $\hat{\mathbf{P}}$ and $\mathbf{P}_c$ ……………$O\{K^3+(2N+1-K)K^2+(2N+1)^2K\}$
(3) Partition $\mathbf{P}_c$ to get $\mathbf{P}_1,\mathbf{P}_2,\mathbf{P}_3,\mathbf{P}_4$ and $\mathbf{P}_y$, obtain $\hat{\beta}_k$ and $\mathbf{A}_1'$ via (10) and (11)…… $O\{2K^2N+3K^3\}$
(4) We obtain $\mathbf{B}_1$ and $\mathbf{B}_2$ from $\mathbf{A}_1'$, obtain $\hat{\alpha}_k$ from $\mathbf{B}_1{}^{+}\mathbf{B}_2$, and finally estimate elevation and azimuth via (13)……. ………………………………………………………………$O\{K^2(4N-2)+(4N-1)K\}$

Remark 1. The elevation angle is assumed to be within [0° 90°], which would not cause ambiguity problems. For the azimuth angle is within [−180° 180°], and we estimate azimuth angle through $\hat{\phi}_k = angle(\hat{\eta}_k)$, which is also within [−180° 180°], so there will be no ambiguity problems, too.

Remark 2. There are some differences between the proposed algorithm and Wu's algorithm. (1) The proposed algorithm can obtain automatically-paired two dimensional angles, while Wu's algorithm requires additional pairing. (2) The proposed algorithm constructs an improved the propagator matrix $\mathbf{P}_c$, which use fully all elements of the array, while the propagator matrix in Wu's algorithm misses some elements. So the proposed algorithm has better angle estimation performance than Wu's algorithm. (3) Wu's algorithm will have angle ambiguity problem, but the proposed algorithm uses (13) to estimate the angles, which will avoid the angle ambiguity problem (referring to Remark 1). (4) The proposed algorithm has lower complexity than Wu's method.

3.2. *Complexity analysis*

The complexity we analyze depends on the matrix complex multiplication. The complexity of the proposed algorithm is $O\{(2N+1)^2(J+K)+3K^3+(8N-1)K^2+(4N-1)K\}$, while Wu's algorithm [8] costs $O\{9N^2(J+K)+5NK^2-K^3+2K(N-K)^2+2(N-K)^3\}$ without considering the pairing match. Fig. 2 shows the complexity comparison between the proposed algorithm and Wu's algorithm. From Fig. 2, we find that Wu's algorithm has heavier computational load than the proposed algorithm.

4. Error analysis and Cramer–Rao bound (CRB)

4.1. *Error analysis*

This section aims at analyzing estimation error. We assume $\overline{\mathbf{P}}_c = \mathbf{P}_c + \partial\mathbf{P}_c$ where $\partial\mathbf{P}_c$ is error estimation matrix. Then $\overline{\mathbf{P}}_1 = \mathbf{P}_1 + \partial\mathbf{P}_1, \overline{\mathbf{P}}_y = \mathbf{P}_y + \partial\mathbf{P}_y$. According to the first-order approximation for $[\mathbf{P}_1+\partial\mathbf{P}_1]^{+}$, we get $\hat{\boldsymbol{\Psi}}_y$

$$\hat{\boldsymbol{\Psi}}_y = \mathbf{A}_1{}^{-1}(\mathbf{I}_K + \mathbf{P}_1^{+}(\partial\mathbf{P}_y - \partial\mathbf{P}_1))\boldsymbol{\Phi}_y\mathbf{A}_1 \tag{14}$$

The kth eigenvalue of $\hat{\boldsymbol{\Psi}}_y$ is $\hat{\beta}_k = \beta_k + \partial\beta_k$, where $\partial\beta_k = \beta_k\mathbf{e}_k^T\mathbf{P}_1^{+}(\partial\mathbf{P}_y - \partial\mathbf{P}_1)\mathbf{e}_k$, and $\mathbf{e}_k$ is a unit vector, in which the kth element is 1, and others are zeros. We also get

$$\overline{\mathbf{A}'}_1 = \mathbf{A}_1' + \partial\mathbf{A}_1' \tag{15}$$

where $\partial\mathbf{A}_1'$ is estimation error of $\mathbf{A}_1'$. According to Eqs. (12) and (15), we have

$$\overline{\mathbf{B}}_1=\begin{bmatrix}\overline{\mathbf{P}}_1\\ \overline{\mathbf{P}}_3\end{bmatrix}\overline{\mathbf{A}}_1'=\begin{bmatrix}\mathbf{P}_1\\ \mathbf{P}_3\end{bmatrix}\mathbf{A}_1'+\begin{bmatrix}\mathbf{P}_1\\ \mathbf{P}_3\end{bmatrix}\partial\mathbf{A}_1'+\begin{bmatrix}\partial\mathbf{P}_1\\ \partial\mathbf{P}_3\end{bmatrix}\mathbf{A}_1'+\begin{bmatrix}\partial\mathbf{P}_1\\ \partial\mathbf{P}_3\end{bmatrix}\partial\mathbf{A}_1' \tag{16a}$$

$$\overline{\mathbf{B}}_2=\begin{bmatrix}\overline{\mathbf{P}}_2\\ \overline{\mathbf{P}}_4\end{bmatrix}\overline{\mathbf{A}}_1'=\begin{bmatrix}\mathbf{P}_2\\ \mathbf{P}_4\end{bmatrix}\mathbf{A}_1'+\begin{bmatrix}\mathbf{P}_2\\ \mathbf{P}_4\end{bmatrix}\partial\mathbf{A}_1'+\begin{bmatrix}\partial\mathbf{P}_2\\ \partial\mathbf{P}_4\end{bmatrix}\mathbf{A}_1'+\begin{bmatrix}\partial\mathbf{P}_2\\ \partial\mathbf{P}_4\end{bmatrix}\partial\mathbf{A}_1' \tag{16b}$$

Using the first-order approximation for $[\mathbf{B}_1+\partial\mathbf{B}_1]^+$, we get

$$\hat{\mathbf{\Phi}}_x=\mathbf{\Phi}_x+\mathbf{B}_1^+(\partial\mathbf{B}_2-\partial\mathbf{B}_1)\mathbf{\Phi}_x$$

We define $\hat{\alpha}_k=\alpha_k+\partial\alpha_k$, where α_k and $\partial\alpha_k$ are the (k,k) element of $\mathbf{\Phi}_x$ and $\mathbf{B}_1^+(\partial\mathbf{B}_2-\partial\mathbf{B}_1)\mathbf{\Phi}_x$.

As $\hat{\eta}_k=-angle(\hat{\alpha}_k)-jangle(\hat{\beta}_k)$, then

$$\partial\eta_k=-\mathrm{Im}\left(\frac{\partial\alpha_k}{\alpha_k}\right)-j\,\mathrm{Im}\left(\frac{\partial\beta_k}{\beta_k}\right) \tag{17}$$

Then the variance of azimuth estimation error is shown as

$$E\left[\partial\phi_k^2\right]=\left[\mathrm{Im}\left(\frac{\partial\eta_k}{\eta_k}\right)\right]^2=\frac{1}{2}\left[E\left\{|\partial\eta_k|^2\right\}-\mathrm{Re}\left\{E\left[(\partial\eta_k)^2(\eta_k^*)^2\right]\right\}\right] \tag{18}$$

And the variance of elevation estimation error is shown as

$$\begin{aligned}E\left[\partial\theta_k^2\right]&=\left[\frac{\lambda}{2\pi d\cos\theta_k}\right]^2\left[\mathrm{Re}\left(\frac{\partial\eta_k}{\eta_k|\eta_k|}\right)\right]^2\\&=\left[\frac{\lambda}{2\pi d\cos\theta_k}\right]^2\frac{1}{2}\left[E\left\{\left|\frac{\partial\eta_k}{\eta_k|\eta_k|}\right|\right\}^2-\mathrm{Im}\left\{E\left[\left(\frac{\partial\eta_k}{\eta_k|\eta_k|}\right)\right]^2\right\}\right]\\&=\frac{1}{2}\left[\frac{\lambda}{2\pi d}\right]^4\left[\frac{1}{\cos\theta_k\sin\theta_k}\right]^2\left\{\left[\frac{\lambda}{2\pi d\sin\theta_k}\right]^2E\left\{|\partial\eta_k|^2\right\}\right.\\&\quad\left.-\mathrm{Im}\{E[(\eta_k^*)^2(\partial\eta_k)^2]\}\right\}\end{aligned} \tag{19}$$

4.2. Cramer–Rao bound

According to [11], we know the CRB for 1-dimensional DOA estimation, and now we derive the CRB for two-dimensional DOA estimation, which is an expansion of 1-dimensional CRB. The parameters which is needed to estimate can be expressed as

$$\boldsymbol{\zeta}=\left[\theta_1,\ldots,\theta_K,\phi_1,\ldots,\phi_K,\mathbf{s}_R{}^T(1),\ldots,\mathbf{s}_R{}^T(J),\mathbf{s}_I{}^T(1),\ldots,\mathbf{s}_I{}^T(J),\sigma^2\right]^T \tag{20}$$

where $\mathbf{s}_R(t),t=1,\ldots,J$ and $\mathbf{s}_I(t),t=1,\ldots,J$ denote the real and imaginary parts of $\mathbf{s}(t)$, respectively.

The output with J snapshots is expressed as

$$\mathbf{y}=\left[\mathbf{x}^T(1),\ldots,\mathbf{x}^T(J)\right]$$

The mean $\boldsymbol{\mu}$ and the covariance matrix $\mathbf{\Gamma}$ of $\mathbf{y}$ are

$$\boldsymbol{\mu}=\begin{bmatrix}\mathbf{A}\mathbf{s}(1)\\ \vdots\\ \mathbf{A}\mathbf{s}(J)\end{bmatrix},\quad \mathbf{\Gamma}=\begin{bmatrix}\sigma^2\mathbf{I} & & 0\\ & \ddots & \\ 0 & & \sigma^2\mathbf{I}\end{bmatrix} \tag{21}$$

From [11], the (i,j) element of the CRB matrix ($\mathbf{P}_{cr}$) is expressed as

$$\left[\mathbf{P}_{cr}^{-1}\right]_{ij}=tr\left[\mathbf{\Gamma}^{-1}\mathbf{\Gamma}_i'\,\mathbf{\Gamma}^{-1}\mathbf{\Gamma}_j'\right]+2\mathrm{Re}\left[\boldsymbol{\mu}_i'*\mathbf{\Gamma}^{-1}\boldsymbol{\mu}_j'\right] \tag{22}$$

where $\mathbf{\Gamma}_i'$ and $\boldsymbol{\mu}_i'$ are the derivative of $\mathbf{\Gamma}$ and $\boldsymbol{\mu}$ on the ith element of $\boldsymbol{\zeta}$, respectively. The covariance matrix is just related to σ^2, so the first part of (22) can be ignored. Then the (i,j) element of the CRB matrix ($\mathbf{P}_{cr}$) can be rewritten as

$$[\mathbf{P}_{cr}^{-1}]_{ij}=2\mathrm{Re}[\boldsymbol{\mu}_i'^*\mathbf{\Gamma}^{-1}\boldsymbol{\mu}_j'] \tag{23}$$

According to (21),

$$\frac{\partial\boldsymbol{\mu}}{\theta_k}=\begin{bmatrix}\frac{\partial\mathbf{A}}{\partial\theta_k}\mathbf{s}(1)\\ \vdots\\ \frac{\partial\mathbf{A}}{\partial\theta_k}\mathbf{s}(J)\end{bmatrix}=\begin{bmatrix}\mathbf{d}_{k\theta}\mathbf{s}_k(1)\\ \vdots\\ \mathbf{d}_{k\theta}\mathbf{s}_k(J)\end{bmatrix},\quad k=1,\ldots,K \tag{24a}$$

$$\frac{\partial\boldsymbol{\mu}}{\phi_k}=\begin{bmatrix}\frac{\partial\mathbf{A}}{\partial\phi_k}\mathbf{s}(1)\\ \vdots\\ \frac{\partial\mathbf{A}}{\partial\phi_k}\mathbf{s}(J)\end{bmatrix}=\begin{bmatrix}\mathbf{d}_{k\phi}\mathbf{s}_k(1)\\ \vdots\\ \mathbf{d}_{k\phi}\mathbf{s}_k(J)\end{bmatrix},\quad k=1,\ldots,K \tag{24b}$$

where $\mathbf{s}_k(t)$ is the kth element of $\mathbf{s}(t)$, and

$$\mathbf{d}_{k\theta}=\frac{\partial\mathbf{a}(\theta_k,\phi_k)}{\partial\theta_k} \tag{25a}$$

$$\mathbf{d}_{k\phi}=\frac{\partial\mathbf{a}(\theta_k,\phi_k)}{\partial\phi_k} \tag{25b}$$

where $\mathbf{a}(\theta_k,\phi_k)$ is the kth column of $\mathbf{A}$.

Let

$$\mathbf{\Delta}\triangleq\begin{bmatrix}\mathbf{d}_{1\theta}\mathbf{s}_1(1) & \cdots & \mathbf{d}_{K\theta}\mathbf{s}_K(1) & \mathbf{d}_{1\phi}\mathbf{s}_1(1) & \cdots & \mathbf{d}_{K\phi}\mathbf{s}_K(1)\\ \vdots & & \vdots & \vdots & & \vdots\\ \mathbf{d}_{1\theta}\mathbf{s}_1(J) & \cdots & \mathbf{d}_{K\theta}\mathbf{s}_K(J) & \mathbf{d}_{1\phi}\mathbf{s}_1(J) & \cdots & \mathbf{d}_{K\phi}\mathbf{s}_K(J)\end{bmatrix} \tag{26}$$

and

$$\mathbf{G}\triangleq\begin{bmatrix}\mathbf{A} & & 0\\ & \ddots & \\ 0 & & \mathbf{A}\end{bmatrix},\quad \mathbf{s}=\begin{bmatrix}\mathbf{s}(1)\\ \vdots\\ \mathbf{s}(J)\end{bmatrix} \tag{27}$$

then $\boldsymbol{\mu}=\mathbf{G}\boldsymbol{s}$, and

$$\frac{\partial\boldsymbol{\mu}}{\partial\mathbf{s}_R{}^T}=G,\quad \frac{\partial\boldsymbol{\mu}}{\partial\mathbf{s}_I{}^T}=jG \tag{28}$$

According to (24)–(28)

$$\frac{\partial\boldsymbol{\mu}}{\partial\boldsymbol{\zeta}^T}=[\mathbf{\Delta},\mathbf{G},j\mathbf{G},0] \tag{29}$$

Eq. (23) can be denoted by

$$2\mathrm{Re}\left\{\frac{\partial\boldsymbol{\mu}^*}{\partial\boldsymbol{\zeta}}\mathbf{\Gamma}^{-1}\frac{\partial\boldsymbol{\mu}}{\partial\boldsymbol{\zeta}^T}\right\}=\begin{bmatrix}\mathbf{J} & 0\\ 0 & 0\end{bmatrix} \tag{30}$$

where

$$\mathbf{J}\triangleq\frac{2}{\sigma^2}\mathrm{Re}\left\{\begin{bmatrix}\mathbf{\Delta}^H\\ \mathbf{G}^H\\ -j\mathbf{G}^H\end{bmatrix}\begin{bmatrix}\mathbf{\Delta} & \mathbf{G} & j\mathbf{G}\end{bmatrix}\right\}$$

So we just consider the elements related to the angles in $\mathbf{J}^{-1}$. Define

$$\mathbf{B} \triangleq \left(\mathbf{G}^H\mathbf{G}\right)^{-1}\mathbf{G}^H\mathbf{\Delta} \tag{31}$$

$$\mathbf{F} \triangleq \begin{bmatrix} \mathbf{I} & 0 & 0 \\ -\mathbf{B}_R & \mathbf{I} & 0 \\ -\mathbf{B}_I & 0 & \mathbf{I} \end{bmatrix} \tag{32}$$

We demonstrate that

$$\begin{bmatrix} \mathbf{\Delta} & \mathbf{G} & j\mathbf{G} \end{bmatrix}\mathbf{F} = \begin{bmatrix} (\mathbf{\Delta}-\mathbf{GB}) & \mathbf{G} & j\mathbf{G} \end{bmatrix} = \begin{bmatrix} \mathbf{\Pi}_{\mathbf{G}}^{\perp}\mathbf{\Delta} & \mathbf{G} & j\mathbf{G} \end{bmatrix} \tag{33}$$

where $\mathbf{\Pi}_{\mathbf{G}}^{\perp} = \mathbf{I} - \mathbf{G}\left(\mathbf{G}^H\mathbf{G}\right)^{-1}\mathbf{G}^H$ and $\mathbf{G}^H\mathbf{\Pi}_{\mathbf{G}}^{\perp} = 0$.

$$\begin{aligned}
\mathbf{F}^T\mathbf{J}\mathbf{F} &= \frac{2}{\sigma^2}\mathrm{Re}\left\{\mathbf{F}^H\begin{bmatrix} \mathbf{\Delta}^H \\ \mathbf{G}^H \\ -i\mathbf{G}^H \end{bmatrix}\begin{bmatrix} \mathbf{\Delta} & \mathbf{G} & i\mathbf{G} \end{bmatrix}\mathbf{F}\right\} \\
&= \frac{2}{\sigma^2}\mathrm{Re}\left\{\begin{bmatrix} \mathbf{\Delta}^H\mathbf{\Pi}_{\mathbf{G}}^{\perp} \\ \mathbf{G}^H \\ -i\mathbf{G}^H \end{bmatrix}\begin{bmatrix} \mathbf{\Pi}_{\mathbf{G}}^{\perp}\mathbf{\Delta} & \mathbf{G} & i\mathbf{G} \end{bmatrix}\right\} \\
&= \frac{2}{\sigma^2}\mathrm{Re}\left\{\begin{bmatrix} \mathbf{\Delta}^H\mathbf{\Pi}_{\mathbf{G}}^{\perp}\mathbf{\Delta} & 0 & 0 \\ 0 & \mathbf{G}^H\mathbf{G} & i\mathbf{G}^H\mathbf{G} \\ 0 & -i\mathbf{G}^H\mathbf{G} & \mathbf{G}^H\mathbf{G} \end{bmatrix}\right\}
\end{aligned} \tag{34}$$

so $\mathbf{J}^{-1}$ is written as

$$\begin{aligned}
\mathbf{J}^{-1} &= \mathbf{F}\left(\mathbf{F}^T\mathbf{J}\mathbf{F}\right)^{-1}\mathbf{F}^T \\
&= \frac{\sigma^2}{2}\begin{bmatrix} \mathbf{I} & 0 & 0 \\ -\mathbf{B}_R & \mathbf{I} & 0 \\ -\mathbf{B}_I & 0 & \mathbf{I} \end{bmatrix}\begin{bmatrix} \mathrm{Re}\left(\mathbf{\Delta}^H\mathbf{\Pi}_{\mathbf{G}}^{\perp}\mathbf{\Delta}\right) & 0 & 0 \\ 0 & \kappa & \kappa \\ 0 & \kappa & \kappa \end{bmatrix}\begin{bmatrix} \mathbf{I} & -\mathbf{B}_R^T & -\mathbf{B}_I^T \\ 0 & \mathbf{I} & 0 \\ 0 & 0 & \mathbf{I} \end{bmatrix} \\
&= \begin{bmatrix} \frac{\sigma^2}{2}\left[\mathrm{Re}\left(\mathbf{\Delta}^H\mathbf{\Pi}_{\mathbf{G}}^{\perp}\mathbf{\Delta}\right)\right]^{-1} & \kappa & \kappa \\ \kappa & \kappa & \kappa \\ \kappa & \kappa & \kappa \end{bmatrix}
\end{aligned}$$

where κ denotes the part we do not concern. Till now, we give the CRB matrix as follows:

$$CRB = \frac{\sigma^2}{2}\left[\mathrm{Re}\left(\mathbf{\Delta}^H\mathbf{\Pi}_{\mathbf{G}}^{\perp}\mathbf{\Delta}\right)\right]^{-1}$$

After further simplification, we rewrite the CRB matrix

$$CRB = \frac{\sigma^2}{2J}\left\{\mathrm{Re}\left[\mathbf{D}^H\mathbf{\Pi}_{\mathbf{A}}^{\perp}\mathbf{D} \odot \hat{\mathbf{P}}_w^{\ T}\right]\right\}^{-1} \tag{35}$$

where $\mathbf{D} = \left[\frac{\partial \mathbf{a}_1}{\partial \theta_1}, \frac{\partial \mathbf{a}_2}{\partial \theta_2}, \ldots, \frac{\partial \mathbf{a}_K}{\partial \theta_K}, \frac{\partial \mathbf{a}_1}{\partial \phi_1}, \frac{\partial \mathbf{a}_2}{\partial \phi_2}, \ldots, \frac{\partial \mathbf{a}_K}{\partial \phi_K}\right]$ with $\mathbf{a}_k$ being the kth column of $\mathbf{A}$; $\hat{\mathbf{P}}_w = \begin{bmatrix} \hat{\mathbf{P}}_s & \hat{\mathbf{P}}_s \\ \hat{\mathbf{P}}_s & \hat{\mathbf{P}}_s \end{bmatrix}$, $\hat{\mathbf{P}}_s = \frac{1}{J}\sum_{t=1}^{J}\mathbf{s}(t)\mathbf{s}^H(t)$, $\mathbf{\Pi}_{\mathbf{A}}^{\perp} = \mathbf{I}_{2N+1} - \mathbf{A}\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H$.

5. Simulation results

Define Root mean square error (RMSE) as

$$RMSE = 1/K\sum_{k=1}^{K}\sqrt{1/1000\sum_{l=1}^{1000}(\hat{\beta}_{k,l}-\beta_k)^2} \tag{36}$$

We define signal to noise ratio (SNR)

$$SNR = \frac{\|\mathbf{AS}\|_F^2}{\|\mathbf{W}\|_F^2}. \tag{37}$$

where $\hat{\beta}_{k,l}$ are the estimation of θ_k/ϕ_k of the lth Monte Carlo trial; We assume that there are $K=2$ sources with DOA $(\theta_1,\phi_1) = (10^\circ,-15^\circ)$, $(\theta_2,\phi_2) = (55^\circ,40^\circ)$. $d=\lambda/2$ is used in the simulations.

Fig. 3 depicts angle estimation result of the proposed improved algorithm for all two sources with $N=10$, $J=100$, SNR=15 dB. It is shown that elevation and azimuth angles can be clearly observed.

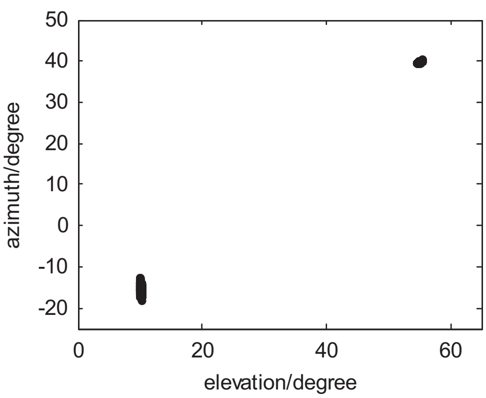

Fig. 3. Angle estimation performance of the proposed algorithm with SNR=15 dB.

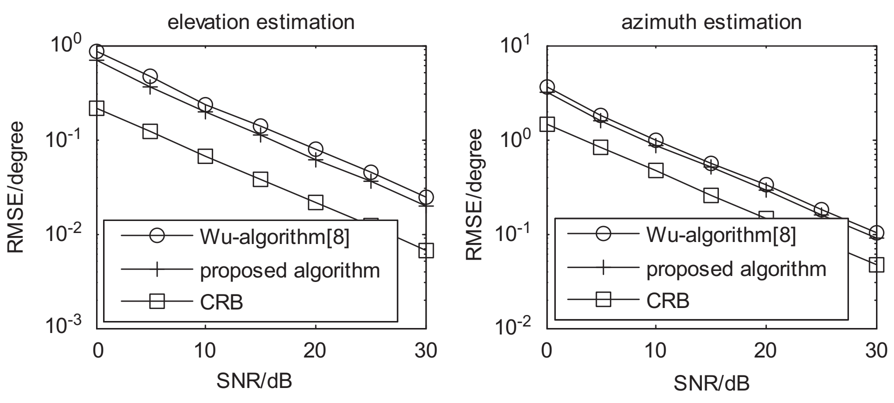

Fig. 4. Angle estimation performance comparison ($N=10$, $J=100$, $K=2$).

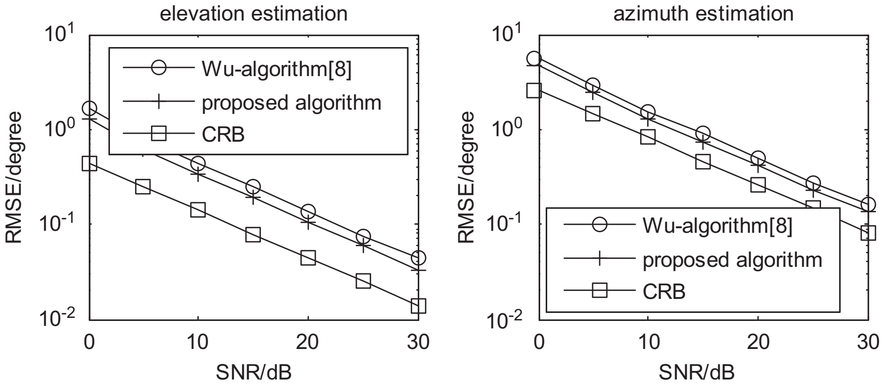

Fig. 5. Angle estimation performance comparison ($N=8$, $J=50$, $K=2$).

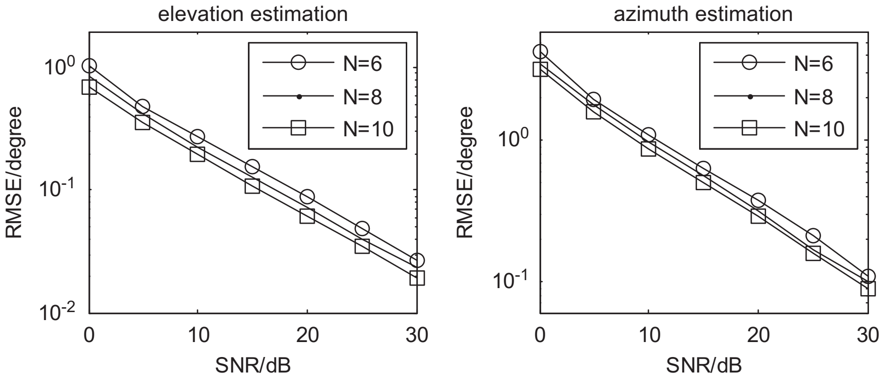

Fig. 6. Angle estimation performance with different values of N.

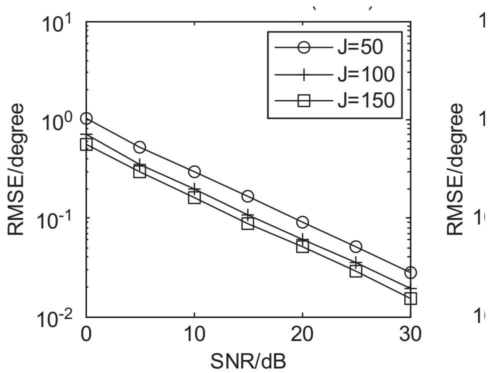

Fig. 7. Angle estimation performance with different values of J.

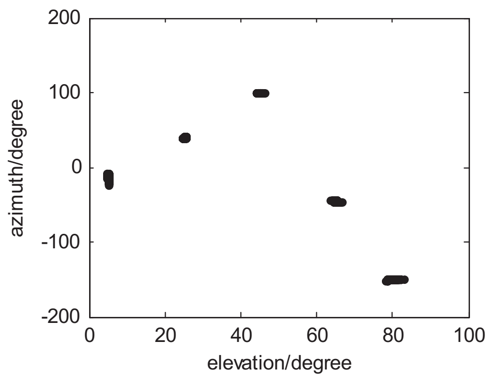

Fig. 8. Angle estimation performance of the proposed algorithm with SNR=15 dB.

We compare the proposed improved algorithm against Wu's algorithm [8] and CRB. Figs. 4 and 5 present the comparisons of the algorithms. From Figs. 4 and 5, we can find that the angle estimation performance of the proposed algorithm is better than Wu's algorithm.

Fig. 6 shows angle estimation performance of the proposed algorithm with $J=100$, $K=2$ and different N. From Fig. 6, the angle estimation performance of the proposed algorithm is improved with the number of antennas increasing.

Fig. 7 presents angle estimation performance of the proposed algorithm with $N=10$, and different values of J. It is indicated in Fig. 7 that the angle estimation performance of the proposed algorithm is improved with J increasing.

Fig. 8 shows angle estimation performance of the proposed algorithm with larger values of K. We assume $K=5$ (elevation angles: $[5^\circ, 25^\circ, 45^\circ, 65^\circ, 80^\circ]$, azimuth

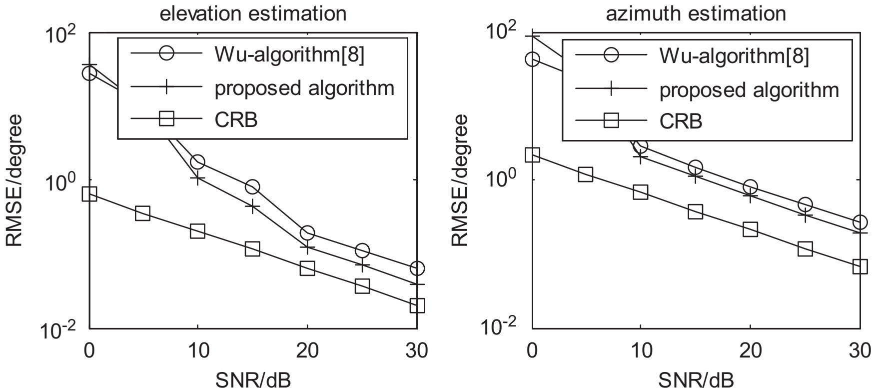

Fig. 9. Angle estimation performance comparison (closely-spaced, $N=10$, $J=100$, $K=2$).

angles: $[-15^{\circ}, 40^{\circ}, 100^{\circ}, -45^{\circ}, -150^{\circ}]$) with $N=10, J=100$, and SNR$=15$ dB. It is shown that elevation and azimuth angles can be clearly observed.

Fig. 9 presents the comparison of the algorithms in the situation of two closely-spaced sources with $(\theta_1,\phi_1) = (10^{\circ},35^{\circ}),(\theta_2,\phi_2) = (15^{\circ},40^{\circ})$. From Fig. 9, we can find that the angle estimation performance of the proposed algorithm is better than Wu's algorithm.

6. Conclusion

In this paper, we have proposed an improved two-dimensional DOA estimation algorithm for two-parallel uniform linear arrays using PM. The proposed algorithm has the following advantages: (1) it can achieve automatically paired two-dimensional estimation of angles, while Wu's algorithm in [8] needs additional pair matching; (2) it has better angle estimation performance than Wu's algorithm; and (3) it has lower complexity than Wu's method.

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