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Entropy-based subspace separation for multiple frequency estimation

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ABSTRACT

A subspace-based algorithm for estimating the order and the frequencies of multiple sinusoids embedded in noise is proposed. The new estimator (referred to as E-MUSIC) uses the entropy of a random variable related to the angles between the signal and noise subspaces as its objective function. Maximizing the entropy tends to achieve uniform angle distribution and thus leads to maximal subspace separation. The entropy-based objective function and the performance of the E-MUSIC algorithm is compared with some reference algorithms in the literature. Simulations which are performed in additive white and colored Gaussian noise show that the E-MUSIC offers an improvement for both model order and multiple frequency estimation. The improvement is more pronounced for high model orders and large SNR values.

1. Introduction

Estimating the order and the frequencies of a mixture of sinusoidal signals buried in noise has applications in many areas including communications, radar and parametric coding of speech and audio signals. For example, smart antenna systems (which are widely used in wireless mobile communications for increasing the channel capacity and coverage) make use of direction of arrival estimation techniques to decipher which emitters are present and what their angular locations are [1]. This information in turn can be used by smart antenna systems to eliminate or combine signals for greater fidelity or suppress interferers to improve capacity. Additionally, a Doppler-sonar can be used to measure the velocity of a ship by exploiting the Doppler effect on acoustic waves (especially their frequencies) [2].

The early estimation methods are based on considerations such as Rissanen's Minimum Description Length (MDL) [3–5], the eigenvalue-based method (EIG) [6], the Akaike Information Criterion (AIC) [7] and Efficient Detection Criteria (EDC) [8]. Phase-based methods were proposed in [9]. Maximum *a posteriori* (MAP)-rule-based methods and subspace-separation-based methods have also been proposed. The former take advantage of the *a priori* knowledge of the probability distribution function of the observation noise; whereas the latter use only an estimate of the covariance matrix of the observation and require no probability model of the noise. Certain eigenvectors of the covariance matrix are then used to form a subspace of noise (referred to as the noise subspace). Parameter estimation can then be carried out by separating the noise subspace from the subspace of the signal of interest (called the signal subspace).

Some of the parameter estimation methods which are based on the idea of subspace separation include the MUltiple SIgnal Classification (MUSIC) algorithm [10], the harmonic MUSIC (HMUSIC) [11], the frequency selective harmonic music (F-HMUSIC) [12], the estimation of signal parameters via rotational invariance technique (ESPRIT) [13], and the propagator method (PM) [14]. One of the recent methods, named as the new-MUSIC, of Christensen et al. [15], achieves the separation by minimizing the approximate average of cosines of the angles between the signal and noise subspaces.

In this paper, we propose a new estimator which is referred to as the E-MUSIC.[1] This new estimator is similar to the new-MUSIC except that it uses a new and more effective cost function. Instead of the approximate average of the angles as in [15], we try to maximize the subspace separation by considering the angles individually. The idea is that while pushing these angles towards a maximum angle of $\pi/2$, we also try to make the angles to be as uniformly distributed as possible. To this end, we propose to use the entropy of a (discrete-time) random variable whose probability mass function (pmf) is given in terms of the angles.

The remainder of the paper is organized as follows. Section 2 introduces the estimation problem and the new-MUSIC algorithm proposed in [15]. Our entropy-based criterion for subspace separation is proposed in Section 3; its properties and relations to new-MUSIC criteria are also discussed. Section 4 presents experimental results under additive white and colored Gaussian noise scenarios using a wide range of SNR values and various model

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[1] The prefix E stands for entropy.

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orders. The results demonstrate the improvement on those reported in [15]. Finally, conclusions are drawn in Section 5.

2. The estimation problem and the new-MUSIC method [15]

A discrete-time signal which is composed of complex sinusoids with frequencies ω_ℓ, amplitudes A_ℓ and phases ϕ_ℓ, corrupted by additive noise w, can be modeled as follows

$$x[n] = \sum_{\ell=1}^{L} A_\ell e^{j(\omega_\ell n + \phi_\ell)} + w[n], \quad n = 0, 1, \ldots, N-1. \tag{1}$$

Here, L represents the unknown model order. The phases for the sinusoids are assumed to be independent and uniformly distributed on the interval $(-\pi, \pi]$. It should be noted that the probability distribution of the additive noise w is not required.

The problem is to estimate the order L and the frequencies ω_ℓ based on the observation samples of the signal x.

The consecutive samples of the observed signal in vector form are given by

$$\mathbf{x}[n] = \begin{bmatrix} x[n] & x[n+1] & \cdots & x[n+M-1] \end{bmatrix}^T \in \mathbb{C}^M \tag{2}$$

where $M < N$. Then the signal subspace $\mathcal{S}$ is spanned by the columns of matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}(\omega_1) & \cdots & \mathbf{a}(\omega_L) \end{bmatrix} \in \mathbb{C}^{M \times L} \tag{3}$$

where

$$\mathbf{a}(\omega) = \begin{bmatrix} 1 & e^{j\omega} \cdots e^{j\omega(M-1)} \end{bmatrix}^T. \tag{4}$$

Denote the covariance matrix as

$$\mathbf{R} = E\{\mathbf{x}[n]\mathbf{x}[n]^H\} \in \mathbb{C}^{M \times M} \tag{5}$$

where E denotes the expectation operator. Let $\mathbf{R}$ have a singular value decomposition (SVD)

$$\mathbf{R} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H. \tag{6}$$

If the entries (all non-negative) of the diagonal matrix $\boldsymbol{\Lambda}$ are arranged in decreasing order, then under the independence assumption of the noise samples, the last $M-L$ columns of $\mathbf{U}$ span the noise subspace $\mathcal{N}$ which is orthogonal to the signal subspace $\mathcal{S}$. Note that in general, subspace-based estimation methods are based on this fact.

The estimation can be carried out effectively by using the angles between $\mathcal{S}$ and $\mathcal{N}$ [15]. There are $K = \min\{L, M-L\}$ principal angles θ_k, $k = 1, 2, \ldots, K$, between the two subspaces that can be characterized by

$$\cos(\theta_k) = \sigma_k \leqslant 1. \tag{7}$$

The σ_k's are the singular values of the product matrix $\boldsymbol{\Pi}_\mathbf{A}\boldsymbol{\Pi}_\mathbf{G}$ with

$$\boldsymbol{\Pi}_\mathbf{A} = \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H, \tag{8}$$

$$\boldsymbol{\Pi}_\mathbf{G} = \mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H \tag{9}$$

the orthogonal projection matrices for the signal and noise spaces $\mathcal{S}$ and $\mathcal{N}$ respectively. Note that if the two subspaces are orthogonal, then $\theta_k = \pi/2$, i.e., $\sigma_k = 0$ for $k = 1, \ldots, K$.

For the covariance matrix $\mathbf{R}$, we can use the following estimate (which is consistent for ergodic processes as stated in [15]):

$$\hat{\mathbf{R}} = \frac{1}{N-M+1} \sum_{n=0}^{N-M} \mathbf{x}[n]\mathbf{x}^H[n]. \tag{10}$$

The new-MUSIC method estimates the order and the distinct frequencies by minimizing a normalized Frobenius norm of $\mathbf{A}^H\mathbf{G}$, i.e.,

$$\{\hat{L}, \hat{\boldsymbol{\omega}}\} = \arg\min_L \Big\{ \min_{\{\omega_k \neq \omega_\ell\}} \{J(\boldsymbol{\omega})\} \Big\}, \quad \text{with } J(\boldsymbol{\omega}) = \frac{1}{MK} \|\mathbf{A}^H\mathbf{G}\|_F^2. \tag{11}$$

It is shown that this cost function is asymptotically (i.e., for large M) related to the angles between the subspaces $\mathcal{S}$ and $\mathcal{N}$ [15] as

$$J(\boldsymbol{\omega}) \approx \frac{1}{K} \sum_{k=1}^{K} \cos^2(\theta_k) = \frac{1}{K} \sum_{k=1}^{K} \sigma_k^2. \tag{12}$$

This implies that, approximately, the estimator (11) maximizes the average angle between the two subspaces.

3. The proposed estimator: E-MUSIC

In this work, we go one step further. Not only do we wish to maximize the average angle, but we also want to force the angles to have a uniform distribution. This way, the estimator would be more robust to the noise, since in the noise-free case, all the K angles are identical to $\pi/2$. To this end, we consider all M singular values σ_m, $m = 1, 2, \ldots, M$, of $\boldsymbol{\Pi}_\mathbf{A}\boldsymbol{\Pi}_\mathbf{G}$ and use them to form a pmf.

Note that there are possibly K non-zero and $M-K$ zero singular values: $\sigma_m = \cos(\theta_k), m = 1, 2, \ldots, K$; and $\sigma_m = 0, m = K, K+1, \ldots, M$. Let

$$s_m = 1 - \sigma_m^2, \quad m = 1, 2, \ldots, K \tag{13}$$

and normalize them to form

$$p_m = \frac{s_m}{S}, \quad m = 1, 2, \ldots, M \tag{14}$$

where $S = \sum_{m=1}^{M} s_m = M - \sum_{k=1}^{K} \sigma_k^2$. Consider a random variable p with a pmf of p_m. Its entropy is then given by

$$H(p) = -\sum_{m=1}^{M} p_m \ln p_m = -\sum_{m=1}^{M} \frac{s_m}{S} \ln \frac{s_m}{S} \tag{15}$$

i.e.

$$H(p) = \ln S - \frac{1}{S} \sum_{k=1}^{K} s_k \ln s_k \tag{16}$$

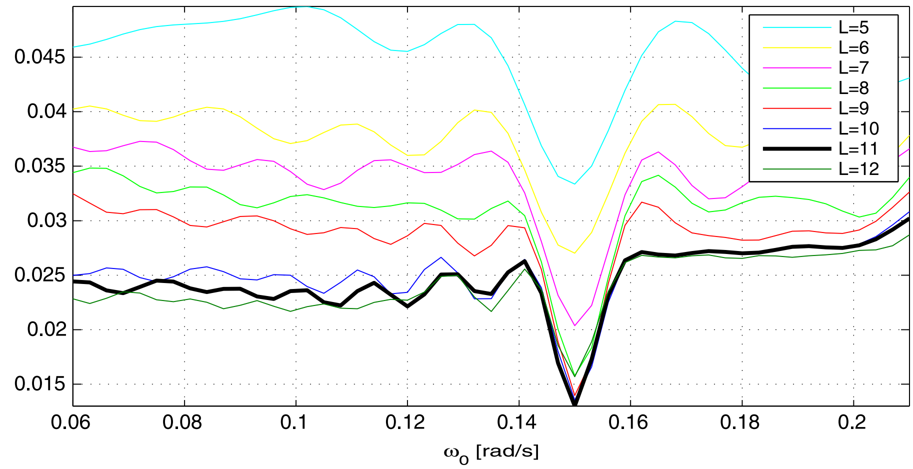
which can also be expressed in terms of σ_m using (13).

It is well known that H is maximized if all the p_m are identical. Maximizing H would lead to s_m, and σ_m, thus the principal angles being as uniformly distributed as possible. Note also that the use of the $M-K$ zero singular values in forming the pmf has the effect of forcing the s_m to approach one (σ_m to zero) and thus the principal angles being close to $\pi/2$ as much as possible. Consequently, the two subspaces would be maximally separated.

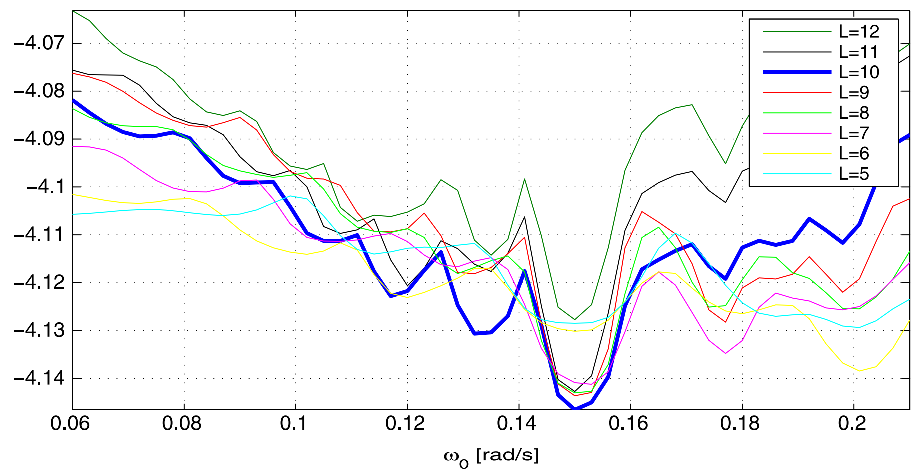
Based on the above discussion, we propose a new estimator, called E-MUSIC, with a cost function given in terms of the entropy of p. Specifically, for a given L and a frequency vector $\boldsymbol{\omega}$ of length L, the cost function to be minimized is $-H(\boldsymbol{\omega})$.[2] Then the order of x and the unknown frequencies are determined by solving the following optimization problem:

$$\{\hat{L}, \hat{\boldsymbol{\omega}}\} = \arg\min_L \Big\{ \min_{\{\omega_k \neq \omega_\ell\}} \{-H(\boldsymbol{\omega})\} \Big\}. \tag{17}$$

[2] For notational simplicity, the same symbol H is used for the cost function.



(a) Cost function J.



(b) Cost function H.

Fig. 1. Cost functions J and H ($L = 6$, SNR $= 5$ dB, $N = 128$, $M = 64$).

We now relate the entropy H to the approximate cost function J in (12). The relation shows that maximizing H indeed has the effect of minimizing J. First, simple derivation verifies

$$e^{-H(\boldsymbol{\omega})} = \prod_{m=1}^{M} \left(\frac{s_m}{S}\right)^{\frac{s_m}{S}} \tag{18}$$

$$= \left(\frac{1}{S}\right)^{\sum_{m=1}^{M} \frac{s_m}{S}} \prod_{k=1}^{K} (s_k)^{\frac{s_k}{S}} \tag{19}$$

$$= \frac{1}{S}\left(\prod_{k=1}^{K} \left(1 - \sigma_k^2\right)^{1-\sigma_k^2}\right)^{\frac{1}{S}}. \tag{20}$$

Then noting that for any $0 < \tau < 1$, the inequality

$$(1-t)^{1-t} \leqslant 1 - at \tag{21}$$

holds for $t \in [0, \tau]$, where $0 < a = (1 - (1-\tau)^{1-\tau})/\tau < 1$, and using the fact that the geometric mean is not more than the arithmetic mean, we have that for small σ_k's (without loss of generality, we suppose that they all are less or equal to some $0 < \tau < 1$)

$$e^{-H(\boldsymbol{\omega})} \leqslant \frac{1}{S}\left(\prod_{k=1}^{K} \left(1 - a\sigma_k^2\right)\right)^{\frac{1}{S}} \tag{22}$$

$$\leqslant \frac{1}{S}\left(\frac{1}{K}\sum_{k=1}^{K} \left(1 - a\sigma_k^2\right)\right)^{\frac{K}{S}} \tag{23}$$

$$= \frac{1}{S}\left(1 - \frac{a}{K}\sum_{k=1}^{K} \sigma_k^2\right)^{\frac{K}{S}}. \tag{24}$$

Consequently,

$$J(\boldsymbol{\omega}) \simeq \frac{1}{K}\sum_{k=1}^{K} \sigma_k^2 \leqslant a^{-1}\left(1 - \left(Se^{-H(\boldsymbol{\omega})}\right)^{\frac{S}{K}}\right) \tag{25}$$

which provides an upper bound for the average of σ_k^2. Since $0 < H(\boldsymbol{\omega}) \leqslant \ln M$, it follows that $1/M \leqslant e^{-H(\boldsymbol{\omega})} < 1$. Also, $S \simeq M$ for any small σ_k's. As a result, inequality (25) implies that maximizing H would have the effect of minimizing J in (11).

We now compare the two cost functions J and H in a numerical example. Consider a signal x of (1) with $L = 10$. For visualization purposes, we assume that $\omega_\ell = \ell\omega_0$, $\ell = 1, 2, \ldots, 10$, for some fundamental frequency ω_0. In this example, we set the fundamental frequency $\omega_0 = 0.15$. The signal is composed of ten sinusoids of random amplitudes, $A_\ell \in [0.1, 1.1]$, $\ell = 1, 2, \ldots, 10$, and w is additive Gaussian noise such that SNR $= 5$ dB. Fig. 1 shows the plots of J and H as functions of ω_0 and for all test orders L from 5 to 12. It is clear that with H, the global minimal value is achieved (correctly) at $L = 10$ and $\omega_0 = 0.15$. On the other hand, optimizing

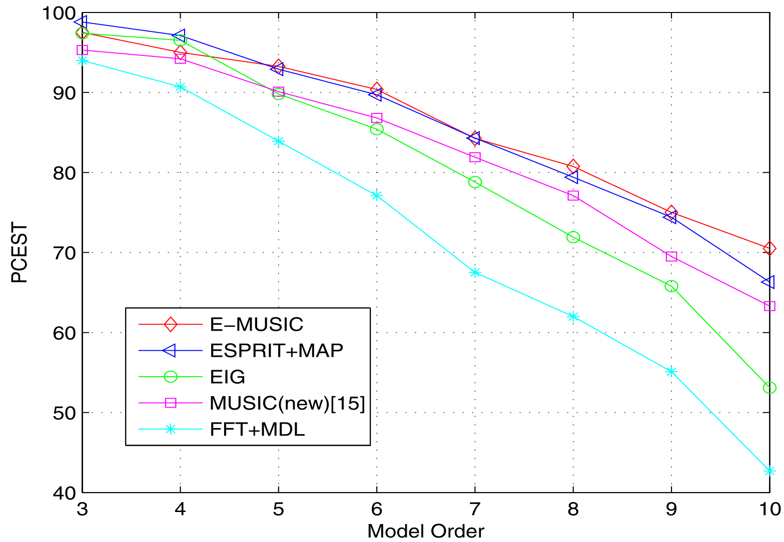


Fig. 2. Performance comparison with other methods for varying order (SNR = 20 dB, $N = 128$, $M = 64$).

Table 1
PCEST as a function of the number of observations (SNR = 5 dB).

N	64	128	256	512
PCEST	44.25	75.43	89.40	92.85

J as in (11) would give an incorrect estimation order $L = 11$. The new entropy-based cost function achieves subspace separation by considering the angles individually and allows more efficient estimation of the model order.

4. Experimental results

To assess the performance of the new estimator, we carried out Monte Carlo simulations in both additive white and colored Gaussian noise. For the AWGN simulations the signal $x[n]$ in Eq. (1) was generated as in [15], with amplitudes set to unity or to differing values, and frequencies and phases generated uniformly in the interval $(-\pi, \pi]$.

We used the entropy-based E-MUSIC algorithm to estimate the true model order and the frequencies. The entropy optimization problem was solved in MATLAB© using the function *fmincon* (used to solve general nonlinear minimization problems) with the sequential quadratic programming option. Additionally, MATLAB's Multistart Optimization Tool was used, which allows to start from multiple sets of initial frequencies. One initial set was obtained by taking the frequencies of the leading magnitudes of the fast Fourier transform of x. The correct rate for the model order and accuracy of the frequency estimates were then evaluated. The correct order was taken as the tested L value that maximizes the entropy. For each run the number of start points were assumed as three times the tested L value and the experiment was repeated 400 times before evaluating the order estimation results for different SNRs. In addition, the mean squared error (MSE) of the frequency estimates was compared with the Cramer–Rao Lower Bound (CRLB) [16].

In order to investigate the effect of the number of observations on the percentage of correct estimation (PCEST) we have taken a similar approach as in [15]. The percentage of correctly estimated model order versus number of observations has been provided for $L = 6$ and SNR = 5 dB. From Table 1, it is seen that PCEST improves as N and remains essentially unchanged after a large enough value for N ($N \geqslant 256$). It must also be noted that in general, large model orders L, require a relatively large number of observation samples for reliable estimation.

Table 2
PCEST for differing amplitudes ($L = 3$, SNR = 20 dB, $N = 128$).

Amplitude sets	A_1	A_2	A_3	PCEST (%)
SET-1	0.95	1.0	1.15	95.75
SET-2	0.7	1.2	1.35	43.00
SET-3	1.0	2.0	4.0	16.24

In Fig. 2, the model order L was varied in the range $3 \leqslant L \leqslant 10$ and the PCEST of the proposed method was compared with other classical algorithms for unity amplitudes and random frequency and phases. At a fixed SNR value of 20 dB, it can be observed that the performance of the E-MUSIC algorithm is the highest, being comparable to the ESPRIT-MAP algorithm [15]. For the estimation of L in the range 3–6, the PCEST is above 90% and drops to 70% at $L = 10$ for the E-MUSIC algorithm. The performance gain is more prominent for higher L values.

The E-MUSIC algorithm's PCEST performance was also obtained with $L = 3$, SNR = 20 dB and $N = 128$ ($M = N/2$) for three differing sets of amplitudes as shown in Table 2.

The results indicate that when the amplitudes are largely different as in SET-3, the corresponding powers of the individual sinusoids become prohibitively different, i.e. the amplitudes vary by a factor of 2 and the corresponding powers vary by 4 where the power of the third sinusoid is 16 times more than the first one. In this case, the tone with the lowest power, experiences a very low SNR (in order to maintain an overall average SNR for the signal which is composed of multiple tones) and its estimation suffers drastically. It is also clear that small variations in the amplitude (i.e. SET-1) do not lead to considerable change in PCEST.

In Fig. 3, the model order is fixed at $L = 6$ and the SNR is varied from -10 to 20 dB. It is observed that the E-MUSIC outperforms some of the classical methods like AIC and MDL for SNR $\geqslant 4$ dB. We also note that it is not possible to estimate the model order by using the EDC algorithm for SNR < 10 dB.

In order to investigate the effects of colored noise on the performance of E-MUSIC, a new set of simulations are performed. In [15], the MUSIC algorithm was tested under colored Gaussian noise for $N = 200$ and $L = 5$ for a range of SNR values. The coloring was obtained by a second-order autoregressive process whose transfer function is

$$H(z) = \frac{1}{1 - 0.25z^{-1} + 0.50z^{-2}}. \tag{26}$$

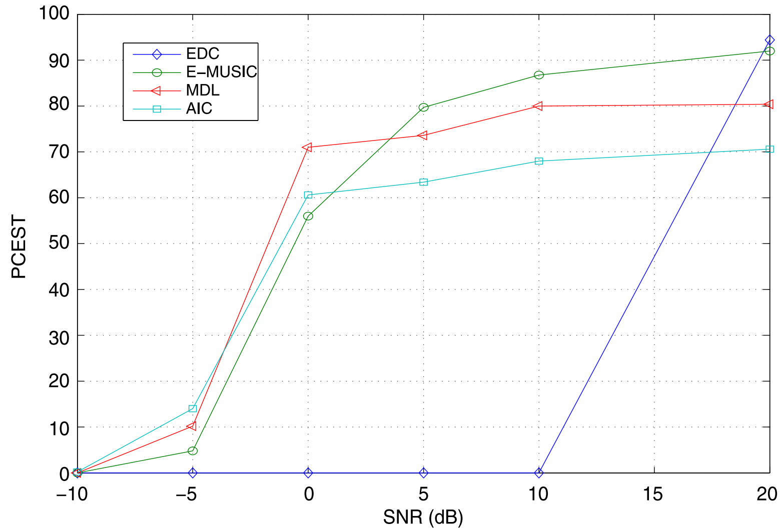


Fig. 3. Performance comparison with classical estimation methods for varying SNR ($L = 6$, $N = 200$, $M = 100$).

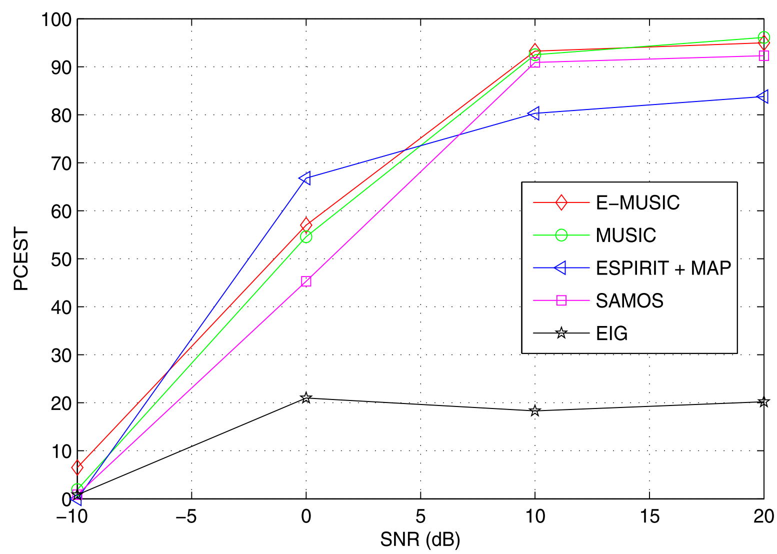


Fig. 4. Performance comparison under colored Gaussian noise (SNR = 20 dB, $L = 5$, $N = 200$, $M = 100$).

The PCEST curve for the proposed algorithm with $L = 5$ and $N = 200$ is obtained and compared against other algorithms in Fig. 4. The results indicate that the E-MUSIC performs similar to the MUSIC and outperforms the others in the high SNR range which shows its robustness in colored noise.

We have also conducted Monte Carlo simulations (300 trials) to obtain the MSE using our E-MUSIC algorithm, for two cases: (a) the estimation of a fundamental frequency, and (b) the estimation of a set of random frequencies. The MSE curves that were obtained for the SNR values in the range $-10 \leqslant \text{SNR} \leqslant 20$ dB are shown in Figs. 5 and 6 respectively.

In Fig. 5, estimates of the fundamental frequency were computed using a brute force approach. For each run, the entropy-based new objective function was computed over a range of frequencies and the frequency maximizing the entropy was selected and compared to the original. Finally, squared errors of the fundamental frequency were averaged. Choosing to optimize in this manner de-couples the effects of objective function and the optimizer. The performance therefore relies solely on the new entropy-based objective function. The MSE for estimating the fundamental frequency for varying SNR was compared with the CRLB bound in [16].

The results in Fig. 6 show the MSE for estimating a random set of independent frequencies $(\omega_1, \omega_2, \omega_3)$ using the nonlinear constrained optimizer *fmincon* with the Multistart Optimization Tool (the number of start points have been set to 40). In this case, the CRLB curve is for a random set of distinct frequencies as in [17, 18] and is different from the one depicted in Fig. 5. It should also be noted that the new cost function H used in E-MUSIC and the cost function J used in [15] are compared in the figure. Since the model order is low ($L = 3$), the performance seems to be similar for both of the cost functions. The new entropy-based cost function does better when the model order is high and the subspace separation is improved by considering the angles individually. In Figs. 5 and 6, the MSE values approach (but do not converge) the CRLB after $\text{SNR} \geqslant -5$ dB and $\text{SNR} \geqslant 0$ dB respectively. This is expected as MUSIC-type algorithms approach but may not converge on the CRLB [20].

The MSE performance of the E-MUSIC algorithm is compared with that of the Iterative Quadratic Maximum Likelihood (IQML) algorithm [19] for a dual tone ($L = 2$) complex signal. In order

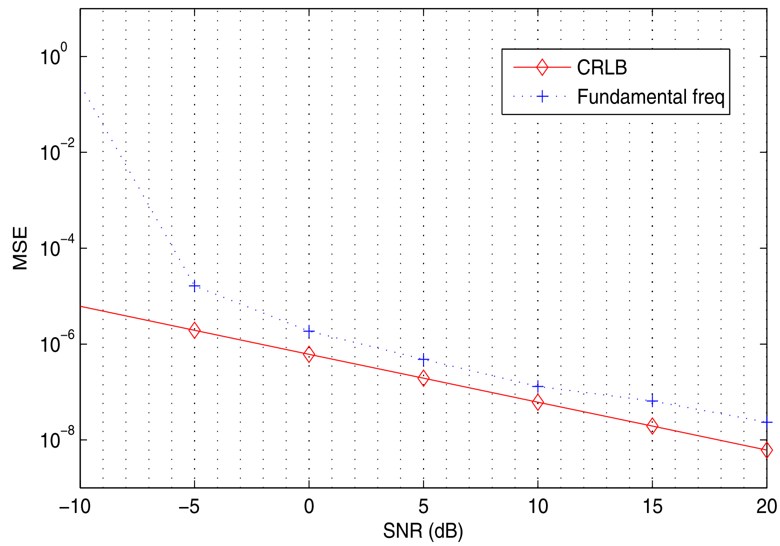


Fig. 5. MSE comparison for the estimation of a fundamental frequency ($L = 3$, $N = 160$, $M = 80$).

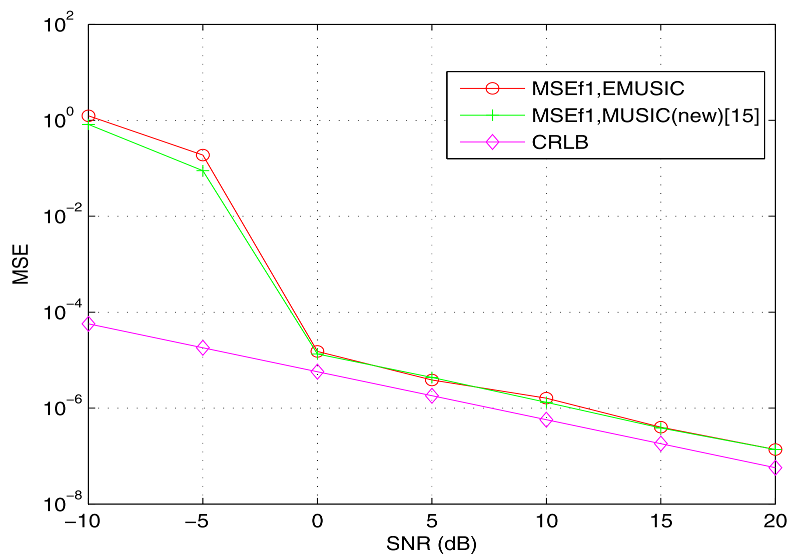


Fig. 6. MSE comparison for the estimation of a set of three random frequencies ($L = 3$, $N = 128$, $M = 64$).

to have a fair comparison, the E-MUSIC algorithm was simulated with the same amplitude, phase and frequency parameters. The parameters are as follows: $A_1 = A_2 = \sqrt{2}$, $\omega_1 = 0.1\pi$, $\omega_2 = 0.9\pi$, $\phi_1 = 0.2\pi$, $\phi_2 = 0.8\pi$ and $N = 20$. Fig. 7 shows the comparison of the performance among IQML, E-MUSIC and CRLB algorithms. It is clear that the MSE of the dual tone E-MUSIC and the IQML are very close to each other for SNR > 5 dB for this case of a very low number of observation samples. It is known that an ML estimator is asymptotically (as the number of observations tends to infinity) efficient under some regularity conditions (the main condition is that the ML is consistent) [18]. Also, the ML estimation method requires the probability distribution of the noise whereas the E-MUSIC algorithm does not. Unlike the ML, the number of observations (N) does not need to be prohibitively large (especially for large model orders) for the proposed algorithm to work effectively.

Finally we consider the computational complexity of our method. In general, the computational complexity is a common issue in all the subspace-based methods and we acknowledge that ours is not an exception. The major computations required in these methods mainly consist of two parts: (1) the evaluation of the cost function, and (2) the process of the optimizer employed. Examining the E-Music algorithm shows that two SVDs (each of size $M \times M$), two matrix inversions ($L \times L$ and $(M - L) \times (M - L)$, respectively) and $(M - L)M + ML$ matrix multiplication operations are required for the evaluation of the cost function and thus the complexity is $O(M^3)$. This complexity is the same as those of many of the subspace-based methods since they all involve SVD of the correlation matrix ($M \times M$) [15]. For example, evaluating the cost function of the method in [15] requires one SVD (of size $M \times M$) in addition to $M(M - L)L + ML$ matrix multiplication operations. Two of the well-known methods, root MUSIC [21] and spectral [10] methods also have similar complexity. In order to estimate the overall complexity of the proposed algorithm, the number of function calls to the above two objective functions within the optimizer are also compared under the same simulation setup (SNR $= 5$ dB, $L = 3$, $N = 128$, $M = 64$, see Fig. 6). The algorithm in [15] requires on the average (over 400 ensemble runs) 52.9 calls and the E-MUSIC requires 57.8 calls. It can be concluded that the overall computational complexity of the E-MUSIC algorithm is similar to those of conventional MUSIC methods.

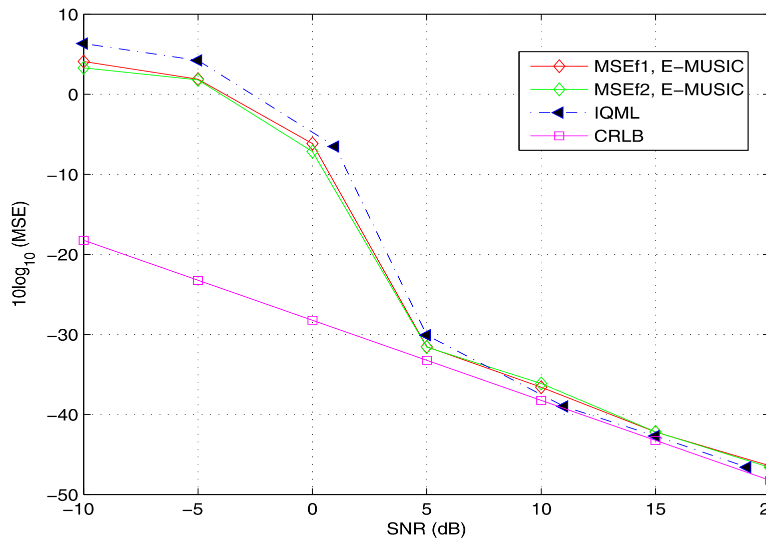


Fig. 7. MSE comparison for E-MUSIC vs IQML (SNR = 20 dB, $L = 2$, $N = 20$).

5. Conclusions

This paper presents a new algorithm for estimating the model order and the frequencies of multiple sinusoids buried in a noisy observation. It is based on the standard subspace separation method but with a new entropy-based cost function. The new function tends to yield maximal subspace separation and results in improved performance. Compared with some classical methods, the proposed algorithm requires a more demanding optimization process but has increased performance in estimating the model order in white and colored Gaussian noise. Additionally, multiple frequencies are estimated with lower MSE especially at high SNR values.

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