

Fast communication

Subspace-based method for joint range and DOA estimation of multiple near-field sources

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Abstract

A subspace-based method for joint range and DOA (direction-of-arrival) estimation of multiple near-field sources is presented. The proposed method uses fourth-order cumulants, and the range and DOA parameters are directly given by the eigenvectors and eigenvalues of a constructed matrix. Compared with some available methods, the 2D parameters are automatically paired and the loss of array aperture is also reduced. Performance evaluation via computer simulations is included to demonstrate the effectiveness of the proposed algorithm.

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1. Introduction

Direction-of-arrival (DOA) estimation is an important research problem in radar, sonar, radio-astronomy, communication, etc, and many classical algorithms have been developed under the assumption of far-field sources, such as Capon, ML (maximum likelihood), MUSIC, ESPRIT [1], etc. When the source is in the Fresnel field of array aperture, however, the plane wavefront approximation to the spherical wavefront is no longer valid and leads to a performance degradation of previous classical algorithm in the presence of near-field sources [2–7]. In recent years, many bearing estimation methods for near-field source have been

proposed in the literature, including ML [2], 2D MUSIC [3,4], ESPRIT-like based on high-order statistics [5], WLP [6] (weighted linear prediction), 1D MUSIC [7], etc. Most of methods proposed in the above literature need multiple-dimension search computation or additional parameters pairing processing, in addition, several existing methods [5,6] without search computation load need use the symmetric structure of array and hence the effective aperture of array is largely reduced.

To overcome the shortcomings aforementioned, in this paper, we introduce a new method for joint range and DOA estimation of multiple narrow-band near-field sources. The proposed method uses the fourth-order cumulants, and the ranges and DOAs parameters are directly obtained from the eigenvectors and eigenvalues of the eigen-decomposition of a constructed matrix. Compared with some available

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Table 1
Comparison of different algorithms for N DOAs estimation

Algorithm	Minimum number of sensors	Search required	Pair required
2D MUSIC [3,4]	$N + 1$	Yes	No
ESPRIT-like [5]	$2N$	No	Yes
WLP method [6]	$2N + 1$	No (iteration required)	Yes
Proposed algorithm	$N + 1$	No	No

Remark: Seen from the above table, the proposed method for estimation of N signal sources need only $N + 1$ sensors, but other methods without search computation need $2N$ sensors at the least.

methods [5,6], the advantages of the proposed method lie in that the 2D parameters pairing is not required and the loss of array aperture is also effectively avoided (see Table 1 for comparison).

2. Proposed method

Consider an uniform linear array of N sensors with inter-element spacing d , assumes that P narrow-band near-field signals impinge on the array. The output of the m th sensor can be approximately expressed by (see [5,6] for details)

$$x_m(t) = \sum_{i=1}^P s_i(t) e^{j(\omega_i m + \phi_i m^2)} + n_m(t) \quad \text{for } m = -1, 0, 1, \dots, N-2. \quad (1)$$

The parameters ω_i and ϕ_i are functions of the azimuth θ_i and range r_i of the i th source, and they are expressed as

$$\omega_i = \frac{-2\pi}{\lambda} d \sin(\theta_i) \quad \text{and} \quad \phi_i = \pi \frac{d^2}{\lambda r_i} \cos^2(\theta_i) \quad (2)$$

For unique estimation of source parameters, the following assumptions will be made

(A1) The P sources $[s_1(t), \dots, s_P(t)]$ are assumed to be zero mean, statistically independent of each other and have nonzero fourth-order cumulants, whereas the additive noise component $n_m(t)$ is zero mean, Gaussian and independent of the source signals.

(A2) The inter-element spacing between the sensors satisfies $d \leq \lambda/4$, here λ denotes the wavelength of the source wavefronts.

The number of sensors requires only $N > P$, which is half the number of sensors required in [5,6] (see Table 1). Let the sensor $m = 0$ be the phase reference point in this paper and this is different from that in [5,6].

The goal is to estimate the parameters $\{\theta_1, \dots, \theta_P, r_1, \dots, r_P\}$ of the P sources from the received array data $[x_{-1}(t), \dots, x_{N-2}(t)]$.

From Eq. (1), the array outputs can be written in matrix form as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \quad t = 1, 2, \dots, M, \quad (3)$$

where

$$\begin{aligned} \mathbf{x} &= [x_{-1} \ x_0 \ x_1 \ \cdots \ x_{N-2}]^T, \\ \mathbf{s} &= [s_1 \ s_2 \ \cdots \ s_P]^T, \\ \mathbf{n} &= [n_{-1} \ n_0 \ \cdots \ n_{N-2}]^T, \quad \mathbf{A} = [\mathbf{a}_1, \dots, \mathbf{a}_P], \end{aligned} \quad (4)$$

$$\begin{aligned} \mathbf{a}_i(\theta_i, r_i) &= [e^{j(-1)\omega_i + j(-1)^2\phi_i} \ 1 \ e^{j(\omega_i + \phi_i)} \ \cdots \ e^{j(N-2)\omega_i + j(N-2)^2\phi_i}]^T. \end{aligned} \quad (5)$$

We first give the fourth-order cumulant of the complex measurements $x_0(t), x_0^*(t), x_u(t), x_v^*(t)$, which is denoted as $\text{cum}(x_0(t), x_0^*(t), x_u(t), x_v^*(t))$. From Eq. (1) and cumulant properties [8], it follows that

$$\text{cum}(x_0(t), x_0^*(t), x_u(t), x_v^*(t)) = \sum_{i=1}^P \gamma_{4,s_i} e^{j(u-v)\omega_i + j(u^2-v^2)\phi_i} \quad (6)$$

where γ_{4,s_i} ($i = 1, 2, \dots, P$) denotes the kurtosis of the i th signal source and is defined as

$$\gamma_{4,s_i} = \text{cum}\{s_i^*(t), s_i(t), s_i^*(t), s_i(t)\} = E\{|s_i(t)|^4\}.$$

The two cumulant matrices \mathbf{C}_1 and \mathbf{C}_2 are then defined as

$$\mathbf{C}_1 \triangleq \text{cum}(x_0(t), x_0^*(t), \mathbf{x}(t), \mathbf{x}^H(t)) = \mathbf{A}\mathbf{C}_s\mathbf{A}^H, \quad (7)$$

$$\mathbf{C}_2 \triangleq \text{cum}(x_1(t), x_{-1}^*(t), \mathbf{x}(t), \mathbf{x}^H(t)) = \mathbf{A}\mathbf{\Psi}\mathbf{C}_s\mathbf{A}^H, \quad (8)$$

where $*$ denotes the complex conjugate and H stands for Hermitian transposition, and

$$\mathbf{C}_s = \text{diag}\{\gamma_{4,s_1}, \dots, \gamma_{4,s_P}\}, \quad \mathbf{\Psi} = \text{diag}\{e^{j2\omega_1}, \dots, e^{j2\omega_P}\}. \quad (9)$$

The first two elements of \mathbf{C}_1 and \mathbf{C}_2 play a role of guiding sensors, i.e., x_{-1}, x_0, x_1 are guiding sensors for constructing \mathbf{C}_1 , \mathbf{C}_2 , respectively. The sensor $m = 0$ is set to be the phase reference point in our method, however, the phase reference point is

located at the symmetry center of array in [5,6]. We can exploit efficiently the symmetry of sensor pair $\{x_1, x_{-1}\}$ to construct the cumulant matrix \mathbf{C}_2 and hence yield the rotational matrix $\mathbf{\Psi}$ which is function of only $\{\omega_i\}$.

Both \mathbf{C}_1 and \mathbf{C}_2 are $N \times N$ matrices with rank of P , therefore, it is easily shown that the two matrices have the following relation:

$$\mathbf{C}_2^H \mathbf{C}_1^\# \mathbf{A} = \mathbf{A} \mathbf{\Psi}, \quad (10)$$

where $\#$ denotes the pseudo-inverse of a matrix, it can be obtained by the eigen-decomposition of \mathbf{C}_1 , that is,

$$\mathbf{C}_1 = \sum_{i=1}^N \sigma_i \mathbf{v}_i \mathbf{v}_i^H, \quad (11)$$

where $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_P > \sigma_{P+1} \dots = \sigma_N$, then we have,

$$\mathbf{C}_1^\# = \sum_{i=1}^P \frac{1}{\sigma_i} \mathbf{v}_i \mathbf{v}_i^H. \quad (12)$$

Let $\mathbf{C} = \mathbf{C}_2^H \mathbf{C}_1^\#$ and it can be decomposed into

$$\mathbf{C} = \sum_{i=1}^N \alpha_i \mathbf{u}_i \mathbf{u}_i^H. \quad (13)$$

Since $\text{rank}(\mathbf{C}) = P$, $\mathbf{\Psi}$ can be estimated by the P nonzero eigenvalues of eigendecomposition of \mathbf{C} , and the DOAs can be obtained by the diagonal elements of matrix $\mathbf{\Psi}$, that is

$$\hat{\theta}_i = \sin^{-1} \left(\frac{-\angle(\alpha_i)}{(4\pi d/\lambda)} \right) \quad i = 1, 2, \dots, P, \quad (14)$$

where α_i is the i th nonzero eigenvalue of \mathbf{C} and $\angle(\alpha_i)$ denotes the phase angle of α_i . On the other hand, (10) and (13) indicate that \mathbf{A} has the same column space with $\mathbf{U} = [\mathbf{u}_1, \dots, \mathbf{u}_P]$, that is, $\text{span}[\mathbf{a}_1, \dots, \mathbf{a}_P] = \text{span}[\mathbf{u}_1, \dots, \mathbf{u}_P]$, therefore, \mathbf{a}_i can be estimated by the associated eigenvector \mathbf{u}_i . Mimicking [9], one may obtain $\hat{\phi}_i$ by minimizing a least squares cost function $\sum_{l=1}^{N-1} (\phi_i - \angle(\hat{\mathbf{u}}_i(k+1)/\hat{\mathbf{u}}_i(k)))^2$, therefore, the estimated $\hat{\phi}_i$ is given by

$$\hat{\phi}_i = \frac{\sum_{k=1}^{N-1} \angle(\hat{\mathbf{u}}_i(k+1)/\hat{\mathbf{u}}_i(k)) - (1/2)(N-1)\angle(\alpha_k)}{(N-1)(N-3)}. \quad (15)$$

Based on the estimations of $\hat{\phi}_i$ and $\hat{\theta}_i$, yielding

$$\hat{r}_i = \frac{2\pi d^2 \cos^2(\hat{\theta}_i)}{\lambda \hat{\phi}_i}. \quad (16)$$

The parameters of each source are automatically paired since the eigenvalues are paired with the corresponding eigenvectors.

Finally, it is helpful to compare the proposed algorithm to the ESPRIT-like one [5]. Both methods need to construct cumulant matrices, but they estimate the DOA and range parameters in different ways. Besides the eigenvalues, the eigenvectors are also used in this paper. Regarding computational complexity, we ignore the same computational load of the two methods that is comparatively small and consider the major part, namely, multiplications involved in calculating the cumulant matrices and in performing the eigen-decompositions, our algorithm requires $18N^2M + \frac{4}{3}N^3$, while the ESPRIT-like one requires $36N^2M + \frac{4}{3}(1.5N)^3$, where M is the number of snapshots and N is the number of sensors. Clearly, the proposed algorithm is computationally more efficient, and in general cases, $M \gg N$, it has the computational load, at least, a half of the ESPRIT-like one [5].

3. Simulation results

To verify the performance of the proposed method, a set of computer simulations is carried

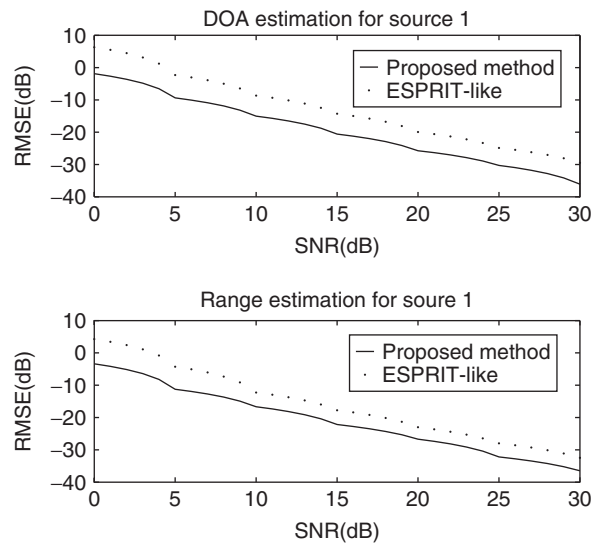


Fig. 1. The RMSEs of estimated DOAs and ranges for source 1 versus the Input SNR. There are six sensors and 2000 snapshots are used.

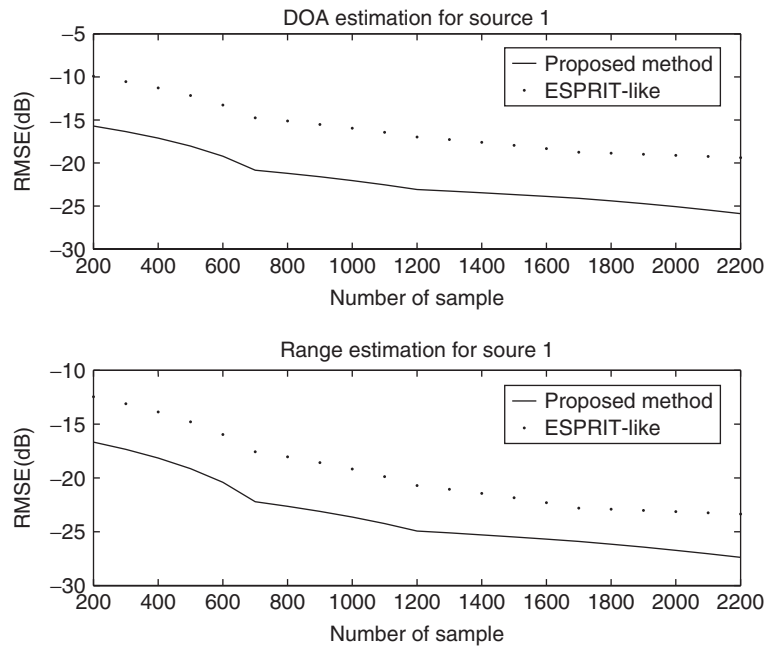


Fig. 2. The RMSEs of estimated DOAs and ranges for source 1 versus the number of sample. There are six sensors and SNR = 20 dB are used.

out in this section. We consider a uniform linear array consisting of $n = 6$ antennas with inter-spacing $d = \lambda/4$. The reference sensor is sensor 0. Two equal power uncorrelated signal sources imping on this array. The estimation performance is measured by the root mean square error (RMSE). The RMSE of range parameter is normalized by the signal wavelength λ . All results provided were averages of 500 independent runs. The first source is located at $\theta_1 = 40^\circ$ and at a range of $r_1 = 5\lambda$, and the other is located at $\theta_2 = 20^\circ$ and at a range of $r_2 = 0.5\lambda$.

In the first experiment, we assume that the sensor noise is additive Gaussian noise. The number of samples is set to $T = 2000$ at each sensor. The results for range and DOA estimates of the two sources are shown in Fig. 1. For comparison, the estimation results using the ESPRIT-like algorithm [5] are also shown in all the figures. Seen from the figures, the estimation accuracy of the proposed method is much higher than that of ESPRIT-like method over all signal-to-noise ratio (SNR), the results for source 2 are also similar and omitted here.

In the second experiment, the input SNR of two sources is set to be SNR = 20 dB. The number of samples is varied from 200 to 2000. The estimation results for the two methods are shown in Fig. 2.

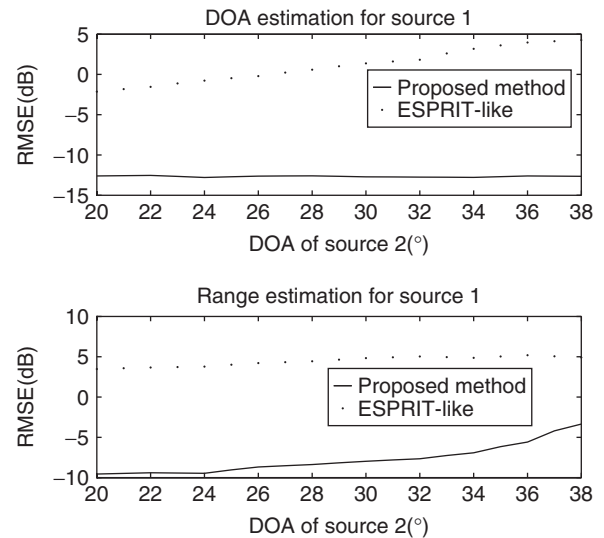


Fig. 3. The RMSEs of estimated DOAs and ranges for source 1 versus DOA of source 2. There are six sensors and SNR = 20 dB, 2000 snapshots are used.

In the third experiment, the performance in resolving capability of the proposed method is also compared with that of ESPRIT-like methods. The input SNR of two equal-power sources is set to be SNR = 20 dB. The number of samples is set to be 2000. The DOA of source 1 is fixed at 40° whereas

the DOA of the second source is varied from 20° to 38° . It is seen from Figs. 2 and 3 that the performance of the proposed method is better than that of ESPRIT-like method, since the effective aperture size of the former method is much larger than that in the ESPRIT-like method.

4. Conclusions

A novel subspace-based method is presented for localization of multiple near-field sources. The proposed algorithm uses effectively the aperture of array, and hence our method can estimate the DOAs of $N - 1$ signal sources using N sensors. The proposed algorithm does not need to search computation and parameters pairing processing. Furthermore, the simulation results are included to demonstrate the performance improvement of the proposed algorithm compared to the existing ESPRIT-like method.

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