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Low Complexity Robust Direction Finding Method for Impulsive Noise in l_p-Space

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Abstract

A robust low complexity direction of arrival (DOA) estimation methodforimpulsive noise is proposed in this paper. The presence of outliers makes it difficult to estimate the subspace accurately, and as a result leads to serious estimation errors. In our method, robust signal subspace is first obtained by iterative re-weight singular value decomposition (IR-SVD) of data matrix. Then subspace rotation operator is calculated via matrix l_p-norm minimization procedureinstead of least squares (LS). Compared to traditional ESPRIT, our proposed method performs more robust in the presence of impulsive noise. Besides, this ESPRIT like method provides close-form solution of DOA, which saves computational load by avoiding grid searching. Simulation results illustrate that proposed method outperforms than several outliers-resistant algorithms in scenario of impulsive noise.

Keywords: impulsive noise, robust DOA, ESPRIT, l_p-space;

1. Introduction

Direction-of-arrival (DOA) estimation is critical in many applications such as radar, sonar, and wireless communication systems. The presence of non-Gaussian noise makes performance of some well-known algorithms degrade, such as MUSIC[1], ESPRIT[2], weight subspace fitting (WSF)[3],and space-alternating generalized expectation maximization (SAGE)[4]. For subspace based methods, the presence of outliers leads to estimation errors of the subspace, and as a result causes serous estimate errors of DOA.

To resist effect of outliers, some improved algorithms have been proposed[5-10]. For example, FLOM-MUSIC[5], ROC-MUSIC[6], FLOC-ESPRIT[7], and FLIC-MUSIC[8] aim to construct a more robust covariancematrix, named low-order moment, which can be used to obtain robust DOA estimations by means ofMUSIC or ESPRIT. However, when the probability density function (PDF) of noise has a heavy tail, this kind of algorithms performs poorly. The algorithm proposed by Yardimic[9] attempts to design an optimal penalty function via maximum likelihood estimation technique according to specific noise model. ZMNL-MUSIC[10] applies zero-memory nonlinear (ZMNL) functions to

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clip the effect of outliers, which can provide more accurate estimations with low computational complexity cost, while the performance may degrade due to rank increase of the signal subspace.

The key step of subspace based methods is to estimate the signal or noise subspace. Under the assumption of Gaussian noise, singular value decomposition (SVD) of data matrix or engine Eigenvalue decomposition (EVD) of covarianceprovide optimal estimation of signal or noise subspace. While the presence of outliers produces estimate errors of subspace, consequently, causes serious DOA estimation error. Different from the above algorithms, another group of methods extend the conventional modus from l_2 to l_p-space, which can produce more robust parameter estimations than above methods, i.e. l_p-MUSIC[11].

The l_p-MUSIC is confirmed outperforms than current outliers-resistant methods[11], while also need grid searching of spectrum, which is an extremely time-consuming process especially in the case of joint azimuth and elevation angle estimation. In order to reduce the computational complexity, a robust ESPRIT like method is proposed which can provide close-form solution of DOA. Robust signal subspace is first obtained by means of IR-SVD of data matrix. Then subspace rotation operator is calculated in l_p-space instead of using least squares (LS). Finally, close-form solution of DOA can be obtained based on above steps. The notations used in this paper are as follows. $|\cdot|$ denotes the modulus of a complex number and $||\cdot||_p$denotes p norm of a vector. $[\cdot]^T$, $[\cdot]^H$and $[\cdot]^*$ denote transpose, Hermitian transpose and conjugate of matrix, respectively.

2. Problem Formulation

2.1. Signal Model

Consider a uniform linear array (ULA) of $M+1$ antennas that consists of two overlapping subarrays of Midentical and omnidirectional antennas each. Let the first subarray is composed of the antennas with the indices $1,2,\cdots,M$ and second one with the indices $2,3,\cdots,M+1$.Assume that Lnarrow-band, far-field uncorrelated sources impinge on the array. The output snapshot vector of first M antennas (first subarray) can be written as

$$\mathbf{y}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{w}(t) \tag{1}$$

where $\boldsymbol{x}(t)$ isincident signal vector, $\mathbf{A}=\left[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\cdots,\mathbf{a}(\theta_L)\right]$ is the array manifold matrix with

$$\mathbf{a}(\theta_i) = [1, e^{-j2\pi d sin(\theta_i)/\lambda},\cdots, e^{-j2\pi(M-1)d sin(\theta_i)/\lambda}]^T \tag{2}$$

$\theta_i$is the direction of arrival and λ denotes wavelength. The inter-element spacing $d<\lambda/2$. $\mathbf{w}(t)$ is the additive non-Gaussian noise.In the rest of this article, we employ discrete time version of (1) with $n = t/T$, where T is the sampling interval. The received data of first subarray can be expressed as matrix form

$$\mathbf{Y}=\mathbf{AX}+\mathbf{W} \tag{3}$$

where $\mathbf{Y}\in\mathbf{C}^{M\times N}$ is the received signal matrix with Nbeing the number of snapshots. Similarly, $\mathbf{X}\in\mathbf{C}^{L\times N}$ is the impinging signal matrix and $\mathbf{W}\in\mathbf{C}^{M\times N}$ is the Non-Gaussian noise matrix.Let $\mathbf{A}'$ denote manifold matrix of the second subarray, where $\mathbf{A}'=\mathbf{A\Phi}$, termed rotational invariance among two subarray steering matrixes with

$$\mathbf{\Phi}= \mathrm{diag}\left[e^{-j2\pi d sin(\theta_1)/\lambda}, e^{-j2\pi d sin(\theta_2)/\lambda},\cdots, e^{-j2\pi d sin(\theta_L)/\lambda}\right]. \tag{4}$$

The diagonal matrix $\mathbf{\Phi}$ is the desired here, termed rotationalmatrix that carries the information of DOA. The expanded version of $\mathbf{Y}$ and $\mathbf{A}$ are defined as

$$\overline{\mathbf{Y}} = \begin{bmatrix} \mathbf{Y} \\ \mathbf{Y}' \end{bmatrix}, \overline{\mathbf{A}} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}' \end{bmatrix} \tag{5}$$

where $\mathbf{Y}$ and $\mathbf{Y}'$ denote received data of first and second subarray, respectively.

3. Robust ESPRIT in l_p-Space

3.1. Robust signal subspace estimation

Let's rewrite free noise received data of array output $\mathbf{Y} = \mathbf{AX}$, where array steering matrix $\mathbf{A}$is full column rank, and incident matrix $\mathbf{X}$is full row rank. Array output matrix $\mathbf{Y}$is linear combination of$L$ vectors, the columns of ,so, $\mathbf{Y}$is the L-dimensional subspace. Performing SVD of free noise received data matrix

$$\mathbf{Y} = \mathbf{U}_s \mathbf{\Sigma}_s \mathbf{V}_s^{\mathbf{H}} = \mathbf{UZ} \tag{6}$$

With and. The matrix consists of the first L columns of the left singular matrix of $\mathbf{Y}$. Similarly, the matrix consists of the first L columns of right singular matrix, and is a$L\times L$ diagonal matrix consists of largest L singular values of $\mathbf{Y}$.Obviously, the space spanned by columns of $\mathbf{A}$is the same as the one spanned by columns of $\mathbf{U}$, termed signal subspace.

When received data is contaminated by noise, the estimation of signal subspace can be obtained by following fitting procedure

$$J(\mathbf{U},\mathbf{Z}) = \| \mathbf{Y} - \mathbf{UZ} \|_p^p \tag{7}$$

where $\| \boldsymbol{x} \|_p^p = \sum_{n=1}^{N} | x(n) |^p$, termed l_p-norm of a vector.As the general definition of l_p-norm, the index p is required $p \geq 1$, when $1 > p > 0$, $\|\boldsymbol{x}\|_p^p$ is the quasi-norm[12].

The functionevaluates the l_p-norm of residual fitting error.Our object is to obtain optimal ($\mathbf{U}$, $\mathbf{Z}$)by minimizing J,which leads to an optimization problem with two matrix variables ($\mathbf{U}$, $\mathbf{Z}$). Besidessolution strategy, let's focus on the other critical problem, the choice of an appropriate p. The chosen of p depends on the noise characteristics, in other words, the distribution of residual fitting error. When the noise satisfies Gaussian distribution, the most general cause, l_2-norm is adopted, which leads to a well know maximal likelihood (ML) problem. The optimal solutions can be obtained by SVD of $\mathbf{Y}$, which imply that$\mathbf{U}$=$\mathbf{U_s}$, andwith $\mathbf{U}_s$and $\mathbf{V}_s$being matrixes consist of left and right singular vectors corresponding to L largest singular values of $\mathbf{Y}$, respectively.

For impulsive noise, l_2-norm is not appropriate anymore due to the more weights are put on the large errors.In order to resist outliers, l_p-norm ($2 > P > 0$)is adopted, which leads to a matrix l_p-norm minimization problem. When $P \geq 1$, it is convex, butnonconvex anymore for $p< 1$. IR-SVD [11]can solve this problem effectively by converting l_p-norm minimization problem to weighted l_2-norm minimization one

$$J(\mathbf{U},\mathbf{Z}) = \| \mathbf{Y} - \mathbf{UZ} \|_p^p = \| \mathbf{D} \odot (\mathbf{Y} - \mathbf{UZ}) \|_2^2 \tag{8}$$

where $\mathbf{D}$is weighting matrix with entry, and is the (m, n)th element of residual fitting error matrix $\mathbf{Y} - \mathbf{UZ}$. The operator $\odot$ denotes Hadamard product of twomatrixes.Primary estimation of subspace **U**can be obtained by performing SVD of weighted data matrix $\mathbf{D} \odot \mathbf{Y}$. Then **D**can be updated by following steps

$$\mathbf{D}^{(k)} := \mathbf{Y} - \mathbf{U}^{(k)}\mathbf{Z}^{(k)}, \text{with} \quad \mathbf{U}^{(k)} := \mathbf{U}_s^{(k)}, \mathbf{Z}^{(k)} := \mathbf{\Sigma}_s^{(k)}(\mathbf{V}_s^{(k)})^H \tag{9}$$

where superscript k denotes the kth iteration, and this procedure will carry on until converge. At the first iteration, $\mathbf{U}^{(0)}$ and $\mathbf{Z}^{(0)}$ are initialized as $\mathbf{U}^{(0)} = \mathbf{U}_s$and $\mathbf{Z}^{(0)} = \mathbf{\Sigma}_s \mathbf{V}_s^H$ which can be calculated via SVD of **Y**, rather than initialized as full rank random matrix adopted in original IR-SVD[11].However, although IR-SVD is effective for this matrix l_p-norm minimization problem, it is possible converge to local minima when p <1. So, we adopt p = 1 to resist impulsive noise, meanwhile, to guarantee converge to global minima of this convex optimization problem.

3.2. Robust DOA estimation via ESPRIT

The robust signal subspaces have been prepared at this point, which can be used to estimate DOA via rotational invariance. The expanded version of **U** and **A** span the same space [2], which imply that $\text{span}\{\bar{\mathbf{U}}\} = \text{span}\{\bar{\mathbf{A}}\}$ .The relationship of $\bar{\mathbf{U}}$ and $\bar{\mathbf{A}}$ describedabove can be further written as

$$\bar{\mathbf{U}} = \bar{\mathbf{A}}\mathbf{T} = \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{AT} \\ \mathbf{A\Phi T} \end{bmatrix} . \tag{10}$$

Here, $\mathbf{U}_1$ and $\mathbf{U}_2$ are the robust subspaces calculated via $\mathbf{Y}$ and $\mathbf{Y}'$ by means of IR-SVD described in Section 3.1, respectively. To obtain $\mathbf{\Phi}$, we introduce following fitting procedure

$$\mathbf{\Phi} = \arg \min_{\mathbf{\Phi}} \left\| \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{bmatrix} - \begin{bmatrix} \mathbf{AT} \\ \mathbf{A\Phi T} \end{bmatrix} \right\|_p^p . \tag{11}$$

A simple strategy to solve this problem is to set $\mathbf{U_1} = \mathbf{AT}$ [13], associate with non-singular assumption of **T**, the rotational invariance of two subspaces can be expressed as

$$\mathbf{\Phi} = \arg \min_{\mathbf{\Phi}} \left\| \mathbf{U}_2 - \mathbf{U}_1 \mathbf{T}^{-1} \mathbf{\Phi} \mathbf{T} \right\|_p^p . \tag{12}$$

For notation simply, equation (12) can be further written as

$$\mathbf{\Psi} = \arg \min_{\mathbf{\Phi}} \left\| \mathbf{U}_2 - \mathbf{U}_1 \mathbf{\Psi} \right\|_p^p \tag{13}$$

where $\mathbf{\Psi} = \mathbf{T}^{-1}\mathbf{\Phi}\mathbf{T}$. To obtain more robust estimation of $\mathbf{\Psi}$, we set $0 < p < 2$, while, there is no close form solution of (13) in this case. However, this problem can be solved via an iteration strategy refer to alternating convex optimization (ACO) method[11]. The matrix l_p-norm minimization problem (13) can be divided into N vector l_p-norm minimization subproblems

$$\boldsymbol{\psi}_n = \arg \min_{\boldsymbol{\psi}} \left\| \mathbf{u}_n - \mathbf{U}_1 \boldsymbol{\psi}_n \right\|_p^p \tag{14}$$

where $\mathbf{u}_n$ and $\boldsymbol{\psi}_n$ are the nth columns of $\mathbf{U}_2$ and $\boldsymbol{\Psi}$, respectively. We can get optimal $\boldsymbol{\Psi}$ by solving these N sub-problems separately, which can be easily solved via Newton's method.

Due to thediagonal matrix $\boldsymbol{\Phi}$consists of eigenvalues of Ψ, and **T**consists of corresponding eigenvectors. Once diagonal $\boldsymbol{\Psi}$ is prepared, $\boldsymbol{\Phi}$can be acquired by means of performing EVD of $\boldsymbol{\Psi}$, and direction of arrival can be calculated by

$$\theta_i = \arcsin\left(-\frac{\lambda}{2\pi d}\arg\left(\varphi_i\right)\right) \tag{15}$$

where $\phi_i$is the ith diagonal element of $\boldsymbol{\Phi}$, and arg(·) denotes operator taking the angle of a complex value. Detailed implementation of IR-SVD and Newton's method are described in Table.1.

Table 1.Implementation of IR-SVD and Newton's method.

Newton's method	IR-SVD
for n = 1,2,N, **do**	**Initialization:** $\mathbf{Z}^{(0)} = \boldsymbol{\Sigma}_s \mathbf{V}_s^H$, $\mathbf{U}^{(0)} = \mathbf{U}_s$
Calculate $\boldsymbol{\psi}_n = \arg\min_{\boldsymbol{\psi}} \left\Vert \mathbf{u}_{2,n} - \mathbf{U}_1 \boldsymbol{\psi}_n \right\Vert_p^p$	**for** k = 0,1,2,…,**do** until converge
for k = 1,2,...,**do**until converge # Newton's method begin	$\mathbf{D}^{(k)} := \mathbf{Y} - \mathbf{U}^{(k)}\mathbf{Z}^{(k)}$
$\boldsymbol{\psi}_n^{(k+1)} = \boldsymbol{\psi}_n^{(k)} + \mu_k \Delta\boldsymbol{\psi}_n^{(k)}$ with	Performing SVD of $\mathbf{D}^{(k)} \odot \mathbf{Y}$
$\Delta\boldsymbol{\psi}_n^{(k)} = -\frac{p}{2}\left(\mathbf{U}_1 \mathbf{W}^{(k)} \mathbf{U}_1\right)^{-1} \mathbf{U}_1 \mathbf{W}^{(k)} \left(\mathbf{U}_1 \boldsymbol{\psi}_n^{(k)} - \mathbf{u}_n\right)$,	$\mathbf{U}^{(k)} := \mathbf{U}_s^{(k)}$
$\mathbf{W}^{(k)} = \mathrm{diag}\left\{\left\vert r_1\right\vert^{p-2}, \left\vert r_2\right\vert^{p-2}, \cdots, \left\vert r_M\right\vert^{p-2}\right\}$,	$\mathbf{Z}^{(k)} := \boldsymbol{\Sigma}_s^{(k)} (\mathbf{V}_s^{(k)})^H$
$r_m^{(k)} = \mathbf{u}_n^{(k)} - \mathbf{U}_1^{(k)} \boldsymbol{\psi}_n^{(k)}$,	**end**
$\mu_k = \arg\min_{\mu>0} \left\Vert \mathbf{U}_1 \left(\boldsymbol{\psi}_n^{(k)} + \mu\Delta\boldsymbol{\psi}_n^{(k)}\right)\right\Vert_p^p$;	
end	
end	

4. Simulation

In this section, we present statistical performance comparison of our proposed method with conventional ESPRIT, FLOC-ESPRIT, and l_p−MUSICin the scenarios of impulsivenoise. A kind of typical impulsive noise is adopted named symmetry α-stable($S\alpha S$) noise, whose PDF is, where α describes the tail of distribution andγ represents scalar. The range of α is $0<\alpha<2$, the smallerα corresponding to the heavier tail of distribution. Since the second orderstatistic (SOS) of the $S\alpha S$ is infinite for impulsive noise, thegeneralized SNR is adopted[11]with. Detection probability and root mean square error (RMSE) areused as measurements with definition

$$RMSE = \sqrt{\frac{1}{Nc}\sum_{n=1}^{N_c}\left(\theta_n - \theta_n\right)^2} \tag{16}$$

where N_c is the number of Monte Carlo trials, andareestimates and true value, respectively. When estimate erroris less than, this time of estimation can be regarded asresolving two sources successfully. In simulation, we adoptl_1-norm as penalty function of residual errors.

Two narrow-band incoherentsignals are set as sources who arrive at directions of and. The 7 elements uniform linear array is adopted asreceiver, and α =1.4 for $S\alpha S$ noise.GSNR increases from -18dB to 45dB, and for each GSNR, 2000 Monte Carlo trials are carriedout. Here, the number of snapshots is 100. Fig.1 and Fig.2 show the

RMSE and detection probabilityversus GSNR, respectively. The presence of impulsivenoise makes performance of traditional ESPRIT and FLOC-ESPRIT degrade, while Robust ESPRIT and l_p-MUSIC cansolve two signals successfully when GSNR > 5dB. The statistical performance ofrobust ESPRIT and l_p-MUSIC is closely. l_p-MUSIC performs a little better under the lower GSNR while our proposed method outperforms in higher GSNR scenario. However, different from l_p-MUSIC,our proposed method provides close-form solution of DOA,therefore, avoid time-consuming procedure of grid searching, in other word, it is low complexity.

5. Conclusion

In order to obtain robust DOA estimation with low computational load, an ESPRIT like method is proposed. Robust signal subspace is calculated at first by means of IR-SVD, then, the subspace rotation operator is obtained via a matrix l_p-norm minimization processing. Estimation of DOA can be acquired from closed solution without performing grid searching, as a result, save its computational load. Numerical simulation results illustrate that our method performs well as l_p–MUSIC dose, sometimes even better. In addition, our method besidesl_p-MUSIC outperform than other outliers-resistant algorithms in impulsive noise.

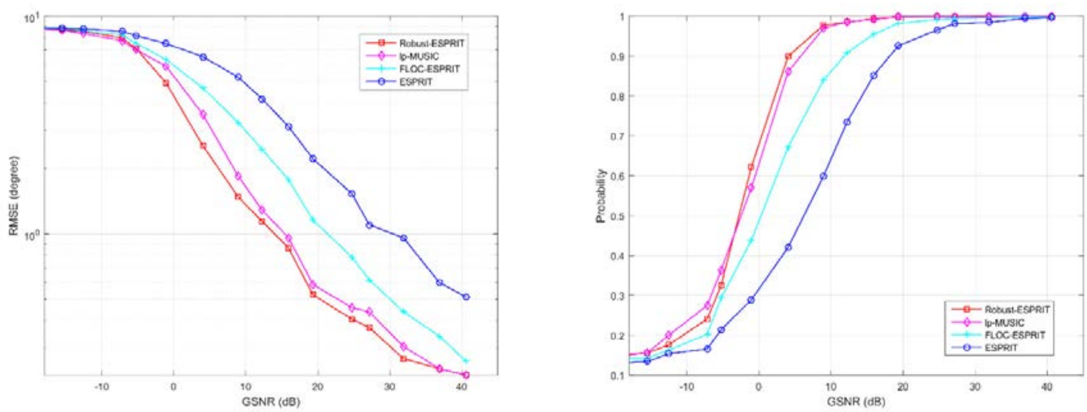

Fig. 1. (a) RMSE of DOA estimates versus GSNR with α = 1.4; (b) Detection probability of DOA estimates versus GSNR with α = 1.4.

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