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Procedia Computer Science 143 (2018) 277–284

Procedia Computer Science

www.elsevier.com/locate/procedia

8th International Conference on Advances in Computing and Communication (ICACC-2018)

Novel Rank Enhancing Algorithm for 2D Sparse Arrays with Contiguous Coarrays

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Abstract

Sparse array is an irregular array with a high degree of freedom in its difference coarray domain. Nested array and coprime array are two important one dimensional sparse arrays. For fixed number of sensors, nested array has a high degree of freedom, resolution and accuracy than coprime array. 1D nested arrays do not provide 2D scanning. 2D nested array which is an extension of 1D nested array provides both 2D scanning as well as high degree of freedom, however direction of arrival (DoA) estimation techniques used for 2D nested array such as 2D unitary ESPRIT and 2D spatial smoothing MUSIC has high computational complexity. This paper shows that spatial smoothing is not necessary to apply 2D MUSIC in the difference coarray domain of 2D nested array. A new Hermitian Toeplitz matrix is constructed instead of spatial smoothing. 2D MUSIC on this Hermitian Toeplitz matrix can resolve the DoA of sources with less computations.

Keywords: DOA estimation, 2D MUSIC, 2D spatial smoothing, difference coarray, 2D nested arrays.

1. Introduction

Direction of arrival estimation using antenna arrays has a wide application in different fields like SONAR, RADAR, communication system, biomedicine and seismology [1], [2] . Uniform linear array (ULA) is the most common array used for DoA estimation which is limited by 1D scanning. However uniform circular array (UCA) and uniform rectangular array (URA) can provide 2D scanning. 2D DoA estimation has attracted a lot of attention and it is required to operate in both azimuthal and elevation angles. High resolution algorithms like 2D MUSIC [3] and 2D ESPRIT [4], [5] can be applied to both URA and UCA. However these algorithms work with the assumptions that the number of

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10.1016/j.procs.2018.10.396

sources are less than the number of sensors and antenna arrays are uniformly spaced. Sparse arrays such as coprime and nested arrays are irregular arrays which can detect more sources than the number of sensors used. These arrays have a high degree of freedom (DoF) in its difference coarray domain. A two level nested array is a chain of two uniform linear arrays, where half of the sensors are spaced with unit distance and the other half of the sensors are spaced directly proportional to the number of sensors in the first half. All the missing sensor locations in the second half of the nested array can be recreated in its difference coarray [6], [7] . Coprime array is a combination of two uniform linear arrays with coprime pair of sensors. It utilizes the properties of coprime numbers. Using 2M+N-1 sensors coprime array can create an effect of MN number of sensors [8], [9] . For fixed number of sensors, nested array has a high degree of freedom, resolution and accuracy than coprime array. Nevertheless coprime array is sparser than nested array.

2D sparse arrays like 2D nested array can provide both 2D scanning as well as a high DoF. 2D nested array is an extension of 1D nested array [10] . Sensor locations of 2D nested array is equivalent to the cross product of two identical 1D nested arrays [11] . The difference coarray of 2D nested array is generated by vectorizing a covariance matrix of the received data. Thus the data in the coarray domain is correlated. This received data at coarray domain act as a single snapshot and the rank of the covariance matrix generated from these data is one. In order to apply high resolution algorithms, high rank matrix is necessary. For this reason high resolution algorithms cannot be directly applied to the 2D difference coarray generated from the 2D nested array. The traditional approach to address this problem is 2D spatial smoothing [12] . 2D spatial smoothed matrix in coarray domain is a positive definite matrix. 2D MUSIC and 2D ESPRIT can be applied to this 2D spatially smoothed matrix.

In this paper it is shown that 2D spatial smoothing matrix is not needed to build a high rank matrix for 2D nested array. Instead of spatial smoothed matrix, a Hermitian Toeplitz matrix can be generated from the coarray received data itself. 1D version of this Toeplitz matrix was proposed in [13] , and the insight provided in this paper is adopted here. This method reduces computational complexity associated with the existing approaches [11], [12] .

2. Signal model

Consider P uncorrelated sources impinging on a 2D sparse array whose sensor locations are at nd, where $n = (n_x, n_y)\ \epsilon\ S$ is an integer valued vectors and $d = \lambda/2$. Sensor locations of 2D nested array is equivalent to the cross product of two identical 1D nested arrays. Sensor locations of a 1D nested array is notated by,

$$S_{1DNested} = \{1, 2, \cdots, N_1, (N_1 + 1), 2(N_1 + 1), \cdots, N_2(N_1 + 1)\} \tag{1}$$

where N_1 and N_2 are the number of sensors in the first and second level of a two level nested array. Sensor locations of 2D nested arrays notated by, $S_{2DNested} = S_{1DNested} * S_{1DNested}$. Sensor outputs can be modelled as,

$$X_S = \Sigma_{i=1}^{P} G_i A_S(\theta_i, \phi_i) + N_S \tag{2}$$

where G_i is the complex amplitude of i^{th} source and N_S is the AWGN noise. The steering vector $A_S(\theta_i, \phi_i)$ corresponds to the sensor location at $(n_x, n_y)\ \ \epsilon\ S$ is given by $e^{j2\pi n_x(\frac{d}{\lambda})sin(\theta_i)cos(\phi_i)+n_y(\frac{d}{\lambda})sin(\theta_i)sin(\phi_i)}$ where $\phi_i\ \epsilon\ (0, 2\pi)$, and $\theta_i\ \epsilon\ (0, \pi)$ are azimuth and elevation angles respectively. The terms $(\frac{d}{\lambda})sin(\theta_i)cos(\phi_i)$ and $(\frac{d}{\lambda})sin(\theta_i)sin(\phi_i)$ can be normalized to $\bar{\theta}$ and $\bar{\phi}$ respectively. The covariance matrix of X_S can be represented as,

$$R_S = E[X_S X_S^H] = \sum_{i=1}^{P} \sigma_i^2 A_S(\bar{\theta}, \bar{\phi}) A_S^H(\bar{\theta}, \bar{\phi}) + \sigma^2 I \tag{3}$$

where σ_i^2 is the signal power and σ^2 is the noise power.

2.1. Difference Coarray

Difference coarray (D) is defined as the array located at positions equal to the pair wise differences between sensor locations of a physical array given by

$$D = n_1 - n_2 \qquad\qquad n_1, n_2 \ \varepsilon \ S \tag{4}$$

where n_1 and n_2 are the different sensor pairs in a physical array. Sensors of difference coarray are now located at md here $m = (m_x, m_y) \ \varepsilon \ D$. The weight function $w(m)$ gives the number of sensor pairs with same separation $m \ \varepsilon \ D$. Weight functions are symmetric $w(m) = w(-m)$, $(n_1, n_2) \ \epsilon \ w(m)$ if and only if $(n_2, n_1) \ \epsilon \ w(m)$ [11] . The uniform rectangular part of difference coarray represents the contigous coarray (U). For 2D nested array, D = U, and such arrays are called hole free arrays. Unlike difference coarray of uniform arrays, difference coarray of sparse arrays has more number of sensors in its difference coarray than its physical array.

Received data corresponding to the difference coarray can be obtained from (3). Since the entries on covariance matrix depends only on the differences between sensor locations, vectorizing this covariance matrix and averaging the entries which has same sensor separations yield the received data on the difference coarray.

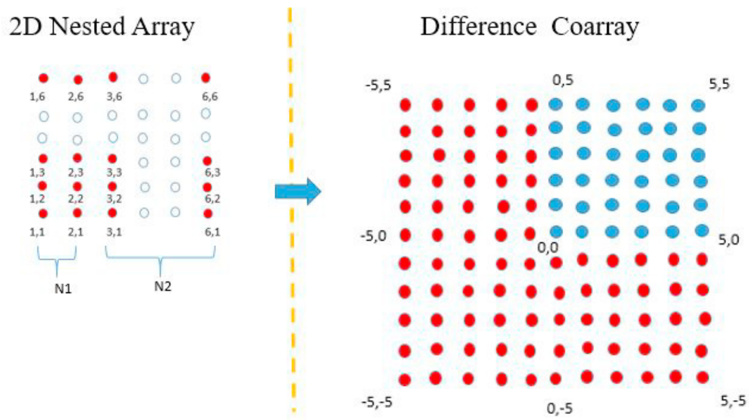


Fig. 1: An illustration of physical array and difference coarray of 2D nested array with 16 sensors, here $N_1 = 2$ and $N_2 = 2$.

$$X_D = \Sigma_{i=1}^{P} \sigma_i^2 A_D(\bar{\theta}, \bar{\phi})_i + \sigma^2 e_d \tag{5}$$

where e_d being a column vector of all zeros except a 1 at the d^{th} position. Difference coarray of the 2D nested array has a uniform rectangular form of size $M \times M$. Here M is equal to the coarray aperture of 1D nested array that is used for the construction of 2D nested array.

Fig. 1 shows an illustration of physical array of 2D nested array and its difference coarray with 16 sensors. Physical array that represents sensor locations of 2D nested array is equivalent to the cross product of two identical two level nested arrays with two sensors in each level. All the missing sensor locations in physical array is recreated in its difference coarray domain. Difference coarray in this example is equivalent to the difference coarray of a rectangular array with 36 sensors. This indicates that using 16 sensors, 2D nested array can virtually create an effect of 36 sensors.

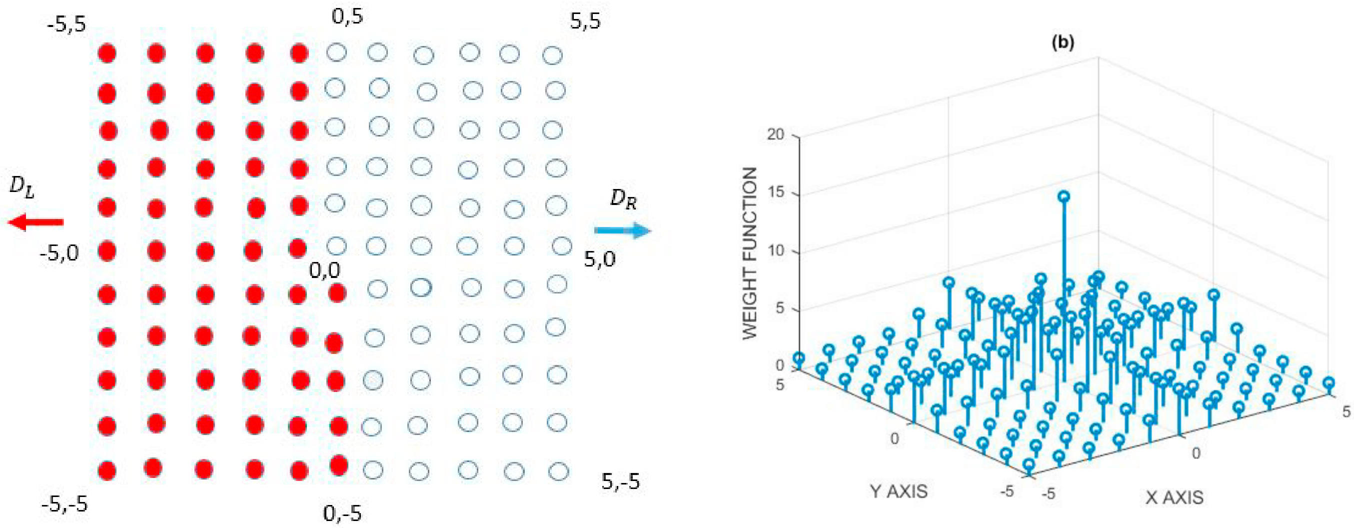


Fig. 2: (a) An illustration of Hermitian symmetry of difference coarray of 2D nested array with 16 sensors, for $N_1 = 2$ and $N_2 = 2$. (b)Weight function of 2D nested array with 16 sensors, for $N_1 = 2$ and $N_2 = 2$.

3. DOA Estimation Based on 2D Spatial Smoothing

High resolution algorithms like 2D MUSIC and 2D ESPRIT cannot be directly applied to the difference coarray generated from the 2D nested array. Since the difference coarray is generated from covariance matrix of the received data, data in the coarray domain is correlated. These data behave like a single snapshot and the rank of the covariance matrix computed from these data is one. Using this covariance matrix multiple source detection is not possible. Spatial smoothing is a traditional approach used to decorrelate correlated sources. Here impinging sources are assumed as uncorrelated, however it acts as correlated when it reaches the coarray domain. To address this problem [11], [12] spatial smoothing technique is adopted to decorrelate the sources at coarray domain. 2D spatial smoothed matrix is constructed by dividing difference coarray into $(M+1)^2/4$ identical subarrays of size $(\frac{M+1}{2}, \frac{M+1}{2})$. 2D spatial smoothed matrix in the coarray domain is a positive definite matrix. 2D spatial smoothed matrix can be modelled as,

$$\hat{R}_S = \frac{1}{\sqrt{\frac{(M+1)^2}{4}}}(A_0 R_{SS} A_O^H \sigma^2 I_{\frac{(M+1)}{2} \times \frac{(M+1)}{2}}) \tag{6}$$

where A_0 is the array manifold for the first subarray matrix and R_{SS} is the signal covariance matrix. 2D MUSIC and ESPRIT can be applied to this spatially smoothed matrix to estimate 2D DoA. Still, 2D spatial smoothing requires high computational time. The method used in [13] is adopted in this paper to reduce this computational complexity associated with 2D spatial smoothing.

4. Novel Rank Enhanced Matrix Without 2D Spatial smoothing

In this section we show that 2D spatial smoothing is not required to apply 2D MUSIC in 2D sparse arrays with hole free coarray. Instead of spatial smoothing a Hermitian Toeplitz matrix can be generated from the coarray received data. 1D version of this matrix is proposed in [13] , and the insight provided in this paper is used here.

In order to generate a new matrix from the coarray received data, such that it produces the exact same MUSIC spectra [12] without implementing 2D spatial smoothing, difference coarray received vector X_D should be Hermitian symmetric. Difference coarray of 2D nested array is a contiguous rectangular array and its weight functions are symmetric $w(m) = w(-m)$ where $m = (m_x, m_y) \epsilon D$. Difference coarray of 2D nested array can be divided into D_L and D_R. D_L ranges from $(-L, -L) \leq (m_x, m_y) \leq (0, 0)$. D_R ranges from $(0, 0) \leq (m_x, m_y) \leq (L, L)$, where $L = \frac{(M-1)}{2}$. D_L and D_R are Hermitian symmetric to each other. Sensor position (0, 0) is common for D_L and D_R, thus difference coarray is symmetric about the center.

For example consider Fig. 2 (a) which shows the Hermitian symmetry of difference coarray of a 2D nested array with 16 sensors. In this figure difference coarray is divided into D_L and D_R. Here D_L ranges from $(-5, -5)$ to $(0, 0)$ and D_R ranges from $(0, 0)$ to $(5, 5)$. Fig. 2 (b) shows the simulation result for the weight function $w(m)$ of corresponding 2D nested array. From the Fig. 2 (a), (b) it is clear that $|w(m)| = |w(-m)|, m \epsilon D$ and D_L is Hermitian symmetric to D_R. Following the Hermitian symmetry between D_L and D_R and the property of weight function, it is obtained as:

$$\langle X_D \rangle_m = \langle X_D \rangle_{-m} \tag{7}$$

The bracket notation indicates the data at the location $m = (m_x, m_y) \epsilon D$. Since X_D follows Hermitian symmetry, difference coarray received vector can be reshaped into Toeplitz matrix $\tilde{R}$ just by placing the received data in D_L as the first row and the received data in D_R as the first column. Hereby without further arithmetic operations a high rank matrix can be obtained as,

$$\tilde{R} = \begin{bmatrix} [X_D]_{0,0} & [X_D]_{0,-1} & \cdots & [X_D]_{-L,-L} \\ [X_D]_{0,1} & [X_D]_{0,0} & \cdots & [X_D]_{-(L-1),-(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ [X_D]_{L,L} & [X_D]_{L-1,L-1} & \cdots & [X_D]_{0,0} \end{bmatrix} \tag{8}$$

where the first row corresponds to the received data in D_L and the first column corresponds to the data in D_R. MUSIC on $\tilde{R}$ resolves L^2 uncorrelated sources. This method reduces the computational complexity associated with the existing approaches. This approach can produce the exactly same MUSIC spectrum [12] without implementing spatial smoothing.

4.1. Computational Savings

For the construction of $\hat{R}_S$ based on 2D spatial smoothing approach, it takes $O(L + 1)^6$ operations. This is because 2D spatial smoothing approach needs to construct $(L + 1)^2$ identical subarrays and each subarray takes $O(L + 1)^4$ multiplication. Thus cost for $\hat{R}_S$ is $O(L + 1)^6$. On the contrary, to construct $\tilde{R}$, no multiplication is required. X_D can be reshaped into $\tilde{R}$ without any arithmetic operation. Hence the proposed method saves $O(L + 1)^6$ computations.

5. Results and Discussions

Fig.3 (a, b) shows the MUSIC spectrum of P = 49 sources based on the proposed Hermitian Toeplitz matrix created from a 2D nested array with 36 sensors for two different values of snapshots T = 100 and T = 1000, where $N_1 = 3, N_2 = 3$. Sources chosen uniformly over $\bar{\theta}\ \varepsilon\ [-0.3, 0.3]^2, \bar{\phi}\ \varepsilon\ [-0.3, 0.3]^2$. Difference coarray of 2D nested array with 36 sensors is equivalent to a rectangular array with 64 virtual sensors. In Fig 3. (a, b) there are 49 distinguishable peaks even though number of sources are greater than number of sensors. The spectrum of the proposed method improves as the number of snapshots increases.

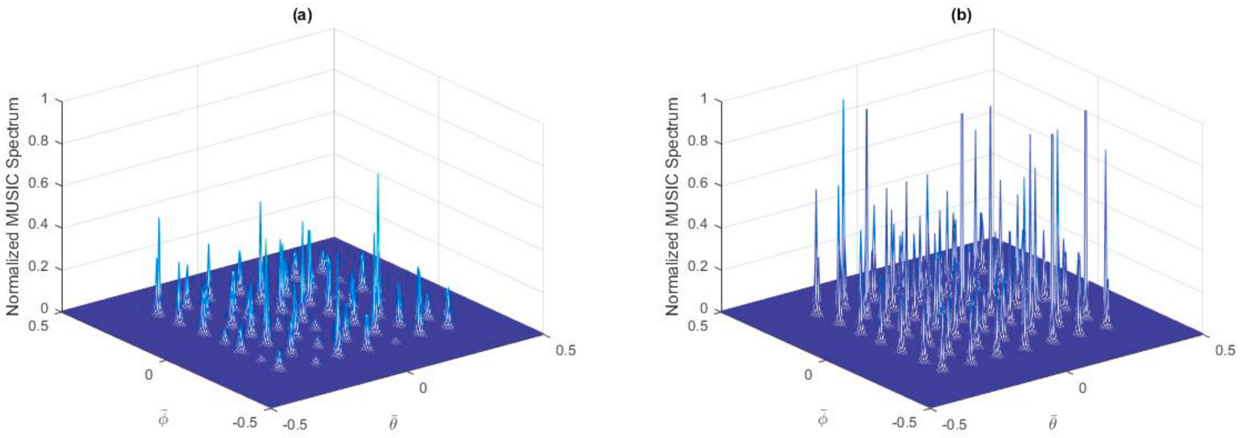


Fig. 3: 2D MUSIC spectrum based on a 2D nested array with 36 sensors, 0 dB SNR and Hermitian Toeplitz matrix $\tilde{R}$. 49 sources chosen uniformly over $(\bar{\theta}, \bar{\phi})\ \varepsilon\ [-0.3, 0.3]^2$; (a)Snapshots T=100. (b)Snapshots T=1000.

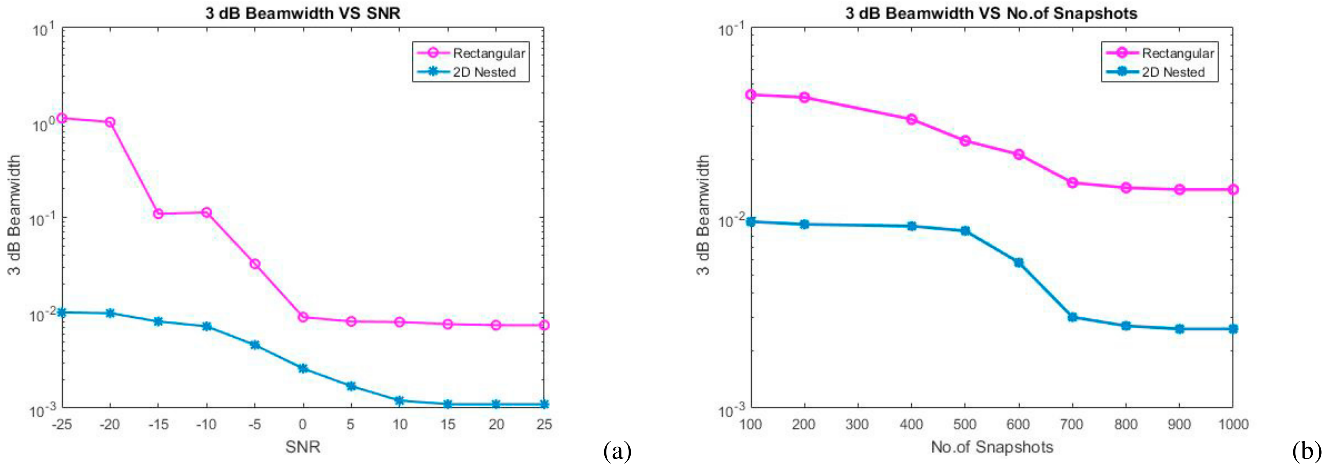
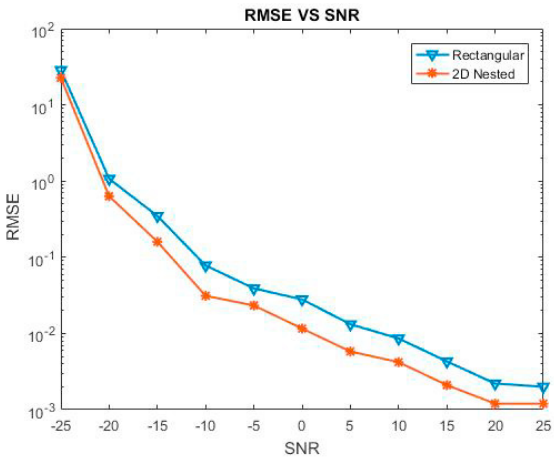


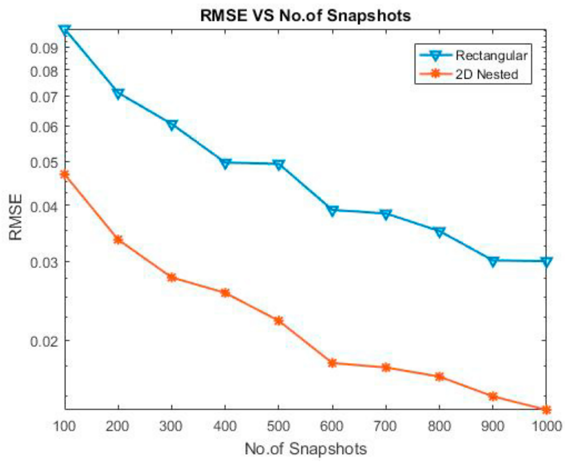
Fig. 4: Resolution (a) 3 dB beamwidth as a function of SNR (b) 3 dB beamwidth as a function of number of snapshots,T.

5.1. Resolution

3 dB beam width has been used to measure the resolution, as a function of SNR and number of snapshots, T. Resolution is inversely proportional to the 3 dB beam width. Fig. 4 (a) depicts the relationship between 3 dB beam width and SNR for URA and 2D nested array and Fig. 4 (b) depicts the relationship between 3 dB beam width and number of snapshots for URA and 2D nested array. Both URA and 2D nested array have 16 number of physical sensors. Default parameter setting is 0 dB SNR and number of snapshots T = 500. In Fig. 4 (a) 3 dB beamwidth of both arrays decreases from -25 dB to 10 dB and remains constant. For all SNR 2D nested array based on Hermitian Toeplitz matrix outperforms URA. In Fig. 4 (b), for both arrays, 3 dB beam width decreases with increase in the number of snapshots T and remains constant after 700 snapshots. For all the values of snapshots, 2D nested array based on Hermitian Toeplitz matrix gives better results.



(a)



(b)

Fig. 5: Accuracy (a) RMSE as a function of SNR (b) RMSE as a function of number of snapshots,T.

5.2. Accuracy

Root Mean Square Error (RMSE) has been used to calculate accuracy as a function of SNR and number of snapshots. For 2D arrays RMSE can be defined as, $RMSE = \sqrt{\frac{1}{P}\sum_{i=1}^{p}[((\bar{\theta}_{est})_i - (\bar{\theta}_{true})_i)^2 + ((\bar{\phi}_{est})_i - (\bar{\phi}_{true}))^2)]}$. Here $\bar{\theta}_{est}, \bar{\phi}_{est}$ are the normalized estimated angles and $\bar{\theta}_{true}, \bar{\phi}_{true}$ are the normalized true angles. RMSE is computed by averaging over 100 Monte Carlo simulations. Default parameter settings is 0 dB SNR, 500 snapshots, total number of physical sensors is 16 and number of uncorrelated sources P = 4. Fig. 5 (a) shows the dependence of the RMSE on SNR. It can be seen that RMSE is decreasing with increase in SNR for both arrays. For all SNR 2D nested array based on Hermitian Toeplitz matrix outperforms URA. Fig. 5 (b) shows the relation of RMSE and number of snapshots. As the number of snapshots increases there is continuous decrease in RMSE for both arrays. Here additionally for each snapshot, 2D nested array based on Hermitian Toeplitz matrix gives better result than URA.

6. Conclusion

2D coarray MUSIC can be obtained more directly from $\tilde{R}$ instead of spatially smoothed matrix $\hat{R}_s$. This method reduces the computational complexity associated with the existing approaches for 2D DoA estimation on 2D sparse arrays. This approach can be applied to any 2D sparse array that has uniform rectangular section in its difference coarray.

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