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Nuclear norm minimization framework for DOA estimation in MIMO radar

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ABSTRACT

In this paper, the direction of arrival (DOA) estimation for noncircular sources in multiple-input multiple-output (MIMO) radar is dealt with by a novel nuclear norm minimization (NNM) framework. The proposed method exploits the noncircular property of signals to extend the data model for doubling the array aperture. Then a block sparse model of the extended data is formulated without the influence of the unknown noncircularity phase, and a novel signal reconstruction algorithm based on nuclear norm minimization is proposed to recover the block-sparse matrix. In addition, a weight matrix based on the reduced dimensional noncircular Capon (RD NC-Capon) spectrum is designed to reweight the nuclear norm minimization for enhancing the sparsity of solution. Finally, the DOA is estimated from the non-zero blocks of the reconstructed matrix. Due to exploiting the extended array aperture and block-sparse information, the proposed method provides superior DOA estimation performance and higher angular resolution. Furthermore, the proposed method has a low sensitivity to the priori information on the number of sources. Simulation results are presented to verify the effectiveness and advantages of the proposed method.

1. Introduction

In array signal precessing, direction of arrival (DOA) estimation is the most fundamental issue for applications of wireless communications, sonar and radar [1]. Recently, a novel array processing configuration, named as multiple input multiple output (MIMO) radar [2], has opened new opportunities in parameter estimation especially for DOA estimation. Compared with the traditional phased-array radar system, MIMO radar has a lot of potential advantages, such as higher resolution, more degree of freedom and better parameter identifiability [3,4]. The MIMO radar configurations can be generally categorized into statistical MIMO radar [3] and colocated MIMO radar [4]. The statistical MIMO radar aims at obtaining spatial diversity gain by equipping with separated antennas in both transmit and receive arrays. The colocated MIMO radar makes use of closely located transmit and receive antennas to provide higher spatial resolution by forming a large aperture of a virtual array. In this paper, we focus on the DOA estimation problem of colocated MIMO radar.

Many DOA estimation algorithms have been proposed with MIMO radar based on subspace technique [5–9]. In [5,6], the multiple signal classification (MUSIC) algorithm and its variations are presented for DOA estimation. However, they need a heavy computational burden due to the requirement of extensive spatial searching. In order to avoid this problem, the estimation of signal parameters via rotational invariance technique (ESPRIT) based algorithms is proposed [7–9]. In addition, considering the special structure of the virtual array in colocated MIMO radar with uniform linear array, the reduced dimensional ESPRIT (RD-ESPRIT) [10] is proposed to estimate DOA. Because these subspace-based algorithms need to estimate the signal or noise subspace, they generally need some necessary conditions, such as the priori information on the number of sources, a sufficiently large number of snapshots and high signal-to-noise ratio (SNR), to obtain desired estimation performance. The sparse representation theory has recently attracted much attention on a new way of DOA estimation. Based on the sparse representation theory, the l_1-SVD for DOA estimation is proposed in [11], which uses the singular value decomposition (SVD) to reduce the dimension of observations and the sensitivity of the measurement noise. Then the sparse representation of the covariance matrix or vector based methods are proposed in [12,13]. On the other hand, several sparse representation-based methods have been proposed for DOA estimation in MIMO radar [14–18]. All the simulation results have shown that compared with the

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subspace-based methods, these sparse representation-based methods have lower sensitivity to the priori information on the number of sources and better angle estimation performance in the case of low SNR.

There exist many complex noncircular signals, such as BPSK, MSK, and UQPSK signals used in practical communication and radar systems [19]. However, DOA estimation algorithms mentioned above have not used the noncircular signal property. In [20–22], the noncircular subspace-based methods have been proposed to show, by both theoretical analysis and simulation results, that noncircular subspace-based methods provide higher angular resolution and better angle estimation performance than the conventional subspace-based methods. In [23], exploiting the noncircular signal property, a reweighted l_1-norm penalty based on NC-MUSIC spectrum is proposed for DOA estimation, which provides better performance than the conventional sparse representation-based methods in [14–18]. However, it exploits the noncircular property of signals by combining the information of the received data and its conjugation, which does not utilize the full aperture of the extended observations.

In this paper, a nuclear norm minimization (NNM) framework is proposed for DOA estimation by exploiting the full aperture of the extended observations. The proposed method firstly uses the noncircular property of signals to extend the data model for doubling the array aperture. Then a novel nuclear norm minimization framework is formulated for DOA estimation via establishing a block sparse model, and the spectrum of RD NC-Capon is used to formulate the weight matrix for reweighting the nuclear norm minimization. The proposed method can perform with the full aperture of the extended observations and use the block-sparse information to achieve higher angular resolution and better angle estimation performance than traditional sparse representation-based methods. Moreover, the proposed method is less sensitive to the priori information on the number of sources.

This paper is organized as follows. The data model and problem formation are introduced in Section 2. A nuclear norm minimization framework is presented in Section 3. Several related issues are discussed and the Cramer-Rao bound (CRB) of DOA estimation for noncircular signal is derived in Section 4. The simulation results are shown and analyzed in Section 5. The conclusion is drawn in Section 6.

Notation: $(\cdot)^{\mathrm{H}}$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{-1}$, $(\cdot)^{*}$, and $(\cdot)^{+}$ denote conjugate-transpose, transpose, inverse, conjugate, and pseudo-inverse, respectively. $\otimes$ and $\odot$denote the Kronecker product and Khatri-Rao product, respectively. $\mathbf{I}_K$ denotes a $K \times K$ dimensional unit matrix and $diag\{\cdot\}$ denotes the diagonal matrix. $\mathbf{A}^{(l_2)}$ denotes a column vector whose qth element equals to the l_2 norm of the qth row of $\mathbf{A}$. $\|\cdot\|_1$ and $\|\cdot\|_F$ denote the l_1 norm and Frobenius norm, respectively.

2. Data model and problem formulation

The signal model and sparse representation approach without considering the noncircularity of signals for DOA estimation are introduced in this section. The noncircular sparse representation framework for DOA estimation with noncircularity of signals is presented in Section 3.

Consider a narrowband colocated MIMO radar system, as shown in Fig. 1, equipped with M transmit antennas and N receive antennas that are located closely. A source located in the far-field can be seen at the same angle (i.e. direction of arrival (DOA)) with respect to the normals of transmit and receive arrays. It is assumed that the colocated MIMO radar uses uniform linear arrays (ULAs) for transmitting and receiving signals, and the antennas in these arrays are half-wavelength spaced. The MIMO radar uses M antennas to emit M orthogonal noncircular waveforms (BPSK, UQSK, or MSK modulation) that have identical bandwidth and central frequency. It is assumed that there exists P sources and the DOA of the pth source is denoted as $\theta_p (p = 1, 2, \ldots, P)$. Exploiting the orthogonality of the noncircular waveforms, the output of the matched filter at the receive array, i.e., the received data vector,

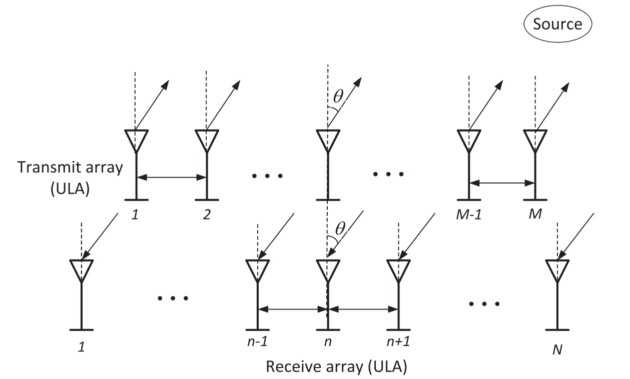

Fig. 1. The configuration of colocated MIMO radar.

$\mathbf{x}(t) \in \mathbb{C}^{MN\times 1}$, can be expressed as [20–22].

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}_c(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{A} = \mathbf{A}_t \odot \mathbf{A}_r \in \mathbb{C}^{MN\times P}$ is the transmit-receive steering matrix, $\mathbf{A}_t = [\mathbf{a}_t(\theta_1), \ldots, \mathbf{a}_t(\theta_P)] \in \mathbb{C}^{M\times P}$ is the transmit steering matrix composed with the transmit steering vector $\mathbf{a}_t(\theta_p) = [1, \exp(j\pi\sin\theta_p), \ldots, \exp(j\pi(M-1)\sin\theta_p)]^{\mathrm{T}}$, and $\mathbf{A}_r = [\mathbf{a}_r(\theta_1), \ldots, \mathbf{a}_r(\theta_P)] \in \mathbb{C}^{N\times P}$ is the receive steering matrix composed with the receive steering vector $\mathbf{a}_r(\theta_p) = [1, \exp(j\pi\sin\theta_p), \ldots, \exp(j\pi(N-1)\sin\theta_p)]^{\mathrm{T}}$, $\mathbf{n}(t) \in \mathbb{C}^{MN\times 1}$ is the Gaussian white noise vector with zero-mean and covariance matrix $\sigma^2\mathbf{I}_{MN}$, and $\mathbf{s}_c(t) \in \mathbb{C}^{P\times 1}$ is the noncircular signal vector, which satisfies with

$$\mathbf{s}_c(t) = \mathbf{\Phi}\mathbf{s}(t) \tag{2}$$

where $\mathbf{\Phi} = \mathrm{diag}\{[\exp(j\psi_1), \ldots, \exp(j\psi_P)]\} \in \mathbb{C}^{P\times P}$ is the noncircularity phase matrix, and $\mathbf{s}(t) \in \mathbb{R}^{P\times 1}$ the real part of complex noncircular signals. When the number of snapshots is L, the received data in Eq. (1) can be rewritten as

$$\mathbf{X} = \mathbf{A}\mathbf{S}_c + \mathbf{N} \tag{3}$$

where $\mathbf{X} = [\mathbf{x}(t_1), \ldots, \mathbf{x}(t_L)] \in \mathbb{C}^{MN\times L}$ is the received data matrix, $\mathbf{S}_c = [\mathbf{s}_c(t_1), \ldots, \mathbf{s}_c(t_L)] = \mathbf{\Phi}\mathbf{S} \in \mathbb{C}^{P\times L}$ is the noncircular signal matrix with $\mathbf{S} = [\mathbf{s}(t_1), \ldots, \mathbf{s}(t_L)] \in \mathbb{R}^{P\times L}$, and $\mathbf{N} = [\mathbf{n}(t_1), \ldots, \mathbf{n}(t_L)] \in \mathbb{C}^{MN\times L}$ is the additive Gaussian white noise matrix.

2.1. Sparse representation based DOA estimation

Considering the configuration of MIMO radar shown in Fig. 1, there exist some redundant elements in the virtual array of MIMO radar, which can be shown as

$$\mathbf{a}_t(\theta) \otimes \mathbf{a}_r(\theta) = \mathbf{G}\mathbf{b}(\theta) \tag{4}$$

where $\mathbf{G}$ and $\mathbf{b}(\theta)$ can be written as

$$\mathbf{G} = [\mathbf{J}_0^{\mathrm{T}}, \mathbf{J}_1^{\mathrm{T}}, \ldots, \mathbf{J}_{M-1}^{\mathrm{T}}]^{\mathrm{T}} \tag{5}$$

$$\mathbf{J}_m = [\mathbf{0}_{N\times m}, \mathbf{I}_N, \mathbf{0}_{N\times(M-m-1)}] \quad m = 0, 1, \ldots, M-1 \tag{6}$$

$$\mathbf{b}(\theta) = [1, \exp(j\pi\sin\theta_p), \ldots, \exp(j\pi(M+N-2)\sin\theta_p)]^{\mathrm{T}}. \tag{7}$$

According to Eq. (4), the number of the effective elements of MIMO radar is $M+N-1$. The reduced dimension transformation technique can be applied to eliminate the redundant elements, and the reduced dimension transformation matrix can be designed as $\mathbf{D} = (\mathbf{G}^{\mathrm{H}}\mathbf{G})^{-(1/2)}\mathbf{G}^{\mathrm{H}} \in \mathbb{C}^{(M+N-1)\times MN}$, which avoids the additional spatial colored noise [10]. Then multiplying $\mathbf{D}$ with the received data matrix $\mathbf{X}$, we have

$$\mathbf{Y} = \mathbf{D}\mathbf{X} = \mathbf{F}^{(1/2)}\mathbf{B}\mathbf{S}_c + \mathbf{D}\mathbf{N} = \mathbf{D}\mathbf{X} = \overline{\mathbf{B}}\mathbf{S}_c + \overline{\mathbf{N}} \tag{8}$$

where $\mathbf{F} = \mathrm{diag}\{[1, 2, \ldots, \underbrace{\min(M,N), \ldots, \min(M,N)}_{|M-N|+1}, \ldots, 2, 1]\}$, $\mathbf{B} = [\mathbf{b}(\theta_1), \ldots, \mathbf{b}(\theta_P)]$ is the new steering matrix after reduced dimension transformation,

$\overline{\mathbf{B}} = \mathbf{F}^{(1/2)}\mathbf{B}$ and $\overline{\mathbf{N}} = \mathbf{DX}$. According to Eq. (8), the data matrix $\mathbf{Y} \in \mathbb{C}^{(M+N-1)\times L}$ corresponds to a virtual array with a weight matrix $\mathbf{F}^{(1/2)}$. By exploiting the sparse representation perspective, the complete dictionary $\boldsymbol{\Omega}_{\overline{\theta}}$ can be constructed as $\overline{\mathbf{B}}_{\overline{\theta}} = \mathbf{F}^{(1/2)}\mathbf{B}_{\overline{\theta}}$, where $\mathbf{B}_{\overline{\theta}} = [\mathbf{b}(\overline{\theta}_1), \ldots, \mathbf{b}(\overline{\theta}_K)]$ and $\{\overline{\theta}_l\}_{l=1}^{K}(K \gg P)$ is a discretized spatial sampling grid of all DOAs of interest. Then the observations in Eq. (8) can be turned into sparse representation model, which is shown as

$$\mathbf{Y} = \overline{\mathbf{B}}_{\overline{\theta}}\mathbf{S}_c^{\overline{\theta}} + \overline{\mathbf{N}} \tag{9}$$

where $\mathbf{S}_c^{\overline{\theta}}$ and $\mathbf{S}_c$ have the same row support, which means that $\mathbf{S}_c^{\overline{\theta}}$ is a P-rows sparse matrix. The nonzero rows of $\mathbf{S}_c^{\overline{\theta}}$ correspond to DOAs in the complete dictionary. Thus, the DOA estimation problem can be converted into finding the position of the nonzero rows of sparse matrix $\mathbf{S}_c^{\overline{\theta}}$. The straightforward recovery of the sparse matrix is by the NP-hard optimization problem based on l_0 norm penalty. The l_1-norm penalty, instead of l_0 norm penalty, can be adopted to solve the convex optimization problem expressed as

$$\min \|\mathbf{S}_c^{\widehat{\theta}\,(l_2)}\|_1, \quad \text{s. t. } \|\mathbf{Y} - \overline{\mathbf{B}}_{\overline{\theta}}\mathbf{S}_c^{\overline{\theta}}\|_F \le \eta \tag{10}$$

where η is the a regularization parameter. After obtaining the sparse matrix $\mathbf{S}_c^{\overline{\theta}}$, the DOA can be estimated by finding the position of nonzero rows of $\mathbf{S}_c^{\overline{\theta}}$. The sparse representation approach in Eq. (10) does not consider the noncircularity of signals. On the other hand, the l_1-norm penalty is a proxy of l_0-norm, and the l_1-norm penalizes larger coefficients more heavily than smaller coefficients, unlike the more democratic penalization of the l_0-norm. Thus, the sparse solution of l_1-norm penalty does not approximate to l_0-norm penalty.

3. Nuclear norm minimization algorithm for DOA estimation

In this section, we propose a nuclear norm minimization framework for DOA estimation by exploiting the full aperture of the extended observations, which can enhance the sparsity of the solution and improve the angle estimation performance. In order to utilize the noncircular characteristic of incident signals, an extended received data vector $\mathbf{Z}$ is given by

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Y} \\ \mathbf{Y}^* \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{B}}\mathbf{S}_c \\ \overline{\mathbf{B}}^*\mathbf{S}_c^* \end{bmatrix} + \begin{bmatrix} \overline{\mathbf{N}} \\ \overline{\mathbf{N}}^* \end{bmatrix}. \tag{11}$$

Based on the noncircular characteristic of signals, i.e., $\mathbf{S}_c = \boldsymbol{\Phi}\mathbf{S}$ and $\mathbf{S}_c^* = \boldsymbol{\Phi}^*\mathbf{S}$, Eq. (11) can be written as

$$\mathbf{Z} = \begin{bmatrix} \overline{\mathbf{B}}\boldsymbol{\Phi} \\ \overline{\mathbf{B}}^*\boldsymbol{\Phi}^* \end{bmatrix}\mathbf{S} + \begin{bmatrix} \overline{\mathbf{N}} \\ \overline{\mathbf{N}}^* \end{bmatrix} = \mathbf{B}_E\mathbf{S} + \mathbf{N}_E \tag{12}$$

where $\mathbf{N}_E$ is the extended noise matrix, and $\mathbf{B}_E = [\mathbf{b}_e(\psi_1, \theta_1), \ldots, \mathbf{b}_e(\psi_1, \theta_1)]$ is the new steering matrix with the steering vector shown as

$$\mathbf{b}_e(\psi, \theta) = [\mathbf{F}^{-1/2}\mathbf{b}(\theta)e^{j\psi})^{\mathrm{T}}, (\mathbf{F}^{-1/2}\mathbf{b}^*(\theta)e^{-j\psi})^{\mathrm{T}}]^{\mathrm{T}}. \tag{13}$$

As shown in Eq. (13), exploiting the noncircular characteristic of signals doubles the aperture of virtual array and can improve the estimation performance. In order to reduce the dimension of the extended data and the sensitivity of the noise, the SVD technique is applied to the extended data $\mathbf{Z}$ [11], which is expressed as

$$\mathbf{Z} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{H}} \tag{14}$$

where $\mathbf{U} \in \mathbb{C}^{2(M+N-1)\times 2(M+N-1)}$ and $\mathbf{V}^{\mathrm{H}} \in \mathbb{C}^{L\times L}$ are composed with left and right singular vectors, and $\boldsymbol{\Sigma} \in \mathbb{C}^{2(M+N-1)\times L}$ is a diagonal matrix whose diagonal elements are $2(M+N-1)$ singular values. Let $\mathbf{V}_s^{\mathbf{H}} \in \mathbb{C}^{P\times L}$ be one part of $\mathbf{V}^{\mathrm{H}}$ corresponding to the P largest singular values. Multiplying the received signal $\mathbf{Z}$ by $\mathbf{V}_s$, we have

$$\mathbf{Z}_{SV} = \mathbf{Z}\mathbf{V}_s = \mathbf{B}_E\mathbf{S}_{SV} + \mathbf{N}_{SV} \tag{15}$$

where $\mathbf{S}_{SV} = \mathbf{S}\mathbf{V}_s$ and $\mathbf{N}_{SV} = \mathbf{N}_E\mathbf{V}_s$. Based on sparse representation viewpoint, the complete dictionary must be constructed according to the structure of the steering matrix $\mathbf{B}_E$ before obtaining the sparse representation model of Eq. (15). Due to the unknown noncircularity phase and its conjugation in the steering matrix, the complete dictionary can not be established directly. In [23], the reweighted l_1-norm penalty based algorithm uses the noncircular characteristic of signals via combining the received data in Eq. (8) and its conjugation. However, it abandons the full aperture corresponding to the extended steering matrix $\mathbf{B}_E$. Noting that the special structure of the steering vector, it can be written as

$$\mathbf{b}_e(\psi, \theta) = \Xi(\theta)\boldsymbol{\phi}(\psi) \tag{16}$$

where

$$\Xi(\theta) = \begin{bmatrix} \mathbf{F}^{-1/2}\mathbf{b}(\theta) & \mathbf{0}_{(M+N-1)\times 1} \\ \mathbf{0}_{(M+N-1)\times 1} & \mathbf{F}^{-1/2}\mathbf{b}^*(\theta) \end{bmatrix} \tag{17}$$

and

$$\boldsymbol{\phi}(\psi) = \begin{bmatrix} e^{j\psi} \\ e^{-j\psi} \end{bmatrix}. \tag{18}$$

Then the steering matrix $\mathbf{B}_E$ can be expressed as

$$\mathbf{B}_E = \boldsymbol{\Xi}\boldsymbol{\Psi} \tag{19}$$

where $\boldsymbol{\Xi} = [\Xi(\theta_1), \ldots, \Xi(\theta_P)] \in \mathbb{C}^{(M+N-1)\times 2P}$, and $\boldsymbol{\Psi} = \text{blkdiag}\{[\boldsymbol{\phi}(\psi_1), \ldots, \boldsymbol{\phi}(\psi_P)]\} \in \mathbb{C}^{2P\times P}$ is the block-diagonal structure. Substituting Eq. (19) into Eq. (15), we achieve

$$\mathbf{Z}_{SV} = \boldsymbol{\Xi}\boldsymbol{\Psi}\mathbf{S}_{SV} + \mathbf{N}_{SV} = \boldsymbol{\Xi}\mathbf{S}_\psi + \mathbf{N}_{SV} \tag{20}$$

where $\mathbf{S}_\psi = \boldsymbol{\Psi}\mathbf{S}_{SV} = [\mathbf{S}_{\psi_1}^{\mathrm{T}}, \ldots, \mathbf{S}_{\psi_P}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{2P\times P}$ is a block structure, which is partitioned into P matrices $\mathbf{S}_{\psi_p} = \boldsymbol{\phi}(\psi_p)\mathbf{s}_p \in \mathbb{C}^{2\times P}$, $p = 1, 2, \ldots, P$, and $\mathbf{s}_p$ is the pth row of $\mathbf{S}_{SV}$. In addition, due to the complex conjugate structure of the noncircularity phase vector $\boldsymbol{\phi}(\psi_p)$, each sub-matrix is rank-one. According to Eq. (19), the matrix $\boldsymbol{\Xi}$ avoids the unknown noncircularity phase, which can be used to formulate the dictionary for the sparse representation of Eq. (20). Based on the discretized spatial sampling grid $\{\overline{\theta}_l\}_{l=1}^{K}(K \gg P)$, the dictionary can be constructed as $\boldsymbol{\Xi}_{\overline{\theta}} = [\Xi(\overline{\theta}_1), \ldots, \Xi(\overline{\theta}_K)] \in \mathbb{C}^{2(M+N-1)\times 2K}$. Then the Eq. (20) can be transformed into a sparse representation model as

$$\mathbf{Z}_{SV} = \boldsymbol{\Xi}_{\overline{\theta}}\widetilde{\mathbf{S}}_\psi + \mathbf{N}_{SV} \tag{21}$$

where $\widetilde{\mathbf{S}}_\psi = [\widetilde{\mathbf{S}}_{\psi_1}^{\mathrm{T}}, \ldots, \widetilde{\mathbf{S}}_{\psi_K}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{2K\times P}$ is a block-sparse signal matrix, and each submatrix satisfies

$$\widetilde{\mathbf{S}}_{\psi_k} = \begin{cases} \mathbf{S}_{\psi_p} & \text{if } \psi_k = \psi_p \\ 0 & \text{else} \end{cases}. \tag{22}$$

It should be highlighted that the matrix $\widetilde{\mathbf{S}}_\psi$ contains two different parts of sparsity: the first part is that $\widetilde{\mathbf{S}}_\psi$ is block-sparse, and the second part is that the blocks $\widetilde{\mathbf{S}}_{\psi_k}$ are rank sparse, i.e., they are either of rank one or rank zeros. In order to exploit twofold sparsity structure for recovering the block-sparse matrix $\widetilde{\mathbf{S}}_\psi$, Eq. (21) can be turned into a matrix rank minimization problem, which is expressed as

$$\begin{aligned} &\min_{\widetilde{\mathbf{S}}_\psi} \sum_{k=1}^{K} \text{Rank}(\widetilde{\mathbf{S}}_{\psi_k}) \\ &\text{s. t.} \quad \|\mathbf{Z}_{SV} - \boldsymbol{\Xi}_{\overline{\theta}}\widetilde{\mathbf{S}}_\psi\|_F \le \beta \end{aligned} \tag{23}$$

where β is the regularization parameters. It is well known that in general the rank minimization problem is non-convex and hard to solve. To make the problem practicable, the convex nuclear norm minimization is used instead of rank minimization, and it has been proved in [24] that the nuclear norm minimization problem leads to a near optimal solution of matrix rank minimization problem. Thus, Eq. (23) can be formulated as the convex nuclear norm minimization problem

$$\min_{\widetilde{\mathbf{S}}_\psi} \sum_{k=1}^{K} \|\widetilde{\mathbf{S}}_{\psi_k}\|_* \text{ s. t. } \|\mathbf{Z}_{SV} - \boldsymbol{\Xi}_{\overline{\theta}}\widetilde{\mathbf{S}}_\psi\|_F \le \beta \tag{24}$$

where the nuclear norm in Eq. (24) is defined by

$$\| \tilde{\mathbf{S}}_{\psi_k} \|_* = \sum_{\ell=1}^{\min(2,P)} \rho_\ell(\tilde{\mathbf{S}}_{\psi_k}) \tag{25}$$

where $\rho_\ell(\tilde{\mathbf{S}}_{\psi_k})$ denotes the ℓth singular value of the submatrix $\tilde{\mathbf{S}}_{\psi_k}$. Since the problem is convex, it can be solved by semidefinite programming [25]. After achieving the block-sparse matrix $\tilde{\mathbf{S}}_\psi$, the DOAs can be estimated from the position of nonzero blocks of $\tilde{\mathbf{S}}_\psi$ or the nonzero $\| \tilde{\mathbf{S}}_{\psi_k} \|_*$. Note that $\sum_{k=1}^{K} \| \tilde{\mathbf{S}}_{\psi_k} \|_*$ can be seen as the l_1-norm of $\| \tilde{\mathbf{S}}_{\psi_k} \|_*$, which has the drawback of the l_1-norm penalty, i.e., the sparse solution of l_1-norm penalty does not approximate to l_0-norm penalty. In order to obtain more sparse solution for improving the performance, a novel RD NC Capon spectrum is formulated to design the weight matrix for reweighting the l_1-norm penalty. According to the extended data in Eq. (12), the two dimensional NC-Capon spectrum function can be constructed as

$$P_{NC-Capon}(\psi, \theta) = \mathbf{b}_e^{\mathrm{H}}(\psi, \theta)\mathbf{R}^{-1}\mathbf{b}_e(\psi, \theta) \tag{26}$$

where $\mathbf{R} = \mathrm{E}(\mathbf{Z}\mathbf{Z}^{\mathbf{H}})$ is the covariance matrix. The heavy computational burden is needed due to two dimensional spatial searching. According to Eq. (16), the NC-Capon spectrum in Eq. (26) can be rewritten as

$$P_{NC-Capon}(\psi, \theta) = \begin{bmatrix} e^{j\psi} \\ e^{-j\psi} \end{bmatrix}^{\mathrm{H}} \mathbf{\Omega}(\theta) \begin{bmatrix} e^{j\psi} \\ e^{-j\psi} \end{bmatrix} \tag{27}$$

and

$$\mathbf{\Omega}(\theta) = \mathbf{\Xi}^{\mathrm{H}}(\theta)\mathbf{R}^{-1}\mathbf{\Xi}(\theta) \tag{28}$$

Obviously, Eq. (27) can be rewritten as

$$P_{NC-Capon}(\psi, \theta) = \begin{bmatrix} 1 \\ e^{-j2\psi} \end{bmatrix}^{\mathrm{H}} \mathbf{\Omega}(\theta) \begin{bmatrix} 1 \\ e^{-2j\psi} \end{bmatrix}. \tag{29}$$

Eq. (29) can be regarded as a quadratic optimization. Then inspired by [26], the RD NC-Capon spectrum function can be obtained by solving the optimization problem shown as follows

$$\min_{\psi,\theta} \mathbf{g}^{\mathrm{H}}(\psi)\mathbf{\Omega}(\theta)\mathbf{g}(\psi) \text{ s. t. } \quad \mathbf{e}^{\mathrm{H}}\mathbf{g}(\psi) = 1 \tag{30}$$

where $\mathbf{g}(\psi) = [1, e^{-j2\psi}]^{\mathrm{T}}$ and $\mathbf{e} = [1, 0]^{\mathrm{T}}$. Using the Lagrange-multiplier method to solve Eq. (30), the RD NC-Capon spectrum function can be obtained as

$$P_{NC-Capon}(\theta) = \mathbf{e}^{\mathrm{H}}(\mathbf{\Omega}(\theta))^{-1}\mathbf{e}. \tag{31}$$

The one-dimensional NC capon spectrum in Eq. (31) is used to formulate the weight matrix. According to the discretized spatial sampling grid $\{\bar{\theta}_l\}_{l=1}^{K}(K \gg P)$, the weight vector $\mathbf{\Upsilon} = [\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_K]$ can be written as

$$\Upsilon_k = \frac{1}{\mathbf{e}^{\mathrm{H}}(\mathbf{\Omega}(\bar{\theta}_k))^{-1}\mathbf{e}}, \quad k = 1, 2, \ldots, K. \tag{32}$$

Then the weight matrix can be designed as

$$\mathbf{W} = \mathrm{diag}\{\mathbf{\Upsilon}/\max(\mathbf{\Upsilon})\}. \tag{33}$$

According to the characteristic of RD NC-Capon function, the elements $\mathbf{W}(i, i) = \Upsilon_i/\max(\mathbf{\Upsilon}) \to 0 (i = 1, 2, \ldots, P)$ corresponding to the possible sources are much smaller than other elements $\mathbf{W}(i, i) = \Upsilon_i/\max(\mathbf{\Upsilon}) (i = 1, 2, \ldots, L - P)$. Thus, the weight matrix $\mathbf{W}$ for reweighting the l_1-norm penalty can achieve the same idea in [15,23,27]. Then the reweighted nuclear norm minimization problem for recovering the block-sparse matrix $\tilde{\mathbf{S}}_\psi$ is formulated as

$$\min_{\tilde{\mathbf{S}}_\psi} \sum_{k=1}^{K} \bar{\Upsilon}_k \| \tilde{\mathbf{S}}_{\psi_k} \|_* \text{ s. t. } \| \mathbf{Z}_{SV} - \mathbf{\Xi}_{\bar{\theta}}\tilde{\mathbf{S}}_\psi \|_F \le \beta \tag{34}$$

where $\bar{\Upsilon}_k$ is the kth diagonal element of $\mathbf{W}$. Eq. (34) can be solved by semidefinite programming [25]. Finally, the DOA can be estimated by plotting the sparse vector $\tilde{\mathbf{V}} = [\| \tilde{\mathbf{S}}_{\psi_1} \|_*, \ldots, \tilde{\mathbf{S}}_{\psi_K} \|_*]$.

4. Related remarks and Cramer-Rao bound

Remark 1. In Eq. (34), the regularization parameter β sets the error impact and plays important role in the final DOA estimation performance. According to [11], the selection of regularization parameter β depends on the probability distribution of the noise matrix. In the proposed framework, it is noticed the noise matrix $\|\mathbf{N}_{SV}\|_F^2$ is asymptotically chi-square distributed with $2(M + N - 1)P$ degrees of freedom. Thus, the regularization parameters β can be chosen as the upper value of $\|\mathbf{N}_{SV}\|_F^2$ with a high probability $1 - \xi$, respectively, where $\xi = 0.01$ is generally enough.

Remark 2. In the proposed framework, the noncircularity of signals is used to extend the virtual array aperture. The block-sparse information and reweighted procedure based the spectrum of RD NC-Capon can enhance the sparsity of the solution. Consequently the angle estimation performance can be improved. In addition, the weight matrix $\mathbf{W}$ is designed by using the methodology of RD NC-Capon in Eq. (33), which does not rely on the prior information on the number of sources. Thus, the proposed framework has a low sensitivity to the priori information on the number of sources.

Cramér-Rao Bound (*CRB*): Based on the received data of noncircular signals in MIMO radar, we derive the CRB of DOA estimation for noncircular signals as follows. Exploiting the property of noncircular signals, the received data in Eq. (3) can be extended as

$$\bar{\mathbf{Y}} = \begin{bmatrix} \mathbf{X} \\ \mathbf{X}^* \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{s}_c(t) \\ \mathbf{A}^*\mathbf{s}_c(t)^* \end{bmatrix} + \begin{bmatrix} \mathbf{N} \\ \mathbf{N}^* \end{bmatrix} = \bar{\mathbf{A}}\mathbf{S} + \underline{\mathbf{N}} \tag{35}$$

where the extended steering matrix $\bar{\mathbf{A}}$ and noise matrix $\underline{\mathbf{N}}$ are

$$\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A}\mathbf{\Phi} \\ \mathbf{A}^*\mathbf{\Phi}^* \end{bmatrix} \quad \underline{\mathbf{N}} = \begin{bmatrix} \mathbf{N} \\ \mathbf{N}^* \end{bmatrix} \tag{36}$$

According to [28], the CRB of DOA estimation based on the extended data in Eq. (35) can be derived as

$$\mathrm{CRB} = \frac{\delta^2}{2J}\{\mathrm{Re}(\mathbf{D}^{\mathrm{H}}\Pi_{\bar{\mathbf{A}}}^{\perp}\mathbf{D} \odot \mathbf{P}_s\}^{-1} \tag{37}$$

where δ^2 is the power of the noise, $\Pi_{\bar{\mathbf{A}}}^{\perp} = \mathbf{I}_{2MN} - \bar{\mathbf{A}}(\bar{\mathbf{A}}^{\mathrm{H}}\bar{\mathbf{A}})^{-1}\bar{\mathbf{A}}^{\mathrm{H}}$, $\mathbf{P}_s = \mathrm{E}[\mathbf{S}\mathbf{S}^{\mathbf{H}}]$, and $\mathbf{D} = [\partial\bar{\mathbf{a}}_1(\theta)/\partial(\theta), \ldots, \partial\bar{\mathbf{a}}_P(\theta)/\partial(\theta)]$ is the matrix whose pth column is given by the derivative of $\bar{\mathbf{a}}_p(\theta)$ with respect to θ_p, and $\bar{\mathbf{a}}_p(\theta)$ denotes the pth column of $\bar{\mathbf{A}}$. According to the expression of the CRB, it is obvious that the parameters, such as the number of snapshots J, the input signal-to-noise ratio (SNR) and DOAs, can effect the CRB. The CRB will become lower when the SNR or/and J increases. In addition, it is well known that the CRB will become lower when the targets are far from each other.

5. Simulation results

In this section, we consider several simulation scenarios to verify the validity of the proposed algorithm. The proposed method is compared with the l_1-SVD algorithm [11], the reweighted l_1-SVD algorithm in [15] (denoted as RW l_1-SVD), the reweighted l_1-norm minimization algorithm considering noncircularity of signals in [23] (denoted as the method in [23]) and CRB in Eq. (37). The root mean square error (RMSE) of DOA estimation is defined as

$$\mathrm{RMSE} = (1/P)\sum_{p=1}^{P}\sqrt{(1/Q)\sum_{i=1}^{Q}(\vartheta_{p,i} - \theta_p)^2} \tag{38}$$

where $\vartheta_{p,i}$ is the estimation of DOA θ_p for the ith Monte Carlo trial, and Q is the number of the Monte Carlo trials.

We consider a narrowband colocated radar system with M transmit antennas and N receive antennas. The transmit and receive arrays are half-wavelength spaced UAs and located closely. The transmit antennas emit M orthogonal noncircular waveforms

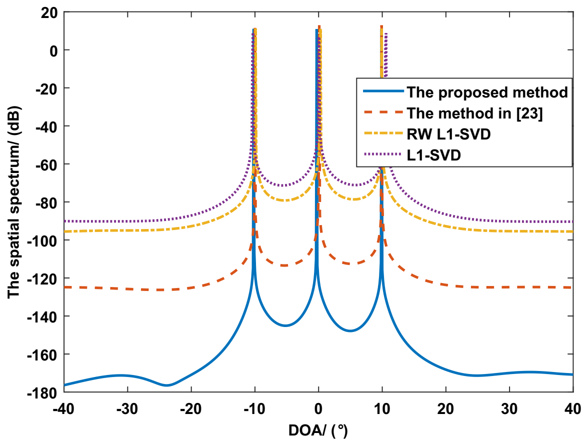

Fig. 2. The spatial spectra for different methods (P=3, M=5, N=6, SNR = 0 dB and L = 100).

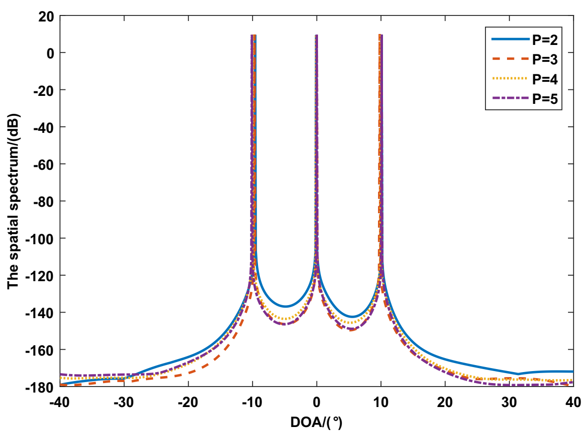

Fig. 3. The sensitivity of the proposed method to the priori information on the number of sources (M = 5, N = 6, SNR = 0 dB and L = 100).

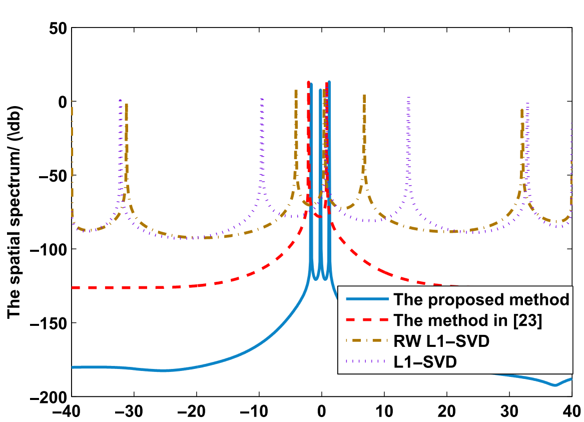

Fig. 4. The spatial spectra of closely located sources for different methods (P = 3, M = 4, N = 4, SNR = 0 dB and L = 100).

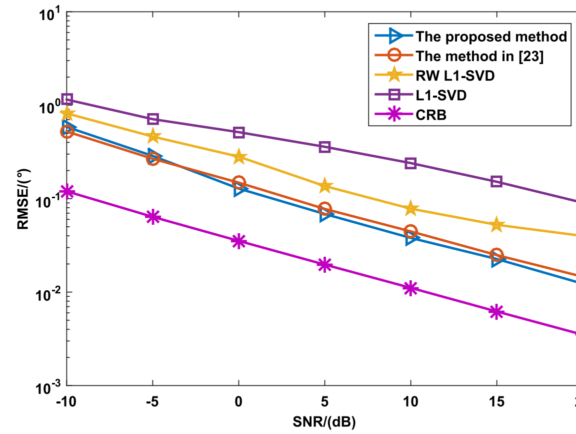

Fig. 5. The RMSE versus SNRs (P = 3, M = 5, N = 6 and L = 100).

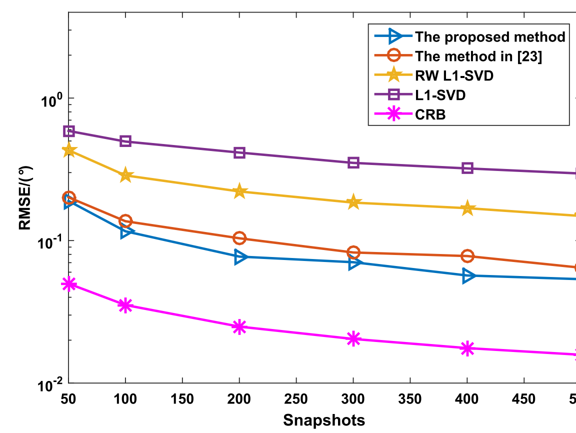

Fig. 6. The RMSE versus snapshots (P = 3, M = 5, N = 6 and SNR = 0 dB).

(BPSK, UQPSK or MSK modulation). Unless otherwise stated in the following simulation scenarios, it is assumed that the number of far-field sources is known as P = 3, and DOAs of uncorrelated three sources are $\theta_1 = -10°$, $\theta_2 = 0°$ and $\theta_3 = 10°$, respectively. The SNR is defined as SNR = $10\log_{10}(\| \mathbf{AS}_c \|_F^2 / \| \mathbf{N} \|_F^2)$. The discretized sampling grids are uniform with 0.01° from −90° to 90° for all methods, and Q = 200 is used for the Monte Carlo trials.

Fig. 2 shows the spatial spectra of all methods. As seen in Fig. 2, the proposed method has lower sidelobes and sharper peaks than l_1-SVD, RW l_1-SVD and the method in [23], which indicates that the proposed method provides higher angular resolution.

Fig. 3 shows the sensitivity of the proposed method to the priori information on the number of sources. It is seen that the proposed method can estimate DOA with the incorrect number of sources, which means that the proposed method has a low sensitivity to the priori information on the number of sources. This is because of designing the weight matrix without requiring the information on the number of sources and the inherent characteristic of sparse representation framework.

Fig. 4 shows the spatial spectra of closely located sources for all methods, where there are P = 3 closely located sources with $\theta_1 = -3°$, $\theta_2 = 0°$ and $\theta_3 = 3°$, respectively. From Fig. 4, it can be seen that the proposed method can resolve three closely located sources, while all the other methods fail to work correctly in this case. It is indicated that the proposed method has higher resolution than the other methods, and can be suitable to estimate the DOA of closely located sources.

Fig. 5 shows the RMSE versus SNRs obtained with different methods, As seen in Fig. 5, the method in [23] can provide superior angle estimation performance than the l_1-SVD and RW l_1-SVD algorithms. The main reason is that the method in [23] uses the noncircularity of signals and reweighted l_1-norm penalty to enhance the sparsity of the solution. On the other hand, the proposed method has better performance than the method in [23] when SNR $\geq$ 0 dB due to exploiting the full aperture of the extended data. However, the performance of the proposed method does not outperform the method in [23] when SNR < 0 dB. The main reason is that due to the limited resolution of RD NC-Capon method when SNR < 0 dB, the weight matrix based on the spectrum of RD NC-Capon can not be effectively used to enhance the sparsity of the solution. Thus, the proposed method has some performance degradation when SNR < 0 dB.

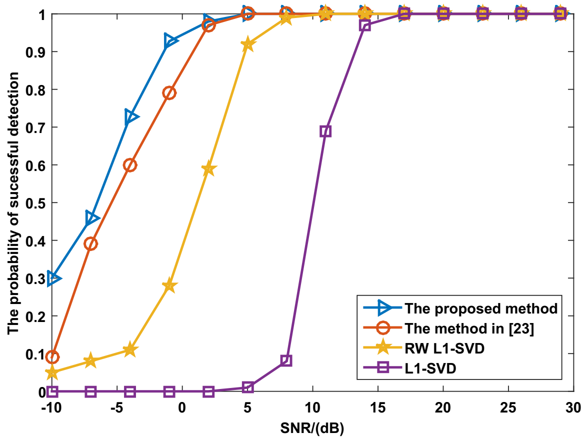

Fig. 7. The probability of successful detection versus SNRs (P = 3, M = 5, N = 6 and L = 100).

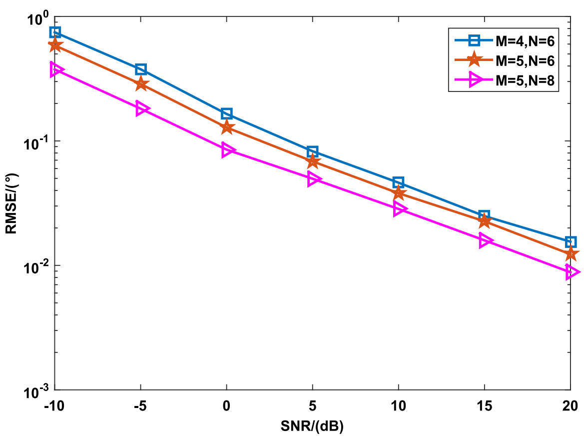

Fig. 8. The RMSE of the proposed method versus different number of elements (P = 3 and L = 100).

Fig. 6 shows the RMSE versus the number of snapshots obtained with different methods. It is seen that the angle estimation performance of all methods can be improved with the increased number of snapshots. On the other hand, the proposed method has better performance than l_1-SVD, RW l_1-SVD and the method in [23] due to exploiting the extended array aperture and block-sparse information.

Fig. 7 shows the probability of successful detection versus SNRs obtained with different methods. All sources can be seen as successful detection when the absolute estimation errors of the DOAs for all sources are within 0.3°. Fig. 7 indicates that all sources can be detected successfully when the SNR is high enough, i.e., all methods have a 100% successful detection. In addition, the proposed method has better probability of successful detection than l_1-SVD, RW l_1-SVD and the method in [23] in low SNR region, i.e., the proposed method has superior resolution than these algorithms.

Fig. 8 shows that the RMSE of the proposed method versus different number of elements. It is observed that the angle estimation performance of the proposed method becomes better with the increased number of transmit elements or/and receive elements. In general, the more elements MIMO radar has, the more diverse gain can be obtained.

6. Conclusion

In this paper, we have proposed a nuclear norm minimization framework for DOA estimation in MIMO radar. The proposed method firstly doubles the array aperture by exploiting the noncircular property of signals. Then a block-sparse model of the extended observations is formulated without the influence of the unknown noncircularity phase, and a novel nuclear norm minimization framework is proposed for DOA estimation. The proposed method can perform with full aperture of the extended observations and utilize the block-sparse information. Thus, the proposed method provides better angle estimation performance and higher resolution than traditional sparse representation based algorithms. In addition, the proposed method has a low sensitivity to the priori information on the number of sources. Simulation results have verified the performances of the proposed method.

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