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Efficient DSPE algorithm for estimating the angular parameters of coherently distributed sources

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Abstract

In this paper, we consider the problem of estimating the direction-of-arrivals (DOAs) and angular spreads of uncorrelated coherently distributed sources. The proposed method enables a decoupled estimation of the DOAs from that of the angular spreads of sources with small angular spread. Therefore, compared with the original DSPE algorithm, the proposed algorithm improves the robustness to the mismodeling of the spatial distribution of sources at least for two popular distribution shapes (Gaussian and uniform). Furthermore, the proposed method works even in the case where the different sources do not have same angular distribution shape.

Keywords: Array signal processing; DOA and angular spread estimation; Coherently distributed sources; DSPE algorithm

1. Introduction

Traditional direction-finding techniques have generally been developed for far-field point sources which travel along a single path to the antenna array. However, in applications such as wireless communications, radar and sonar, the effect of angular spread can no longer be ignored, due to multipath phenomena, a distributed source model will be more appropriate [1]. Depending on the environment of the mobile, the base station-mobile distance and the base station height, angular spreads up to 10° can be observed in practice [2]. On the other hand, depending on the relationship between the channel coherency time and the observation period, the scattered sources can be viewed either as coherently distributed (CD) or incoherently distributed (ID). More precisely, for a CD source, the signal components arriving from different directions are replicas of the same signal [3]. Whereas for an ID source, the channel coherency time being much smaller than the observation period (or the channel coherency time and the sample period are of the same order of values), in this case, all signals coming from different directions are assumed to be uncorrelated [3].

In ID sources case, the rank of the noise-free covariance matrix increases as the angular spread increases. Therefore, most classical techniques based on the point source assumption cannot be easily generalized to this situation. For example, the use of subspace based methods is not convenient practically because the choice of the effective dimension of the pseudo-signal subspace depends on the unknown parameters [5]. Due to the

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difficulty of the subspace based methods in ID sources case, an interesting alternative consists on the use of the beamforming methods. In [6], a generalization of Capon method [7] for estimating multiple ID sources has been proposed. However, this algorithm assumes that the multiple sources must have identical and known angular distribution. Another interesting way consists of using the covariance fitting methods. For example, the so-called COMET-EXIP [8] computes the DOA and angular spread of a single ID source by using two successive one-dimensional (1D) steps. However, this technique suffers from an ambiguity problem that limits its utilization in practice. This ambiguity problem has been identified and solved in [9].

On the other hand, in CD source case, the rank of the noise-free covariance matrix is equal to the number of sources. Consequently, most classical direction-finding methods, which are based on the point source assumption, can be easily extended to the CD source case (see [3,10–13] and references therein). In [3], the MUSIC algorithm [4] has been generalized from point source case to distributed source case. The resulting method (called DSPE[1]) assumes that all sources have an identical and known angular distribution shape. Some generalized beamforming techniques, which suffer from these same drawbacks have been presented in [10]. The method proposed in [11] is a low-complexity sequential 1D searching algorithm (called SOS). It allows the estimation of the nominal azimuth and elevation DOAs of CD sources by using a pair of uniform circular arrays. This method is based on the subspace principle and uses the classical TLS-ESPRIT. However, this technique does not estimate the angular spreads of sources. In addition, this method is also limited to the sources with identical and known angular distribution shape. In [13], a fast 1D DOA estimator using a uniform linear array has been proposed. This method does not require *a priori* knowledge on the angular distribution shape and is applicable to CD sources with different forms of angular distribution. However, it cannot estimate the angular spreads of the sources and suffers from the ambiguity problems.

The aim of this paper is to improve the DSPE algorithm presented in [3] in the case of small angular spread. Compared with the original DSPE estimator, the proposed estimator assumes that the sources have the Gaussian and/or the uniform distributions and improves the robustness to the mismodeling of the spatial distribution.

The paper is organized as follows. In Section 2, we present the used CD source model. In Section 3, we show how the estimation of the DOAs can be decoupled from that of the angular spreads. Based on this idea, the proposed method is developed in the same section. Computer simulation results are presented in Section 4, and conclusions are drawn in Section 5.

[1]Distributed source parameter estimator.

2. Problem formulation

Consider a uniform linear array (ULA) of m sensors, the distance between two adjacent sensors is denoted by d. Suppose that q electromagnetic scattered waves are impinging on the array from angular directions θ_i for $i = 1, 2, \ldots, q$. We assume that the inverse of the travel time of the scattered waves across the antenna array is large compared with the bandwidth of the sources; so that, the narrowband assumption continues to be valid, even in the presence of scattering [14]. For simplicity, we assume that the sensors and sources are on the same plane. The baseband signals received at the antenna array are collected in the observation vector $\boldsymbol{x}(t) = [x_1(t), \ldots, x_m(t)]^{\mathrm{T}}$, where $(.)^{\mathrm{T}}$ denotes the transpose operator, this vector can be modeled as [3]

$$\boldsymbol{x}(t) = \sum_{i=1}^{q} \int_{\phi \in \Theta} \boldsymbol{a}(\phi)\gamma_i(\phi, t; \boldsymbol{\eta}_i)\,\mathrm{d}\phi + \boldsymbol{n}(t), \tag{1}$$

where $\gamma_i(\phi, t; \boldsymbol{\eta}_i)$ is the angular signal distribution of the ith source, $\boldsymbol{\eta}_i = [\theta_i \sigma_i]^{\mathrm{T}}$ is the corresponding unknown parameter vector, θ_i is the nominal DOA of the ith source and σ_i is the corresponding angular spread. Θ is the angular field of view, $\boldsymbol{n}(t)$ is the noise vector and $\boldsymbol{a}(\phi)$ is the steering vector for a point source at DOA ϕ.

Throughout the paper, we will consider the CD source model presented in [3,10–13]. That is, a coherently distributed source, the shape of the angular distribution does not change temporally and the received signal components from this source at different directions are fully correlated. Thus, the angular signal distribution, associated with the ith CD source, can be expressed as

$$\gamma_i(\phi, t; \boldsymbol{\eta}_i) = s_i(t)g_i(\phi; \boldsymbol{\eta}_i), \tag{2}$$

where $s_i(t)$ is the ith complex, random signal source and $g_i(\phi; \boldsymbol{\eta}_i)$ is the corresponding deterministic angular weighting function. The sources are assumed to be uncorrelated with each other, and the deterministic angular weighting function $g_i(\phi;\boldsymbol{\eta}_i)$ is assumed to be non-vanishing only around the actual DOA θ_i. Thus, the distributed source model (1) can be rewritten as

$$\begin{aligned}\boldsymbol{x}(t) &= \sum_{i=1}^{q} s_i(t)\int_{-\infty}^{+\infty}\boldsymbol{a}(\phi)g_i(\phi;\boldsymbol{\eta}_i)\mathrm{d}\phi + \mathbf{n}(t)\\ &= \mathbf{C}\boldsymbol{s}(t) + \boldsymbol{n}(t),\end{aligned} \tag{3}$$

where the vector $\mathbf{s}(t) = [s_1(t), \ldots, s_q(t)]^{\mathrm{T}}$ contains temporal signals transmitted by the q distributed sources. The matrix $\mathbf{C} = [\boldsymbol{c}(\boldsymbol{\eta}_1), \ldots, \boldsymbol{c}(\boldsymbol{\eta}_q)]$ contains the generalized steering vectors of the impinging sources where the generalized steering vector of the ith CD source is given by

$$\boldsymbol{c}(\boldsymbol{\eta}_i) = \int_{-\infty}^{+\infty}\boldsymbol{a}(\phi)g_i(\phi;\boldsymbol{\eta}_i)\,\mathrm{d}\phi. \tag{4}$$

In the literature the Gaussian and uniform shapes are usually used for modeling the deterministic angular weighting function $g(\phi;\boldsymbol{\eta})$. It has been shown in [10–12] that, for small angular spread, the vector $\boldsymbol{c}(\boldsymbol{\eta}_i)$ given by Eq. (4) can be written approximately as

$$\boldsymbol{c}(\boldsymbol{\eta}_i) \simeq \boldsymbol{\Phi}(\theta_i)\boldsymbol{b}(\boldsymbol{\eta}_i), \tag{5}$$

where $\boldsymbol{\Phi}(\theta_i) = \mathrm{diag}(\boldsymbol{a}(\theta_i))$ and $\boldsymbol{b}(\boldsymbol{\eta}_i)$ is real-valued vector that depends on the array geometry and the shape of the angular weighting function $g_i(\phi;\boldsymbol{\eta}_i)$. Indeed, for a uniform linear array, the $(k+1)_{0\leqslant k\leqslant m-1}$th element of the vector $\boldsymbol{b}(\boldsymbol{\eta}_i)$ can be expressed, in the case of Gaussian and uniform shapes, as

$$\begin{aligned}&[\boldsymbol{b}(\boldsymbol{\eta}_i)]_{k+1}^{\mathrm{Gauss}} = \exp\left[-\frac{1}{2}\left(\frac{2\pi d}{\lambda}k\sigma_i\cos\theta_i\right)^2\right]\\ &\quad\text{and}\quad [\boldsymbol{b}(\boldsymbol{\eta}_i)]_{k+1}^{\mathrm{unif}} = \mathrm{sinc}\left[\sqrt{3}\frac{2\pi d}{\lambda}k\sigma_i\cos\theta_i\right].\end{aligned} \tag{6}$$

Assuming that the noise and the signals are uncorrelated and that the noise is spatially and temporally white, the data model (3) allows us to write the covariance matrix of the array measurements as

$$\mathbf{R} = E[\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)] = \mathbf{R}_s + \sigma_n^2\mathbf{I}, \tag{7}$$

where $E[.]$ denotes the expectation, $(.)^{\mathrm{H}}$ denotes the complex conjugate transpose operator and $\mathbf{R}_s = \mathbf{C}\boldsymbol{\Gamma}_s\mathbf{C}^{\mathrm{H}}$ is the noise-free covariance matrix, with $\boldsymbol{\Gamma}_s = E[\mathbf{s}(t)\mathbf{s}^{\mathrm{H}}(t)]$ the diagonal emitted signal covariance matrix and $\sigma_n^2\mathbf{I}$ the noise covariance matrix.

3. Parameter estimation

3.1. Original DSPE algorithm

It is shown in [3] that the MUSIC algorithm can be generalized for CD source case as

$$\hat{\boldsymbol{\eta}} = \arg\min_{\boldsymbol{\eta}}\frac{\boldsymbol{c}^{\mathrm{H}}(\boldsymbol{\eta})\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^{\mathrm{H}}\boldsymbol{c}(\boldsymbol{\eta})}{\boldsymbol{c}^{\mathrm{H}}(\boldsymbol{\eta})\boldsymbol{c}(\boldsymbol{\eta})}, \tag{8}$$

where $\hat{\mathbf{E}}_n$ is the estimated noise subspace spanned by the $(m-q)$ orthonormal eigenvectors associated with the $(m-q)$ smallest eigenvalues of the sample covariance matrix $\hat{\mathbf{R}} = (1/T)\sum_{t=1}^{T}\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)$.

3.2. Limitations

As we can observe, the above DSPE estimator (8) has two drawbacks:

- shape of the angular distribution of different sources has to be identical,
- distribution shape has to be known.

3.3. Proposed improvements

In this section, we consider that all sources have the same (Gaussian or uniform) but unknown angular distribution shape. To improve the DSPE estimator (8), we use the fact that $\boldsymbol{b}(\boldsymbol{\eta})$ in (5) is real valued vector and the matrix $\boldsymbol{\Phi}^*\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^{\mathrm{H}}\boldsymbol{\Phi}$ is Hermitian.

In addition, the cost function $\boldsymbol{c}^{\mathrm{H}}\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^{\mathrm{H}}\boldsymbol{c}/\boldsymbol{c}^{\mathrm{H}}\boldsymbol{c} = \boldsymbol{b}^{\mathrm{T}}\boldsymbol{\Phi}^*\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^{\mathrm{H}}\boldsymbol{\Phi}\boldsymbol{b}/\boldsymbol{b}^{\mathrm{T}}\boldsymbol{\Phi}^*\boldsymbol{\Phi}\boldsymbol{b}$ is real positive valued, therefore, (8) can then be rewritten as

$$\hat{\boldsymbol{\eta}} = \arg\min_{\boldsymbol{\eta}}\frac{\boldsymbol{b}^{\mathrm{T}}\mathrm{Re}(\boldsymbol{\Phi}^*\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^{\mathrm{H}}\boldsymbol{\Phi})\boldsymbol{b}}{\boldsymbol{b}^{\mathrm{T}}\boldsymbol{b}}, \tag{9}$$

where Re(.) denotes the real part.

We have then an objective function $\boldsymbol{b}^{\mathrm{T}}\mathrm{Re}(\boldsymbol{\Phi}^*\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^{\mathrm{H}}\boldsymbol{\Phi})\boldsymbol{b}/\boldsymbol{b}^{\mathrm{T}}\boldsymbol{b}$ to minimize, this minimization problem can be done in an efficient manner by taking into account some properties of the vector $\boldsymbol{b} = [b_1, b_2, \ldots, b_m]^{\mathrm{T}}$. Indeed, according to formula (6), the elements $[b_k]_{k=1,\ldots,m}$ of the vector $\boldsymbol{b}(\boldsymbol{\eta})$ for the Gaussian distribution have to be positive and satisfy the constraints: $1 = b_1 \geqslant b_2 \geqslant \cdots \geqslant b_m \geqslant 0$. Note that, the same properties are also true in the case of small angular spread (e.g. $\sigma \leqslant 1/(m-1)\sqrt{3} \simeq 6.61^\circ$ for $m = 6$ sensors) for a uniform distribution.

From these results, the estimation of the DOA in (9) can be replaced by the following constrained problem:

$$\hat{\theta} = \arg\min_{\theta}\left[\min_{\boldsymbol{x}} \frac{\boldsymbol{x}^{\mathrm{T}}\mathrm{Re}(\boldsymbol{\Phi}^{*}\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^{\mathrm{H}}\boldsymbol{\Phi})\boldsymbol{x}}{\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}} \quad \text{subject to} \quad 0 \leqslant x_m \leqslant \cdots \leqslant x_2 \leqslant x_1 \quad \text{and} \quad x_1 = 1\right], \tag{10}$$

where $\boldsymbol{x} = [x_1, x_2, \ldots, x_m]^{\mathrm{T}}$. This constrained optimization programming problem can be rewritten in the following matrix form:

$$\hat{\theta} = \arg\min_{\theta}\left[\min_{\check{\boldsymbol{x}}} \frac{[1\check{\boldsymbol{x}}^{\mathrm{T}}]\mathrm{Re}(\boldsymbol{\Phi}^{*}\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^{\mathrm{H}}\boldsymbol{\Phi})[1\check{\boldsymbol{x}}^{\mathrm{T}}]^{\mathrm{T}}}{\check{\boldsymbol{x}}^{\mathrm{T}}\check{\boldsymbol{x}}+1} \quad \text{subject to } \mathbf{J}\check{\boldsymbol{x}} \leqslant \boldsymbol{e}\right], \tag{11}$$

where $\check{\boldsymbol{x}} = [x_2, \ldots, x_m]^{\mathrm{T}}$, $\boldsymbol{e} = [1, 0, \ldots, 0]^{\mathrm{T}}$ and $\mathbf{J}$ is $m \times (m-1)$ matrix such as

$$\mathbf{J} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -1 & 1 & \ddots & \vdots \\ 0 & -1 & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & -1 \end{bmatrix}.$$

The minimization problem under brackets (11) can be solved using a sequential quadratic programming (SQP) method [15–19]. In our simulations, the Matlab function fmincon, based on the SQP method is used to solve this problem. Note that, a full description of this algorithm can be founded in www.mathworks.com.

After the DOA search procedure in (11), the angular spread of the ith source can then be estimated by using the original DSPE estimator as

$$\hat{\sigma}_i = \arg\min_{\sigma} \frac{\boldsymbol{b}_i^{\mathrm{T}}(\hat{\theta}_i, \sigma)\mathrm{Re}(\boldsymbol{\Phi}^{*}(\hat{\theta}_i)\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^{\mathrm{H}}\boldsymbol{\Phi}(\hat{\theta}_i))\boldsymbol{b}_i(\hat{\theta}_i, \sigma)}{\boldsymbol{b}_i^{\mathrm{T}}(\hat{\theta}_i, \sigma)\boldsymbol{b}_i(\hat{\theta}_i, \sigma)}, \tag{12}$$

where $\hat{\theta}_i$ is the estimated DOA of the ith source given by (11). Note that, it is possible to employ the EXIP principle [8] to estimate the spread angle σ_i by matching the resulting vector $\hat{\boldsymbol{x}}_i$ associated with the estimated DOA $\hat{\theta}_i$ in (11) to the theoretical vector $\boldsymbol{b}(\boldsymbol{\eta})$. Unfortunately, this is not very interesting for this application since it requires the knowledge of the power of each source as well as the noise power.

Remarks

- The formula (11) which estimates the DOAs is independent of the vector $\boldsymbol{b}$ in (5). Thus, concerning the DOA estimation, the proposed method can be applied, without distinction, for both uniform and Gaussian shapes. Additionally, concerning the spread estimation, the proposed technique can be applied even when the sources have different but known angular distributions.
- The proposed estimator is not limited to the case of ULA, but can also be applied for other antenna geometries (e.g. a uniform circular array). However, it becomes computationally intensive when the number of antenna elements is large.

4. Simulation results

In this section we present some simulation results that illustrate the performance of the proposed method, called decoupled distributed signal parameter estimator (DDSPE). Consider a uniform linear array of six sensors separated by a half wavelength of incoming signals. In these simulations the proposed estimator is compared with MUSIC [4] and DSPE [3] estimators.

The performance of all estimators are obtained from 200 Monte-Carlo simulations by calculating the root-mean-square error (RMS error[2]) of DOA and spread estimates.

The sources emit BPSK modulated signals with a raised cosine pulse shape filter, the roll-off factor is equal to 0.22 and the bit rate is 3.84 Mb/s. The incoming signals are sampled with frequency 38.4 MHz during approximately 8 μs which gives 300 data samples. The proposed methods are initialized with $\check{\boldsymbol{x}}^{(0)} = (1/m)[m-1, \ldots, 1]^{\mathrm{T}}$.

4.1. Sources with identical distribution shape

To examine the performance of the compared methods, we assume here that the various sources have an identical distribution shape.

In the first example, the influence of the signal to noise ratio (SNR) is examined. The parameter vectors of the two CD Gaussian sources are $\boldsymbol{\eta}_1 =$

[2]The RMS error corresponding to the kth element of $\boldsymbol{\eta} = [\theta\sigma]^{\mathrm{T}}$ is calculated as RMS error of η_k estimates $= \sqrt{\frac{1}{200}\sum_{n=1}^{200}[\hat{\eta}_k(n) - \eta_k]^2}$.

$[-10°, 7°]^T$ and $\boldsymbol{\eta}_2 = [10°, 6°]^T$. In this simulation, the actual distribution shape is assumed to be known for the DSPE estimator. It can be observed in Fig. 1, that the proposed DDSPE estimator has performance similar to that of the original DSPE method. In fact, the difference between the results provided by these two estimators increases as the SNR decreases but this increase remains small even for the small values of SNR. For example, for SNR $= -10$ dB, this difference equal $0.1°$ for the DOA estimation and $0.05°$ for the spread estimation.

In the second example, we illustrate the behaviors of the estimators [MUSIC, DSPE and DDSPE] when the spread angle is varied between $1°$ and $10°$. Two Gaussian CD sources are simulated with DOAs $\theta_1 = -10°$ and $\theta_2 = 10°$, the SNR is equal to 10 dB. Here also the DSPE estimator assumes that the distribution shape is known. As we can show in Fig. 2, the proposed DDSPE estimator presents a good alternative for estimating the angular parameters of the distributed sources. Note that, the particular behavior (with a minimum at about $5°$) of the RMS error of angular spread can be explained by the fact that the two estimators are trying to fit an angular dispersion to cope with the noise. In fact, the position of the minimum decreases as the SNR increases.

In the third example, we compare the estimators DSPE and DDSPE when two sources have a same but unknown distribution shape. For simplicity, we assume that the angular spread is known for the DSPE estimator. Actually, the sources have the Gaussian shape, but we consider in the search procedure that they have a uniform shape. In this simulation, the theoretical covariance matrix is used and the angular parameters of the two equipower CD sources are $\boldsymbol{\eta}_1 = [-10°, 7°]^T$ and $\boldsymbol{\eta}_2 = [10°, 7°]^T$. As we can show in Fig. 3 that, the proposed DDSPE estimator is robust to the mismodeling of the angular distribution, whereas the DSPE estimator fails to detect the two sources.

4.2. Sources with different distribution shape

Moreover, it is interesting to examine the behaviors of the two estimators (DSPE and DDSPE) when different CD sources do not necessarily have the same angular distribution shape. In our last example, we consider two equipower CD sources with different angular distribution shape. The shape

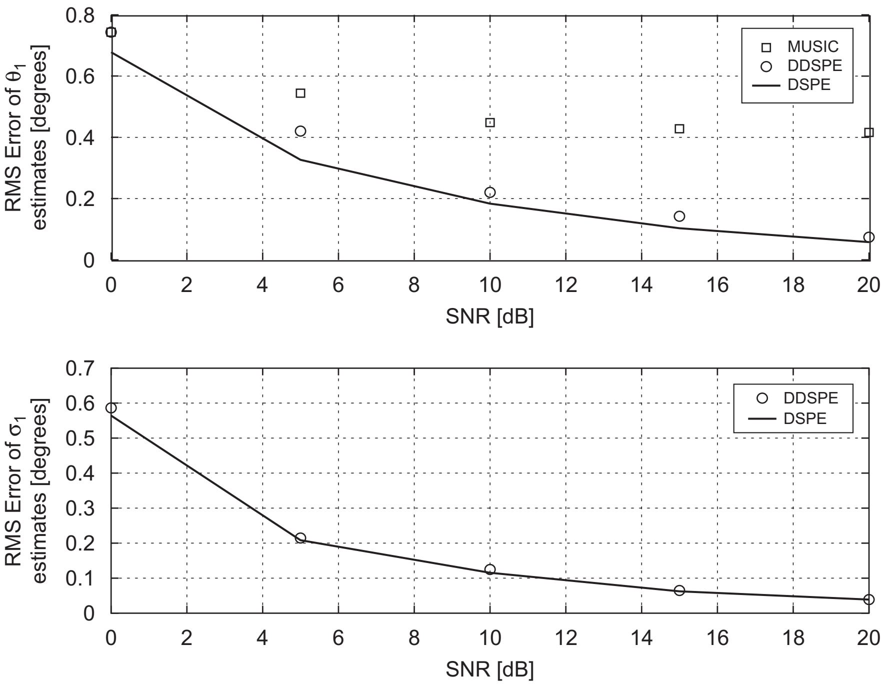

Fig. 1. RMS error of θ_1 and σ_1 estimate versus the signal to noise ratio (SNR), $\eta_1 = [-10°7°]^T$, $\eta_2 = [10°6°]^T$, $T = 300$ snapshots, $m = 6$ sensors (Gaussian distribution).

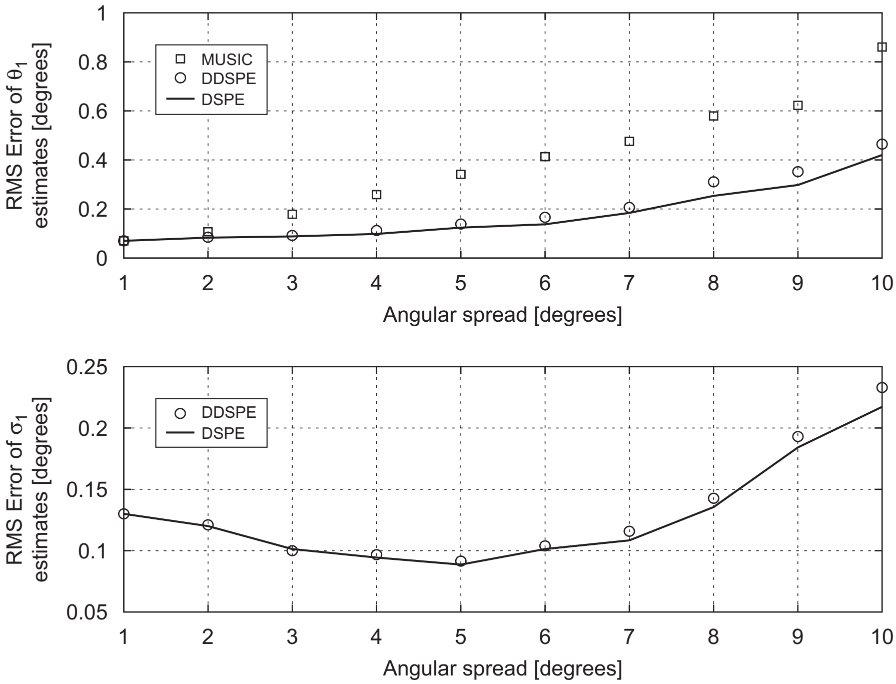

Fig. 2. RMS error of θ_1 and σ_1 estimate versus the true angular spread, $\theta_1 = -10^\circ$, $\theta_2 = 10^\circ$, SNR $= 10$ dB, $T = 300$ snapshots, $m = 6$ sensors (Gaussian distribution).

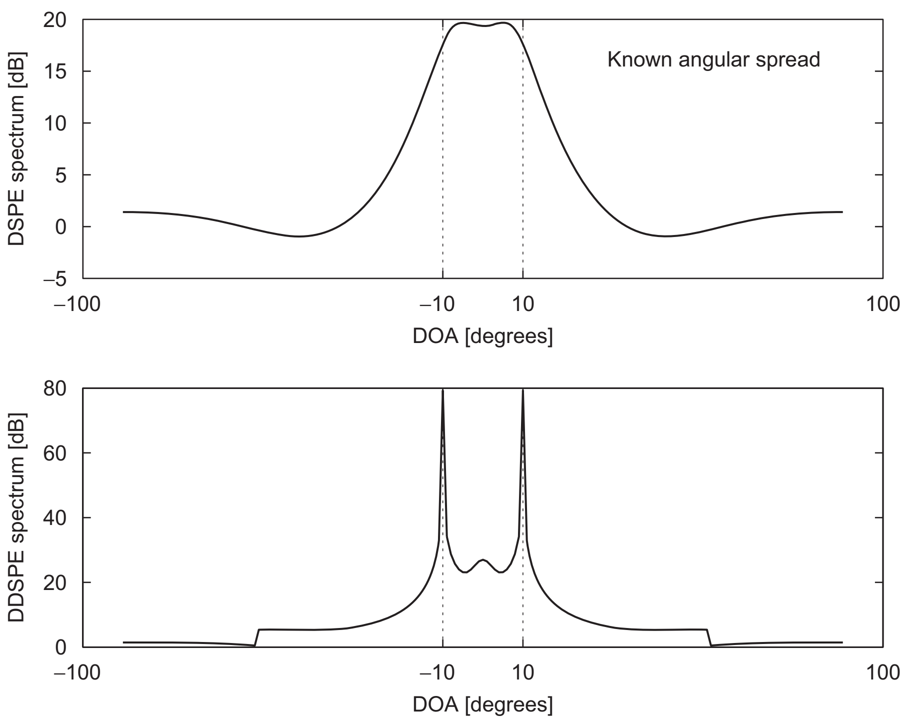

Fig. 3. Spatial spectra of the DSPE (11) and DDSPE (8) estimators in two sources case with same but unknown angular distribution shape, $\eta_1 = [-10^\circ 7^\circ]^T$ (Gaussian distribution), $\eta_2 = [10^\circ 7^\circ]^T$ (Gaussian distribution), theoretical covariance matrix, $m = 6$ sensors.

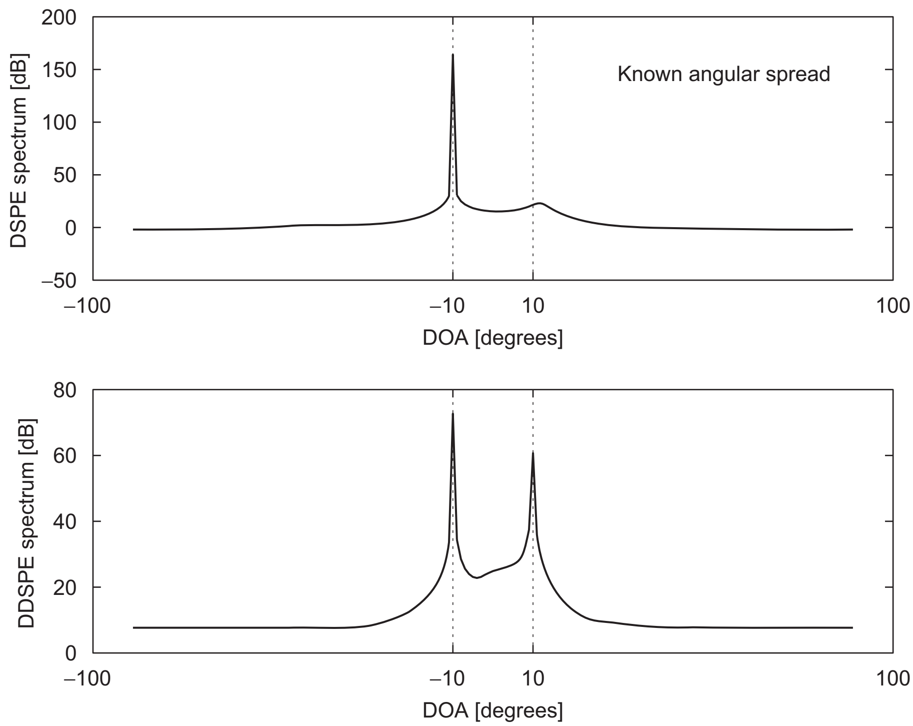

Fig. 4. Spatial spectra of the DSPE (11) and DDSPE (8) estimators in two sources *case* with different angular distribution shapes, $\eta_1 = [-10^\circ 6^\circ]^T$ (Gaussian distribution), $\eta_2 = [10^\circ 6^\circ]^T$ (uniform distribution), theoretical covariance matrix, $m = 6$ sensors.

of the first source is assumed to be Gaussian with $\boldsymbol{\eta}_1 = [-10^\circ, 6^\circ]^T$. The shape of the second source is assumed to be uniform with $\boldsymbol{\eta}_2 = [10^\circ, 6^\circ]^T$. In the search procedure, it is assumed that the two sources have uniform shape. As it can be seen in Fig. 4, the proposed DDSPE estimator is robust to the mismodeling of the spatial distribution of the sources and may be used when different sources have different angular distribution shapes.

5. Conclusion

In this paper, we have presented a new unambiguous algorithm for estimating the DOAs and angular spreads of coherently distributed sources. By decoupling the estimation of the DOAs from that of the angular spreads in the case of Gaussian and/or uniform distributions, it has been shown that the performance of the original DSPE method can be significantly improved. Numerical results show that the proposed method presents a good alternative for estimating the angular parameters and can be applied in case where the Gaussian and uniform distributions coexist.

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