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A new algorithm for 2-D DOA estimation

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Abstract

In this paper we present a new algorithm to estimate the 2-D direction of arrival (DOA) of narrowband sources lying in the far field of the array. The array consists of matched co-directional triplets, and can be considered as an extension of the 1-D ESPRIT scenario to 2-D. The proposed approach is simple and direct and does not require a search procedure or initialization. Existing algorithms require a search to match the correct elevation and azimuth angles and are computationally more expensive. This technique automatically pairs the azimuth and elevation angles by marking them. The computational complexity is twice that of 1-D ESPRIT. Simulation results and comparisons with other existing algorithms are presented to demonstrate the performance of the proposed technique. © 1997 Elsevier Science B.V.

Zusammenfassung

In diesem Artikel stellen wir einen neuen Algorithmus zur Schätzung der zweidimensionalen Welleneinfallsrichtung (DOA) vor, wobei von der Präsenz schmalbandiger Quellen ausgegangen wird, die im Fernfeld der Empfangsantenne liegen. Die Sensorgruppe besteht aus angepaßten kodirektionalen Triplets und kann als eine zweidimensionale Erweiterung des eindimensionalen ESPRIT-Scenariums angesehen werden. Der vorgestellte Ansatz ist einfach, direkt und kommt ohne Suchprodezuren oder Initialisierungen aus. Vorhandene Algorithmen benötigen eine Suche zur Bestimmung (*match*) der tatsächlichen Elevations- und Azimutwinkel und sind somit rechenaufwendiger. Das vorgestellte Verfahren führt durch ihre Markierung automatisch Paarbildungen von Azimut- und Elevationswinkeln durch. Der Rechenaufwand ist dabei doppelt so hoch wie beim 1-D ESPRIT Algorithmus. Zur Demonstration der Leistungsfähigkeit der vorgestellten Technik werden Simultationsergebnisse und Vergleiche mit anderen existierenden Algorithmen vorgestellt. © 1997 Elsevier Science B.V.

Résumé

Dans ce papier, on présente un nouvel algorithme pour l'estimation de la direction d'arrivée (DOA) 2-D de sources à bande étroite situées loin du réseau. Le réseau est constitué de triplets codirectionnels appareillés et peut être considéré comme une extension 2-D du scénario ESPRIT 1-D. L'approche proposée est simple et directe et ne nécessite aucune procédure de recherche ou d'initialisation. Les algorithmes existants requièrent une recherche pour adapter les angles d'élévation et azimuthal corrects et sont bien plus coûteux en temps de calcul. La méthode proposée ici paire les angles d'élévation et azimuthal en les marquant. La complexité algorithmique est double de celle de l'approche ESPRIT 1-D. Des résultats de simulations et des comparaisons avec d'autres algorithmes existants sont présentés afin de mettre en évidence les performances de la méthode proposée. © 1997 Elsevier Science B.V.

Keywords: Array signal processing; 2-D direction of arrival

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1. Introduction

The central problem in array signal processing is the localization of multiple sources, i.e., determination of both the bearings, viz., the azimuth and elevation, of each source. ESPRIT [6] and ESPRIT like high resolution DOA estimation algorithms, are usually designed to determine the DOA of narrow-band, non-coherent signals that lie in a single plane. The standard ESPRIT algorithm can be used to estimate the azimuths and elevations of all the sources but the problem of recombining the decoupled estimates to obtain the (θ, ϕ) pairs for each source is a non-trivial problem.

Van der Veen et al. [9] have proposed an approach that approximates the data matrix by adding small perturbation matrices such that the two matrices for 2-D DOA estimation have equal eigenvectors. Swindlehurst and Kailath proposed a similar approach [7, 8] to handle 2-D DOA estimation in which one of the two data matrices is perturbed such that its eigenvectors coincide with the eigenvectors of the other matrix. However, this algorithm requires reasonably accurate initial estimates of the DOAs and requires a search procedure. More recently Cheng and Hua [3] have proposed a pencil MUSIC approach to solve this problem, which requires a rectangular array of minimum size $2d - 1 \times 2d - 1$ where d is the number of sources, and the eigenvalue decomposition of the covariance matrix of minimum size $d^2 \times d^2$, hence is computationally unattractive.

Chen and Chen [2] have proposed an approach to pair DOAs and center frequency using the marked subspace approach, which is based on the assumption that the matrix product of the signal steering matrix and the signal subspace is diagonal, which is not always valid. Table 1 gives a comparison of various techniques in terms of the required array dimension and complexity of the problem solution.

We present a new approach to solve the 2-D DOA estimation problem. The underlying idea behind this approach is that the two matrix pencils obtained in 2-D ESPRIT have the same set of eigenvectors. We call this set of eigenvectors (the eigenvectors corresponding to the signal eigenvalues) as the signal subspace. We construct a marker matrix which is a diagonal matrix with distinct, non-zero diagonal elements. Using the original covariance matrices, the signal subspace and the marker matrix we construct marked covariance matrices. Solving the matrix pencils of the marked matrices results in the correct eigenvalue pairing. The correct eigenvalue pairs can be obtained as the ones lying on the same circle, each pair lying on a circle of different radius. The restrictions of planar wavefronts and matched, co-directional triplets apply.

The proposed approach is direct and requires no initial estimates or search procedures to obtain estimates of azimuth/elevation angles. The minimum number of elements required is $2d + 1$. Further this approach can easily be extended for estimating higher dimensional parameter vectors (e.g., simultaneous estimation of azimuth, elevation and frequency. Also using spatial smoothing along with the proposed approach, the technique can be extended to the situations of coherent multipath propagation [1].

The paper is organized as follows. In Section 2 we present the data model. The basic definitions and the proposed technique are discussed in Section 3, followed by simulation results and the conclusion in Sections 4 and 5, respectively.

Table 1
Comparison of 2-D DOA estimation algorithms

Algorithm	Min. array dimension	Eigen-decomp. required	Search required	Initialization required	Effective for coherent sources
Pencil MUSIC [5]	$d \times d$	$d^2 \times d^2$	No	No	Yes
Van der Veen et al. [9]	$2 \times d$	$d \times d$	Yes	Yes	No
Swindlehurst et al. [8]	$2 \times d$	$d \times d$	Yes	Yes	Yes
Chen and Chen [7]	$2 \times d$	$d \times d$	No	No	No
Proposed algorithm	$2 \times d$	$d \times d$	No	No	Yes

2. Data model

Consider three arrays of sensors located on the X–Y plane at the positions (x_{xi}, y_{yi}), $(x_{xi}+u_y, y_{yi}+v_y)$ and $(x_{xi}+u_z, y_{yi}+v_z)$, respectively, where $1 \leqslant i \leqslant N$. The displacements of the latter two arrays given by (u_y, v_y) and (u_z, v_z), respectively, are assumed to be linearly independent vectors in the X–Y plane. Let there be d narrow-band sources with center frequency ω_0 emitting plane wave signals $s_k(t)$, such that the kth source has an elevation angle θ_k and azimuth angle ϕ_k. The observed signals at the sensors located at (x_{xi}, y_{yi}), $(x_{xi}+u_y, y_{yi}+v_y)$ and $(x_{xi}+u_z, y_{yi}+v_z)$ are then given by

$$x_i(t) = \sum_{k=1}^{d} \mathrm{e}^{\mathrm{j}\omega_0(\tau_i(\theta_k,\phi_k))} s_k(t) + n_{x,i}(t), \tag{1}$$

$$y_i(t) = \sum_{k=1}^{d} \mathrm{e}^{\mathrm{j}\omega_0(T_y(\theta_k,\phi_k))} \mathrm{e}^{\mathrm{j}\omega_0(\tau_i(\theta_k,\phi_k))} s_k(t) + n_{y,i}(t), \tag{2}$$

$$z_i(t) = \sum_{k=1}^{d} \mathrm{e}^{\mathrm{j}\omega_0(T_z(\theta_k,\phi_k))} \mathrm{e}^{\mathrm{j}\omega_0(\tau_i(\theta_k,\phi_k))} s_k(t) + n_{z,i}(t), \tag{3}$$

where

$$\tau_i(\theta,\phi) = (x_{xi}\cos\phi\sin\theta + y_{yi}\sin\phi\sin\theta)/c, \quad 1 \leqslant i \leqslant N,$$

$$T_k(\theta,\phi) = (u_k\cos\phi\sin\theta + v_k\sin\phi\sin\theta)/c, \quad k = y, z.$$

These equations can be expressed in matrix form as

$$\boldsymbol{x}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{s}(\boldsymbol{t}) + \boldsymbol{n}_x(\boldsymbol{t}), \tag{4}$$

$$\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{\Phi}_y\boldsymbol{s}(\boldsymbol{t}) + \boldsymbol{n}_y(\boldsymbol{t}), \tag{5}$$

$$\boldsymbol{z}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{\Phi}_z\boldsymbol{s}(\boldsymbol{t}) + \boldsymbol{n}_z(\boldsymbol{t}), \tag{6}$$

where

$$\boldsymbol{x}(\boldsymbol{t}) = [x_1(t)\ x_2(t)\ \ldots\ x_N(t)]^{\mathrm{T}},$$

$$\boldsymbol{y}(\boldsymbol{t}) = [y_1(t)\ y_2(t)\ \ldots\ y_N(t)]^{\mathrm{T}},$$

$$\boldsymbol{z}(\boldsymbol{t}) = [z_1(t)\ z_2(t)\ \ldots\ z_N(t)]^{\mathrm{T}},$$

$$\boldsymbol{s}(\boldsymbol{t}) = [s_1(t)\ s_2(t)\ \ldots\ s_d(t)]^{\mathrm{T}},$$

$$\boldsymbol{\Phi}_k = \mathrm{diag}[\mathrm{e}^{\mathrm{j}\omega_0(T_k(\theta_1,\phi_1))} \ldots \mathrm{e}^{\mathrm{j}\omega_0(T_k(\theta_d,\phi_d))}], \quad k = y, z,$$

$$\boldsymbol{A} = [\boldsymbol{a}_1 \ldots \boldsymbol{a}_d],$$

where

$$\boldsymbol{a}_i(\theta_i,\phi_i) = [\mathrm{e}^{\mathrm{j}\omega_0(\tau_1(\theta_i,\phi_i))} \ldots \mathrm{e}^{\mathrm{j}\omega_0(\tau_N(\theta_i,\phi_i))}]^{\mathrm{T}}$$

and

$$\boldsymbol{n}_k = [n_{k,1}(t) \ldots n_{k,N}(t)], \qquad \boldsymbol{k} = \boldsymbol{x}, \boldsymbol{y}, z.$$

The columns of $\boldsymbol{A}$ are assumed to be linearly independent. This amounts to assuming that the signal subspace has dimension d. It is assumed that $\boldsymbol{P} = E[\boldsymbol{s}(\boldsymbol{t})\boldsymbol{s}(\boldsymbol{t})^{\mathrm{H}}]$ is non-singular, i.e., sources are not fully correlated.
$E[\boldsymbol{s}(\boldsymbol{t})\boldsymbol{n}_k(\boldsymbol{t})^{\mathrm{H}}] = \boldsymbol{0}$ for $k = x, y, z$, i.e., signal and noise are uncorrelated.

$$E[\boldsymbol{n}_x(\boldsymbol{t})\boldsymbol{n}_x(\boldsymbol{t})^{\mathrm{H}}] = \sigma^2\boldsymbol{I}_N,$$

$$E[\boldsymbol{n}_x(\boldsymbol{t})\boldsymbol{n}_y(\boldsymbol{t})^{\mathrm{H}}] = \sigma^2\boldsymbol{N}_{xy},$$

$$E[\boldsymbol{n}_x(\boldsymbol{t})\boldsymbol{n}_z(\boldsymbol{t})^{\mathrm{H}}] = \sigma^2\boldsymbol{N}_{xz},$$

where $\boldsymbol{I}_N$ is the $N \times N$ identity matrix, $\boldsymbol{N}_{xy}$ and $\boldsymbol{N}_{xz}$ are $N \times N$ matrices with ones along one of the super diagonals for a linearly displaced array whose place in the matrix depends on the array overlap. If the X and Y (Z) arrays do not overlap then $\boldsymbol{N}_{xy}(\boldsymbol{N}_{xz}) = \boldsymbol{0}$.

The source location pairs (θ_i, ϕ_i) are assumed to be distinct points in $[0, \pi/2] \times [0, 2\pi]$. The plane wave condition $(u_k^2 + v_k^2)^{1/2} < 2\pi c/\omega_0$, $k = x, y$ is imposed.

It is seen that the ordered pairs $(T_y(\theta_i,\phi_i), T_z(\theta_i,\phi_i))$, $1 \leqslant i \leqslant d$, are all distinct [5]. From this result it follows that the ordered pairs $(\gamma_{y,i}, \gamma_{z,i})$, $1 \leqslant i \leqslant d$, where $\gamma_{y,i}, \gamma_{z,i}$ are, respectively, the ith diagonal entries of $\boldsymbol{\Phi}_y$ and $\boldsymbol{\Phi}_z$ are all distinct.

3. Problem solution

3.1. Definitions

We define the auto and cross covariance matrices as follows:

$$\boldsymbol{R}_{xx} = E[\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}], \tag{7}$$

$$\boldsymbol{R}_{xy} = E[\boldsymbol{x}\boldsymbol{y}^{\mathrm{H}}], \tag{8}$$

$$\boldsymbol{R}_{xz} = E[\boldsymbol{x}\boldsymbol{z}^{\mathrm{H}}]. \tag{9}$$

We then have

$$\boldsymbol{R}_{xx} = \boldsymbol{A}\boldsymbol{P}\boldsymbol{A}^{\mathrm{H}} + \sigma^2 \boldsymbol{I}_N, \tag{10}$$

$$\boldsymbol{R}_{xk} = \boldsymbol{A}\boldsymbol{P}\boldsymbol{\Phi}_k^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}} + \sigma^2 \boldsymbol{N}_{xk}, \quad \boldsymbol{k} = \boldsymbol{y}, \boldsymbol{z}. \tag{11}$$

Now $\boldsymbol{R}_{xx}$ is a Hermitian positive semidefinite matrix whose minimum eigenvalue is σ^2 and has multiplicity $N-d$ [6]. We can obtain an estimate of the noiseless auto and cross covariance matrices by the relations

$$\boldsymbol{C}_{xx} = \boldsymbol{R}_{xx} - \sigma^2 \boldsymbol{I}_N = \boldsymbol{A}\boldsymbol{P}\boldsymbol{A}^{\mathrm{H}}, \tag{12}$$

$$\boldsymbol{C}_{xk} = \boldsymbol{R}_{xk} - \sigma^2 \boldsymbol{N}_{xk} = \boldsymbol{A}\boldsymbol{P}\boldsymbol{\Phi}_k^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}, \quad \boldsymbol{k} = \boldsymbol{y}, \boldsymbol{z}. \tag{13}$$

It is seen that $\boldsymbol{C}_{xx}$ is a positive semidefinite matrix of rank d [6]. Thus $\boldsymbol{C}_{xx}$ has d positive eigenvalues and $N-d$ eigenvalues equal to zero. Solving for the eigenvalues of the matrix pencil $(\boldsymbol{C}_{xx}, \boldsymbol{C}_{xk})$, $\boldsymbol{k} = \boldsymbol{y}, \boldsymbol{z}$ we obtain

$$|\boldsymbol{C}_{xx} - \lambda_k \boldsymbol{C}_{xk}| = 0 \tag{14}$$

$$\Rightarrow \quad |\boldsymbol{A}\boldsymbol{P}\boldsymbol{A}^{\mathrm{H}} - \lambda_k \boldsymbol{A}\boldsymbol{P}\boldsymbol{\Phi}_k^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}| = 0 \tag{15}$$

$$\Rightarrow \quad \boldsymbol{A}\boldsymbol{P}|\boldsymbol{I} - \lambda_k \boldsymbol{\Phi}_k^{\mathrm{H}}|\boldsymbol{A}^{\mathrm{H}} = 0. \tag{16}$$

The eigenvalues of the above matrix pencils $\lambda_{k,i}$, $1 \leqslant i \leqslant d$, give the diagonal values of $\boldsymbol{\Phi}_y$ for $\boldsymbol{k} = \boldsymbol{y}$ and $\boldsymbol{\Phi}_z$ for $\boldsymbol{k} = \boldsymbol{z}$, respectively. The problem now is to pair the right $\lambda_{y,i}$ to $\lambda_{z,j}$, $1 \leqslant i, j \leqslant d$, to obtain the correct pairing of DOAs.

3.2. Proposed approach

We define the signal subspace $\boldsymbol{E}_s$ as the set of eigenvectors corresponding to the generalized eigenvalues of the matrix pencil $(\boldsymbol{C}_{xx}, \boldsymbol{C}_{xy})$. Hence $\boldsymbol{E}_s = [\boldsymbol{v}_1 \boldsymbol{v}_2 \dots \boldsymbol{v}_d]$ where $\boldsymbol{v}_i$, $1 \leqslant i \leqslant d$, is the eigenvector corresponding to the ith eigenvalue of the matrix pencil $(\boldsymbol{C}_{xx}, \boldsymbol{C}_{xy})$. Assume that all the eigenvalues are arranged in the descending order.

We define a new marker matrix $\boldsymbol{D}$ as

$$\boldsymbol{D} = \mathrm{diag}[r_1, r_2, \dots, r_d],$$
$$r_i \neq r_j, \quad \text{for} \quad i \neq j, \quad 1 \leqslant i, \quad j \leqslant d. \tag{17}$$

We now construct the marked auto covariance matrix $\boldsymbol{C}_{xxm}$ as

$$\boldsymbol{C}_{xxm} = \boldsymbol{E}_s^{\mathrm{H}} \boldsymbol{C}_{xx} \boldsymbol{E}_s \boldsymbol{D} \tag{18}$$

and marked cross covariance matrices

$$\boldsymbol{C}_{xkm} = \boldsymbol{E}_s^{\mathrm{H}} \boldsymbol{C}_{xk} \boldsymbol{E}_s, \quad \boldsymbol{k} = \boldsymbol{y}, \boldsymbol{z}. \tag{19}$$

Solving for the eigenvalues of the matrix pencils $(\boldsymbol{C}_{xxm}, \boldsymbol{C}_{xkm})$, $\boldsymbol{k} = \boldsymbol{y}, \boldsymbol{z}$, we get

$$|\boldsymbol{C}_{xxm} - \lambda_k \boldsymbol{C}_{xkm}| = 0, \quad \boldsymbol{k} = \boldsymbol{y}, \boldsymbol{z} \tag{20}$$

$$\Rightarrow \quad |\boldsymbol{E}_s^{\mathrm{H}} \boldsymbol{C}_{xx} \boldsymbol{E}_s \boldsymbol{D} - \lambda_k \boldsymbol{E}_s^{\mathrm{H}} \boldsymbol{C}_{xk} \boldsymbol{E}_s| = 0 \tag{21}$$

$$\Rightarrow \quad |\boldsymbol{E}_s^{\mathrm{H}} \boldsymbol{A}\boldsymbol{P}\boldsymbol{A}^{\mathrm{H}} \boldsymbol{E}_s \boldsymbol{D} - \lambda_k \boldsymbol{E}_s^{\mathrm{H}} \boldsymbol{A}\boldsymbol{P}\boldsymbol{\Phi}_k^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}} \boldsymbol{E}_s| = 0. \tag{22}$$

To simplify the above equation we use the following two lemmas.

Lemma 1. *Let $\boldsymbol{M} = \boldsymbol{A}^{\mathrm{H}}\boldsymbol{E}_s$, then $\boldsymbol{M}$ is a column permutation of a diagonal matrix.*

Proof. According to the definition of the GEVD, the relationship between the generalized eigenvalues and generalized eigenvectors of the matrix pencil $(\boldsymbol{C}_{xx}, \boldsymbol{C}_{xy})$ is given by

$$\boldsymbol{C}_{xx}\boldsymbol{v}_i = \lambda_{y,i} \boldsymbol{C}_{xy} \boldsymbol{v}_i, \quad i = 1, 2, \dots, d, \tag{23}$$

where $\lambda_{y,i}$, $1 \leqslant i \leqslant d$, are the eigenvalues of the matrix pencil $(\boldsymbol{C}_{xx}, \boldsymbol{C}_{xy})$. The above equation can be rewritten as

$$\boldsymbol{A}\boldsymbol{P}(\boldsymbol{I} - \lambda_{y,i}\boldsymbol{\Phi}_y^{\mathrm{H}})\boldsymbol{A}^H \boldsymbol{v}_i = \boldsymbol{0}_{N\times 1}, \tag{24}$$

where $\boldsymbol{0}_{N\times 1}$ denotes a vector of zeros of dimension $N \times 1$.

Let $\boldsymbol{a}_k$ be the kth column of $\boldsymbol{A}$, $\boldsymbol{p}_j$ be the jth column of $\boldsymbol{P}$, and $m_{ij} = \boldsymbol{a}_j{}^{\mathrm{H}}\boldsymbol{v}_i$, $j = 1, 2, \dots, d$. The above equation can be rewritten as

$$\sum_{k=1}^{d} \boldsymbol{a}_k \sum_{j=1}^{d} \boldsymbol{p}_j (1 - \lambda_{y,i}\gamma_{y,j}^{\mathrm{H}}) m_{ij} = \boldsymbol{0}_{N\times 1}. \tag{25}$$

As the columns of $\boldsymbol{A}$, viz., $\boldsymbol{a}_k$, $k = 1, 2, \dots, d$, are linearly independent and under the assumption that the sources are uncorrelated, $\boldsymbol{p}_j$, $j = 1, 2, \dots, d$, are linearly independent, one can show

$$(1 - \lambda_{y,i}\gamma_{y,j}^{\mathrm{H}}) m_{ij} = 0, \quad j = 1, 2, \dots, d. \tag{26}$$

Let $\lambda_{y,i} = \gamma_{y,j}$ for $j = k_i$. As all eigenvalues of the matrix pencil $(\boldsymbol{C}_{xx}, \boldsymbol{C}_{xy})$ are distinct, therefore $\lambda_{y,i} \neq \gamma_{y,j}$ for $j \neq k_i$. This implies

$$\boldsymbol{a}_i^{\mathrm{H}} \boldsymbol{v}_j = m_{ij} = 0, \quad j = 1, 2, \dots, d \quad \text{and} \quad j \neq k_i, \tag{27}$$

and the k_i's for $i=1,2,\dots,d$ are distinct. Therefore $\boldsymbol{M}=\boldsymbol{A}^{\mathrm{H}}\boldsymbol{E}_s$ is a column permutation of a diagonal matrix. □

Lemma 2. *Consider matrix $\boldsymbol{M}$ a column permutation of a diagonal matrix and $\boldsymbol{D}$ a diagonal matrix. Then $\boldsymbol{MD}=\boldsymbol{D}_1\boldsymbol{M}$ where $\boldsymbol{D}_1$ is the matrix $\boldsymbol{D}$, with its diagonal entries shuffled. Note that $\boldsymbol{D}_1$ is also a diagonal matrix. The proof is trivial and can be obtained by simple matrix manipulations.*

Using the above two lemmas

$$\boldsymbol{A}^{\mathrm{H}}\boldsymbol{E}_s\boldsymbol{D}=\boldsymbol{D}_1\boldsymbol{A}^{\mathrm{H}}\boldsymbol{E}_s, \tag{28}$$

where $\boldsymbol{D}_1$ is the $\boldsymbol{D}$ matrix with its diagonal entries shuffled. Therefore Eq. (22) becomes

$$|\boldsymbol{E}_s^{\mathrm{H}}\boldsymbol{APD}_1\boldsymbol{A}^{\mathrm{H}}\boldsymbol{E}_s-\lambda_k\boldsymbol{E}_s^{\mathrm{H}}\boldsymbol{AP\Phi}_k^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{E}_s|=0 \tag{29}$$

$$\Rightarrow\ \boldsymbol{E}_s^{\mathrm{H}}\boldsymbol{AP}|\boldsymbol{D}_1-\lambda_k\boldsymbol{\Phi}_k^{\mathrm{H}}|\boldsymbol{A}^{\mathrm{H}}\boldsymbol{E}_s=0. \tag{30}$$

Let the eigenvalues obtained be λ_{km}, $k=y,z$.

From the structure of Eq. (30) it is seen that

$$\lambda_{ym,i}=d_{1j}\gamma_{y,j},\quad 1\leqslant i,j\leqslant d, \tag{31}$$

$$\lambda_{zm,i}=d_{1j}\gamma_{z,j},\quad 1\leqslant i,j\leqslant d, \tag{32}$$

where d_{1j}, $1\leqslant j\leqslant d$, is the jth diagonal entry of D_1. $\gamma_{y,j}$ and $\gamma_{z,j}$, $1\leqslant j\leqslant d$, are the jth diagonal term of $\boldsymbol{\Phi}_y$ and $\boldsymbol{\Phi}_z$, respectively.

Hence we see that the correct eigenvalue pairs are multiplied by the same and unique entry from $\boldsymbol{D}_1$. As all the entries of $\boldsymbol{D}_1$ are different and magnitude of $\gamma_{k,j}=1$ for $k=y,z$ and $1\leqslant j\leqslant d$, the correct $\gamma_{y,i}$ and $\gamma_{z,i}$, $1\leqslant i,j\leqslant d$, pairs can be obtained by comparing the absolute values of λ_{ym} and λ_{zm}. Dividing the marked eigenvalue pairs by their magnitude, we obtain the correct $(\gamma_{y,i},\gamma_{z,i})$ pairs for $1\leqslant i\leqslant d$.

The DOA of the sources can now be estimated from the following equations:

$$\phi_i=\tan^{-1}\left[\frac{u_z\alpha_i-u_y}{v_y-v_z\alpha_i}\right],\quad 1\leqslant i\leqslant d, \tag{33}$$

$$\theta_i=\sin^{-1}\left[\frac{\alpha_i}{u_y\cos\phi+v_y\sin\theta}\right],\quad 1\leqslant i\leqslant d, \tag{34}$$

where

$$\alpha_i=\frac{\log(\gamma_{y,i})}{\log(\gamma_{z,i})},\quad 1\leqslant i\leqslant d.$$

4. Simulation results

Simulations were carried out to test the performance of the proposed technique. Sensors were assumed to be lying along a linear array corresponding to $y_{yi}=0$, $1\leqslant i\leqslant N$, and $x_{xi}=\delta*i$, $1\leqslant i\leqslant N$, where δ is the sensor displacement, which is taken to be half the wavelength of the signal waves. $(u_y,v_y)=(\delta,0)$ and $(u_z,v_z)=(0,\delta)$. Number of snapshots per experiment is taken to be 200. True DOAs are $(30°,60°)$, $(60°,30°)$ and $(45°,45°)$. Number of sensors in each subarray taken was 16 which requires a total number of 33 sensors. The results of the DOA estimations for 50 trial runs using the proposed approach is shown in Fig. 1 for SNR = 10 dB and Fig. 2 for SNR = 20 dB. It is seen that the proposed algorithm is able to correctly pair the azimuth and elevation angles and obtain high resolution DOAs.

Performance evaluation was done by studying the effect of reducing SNR on the mean square error. Fig. 3 shows the variation of the average mean square error in estimating the azimuth angle as a function of SNR.

Table 2 gives a comparison of the various algorithms in terms of MSE obtained as a function of size of the array, number of snapshots and SNR. It is seen that the proposed algorithm gives low MSE compared to the other techniques. Here the comparison is based on different data lengths, as some of the values have been taken from the graphs provided by the respective papers. It conveys that the proposed technique gives better performance compared to the existing algorithms, both in terms of SNR and computational complexity.

5. Conclusions

In this paper, we have proposed a simple technique to handle the problem of 2-D DOA estimation. It is a direct approach which obtains high resolution DOA estimates without a search procedure. The idea used is that the two eigenvalue matrix pencils obtained share the same set of eigenvectors. This idea can be easily extended to estimate higher dimensional parameter spaces.

Comparisons, summarized in Tables 1 and 2, show that the performance of this algorithm is better than

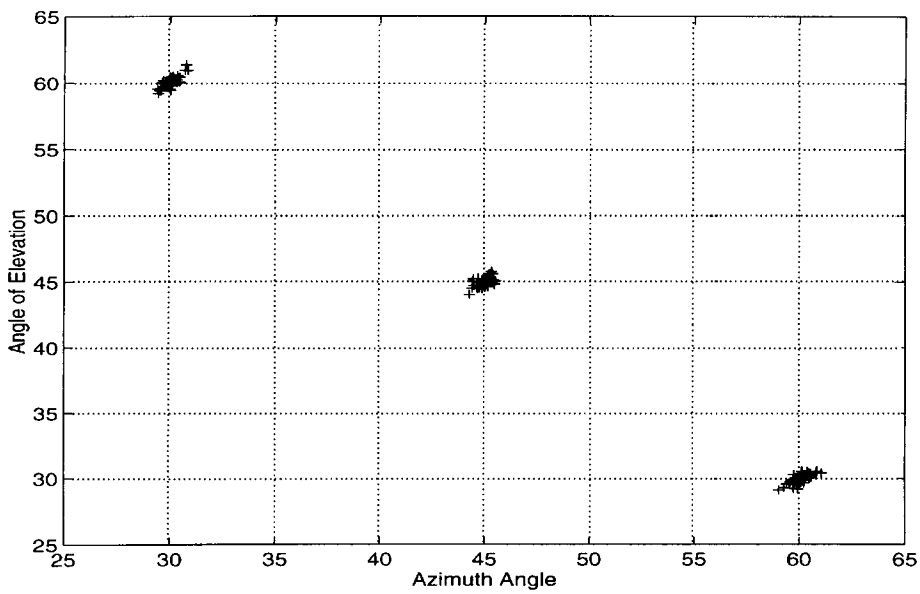

Fig. 1. Simulation result for 50 runs. SNR = 10 dB, number of sensors = 16, actual DOAs are (30°, 60°), (45°, 45°) and (60°, 30°).

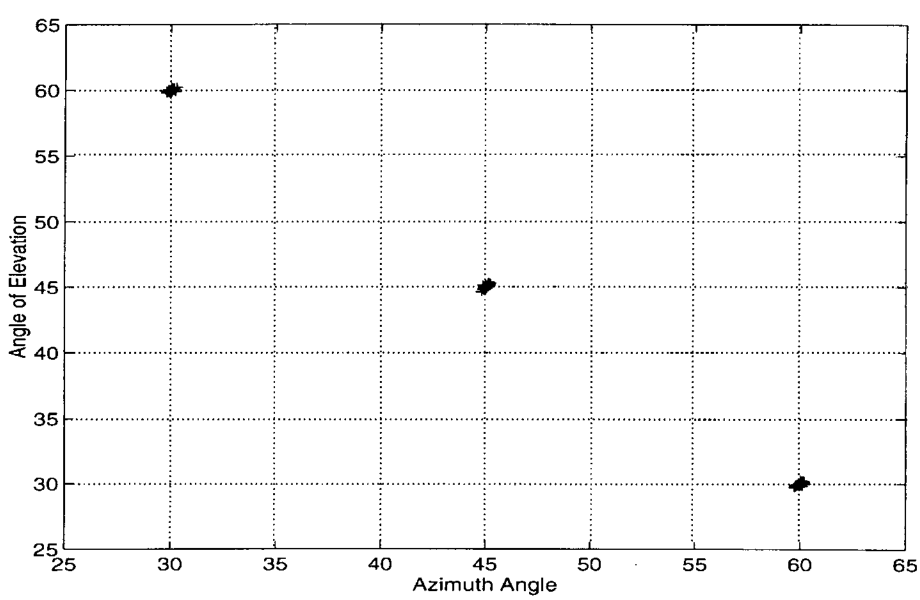

Fig. 2. Simulation result for 50 runs. SNR = 20 dB, number of sensors = 16, actual DOAs are (30°, 60°), (45°, 45°) and (60°, 30°).

that of the algorithms given in [3, 8, 9] computationally and/or in the number of sensors required with comparable or better mean square error under similar conditions. Extension of this approach to the situation of coherent multipath propagation utilizing spatial smoothing and combined forward backward smoothing has also been worked out and is being communicated separately.

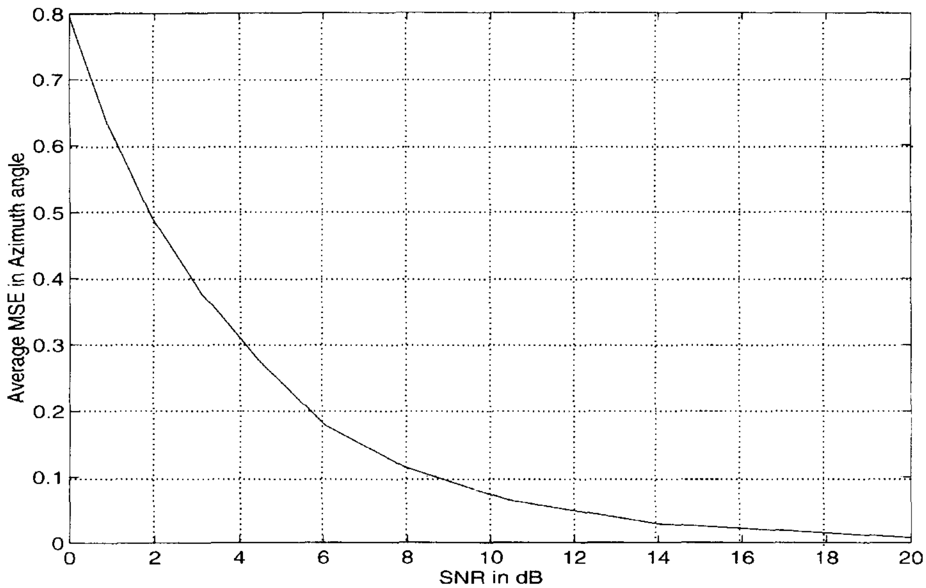

Fig. 3. Average MSE for varying SNR. Number of sensors = 16. Three sources with actual DOAs (30°, 60°), (45°, 45°) and (60°, 30°).

Table 2
Comparison of simulation results of 2-D DOA estimation algorithms

Algorithm	Array dim.	Eigendecomposition of matrix of size	No. of sources	No. of snaps	SNR in dB	MSE obtained
Pencil MUSIC [1]	20 × 20	49 × 49	2	200	10	0.01
Van der Veen et al. [9]	10 × 10	81 × 81	4	250	12	1.00
Swindlehurst et al. [8]	5 × 5	16 × 16	2	250	10	0.20
Proposed algorithm	2 × 16	16 × 16	3	200	10	0.06

Note: Some of the MSE values are approximate as they have been taken from graphs provided by the respective papers.

Notations

$\boldsymbol{A}$	capital alphabet in bold italics is a matrix
$\boldsymbol{a}$	small alphabet in bold italics is a vector
$a_{i,j}$	element on ith row and jth column of matrix $\boldsymbol{A}$
$\boldsymbol{A}^{\mathrm{H}}$	complex conjugate transpose of $\boldsymbol{A}$
$\lvert\boldsymbol{A}\rvert$	determinant of matrix $\boldsymbol{A}$
$E[a(t)]$	expected value of the time varying variable $a(t)$
d	number of sources (signals)
N	number of sensors in each array
$\boldsymbol{A}$	signal direction matrix
$\boldsymbol{P}$	signal covariance matrix
$\boldsymbol{N}_{kk}$	additive noise
$\boldsymbol{\Phi}_k$	rotation matrix due to displacement between X and K arrays
θ	angle of elevation
ϕ	azimuth angle
$\gamma_{k,i}$	diagonal entries of $\boldsymbol{\Phi}_k$
$\boldsymbol{R}_{ij}$	covariance matrix between vectors $\boldsymbol{i}$ and $\boldsymbol{j}$, with noise
$\boldsymbol{C}_{ij}$	noiseless covariance matrix between vectors $\boldsymbol{i}$ and $\boldsymbol{j}$
$\boldsymbol{C}_{ij\mathrm{m}}$	noiseless marked covariance matrix between vectors $\boldsymbol{i}$ and $\boldsymbol{j}$
$\boldsymbol{D}$	diagonal matrix with all different entries

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