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Analysis of low frequency oscillations in power system using EMO ESPRIT

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A B S T R A C T

Identification of poorly damped low frequency oscillations present in the densely interconnected power system is of paramount importance to maintain its stable operation. Estimation of signal parameters via rotational invariance technique (ESPRIT) is a parametric method used for analysing such signals even under noisy conditions. However, this method requires precise information about the number of modes present in the signal. Hence, this work uses a combination of Exact Model Order (EMO) algorithm and ESPRIT for analysing these low frequency oscillations. The performance of the proposed method is tested using various synthetic signals with different levels of noise and PMU reporting rates. Further, the robustness of the proposed method towards noise resistance is compared with modified Prony, TLS-ESPRIT and ARMA methods. Finally, the proposed method is tested using real time probing test data obtained from Western Electricity Coordinating Council (WECC) network. Results reveal that the proposed method is accurate, precise and outperforms the other methods.

1. Introduction

The modern day power systems are highly interconnected for effectively sharing the increasing load demand. These complex interconnected networks pose many challenges to the power system engineers such as monitoring and detection of poorly damped low frequency oscillations [1]. Traditionally, these low frequency modes are identified through Eigen value analysis by linearizing the power system model around the operating point of the system [2]. The main disadvantage of this method is that it is inaccurate and requires extensive information about the modelling of the system. With the advent of Phasor Measurement Unit (PMU) and Wide Area Measurement System (WAMS), it is possible to estimate and monitor these modes based on the data from the Phasor Data Concentrator (PDC). Such methods are collectively known as measurement based methods [3,4].

Some of the common measurement based methods used for small signal estimation are Prony analysis [5–10], ARMA [11], Fast Fourier Transform (FFT) [12], Continuous Wavelet Transform (CWT) [13,14], Discrete Wavelet Transform (DWT) [15], Hilbert-Huang transform [16] ESPRIT [17–19], Matrix Pencil [20] and Complex-Singular Value Decomposition (C-SVD) [21].

Among these methods, Prony method [5–8] estimates the frequency and damping of signals accurately when noise content of the signal is low but fails under highly noisy conditions. Modified Prony methods are proposed in [9,10] but the issue of sensitivity to noise is not fully removed. Times series techniques like AR, ARMA [11,22] assumes a predefined time series order and fits its free parameters such that the difference between the original signal and the time series is minimized. The practical application of these methods are limited as they cannot estimate closely spaced modes present in a signal. Moreover, these methods are computationally intensive. FFT based methods [12] extract only the frequency information of the modes present in the signal whereas damping ratio of these modes cannot be estimated. Wavelet techniques like CWT, DWT [13–15] are based on multi-resolution analysis where wavelets of variable sizes are used to extract the modal information present in the signal. These methods are easy to implement and can extract modal information of non stationary signals effectively but their accuracy is dependent on the shape of the mother wavelet and decomposition levels. HHT based methods [16] use a combination of EMD and Hilbert's transform for estimating the modes in the low frequency oscillations. The frequency and damping estimation of this method is accurate only if the modes extracted using the EMD are mono frequency components.

Of late, ESPRIT based methods are used for low frequency mode identification in [17–19,23]. These methods decompose the autocorrelation matrix into signal and noise subspaces and the modes

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are identified from the signal subspace. ESPRIT based methods has the ability to detect close modes and provide good accuracy for modal estimation especially in noisy environments. However, they require precise information about model order or the number of modes present in the signal for successful modal estimation, which is one of their drawbacks.

From the aforementioned literature, it can be concluded that the ESPRIT coupled with a good model order algorithm could overcome most of the drawbacks of other measurement based methods. The ESPRIT models in [17,18] uses a method based on singular value of autocorrelation matrix for estimating the model order. Though this method is simple, it provides inaccurate estimates of model order when the number of modes present in the signal is less or modes are closely spaced. A detailed explanation of the same is given in Section 2. Considering these facts, an attempt has been made in this paper to develop an ESPRIT based modal estimation method with a better model order algorithm to accurately estimate low frequency modes at varying levels of noise and PMU reporting rates. The low frequency oscillations under consideration are inter area (<1 Hz), intra plant (2–3 Hz), local plant(1–2 Hz) and torsional and control modes. The proposed method uses Exact Model Order algorithm [24] to estimate the number of frequency components present in the signal. The effectiveness of this method is tested using synthetic test signals and the results are compared with modified Prony method [9], ARMA based method [11] and TLS-ESPRIT [17] method. Further, the proposed method is used to estimate the modes of a practical probing test data of the WECC system.

2. ESPRIT and model order estimation

ESPRIT is a signal processing technique which decomposes complex signals into sum of sinusoids using a subspace based approach. It can be mathematically represented as

$$x(t) = \sum_{i=1}^{K} a_i \cos(2\pi * f_i t + \phi_i) + w_t \tag{1}$$

Here, $x(t)$ is the signal to be decomposed. In this paper, it is assumed that x(t) is obtained from different PMUs placed in the power system. f_i and ϕ_i are the frequency and phase of the i^{th} sinusoidal component decomposed from $x(t)$. w_t represents the white Gaussian noise present in the signal and K is the total number of frequency components present in the signal. For exact estimation of frequency components, this technique demands prior information of the number of modes present in the signal. It can be achieved using a model order estimation algorithm, which is described in the next sub-section.

2.1. Model order estimation

The model order estimation considers the dominant singular values of the autocorrelation matrix generated from the signal. A commonly used method for model order estimation was proposed in [17]. In this method, the auto correlation matrix is formed using the data from the PMU. Singular Value Decomposition (SVD) is performed on this matrix. The singular values are arranged in decreasing order and K(i), which is an index used for separating the signal and noise subspace is obtained using the following equation

$$K(i) = \frac{\rho_1 + \rho_2 + \rho_3 + \dots \rho_i}{\rho_1 + \rho_2 + \rho_3 + \dots \rho_l} \tag{2}$$

Here, ρ_i is the i^{th} singular value of the auto correlation matrix and l is the total number of singular values of the autocorrelation matrix. The value of i for which $K(i)$ is closest to one is selected as the model order of the system. The main limitation of this method is that it cannot accurately estimate the model order when the number of modes present in the signal is less or modes are closely spaced. This can be explained with the help of an example.

Let us consider a power system signal as in the following equation.

$$\begin{aligned} x1 = {} & ((1\cos(2\pi * 0.4t)\exp(-0.0909t)) + (0.9\cos(2\pi * 0.5t) \\ & \times \exp(-0.35t)) + (0.7\cos(2\pi * 0.6t + (\pi/6)) \\ & \times \exp(-0.2001t)) + (0.4\cos(2\pi * 1.1t + (\pi/4)) \\ & * \exp(-0.666t))); \end{aligned} \tag{3}$$

This signal is sampled at 50 Hz. It is corrupted by adding white Gaussian noise at 40 dB. The value of $K(i)$ of the signal is determined from the singular values of its autocorrelation matrix and $K(i)$ vs i graph is plotted. It is shown in Fig. 1.

It is observed that the value of $K(i)$ is closest to one at $i = 5$. So the model order is estimated as 5. But this estimation is incorrect as the signal in (3) has only four frequency components. Also the estimate further deviates if the signal is highly noise contaminated and contains close frequency components. Therefore, this paper employs the Exact Model Order algorithm proposed in [24] for precise estimation of number of modes.

The proposed algorithm is based on the fact that there will be considerable difference between the eigen values of the signal and noise subspace. This property is used for effectively separating the autocorrelation matrix into signal and noise subspaces. This algorithm can be summarized in the following steps.

1. Arrange the eigen values (λ_i) of the autocorrelation matrix in the increasing order.
2. Find relative difference (RD) vector using the following equation. Plot the value of RD against i, which is called RD index (RDI).

$$RD = \frac{\lambda_i - \lambda_{i-1}}{\lambda_{i-1}} \quad for\ i = 2, 3, 4 \dots M \tag{4}$$

Here, M is the total number of Eigen values of the autocorrelation matrix.

3. Select the five highest peaks in the RD vs RDI plot. The highest value of RDI among these highest peaks is selected as the first estimate of model order.
4. Check whether the eigenvalue corresponding to the selected RDI has more energy than the noise subspace. This is done using the following equation.

$$\lambda_\$ \geq \alpha * \frac{\lambda_{\$+1} + \lambda_{\$+2} + \lambda_{\$+3} + \dots \lambda_M}{M - \$} \tag{5}$$

Here \$ represent the preliminary estimate of the model order. The value of α is between two and five.

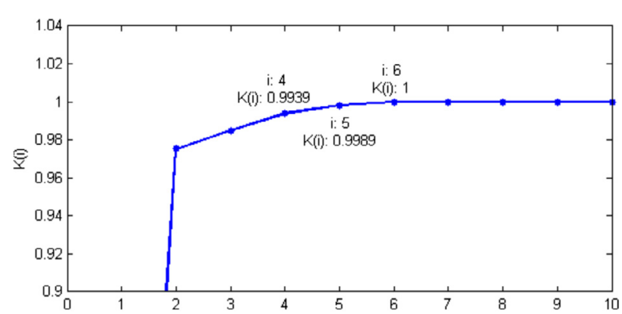

Fig. 1. K(i) vs i plot.

5. If the above equation is satisfied, then model order is estimated as $RDI/2$. Else, the next lower value of RDI among the five peaks is selected as the next estimate. The process continues till the above equation is satisfied.

To prove the effectiveness of this method, the signal in (3) is chosen for testing this method. The RD vs RDI graph of this signal is shown in Fig. 2. It is observed that peaks of RD occurs at RDI values of 4, 6 and 8. Therefore, according to step 3, the $RDI = 8$ is selected as the preliminary model order estimate and verified using (5). This value satisfies (5) and thereby the obtained model order is $RDI/2 = 4$ which is the true value.

2.2. ESPRIT algorithm

The main steps for estimation of frequency using ESPRIT algorithm is as follows [23,24].

Step 1: Form auto correlation matrix $\boldsymbol{R}_x$ from the given set of data point received from the PMU.

$$\boldsymbol{R}_x = \frac{1}{N-M}\boldsymbol{X}^H \cdot \boldsymbol{X} \tag{6}$$

where $\boldsymbol{X}$ is the Hankel matrix of order M and N is the length of the signal under consideration. It is constructed from the signal x(t) as shown below

$$\boldsymbol{X} = \begin{bmatrix} x(0) & x(1) & .. & x(M-1) \\ x(1) & x(2) & .. & x(M) \\ . & . & .. & . \\ . & . & .. & . \\ x(N-M) & x(N-M+1) & .. & x(N-1) \end{bmatrix}$$

Step 2: Decompose the auto correlation matrix using Eigen Value Decomposition. The eigen values are arranged in the decreasing order and the first n eigen values are selected where n is the model order of the signal obtained using EMO algorithm. The eigen vectors corresponding to these eigen values form the signal subspace $\boldsymbol{R}_{xs}$.

Step 3: Two shifted submatrices $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are generated from the signal subspace $\boldsymbol{R}_{xs}$ using the following formula

$$\boldsymbol{R}_i = \boldsymbol{S}_i * \boldsymbol{R}_{xs},\ i = 1,2 \tag{7}$$

where $\boldsymbol{S}_1 = [\boldsymbol{I}_{N_s} 0_{d_s}]$ and $\boldsymbol{S}_2 = [0_{d_s} \boldsymbol{I}_{N_s}]$. $\boldsymbol{I}_{N_s}$ is an identity matrix of size $N_s \times N_s$ where $N_s = M - d_s$. d_s is the distance between two sub matrices which is usually kept as 1.

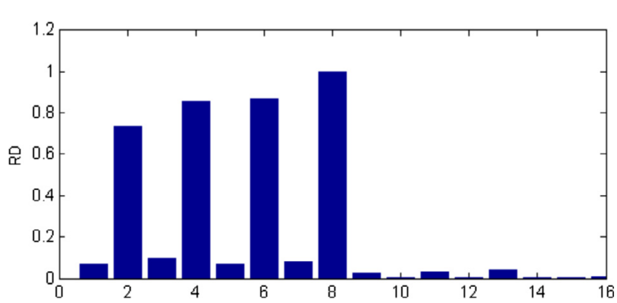

Fig. 2. RD vs RDI plot.

Step 4: The shifted submatrices $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ can be related through a matrix ψ using shift invariance property such that $\boldsymbol{R}_2 = \boldsymbol{R}_1 \cdot \psi$. The matrix ψ is found out using the following least square estimate

$$\psi = \left(\boldsymbol{R}_1^H \boldsymbol{R}_1\right)^{-1} \boldsymbol{R}_1^H \boldsymbol{R}_2 \tag{8}$$

Step 5: The frequency (f_k) and attenuation factor (AF_k) of the components of the signal is obtained from the eigen values of the matrix ψ.

$$f_k = f_s * \frac{imag(\log(\lambda_{\psi k}))}{(2\pi)} \quad \forall k = 1,2,3\ldots 2n \tag{9}$$

$$AF_k = f_s * real(\log(\lambda_{\psi k})) \quad \forall k = 1,2,3\ldots 2n \tag{10}$$

Here, f_s is the sampling frequency of the signal and $\lambda_{\psi k}$ is the eigenvalue of the matrix ψ. The damping coefficient (ζ) of a frequency component is obtained from its frequency and attenuation factor using the following equation [13].

$$\zeta_k = -\frac{AF_k}{\sqrt{(AF_k)^2 + (\omega_k)^2}} \quad \forall k = 1,2,3\ldots 2n \tag{11}$$

3. Simulation results and discussion

3.1. Simulated test signals

To validate the performance of the proposed method, its results are compared with that of ARMA block processing [11], TLS-ESPRIT [17] and modified Prony methods [9]. The performance of the ARMA method [11] is poor under noisy conditions. So a low pass filter is used to separate the low frequency modes of the signal and to reduce the effect of measurement noise. The specifications of this filter are (i) 2 Hz pass-band corner frequency, (ii) 5 Hz stop-band corner frequency, (iii) 0.2 dB ripple in the pass-band, and (iv) 20 dB ripple in the stop-band. This section presents the results of three different test signals with varying noise level. The test signals considered for illustration are

$$Signal\ 1 : (2\cos(2\pi * 0.2t + 1.5\pi))\exp(-0.17t) + 2\cos(2\pi * 0.75t + (0.5\pi))\exp(-0.13t) \tag{12}$$

$$Signal\ 2 : 2\cos(2\pi * 0.2t + (1.5\pi)(\exp(-0.17t) + 2\cos(2\pi * 0.28t + (0.5\pi))\exp(-0.05t) + (2\cos(2\pi * 0.75t + (4.5\pi))\exp(-0.13t))); \tag{13}$$

$$Signal\ 3 : (2\cos(2\pi * 0.25t + (1.5\pi))\exp(-0.17t)) + (2\cos(2\pi * 0.33t + (1.5\pi))\exp(-0.12t)) + (2cos(2\pi * 0.78t + (0.5\pi))\exp(-0.13t)) + (2cos(2\pi * 0.87t + (0.5\pi))\exp(-0.0702t))); \tag{14}$$

These signals are simulated in MATLAB and white Gaussian noise is added to them. The modal parameters of these corrupted signals are estimated using the proposed method, ARMA, TLS-ESPRIT and modified Prony method. The results obtained are tabulated in Tables 1–4.

Table 1 compares the estimated frequencies and attenuation factors of the test signals using the proposed method, ARMA [11] and modified Prony method [9]. In this table, *freq* refers to the frequency of the mode, *AF* is attenuation factor and *Std* is standard deviation. The frequencies and attenuation factors of a signal at a particular SNR is obtained by taking the mean of the estimated values obtained from fifty independent simula-

Table 1
Sensitivity analysis of the proposed method with that of modified Prony method [9] and ARMA method [11].

Method	SNR = 5				SNR = 10				SNR = 15			
	Estimated		Std (%)		Estimated		Std (%)		Estimated		Std (%)	
	Freq (Hz)	AF	Freq (Hz)	AF	Freq (Hz)	AF	Freq (Hz)	AF	Freq (Hz)	AF	Freq (Hz)	AF
					Signal 1							
True	0.200	0.170			0.200	0.170			0.200	0.170		
value	0.750	0.130			0.750	0.130			0.750	0.130		
EMO	0.199	0.171	0.24	1.53	0.200	0.171	0.15	0.63	0.199	0.169	0.06	0.42
ESPRIT	0.750	0.129	0.16	0.79	0.750	0.130	0.08	0.62	0.750	0.131	0.06	0.29
Modified	0.201	0.178	0.36	2.73	0.201	0.176	0.18	1.25	0.199	0.172	0.11	0.58
Prony	0.751	0.142	0.45	1.24	0.751	0.133	0.13	0.80	0.749	0.132	0.08	0.38
ARMA	0.201	0.184	0.34	2.80	0.199	0.178	0.12	0.37	0.201	0.175	0.08	0.72
	0.750	0.143	0.26	1.02	0.751	0.131	0.37	0.53	0.749	0.132	0.06	0.35
					Signal 2							
True	0.200	0.170			0.200	0.170			0.200	0.170		
Value	0.280	0.050			0.280	0.050			0.280	0.050		
	0.750	0.130			0.750	0.130			0.750	0.130		
EMO	0.200	0.174	0.90	2.57	0.200	0.172	0.37	1.63	0.199	0.171	0.17	0.92
ESPRIT	0.280	0.052	0.23	0.95	0.280	0.052	0.08	0.46	0.280	0.050	0.04	0.26
	0.749	0.132	0.17	0.09	0.750	0.131	0.09	0.62	0.749	0.131	0.04	0.34
	0.211	0.266	0.57	5.59	0.209	0.202	1.73	1.50	0.202	0.170	0.15	3.98
Prony	0.278	0.079	0.58	1.98	0.309	0.059	10.57	0.95	0.279	0.059	0.38	2.30
	0.749	0.140	0.29	1.71	0.744	0.136	2.08	0.93	0.749	0.134	0.14	0.44
ARMA					0.216	0.202	0.86	3.66	0.214	0.206	1.80	2.51
					0.284	0.053	0.22	0.69	0.297	0.058	5.69	0.46
					0.749	0.135	0.11	1.29	0.749	0.135	0.10	0.61
					Signal 3							
True	0.250	0.170			0.250	0.170			0.250	0.170		
value	0.330	0.120			0.330	0.120			0.330	0.120		
	0.780	0.130			0.780	0.130			0.780	0.130		
	0.870	0.070			0.870	0.070			0.870	0.070		
EMO	0.249	0.175	0.76	3.24	0.250	0.169	0.37	1.77	0.250	0.169	0.19	0.82
ESPRIT	0.330	0.119	0.42	2.57	0.330	0.122	0.24	1.18	0.330	0.119	0.24	1.18
	0.782	0.131	0.36	1.79	0.780	0.129	0.18	0.09	0.780	0.129	0.08	0.61
	0.870	0.071	0.19	0.09	0.869	0.071	0.11	0.49	0.870	0.070	0.07	0.29
Modified	0.252	0.162	0.19	2.29	0.249	0.177	0.26	0.15	0.248	0.172	0.24	1.36
Prony	0.330	0.123	0.17	1.14	0.331	0.122	0.25	1.00	0.324	0.126	0.11	0.73
	0.780	0.114	0.16	7.43	0.780	0.131	0.11	0.70	0.779	0.137	0.12	0.63
	0.869	0.070	0.04	0.63	0.870	0.069	0.05	0.18	0.871	0.071	0.04	0.30
ARMA					0.223	0.150	1.50	4.41	0.235	0.186	1.48	2.56
					0.341	0.152	0.85	3.77	0.337	0.122	0.66	2.51
					0.785	0.102	0.69	4.07	0.780	0.118	0.38	2.19
					0.868	0.081	0.32	2.27	0.870	0.078	0.13	0.86

ARMA method fails to estimate the modal parameters of Signals 2 and 3 with 5 dB SNR. So the columns are left blank.

tions performed. It is observed that for signals which do not have closely spaced modes like Signal 1, the error in frequency estimation is negligible for all the methods. But the proposed method provides better estimation of attenuation factor than the other two methods. For instance, while estimating Signal 1 at 5 dB SNR, the error in attenuation factor estimation of 0.75 Hz mode using the proposed method is 0.001 while that of ARMA and modified Prony method is 0.112 and 0.113 respectively. In terms of percentage error, this amounts to 0.23% for the proposed method while for ARMA and modified Prony methods, it is approximately 9%.

It is also noted that when the modes of the signal are closer to each other, as in Signal 2, the performance of the proposed method is much better than that of the modified Prony and ARMA methods. This is evident from Figs. 3 and 4 where absolute error in frequency and attenuation factor estimation of Signal 2 obtained through ARMA, the modified Prony and the proposed method are plotted at different SNR levels. It is noticed that, the absolute error in frequency and attenuation factor estimation is comparatively lesser for the proposed method when compared to that of other two methods. The maximum absolute error in the frequency and attenuation factor estimation of Signal 2 with 5 dB SNR is 7.7% and 27.6% for modified Prony method whereas that of the proposed model is only 0.2% and 3.8%. ARMA method is not considered for this comparison as it fails to detect all the modes present in this signal. Moreover, the standard deviation of the proposed method is very small which implies that almost all the estimations are similar. Hence, it can be inferred that, when compared to the modified Prony and ARMA methods, the proposed method is better suited for estimating the signal parameters especially when the signal has high noise contamination.

Table 2 shows the comparison of modes estimated by TLS-ESPRIT and EMO ESPRIT methods. In this table, MO represents the model order of the system. As explained in the Section 2.1, the model order of EMO ESPRIT method is found using EMO algorithm whereas that of TLS-ESPRIT method is estimated using K(i) index obtained from (2). Three values of K(i) which are closer to one i.e. 0.935, 0.99, 0.995 are used for this comparison. It is observed that while estimating signals with less number of frequency components, fictitious modes are present in the estimated results of TLS-ESPRIT method due to over estimation of model order. For instance, while analysing Signal 1 with 5 dB SNR, TLS-ESPRIT method gave four modes of which two are fictitious (9.226 Hz and 17.847 Hz) at K(i) = 0.935. The results are worse in case of K(i) = 0.99 and K(i) = 0.995 as the number of modes estimated are 46 and 90 respectively. This is due to the usage of incorrect value of K(i). Fig. 5 shows the plot of K(i) vs i of Signal 1 with 5 dB SNR. It can be noticed that for accurate estimation of the modes

Table 2
Sensitivity analysis of the proposed method with TLS-ESPRIT method for different signals [17].

SNR	TLS-ESPRIT									EMO ESPRIT		
	k(i) = 0.935			k(i) = 0.99			k(i) = 0.995					
	MO	Freq	AF	MO	Freq	AF	MO	Freq	AF	MO	Freq	AF
						Signal 1						
	4	0.1999	0.1683		0.1992	0.1763	90	0.1981	AF	2	0.1990	0.1743
5		0.7500	0.1304	46	0.7475	0.1279		0.7525	AF		0.7490	0.1281
		9.226	0.0122		+44 other components			+ 88 other components				
		17.847	0.0733									
	4	0.2004	0.1712	4	0.1999	0.1716	4	0.2000	0.1717	2	0.2001	0.1708
10		0.7419	0.1193		0.7462	0.1272		0.7500	0.1299		0.7500	0.1301
		14.6867	-0.0002		10.312	0.0076		7.6169	0.0242			
					18.799	-0.0321		16.9532	0.0499			
	3	0.1999	0.1701	4	0.2000	0.1693	4	0.2000	0.1689	2	0.1998	0.1698
15		0.7500	0.1301		0.7500	0.1301		0.7499	0.1301		0.7500	0.1311
		14.224	0.0063		10.7566	−0.0011		10.6568	0.0074			
					8.5599	0.0003		19.2177	-0.0107			
						Signal 2						
	2	0.2613	0.0177	4	0.2007	0.1717	4	0.2015	0.1732	3	0.2004	0.1744
5		0.7374	0.0855		0.2805	0.0522		0.2804	0.0512		0.2803	0.0509
					0.7500	0.1313		0.7503	0.1324		0.7497	0.1322
					12.6422	0.0014		13.573	0.0014			
	2	0.2612	0.0177	4	0.2014	0.1687	4	0.2005	0.1697	3	0.2002	0.1721
10		0.7371	0.0830		0.2802	0.0512		0.2802	0.0504		0.2800	0.0509
					0.7499	0.1296		0.7501	0.1308		0.7500	0.1311
					14.217	0.0071		14.487	0.0182			
	2	0.2611	0.0177	4	0.2002	0.1726	4	0.2003	0.1694	3	0.1999	0.1706
15		0.7375	0.0834		0.2800	0.0508		0.2801	0.0500		0.2800	0.0500
					0.7500	0.1302		0.7500	0.1308		0.7499	0.1310
					13.886	−0.0006		15.103	−0.6663			
						Signal 3						
	3	0.3025	0.2699	4	0.2478	0.1714	6	0.2490	0.1771	4	0.2492	0.1745
5		0.7224	0.0697		0.3314	0.1235		0.3311	0.1139		0.3300	0.1189
		0.8641	0.1127		0.7810	0.1295		0.7803	0.1346		0.7817	0.1305
					0.8697	0.0714		0.8698	0.0701		0.8703	0.0706
								8.2821	0.0067			
								16.469	−0.0015			
	3	0.3025	0.2731	4	0.2504	0.1724	5	0.2496	0.1695	4	0.2503	0.1694
10		0.7217	0.0696		0.3302	0.1174		0.3296	0.1234		0.3301	0.1216
		0.8643	0.1130		0.7799	0.1296		0.7802	0.1281		0.7801	0.1291
					0.8702	0.0710		0.8702	0.0707		0.8699	0.0707
								15.826	0.0071			
15	3	0.3027	0.2729	4	0.2497	0.1714	5	0.2503	0.1705	4	0.2501	0.1699
		0.7228	0.0680		0.3301	0.1201		0.3299	0.1191		0.3301	0.1193
		0.8642	0.1130		0.7800	0.1300		0.7798	0.1303		0.7801	0.1291
					0.8700	0.0705		0.8701	0.0700		0.8699	0.0703
								10.944	−0.002			

Table 3
Comparison of the true mode extraction capability of the proposed method with the methods in [9,17,11].

SNR	Signal 2								Signal 3							
	TLS-ESPRIT		EMO ESPRIT		PRONY		ARMA		TLS-ESPRIT		EMO ESPRIT		PRONY		ARMA	
	γ_1	γ_2	γ_1	γ_2	γ_1	γ_2	γ_1	γ_2	γ_1	γ_2	γ_1	γ_2	γ_1	γ_2	γ_1	γ_2
5	49	1	49	1	17	33	8	42	42	8	44	6	17	33	-	-
10	50	0	50	0	18	32	11	39	47	3	49	1	19	31	10	40
15	50	0	50	0	20	30	12	38	50	0	50	0	22	28	11	39
20	50	0	50	0	25	25	14	36	50	0	50	0	24	26	13	37

γ_1 Number of times all the modes in the signal are estimated.
γ_2 Number of times only some modes in the signal are estimated.

of Signal 1, which has only two frequency components, the optimum value of K(i) should be around 0.75. The usage of higher value of K(i) leads to incorrect model order estimations resulting in fictitious modes.

It is also noticed that, while analysing signals with higher number of frequency components, like Signal 3, using the TLS-ESPRIT method, there are chances of under estimation of model order for the chosen value of K(i). Fig. 6 shows the K(i) vs i plots of Signal 3 with 15 dB SNR. From the graph, it can be concluded that the optimum value of K(i) for accurate estimation of MO of this signal is around 0.99. If the value of K(i) used in the TLS-ESPRIT method is less than this optimum value, then one or more modes present in the signal are not estimated. Moreover, the estimated modes are inaccurate with a high degree of error. This is evident from the estimated results of Signal 3 using TLS-ESPRIT method when K(i) is set as 0.935. In this case, only three modes out of four of Signal

Table 4
Performance of the proposed method for Signal 3 with change in PMU reporting rate.

PMU reporting rate	γ_1	γ_2	Estimated		Standard deviation		Absolute percentage Error	
			Freq (Hz)	AF	Freq (Hz)	AF	Freq (Hz)	AF
10	50	0	0.2507	0.1729	0.0054	0.0207	0.2800	1.712
			0.3300	0.1187	0.0027	0.0133	0	1.086
			0.7798	0.1303	0.0024	0.0116	0.0256	0.238
			0.8700	0.0699	0.0012	0.0057	0	0.7112
15	50	0	0.2499	0.1735	0.0040	0.0147	0.0413	2.05
			0.3311	0.1188	0.0020	0.0119	0.3369	1.00
			0.7793	0.1291	0.0019	0.0102	0.0891	0.69
			0.8703	0.0706	0.00094	0.0051	0.0345	0.56
20	50	0	0.2496	0.1708	0.0032	0.0157	0.0112	0.47
			0.3308	0.1193	0.0021	0.0112	0.2446	0.58
			0.7800	0.1294	0.0016	0.0071	0	0.46
			0.8700	0.0707	0.00068	0.0043	0	0.426
30	50	0	0.2508	0.1725	0.0027	0.0117	0.32	1.47
			0.3298	0.1182	0.0018	0.00085	0.0606	1.50
			0.7797	0.1293	0.0014	0.0073	0.0384	0.53
			0.8701	0.0708	0.00061	0.0040	0.011	0.56
60	50	0	0.2499	0.1703	0.0017	0.0080	0.04	0.1764
			0.3302	0.1197	0.0011	0.0060	0.0606	0.2546
			0.7801	0.1300	0.00071	0.0049	0.0128	0
			0.8700	0.0708	0.00046	0.0022	0	0.5612
100	50	0	0.2500	0.1703	0.0014	0.0065	0	0.171
			0.3300	0.1210	0.00086	0.0046	0	0.833
			0.7801	0.1296	0.00065	0.0038	0.0128	0.3243
			0.8700	0.0703	0.00038	0.0016	0	0.1412

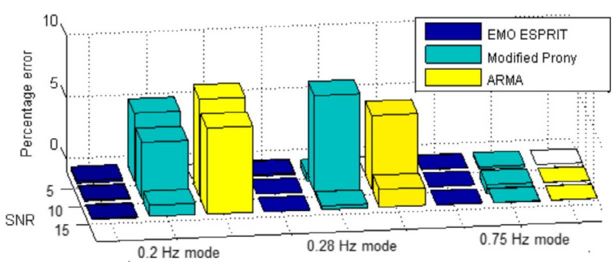

Fig. 3. Absolute error in the frequency estimation of modes of Signal 2 with different methods at different SNR levels.

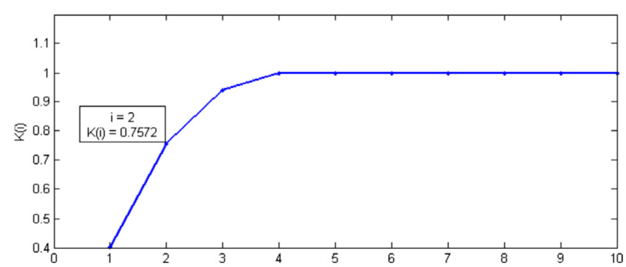

Fig. 5. K(i) vs i plot of Signal 1 at 5 dB SNR.

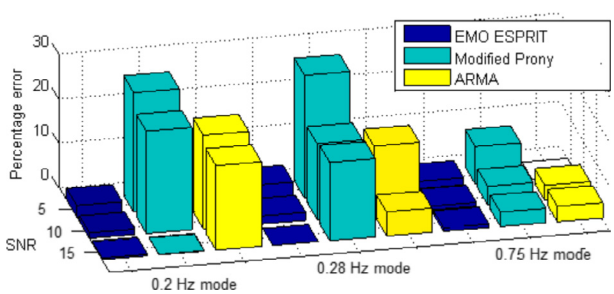

Fig. 4. Absolute error in the attenuation factor estimation of modes of Signal 2 with different methods at different SNR levels.

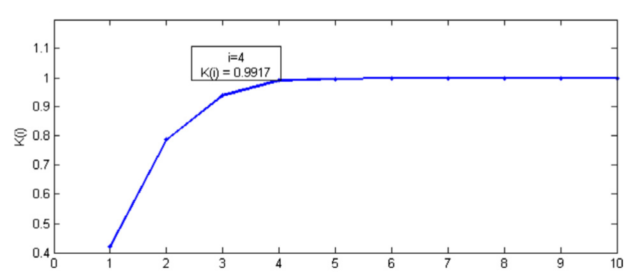

Fig. 6. K(i) vs i plot of Signal 3 at 15 dB SNR.

3 are identified and the error in the frequency and damping estimation of these modes are quite high. This issue can be prevented by setting the value of K(i) to a higher value, but it may cause over estimation of model order resulting in fictitious modes.

So, it can be inferred that accurate estimation of model order in TLS-ESPRIT method is difficult as the optimum value of K(i) varies according to the signal considered. It will be closer to one for signals with more number of frequency components and less for signals with less frequency components as evident from Figs. 5 and 6. Hence, using a common K(i) value for analysing all the signals will result in under estimation or over estimation of model order for most of the signals. Furthermore, it is observed that irrespective of the signal analysed, the estimated results of the TLS-ESPRIT method changes with change in the value of K(i). However, all these issues are not present in EMO ESPRIT method as its model order is estimated using a completely different algorithm making it best suited for analysing low frequency oscillations than TLS-ESPRIT method.

Table 3 shows the comparison of the true mode extraction capability of the proposed method with that of the TLS-ESPRIT method

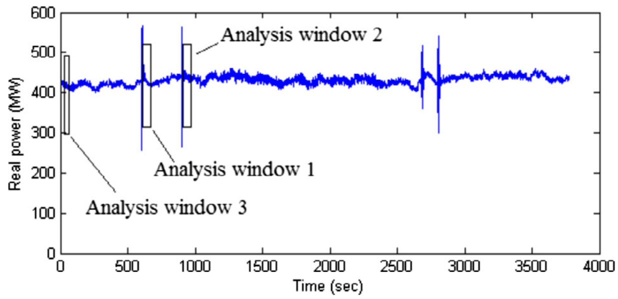

Fig. 7. Probing data corresponding to flow of real power [26].

[17], modified Prony method [9] and ARMA method [11]. Fifty independent simulations were performed on each signal at different SNRs for this purpose. It is observed that, in case of signals with closely separated modes, the proposed method outperforms the other methods with higher prediction rate except TLS-ESPRIT. For instance, while estimating the modal parameters of the Signal 3 with 5 dB SNR, the percentage of successful estimation of all modes in the signal is 88% for the proposed method while that of the modified Prony method is only 34%. The ARMA method is not able to detect all the modes in this case. It is also noticed that while estimating the true modes of the signals using modified Prony and ARMA methods, the simulated results have some fictitious modes present in it along with the true modes of the signal. This is true for TLS-ESPRIT method also when the model order estimation is not accurate. If prior information is not available about the modes of the signal being estimated, then accurate estimation of these modes may not be possible. So it can be inferred that the proposed model has better true mode extraction capability than the models in [9,17,11].

In practical scenario, the PMU reporting rate vary from 10 Hz to 100 Hz. Therefore, simulations have been carried out to verify the performance of the proposed method with change in PMU reporting rate and the results obtained are listed in Table 4. Fifty independent simulation of Signal 3 at each reporting rate is performed for this purpose and the frequency and damping is estimated by taking the means of these fifty simulations. It is observed that irrespective of the reporting rates of the PMU, all the modes were accurately estimated even though the signal has close modes. It is also noted that the error in the frequency estimation is negligible even at low reporting rates. For instance, when the PMU reporting rate is 10 Hz, the absolute error in the frequency and damping estimation is only 0.28% and 2% respectively. It is also observed that, at higher reporting rates, the results of the estimation are better than that of low rates. The standard deviation of the measurements of the modal parameters is also quite small as expected. Therefore, it can be concluded that the proposed method is quite robust even under low reporting rate conditions and high noise levels.

3.2. Western Electricity Coordinating Council (WECC) system

The performance validation of the proposed method is carried out using the probe test data from WECC system obtained on 14^{th} September 2005. Three windows with window lengths of 8.6 s as shown in Fig. 7 are used for this purpose. Analysis window 1 (20:10:11.993–20:10:20.526 UTC) and 2 (20:15:13.324–20:15:21.857 UTC) corresponds to data acquired after first and second sequential probing of $\pm$125 MW respectively whereas analysis window 3 (20:00:03.333–20:00:11.866 UTC) corresponds to ambient data [25,26]. Each analysis window is corrupted by adding white Gaussian noise of 30 dB SNR. The modal estimation of the signals corresponding to these analysis windows are carried out using the proposed method, TLS-ESPRIT method [17], modified Prony method [9] and ARMA method [11]. The frequencies and damping coefficients obtained through these methods are tabulated in Tables 5 and 6.

It is observed from Table 5 that the estimated mode (0.3207 Hz and 8.30% damping) corresponding to analysis window 1 with the proposed method is almost the same as the value reported in [25] whereas ARMA, the modified Prony and the TLS-ESPRIT methods gave less accurate estimation of this mode. Similar observation are made for analysis window 2 and 3 where the proposed method gave a good estimate of the frequency and damping of the modes present in these windows. Table 6 shows that the proposed method can distinguish the modal parameters accurately even under highly noisy conditions. Thus, it can be inferred that, the proposed method is better suited for the estimation of real time signals. However, this algorithm is slightly more computationally intensive than similar methods and hence, its implementation requires a powerful processor.

4. Conclusion

In this paper, an ESPRIT based technique for identification of low frequency oscillations in power systems is proposed. This method requires accurate information about the model order of the system which is obtained using Exact Model Order algorithm. The proposed method estimates the frequency and damping of the signals accurately even under high levels of noise and low PMU reporting rates. The effectiveness of the proposed method is

Table 5
Frequencies and damping ratios of WECC system probe data.

Window	Estimated value from [25]		TLS-ESPRIT		EMO ESPRIT		Modified Prony		ARMA	
	Frequency (Hz)	ζ	Frequency (Hz)	ζ	Frequency (Hz)	ζ	Frequency (Hz)	ζ	Frequency (Hz)	ζ
Window 1	0.318	8.30	0.3259	6.86	0.3207	8.30	0.3167	8.87	0.3626	12.54
Window 2	–	–	0.3151	7.78	.3149	7.88	0.3143	8.25	0.4010	6.78
Window 3	–	–	0.2991	2.43	0.2720	2.81	0.2635	2.74	.02826	3.49

Table 6
Frequencies and damping ratios of WECC system probe data under different noise levels.

Window	SNR = 5		SNR = 10		SNR = 15		SNR = 20	
	Frequency (Hz)	ζ	Frequency (Hz)	ζ	Frequency (Hz)	ζ	Frequency (Hz)	ζ
Window 1	0.3207	8.28	0.3208	8.32	0.3207	8.30	0.3207	8.29
Window 2	0.3157	7.95	0.3149	7.97	0.3143	8.00	0.3146	8.00
Window 3	0.2747	2.78	0.2799	2.83	0.2722	3.07	0.2719	2.84

compared with an ARMA, the TLS-ESPRIT and the modified Prony methods in the literature using synthetic signals as well as test probe data from WECC. Simulation results prove that, irrespective of the noise contamination, presence of close modes in the signal and low PMU reporting rates, the proposed method consistently provides accurate estimation of the frequency and damping of the signal under consideration without the presence of fictitious modes.

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