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Direction-of-arrival estimation based on spatial–temporal statistics without knowing the source number

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ABSTRACT

Direction-of-arrival (DOA) estimation is a central problem in array processing and has a variety of applications. In this paper, a new algorithm for finding DOAs of multiple temporally correlated signals is devised. The proposed approach is based on the joint diagonalization structure of a set of spatio-temporal correlation matrices. Unlike the subspace-based DOA estimators, it is not necessary to estimate the noise or signal subspace explicitly. Moreover, the proposed method can provide the spatial spectrum and estimate the DOAs even when the number of sources is not known *a priori*. Interestingly, it is revealed that the well-known MUSIC method is a special case of our algorithm. Simulation results validate that the developed approach is superior to conventional DOA estimators in terms of resolution capability, estimation accuracy, and robustness against array model errors.

1. Introduction

Direction-of-arrival (DOA) estimation is a basic technique widely used in sensor arrays for localizing radiating sources. This problem can be solved by the parametric methodology which is able to resolve closely spaced DOAs [1]. Conventional high-resolution DOA estimation methods such as maximum likelihood (ML) [2], MUSIC [3,4], ESPRIT [5], and weighted subspace fitting (WSF) [6] estimators require that the number of sources is known *a priori*. Unfortunately, the source number is usually not available in practice. Source enumeration is not an easy task. The information theoretic criteria based techniques for determining the number of sources, such as Akaike's AIC and Rissanen's MDL criteria [7], are effective only when the noise is spatially white and they are very sensitive to spatially correlated noise [8]. Detection of source number with high correct rate in correlated noise is a more difficult task. Numerical analysis rooted rank detection techniques, such as the one proposed in [9], seem to be more accurate and robust than the classical information theoretic ones in the presence of spatially correlated noise. It is worth noting that the rank detection method for source enumeration of [10] is considered in a multiarray setting while this paper considers one array of sensors.

Another assumption that most DOA estimation methods require is the spatial whiteness of the additive measurement noise. However, in practice, the noise is resulted from a number of different phenomena, and it is often correlated across the array [11,12]. Therefore the performance of these methods may be significantly degraded [13]. It is possible to prewhiten the noise after estimating the covariance matrix of the background noise from data collected in the absence of waveform arrivals, nevertheless, such signal-free data are often unavailable. Techniques have been developed for mitigating the effects of the unknown noise field [14–16,18]. The algorithm in [14] incorporates a parametric model for the noise covariance matrix into the estimation process. The method in [15] relies on the assumption that

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the data are received by two well spatially separated arrays so that their noise outputs are uncorrelated. The technique of [16,18] takes advantage of the temporal correlation matrices that are immune to noise. A common limitation of these algorithms is that they require the number of sources to be known.

Moreover, conventional subspace DOA estimation methods strongly rely on the assumption of perfectly calibrated arrays, and hence their performance may be seriously degraded in the presence of array model errors. Indeed, the issue of array calibration has attracted much attention [19]. However, even if we have a perfectly calibrated array, the model errors caused by unideal propagation due to the anisotropy of medium and the reflection and diffraction of environment are generally out of our control. The fact that many realistic sources are spatially distributed rather than point sources also accounts for the model errors. In recent years, the beamforming techniques which are robust against several kinds of uncertainties, including the model errors, are developed [20,21]. However, these robust methods are designed for beamforming rather than DOA estimation. The robust beamformer needs DOA estimates. Therefore a DOA estimation algorithm robust against array model errors is highly desirable.

The purpose of this paper is to devise a non-parametric DOA estimation method in correlated noise field to overcome the aforementioned shortcomings of the existing DOA estimators. By harnessing the joint diagonalization structure of a combined set of spatio-temporal correlation matrices, a new spatial spectrum is derived and the DOAs can be estimated from it subsequently. Compared with the conventional schemes, the proposed algorithm has three advantages. First, it is robust against spatially correlated noise and array model errors. Second, it has no assumptions on the models of noises and signals, and sparsity of the sensor arrays. Third, unlike the subspace-based methods, it is not necessary to determine the number of sources before computing the spatial spectrum. Experimental performance comparison of the proposed technique with the instrumental variable MUSIC (IV-MUS) [16] and joint block diagonalization MUSIC (JBD-MUSIC) [18] schemes, all of which can take advantage of the spatio-temporal correlation matrices, is presented.

The remainder of this paper is organized as follows. In Section 2, the array processing model for DOA estimation is given. In Section 3, we present the new DOA estimation algorithm. Computer simulations are conducted to demonstrate the effectiveness of the proposed algorithm in Section 4. Finally, conclusions are drawn in Section 5.

The notations used in this paper are given as follows. Bold upper-case and lower-case letters denote matrices and vectors, respectively. The superscripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, $(\cdot)^{-1}$, and $(\cdot)^\dagger$ represent the transpose, complex conjugate, Hermitian transpose, inverse, and Moore-Penrose pseudoinverse, respectively. The $\mathrm{j}=\sqrt{-1}$ is the imaginary unit and $\mathbb{C}$ denotes the set of complex numbers. $\|\cdot\|$, $\|\cdot\|_F$, and $\mathrm{tr}(\cdot)$ represent the Euclidean norm of a vector, Frobenius norm, and trace of a matrix, respectively. The operator $\mathrm{E}\{\cdot\}$ denotes expectation and $\boldsymbol{I}$ is the identity matrix.

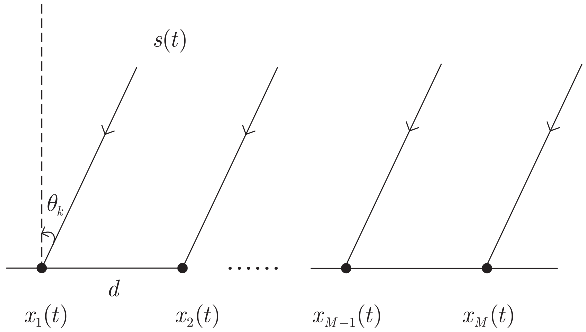

Fig. 1. A ULA of M sensors with inter-element spacing d.

2. Signal model

As shown in Fig. 1, we consider a uniform linear array (ULA) of M sensors with inter-element spacing d, and N $(N<M)$ far-field independent narrowband radiating sources. Let the location of the first sensor be the phase reference point. The signal received by the mth sensor is expressed as

$$x_m(t)=\sum_{n=1}^{N} s_n(t)e^{\mathrm{j}2\pi(m-1)d\ \sin(\theta_n)/\lambda}+w_m(t) \tag{1}$$

where $s_n(t)$ is the nth source signal, λ denotes the signal wavelength, θ_n is the DOA of the nth source, and $w_m(t)$ is the additive noise. Eq. (1) can be rewritten in matrix form compactly as

$$\boldsymbol{x}(t)=\boldsymbol{A}\boldsymbol{s}(t)+\boldsymbol{w}(t) \tag{2}$$

where

$$\begin{aligned}\boldsymbol{x}(t)&=[x_1(t),\ldots,x_M(t)]^T\\ \boldsymbol{s}(t)&=[s_1(t),\ldots,s_N(t)]^T\\ \boldsymbol{w}(t)&=[w_1(t),\ldots,w_M(t)]^T,\end{aligned} \tag{3}$$

and the array manifold matrix $\boldsymbol{A}$ is

$$\boldsymbol{A}=[\boldsymbol{a}(\theta_1),\ldots,\boldsymbol{a}(\theta_N)] \tag{4}$$

with the steering vector

$$\boldsymbol{a}(\theta)=[1,e^{\mathrm{j}2\pi d\ \sin(\theta)/\lambda},\ldots,e^{\mathrm{j}2\pi(M-1)d\ \sin(\theta)/\lambda}]^T. \tag{5}$$

We aim at estimating the DOAs of the N sources $\{\theta_n\}_{n=1}^N$ using the received signals of the array. The conventional subspace method requires that the number of sources must be strictly less than the number of sensors, i.e., $N<M$. In our proposed approach, it only needs $N\le M$. In other words, the number of sources can be equal to the number of sensors. Moreover, our method does not need to determine the number of sources in advance.

Herein we give two common assumptions about the properties of the source and noise:

(A1) The sources are zero mean, mutually uncorrelated, and temporally correlated;
(A2) The noises are zero mean, temporally white, and can be spatially correlated.

3. The proposed DOA estimation algorithm

In this section, we develop the algorithm for DOA estimation of multiple temporally correlated signals without knowing the source number.

3.1. Joint diagonalization structure of spatio-temporal correlation matrices

The spatio-temporal correlation matrix of $\boldsymbol{x}(t)\in\mathbb{C}^M$ at time lag τ is defined as

$$\boldsymbol{R_x}(\tau)=\mathrm{E}\{\boldsymbol{x}(t)\boldsymbol{x}^H(t-\tau)\}. \tag{6}$$

Since the noise is temporally white, it follows that

$$\boldsymbol{R_w}(\tau)=\mathrm{E}\{\boldsymbol{w}(t)\boldsymbol{w}^H(t-\tau)\}=\begin{cases}\boldsymbol{0}, & \text{for } \tau\neq 0\\ \boldsymbol{R_w}(0) & \text{for } \tau=0\end{cases}. \tag{7}$$

If the time lag is taken as $\tau\neq 0$, then we have

$$\boldsymbol{R_x}(\tau)=\boldsymbol{A}\boldsymbol{R_s}(\tau)\boldsymbol{A}^H,\quad \forall\ \tau\neq 0 \tag{8}$$

where $\boldsymbol{R_s}(\tau)=\mathrm{E}\{\boldsymbol{s}(t)\boldsymbol{s}^H(t-\tau)\}$ is the spatio-temporal correlation matrix of the sources. Note that $\boldsymbol{R_x}(\tau)$ is immune to noise for $\tau\neq 0$ due to Assumption (A2).

Invoking Assumption (A1), $\boldsymbol{R_s}(\tau)$ is diagonal since the sources are mutually uncorrelated, i.e., $\boldsymbol{R_s}(\tau)$ can be written as

$$\boldsymbol{R_s}(\tau)=\mathrm{diag}\{r_1(\tau),\dots,r_N(\tau)\} \tag{9}$$

where

$$r_n(\tau)=\mathrm{E}\{s_n(t)s_n^*(t-\tau)\},\quad n=1,\dots,N \tag{10}$$

is the autocorrelation function of the nth source with time lag τ. Construct K spatio-temporal correlation matrices $\boldsymbol{R_x}(\tau_k)$ $(1\le k\le K)$ at K different time lags. They have the same decomposition form

$$\boldsymbol{R_x}(\tau_k)=\boldsymbol{A}\boldsymbol{R_s}(\tau_k)\boldsymbol{A}^H=\sum_{n=1}^{N} r_n(\tau_k)\boldsymbol{a}(\theta_n)\boldsymbol{a}^H(\theta_n). \tag{11}$$

Eq. (11) means that the K spatio-temporal correlation matrices have the joint diagonalization structure and span the same range space of $\boldsymbol{A}$, i.e.,

$$\mathrm{range}(\boldsymbol{R_x}(\tau_k))=\mathrm{range}(\boldsymbol{A}),\quad k=1,\dots,K. \tag{12}$$

Therefore, we can utilize these multiple spatio-temporal correlation matrices to identify the range space of the array manifold matrix $\boldsymbol{A}$ and estimate the DOA parameters.

The IV-MUS method [16] exploits the K spatio-temporal correlation matrices for DOA estimation. It estimates the signal and noise subspaces by performing the singular value decomposition (SVD) of the M-by-MK matrix $[\boldsymbol{R_x}(\tau_1),\dots,\boldsymbol{R_x}(\tau_K)]$. The number of sources must be known for separating the noise and signal subspaces. In the next subsection, we will develop a new spatial spectrum for DOA estimation which can avoid the problem of determining the number of sources by exploiting the special joint diagonalization structure of $\{\boldsymbol{R_x}(\tau_k)\}_{k=1}^K$ in (11).

3.2. Cost function design

We derive the cost function for DOA estimation based on (11). For the pth $(p=1,\dots,N)$ source, we define a vector $\boldsymbol{b}_p\in\mathbb{C}^M$ that is orthogonal to the range space spanned by the steering vectors except for $\boldsymbol{a}(\theta_p)$, i.e.,

$$\boldsymbol{b}_p\perp\mathrm{range}\{\boldsymbol{a}(\theta_1),\dots,\boldsymbol{a}(\theta_{p-1}),\boldsymbol{a}(\theta_{p+1}),\dots,\boldsymbol{a}(\theta_N)\}. \tag{13}$$

In other words, we have

$$\boldsymbol{a}^H(\theta_n)\boldsymbol{b}_p=\begin{cases}\boldsymbol{a}^H(\theta_p)\boldsymbol{b}_p, & n=p\\ 0, & n\neq p.\end{cases} \tag{14}$$

Since we have assumed $N\le M$, there must exist nonzero vector $\boldsymbol{b}_p$ which satisfies (14). Substituting (14) into (11), we obtain

$$\boldsymbol{R_x}(\tau_k)\boldsymbol{b}_p=\sum_{n=1}^{N} r_n(\tau_k)\boldsymbol{a}(\theta_n)\boldsymbol{a}^H(\theta_n)\boldsymbol{b}_p=d_k\boldsymbol{a}(\theta_p) \tag{15}$$

where $d_k=r_n(\tau_k)\boldsymbol{a}^H(\theta_p)\boldsymbol{b}_p$ is a scalar. The geometric interpretation of (15) is that there exists a vector $\boldsymbol{b}$ that makes $\boldsymbol{R_x}(\tau_k)\boldsymbol{b}$ and $\boldsymbol{a}(\theta)$ collinear when θ equals one of the true DOAs, i.e.,

$$\boldsymbol{R_x}(\tau_k)\boldsymbol{b}=d_k\boldsymbol{a}(\theta),\quad k=1,\dots,K \tag{16}$$

Collect the unknown parameters $\{d_k\}_{k=1}^K$ into a vector $\boldsymbol{d}=[d_1,\dots,d_K]^T$. Since for all $1\le k\le K$, (16) holds true, it is natural to propose the following cost function for finding the azimuth θ:

$$\begin{aligned}\min\quad & J(\theta,\boldsymbol{b},\boldsymbol{d})=\sum_{k=1}^{K}\|\boldsymbol{R_x}(\tau_k)\boldsymbol{b}-d_k\boldsymbol{a}(\theta)\|^2\\ \text{s.t.}\quad & \|\boldsymbol{d}\|^2=1.\end{aligned} \tag{17}$$

where $\boldsymbol{a}(\theta)$ is the steering vector with parameter θ to be optimized. Since a trivial solution of (17) is $\{\boldsymbol{b}=\boldsymbol{0},\boldsymbol{d}=\boldsymbol{0}\}$, the constraint $\|\boldsymbol{d}\|^2=1$ is needed to avoid this trivial solution. Note that $\boldsymbol{b}$ and $\boldsymbol{d}$ are two unknown vectors to be optimized. Only the DOA parameter θ is of interest, while $\boldsymbol{b}$ and $\boldsymbol{d}$ are nuisance parameters.

3.3. Cost function minimization

The cost function of (17) is a complicated multidimensional nonlinear optimization problem. Since $\boldsymbol{b}$ and $\boldsymbol{d}$ are unknown nuisance parameters to be optimized, it is difficult to use (17) directly to search for the DOAs. In our optimization strategy, the nuisance parameters $\{\boldsymbol{b},\boldsymbol{d}\}$ are eliminated to obtain a simple cost function of θ only. The optimization problem in (17) can be rewritten compactly as

$$\begin{aligned}\min\quad & J(\theta,\boldsymbol{b},\boldsymbol{d})=\boldsymbol{b}^H\boldsymbol{F}\boldsymbol{b}-\boldsymbol{b}^H\boldsymbol{G}(\theta)\boldsymbol{d}-\boldsymbol{d}^H\boldsymbol{G}^H(\theta)\boldsymbol{b}+M\\ \text{s.t.}\quad & \|\boldsymbol{d}\|^2=1\end{aligned} \tag{18}$$

where the M-by-M matrix $\boldsymbol{F}$ is

$$\boldsymbol{F}=\sum_{k=1}^{K}\boldsymbol{R_x}^H(\tau_k)\boldsymbol{R_x}(\tau_k) \tag{19}$$

and $\boldsymbol{G}(\theta)\in\mathbb{C}^{M\times K}$ is

$$\boldsymbol{G}(\theta)=[\boldsymbol{R_x}^H(\tau_1)\boldsymbol{a}(\theta),\dots,\boldsymbol{R_x}^H(\tau_K)\boldsymbol{a}(\theta)]. \tag{20}$$

First, we consider to eliminate the nuisance parameter $\boldsymbol{b}$. For fixed θ and $\boldsymbol{d}$, the optimal $\boldsymbol{b}$ satisfies the following first-order condition:

$$\nabla_{\boldsymbol{b}}J(\theta,\boldsymbol{b},\boldsymbol{d})=\frac{\partial J(\theta,\boldsymbol{b},\boldsymbol{d})}{\partial\boldsymbol{b}^*}=\boldsymbol{F}\boldsymbol{b}-\boldsymbol{G}(\theta)\boldsymbol{d}=\boldsymbol{0} \tag{21}$$

which gives

$$\boldsymbol{b}^{[\mathrm{opt}]} = \boldsymbol{F}^{\dagger}\boldsymbol{G}(\theta)\boldsymbol{d} \tag{22}$$

In the absence of noise, if only one spatio-temporal correlation matrix is adopted, i.e., $K=1$, the rank of $\boldsymbol{F}$ is $N \le M$. In this case and when the number of sources is strictly less than the number of sensors ($N<M$), the matrix $\boldsymbol{F}$ is singular. The optimal $\boldsymbol{b}$ satisfying the optimal condition of (21) is not unique. We can use the minimum norm solution of (21) given in (22) (the pseudoinverse $\boldsymbol{F}^{\dagger}$ is used) as the optimal $\boldsymbol{b}$. In practice, the estimates of spatio-temporal correlation matrices are always noisy, and $K \ge 2$. The matrix $\boldsymbol{F}$ has full rank and the optimal $\boldsymbol{b}$ is unique. Hence the pseudoinverse $\boldsymbol{F}^{\dagger}$ can be replaced with the inverse $\boldsymbol{F}^{-1}$.

By substituting $\boldsymbol{b}^{[\mathrm{opt}]}$ back into (18), the optimization problem with $\boldsymbol{b}$ being eliminated is simplified to

$$\begin{aligned} \min \quad & J(\theta,\boldsymbol{d}) = M - \boldsymbol{d}^H\boldsymbol{G}^H(\theta)\boldsymbol{F}^{\dagger}\boldsymbol{G}(\theta)\boldsymbol{d} \\ \text{s.t.} \quad & \|\boldsymbol{d}\|^2 = 1 \end{aligned} \tag{23}$$

By observing (23), it is clear that for fixed θ, the optimal $\boldsymbol{d}$ is given by the unit-norm eigenvector of $\boldsymbol{G}^H(\theta)\boldsymbol{F}^{\dagger}\boldsymbol{G}(\theta)$ corresponding to its maximum eigenvalue. Hence the estimation of θ is reduced to

$$\min \quad J(\theta) = M - \lambda_{\max}(\boldsymbol{G}^H(\theta)\boldsymbol{F}^{\dagger}\boldsymbol{G}(\theta)) \tag{24}$$

where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of a square matrix. Therefore we introduce a new "spatial spectrum" of the proposed algorithm

$$P(\theta) = \frac{1}{M - \lambda_{\max}(\boldsymbol{G}^H(\theta)\boldsymbol{F}^{\dagger}\boldsymbol{G}(\theta))}. \tag{25}$$

Thus we can detect and estimate the DOAs by searching for the maxima of $P(\theta)$. It is noteworthy that there is no need to determine the number of sources in advance for the proposed method. After plotting the spatial spectrum, the number of sources can be determined by counting the number of peaks in $P(\theta)$.

The main steps for implementing the proposed algorithm for DOA estimation are summarized in Algorithm 1.

Algorithm 1. Proposed DOA estimation algorithm.

Input: Received signals.
Output: DOA estimates.

1. Parameter selection:
   Choose appropriate time lags $\{\tau_1,\ldots,\tau_K\}$ so that the spatio-temporal correlation matrix at time lag τ_k is immune to noise.
2. Estimate the K spatio-temporal correlation matrices $\widehat{\boldsymbol{R}}_{\boldsymbol{x}}(\tau_k)$ at K distinct time lags τ_k $(1 \le k \le K)$.
3. Calculate $\boldsymbol{F}$ according to (19).
4. Plot the spatial spectrum $P(\theta)$ of (25), where $\boldsymbol{G}(\theta)$ is defined in (20).
5. Detect and estimate the DOAs by searching for the maxima of $P(\theta)$.

Remark 1. Selection of appropriate time lag parameters $\{\tau_k\}_{k=1}^K$ is an important issue for the proposed algorithm. First, the nonzero time lag τ_k should be chosen close to the origin so that $\boldsymbol{R_s}(\tau_k)$ has a relatively large magnitude. Therefore we have $\tau_k = k$ $(1 \le k \le K)$. Intuitively, more spatio-temporal correlation matrices with different lags will contain more statistical information. However, in practice, as the correlation matrices have to be estimated from a finite length of snapshots, there exists estimation perturbation. If K is too large, the magnitude of the correlation function will become weak. The estimation error caused by the perturbation of finite length of snapshots and the noise will bring more adverse effect to the relatively weak correlation function. In numerical simulations, we find that the value of K from 3 to 5 can provide a satisfactory performance.

Remark 2. Note that the proposed method depends on the joint diagonal structure shown in (8) and (10), where the source correlation matrix $\boldsymbol{R}_s(\tau)$ is a diagonal matrix. This requires that the sources are mutually uncorrelated. However, it is often confronted with the correlated sources due to multipath propagation. In numerical simulation of Section 4, we find that the proposed method is quite robust to correlated multipath sources.

3.4. Relation between proposed algorithm and MUSIC

In this subsection, we present an interesting result, namely, the MUSIC method is a special case of the proposed algorithm if only one single spatio-temporal correlation matrix $\boldsymbol{R_x}(\tau_1)$ is adopted. For this case, the matrices $\boldsymbol{F}$ of (19) and $\boldsymbol{G}(\theta)$ of (20) are reduced to

$$\boldsymbol{F} = \boldsymbol{R}_{\boldsymbol{x}}^H(\tau_1)\boldsymbol{R}_{\boldsymbol{x}}(\tau_1) \tag{26}$$

$$\boldsymbol{G}(\theta) = \boldsymbol{R}_{\boldsymbol{x}}^H(\tau_1)\boldsymbol{a}(\theta) \tag{27}$$

and we obtain

$$\begin{aligned} \boldsymbol{G}^H(\theta)\boldsymbol{F}^{\dagger}\boldsymbol{G}(\theta) &= \boldsymbol{a}^H(\theta)\boldsymbol{R}_{\boldsymbol{x}}(\tau_1)(\boldsymbol{R}_{\boldsymbol{x}}^H(\tau_1)\boldsymbol{R}_{\boldsymbol{x}}(\tau_1))^{\dagger}\boldsymbol{R}_{\boldsymbol{x}}^H(\tau_1)\boldsymbol{a}(\theta) \\ &= \boldsymbol{a}^H(\theta)\boldsymbol{P}_{\boldsymbol{R_x}}\boldsymbol{a}(\theta) \end{aligned} \tag{28}$$

where

$$\boldsymbol{P}_{\boldsymbol{R_x}} = \boldsymbol{R_x}(\tau_1)\left(\boldsymbol{R}_{\boldsymbol{x}}^H(\tau_1)\boldsymbol{R_x}(\tau_1)\right)^{\dagger}\boldsymbol{R}_{\boldsymbol{x}}^H(\tau_1) \tag{29}$$

is the projection matrix [17] onto the range space of $\boldsymbol{R_x}(\tau_1)$. Then the spatial spectrum of the proposed method shown in (25) is reduced to

$$P(\theta) = \frac{1}{M - \boldsymbol{a}^H(\theta)\boldsymbol{P}_{\boldsymbol{R_x}}\boldsymbol{a}(\theta)}. \tag{30}$$

Note that rank $(\boldsymbol{R_x}(\tau_1)) = N \le M$. When $N=M$, $\boldsymbol{R_x}(\tau_1)$ has full rank and spans the whole space of $\mathbb{C}^M$. This results in $\boldsymbol{P}_{\boldsymbol{R_x}} = \boldsymbol{I}$ and the spatial spectrum in (30) is always infinite. When $N<M$, $\boldsymbol{R_x}(\tau_1)$ is rank deficient and $\boldsymbol{P}_{\boldsymbol{R_x}} \ne \boldsymbol{I}$. The eigenvalue decomposition (EVD) of $\boldsymbol{R_x}(\tau_1)$ is

$$\boldsymbol{R_x}(\tau_1) = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H = \boldsymbol{U}_s\boldsymbol{\Lambda}_s\boldsymbol{U}_s^H + \sigma^2\boldsymbol{U}_n\boldsymbol{U}_n^H \tag{31}$$

where $\boldsymbol{U} = [\boldsymbol{U}_s, \boldsymbol{U}_n]$, $\boldsymbol{\Lambda}_s = \mathrm{diag}\{\lambda_1,\ldots,\lambda_N\}$ is a diagonal matrix containing the N principal eigenvalues in descending order and $\boldsymbol{U}_s = [\boldsymbol{u}_1,\ldots,\boldsymbol{u}_N]^T$ contains the corresponding orthonormal eigenvectors. The $\boldsymbol{U}_n$ contains the remaining $(M-N)$ orthonormal eigenvectors associated with eigenvalue σ^2. Notice that for nonzero time lag $\tau_1 \ne 0$, the eigenvalue associated with the noise subspace will be zero, i.e., $\sigma=0$. In this case, the noise subspace is totally equivalent to the null space. The range space spanned by $\boldsymbol{U}_s$ is known as signal subspace and its orthogonal complement, called noise subspace, is spanned by $\boldsymbol{U}_n$. The MUSIC method is based on the fact that the range space spanned by the array manifold matrix $\boldsymbol{A}$ is orthogonal to the noise subspace. Therefore it adopts the following spatial spectrum:

$$P_{\mathrm{MUSIC}}(\theta)=\frac{1}{\boldsymbol{a}^H(\theta)\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{a}(\theta)}. \tag{32}$$

For known N, the columns of $\boldsymbol{R}_{\boldsymbol{x}}(\tau_1)$ span the signal subspace

$$\mathrm{range}(\boldsymbol{R}_{\boldsymbol{x}}(\tau_1))=\mathrm{range}(\boldsymbol{U}_s). \tag{33}$$

Therefore the projection matrix onto the range space of $\boldsymbol{R}_{\boldsymbol{x}}(\tau_1)$ can be expressed as

$$\boldsymbol{P}_{\boldsymbol{R}_{\boldsymbol{x}}}=\boldsymbol{U}_s(\boldsymbol{U}_s^H\boldsymbol{U}_s)^{-1}\boldsymbol{U}_s^H=\boldsymbol{U}_s\boldsymbol{U}_s^H. \tag{34}$$

Substituting (34) to (30), the spatial spectrum of the proposed method is written as

$$\begin{aligned}P(\theta)&=\frac{1}{M-\boldsymbol{a}^H(\theta)\boldsymbol{U}_s\boldsymbol{U}_s^H\boldsymbol{a}(\theta)}\\&=\frac{1}{\boldsymbol{a}^H(\theta)\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{a}(\theta)}\\&=P_{\mathrm{MUSIC}}(\theta)\end{aligned} \tag{35}$$

where we have used the fact that the signal subspace is orthogonal to the noise subspace, i.e., $\boldsymbol{I}-\boldsymbol{U}_s\boldsymbol{U}_s^H=\boldsymbol{U}_n\boldsymbol{U}_n^H$, and the property of $M=\boldsymbol{a}^H(\theta)\boldsymbol{a}(\theta)$. It is clear that the MUSIC method is a special case of the proposed algorithm. The latter reduces to the MUSIC if only one single spatio-temporal correlation matrix is adopted.

4. Simulation results

The performance of the proposed algorithm is compared to the IV-MUS [16] method, which is a subspace-based DOA estimator that also exploits multiple spatio-temporal correlation matrices. For this purpose, we consider a ULA of $M=5$ sensors separated by half a wavelength and receiving signals from $N=2$ sources. The sources are generated by filtering a complex white Gaussian process by a 7th order moving average (MA) model with coefficients randomly drawn from zero mean, unit variance complex Gaussian distribution. The noise used in this simulation is zero mean, Gaussian distributed, temporally white and spatially correlated. The noise covariance matrix is modeled, as in [16], to have (k,l)th element $[\boldsymbol{R}_{\boldsymbol{w}}]_{k,l}=\sigma^2 0.9^{|k-l|}$, where the noise power level σ^2 is adjusted to give the desired signal-to-noise ratio (SNR) defined as

$$\mathrm{SNR}=10\log_{10}\frac{\mathrm{tr}[\boldsymbol{A}\boldsymbol{R}_{\boldsymbol{s}}(0)\boldsymbol{A}^H]}{\mathrm{tr}[\boldsymbol{R}_{\boldsymbol{w}}(0)]}$$

For both the proposed and IV-MUS algorithms, $K=4$ spatio-temporal correlation matrices at the first four lags are considered, i.e., $\tau_k\in\{1,2,3,4\}$. The correlation matrices are estimated by

$$\widehat{\boldsymbol{R}}_{\boldsymbol{x}}(\tau_k)=\frac{1}{T}\sum_{t=\tau_k+1}^{T}\boldsymbol{x}(t)\boldsymbol{x}^H(t-\tau_k)$$

where T is the number of snapshots.

Experiment 1: In this example, the incident angles of the sources are $\theta_1=10^\circ$ q and $\theta_2=30^\circ$, respectively. The number of snapshots is $T=1000$. The power of the second source is 5 dB lower than that of the first source. The SNR is 5 dB. For the VI-MUS scheme, the source number, i.e., the effective rank of matrix $[\widehat{\boldsymbol{R}}(1),\ldots,\widehat{\boldsymbol{R}}(4)]$, is determined by the rank-detection criterion in [9]. Fig. 2 displays 10 typical spatial spectrum estimates, which shows that the VI-MUS spatial spectrum estimates highly rely on the accuracy of the source enumeration. The spatial spectrum estimates of the proposed algorithm always have two distinct peaks, which suggests that the number of sources is $N=2$.

In the following simulations, we always assume that the number of sources is known for the IV-MUS method.

For the same experiment settings, Fig. 3 displays the probability of resolution (PR) and the root mean squared errors (RMSEs) of the DOA estimates versus SNR for 500 Monte-Carlo runs. The sources are considered as resolved if both the errors of $\widehat{\theta}_1$ and $\widehat{\theta}_2$ are less than 5°. The RMSE is averaged over the runs that the sources are successfully resolved. This figure shows that the PR and RMSE performance of the proposed algorithm is better than that of the VI-MUS method when SNR ≤ 0 dB. The performance of the two algorithms is comparable when SNR$\ge$5 dB.

Experiment 2: In this example, the two sources are closely spaced with $\theta_1=-5^\circ$ and $\theta_2=5^\circ$. The number of snapshots is 800. Both sources have identical variance. Figs. 4 and 5 show the PR and RMSE of the DOA estimates versus SNR for 1000 Monte-Carlo runs. The sources are

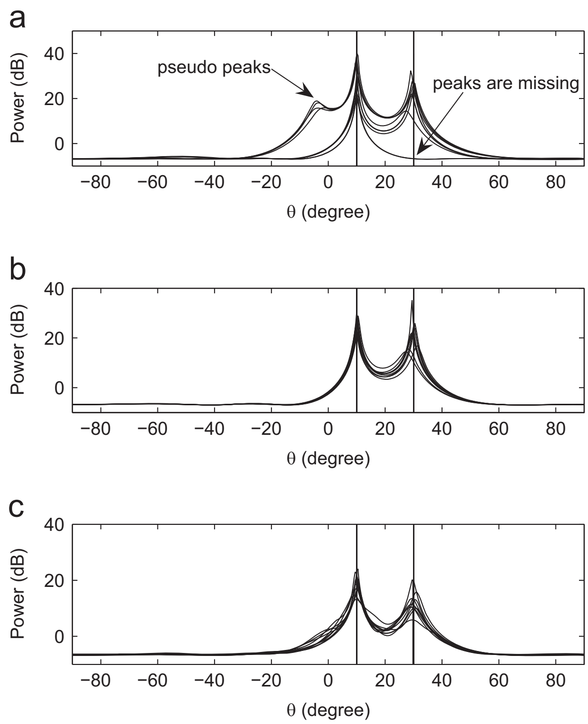

Fig. 2. Plots of 10 typical spectrum estimates. Vertical lines show the true DOAs. The number of sources $\widehat{N}$ is estimated by the rank-detection criterion in [9]. When $\widehat{N}=3$, pseudo peaks arise in the spatial spectrum estimates of VI-MUS, as shown in the top panel of Fig. 2. When $\widehat{N}=1$, only one peak indicating the DOA estimate of first source, which has higher power, arises in the spatial spectrum estimate of VI-MUS. VI-MUS has sufficiently good performance only if N is exactly known, as shown in the middle panel of Fig. 2. The number and DOA estimates of sources are clearly shown in the spatial spectrum estimates of the proposed algorithm, as shown in the bottom panel of Fig. 2. (a) IV-MUS (N is estimated), (b) IV-MUS (N is Known) and (c) Proposed algorithm (N is Unknown)

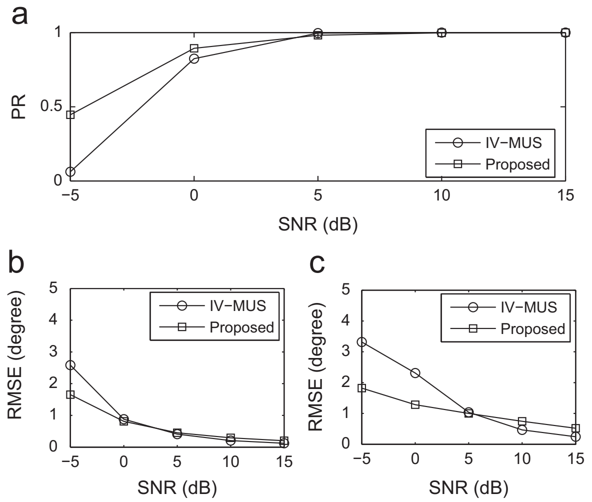

Fig. 3. PR and RMSE versus SNR for two sources. The power of source 2 is 5 dB lower than that of source 1. (a) Probability of resolution, (b) θ_1 and (c) θ_2

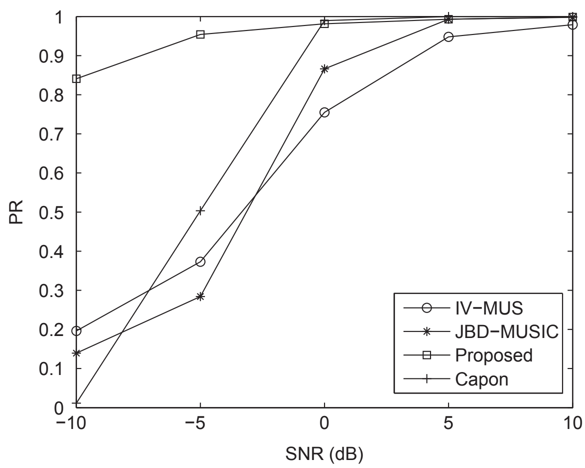

Fig. 4. PR versus SNR.

considered as resolved if both the errors of $\widehat{\theta}_1$ and $\widehat{\theta}_2$ are less than 5°. It shows that the PR and RMSE performances of the proposed algorithm outperform those of VI-MUS and JBD-MUSIC methods at low SNR. They have comparable performance at high SNR.

Experiment 3: The experiment is devised to compare the robustness against the array model errors of the three DOA estimation algorithms. The additive array model error of $\boldsymbol{A}$ can be expressed as

$$\boldsymbol{A} = \boldsymbol{A}_0 + \Delta\boldsymbol{A} \tag{36}$$

where $\boldsymbol{A}_0$ satisfies the ideal model in (4), and $\Delta\boldsymbol{A}$ is a perturbation term. We assume $\Delta\boldsymbol{A}$ is time invariant since the number of snapshots for DOA estimation is small in our simulations. In practice, many factors account for the model errors of A. These factors include the array errors (including pattern error, channel gain and phase error, position error and cross coupling), the unideal propagation due to the anisotropy of medium and the reflection and diffraction of environment, the spatially distributed

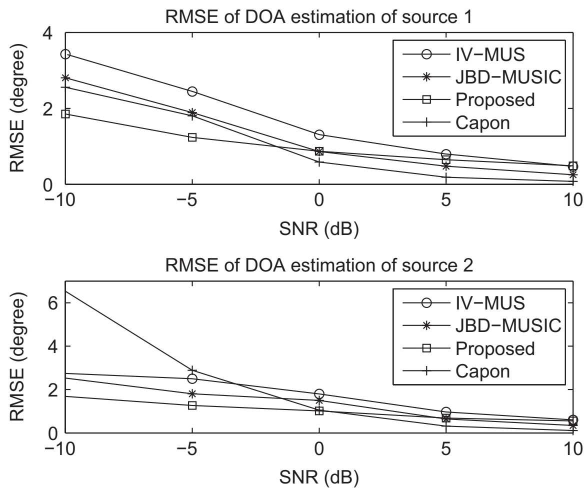

Fig. 5. RMSEs versus SNR.

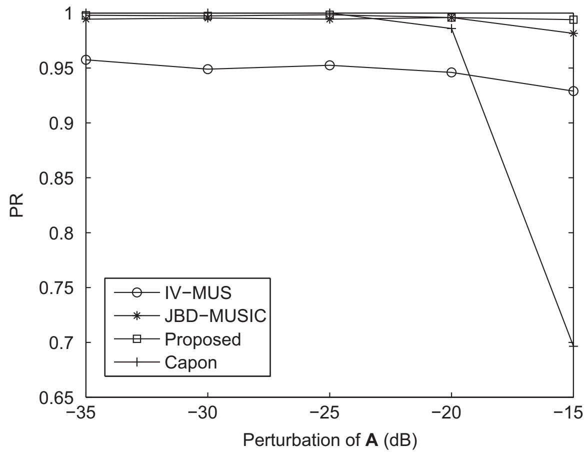

Fig. 6. PR versus the array model perturbation $\delta\boldsymbol{A}$.

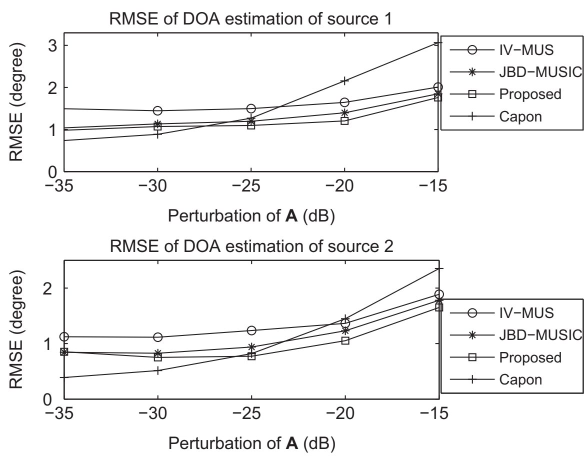

Fig. 7. RMSEs versus the array model perturbation $\delta\boldsymbol{A}$.

property of realistic sources, and the nonlinearity of amplifiers, etc. According to the laws of large number, it is reasonable to assume that both the real and imaginary parts of all the elements of $\Delta \boldsymbol{A}$ are Gaussian distributed with zero mean and identical variance. We define the relative perturbation of $\boldsymbol{A}$ in dB as

$$\delta \boldsymbol{A} = 10 \log_{10}\left(\frac{\|\Delta \boldsymbol{A}\|_F^2}{\|\boldsymbol{A}_0\|_F^2}\right). \tag{37}$$

The DOAs of the two sources are set to $\theta_1 = -10^\circ$ and $\theta_2 = 10^\circ$ and the number of snapshots is 800. The other experiment parameters are the same as Experiment 1. Figs. 6 and 7 illustrate the PR and RMSEs of the DOA estimates versus the relative array model perturbation $\delta \boldsymbol{A}$. It can be seen that the proposed method is more robust against the array model errors than the IV-MUS and JBD-MUSIC methods.

Experiment 4: This experiment is designed to validate the robustness to the correlated sources. The DOAs of two correlated sources are $\theta_1 = -6^\circ$ and $\theta_2 = 6^\circ$ and SNR = 5 dB. The other simulation parameters are the same as Experiment 1. The normalized correlation coefficient of the two sources is defined as

$$\rho = \frac{|\mathrm{E}\{s_1(t)s_2^*(t)\}|}{\sqrt{\mathrm{E}\{|s_1(t)|^2\}\mathrm{E}\{|s_2(t)|^2\}}}. \tag{38}$$

The two sources are uncorrelated when $\rho = 0$. When $\rho = 1$, they are 100% correlated (coherent). The larger value ρ, the higher correlation the two sources have. Figs. 8 and 9 show the PR and RMSEs of the DOA estimates versus ρ, which varies from 0 to 1. As can be seen, all of the three methods degrade if the sources are correlated, especially in the case of coherent ($\rho = 1$) sources. The proposed algorithm has certain robustness to sources correlation and outperforms the IV-MUS and JBD-MUSIC methods. Therefore it can be applied to DOA estimation of correlated sources with moderate correlation coefficient.

5. Conclusion

This paper proposes a new DOA estimation algorithm in correlated noise field. An interesting feature of the proposed algorithm is that it is not necessary to determine the number of sources before plotting the spatial spectrum. The number of sources is determined by counting the number of dominant peaks of the spatial spectrum. Such a feature is highly desirable for practical applications, because determination of the number of sources is not an easy task. In addition, we have revealed that the MUSIC method is a special case of our algorithm if only one single spatio-temporal correlation matrix is adopted. Simulation results confirm the effectiveness of the proposed algorithm.

Although it is not necessary to determine the number of sources for plotting the spectrum, we should determine the number of sources by inspecting the spectrum $P(\theta)$ in the proposed method. In our simulations, N can be easily determined. Our future work is to develop a robust statistical detection method for deciding the number of sources.

In addition, our algorithm requires an exhaustive spectral search over the DOA parameter, which is computationally demanding. Developing a spectral search-free version of the proposed algorithm taking example by the root-MUSIC [22] and other search-free techniques [23], is highly desirable.

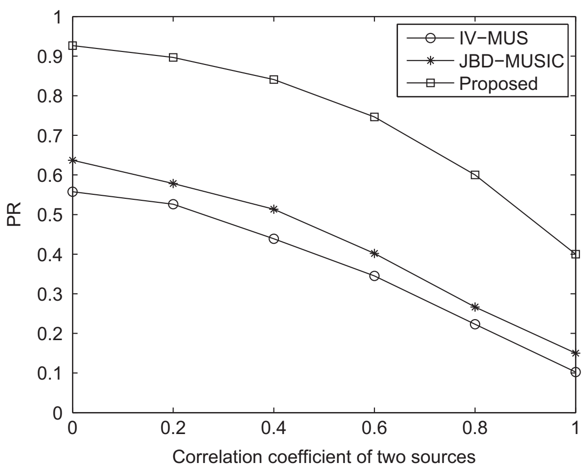

Fig. 8. PR versus normalized correlation coefficient ρ of the two sources.

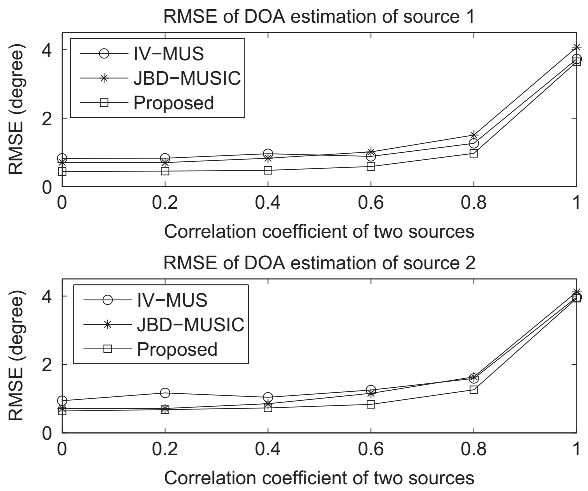

Fig. 9. RMSEs versus normalized correlation coefficient ρ of the two sources.

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