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Fast communication

DOA estimation for noncircular signals in the presence of mutual coupling ☆

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ABSTRACT

This communication considers the problem of direction-of-arrival (DOA) estimation for multiple uncorrelated noncircular signals in the presence of mutual coupling. A method is proposed to eliminate the effect of mutual coupling for uniform linear arrays (ULAs). By taking advantage of the special structure of the mutual coupling matrix, the mutual coupling free covariance matrix and elliptic covariance matrix, the mutual coupling coefficients can be estimated using an iterative algorithm. The estimated mutual coupling coefficients can be embedded within any DOA estimation algorithm for circular signals or noncircular signals. Simulation results demonstrate the effectiveness of the proposed method.

1. Introduction

In the fields of radar, sonar and mobile communication, the problem of estimating the directions-of-arrival (DOAs) of plane waves impinging on a sensor array has been an important topic. Many high-resolution methods have been proposed [1–3]. However, most of these methods rely on exact knowledge of the array manifold. But the array manifold is often affected by unknown array characteristics such as mutual coupling between sensors.

To eliminate or reduce the effect of mutual coupling, a lot of calibration methods have been developed over the last decades. In [4], Friedlander et al. proposed an iterative method to compensate mutual coupling as well as the perturbation of gain and phase. Although this method can calibrate different types of array error, it requires a preliminary estimate of the array perturbations. In the literature [5–7], a class of non-sensitive DOA estimation methods compensating the effects of mutual coupling are presented. By setting a group of sensors as auxiliary sensors, the DOAs can be directly estimated without compensating for mutual coupling. The main advantage of these methods is that no iteration is required but aperture loss is caused by setting auxiliary sensors. In [8], Sellone et al. proposed another iterative method, which alternatingly minimizes a cost function with respect to two complex valued symmetric Toeplitz matrices and a complex Hermitian Toeplitz matrix. Compared with the method proposed in [4], this method is less sensitive to the array perturbations and can get an effective DOA estimation without preliminary estimate in many cases.

All methods mentioned above just rely on the positive definite Hermitian covariance matrix of the array output. In mobile communications, the baseband signals such as binary-phase-shift-keying (BPSK) and offset-quadrature-phase-shift-keying (OQPSK) modulated signals are complex

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noncircular. Previous literature has shown that exploitation of the noncircularity helps to improve the performance of DOA estimation and many DOA estimation methods based on both the Hermitian covariance matrix and the complex valued symmetric elliptic covariance matrix have been proposed [9–11]. Similarly, the elliptic covariance matrix can also be exploited to calibrate the array perturbations. However, very few contributions have been devoted to the array calibration for noncircular signals.

In this communication, we present an extension of the work in [8] to mutual coupling calibration utilizing the noncircularity of the signals. Utilizing the special structure of the elliptic covariance matrix, a new cost function is defined based on the Hermitian covariance matrix and the symmetric elliptic covariance matrix. Then the mutual coupling matrix (MCM) is estimated by minimizing the cost function alternatingly with respect to two complex valued symmetric Toeplitz matrices (the mutual coupling matrices), a Hermitian Toeplitz matrix (the mutual coupling free covariance matrix) and an anti-Toeplitz matrix (the mutual coupling free elliptic covariance matrix). Then the estimated mutual coupling matrix can be embedded within any DOA estimation algorithm such as the MUSIC [2] for circular signals and NC-MUSIC [11] for noncircular signals.

Notation: upper (lower) bold face letters are used to denote matrices (column vectors). $\{\cdot\}^*$, $\{\cdot\}^T$, $\{\cdot\}^H$, $\otimes$, diag$\{\cdot\}$ and $E\{\cdot\}$ denote conjugate, transpose, conjugate transpose, Kronecker product, diagonalization and expectation, respectively. $[\cdot]_i$ and $[\cdot]_{i,j}$ denote the ith element of a vector and the element of matrix in the ith row and the jth column, respectively.

2. ARRAY model

Consider K narrowband uncorrelated noncircular signals $s_k(t)$, $k=1,2,\cdots,K$ impinging on a uniform linear array (ULA) with M omni-directional sensors. The DOA of the kth signal is denoted by θ_k. Assume $\mathbf{C}$ denotes the MCM of this ULA. Then the output of the array can be written as

$$\mathbf{x}(t)=\mathbf{CA}(\boldsymbol{\theta})\mathbf{s}(t)+\mathbf{n}(t), \tag{1}$$

where $\mathbf{x}(t)=[x_1(t),x_2(t),\cdots,x_M(t)]^T$, $\mathbf{A}(\boldsymbol{\theta})=[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\cdots,\mathbf{a}(\theta_K)]$, $\mathbf{s}(t)=[s_1(t),s_2(t),\cdots,s_K(t)]^T$ and $\mathbf{n}(t)=[n_1(t),n_2(t),\cdots,n_M(t)]^T$ denote the received signal vector, the ideal steering matrix, the source signal vector, and the additive noise vector, respectively. The steering vector can be expressed as $\mathbf{a}(\theta)=[1,e^{j2\pi d\ \sin\ \theta/\lambda},\cdots,e^{j2\pi(M-1)d\ \sin\ \theta/\lambda}]^T$, where d and λ denote the inter-sensor spacing and the signal carrier wavelength, respectively.

Since the mutual coupling coefficient between two sensors is inversely proportional to their distance, it is reasonable to approximate the mutual coupling coefficient as zero between two sensors that are far away from each other. For ULAs, the MCM $\mathbf{C}\in\mathbb{C}^{M,M}$ can be expressed as a banded symmetric Toeplitz matrix [4,8]

$$[\mathbf{C}]_{i,j}=\begin{cases} c_{|i-j|} & |i-j|<P \\ 0 & \text{otherwise} \end{cases}, \tag{2}$$

The relationship of (2) can be denoted as $\mathbf{C}=\text{Stoeplitz}(\mathbf{c})$, where $\mathbf{c}=[c_0,c_1,\cdots,c_{P-1}]^T$, $P\le M$ and $1=c_0>|c_1|>|c_2|\cdots>|c_{P-1}|$. Assume the noise $\mathbf{n}(t)$ is complex circular, zero-mean, spatially white and independent with the signals while the signal $\mathbf{s}(t)$ is complex noncircular and zero-mean. In this communication, only the circularity at the order 2 is considered. A signal $s(t)$ is said to be noncircular, if the elliptic covariance $\text{E}\{s^2(t)\}$ does not equal zero [12]. Then the covariance and elliptic covariance matrix of the array output can be expressed as follows:

$$\mathbf{R}_x=\text{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\}=\mathbf{CAR}_s\mathbf{A}^H\mathbf{C}^H+\sigma_n^2\mathbf{I}_M, \tag{3}$$

$$\mathbf{R}'_x=\text{E}\{\mathbf{x}(t)\mathbf{x}^T(t)\}=\mathbf{CAR}'_s\mathbf{A}^T\mathbf{C}^T, \tag{4}$$

where $\mathbf{R}_s=\text{diag}([\sigma_1^2,\sigma_2^2,\cdots,\sigma_K^2]^T)$, $\mathbf{R}'_s=\text{diag}([\rho_1 e^{j\phi_1}\sigma_1^2,\rho_2 e^{j\phi_2}\sigma_2^2,\cdots,\rho_K e^{j\phi_K}\sigma_K^2]^T)$, with σ_k^2, ρ_k and ϕ_k to be the power, noncircularity rate and noncircularity phase of the kth signal, σ_n^2 denotes the power of the additive noise. In practice, $\mathbf{R}_x$ and $\mathbf{R}'_x$ can be estimated as $\hat{\mathbf{R}}_x=(1/N_{sample})\Sigma_{t=1}^{N_{sample}}\mathbf{x}(t)\mathbf{x}^H(t)$ and $\hat{\mathbf{R}}'_x=(1/N_{sample})\Sigma_{t=1}^{N_{sample}}\mathbf{x}(t)\mathbf{x}^T(t)$, where N_{sample} is the number of samples. Denote $\mathbf{S}=\mathbf{AR}_s\mathbf{A}^H$ and $\mathbf{S}'=\mathbf{AR}'_s\mathbf{A}^T$. It is easy to deduce that $[\mathbf{S}]_{p,q}=\Sigma_{k=1}^K\sigma_k^2 e^{j2\pi(p-q)d\ \sin\ \theta_k/\lambda}$ and $[\mathbf{S}']_{p,q}=\Sigma_{k=1}^K\rho_k e^{j\phi_k}\sigma_k^2 e^{j2\pi(p+q-2)d\ \sin\ \theta_k/\lambda}$, which means that $\mathbf{S}$ is a Hermitian Toeplitz matrix and $\mathbf{S}'$ is an anti-Toeplitz matrix.

Based on the above observations, the MCM $\mathbf{C}$ can be estimated by solving the following optimization problem given the estimates of $\mathbf{R}_x$, $\mathbf{R}'_x$ and the power of noise σ_n^2 ($\mathbf{R}_x,\mathbf{R}'_x,\sigma_n^2$ are used instead of their estimates for convenience of notation).

$$\{\mathbf{C},\mathbf{S},\mathbf{S}'\}_{est}=\arg\min_{\mathbf{C},\mathbf{S},\mathbf{S}'}||\mathbf{R}_x-\sigma_n^2\mathbf{I}_M-\mathbf{CSC}^H||_F^2+||\mathbf{R}'_x-\mathbf{CS}'\mathbf{C}^T||_F^2$$
$$s.t.\mathbf{C}\in\boldsymbol{T}_{s,M,P},\mathbf{S}\in\boldsymbol{T}_{h,M},\mathbf{S}'\in\boldsymbol{T}_{a,M}, \tag{5}$$

where $\boldsymbol{T}_{s,M,P}$, $\boldsymbol{T}_{h,M}$ and $\boldsymbol{T}_{a,M}$ denote the set of $M\times M$ complex valued banded symmetric Toeplitz matrices satisfying $[\cdot]_{i,j}=0$ when $|i-j|\ge P$, the set of $M\times M$ complex Hermitian Toeplitz matrices and the set of $M\times M$ complex anti-Toeplitz matrices, respectively.

3. The proposed method

In this section, we propose a method for MCM estimation by solving the problem (5). Then the MCM can be embedded within any method of DOA estimation for circular signals or noncircular signals, such as MUSIC [2] and NC-MUSIC [11]. Before developing the proposed method, some useful lemmas are given first.

3.1. Introductory Lemmas

Lemma 1. *Let* $\mathbf{b}_s=[b_0,b_1,\cdots,b_{P-1}]^T$, $\mathbf{B}_s\in\mathbb{C}^{M,M}$, $M\ge P$, $\mathbf{B}_s=\text{Stoeplitz}(\mathbf{b}_s)$ *defined as* $[\mathbf{B}_s]_{i,j}=[\mathbf{b}_h]_{|i-j|}$ *for* $|i-j|<P$ *and* $[\mathbf{B}_s]_{i,j}=0$ *for* $|i-j|\ge P$, *then for any* $\mathbf{x}\in\mathbb{C}^{M,1}$, *we have*

$$\mathbf{B}_s\mathbf{x}=\mathbf{Q}_s(\mathbf{x})\mathbf{b}_s, \tag{6}$$

where $\mathbf{Q}_s(\mathbf{x})$ is the sum of the following two $M\times P$ matrices

$$[\mathbf{Q}_{s1}(\mathbf{x})]_{i,j}=\begin{cases} [\mathbf{x}]_{i+j-1} & i+j\le M+1 \\ 0 & \text{otherwise} \end{cases}, \tag{7}$$

$$[\mathbf{Q}_{s2}(\mathbf{x})]_{i,j}=\begin{cases}[\mathbf{x}]_{i-j+1} & 2\le j\le i\\ 0 & \text{otherwise}\end{cases}. \tag{8}$$

Lemma 2. *Let* $\mathbf{b}_h=[b_{M-1}^*,\cdots,b_1^*,b_0,b_1,\cdots,b_{M-1}]^T$, $\mathbf{B}_h\in\mathbb{C}^{M,M}$, $\mathbf{B}_h=$ Htoeplitz$(\mathbf{b}_h)$ *defined as* $[\mathbf{B}_h]_{i,j}=[\mathbf{b}_h]_{j-i+M}$, *then for any* $\mathbf{x}\in\mathbb{C}^{M,1}$, *we have*

$$\mathbf{B}_h\mathbf{x}=\mathbf{Q}_h(\mathbf{x})\mathbf{b}_h, \tag{9}$$

where $\mathbf{Q}_h(\mathbf{x})$ is the sum of the following two $M\times(2M-1)$ matrices

$$[\mathbf{Q}_{h1}(\mathbf{x})]_{i,j}=\begin{cases}[\mathbf{x}]_{i+j-M} & i+j\le 2M,\ \text{and } j\ge M\\ 0 & \text{otherwise}\end{cases}, \tag{10}$$

$$[\mathbf{Q}_{h2}(\mathbf{x})]_{i,j}=\begin{cases}[\mathbf{x}]_{i+j-M} & i+j\ge M+1,\ \text{and } j\le M-1\\ 0 & \text{otherwise}\end{cases}. \tag{11}$$

Lemma 3. *Let* $\mathbf{b}_a=[b_1,b_2,\cdots,b_{2M-1}]^T$, $\mathbf{B}_a\in\mathbb{C}^{M,M}$, $\mathbf{B}_a=$ Atoeplitz$(\mathbf{b}_a)$, *defined as* $[\mathbf{B}_a]_{i,j}=[\mathbf{b}_a]_{i+j-1}$, *then for any* $\mathbf{x}\in\mathbb{C}^{M,1}$, *we have*

$$\mathbf{B}_a\mathbf{x}=\mathbf{Q}_a(\mathbf{x})\mathbf{b}_a, \tag{12}$$

where $\mathbf{Q}_a(\mathbf{x})$ is defined as

$$[\mathbf{Q}_a(\mathbf{x})]_{i,j}=\begin{cases}[\mathbf{x}]_{j-i+1} & i\le j\le i+M-1\\ 0 & \text{otherwise}\end{cases}. \tag{13}$$

Lemma 4. *Let* $\mathbf{D}_l\in\mathbb{C}^{N,M}$, $\mathbf{G}_l\in\mathbb{C}^{N,MP}$, $\mathbf{b}\in\mathbb{C}^{P,1}$, *then we have*

$$\begin{aligned}\mathbf{b}_{opt}&=\underset{\mathbf{b}}{\operatorname{argmin}}\sum_{l=1}^{L}||\mathbf{D}_l-\mathbf{G}_l(\mathbf{I}_M\otimes\mathbf{b})||_F^2\\&=\{\sum_{l=1}^{L}[\text{vecb}(\mathbf{G}_l,P)]^H\text{vecb}(\mathbf{G}_l,P)\}^{-1}\\&\quad\{\sum_{l=1}^{L}[\text{vecb}(\mathbf{G}_l,P)]^H\text{vec}(\mathbf{D}_l)\},\end{aligned} \tag{14}$$

where vec$(\cdot)$ denotes the stack operation which places in order the columns of a matrix on the top of one another. For matrices $\mathbf{G}_l\in\mathbb{C}^{N,MP}, l=1,2,\cdots,L$, denote $\mathbf{G}_l=[\mathbf{G}_{l,1},\mathbf{G}_{l,2},\cdots,\mathbf{G}_{l,M}]$, with $\mathbf{G}_{l,m}\in\mathbb{C}^{N,P}, m=1,2,\cdots,M$, then vecb$(\cdot)$ is defined as vecb$(\mathbf{G}_l,P)=[\mathbf{G}_{l,1}^T,\mathbf{G}_{l,2}^T,\cdots,\mathbf{G}_{l,M}^T]^T$.

The proofs of Lemmas 1 and 2 are given in references [4,8], respectively. The proof of Lemma 3 can be easily verified. The proof of Lemma 4 can be deduced using the method of conjugate gradient, which is omitted here.

Besides these above lemmas, we also give a useful definition. For $\mathbf{F}=[\mathbf{f}_1,\mathbf{f}_2,\cdots,\mathbf{f}_M]\in\mathbb{C}^{M,M}$, $\mathbf{Q}_s(\mathbf{F})$ is defined as $\mathbf{Q}_s(\mathbf{F})=[\mathbf{Q}_s(\mathbf{f}_1),\mathbf{Q}_s(\mathbf{f}_2),\cdots,\mathbf{Q}_s(\mathbf{f}_M)]$. In the same way, we can define $\mathbf{Q}_h(\mathbf{F})$ and $\mathbf{Q}_a(\mathbf{F})$.

3.2. MCM estimation

The power of noise can be estimated as the mean of the $2M-K$ smallest eigenvalues of extended covariance matrix defined as $\tilde{\mathbf{R}}=\begin{bmatrix}\mathbf{R}_x & \mathbf{R}_x'\\ \mathbf{R}_x'^* & \mathbf{R}_x^*\end{bmatrix}$. For the sake of concise presentation we make a slight abuse of notation and express $\mathbf{R}_x-\sigma_n^2\mathbf{I}_M$ by $\mathbf{R}_x$. From (5), it is easy to note that the cost function is fourth-order dependent on the mutual coupling coefficients. In order to make the problem easy to solve, similar to [8], the matrices $\mathbf{C}$ and $\mathbf{C}^T$ are treated as two independent matrices $\mathbf{C}_L$ and $\mathbf{C}_R^T$. Then the cost function can be written as follows:

$$J(\mathbf{S},\mathbf{S}',\mathbf{C}_L,\mathbf{C}_R)=||\mathbf{R}_x-\mathbf{C}_L\mathbf{S}\mathbf{C}_R^H||_F^2+||\mathbf{R}_x'-\mathbf{C}_L\mathbf{S}'\mathbf{C}_R^T||_F^2. \tag{15}$$

Denote $\mathbf{S}=$ Htoeplitz$(\mathbf{b}_h)$, $\mathbf{S}'=$ Atoeplitz$(\mathbf{b}_a)$, $\mathbf{C}_L=$ Stoeplitz$(\mathbf{c}_L)$, $\mathbf{C}_R=$ Stoeplitz$(\mathbf{c}_R)$, minimization of $J(\mathbf{S},\mathbf{S}',\mathbf{C}_L,\mathbf{C}_R)$ can be implemented using the following alternating iterative procedure.

1) Minimize J with respect to $\mathbf{b}_h$
Only the first term of J is dependent on $\mathbf{b}_h$. So we just need to minimize $J_1=||\mathbf{R}_x-\mathbf{C}_L\mathbf{S}\mathbf{C}_R^H||_F^2$ with respect to $\mathbf{b}_h$. Using Lemma 2, it is easy to write J_1 as

$$J_1(\mathbf{b}_h)=||\mathbf{R}_x-\mathbf{C}_L\mathbf{Q}_h(\mathbf{C}_R^H)(\mathbf{I}_M\otimes\mathbf{b}_h)||_F^2. \tag{16}$$

Using Lemma 4, $\mathbf{b}_h$ should be updated as

$$\mathbf{b}_h=[\mathbf{W}_h^H\mathbf{W}_h]^{-1}\mathbf{W}_h^H\text{vec}(\mathbf{R}_x). \tag{17}$$

where $\mathbf{W}_h=$ vecb$(\mathbf{C}_L\mathbf{Q}_h(\mathbf{C}_R^H),2M-1)$.

2) Minimize J with respect to $\mathbf{b}_a$
Only the second term of J is dependent on $\mathbf{b}_a$. So we just need to minimize $J_2=||\mathbf{R}_x'-\mathbf{C}_L\mathbf{S}'\mathbf{C}_R^T||_F^2$ with respect to $\mathbf{b}_a$. Using Lemma 3, it is easy to write J_2 as

$$J_2(\mathbf{b}_a)=||\mathbf{R}_x'-\mathbf{C}_L\mathbf{Q}_a(\mathbf{C}_R^T)(\mathbf{I}_M\otimes\mathbf{b}_a)||_F^2. \tag{18}$$

Using Lemma 4, $\mathbf{b}_a$ should be updated as

$$\mathbf{b}_a=(\mathbf{W}_a^H\mathbf{W}_a)^{-1}\mathbf{W}_a^H\text{vec}(\mathbf{R}_x'). \tag{19}$$

where $\mathbf{W}_a=$ vecb$(\mathbf{C}_L\mathbf{Q}_a(\mathbf{C}_R^T),2M-1)$.

3) Minimize J with respect to $\mathbf{c}_L$
Denote $\mathbf{F}_L=\mathbf{S}\mathbf{C}_R^H$, $\mathbf{F}_L'=\mathbf{S}'\mathbf{C}_R^T$, using Lemma 1, the cost function J can be written as

$$\begin{aligned}J(\mathbf{c}_L)=&||\mathbf{R}_x-\mathbf{Q}_s(\mathbf{F}_L)(\mathbf{I}_M\otimes\mathbf{c}_L)||_F^2+||\mathbf{R}_x'-\mathbf{Q}_s(\mathbf{F}_L')\\&(\mathbf{I}_M\otimes\mathbf{c}_L)||_F^2.\end{aligned} \tag{20}$$

Using Lemma 4, $\mathbf{c}_L$ should be updated as

$$\mathbf{c}_L=(\mathbf{W}_{L,1}^H\mathbf{W}_{L,1}+\mathbf{W}_{L,2}^H\mathbf{W}_{L,2})^{-1}(\mathbf{W}_{L,1}^H\text{vec}(\mathbf{R}_x)+\mathbf{W}_{L,2}^H\text{vec}(\mathbf{R}_x')). \tag{21}$$

where $\mathbf{W}_{L,1}=$ vecb$(\mathbf{Q}_s(\mathbf{F}_L),P)$, $\mathbf{W}_{L,2}=$ vecb$(\mathbf{Q}_s(\mathbf{F}_L'),P)$.

4) Minimize J with respect to $\mathbf{c}_R$
Note that $\mathbf{R}_x,\mathbf{S},\mathbf{S}'$ and $\mathbf{R}_x'$ satisfy $\mathbf{R}_x=\mathbf{R}_x^H,\mathbf{S}=\mathbf{S}^H$, $\mathbf{S}'=\mathbf{S}'^T$ and $\mathbf{R}_x'=(\mathbf{R}_x')^T$. Using the fact that $||\mathbf{B}||_F=||\mathbf{B}^T||_F=||\mathbf{B}^H||_F$, the cost function J can be written as

$$J=||\mathbf{R}_x-\mathbf{C}_R\mathbf{S}\mathbf{C}_L^H||_F^2+||\mathbf{R}_x'-\mathbf{C}_R\mathbf{S}'\mathbf{C}_L^T||_F^2. \tag{22}$$

Compared with (15), (22) just exchanges the MCM $\mathbf{C}_L$ and $\mathbf{C}_R$. Similar to (21), $\mathbf{c}_R$ should be updated as

$$\mathbf{c}_R=(\mathbf{W}_{R,1}^H\mathbf{W}_{R,1}+\mathbf{W}_{R,2}^H\mathbf{W}_{R,2})^{-1}(\mathbf{W}_{R,1}^H\text{vec}(\mathbf{R}_x)+\mathbf{W}_{R,2}^H\text{vec}(\mathbf{R}_x')). \tag{23}$$

where $\mathbf{W}_{R,1}=$ vecb$(\mathbf{Q}_s(\mathbf{F}_R),P)$,$\mathbf{F}_R=\mathbf{S}\mathbf{C}_L^H$, $\mathbf{W}_{R,2}=$ vecb $(\mathbf{Q}_s(\mathbf{F}_R'),P)$ and $\mathbf{F}_R'=\mathbf{S}'\mathbf{C}_L^T$.
Repeat steps (1)–(4) until convergence. Then the MCM can be estimated as $\mathbf{C}=(\mathbf{C}_L+\mathbf{C}_R)/2$. For the first iteration, $\mathbf{C}_L$ and $\mathbf{C}_R$ can be initialized as the $M\times M$

identity matrix. As the cost function does not increase with the updating of $\mathbf{b}_h, \mathbf{b}_a, \mathbf{c}_L$, and $\mathbf{c}_R$, the iterative method will always converge to a minimum. Although consistency of the proposed estimator is not investigated in this paper, the proposed method will provide estimates of reasonable accuracy in most scenarios, which will be demonstrated by simulation experiments in Section 4.

4. Simulation

In this section, we illustrate the performance of the proposed method through simulations. In the following simulations, a ULA with 8 omnidirectional sensors spaced half a wavelength apart is used. Four BPSK signals with equal power σ_s^2 and random noncircularity phase are coming from directions of $[-41^\circ, -29^\circ, 23^\circ$ and $37^\circ]$. The number of signals $K=4$ is assumed to be known to the receiver. The additive noise is complex white the Gaussian random processes with zero mean. The signal to noise ratio is defined as $SNR = 10\log_{10}(\sigma_s^2/\sigma_n^2)$. Assume that the length of mutual coupling is 3, i.e. $P=3$, and the mutual coupling coefficients are set to $c_0=1$, $c_1=0.6+0.4j$ and $c_2=0.1-0.2j$.

We compare the proposed method with the method proposed in [8]. As mentioned in Section 1, the method proposed in [4] needs effective initial estimate. However, for the scenario given here, DOA estimation methods do not consider the mutual coupling such as MUSIC cannot resolve these four signals. In other words, MUSIC cannot provide effective initial estimate. The methods using auxiliary sensors [5–7] perform well when the number of sensors is obviously larger than the length of mutual coupling and the number of signals (at least larger than $2P-2+K$). However, these methods are not suitable for the scenario given here.

The root mean square error (RMSE) is selected to demonstrate the estimation performance. RMSE of MCM and DOA estimation are defined as

$$\mathrm{RMSE}_{\mathrm{MCM}} = \sqrt{\left(\sum_{l=1}^{L} \|\hat{\mathbf{C}}_l - \mathbf{C}\|_F^2 / \|\mathbf{C}\|_F^2\right)/L}, \tag{24}$$

$$\mathrm{RMSE}_{\mathrm{DOA}} = \sqrt{\left(\sum_{l=1}^{L} \sum_{k=1}^{K} (\hat{\theta}_{k,l} - \theta_k)^2\right)/(KL)}, \tag{25}$$

where $\hat{\mathbf{C}}_l$ and $\hat{\theta}_{k,l}$ are the estimates of $\mathbf{C}$ and θ_k in the lth Monte Carlo trial, respectively. L is the number of Monte Carlo trials. In the following simulations, we perform 500 Monte Carlo trials for each experiment. For the DOA estimation, the estimated MCMs are embedded with the MUSIC method. To utilize the elliptic covariance matrix, all the estimated MCMs are also embedded within the NC-MUSIC method [11].

Fig. 1 shows the RMSE of MCM versus SNR with the fixed 500 samples, while Fig. 2 shows the RMSE of MCM versus the number of samples with the SNR=0 dB. From Figs. 1 and 2, it is shown that both methods perform better as the SNR or the number of samples increases. In addition, the proposed method can provide a more accurate MCM estimate, especially for low SNR. This conclusion is expected since more information (from the elliptic covariance matrix) is exploited by the proposed method.

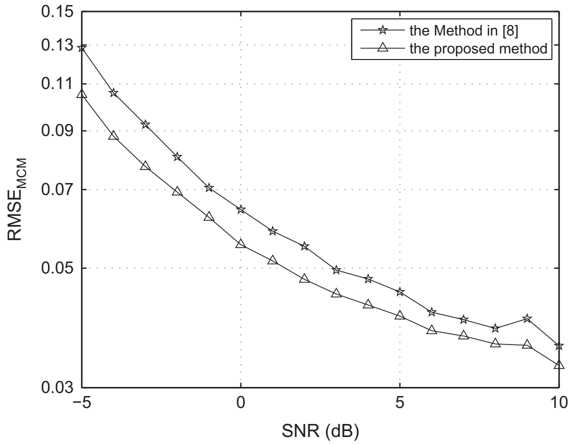

Fig. 1. RMSE of MCM estimates versus SNR.

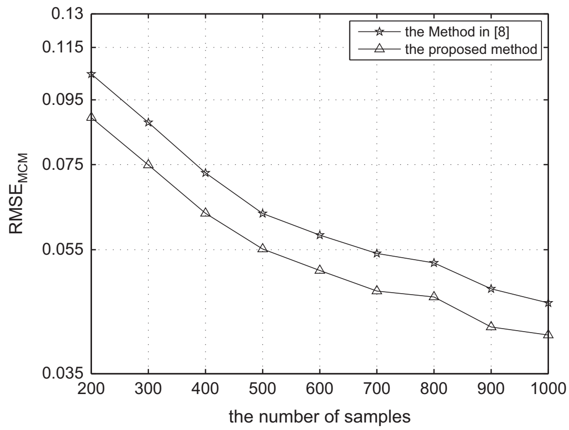

Fig. 2. RMSE of MCM estimates versus the number of samples.

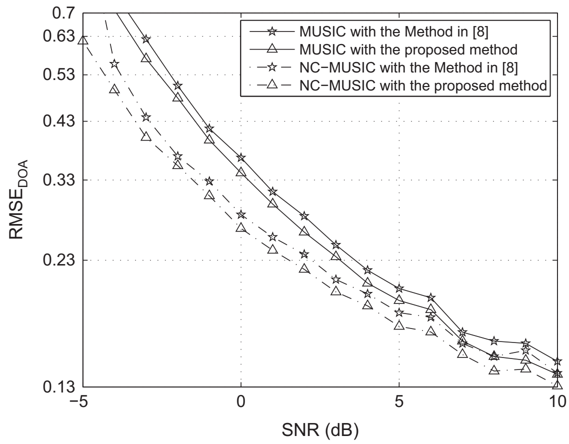

Fig. 3. RMSE of DOA estimates versus SNR.

Fig. 3 shows the RMSE of DOA versus SNR with the fixed 500 samples, while Fig. 4 shows the RMSE of DOA versus the number of samples with the SNR=0 dB. Figs. 3

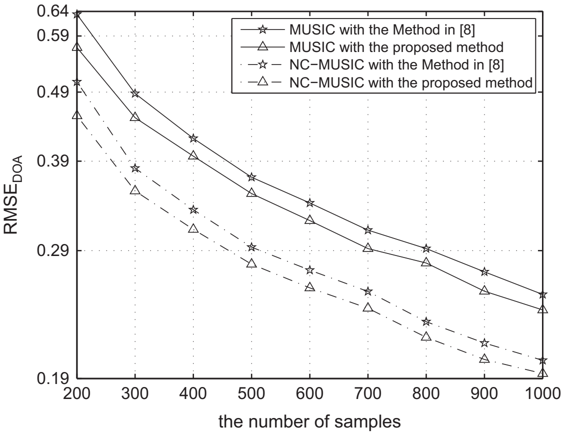

Fig. 4. RMSE of DOA estimates versus the number of samples.

and 4 show that NC-MUSIC with estimated MCM outperforms MUSIC with estimated MCM obviously. On the other hand, it is shown that MUSIC combined with the proposed method outperforms MUSIC combined with the method proposed in [8], NC-MUSIC combined with the proposed method outperforms NC-MUSIC combined with the method proposed in [8], which is consistent with the conclusion that the proposed method can provide a more accurate estimation of MCM. From Figs. 1 and 3, we can also note that all curves decrease slowly when SNR is high. The reason is that both MCM estimation methods assume the source signals be independent. However, this assumption is just approximate when the number of samples is limited.

5. Conclusion

In this communication, the problem of DOA estimation for noncircular signals in the presence of unknown mutual coupling has been addressed. The method proposed in [8] has been extended to the case of noncircular signals. The extended method takes advantage of the noncircularity of the signals to estimate the mutual coupling coefficients more accurately. The anti-Toeplitz structure of the elliptic covariance matrix is utilized to define a new cost function and then the MCM can be estimated by minimizing this cost function using an iterative procedure. Similar to the method developed in [8], the proposed method does not require the presence of calibration sources and the estimated mutual coupling coefficients can be embedded within any super-resolution DOA estimation method. Numerical simulations showed that the proposed method can improve the MCM estimation accuracy, which leads to a more accurate DOA estimation using the estimated MCM.

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