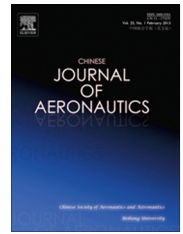




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DOA estimation and mutual coupling calibration with the SAGE algorithm



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Abstract In this paper, a novel algorithm is presented for direction of arrival (DOA) estimation and array self-calibration in the presence of unknown mutual coupling. In order to highlight the relationship between the array output and mutual coupling coefficients, we present a novel model of the array output with the unknown mutual coupling coefficients. Based on this model, we use the space alternating generalized expectation-maximization (SAGE) algorithm to jointly estimate the DOA parameters and the mutual coupling coefficients. Unlike many existing counterparts, our method requires neither calibration sources nor initial calibration information. At the same time, our proposed method inherits the characteristics of good convergence and high estimation precision of the SAGE algorithm. By numerical experiments we demonstrate that our proposed method outperforms the existing method for DOA estimation and mutual coupling calibration.

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1. Introduction

The problem of estimating the direction of arrival (DOA) of multiple narrowband signals plays an important role in many fields, including radar, wireless communications, seismology, sonar and so on. Most of the existing DOA estimation methods rely heavily on perfect knowledge of the array manifold. However, the array manifold is often affected by array imperfections in practice, such as mutual coupling, gain/phase uncertainty and sensor location error. The performance of

the DOA estimation methods may deteriorate significantly without array manifold calibration.^{1–6}

In this paper, we focus on the mutual coupling calibration and DOA estimation problem for deterministic signals. Various array calibration methods have been proposed with respect to the mutual coupling effect.^{7–14} In Ref. ⁷, an iterative least-mean-square algorithm is proposed to estimate the calibration matrix, but it requires preliminary calibration. In Ref. ⁸, Fabrizio and Alberto present an online mutual coupling compensation algorithm for uniform and linear arrays, which could estimate the coupling coefficients through an alternating minimization technique, but the convergence is not well guaranteed. The iterative algorithm in Ref. ⁹ compensates the mutual coupling effect and the gain/phase perturbation simultaneously. However, the convergence rate is low, and the computational cost is very expensive. In Ref. ¹⁰, a maximum-likelihood algorithm is presented for array calibration, which takes into consideration the array imperfections of mutual coupling,

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gain/phase perturbation, as well as sensor position errors. However, the method requires a set of calibration sources with known locations, which is hardly available in practice. Compared with the methods that require calibration sources or initial calibration information, self-calibration methods are more attractive under some circumstances. Liu et al.¹¹ present a novel DOA estimation algorithm with uniform linear arrays in the presence of mutual coupling via blind calibration. Weiss and Friedlander¹² present a self-calibration algorithm that compensates for mutual coupling and gain/phase perturbation. However, they require multi-dimensional parameter searching, and the convergence is not well guaranteed. In Ref.¹³, an array self-calibration method to compensate for sensor position perturbation is presented by the SAGE algorithm. It remains robust in critical scenarios including large sensor position errors and closely located signals. But it only focuses on sensor position perturbations. In Ref.¹⁴, a unified framework is proposed for DOA estimation and array self-calibration to all kinds of array perturbations, which can achieve good performance in scenarios with low SNR, limited snapshots and spatially adjacent signals, but it only aims at stochastic signals.

On the other hand, among the existing DOA estimation methods, the maximum-likelihood (ML) ones achieve the best asymptotic performance. The main drawback of the ML approach is the high computational complexity caused by maximization of the likelihood function. To overcome this drawback, the expectation maximization (EM) algorithm has been derived for both deterministic and stochastic signal models.^{15–18} The SAGE algorithm is a variation of the widely used EM algorithm, which updates subsets of parameters sequentially in one iteration. Through the augmentation scheme specified by the EM or SAGE algorithm, the complicated multi-dimensional searching involved in maximizing the likelihood functions can be simplified to one-dimensional searching. It has been proved in Ref.¹⁹ that SAGE converges faster than EM while retaining the advantages in numerical simplicity and stability.

This paper develops a novel calibration method to compensate for unknown mutual coupling in uniform linear arrays. The method jointly estimates the DOA parameters and the coupling coefficients by using the SAGE algorithm. First, an array output model with unknown mutual coupling is presented to highlight the relationship between the array output and the mutual coupling coefficients. Then we derive a SAGE-based calibration algorithm that updates the DOA parameters and the coupling coefficients sequentially based on this model. Compared to the existing methods, our algorithm requires neither calibration sources nor initial calibration information. At the same time, simulation results demonstrate that it can achieve higher parameter estimation precision.

2. Perturbed array output formulation

Suppose that K unknown and deterministic narrowband signals impinge onto an M element uniform linear array (ULA) from directions of $\boldsymbol{\vartheta} = [\vartheta_1, \vartheta_2, \dots, \vartheta_K]^T$, N snapshots are collected, and the output of the accurately calibrated array at time t is

$$\mathbf{x}^{\text{pf}}(t) = \sum_{k=1}^K \mathbf{a}(\vartheta_k) s_k(t) + \mathbf{v}(t) = \mathbf{A}(\boldsymbol{\vartheta}) \mathbf{s}(t) + \mathbf{v}(t) \quad (1)$$

where $\mathbf{A}(\boldsymbol{\vartheta}) = [\mathbf{a}(\vartheta_1), \mathbf{a}(\vartheta_2), \dots, \mathbf{a}(\vartheta_K)]$ and $\mathbf{a}(\vartheta_k) = [e^{j\varphi_{k,1}}, e^{j\varphi_{k,2}}, \dots, e^{j\varphi_{k,M}}]^T$ are the array responding matrix and vector, respectively, $\varphi_{k,m} = (2\pi d_m \sin \vartheta_k) / \lambda$, where λ represents the signal wavelength and d_1, d_2, \dots, d_M are the distances of the M array sensors from the reference located on the array axes. $s_k(t)$ is the waveform of the k th signal at time t , $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T$, $\mathbf{v}(t) = [v_1(t), v_2(t), \dots, v_M(t)]^T$ is the noise vector with variance $\sigma^2 \mathbf{I}$, $\mathbf{x}^{\text{pf}}(t) = [x_1^{\text{pf}}(t), x_2^{\text{pf}}(t), \dots, x_M^{\text{pf}}(t)]^T$ and the superscript “pf” is the short of “perturbation-free”.

When mutual coupling effects are taken into consideration, the array output is given as follows:

$$\mathbf{x}(t) = \sum_{k=1}^K \mathbf{a}'(\vartheta_k) s_k(t) + \mathbf{v}(t) = \mathbf{A}'(\boldsymbol{\vartheta}) \mathbf{s}(t) + \mathbf{v}(t) \quad (2)$$

where $\mathbf{A}'(\boldsymbol{\vartheta}) = [\mathbf{a}'(\vartheta_1), \mathbf{a}'(\vartheta_2), \dots, \mathbf{a}'(\vartheta_K)]$, and $\mathbf{a}'(\vartheta)$ is the perturbed array responding vector, $\mathbf{a}'(\vartheta) = \mathbf{C} \mathbf{a}(\vartheta)$ with \mathbf{C} denoting the mutual coupling matrix. The mutual coupling matrix can be written more explicitly as $\mathbf{C} = \text{toeplitz}\left(\left[1, c_1, c_2, \dots, c_P, \mathbf{0}_{(M-P-1) \times 1}^T\right]^T\right)$ in ULA,¹⁰ where

c_p is the coupling coefficient of the two sensors displaced by $(p-1)$ times the inter-element spacing of the ULA, and c_p is very small when $p > P$ and c_p is thus neglected.

Although the directional information of the incident signals is still reserved in the perturbed array outputs, it can hardly be extracted directly based on the array output formulation in Eq. (2). In order to highlight the perturbation-free signal components, and also make clearer the relationship between the array output and the perturbation parameters, the following model can be established for the perturbed array output.

$$\mathbf{x}(t) = \mathbf{A}'(\boldsymbol{\vartheta}) \mathbf{s}(t) + \mathbf{v}(t) = \mathbf{A}(\boldsymbol{\vartheta}) \mathbf{s}(t) + \mathbf{Q}(t) \mathbf{c} + \mathbf{v}(t) \quad (3)$$

where $\mathbf{c} = [c_1, c_2, \dots, c_P]^T$ stands for the coupling coefficient vector. And the following equation holds for $\mathbf{Q}(t)$ and \mathbf{c} .

$$\mathbf{Q}(t) \mathbf{c} = [\mathbf{A}'(\boldsymbol{\vartheta}) - \mathbf{A}(\boldsymbol{\vartheta})] \mathbf{s}(t) = (\mathbf{C} - \mathbf{I}_M) \mathbf{A}(\boldsymbol{\vartheta}) \mathbf{s}(t) \quad (4)$$

where \mathbf{I}_M stands for M dimension unitary matrix. The matrix of $\mathbf{Q}(t)$ can be expressed explicitly according to $[\mathbf{Q}(t)]_{:,p} = \mathbf{G}_p \mathbf{A}(\boldsymbol{\vartheta}) \mathbf{s}(t)$ and $\mathbf{G}_p = \partial \mathbf{C} / \partial c_p$ contains nonzero elements of 1 only on the $\pm p$ diagonals. Such an expression of $\mathbf{Q}(t)$ is concluded by taking the differentiations of both sides of Eq. (4) with respect to c_p .

The perturbed array output formulation given in Eq. (3) reveals the relationship between the array output and the coupling coefficients, and such a relationship can be exploited to estimate the coupling coefficients by maximizing the log-likelihood of the array output $\mathbf{x}(t)$. Meanwhile, for a particular coupling coefficients estimate, the log-likelihood can be maximized to determine the signal directions. In the next section, we follow this guideline to calibrate the array imperfection and estimate the signal directions iteratively.

3. SAGE-based algorithm for DOA estimation and array calibration

Both the EM and SAGE algorithms have been applied to the problem of DOA estimation without array imperfections, and they show good convergence properties. The EM algorithm is a general numerical method for finding the maximum-likelihood estimates which is characterized with simple implementation and stable convergence. The SAGE algorithm is a

variation of the EM algorithm, which follows a more flexible optimization scheme and converges faster than the EM algorithm. Instead of estimating all the parameters at once in EM, SAGE breaks up the problem into several smaller ones and uses EM to update the parameter subsets associated with each decomposed problem. In order to solve the DOA estimation problem in the presence of mutual coupling, this section extends the existing research to calibrate the mutual coupling effect together with estimating the source directions by the SAGE algorithm.

Similar to the DOA estimation problem in calibrated arrays, we construct an augmented data vector as follows to separate different signal components contained in the array output based on the model of the perturbed array output in Eq. (3).

$$\begin{aligned} \mathbf{y}_k(t) &= \mathbf{a}'(\vartheta_k)s_k(t) + \mathbf{v}(t) \\ &= \mathbf{a}(\vartheta_k)s_k(t) + \mathbf{Q}_k(t)\mathbf{c} + \mathbf{v}(t) \quad k = 1, 2, \dots, K \end{aligned} \quad (5)$$

where $\mathbf{Q}_k(t)\mathbf{c} = [\mathbf{a}(\vartheta_k) - \mathbf{a}'(\vartheta_k)]s_k(t)$, $[\mathbf{Q}_k(t)]_{:,p} = \frac{\partial}{\partial c_p}[\mathbf{a}'(\vartheta_k) - \mathbf{a}(\vartheta_k)]s_k(t) = \mathbf{G}_{k,p}\mathbf{a}(\vartheta_k)s_k(t)$, $\mathbf{G}_{k,p}$ is a variation of \mathbf{G}_p corresponding to the k th signal component, and the noise vector $\mathbf{v}(t) \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_M)$.

In the framework of the SAGE algorithm, the likelihood of $\mathbf{y}_1(t), \mathbf{y}_2(t), \dots, \mathbf{y}_K(t)$ is given by

$$p(\mathbf{y}_k(t) | \Omega_k(t)) = |\pi\sigma^2 \mathbf{I}_M|^{-1} \exp \left\{ -\frac{1}{\sigma^2} \|\mathbf{y}_k(t) - \mathbf{a}'(\vartheta_k)s_k(t)\|_2^2 \right\} \quad (6)$$

where $\Omega_k(t)$ is the unknown parameter set associated with $\mathbf{y}_k(t)$, and it is a subset of $\Omega = \{\vartheta, \{s(t_n)\}_{n=1}^N, \mathbf{c}, \sigma^2\}$. Obviously, $\mathbf{y}_1(t), \mathbf{y}_2(t), \dots, \mathbf{y}_K(t)$ are normally distributed with the same variances and different means. The log-likelihood of $\{\mathbf{y}_1(t_n), \mathbf{y}_2(t_n), \dots, \mathbf{y}_K(t_n)\}_{n=1}^N$ can be obtained by calculating the mean of the log-likelihoods of $\{\mathbf{y}_k(t_n)\}_{n=1}^N$ for different k .

$$\begin{aligned} \mathcal{L}(\{\mathbf{y}_1(t_n), \mathbf{y}_2(t_n), \dots, \mathbf{y}_K(t_n)\}_{n=1}^N) &= \frac{1}{K} \sum_{k=1}^K \mathcal{L}(\{\mathbf{y}_k(t_n)\}_{n=1}^N) \\ &= MN \ln \sigma^2 + \frac{1}{K\sigma^2} \sum_{n=1}^N \sum_{k=1}^K \|\mathbf{y}_k(t_n) - \mathbf{a}'(\vartheta_k)s_k(t_n)\|_2^2 \end{aligned} \quad (7)$$

According to Eq. (7), the log-likelihood of $\{\mathbf{y}_1(t_n), \mathbf{y}_2(t_n), \dots, \mathbf{y}_K(t_n)\}_{n=1}^N$ can be written as

$$\begin{aligned} \mathcal{L}(\{\mathbf{y}_1(t_n), \mathbf{y}_2(t_n), \dots, \mathbf{y}_K(t_n)\}_{n=1}^N) &= KMN \ln \sigma^2 + \frac{1}{\sigma^2} \sum_{n=1}^N \sum_{k=1}^K \|\mathbf{y}_k(t_n) - \mathbf{a}'(\vartheta_k)s_k(t_n)\|_2^2 \\ &= KMN \ln \sigma^2 + \frac{1}{\sigma^2} \sum_{n=1}^N \sum_{k=1}^K \|\mathbf{y}_k(t_n) - \mathbf{a}(\vartheta_k)s_k(t_n) - \mathbf{Q}_k(t_n)\mathbf{c}\|_2^2 \end{aligned} \quad (8)$$

The second expression of $\mathcal{L}(\{\mathbf{y}_1(t_n), \mathbf{y}_2(t_n), \dots, \mathbf{y}_K(t_n)\}_{n=1}^N)$ in Eq. (8) will be used for optimizing \mathbf{c} .

The SAGE algorithm updates the unknown parameters with an iterative guideline. Each iteration of the SAGE algorithm consists of an E-step and an M-step. Given the estimate of the $(q-1)$ th iteration Ω^{q-1} , the q th iteration consists of the following steps.

Step 1. E-step

Calculate

$$F(\Omega^q; \Omega^{q-1}) = E[\mathcal{L}(\{\mathbf{y}_1(t_n), \mathbf{y}_2(t_n), \dots, \mathbf{y}_K(t_n)\}_{n=1}^N) | \{\mathbf{x}(t_n)\}_{n=1}^N, \Omega^{q-1}] \quad (9)$$

where the superscript q stands for the iteration index, and Eq. (9) is equivalent to computing the following conditional expectations,

$$\begin{aligned} \mathbf{y}_k^q(t) &= E[\mathbf{y}_k(t) | \mathbf{x}(t), \Omega^{q-1}] \\ &= \mathbf{a}'(\vartheta_k^{q-1})s_k^{q-1}(t) + \mathbf{x}(t) - \mathbf{A}'(\mathbf{c}^{q-1}, \vartheta^{q-1})s^{q-1}(t) \end{aligned} \quad (10)$$

$$\begin{aligned} \mathbf{R}_{y_k}^q &= E \left[\frac{1}{N} \sum_{n=1}^N \mathbf{y}_k(t_n) \mathbf{y}_k^H(t_n) | \{\mathbf{x}(t_n)\}_{n=1}^N, \Omega^{q-1} \right] \\ &= \frac{1}{N} \sum_{n=1}^N \mathbf{y}_k^q(t_n) (\mathbf{y}_k^q(t_n))^H \end{aligned} \quad (11)$$

In order to emphasize the dependence on the array perturbation parameters, we use $\mathbf{A}'(\mathbf{c}^{q-1}, \vartheta^{q-1})$ to denote the perturbed responding matrix in Eq. (10), what relies on the current estimates of the mutual coupling coefficients.

Step 2. M-step

The unknown parameters contained in Ω can be updated by maximizing $F(\Omega^q; \Omega^{q-1})$, and the updating schemes, which are derived by setting the differentiations of $F(\Omega^q; \Omega^{q-1})$ to those parameters to zero, are given as follows:

$$\vartheta_k^q = \arg \max_{\vartheta} \frac{(\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta))^H \mathbf{R}_{y_k}^q \mathbf{a}'(\mathbf{c}^{q-1}, \vartheta)}{\|\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta)\|_2^2} \quad (12)$$

$$s_k^q(t_n) = \frac{(\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q))^H \mathbf{y}_k^q(t_n)}{\|\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q)\|_2^2} \quad (13)$$

$$(\sigma^2)^q = \frac{1}{KMN} E \left[\sum_{n=1}^N \sum_{k=1}^K \|\mathbf{y}_k(t_n) - \mathbf{a}'(\vartheta_k)s_k(t_n)\|_2^2 \right] \quad (14)$$

$$\begin{aligned} \mathbf{c}^q &= \left\{ E \left[\sum_{n=1}^N \sum_{k=1}^K \mathbf{Q}_k^H(t_n) \mathbf{Q}_k(t_n) \right] \right\}^{-1} \\ &\quad \times E \left[\sum_{n=1}^N \sum_{k=1}^K \mathbf{Q}_k^H(t_n) (\mathbf{y}_k(t_n) - \mathbf{a}(\vartheta_k)s_k(t_n)) \right] \end{aligned} \quad (15)$$

As can be seen from Eq. (12) to Eq. (15), the update processes of ϑ_k and $\{s_k(t_n)\}_{n=1}^N$ mainly depend on the corresponding augmented data $\{\mathbf{y}_k(t_n)\}_{n=1}^N$, and they do not matter with $\{\mathbf{y}_1(t_n), \mathbf{y}_2(t_n), \dots, \mathbf{y}_{k-1}(t_n), \mathbf{y}_{k+1}(t_n), \dots, \mathbf{y}_K(t_n)\}_{n=1}^N$. As the parameters of \mathbf{c} and σ^2 exist in the models of all the signal components, their update processes rely on $\{\mathbf{y}_1(t_n), \mathbf{y}_2(t_n), \dots, \mathbf{y}_K(t_n)\}_{n=1}^N$. The iteration process can be divided into two parts, the first one contains the sequential updating of ϑ_k and $\{s_k(t_n)\}_{n=1}^N$ for different k , and the second one updates \mathbf{c} and σ^2 by using all the augmented data. Straightforward formulation of $\mathbf{Q}_k(t)$ in the iterations is very complicated, thus we carry out some deeper analysis in the Appendix A on the update processes associated with this matrix to simplify the implementation of the iteration.

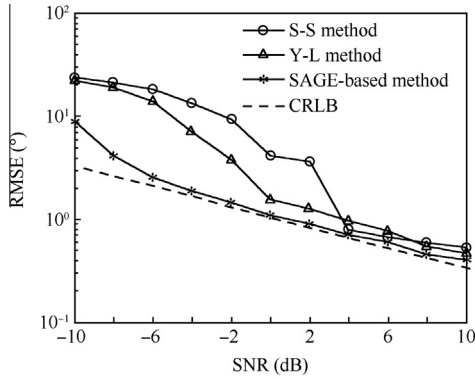
4. Simulation results

The simulations in this section will be carried out to demonstrate the performance of the proposed algorithms. The existing methods of the S-S method⁸ and the Y-L method,²⁰ as well as the Cramer-Rao lower bound (CRLB),²¹ are also implemented for performance comparison.

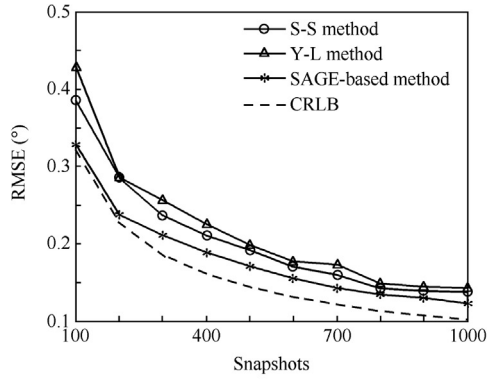
We consider a uniform linear array of 5 sensors with inter-element spacing of half a wavelength of the incident signals. Suppose that two equal-power independent signals impinge onto the array from directions of -9° and 25° . The coupling coefficient vector is $\mathbf{c} = [0.4 + 0.3j, 0.3 + 0.1j]^T$. The required initial DOA estimate $\boldsymbol{\vartheta}^0$ can be initialized by subspace methods such as MUSIC or the standard ML approach. The initial coupling coefficient vector estimate is chosen to be $\mathbf{c}^0 = [0, 0]^T$. And $\mathbf{s}^0(t)$ is obtained by computing the least squares estimation for $\boldsymbol{\vartheta} = \boldsymbol{\vartheta}^0$ and $\mathbf{c} = \mathbf{c}^0$. The algorithm is terminated if the relative increase in the data likelihood function is smaller than 10^{-4} or the number of iterations achieves 50.

The average root-mean-square error (RMSE) of the DOA estimates of the two signals is considered for statistical direction estimation precision evaluation, which is defined as

$$\text{RMSE}_\vartheta = \sqrt{\sum_{w=1}^W \|\hat{\boldsymbol{\vartheta}}^w - \boldsymbol{\vartheta}^w\|^2 / (KW)} \quad (16)$$

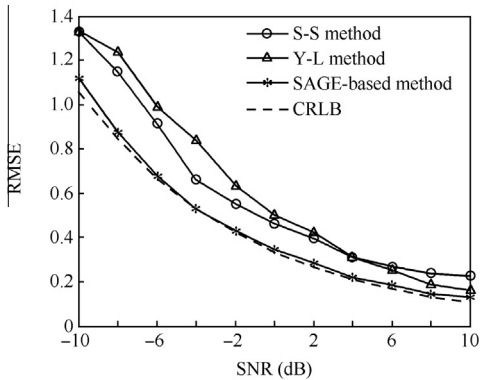


(a) RMSE vs SNR

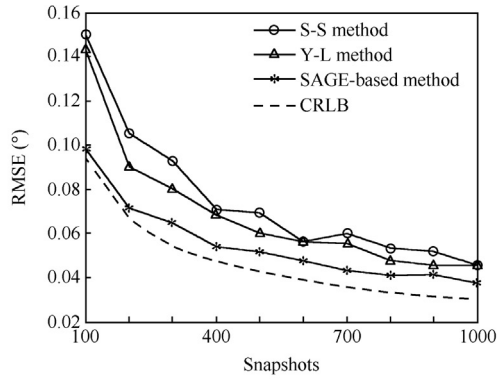


(b) RMSE vs snapshots

Fig. 1 RMSE of DOA estimates of different methods.



(a) RMSE vs SNR



(b) RMSE vs snapshots

Fig. 2 RMSE of coupling coefficient estimates of different methods.

where W denotes the number of Monte Carlo experiments. $\hat{\boldsymbol{\vartheta}}^w$ and $\boldsymbol{\vartheta}^w$ are the estimated and true directions in the w th simulation.

Fig. 1(a) shows the RMSE of the DOA estimates of different methods against SNR computed via 500 Monte Carlo runs for each SNR (the number of snapshots is fix to 50). Fig. 1(b) shows the RMSE of the DOA estimates of different methods versus the number of snapshots (with SNR = 5 dB) computed via 500 Monte Carlo runs for each number of snapshots. We can observe that the proposed SAGE-based method can achieve higher estimation accuracy than S-S method and Y-L Method, and it improves with a similar speed as that of the CRLB.

The RMSE of the mutual coupling coefficients is used for array calibration precision evaluation, which is defined as

$$\text{RMSE}_c = \frac{1}{\|\mathbf{c}\|_2} \times \sqrt{\sum_{w=1}^W \|\hat{\mathbf{c}}^w - \mathbf{c}\|_2^2 / W} \quad (17)$$

where \mathbf{c} keeps constant in each scenario, and $\hat{\mathbf{c}}^w$ is the estimated coupling coefficient vector in the w th simulation.

Fig. 2(a) shows the RMSE of the mutual coupling coefficients versus SNR in the same simulations (the number of snapshots is fix to 50). Fig. 2(b) shows the RMSE of the mutual coupling coefficients estimates of different methods versus the number of snapshots (with SNR = 5 dB) computed

via 500 Monte Carlo runs for each number of snapshots. The results illustrate that SAGE-based Method achieves the best performance among the three methods, and its RMSE is very close to the CRLB in the considered scenarios.

5. Conclusion

In this letter, we propose a SAGE-based algorithm for DOA estimation and mutual coupling calibration in the case of uniform linear arrays and unknown determined signals. The implementation of the proposed method follows the standard SAGE iterations and thus has guaranteed convergence. The simulation results show that compared with the existing methods, the proposed one performs much better in both DOA estimation and array calibration precisions, especially when the SNR is low. The guideline that the new method follows can also be extended to the calibration of other kinds of array imperfections, such as gain/phase perturbation and sensor location error, but the straightforward extensions are excluded in this letter due to space limitation.

Acknowledgment

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Appendix A

According to Eqs. (10), (11) and (13), one can obtain that

$$\begin{aligned} \mathbf{R}_{s_k}^q &= E \left[\frac{1}{N} \sum_{n=1}^N s_k(t_n) s_k^H(t_n) \{ \mathbf{x}(t_n) \}_{n=1}^N, \Omega^{q-1} \right] \\ &= \frac{(\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^{q-1}))^H \mathbf{R}_{y_k}^q \mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q)}{\|\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q)\|_2^4} \end{aligned} \quad (\text{A1})$$

$$\mathbf{R}_{y_k, s_k}^q = E \left[\frac{1}{N} \sum_{n=1}^N y_k(t_n) s_k^H(t_n) \{ \mathbf{x}(t_n) \}_{n=1}^N, \Omega^{q-1} \right] = \frac{\mathbf{R}_{y_k}^q \mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q)}{\|\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q)\|_2^2} \quad (\text{A2})$$

Moreover, according to Eqs. (12), (13), (A1) and (A2), one can get that

$$\begin{aligned} & \frac{1}{N} E \left[\sum_{n=1}^N \|\mathbf{y}_k(t_n) - \mathbf{a}'(\vartheta_k) s_k(t_n)\|_2^2 \right] \\ &= \text{tr} \left[\mathbf{R}_{y_k}^q - \mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q) (\mathbf{R}_{y_k, s_k}^q)^H - \mathbf{R}_{y_k, s_k}^q (\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q))^H \right. \\ & \quad \left. + \mathbf{R}_{s_k}^q \mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q) (\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q))^H \right] \\ &= \text{tr} \left\{ \left[\mathbf{I}_M - \frac{\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q) (\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q))^H}{\|\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q)\|_2^2} \right] \mathbf{R}_{y_k}^q \right\} \end{aligned} \quad (\text{A3})$$

Denote $\mathbf{h}_k = \frac{1}{N} E \left[\sum_{n=1}^N \mathbf{Q}_k^H(t_n) (\mathbf{y}_k(t_n) - \mathbf{a}(\vartheta_k) s_k(t_n)) \right]$ and $\Sigma_k = \frac{1}{N} E \left[\sum_{n=1}^N \mathbf{Q}_k^H(t_n) \mathbf{Q}_k(t_n) \right]$, then the elements of the vector and the matrix can be computed according to $[\mathbf{Q}_k(t)]_{:,p} = \mathbf{G}_{k,p} \mathbf{a}(\vartheta_k) s_k(t)$ as follows:

$$\begin{aligned} (\mathbf{h}_k)_p &= \mathbf{a}^H(\vartheta_k) \mathbf{G}_{k,p}^H \frac{1}{N} E \left[\sum_{n=1}^N (\mathbf{y}_k(t_n) s_k^H(t_n) - \mathbf{a}(\vartheta_k) s_k(t_n) s_k^H(t_n)) \right] \\ &= \mathbf{a}^H(\vartheta_k) \mathbf{G}_{k,p}^H \sum_{n=1}^N (\mathbf{R}_{y_k, s_k}^q - \mathbf{a}(\vartheta_k) \mathbf{R}_{s_k}^q) \\ &= \mathbf{a}^H(\vartheta_k) \mathbf{G}_{k,p}^H \left[\mathbf{I}_M - \frac{\mathbf{a}(\vartheta_k^q) (\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q))^H}{\|\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q)\|_2^2} \right] \frac{\mathbf{R}_{y_k}^q \mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q)}{\|\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q)\|_2^2} \end{aligned} \quad (\text{A4})$$

$$\begin{aligned} \left(\sum_k \right)_{p_1, p_2} &= \frac{1}{N} E \left[\sum_{n=1}^N s_k^*(t) \mathbf{a}^H(\vartheta_k) \mathbf{G}_{k,p_1}^H \mathbf{G}_{k,p_2} \mathbf{a}(\vartheta_k) s_k(t) \right] \\ &= \frac{1}{N} E \left[\sum_{n=1}^N s_k(t) s_k^*(t) \right] \mathbf{a}^H(\vartheta_k) \mathbf{G}_{k,p_1}^H \mathbf{G}_{k,p_2} \mathbf{a}(\vartheta_k) \\ &= \frac{(\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q))^H \mathbf{R}_{y_k}^q \mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q)}{\|\mathbf{a}'(\mathbf{c}^{q-1}, \vartheta_k^q)\|_2^4} \mathbf{a}^H(\vartheta_k) \mathbf{G}_{k,p_1}^H \mathbf{G}_{k,p_2} \mathbf{a}(\vartheta_k) \end{aligned} \quad (\text{A5})$$

where $(\mathbf{h}_k)_p$ represents the p th element of \mathbf{h}_k and $(\Sigma_k)_{p_1, p_2}$ stands for the (p_1, p_2) th element of Σ_k .

Finally, Eqs. (10), (11) and Eqs. (A3)–(A5) can be substituted into Eqs. (12)–(15) to yield more convenient steps for updating the unknown parameters and implementing the iterations.

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