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Covariance sparsity-aware DOA estimation for nonuniform noise

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ABSTRACT

This paper reformulates the problem of direction-of-arrival (DOA) estimation for unknown nonuniform noise by exploiting a sparse representation of the array covariance vectors. In the proposed covariance sparsity-aware DOA estimator, the unknown noise variances can be eliminated by a linear transformation, and DOA estimation is reduced to a sparse reconstruction problem with nonnegativity constraint. The proposed method not only obtains an extended-aperture array with increased degrees of freedom which enables us to handle more sources than sensors, but also provides superiority in performance and robustness against nonuniform noise. Numerical examples under different conditions demonstrate the effectiveness of the proposed method.

1. Introduction

The problem of direction-of-arrival (DOA) estimation frequently arises in a variety of applications, such as radar, sonar, radio astronomy and so on [1]. A large number of excellent solutions have been provided to this problem during the past several decades. Recently, an advanced topic, namely, sparse signal representation (SSR) framework, has attracted a significant interest in DOA estimation, mainly due to the key observation that the DOAs of signals are substantially sparse in all the spatial domain. The idea of utilizing SSR, which is intrinsically different from the subspace-based methods like MUSIC [2] and Root-MUSIC [3], provides a new sparse signal reconstruction perspective for DOA estimation, and has been well studied in various contexts. The ℓ_1-based singular value decomposition (L1-SVD) algorithm [4] and its varieties [5–7], mainly address the DOA estimation problem by directly representing the array output in time domain with an overcomplete basis from the array response vector. To make use of the second-order statistics of the array output, another kind of methods [8–11] tackles the underlying DOA parameter estimation problem by using a sparse representation of the array covariance matrix/vectors and offers clear-cut guidelines for the selection of the regularization parameter.

Despite some attractive features of these aforementioned SSR-based methods, all of them explicitly or implicitly assume that the sensor noises are spatially uniform/homogeneous and with a common variance. Indeed, they rest upon the uniform noise assumption and the knowledge of noise variance in sparse representation modeling to the benefit of choosing an appropriate regularization parameter for robustness guarantee. In some practical applications, the uniform white noise assumption, however, might be unrealistic since the noise levels among all sensors exhibit some disturbances due to the nonidealities of the practical arrays, such as the nonuniformity of the sensor response, the nonideality of the receiving channel and the mutual coupling between sensors [12,13]. Therefore, the sensor noises should be, in general, considered as the case of nonuniform noise levels with an arbitrary diagonal covariance matrix. When such a deviation from the spatially uniform noise assumption occurs, the conventional SSR-based DOA estimation techniques will mismodel the noise and their performance may thereby degrade severely. While the recent sparse iterative covariance-based estimation (SPICE) method [14], based on a sparse covariance fitting criterion, takes account of the noise in a natural manner against the uniform noise assumption, it still relies on the knowledge of the noise variances which need to be estimated in each iteration. This motivates us to explore another new direction finding techniques for the nonuniform noise case.

In this paper, from the perspective of the practical applications and in the SSR framework, a new covariance sparsity-aware DOA estimation method is proposed for the nonuniform noise case. Through vectorizing the covariance matrix, the unknown noise variances can be removed by a linear transformation. Then a novel but noise-free sparse representation, with measurement matrix from extended steering vectors which provide an increased degrees

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of freedom of linear arrays, is obtained, and the DOA estimation can be cast as a problem of recovering a nonnegative sparse vector. As we shall show in Section 3, the proposed covariance sparsity-aware DOA estimator does not require the *a priori* knowledge of the source number, and is capable of handling more sources than sensors. Additionally, due to the extended-aperture of linear arrays and the elimination of noise variances, the proposed method can provide high resolution as well as robustness against nonuniform noise.

2. Problem formulation

Consider K narrowband far-field signals from distinct directions $\{\theta_k\}_{k=1}^K$ impinging on an arbitrary linear array of M $(M>1)$ sensors. The $M\times 1$ array output vector with N snapshots can be modeled as

$$\mathbf{x}(t)=\mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t)+\mathbf{n}(t),\quad t=1,\dots,N \tag{1}$$

where $\boldsymbol{\theta}=[\theta_1,\dots,\theta_K]^T$, $\mathbf{s}(t)=[s_1(t),\dots,s_K(t)]^T$ and $\mathbf{n}(t)=[n_1(t),\dots,n_M(t)]^T$ are, respectively, the unknown parameter vector of DOAs, the $K\times 1$ signal vector and the $M\times 1$ noise vector. Here, $(\cdot)^T$ denotes the transpose, $\mathbf{A}(\boldsymbol{\theta})=[\mathbf{a}(\theta_1),\dots,\mathbf{a}(\theta_K)]\in\mathbb{C}^{M\times K}$ is the array manifold matrix whose kth steering vector is given by

$$\mathbf{a}(\theta_k)=\left[1,e^{-j2\pi d_2\sin\theta_k/\lambda},\dots,e^{-j2\pi d_M\sin\theta_k/\lambda}\right]^T \tag{2}$$

where $\theta_k\in(-\pi/2,\pi/2)$, λ and d_m $(d_1=0)$ represent the signal wavelength and the position of the mth sensor corresponding to the first sensor, respectively. Some statistical assumptions on the source signal and noise are made as follows:

(1) The source signals are spatially uncorrelated, temporally white, and zero-mean.
(2) The noises $\{n_m(t)\}_{m=1}^M$ are zero-mean, complex circular Gaussian, and with variances $\{\sigma_m^2\}_{m=1}^M$, respectively.
(3) The noise is statistically independent of all the signals.

With these assumptions, we have the following covariance matrix

$$\mathbf{R}=E\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\}=\mathbf{A}(\boldsymbol{\theta})\mathbf{P}\mathbf{A}^H(\boldsymbol{\theta})+\mathbf{Q} \tag{3}$$

where $E(\cdot)$ and $(\cdot)^H$, respectively, denote the expectation operator and conjugate transpose, $\mathbf{P}=E\{\mathbf{s}(t)\mathbf{s}^H(t)\}=\operatorname{diag}(\mathbf{p})$ is the source waveform covariance matrix with diagonal elements $\mathbf{p}=[p_1^2,\dots,p_K^2]^T$ being the signal power vector, and $\mathbf{Q}=E\{\mathbf{n}(t)\mathbf{n}^H(t)\}=\operatorname{diag}(\boldsymbol{\sigma})$ is the noise covariance matrix with $\boldsymbol{\sigma}=[\sigma_1^2,\dots,\sigma_M^2]^T$ being the noise power vector. The work is to estimate the DOAs $\{\theta_k\}_{k=1}^K$ from (3) without any knowledge of noise powers $\{\sigma_m^2\}_{m=1}^M$ and the source number K.

3. DOA estimation in unknown nonuniform noise environment

3.1. Proposed covariance-based sparse representation

Following [15] and vectorizing $\mathbf{R}$ in (3), we then have

$$\begin{aligned}\mathbf{y}&\triangleq\operatorname{vec}(\mathbf{R})=\operatorname{vec}\left[\sum_{k=1}^K p_k^2\mathbf{a}(\theta_k)\mathbf{a}^H(\theta_k)\right]+\operatorname{vec}(\mathbf{Q})\\&=\left[\mathbf{A}^*(\boldsymbol{\theta})\odot\mathbf{A}(\boldsymbol{\theta})\right]\mathbf{p}+\mathbf{1}_n\end{aligned} \tag{4}$$

where $\mathbf{1}_n=[\sigma_1^2\mathbf{e}_1^T,\dots,\sigma_M^2\mathbf{e}_M^T]^T$ with $\mathbf{e}_m$ $(m=1,\dots,M)$ being an $M\times 1$ column vector of all zeros except a 1 in the mth entry, $\mathbf{A}^*(\boldsymbol{\theta})\odot\mathbf{A}(\boldsymbol{\theta})\triangleq[\mathbf{a}^*(\theta_1)\otimes\mathbf{a}(\theta_1),\dots,\mathbf{a}^*(\theta_K)\otimes\mathbf{a}(\theta_K)]\in\mathbb{C}^{M^2\times K}$ in which $(\cdot)^*$, $\odot$ and $\otimes$ represent the complex conjugate, Khatri–Rao product and Kronecker product, respectively. It is interesting to observe that $\mathbf{y}$ in (4), similar to (1), can be taken as the array output of single snapshot where $\mathbf{A}^*(\boldsymbol{\theta})\odot\mathbf{A}(\boldsymbol{\theta})$, $\mathbf{p}$, and $\mathbf{1}_n$, are the virtual manifold matrix, equivalent source vector, and noise vector, respectively. Notice that the vector $\mathbf{1}_n$ has only M nonzero elements which equal to $\{\sigma_m^2\}_{m=1}^M$. Based on this observation, we can remove these elements of $\mathbf{y}$ corresponding to the positions of $\{\sigma_m^2\}_{m=1}^M$ in $\mathbf{1}_n$; this also means the rest $M(M-1)$ entries of $\mathbf{y}$ corresponding to these positions of zero elements in $\mathbf{1}_n$ are preserved. Mathematically, this operation can be formulated as

$$\mathbf{z}\triangleq\mathbf{J}\mathbf{y}=\mathbf{J}\left[\mathbf{A}^*(\boldsymbol{\theta})\odot\mathbf{A}(\boldsymbol{\theta})\right]\mathbf{p}=\mathbf{B}(\boldsymbol{\theta})\mathbf{p}. \tag{5}$$

Here, $\mathbf{J}$ is an $M(M-1)\times M^2$ selecting matrix and can be represented as

$$\mathbf{J}^T=[\mathbf{J}_1,\mathbf{J}_2,\dots,\mathbf{J}_{M-1}] \tag{6}$$

where

$$\mathbf{J}_m=[\tilde{\mathbf{e}}_{(m-1)(M+1)+2},\tilde{\mathbf{e}}_{(m-1)(M+1)+3},\dots,\tilde{\mathbf{e}}_{m(M+1)}]\in\mathbb{R}^{M^2\times M},\quad m=1,\dots,M-1, \tag{7}$$

$\tilde{\mathbf{e}}_i$ $(i=(m-1)(M+1)+2,\dots,m(M+1))$ is an $M^2\times 1$ column vector with 1 at the ith position and 0 elsewhere, $\mathbf{B}(\boldsymbol{\theta})\triangleq\mathbf{J}[\mathbf{A}^*(\boldsymbol{\theta})\odot\mathbf{A}(\boldsymbol{\theta})]\in\mathbb{C}^{M(M-1)\times K}$ is the new steering matrix. This elimination operation avoids the estimation of noise variances, facilitating a noise-free sparse representation.

Typically, the underlying SSR-based DOA estimation methods involve a grid sampling over the potential space from $-\pi/2$ to $\pi/2$, which forms the grid set $\boldsymbol{\Theta}=\{\tilde{\theta}_1,\dots,\tilde{\theta}_{\tilde{N}}\}$ where we assume that the true DOAs are exactly on the sampling grid $\boldsymbol{\Theta}$ and $\tilde{N}\gg M(M-1)$. As a result, $\mathbf{z}$ can be rewritten as the following noise-free sparse representation

$$\mathbf{z}=\mathbf{B}(\boldsymbol{\Theta})\boldsymbol{\eta}' \tag{8}$$

where $\mathbf{B}(\boldsymbol{\Theta})=\mathbf{J}[\mathbf{A}^*(\boldsymbol{\Theta})\odot\mathbf{A}(\boldsymbol{\Theta})]\in\mathbb{C}^{M(M-1)\times\tilde{N}}$ is an over-complete dictionary, $\boldsymbol{\eta}'$ is ideally a K-sparse vector (viz., $\|\boldsymbol{\eta}'\|_0=K$, $\|\cdot\|_0$ denotes the ℓ_0-norm) with nonzero values $\{p_k^2\}_{k=1}^K$ whose positions are indexed by the corresponding signal directions $\{\theta_k\}_{k=1}^K$ in $\boldsymbol{\Theta}$. That is, we can infer that there exists a corresponding source from $\tilde{\theta}_{\tilde{k}}$ $(\tilde{k}=1,\dots,\tilde{N})$ if for some $\tilde{k}$ the $\tilde{k}$th element of $\boldsymbol{\eta}'$ to be estimated is deemed to be nonzero. Therefore, the DOA estimation problem turns out to be that of recovering the sparse vector $\boldsymbol{\eta}'$ and detecting the locations of nonzero elements of this vector.

3.2. Separable signal number

We note that the dimension of the virtual manifold matrix $\mathbf{B}(\boldsymbol{\theta})$ is $M(M-1)$, which significantly enhances the degrees of freedom (DOF) of the original linear array with M degrees of freedom and opens up the possibility of handling the case of more sources than sensors $(K\geqslant M)$, i.e., the so-called underdetermined DOA estimation [15,16]. This motivates us to discuss the problem of separable signal number of the proposed method based on (8). This problem naturally relates to the sparsity of the vector $\boldsymbol{\eta}'$ since, as is shown in (8), the DOA estimation problem relies on the sparse estimation of this vector. The following proposition provides the key result on the separable signal number.

Proposition 1. *If any M-element array (possibly nonlinear) meets unambiguity, then the maximum separable signal number is $[M(M-1)]/2$. For 2-level nested array with M_1 and M_2 $(M_1+M_2=M)$ elements respectively in each level, this number is $M_2(M_1+1)-1$. For uniform linear array (ULA), this number is $M-1$. Instead, if the array structure suffers from ambiguity, then this number is 0.*

The proof of Proposition 1 is given in Appendix A. It follows from Proposition 1 that it is possible to tackle the underdetermined case of $K \geqslant M$ if we exploit the class of nonuniform arrays such as nested arrays [16] to increase the DOF. However, the proposed method cannot handle the manifold ambiguity (or spatial aliasing) problem. Note that in wideband signal scenario, the ambiguity problem might be avoided by using multiple dictionaries to jointly recover the sparse vectors, as has been elaborated in [9,17]. Actually, the essential idea in [9,17] is to employ the frequency diversity to handle the ambiguity problem. As a result, one possible approach to deal with this issue is to exploit the frequency diversity of the wideband signals. This will be addressed as our future work. Interestingly, for case of 2-level nested array with a ULA in each level, the proposed approach has the same identifiability condition on the separable signal number as that of the spatial smoothing based MUSIC (SS-MUSIC) approach [16]. Nevertheless, their derivations are essentially different.

Remark. Intuitively, the deleting operation in (5) may result in the loss of large amounts of information since we have removed M elements of $\mathbf{y}$. However, this removing operation results in the loss of only one degree instead of M degrees of freedom (see Appendix A), which indicates that there is almost no loss of effect or information on the performance because in Appendix A we have proved that $\mathrm{DOF}^{\mathrm{B}}_{\max} = \mathrm{DOF}^{\mathrm{A}}_{\max} - 1 = M(M-1) \in \mathcal{O}(M^2)$ where $\mathrm{DOF}^{\mathrm{A}}_{\max} = M(M-1) + 1 \in \mathcal{O}(M^2)$ [16], i.e., $\mathrm{DOF}^{\mathrm{A}}_{\max} \approx \mathrm{DOF}^{\mathrm{B}}_{\max}$.

3.3. DOA estimation

As just described previously, the DOA estimation problem can be cast as a problem of finding the sparsest solution of underdetermined linear system $\mathbf{z} = \mathbf{B}(\boldsymbol{\Theta})\boldsymbol{\eta}$. Naturally, we should choose the ℓ_0-norm as an ideal measure of sparsity. However, the ℓ_0-norm minimization problem is nonconvex, NP-hard and thereby cannot be solved. Therefore following [4–11] and together with the nonnegativeness of $\boldsymbol{\eta}'$, we can relax this problem to a simpler ℓ_1-norm minimization problem, where we equivalently seek to

$$\min_{\boldsymbol{\eta}} \mathbf{1}^T\boldsymbol{\eta} \quad \text{s.t.} \quad \mathbf{z} = \mathbf{B}(\boldsymbol{\Theta})\boldsymbol{\eta}, \quad \boldsymbol{\eta} \succcurlyeq \mathbf{0} \tag{9}$$

where $\mathbf{1}$ and $\mathbf{0}$ are column vectors composed of 1 and 0, and $\succcurlyeq$ is $\geqslant$ with an elementwise operation.

In practice, only finite samples are available. So the unknown $\mathbf{y}$ is estimated from the N snapshots, viz., $\hat{\mathbf{y}} = \mathrm{vec}(\hat{\mathbf{R}}) = \mathbf{y} + \Delta\mathbf{y}$ where $\hat{\mathbf{R}} = (1/N)\sum_{t=1}^{N} \mathbf{x}(t)\mathbf{x}^H(t)$, and $\Delta\mathbf{y} = \hat{\mathbf{y}} - \mathbf{y}$ is the estimate error. The corresponding estimate error of $\mathbf{z}$ is

$$\Delta\mathbf{z} = \hat{\mathbf{z}} - \mathbf{z} = \mathbf{J}\Delta\mathbf{y} \tag{10}$$

where $\hat{\mathbf{z}} = \mathbf{J}\hat{\mathbf{y}}$ denotes the estimate of $\mathbf{z}$. Hence, problem (9) can be converted into the following quadratic error-constrained minimization problem

$$\min_{\boldsymbol{\eta}} \mathbf{1}^T\boldsymbol{\eta} \quad \text{s.t.} \quad \left\|\hat{\mathbf{z}} - \mathbf{B}(\boldsymbol{\Theta})\boldsymbol{\eta}\right\|_2^2 \leqslant \beta, \quad \boldsymbol{\eta} \succcurlyeq \mathbf{0} \tag{11}$$

where $\|\cdot\|_2$ denotes the ℓ_2-norm, β is a critical threshold parameter on the upper bound of the error $\Delta\mathbf{z}$. Another approach to solve this problem is the Lagrangian form, i.e.,

$$\min_{\boldsymbol{\eta}} \left\|\hat{\mathbf{z}} - \mathbf{B}(\boldsymbol{\Theta})\boldsymbol{\eta}\right\|_2^2 + \xi\mathbf{1}^T\boldsymbol{\eta} \quad \text{s.t.} \quad \boldsymbol{\eta} \succcurlyeq \mathbf{0} \tag{12}$$

where ξ is a regularization parameter which balances the sparsity of $\boldsymbol{\eta}$ and the residual noise of $\Delta\mathbf{z}$. The estimator in (12) appears similar to the sparse spectrum fitting (SpSF) algorithm [11] where the best choice of ξ has been well elaborated. However, the SpSF estimator is only applicable to the uniform noise model because the best regularization parameter relies on the noise variance. In fact, problems (11) and (12) are, of course, equivalent, provided that some special relationships between β and ξ are satisfied. Usually, it is more natural to determine an appropriate β rather than ξ if some *a priori* knowledge on the residual noise is available. Therefore, the formulation (11) is preferred in this paper because a reasonable selection of β can be easily obtained through the asymptotic property of $\Delta\mathbf{y}$, as will be shown in the following.

By referring to [19], the error $\Delta\mathbf{y}$ is asymptotically normal (AsN) distribution, viz., $\Delta\mathbf{y} = \hat{\mathbf{y}} - \mathbf{y} = \mathrm{vec}(\hat{\mathbf{R}} - \mathbf{R}) \sim \mathrm{AsN}(\mathbf{0}_{M^2,1}, (\mathbf{R}^T \otimes \mathbf{R})/N)$ which along with (10) yields

$$\Delta\mathbf{z} \sim \mathrm{AsN}\left(\mathbf{0}_{M\times(M-1),1}, \frac{1}{N}\mathbf{J}\left(\mathbf{R}^T \otimes \mathbf{R}\right)\mathbf{J}^T\right). \tag{13}$$

Combining (10) and (13), we get

$$\mathbf{W}^{-\frac{1}{2}}\left[\hat{\mathbf{z}} - \mathbf{B}(\boldsymbol{\Theta})\boldsymbol{\eta}\right] \sim \mathrm{AsN}(\mathbf{0}_{M(M-1),1}, \mathbf{I}_{M(M-1)}) \tag{14}$$

where the weighted matrix $\mathbf{W}^{-1/2}$ is the Hermitian square root of $\mathbf{W}^{-1}$, $\mathbf{W} = \mathbf{J}(\mathbf{R}^T \otimes \mathbf{R})\mathbf{J}^T/N$, $\mathbf{I}_{M(M-1)}$ denotes the $M(M-1) \times M(M-1)$ identity matrix. From (14), we can further deduce that

$$\left\|\mathbf{W}^{-\frac{1}{2}}\left[\hat{\mathbf{z}} - \mathbf{B}(\boldsymbol{\Theta})\boldsymbol{\eta}\right]\right\|_2^2 \sim \mathrm{As}\chi^2\left[M(M-1)\right] \tag{15}$$

where $\mathrm{As}\chi^2[M(M-1)]$ represents the asymptotic chi-square distribution with $M(M-1)$ degrees of freedom. Following (15), how to select the parameter β is now quite obvious. We should introduce the parameter β such that the inequality $\|\mathbf{W}^{-1/2}[\hat{\mathbf{z}} - \mathbf{B}(\boldsymbol{\Theta})\boldsymbol{\eta}]\|_2^2 \leqslant \beta$ is satisfied with a high probability $\tilde{p}$, that is

$$\Pr\left\{\chi^2_{M(M-1)} \leqslant \beta\right\} = \tilde{p}, \quad \beta = \chi^2_{\tilde{p}}\left[M(M-1)\right] \tag{16}$$

where $\Pr(\cdot)$ stands for the probability distribution. Let $\hat{\mathbf{W}} = \mathbf{J}(\hat{\mathbf{R}}^T \otimes \hat{\mathbf{R}})\mathbf{J}^T/N$ be the estimate of $\mathbf{W}$. Then a statistically robust problem for DOA estimation can be reduced as

$$\min_{\boldsymbol{\eta}} \mathbf{1}^T\boldsymbol{\eta} \quad \text{s.t.} \quad \left\|\hat{\mathbf{W}}^{-\frac{1}{2}}\left[\hat{\mathbf{z}} - \mathbf{B}(\boldsymbol{\Theta})\boldsymbol{\eta}\right]\right\|_2 \leqslant \sqrt{\beta}, \quad \boldsymbol{\eta} \succcurlyeq \mathbf{0}. \tag{17}$$

In a nutshell, the idea behind the proposed method is to find the sparsest spatial spectrum estimation of the signals to best match or fit the covariance vector $\hat{\mathbf{z}}$. Particularly in (17), the quadratic constrained part can be viewed as the weighted least square fitting criteria which is an alternative to the maximum likelihood estimation and can asymptotically (in N) attain the Cramér–Rao Lower Bound (CRLB) [19]. Therefore, the proposed estimator in (17) has a certain statistical significance in achieving better DOA estimation. Simulation results in the subsequent section will confirm this point as well, including the robustness against nonuniform noise. Additionally, through our previous analysis, the number of source and the noise variances are not needed in the proposed method, which, however, is not shared by the existing SSR-based methods because, for example, they require the source number for subspace separation (see, e.g., [4]) or for choosing an appropriate regularization parameter (see, e.g., [6,9]), and the noise variance, whose estimate also relies on the source number, for establishing a sparse representation model (see, e.g., [8,10]) or also for the selection of the regularization parameter (see, e.g., [9,11]). Obviously, problem (17) is a second-order cone program problem and can be efficiently solved by off-the-shelf optimization softwares such as CVX [21].

4. Simulation results

In this section, a series of numerical experiments under different conditions are conducted to examine the performance of the proposed estimator. Throughout this section, the sensor noises are generated from a zero-mean complex Gaussian distribution. The

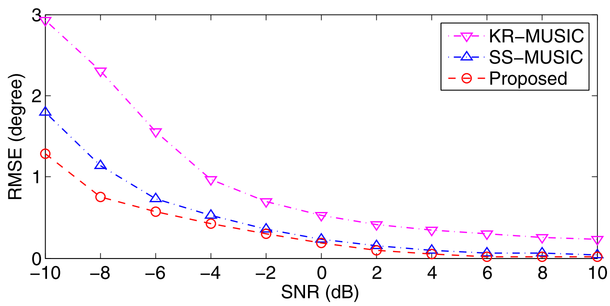


Fig. 1. RMSE versus SNR for quasi-stationary signals in the underdetermined case. $M = 6$, $K = 8$, $N = 3200$.

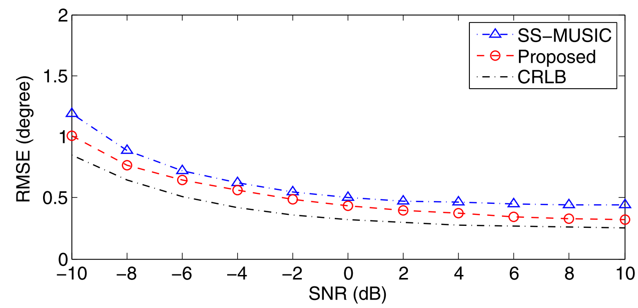


Fig. 2. RMSE versus SNR for stationary signals in the underdetermined case. $M = 6$, $K = 7$, $N = 300$.

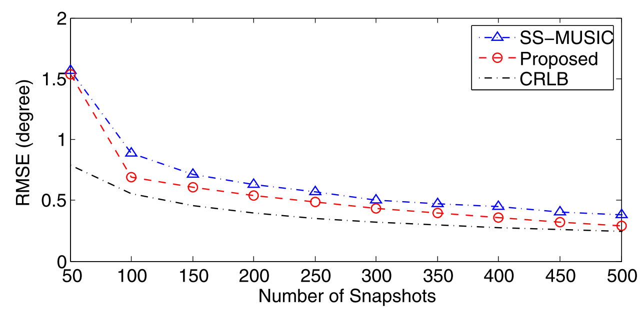


Fig. 3. RMSE versus the number of snapshots for stationary signals in the underdetermined case. $M = 6$, $K = 7$, SNR = 0 dB.

probability $\tilde{p}$ for β in the proposed algorithm is set as 0.999. All the numerical results are obtained from 1000 independent trials and the corresponding root mean square error (RMSE) is defined as

$$\mathrm{RMSE} = \left[\frac{1}{1000K}\sum_{k=1}^{K}\sum_{r=1}^{1000}\left(\hat{\theta}_k^r - \theta_k\right)^2\right]^{1/2}$$

where $\hat{\theta}_k^r$ denotes the estimate of θ_k in the rth trial. Additionally, to measure the variation of the noise environment, we define the worst noise power ratio (WNPR) as $\mathrm{WNPR} = \sigma_{\max}^2/\sigma_{\min}^2$ [12], where $\sigma_{\max}^2$ and $\sigma_{\min}^2$ denote the maximum and the minimum noise variance, respectively.

4.1. Underdetermined DOA estimation

We consider a 2-level nested array of a total of 6 sensors, whose element positions are $\{0, d, 2d, 3d, 7d, 11d\}$ where $d = \frac{\lambda}{2}$, with a ULA structure of 3 sensors in each level. Meanwhile, eight narrowband Gaussian sources from directions $[-60°, -42°, -30°, -15°, 0°, 10°, 22°, 35°]$ impinge on this nested array. In order to compare with KR-MUSIC [15], all the sources are assumed to be quasi-stationary and divided into 16 frames (intervals) with 200 snapshots in each frame, i.e., a total of 3200 snapshots. The input signal-to-noise ratio (SNR) is defined as $10\log_{10}\{(1/N)\cdot(\sum_{t=1}^{N} E[\|\mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t)\|^2])/(\sum_{m=1}^{M}\sigma_m^2)\}$ dB [15]. The noise covariance matrix and the candidate direction grid are set as $\mathbf{Q} = \mathrm{diag}\{10.2, 5.6, 8.5, 11.2, 7.8, 9.5\}$ (i.e., WNPR = 2) and 1° interval over $[-90°, 90°]$, respectively. Based on these experimental conditions, the simulation results on the RMSE versus SNR are depicted in Fig. 1. Meanwhile, the SS-MUSIC approach [16], based on a spatial smoothing processing, is also included for comparison. As is shown in Fig. 1, the proposed approach performs the best over all the range of SNR values. The SS-MUSIC algorithm is somewhat inferior to the proposed method mainly because the spatial smoothing processing will lead to aperture loss.

In what follows, we investigate the performance of the proposed method for the underdetermined DOA estimation of stationary Gaussian signals. To this end, seven stationary sources with equal power p^2, arriving from $[-60°, -42°, -30°, -15°, 20°, 35°, 55°]$, are considered. The input SNR is defined as [12]

$$\mathrm{SNR} = 10\log_{10}\left[\frac{p^2}{M}\sum_{m=1}^{M}\left(1/\sigma_m^2\right)\right]\mathrm{dB}. \tag{18}$$

The other experimental conditions including the noise covariance and the choice of dictionary grid are the same as the previous example. Figs. 2 and 3 show the RMSEs, respectively, versus the SNR with $N = 300$ and versus the number of snapshots with SNR = 0 dB. Also, we plot the relevant stochastic CRLB for the stationary Gaussian signals in nonuniform noise, which is derived in Appendix B. It can be seen from Figs. 2 and 3 that the proposed approach still outperforms the SS-MUSIC whenever the SNR or the number of snapshots is high or small.

4.2. Overdetermined DOA estimation

In the following examples, we consider two stationary Gaussian sources with equal power levels impinging upon an 8-element ULA with half-wavelength inter-sensor spacing, from fixed directions $[-5°, 5°]$ except for the bias analysis. The proposed method is compared with those of the state-of-the-art methods, including the MUSIC [2], Root-MUSIC [3], L1-SVD [4], L1-SRACV [8], SPICE [14] and SpSF [11]. Meanwhile, the conventional CRLB (which is here referred to as CCRLB) [12] for nonuniform noise is also considered to further evaluate their performances. It should be noticed that the CRLB in this paper should be lower than the CCRLB since the implicit *a priori* knowledge of the signal uncorrelation is incorporated into the estimator [20]. However, the CCRLB cannot be applied to the case of underdetermined DOA estimation because this will lead to the noninvertibility of the Fisher information matrix. This is why we did not plot the CCRLB in the previous example. The SNR is defined as the same as (18). In view of the requirement of noise variance in L1-SVD, L1-SRACV and SpSF, the estimate of noise variance is given by the arithmetic mean of the $M - K$ smallest eigenvalues of $\hat{\mathbf{R}}$, provided that the source number K is known *a priori*. However, this requirement is not needed in our proposed approach. In order to achieve better precision, the adaptive grid refinement method [4] is considered for RMSE analysis, where the number of iterations is set as 5, and in each iteration, a 2-point locally uniform grid is distributed symmetrically around each spectral peak, with one-third shrinking and spacing initialized to 1° interval.

Firstly, we consider the uniform noise case, i.e., a special case for nonuniform noise when WNPR = 1. In this experiment, the noise variances and the number of snapshots are fixed at 10 and 400, respectively. Accordingly, the result of the empirical probability of resolution versus the SNR is shown in Fig. 4, where

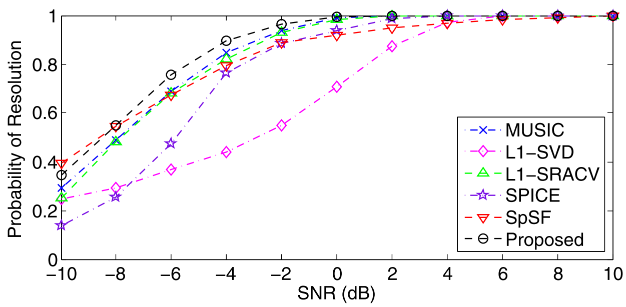
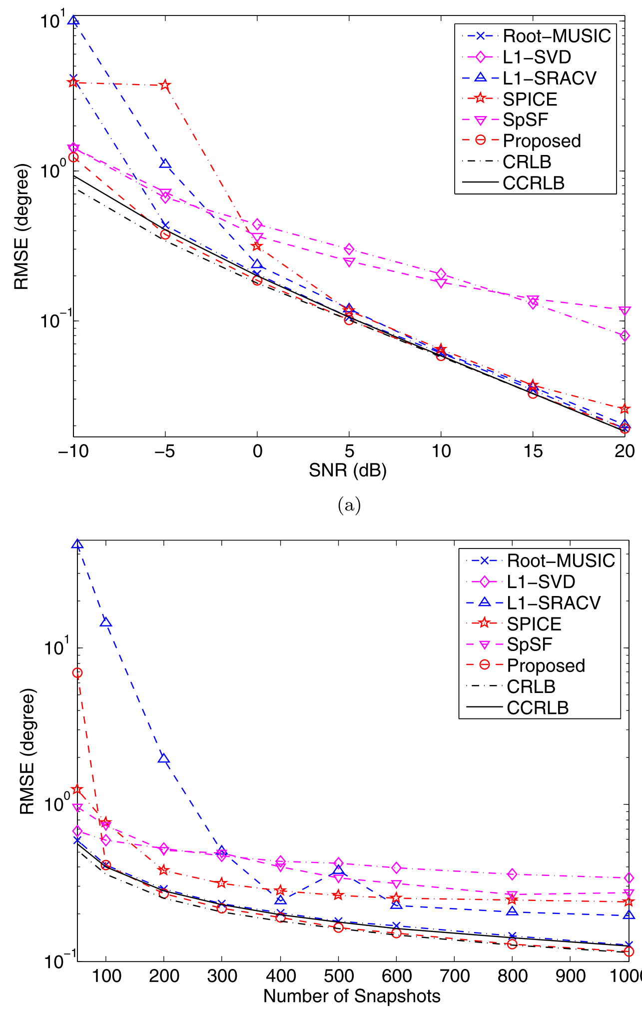


Fig. 4. Probability of resolution versus SNR for uniform noise in the overdetermined case. $M = 8$, $K = 2$, $N = 400$.

the DOAs are considered to be resolved within 1° estimate error. It can be observed that the proposed approach is superior to the other compared methods when SNR $\geqslant -8$ dB, but slightly less accurate than that of SpSF when SNR < -8 dB. However, the resolution probability of SpSF is barely able to attain one even at high SNRs, and as reported in the following experiments, its performance will degrade more seriously for the nonuniform noise environment.

Now, we consider the nonuniform noise case where the noise covariance matrix is fixed at $\mathbf{Q} = \mathbf{Q}_1 = \text{diag}\{10.5, 9.0, 10.0, 8.0, 12.0, 8.5, 6.0, 10.0\}$, i.e., WNPR = 2. Figs. 5(a) and 5(b) depict the RMSE versus the SNR with $N = 400$, and the RMSE versus the number of snapshots with SNR = 0 dB, respectively. From Fig. 5(a), we observe that the proposed method has the lowest RMSE among all the algorithms, particularly when the SNR is greater than 0 dB. Moreover, it approaches the CRLB well. However, the L1-SVD and SpSF provide somewhat poor performance mainly due to the substantial bias for closely spaced sources and the underestimated estimate of the noise variance. From Fig. 5(b), it is seen that our technique can still agree well with the CRLB when the number of snapshots is greater than about 200. Nevertheless, a performance degradation appears in a small number of snapshots (e.g., $N < 100$) mainly thanks to the fact that a consistent estimate of $\hat{\mathbf{R}}$ is guaranteed only for sufficiently large samples. Other SSR-based methods, however, have relatively large errors in nonuniform noise environment, especially in low SNRs and for large samples.

Next, we test the performance of the proposed algorithm in the worst noise case with a large variation of different sensor noise variances, where $\mathbf{Q} = \mathbf{Q}_2 = \text{diag}\{10.0, 1.2, 3.5, 18.0, 2.0, 8.5, 24.0, 6.5\}$ and WNPR = 20. The curves of bias versus the angle separation are plotted in Fig. 6(a) where the first source, SNR and number of snapshots are fixed at −25°, 5 dB and 400, respectively. Meanwhile, the case of $\mathbf{Q} = \mathbf{Q}_1$ is also plotted. It is seen from Fig. 6(a) that the biases in the two different noise cases are somewhat similar. In Fig. 6(b), we plot the RMSE versus the input SNR, and the other experimental parameters are the same as those in Fig. 5(a). We see that our technique still has a similar performance to that of the example shown in Fig. 5(a), and can still yield more competitive estimate than others, which becomes readily apparent at low SNRs. The other methods, however, provide a larger error. Additionally, to test how the parameter β affects the behavior of the proposed method, in Fig. 7 we plot the empirical probability of resolution versus the SNR under various probability values near one of $\tilde{p}$. It can be observed from Fig. 7 that a relatively small value of $\tilde{p}$ (e.g., $\tilde{p} = 0.9$) is slightly better at low SNRs, whereas fails to reach one completely in probability of resolution even at sufficiently large SNRs (see the enlargement of the part of graph in Fig. 7). In contrast, the scheme of choosing a higher $\tilde{p}$ can tend to one in probability of resolution for large SNRs, while in small SNRs result in relatively poor performance. Therefore, choosing the probability $\tilde{p}$ as 0.999 would be an acceptable compromise, which is also the reason that we did so in all other experiments.



(b)

Fig. 5. RMSE of various algorithms for nonuniform noise in the overdetermined case with $M = 8$, $K = 2$. (a) N fixed at 400. (b) SNR fixed at 0 dB.

Finally, to further validate the robustness of the proposed algorithm against the nonuniform noise model, we depict the RMSE versus the WNPR in Fig. 8, where the WNPR is varied from 20 to 200 under the case of $N = 400$, $\mathbf{Q} = \mathbf{Q}_2$. It can be seen from this figure that as the increment of SNR, the CRLB and CCRLB become very close, especially for SNR = 20 dB. The RMSE of the proposed technique is very close to the CRLB (also for CCRLB). This indicates again that the proposed approach is robust against the nonuniform noise.

5. Conclusion

In this work, we have presented a new DOA estimation method using a covariance-based sparse representation in the presence of unknown nonuniform noise. Our investigation has shown that, with moderate snapshots, the proposed method is able to yield reasonably accurate DOA estimation and agree well with the CRLB. Meanwhile, it should be noted that the proposed approach does not rely on the source number and the noise variances. Simulation results are in line with the theoretical analysis.

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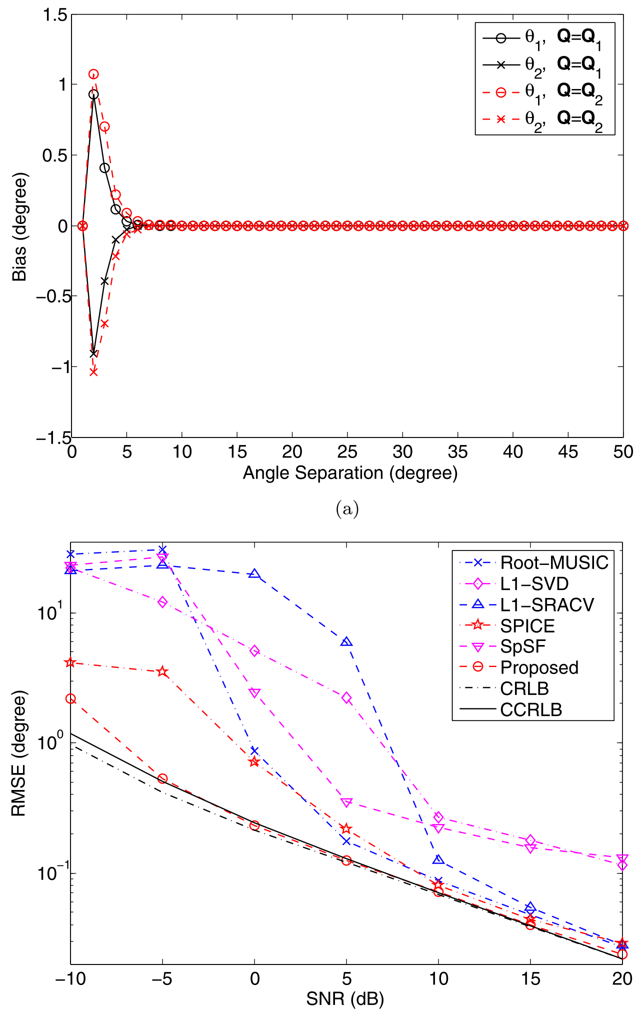

Fig. 6. Results of overdetermined DOA estimation for nonuniform noise with a large variation. $M = 8$, $K = 2$. (a) Estimation bias versus angle separation. (b) RMSE versus SNR with N fixed 400.

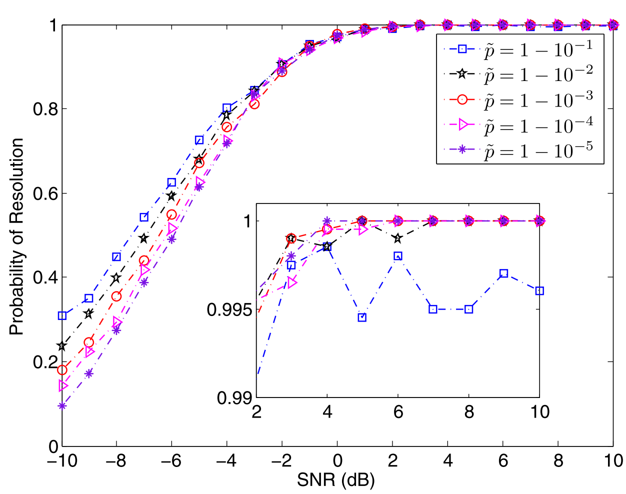

Fig. 7. Probability of resolution versus the SNR under various values of $\tilde{p}$. $M = 8$, $K = 2$, $N = 400$.

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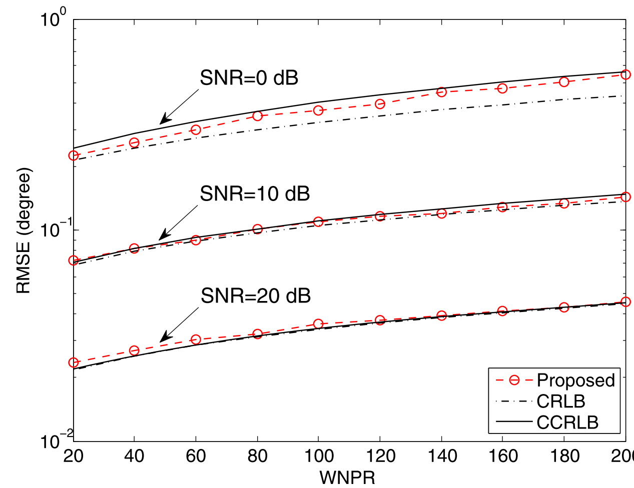

Fig. 8. RMSE versus WNPR. $M = 8$, $K = 2$, $N = 400$.

Appendix A. Proof of Proposition 1

According to the co-array principle [16], the maximum DOF achievable in $\mathbf{A}^*(\boldsymbol{\theta}) \odot \mathbf{A}(\boldsymbol{\theta})$ is $\mathrm{DOF}^{\mathrm{A}}_{\max} = M(M-1)+1$. Note that these row vectors of $\mathbf{A}^*(\boldsymbol{\theta}) \odot \mathbf{A}(\boldsymbol{\theta})$ which we have been removed are all composed of 1 and linearly dependent (M occurrences). Thus pre-multiplying $\mathbf{J}$ to $\mathbf{A}^*(\boldsymbol{\theta}) \odot \mathbf{A}(\boldsymbol{\theta})$ will give rise to a degree 1 deficiency, namely, the maximum DOF achievable of $\mathbf{B}(\boldsymbol{\theta})$ is $\mathrm{DOF}^{\mathrm{B}}_{\max} = M(M-1)$. With nonambiguity in the array structure, we have $\mathrm{Spark}[\mathbf{B}(\boldsymbol{\Theta})] = M(M-1)+1$ where $\mathrm{Spark}[\mathbf{B}(\boldsymbol{\Theta})]$ denotes the smallest possible integer of columns of $\mathbf{B}(\boldsymbol{\Theta})$ that are linearly dependent. Based on Corollary 1 of [18], there exists a unique sparsest representation $\boldsymbol{\eta}$ such that $\mathbf{z} = \mathbf{B}(\boldsymbol{\Theta})\boldsymbol{\eta}$ if and only if

$$\|\boldsymbol{\eta}\|_0 < \frac{\mathrm{Spark}[\mathbf{B}(\boldsymbol{\Theta})]}{2} = \frac{M(M-1)+1}{2} \tag{A.1}$$

which implies that

$$\|\boldsymbol{\eta}\|_0 \leqslant \big[M(M-1)\big]/2. \tag{A.2}$$

Thus, the maximum separable signal number is $M(M-1)/2$. If a 2-level nested array [16] with M_1 and M_2 elements respectively in each level is considered, we can obtain $\mathrm{Spark}[\mathbf{B}(\boldsymbol{\Theta})] = 2M_2(M_1+1)-1$ because $\mathrm{DOF}^{\mathrm{B}}_{\max} = 2M_2(M_1+1)-2$ again from the co-array principle. Then according to (A.1), the maximum separable signal number is $M_2(M_1+1)-1$. Similarly, it is easy to obtain that this number in ULA structure is $M-1$ since the DOF of $\mathbf{B}(\boldsymbol{\theta})$ in ULA structure is $2M-2$. Obviously, if the array structure suffers from ambiguity, then $\mathrm{Spark}[\mathbf{B}(\boldsymbol{\Theta})] = 2$, which directly leads to the uniqueness condition $\|\boldsymbol{\eta}\|_0 = 0$ from (A.1). This completes the proof.

Appendix B. Stochastic CRLB for nonuniform noise

In the unconditional stochastic case, the vector of unknown parameters is defined as

$$\boldsymbol{\Psi} = \big[\boldsymbol{\theta}^T, \boldsymbol{\delta}^T\big]^T, \quad \boldsymbol{\delta}^T = \big[\mathbf{p}^T, \boldsymbol{\sigma}^T\big]^T. \tag{B.1}$$

Recall that the CRLB is the inverse of the Fisher information matrix (FIM). As a result, using the formula that $\mathrm{trace}\{\mathbf{WXYZ}\} = \mathrm{vec}^H(\mathbf{X}^H)(\mathbf{W}^T \otimes \mathbf{Y})\mathrm{vec}(\mathbf{Z})$, the (q,l)th element of the FIM associated with $\mathbf{R}$ is given by [20,22]

$$\begin{aligned}[\mathbf{F}]_{q,l} &= N\,\mathrm{Trace}\left\{\mathbf{R}^{-1}\frac{\partial \mathbf{R}}{\partial \psi_q}\mathbf{R}^{-1}\frac{\partial \mathbf{R}}{\partial \psi_l}\right\} \\ &= N\mathbf{r}_q^H\mathbf{W}_{\mathbf{R}}\mathbf{r}_l \quad \text{for } q,l = 1,\ldots,2K+M \end{aligned} \tag{B.2}$$

where $\mathbf{W_R} = \mathbf{R}^{-T} \otimes \mathbf{R}^{-1}$, $\mathbf{r}_q = \mathrm{vec}(\partial\mathbf{R}/\partial\psi_q)$, ψ_q is the qth element of the parameter vector $\boldsymbol{\Psi}$, and

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{\theta\theta} & \mathbf{F}_{\theta\delta} \\ \mathbf{F}_{\delta\theta} & \mathbf{F}_{\delta\delta} \end{bmatrix}. \tag{B.3}$$

Notice that the covariance matrix can be reformulated as

$$\mathbf{R} = \sum_{k=1}^{K} \mathbf{a}(\theta_k) p_k^2 \mathbf{a}^H(\theta_k) + \sum_{m=1}^{M} \sigma_m^2 \mathbf{e}_m \mathbf{e}_m^T \tag{B.4}$$

where $\mathbf{e}_m$ is defined in (4). According to (B.4) and using the fact that $\mathrm{vec}(\mathbf{b}\mathbf{a}^T) = \mathbf{a} \otimes \mathbf{b}$, we obtain

$$\begin{aligned}
\mathbf{r}_{\theta_k} &= \mathrm{vec}(\partial\mathbf{R}/\partial\theta_k) = p_k^2 \,\mathrm{vec}\big[\mathbf{a}'(\theta_k)\mathbf{a}^H(\theta_k) + \mathbf{a}(\theta_k)\big(\mathbf{a}'(\theta_k)\big)^H\big] \\
&= p_k^2\big[\mathbf{a}^*(\theta_k) \otimes \mathbf{a}'(\theta_k) + \big(\mathbf{a}'(\theta_k)\big)^* \otimes \mathbf{a}(\theta_k)\big] \\
\mathbf{r}_{p_k^2} &= \mathrm{vec}\big(\partial\mathbf{R}/\partial p_k^2\big) = \mathrm{vec}\big[\mathbf{a}(\theta_k)\mathbf{a}^H(\theta_k)\big] = \mathbf{a}^*(\theta_k) \otimes \mathbf{a}(\theta_k) \\
\mathbf{r}_{\sigma_m^2} &= \mathrm{vec}\big(\partial\mathbf{R}/\partial\sigma_m^2\big) = \mathrm{vec}\big(\mathbf{e}_m\mathbf{e}_m^T\big) = \mathbf{e}_m \otimes \mathbf{e}_m
\end{aligned} \tag{B.5}$$

where $\mathbf{a}'(\theta_k) = \partial\mathbf{a}(\theta_k)/\partial\theta_k$. Now combining (B.2), (B.5) and the Khatri–Rao product, we can calculate the submatrices of $\mathbf{F}$ as

$$\begin{aligned}
\mathbf{F}_{\theta\theta} &= N\mathbf{D}_\theta^H \mathbf{W_R} \mathbf{D}_\theta, \qquad \mathbf{F}_{\theta\delta} = N\mathbf{D}_\theta^H \mathbf{W_R} \mathbf{D}_\delta \\
\mathbf{F}_{\delta\delta} &= N\mathbf{D}_\delta^H \mathbf{W_R} \mathbf{D}_\delta, \qquad \mathbf{F}_{\delta\theta} = N\mathbf{D}_\delta^H \mathbf{W_R} \mathbf{D}_\theta = \mathbf{F}_{\theta\delta}^H
\end{aligned} \tag{B.6}$$

where $\mathbf{D}_\theta = [\mathbf{r}_{\theta_1}, \ldots, \mathbf{r}_{\theta_K}] = [\mathbf{A}^*(\boldsymbol{\theta}) \odot \mathbf{A}'(\boldsymbol{\theta}) + (\mathbf{A}'(\boldsymbol{\theta}))^* \odot \mathbf{A}(\boldsymbol{\theta})]\mathbf{P}$, $\mathbf{A}'(\boldsymbol{\theta}) = [\mathbf{a}'(\theta_1), \ldots, \mathbf{a}'(\theta_K)]$, $\mathbf{D}_\delta = [\mathbf{r}_{p_1^2}, \ldots, \mathbf{r}_{p_K^2}, \mathbf{r}_{\sigma_1^2}, \ldots, \mathbf{r}_{\sigma_M^2}] = [\mathbf{A}^*(\boldsymbol{\theta}) \odot \mathbf{A}(\boldsymbol{\theta}), \mathbf{I}_M \odot \mathbf{I}_M]$. Now we have all the submatrices in $\mathbf{F}$ for CRLB calculation. Invoking the matrix inverse lemma of partitioned matrices, we eventually get

$$\mathrm{CRLB}_{\theta\theta} = \big\{\mathbf{F}_{\theta\theta} - \mathbf{F}_{\theta\delta}\mathbf{F}_{\delta\delta}^{-1}\mathbf{F}_{\delta\theta}\big\}^{-1}. \tag{B.7}$$

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