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Polynomial root finding technique for joint DOA DOD estimation in bistatic MIMO radar

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ABSTRACT

In this paper, we propose a new technique to transform the 2-D direction finding in the bistatic MIMO radar into a double 1-D direction finding procedure. Firstly, a search based 2-D MUSIC method to estimate the joint DOA (direction of arrival) and DOD (direction of departure) in multi-target situation is presented. Then, we propose an algorithm based on double polynomial root finding procedure to estimate the DOA and DOD. The proposed method allows an efficient estimation of the target DOA and DOD with automatic pairing. The simulation results of the proposed algorithm are presented and the performances are investigated and discussed.

1. Introduction

It has been shown that the multi-input multi-output (MIMO) antenna systems have the potential to improve dramatically the performance of communication and radar systems over single antenna systems. MIMO radar uses multiple antennas to transmit simultaneously several linearly orthogonal waveforms and also uses multiple antennas to receive the reflected signals [1–5].

The MIMO radar can be subdivided into two classes according to their antennas configurations. One class is the conventional radar array, in which both the transmitting elements and the receiving elements are closely spaced. The other class is the diverse antenna configuration, where the transmitting and the receiving elements are spaced far away from each other to achieve the diversity gain [1,5].

Fishler et al. [2] have investigated the problem of statistical MIMO radar, in which sparse antenna arrays are used to obtain uncorrelated received signal. This scheme is shown to be useful for solving the target scintillation problem [2].

An important feature of MIMO radar is the use of orthogonal transmitted waveforms. When the signals are orthogonal, thus separable, at the receiver, MIMO processing may be performed to achieve spatial and signal waveform diversity [5].

Based on a configuration where the transmitters are widely separated, Tabrikian et al. [6] have proposed a multi-target 1-D localization technique using the MUSIC (MUltiple SIgnal Classification) algorithm for estimating the directions of arrival (DOA). For a MIMO radar with transmitters widely spaced and the receivers half wavelength spaced, in [7], an extended signal model to the multi-target case is proposed. Capon method and APES (Amplitude and Phase EStimation) are then used to estimate the directions of arrival and to estimate the fading coefficients, respectively.

Recently, a bistatic MIMO radar scheme has been proposed in [8], where a two-dimensional (2-D) spatial spectrum estimation technique based on the Capon method is developed. The maximum number of targets localizable by this method is the product of the number of receive and transmit elements minus one. The proposed method needs an exhaustive search through all the 2-D space to find the DOA and DOD of the targets.

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Exploiting the same MIMO radar configuration, Jin et al. [9] have proposed a joint DOA and DOD estimation approach based on the ESPRIT technique developed in [10]. A closed form of the target DOA and DOD is obtained and automatically paired. However, the number of targets which can be localized by this method is smaller than the number of receivers.

With the same architecture of the bistatic MIMO radar, another ESPRIT-based method is proposed in [11] by exploiting the invariance property of both of the transmitting array and the receiving array to estimate the DOA and DOD of targets. The pairing is not obtained automatically by this algorithm, but a solution is given to this problem.

In this paper, we propose a new technique to transform the 2-D direction finding in bistatic MIMO radar, with the same configuration and hypotheses as in [8,9,11], into a double 1-D direction finding procedure. Firstly, a search based 2-D MUSIC method to estimate the joint DOA and DOD in multi-target situation is presented. Then, we propose an algorithm based on double polynomial root finding procedure to estimate the DOA and the DOD of targets. The pairing between the DOA and DOD is automatically obtained.

This paper is organized as follows. The signal model is presented in Section 2. The proposed root finding algorithm is described in Section 3 where the analytical expressions are given. The simulation results of the proposed algorithm are provided in Section 4. Finally, Section 5 concludes this paper.

2. Signal model

We consider a bistatic MIMO radar system with M closely spaced transmit antennas and N closely spaced receive antennas (Fig. 1). Both of them are uniform linear arrays (ULA) and all the elements are omnidirectional. Δ_t and Δ_r are the inter-element spacing at the transmitter and the receiver, respectively. The signal model proposed in [8,9,11] is adopted in this paper. It is assumed that the Doppler frequencies have almost no effect on the orthogonality of the signals. Therefore, the variety of the phase within pulses caused by Doppler frequency can be ignored [9]. The targets range is assumed much larger than the aperture of transmit array and receive array. Each element of the transmitter transmits orthogonal waveforms $\mathbf{s}_m$, $m=1\ldots M$. These signals are reflected by a target assumed at position (θ_r,θ_t) with θ_t denoting the DOD and θ_r denoting the DOA. The received signal can be written as [8,9,11]

$$\mathbf{x}(l,t)=\alpha\mathbf{a}(\theta_r)\mathbf{b}^T(\theta_t)\begin{bmatrix} s_1(l) \\ \vdots \\ s_M(l)\end{bmatrix} e^{j2\pi f_d t}+\mathbf{w}(l,t) \tag{1}$$

where $\mathbf{a}(\theta_r)=[1\ e^{j2\pi\Delta_r\sin(\theta_r)/\lambda}\ \ldots\ e^{j2\pi(N-1)\Delta_r\sin(\theta_r)/\lambda}]^T\in C^{N\times 1}$ is the receiver steering vector; $\mathbf{b}(\theta_t)=[1\ e^{j2\pi\Delta_t\sin(\theta_t)/\lambda}\ \ldots\ e^{j2\pi(M-1)\Delta_t\sin(\theta_t)/\lambda}]^T\in C^{M\times 1}$ is the transmitter steering vector; α is the reflection coefficient depending on the target radar cross section; $e^{j2\pi f_d t}$ is due to the target Doppler shift; $\mathbf{w}(l,t)$ is the noise vector, whose elements are assumed to be independent, zero mean, complex and Gaussian distributed; l represents the time within the pulse (fast time) and t indicates the index of radar pulse (slow time) [5].

Since the transmitted waves are orthogonal ($\langle\mathbf{s}_i,\mathbf{s}_j\rangle=0$ and $\|\mathbf{s}_i\|^2=1$, $i\neq j=1\ldots M$), the output of the receiver matched filters can be written as [8,9,11,12]

$$\mathbf{y}_m(t)=\alpha\mathbf{a}(\theta_r)\mathbf{b}^T(\theta_t)\begin{bmatrix} 0 \\ \vdots \\ e^{j2\pi f_d t} \\ \vdots \\ 0\end{bmatrix}+\mathbf{w}_m(t) \tag{2}$$

$$\mathbf{y}_m(t)=\alpha\mathbf{a}(\theta_r)b_m(\theta_r)e^{j2\pi f_d t}+\mathbf{w}_m(t) \tag{3}$$

where $b_m(\theta_r)$ denotes the m th element of the transmitter steering vector and $\mathbf{w}_m$ is the noise vector after m th matched filter.

Therefore, we obtain an observation matrix of $M\times N$ elements.

Let $\mathbf{z}(t)\in C^{MN\times 1}$ be the output of all the received signal

$$\mathbf{z}(t)=[\mathbf{y}_1(t)^T,\ldots,\mathbf{y}_M(t)^T]^T \tag{4}$$

$\mathbf{z}(t)$ can be also written as

$$\mathbf{z}(t)=\alpha[b_1(\theta_t),\ldots,b_M(\theta_t)]^T\otimes\mathbf{a}(\theta_r)e^{j2\pi f_d t}+\mathbf{n}(t) \tag{5}$$

$$\mathbf{z}(t)=\mathbf{c}(\theta_r,\theta_t)s(t)+\mathbf{n}(t) \tag{6}$$

where $\mathbf{c}(\theta_r,\theta_t)=\mathbf{b}(\theta_r)\otimes\mathbf{a}(\theta_r)$ is the direction vector, $\mathbf{n}(t)$ is the additive white Gaussian noise, $s(t)=\alpha e^{j2\pi f_d t}$ and $\otimes$ is the Kronecker operator.

In the case of P targets, the received signal model can be written as

$$\mathbf{z}(t)=\mathbf{Cs}(t)+\mathbf{n}(t) \tag{7}$$

with the matrix of the target directional vectors

$$\mathbf{C}=[\mathbf{c}(\theta_r^1,\theta_t^1),\ldots,\mathbf{c}(\theta_r^P,\theta_t^P)]_{MN\times P} \tag{8}$$

$$\mathbf{c}(\theta_r^p,\theta_t^p)=\mathbf{b}(\theta_t^p)\otimes\mathbf{a}(\theta_r^p) \tag{9}$$

$$\mathbf{s}(t)=\begin{bmatrix}\alpha_1 e^{j2\pi f_{d1}t} \\ \vdots \\ \alpha_P e^{j2\pi f_{d_P}t}\end{bmatrix} \tag{10}$$

Here, we assume that the different targets have different Doppler frequencies and all the P targets are in

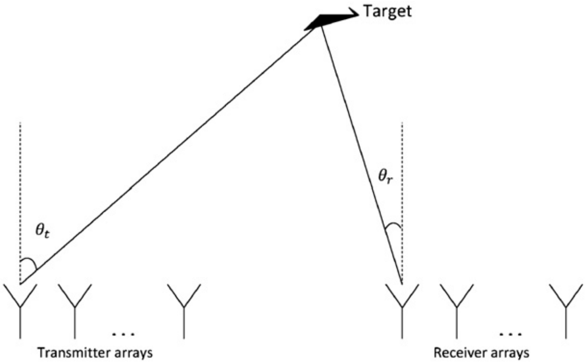

Fig. 1. Bistatic radar configuration.

the same range bin. Therefore, the components of signal vector $\mathbf{s}(t)$ are not fully correlated. In the case where several targets have a same Doppler shift, the well known spatial smoothing [13] can be used to decorrelate the signals.

3. Joint DOA and DOD estimation

The MUSIC algorithm is based on the orthogonality between the noise subspace and the signal subspace. These subspaces can be obtained by the eigendecomposition of the covariance matrix of the received signal

$$\mathbf{R} = \mathbf{E}[\mathbf{z}\mathbf{z}^H] = \mathbf{C}\mathbf{R}_{ss}\mathbf{C}^H + \sigma_n^2 \mathbf{I}_{MN} \tag{11}$$

which can be estimated by

$$\hat{\mathbf{R}} = \frac{1}{L}\sum_{t=1}^{L} \mathbf{z}(t)\mathbf{z}^H(t) \tag{12}$$

Since the actual target directional vectors span the signal subspace, the eigenvectors associated with the $MN - P$ smallest eigenvalues (noise subspace) are orthogonal to the actual target directional vectors.

The eigendecomposition of the covariance matrix can be written as

$$\mathbf{R} = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^H + \mathbf{U}_n\mathbf{\Lambda}_n\mathbf{U}_n^H = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^H + \sigma^2\mathbf{U}_n\mathbf{U}_n^H \tag{13}$$

where $\mathbf{U}_s$ is the matrix of eigenvectors in the signal subspace, $\mathbf{\Lambda}_s = \mathrm{diag}[\lambda_1, \ldots, \lambda_P]$ is a diagonal matrix containing the P largest eigenvalues associated to the columns of $\mathbf{U}_s$; $\mathbf{U}_n$ is the noise eigenvectors matrix, $\mathbf{\Lambda}_n = \mathrm{diag}[\lambda_{P+1}, \ldots, \lambda_{MN}]$ is a diagonal matrix with diagonal elements equal to the eigenvalues corresponding to the noise variance σ^2.

From the orthogonality of the two subspaces, we have

$$\|\mathbf{U}_n^H\mathbf{c}(\theta_r^{(p)}, \theta_t^{(p)})\|^2 = 0, \quad p = 1, \ldots, P \tag{14}$$

To estimate both the DOA and DOD, the classical solution is to find the maxima of the pseudo-spectrum given by

$$P_{MUSIC}(\theta_r, \theta_t) = \frac{1}{\mathbf{c}(\theta_r, \theta_t)^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{c}(\theta_r, \theta_t)} \tag{15}$$

This method allows the direction estimation of $MN-1$ targets but it needs an exhaustive search through all the 2-D space.

Instead of the exhaustive search, we propose to use a double polynomial root finding procedure for estimating the target directions. The polynomial root finding can be done with many fast and efficient algorithms.

Denoting

$$z_r = e^{j2\pi\Delta_r \sin(\theta_r)/\lambda}, \quad z_t = e^{j2\pi\Delta_t \sin(\theta_t)/\lambda} \tag{16}$$

we can rewrite the directional vectors as

$$\mathbf{a}(z_r) = [1, z_r, z_r^2, \ldots, z_r^{N-1}]^T \tag{17}$$

$$\mathbf{b}(z_t) = [1, z_t, z_t^2, \ldots, z_t^{M-1}]^T \tag{18}$$

Using the above expressions, the expression (9) is then rewritten as

$$\mathbf{c}(z_r, z_t) = [\mathbf{a}(z_r)^T, z_t\mathbf{a}(z_r)^T, z_t^2\mathbf{a}(z_r)^T, \ldots, z_t^{M-1}\mathbf{a}(z_r)^T]^T \tag{19}$$

It is known that if θ_r and θ_t correspond to the directions of an actual target, then the directional vector is orthogonal to the noise subspace

$$\mathbf{c}(\theta_r, \theta_t)^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{c}(\theta_r, \theta_t) = 0 \tag{20}$$

$$\Updownarrow$$

$$\mathbf{c}(z_r^{-1}, z_t^{-1})^T\boldsymbol{\Pi}_n\mathbf{c}(z_r, z_t) = 0 \tag{21}$$

where

$$\boldsymbol{\Pi}_n = \mathbf{U}_n\mathbf{U}_n^H \tag{22}$$

$\boldsymbol{\Pi}_n$ can be partitioned as

$$\boldsymbol{\Pi}_n = \begin{bmatrix} \boldsymbol{\Pi}_{11} & \cdots & \boldsymbol{\Pi}_{1M} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\Pi}_{M1} & \cdots & \boldsymbol{\Pi}_{MM} \end{bmatrix} \quad \text{with } \boldsymbol{\Pi}_{ij}|_{i,j=1,\ldots,M} \in C^{N\times N} \tag{23}$$

Therefore, we can rewrite expression (21) as

$$\mathbf{a}(z_r^{-1})^T\left[\sum_{i,j=1}^{M} z_t^{j-i}\boldsymbol{\Pi}_{ij}\right]\mathbf{a}(z_r) = 0 \tag{24}$$

Therefore, the estimation of the set of θ_r and θ_t which minimizes the projection of the directional vector on the noise subspace is equivalent to finding the roots of the above polynomial function.

If z_t does not correspond to one of DOD, and if

$$Rank(\boldsymbol{\Pi}_n) = MN - P \geq N \Rightarrow P \leq M(N-1) \tag{25}$$

then the matrix $[\sum_{j,i=1}^{M} z_t^{j-i}\boldsymbol{\Pi}_{ij}]$ is invertible, and its determinant is not null.

Therefore, to solve the set in expression (24) we can find firstly z_t satisfying

$$D(z_t) = \det\left[\sum_{j,i=1}^{M} z_t^{j-i}\boldsymbol{\Pi}_{ij}\right] = 0 \tag{26}$$

The formula of the polynomial in (26) is given in Appendix A.

The P roots inside and closest to the unitary circle of the polynomial $D(z_t)$, $\check{z}_t^{(p)}|_{p=1,\ldots,P}$, allow to estimate the DOD angles, given by

$$\theta_t^{(p)} = \arcsin\left(\frac{\lambda}{2\pi\Delta_t}\arg(\check{z}_t^{(p)})\right) \tag{27}$$

By substituting the obtained roots $\check{z}_t^{(p)}$ in the global expression (24), we constitute the following equation systems:

$$\mathbf{a}(z_r^{-1})^T\check{\boldsymbol{\Pi}}_p\mathbf{a}(z_r) = 0|_{p=1,\ldots,P} \tag{28}$$

where

$$\check{\boldsymbol{\Pi}}_p = \sum_{j,i=1}^{M} (\check{z}_t^{(p)})^{j-i}\boldsymbol{\Pi}_{ij}, \quad p = 1, \ldots, P \tag{29}$$

To find the roots of the obtained polynomial (28), we can again use the root finding technique. From the principle of MUSIC, we know that (24) becomes zero if and only if (z_r, z_t) corresponds to the target location. Then, for each $\check{z}_t^{(p)}$, this polynomial of degree $2N-2$ has q roots inside and close to the unit cycle where q is the number of targets having the same DOD $=\check{z}_t^{(p)}$. Among the q roots, each one must be chosen only once.

The roots $\check{z}_r^{(p)}$, of the polynomial (28), inside and closest to the unit circle determine the DOA angle given by

$$\theta_r^p = \arcsin\left(\frac{\lambda}{2\pi\Delta_r}\arg(\check{z}_r^{(p)})\right) \tag{30}$$

Using (16), (24) and (26), this decomposition of the 2-D direction finding into a double 1-D root finding procedure can be also used to determine the directions based on double 1-D MUSIC pseudo-spectrum.

We note that the pairing is automatically obtained between the DOD and DOA angles, which avoids the traditional bistatic radar problem of synchronization.

This decomposition of the 2-D direction finding into a double 1-D direction finding procedure (polynomial root finding or double 1-D MUSIC pseudo-spectrum) allows a significant reduction in computational complexity.

Obviously, the searching for the maximum of the MUSIC pseudo-spectrum in the 2-D angle space requires an important number of iterations which depends on the size of the searching space and the searching resolution. For instance, we must estimate the MUSIC pseudo-spectrum $(E/r)^2$ times, where E is the searching angle space and r is the searching resolution, which may be very important if we want a good searching resolution. Otherwise, for the proposed algorithm, we solve one polynomial of degree $2NM-2$ and P polynomials of degree $2N-2$ and the obtained results have infinite resolution.

In expression (26), the maximum number of targets which can be localized by the proposed root finding algorithm is given by the condition in expression (25). This maximum number of localizable targets depends on the way to constitute the received vector (4). Since the constituted received vector $\mathbf{z}(t)$ can be also reshaped as

$$\mathbf{z}(t) = \alpha\mathbf{a}(\theta_r) \otimes \mathbf{b}(\theta_r)e^{j2\pi f_d t} + \mathbf{n}(t) \tag{31}$$

the proposed method can localize $N(M-1)$ targets.

Therefore, the optimal constitution of $\mathbf{z}(t)$ can be made according to the numbers of transmitters and receivers.

4. Simulation results

In this section, we present the simulation results in order to illustrate the performance of the proposed algorithms.

First step, the considered bistatic MIMO radar is composed of M=3 transmitter elements and N=3 elements in receiver array spaced by a half wavelength. The number of the samples L=256 and the SNR=15 dB. Here, the number of targets is P=4 located at the angles $(\theta_r,\theta_t) = (10^\circ,45^\circ)$, $(40^\circ,5^\circ)$, $(55^\circ,70^\circ)$, $(80^\circ,30^\circ)$. The number of Monte-Carlo iterations is fixed at 200.

The result of the 2-D MUSIC pseudo-spectrum given by expression (15) is shown in Fig. 2. We can see that the 2-D MUSIC pseudo-spectrum can localize the targets in the DOA-DOD space.

Fig. 3 shows the obtained result by using the double 1-D MUSIC pseudo-spectrum algorithm. We can observe that the target directions are well localized and these directions are automatically paired.

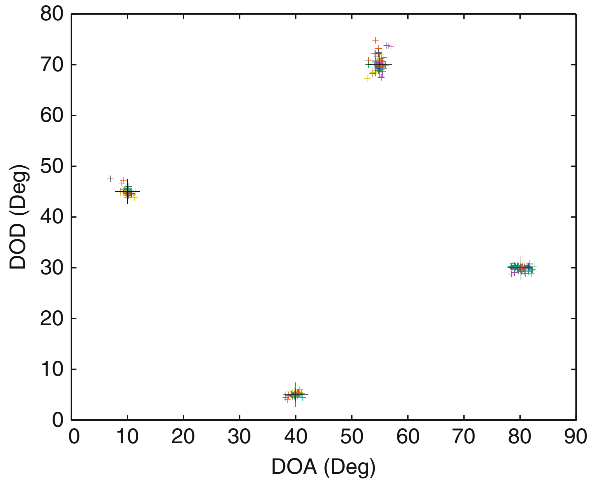

Fig. 2. 2-D MUSIC pseudo-spectrum for P=4 targets, N=M=3, L=256 samples, and SNR=15 dB.

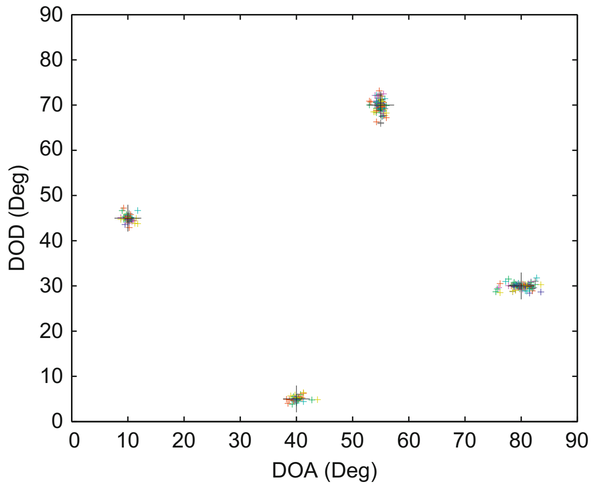

Fig. 3. Double 1-D MUSIC pseudo-spectrum for P=4 targets, N=M=3, L=256 samples and SNR=15 dB.

With the same configuration of the simulation as before, Fig. 4 shows the obtained results by using the double 1-D root finding algorithm. It is clear that the directions are estimated and automatically paired. From the comparison between Figs. 2, 3 and 4, we can notice that the root finding method has better performance on precision than the MUSIC pseudo-spectrum algorithms. This result is not surprising. It is well known that the root-MUSIC method has better performance than the spectral MUSIC method.

For the root finding algorithm, in Figs. 5 and 6 the curves present the root mean square error (RMSE) of the DOD and the DOA estimation versus SNR, respectively. We observe that the proposed root-finding-based algorithm has a good performance in the estimation of DOD and DOA.

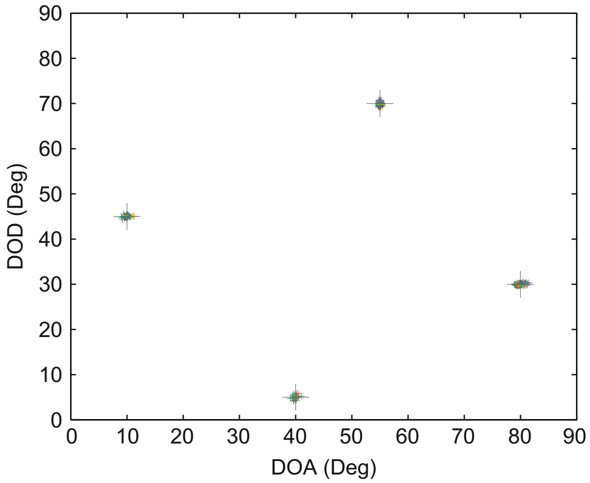

Fig. 4. Double 1-D root finding for P=4 targets, N=M=3, L=256 samples and SNR=15 dB.

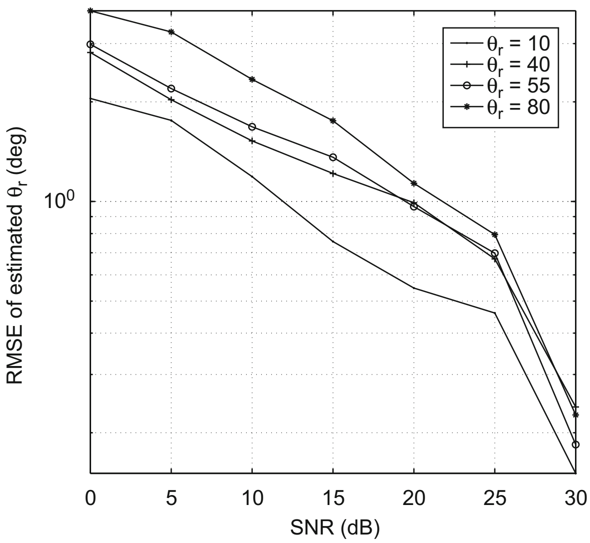

Fig. 5. RMSE in DOA estimation versus SNR by double 1-D root-finding for P=4 targets, N=M=3, L=256 samples.

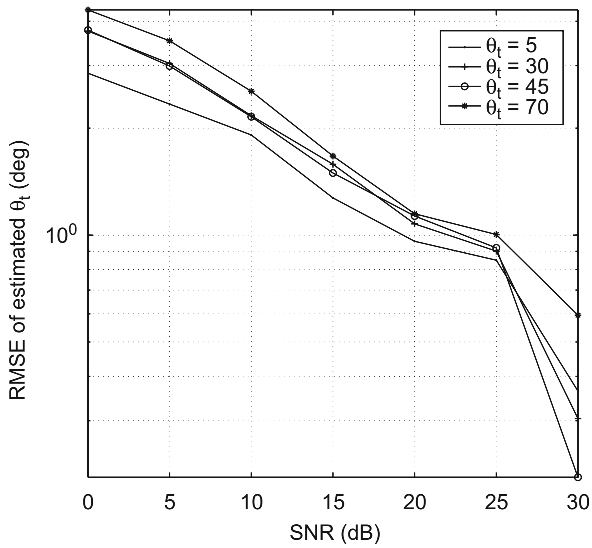

Fig. 6. RMSE in DOD estimation versus SNR by double 1-D root-finding for P=4 targets, N=M=3, L=256 samples.

Second step, we consider the same simulation conditions as in [11], i.e. the bistatic MIMO radar is constituted of M=8 transmitters and N=6 receivers. The number of samples L=100 and the number of targets P=3 located at the angles $(\theta_r,\theta_t)=(10^{\circ},20^{\circ})$, $(-8^{\circ},30^{\circ})$,$(0^{\circ},45^{\circ})$ are considered and 300 Monte-Carlo iterations are used.

Figs. 7(a) and (b) present the average of RMSE of the estimated target DOA and DOD by the proposed algorithm, the 2-D MUSIC pseudo-spectrum, the double 1-D MUSIC pseudo-spectrum and the ESPRIT algorithm proposed in [11]. We observe that the robustness to noise is slightly better for the proposed root finding algorithm than the ESPRIT algorithm. Furthermore, in our proposition the pairing is automatically obtained, but an additional pairing operation is necessary for the ESPRIT algorithm. The 2-D MUSIC pseudo-spectrum is strongly degraded in low SNR.

The root finding based method has better performances than the spectral MUSIC, because the spectral MUSIC method finds the maximum only on the unit circle while the root finding method allows to find the roots close to the unit circle.

Third step, we compare the angular resolution performance of the proposed algorithm to the ESPRIT algorithm proposed in [11]. We consider a bistatic MIMO radar constituted of M=3 transmitters and N=3 receivers. The number of samples is L=256 and SNR=20 dB. This system is used to localize two targets at $(\theta_r,\theta_t)=(70^{\circ},50^{\circ})$ and $(70^{\circ}-Df^{\circ}, 50^{\circ}-Df^{\circ})$, where Df (varied from 2.5° to 20°) is the separation between the targets in DOA (DOD). K=300 Monte-Carlo iterations are used.

Figs. 8(a) and (b) present the averaged RMSE, given by the following expressions, versus the separation between the targets DOAs (DODs)

$$RMSE_{DOA}=\frac{\sqrt{\left[\frac{1}{K}\sum_{k=1}^{K}(\hat{\theta}^{(1)}_{r(k)}-\theta^{(1)}_{r(k)})^2\right]}+\sqrt{\left[\frac{1}{K}\sum_{k=1}^{K}(\hat{\theta}^{(2)}_{r(k)}-\theta^{(2)}_{r(k)})^2\right]}}{2}$$

$$RMSE_{DOD}=\frac{\sqrt{\left[\frac{1}{K}\sum_{k=1}^{K}(\hat{\theta}^{(1)}_{t(k)}-\theta^{(1)}_{t(k)})^2\right]}+\sqrt{\left[\frac{1}{K}\sum_{k=1}^{K}(\hat{\theta}^{(2)}_{t(k)}-\theta^{(2)}_{t(k)})^2\right]}}{2} \quad (32)$$

In these figures, we can observe that both algorithms provide approximately the same resolution in DOA and DOD estimation because both algorithms are based on subspace decomposition.

Fourth step, we investigate the computational complexity of the proposed algorithm.

Fig. 9 presents an evaluation of the computational complexity using the CPUTIME instruction in MATLAB for

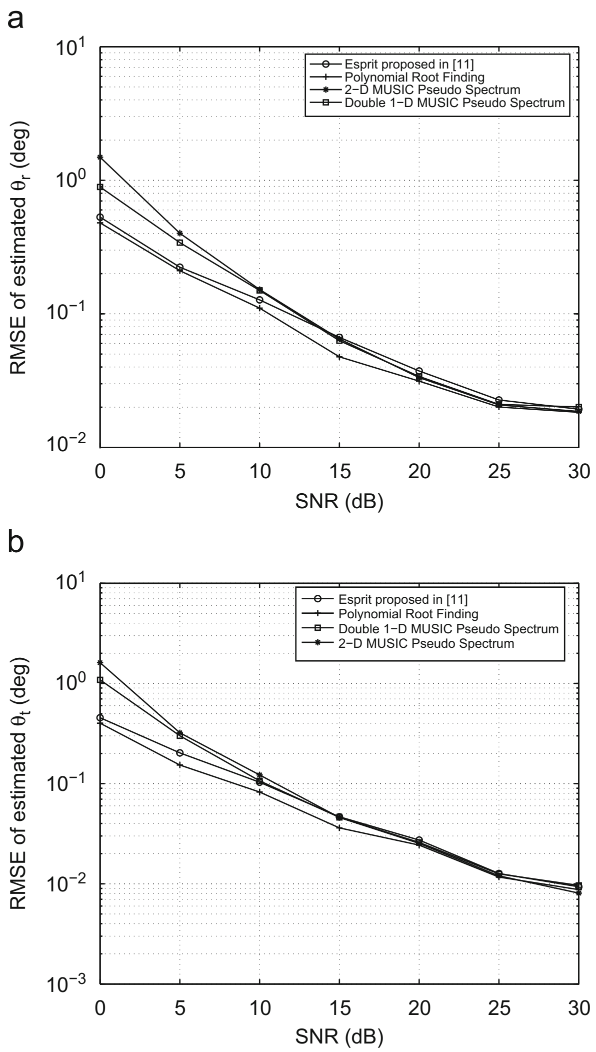

Fig. 7. RMSE of estimation versus SNR of (a) averaged DOAs and (b) averaged DODs for M=8, N=6, P=3 and L=100 samples.

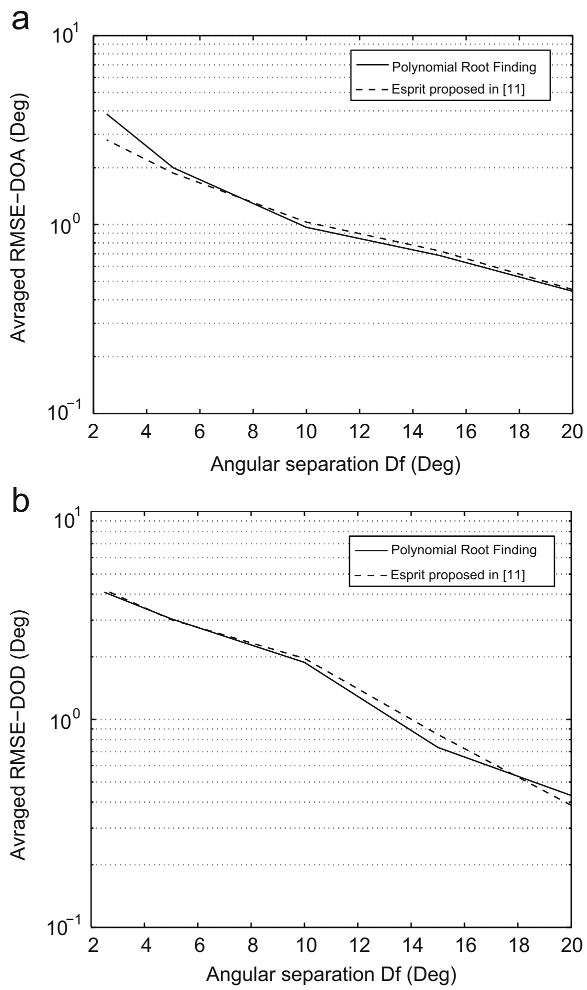

Fig. 8. RMSE of estimation versus (a) DOA distance and (b) DODs distance, for SNR=20 dB, M=3, N=3, P=2 and L=256 samples.

the proposed algorithm, the 2-D MUSIC pseudo-spectrum and the double 1-D MUSIC pseudo-spectrum. The CPUTIME returns the total CPU time (in seconds) used by MATLAB from the time it was started. The CPU time is plotted versus the number of antennas. To facilitate our presentation, we consider M=N in the MIMO system. The MUSIC pseudo-spectrum algorithms are computed for an angle space of [0°, 179°] and searching resolution of 0.5°. The number of targets is fixed to P=4. We can observe that the CPU time of the proposed algorithm is less than the CPU time for the other algorithms.

Last step, we study the influence of the Doppler frequency in direction estimation.

Let us define Ds as the difference of normalized Doppler frequency between two targets located at the angles $(\theta_r,\theta_t)=(30^\circ,30^\circ)$, $(50^\circ,50^\circ)$. The effect of Ds on directions estimation is shown in Figs. 10(a) and (b) for M=3, N=3, P=2 and L=256 samples. The normalized Doppler frequency is defined as $f_{nd}=f_d/f_s$, where f_s is the sampling frequency. Figs. 10(a) and (b) present the RMSE

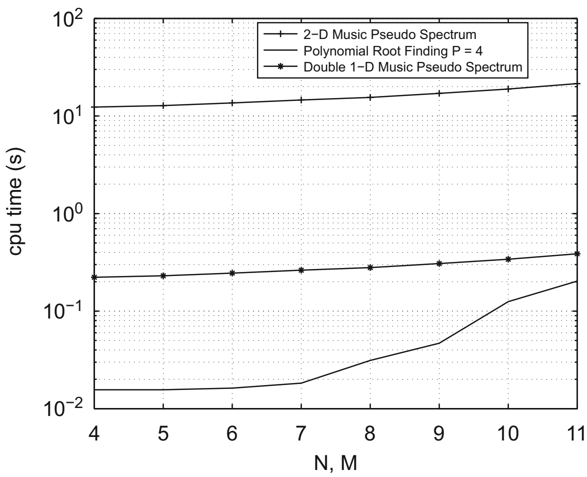

Fig. 9. MATLAB CPUTIME versus N=M with P=4 targets. Searching space [0°,179°] with 0.5° searching resolution.

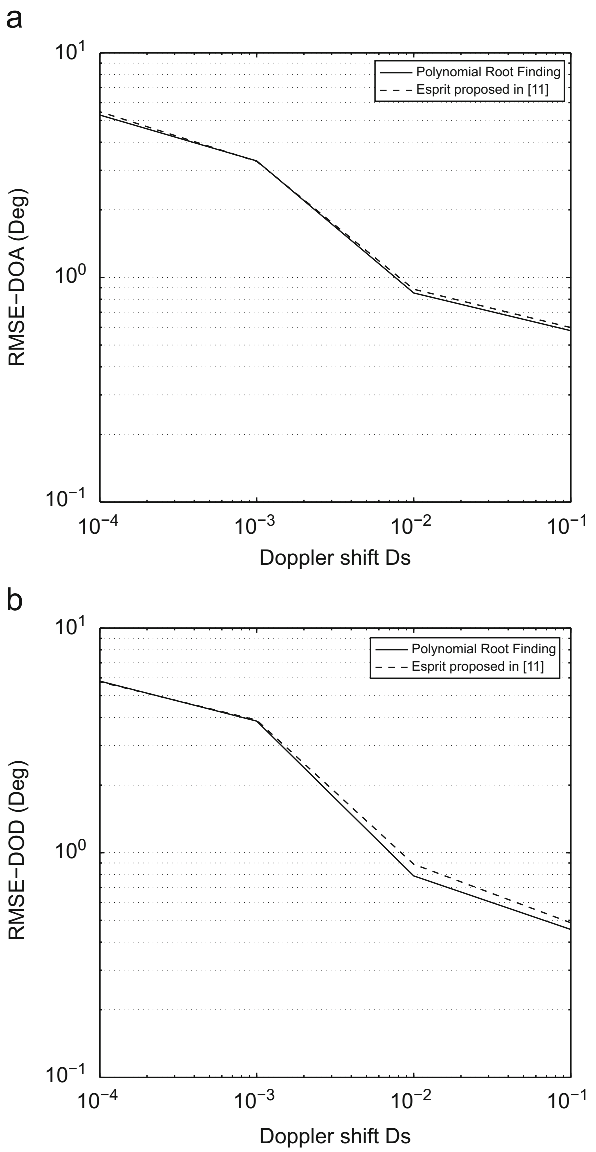

Fig. 10. RMSE of estimation versus Doppler shift of DOAs (a) and DODs (b), for M=3, N=3, P=2, SNR=20 dB, and L=256 samples.

of DOA and DOD, respectively, of the targets versus Ds for a fixed SNR=20 dB using the proposed algorithm and the algorithm developed in [11]. We can observe that the RMSE of the direction estimation becomes more important when Ds decreases. This simulation shows also that the effect of Doppler frequency is almost the same for both algorithms.

5. Conclusion

In this paper, we have proposed a new technique to transform the 2-D direction finding in bistatic MIMO radar into a double 1-D direction finding procedure. We have also proposed a polynomial root finding algorithm for the joint DOA-DOD estimation. The simulation results show that the proposed algorithm provides good performances in angle estimation. In addition, this proposed technique allows an automatic pairing and a high computational efficiency.

Appendix A

The matrix of the noise subspace Π_n can be written as

$$\boldsymbol{\Pi}_n = \begin{bmatrix} \alpha_{1,1} & \cdots & \alpha_{1,N} & \alpha_{1,N+1} & \cdots & \alpha_{1,NM} \\ \vdots & & \vdots & \vdots & \vdots & \vdots \\ \alpha_{N,1} & \cdots & \alpha_{N,N} & \alpha_{N,N+1} & \cdots & \alpha_{N,NM} \\ \alpha_{N+1,1} & \cdots & \alpha_{N+1,N} & \alpha_{N+1,N+1} & \cdots & \alpha_{N,NM} \\ \vdots & & \vdots & \vdots & \vdots & \vdots \\ \alpha_{NM,1} & \cdots & \alpha_{NM,N} & \alpha_{NM,N+1} & \cdots & \alpha_{NM,NM} \end{bmatrix} \tag{33}$$

Rewriting the determinant $D(z_t)$ in (26), we have

$$\det\left[\sum_{j,i=1}^{M} z_t^{j-i}\boldsymbol{\Pi}_{ij}\right] = \begin{vmatrix} \sum_{j,i=1}^{M} z_t^{j-i}\alpha_{[(i-1)N+1,(j-1)N+1]} & \cdots & \sum_{j,i=1}^{M} z_t^{j-i}\alpha_{[(i-1)N+1,jN]} \\ \vdots & \ddots & \vdots \\ \sum_{j,i=1}^{M} z_t^{j-i}\alpha_{[iN,(j-1)N+1]} & \cdots & \sum_{j,i=1}^{M} z_t^{j-i}\alpha_{[iN,jN]} \end{vmatrix} \tag{34}$$

Using the determinant proprieties, we obtain

$$D(z_t) = \sum_{j_1,i_1=1}^{M} \cdots \sum_{j_N,i_N=1}^{M} \begin{vmatrix} z_t^{j_1-i_1}\alpha_{[(i_1-1)N+1,(j_1-1)N+1]} & \cdots & z_t^{j_1-i_1}\alpha_{[(i_1-1)N+1,j_1N]} \\ \vdots & \ddots & \vdots \\ z_t^{j_N-i_N}\alpha_{[i_NN,(j_N-1)N+1]} & \cdots & z_t^{j_N-i_N}\alpha_{[i_NN,j_NN]} \end{vmatrix} \tag{35}$$

from which the polynomial in (26) is obtained as

$$D(z_t) = \sum_{j_1,i_1=1}^{M} \cdots \sum_{j_N,i_N=1}^{M} \sum_{k_1,\ldots,k_N}^{N} (-1)^{f(k_1,k_2,\ldots,k_N)} \prod_{l=1}^{N} z_t^{j_l-i_l}\alpha_{[(i_1-1)N+l,(j_1-1)N+k_l]} \tag{36}$$

where the last sum is computed over all permutations of the numbers 1, 2,…,N and $f(k_1,k_2,\ldots,k_N)$ is the number of permutations that re-orders this set of integers.

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