Available online at www.sciencedirect.com

ScienceDirect

Procedia Computer Science 131 (2018) 918–924

Procedia Computer Science

www.elsevier.com/locate/procedia

8th International Congress of Information and Communication Technology (ICICT-2018)

Central Direction-of-arrival Estimation of Coherently Distributed Sources Based on Rank-reduction Theory

Zheng Dai[a]一, Yao Shan[a], Yan Liu[a]

[a]*The 28th Research Institute of China Electronics Technology Group Corporation, Nanjing, 210000, China*

Abstract

Since the location information of CD source is determined by central DOA and angular spread, the problem of parameter estimation is usually complicated. Using a simple uniform linear array (ULA), the symmetry of angular signal distributed weight (ASDW) vector is proved. We perform a spectrum peak searching based on rank-reduction (RARE) theory. Moreover, the spectrum searching can be replaced by the search-free rooting. The advantage of this algorithm lies in estimating the central DOA without the information about deterministic angular distributed function (DADF).

Keywords: DOA estimation; coherently distributed source; central DOA; rank-reduction theory

1. Introduction

Many high resolution algorithms have been proposed for direction-finding [1-3]. These direction-finding algorithms have good performance in the case of a point source model. However, because of the complicated communication channel, the angle of targets may be extended in many practical scenes. In this case, it is more appropriate to consider the distributed source [4-9].

Among the DOA estimation algorithm of CD sources, the algorithm based on subspace theory is the most widely studied one. In [5] and [10], the authors first propose the concept of spatially distributed source, but also apply the MUSIC algorithm to the direction-finding of CD source, and propose a distributed signal parameter estimator (DSPE), however, the algorithm requires 2D spatial spectrum searching. In [11], when the angular spread of CD source is small, the generalized array manifold (GAM) vectors of the two ULAs have the approximately rotational

* Corresponding author.
E-mail address: lyip1022@126.com

1877-0509 © 2018 The Authors. Published by Elsevier Ltd.

10.1016/j.procs.2018.04.222

invariance relation. Based on this rotational invariance relation, low computational complexity is obtained, but the special array is required to be constructed.

In this paper, we use a ULA with symmetric structure. Firstly, it is proved that the ASDW vector of this array has a symmetric property. Based on this property, the array manifold can be rewritten. After substituting the expression about the subspace theory, the matrix RARE theory can be used to construct the spatial spectrum, the central DOA of CD sources are estimated by spectrum searching. In addition, the spectrum searching can be replaced by root seaking.

2. Data Model

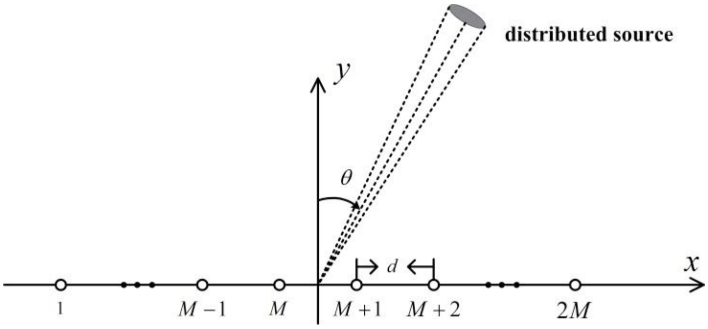


Fig. 1. A uniform linear array.

As shown in Fig. 1, consider a ULA consisting of $2M$ identical sensor elements as Fig. 1 shows, the distance between the adjacent sensors is d . The array receives the signal as

$$\mathbf{x}(t)=\sum_{i=1}^{D}\int\mathbf{a}(\theta)s_i(\theta,t;\boldsymbol{\psi}_i)d\theta+\mathbf{n}(t) \qquad \text{* MERGEFORMAT (1)}$$

where $s_i(\theta,t;\boldsymbol{\psi}_i)$ is the angular signal density function (ASDF), the vector $\boldsymbol{\psi}_i=(\theta_i,\sigma_{\theta_i})$, in which θ_i denotes the central DOA and σ_{θ_i} denotes the angular spread. $\mathbf{n}(t)$ is the noise. $\mathbf{a}(\theta)$ is the array manifold vector, D is the number of CD sources. The ASDF can be written as

$$s_i(\theta,t;\boldsymbol{\psi}_i)=s_i(t)\rho(\theta;\boldsymbol{\psi}_i) \qquad \text{* MERGEFORMAT (2)}$$

Here, $\rho(\theta;\boldsymbol{\psi}_i)$ is the DADF. The observation vector is expressed as

$$\mathbf{x}(t)=\sum_{i=1}^{D}\int\mathbf{a}(\theta)s_i(t)\rho(\theta;\boldsymbol{\psi}_i)d\theta+\mathbf{n}(t) \qquad \text{* MERGEFORMAT (3)}$$

Then the GAM vector is defined as

$$\mathbf{b}(\boldsymbol{\psi}_i)=\int\mathbf{a}(\theta)\rho(\theta;\boldsymbol{\psi}_i)d\theta \qquad \text{* MERGEFORMAT (4)}$$

which can also be expressed as [11-13],

$$\mathbf{b}(\boldsymbol{\psi}_i)=\mathbf{a}(\theta_i)\otimes\mathbf{g}(\boldsymbol{\psi}_i) \qquad \text{* MERGEFORMAT (5)}$$

where $\otimes$ stands for the element-by-element product. $\mathbf{g}(\boldsymbol{\psi}_i)$ is the ASDW vector. Thus, the observation vector can be simplified as

$$\mathbf{x}(t)=\mathbf{B}(\boldsymbol{\psi})\mathbf{s}(t)+\mathbf{n}(t) \qquad \text{* MERGEFORMAT (6)}$$

$\mathbf{B}(\boldsymbol{\psi})=[\mathbf{b}(\boldsymbol{\psi}_1),\mathbf{b}(\boldsymbol{\psi}_2),\ldots,\mathbf{b}(\boldsymbol{\psi}_D)]$ is the $2M\times D$ GAM matrix and $\mathbf{s}(t)=[s_1(t),s_2(t),\ldots,s_D(t)]^{\mathrm{T}}$ consists of D signals.

3. The proposed algorithm

For the array in Fig. 1, $a(\theta)$ is expressed as

$$\mathbf{a}(\theta)=[e^{-j2\pi(d/\lambda)(M-0.5)\sin\theta},e^{-j2\pi(d/\lambda)(M-1.5)\sin\theta},\ldots,e^{j2\pi(d/\lambda)(M-0.5)\sin\theta}]^{\mathrm{T}}$$ * MERGEFORMAT (7)

In the case of small angular extension, define $\theta=\theta_i+\tilde{\theta}$, using the Taylor series approximation, we have $\sin\theta\approx\sin\theta_i+\tilde{\theta}\cos\theta_i$, the elements of array manifold vector $a(\theta)$ can be expressed as

$$\begin{aligned}[\mathbf{a}(\theta)]_m &= e^{j2\pi(d/\lambda)(-M+m-0.5)\sin\theta}\\ &\approx e^{j2\pi(d/\lambda)(-M+m-0.5)\left(\sin\theta_i+\tilde{\theta}\cos\theta_i\right)}\end{aligned}$$ * MERGEFORMAT (8)

where $m=1,2,\ldots,2M$, $[\mathbf{a}(\theta)]_m$ is the m-th element of $\mathbf{a}(\theta)$. Inserting Eq. (8) into Eq. (6),

$$\begin{aligned}[\mathbf{b}(\boldsymbol{\psi}_i)]_m &= \int[\mathbf{a}(\theta)]_m\,\rho(\theta;\boldsymbol{\psi}_i)\,d\theta\\ &=\int e^{j2\pi(d/\lambda)(-M+m-0.5)\left(\sin\theta_i+\tilde{\theta}\cos\theta_i\right)}\rho\left(\tilde{\theta};\boldsymbol{\psi}_i\right)d\tilde{\theta}\\ &=[\mathbf{a}(\theta_i)]_m\,[\mathbf{g}(\boldsymbol{\psi}_i)]_m\quad(m=1,2,\ldots,2M)\end{aligned}$$ * MERGEFORMAT (9)

Therefore, the GAM vector can be expressed as

$$\mathbf{b}(\boldsymbol{\psi}_i)=\mathbf{a}(\theta_i)\otimes\mathbf{g}(\boldsymbol{\psi}_i)$$ * MERGEFORMAT (10)

Here, the ASDW vector is

$$[\mathbf{g}(\boldsymbol{\psi}_i)]_m=\int e^{j2\pi(d/\lambda)(-M+m-0.5)\tilde{\theta}\cos\theta_i}\rho\left(\tilde{\theta};\boldsymbol{\psi}_i\right)d\tilde{\theta}$$ * MERGEFORMAT (11)

Define $\xi_m=2\pi\left(d/\lambda\right)\left(-M+m-0.5\right)\tilde{\theta}\cos\theta_i$, thus, the ASDW vector can be rewritten as

$$\begin{aligned}[\mathbf{g}(\boldsymbol{\psi}_i)]_m &= \int e^{j\xi_m}\rho\left(\tilde{\theta};\boldsymbol{\psi}_i\right)d\tilde{\theta}\\ &=\int(\cos\xi_m+j\sin\xi_m)\rho\left(\tilde{\theta};\boldsymbol{\psi}_i\right)d\tilde{\theta}\\ &=\int\cos\xi_m\cdot\rho\left(\tilde{\theta};\boldsymbol{\psi}_i\right)d\tilde{\theta}+j\int\sin\xi_m\cdot\rho\left(\tilde{\theta};\boldsymbol{\psi}_i\right)d\tilde{\theta}\end{aligned}$$ * MERGEFORMAT (12)

Respecting the fact that $\rho\left(\tilde{\theta};\boldsymbol{\psi}_i\right)$ is an even function, we have

$$[\mathbf{g}(\boldsymbol{\psi}_i)]_m=\int\cos\xi_m\cdot\rho\left(\tilde{\theta};\boldsymbol{\psi}_i\right)d\tilde{\theta}$$ * MERGEFORMAT (13)

Similarly,

$$\begin{aligned}[\mathbf{g}(\boldsymbol{\psi}_i)]_{2M+1-m} &= \int e^{-j2\pi(d/\lambda)(-M+m-0.5)\tilde{\theta}\cos\theta_i}\rho\left(\tilde{\theta};\boldsymbol{\psi}_i\right)d\tilde{\theta}\\ &=\int e^{-j\xi_m}\rho\left(\tilde{\theta};\boldsymbol{\psi}_i\right)d\tilde{\theta}\\ &=\int(\cos(-\xi_m)+j\sin(-\xi_m))\rho\left(\tilde{\theta};\boldsymbol{\psi}_i\right)d\tilde{\theta}\\ &=\int\cos(-\xi_m)\cdot\rho\left(\tilde{\theta};\boldsymbol{\psi}_i\right)d\tilde{\theta}\end{aligned}$$ * MERGEFORMAT (14)

According to Eq. (13) and Eq. (14),

$$[\mathbf{g}(\boldsymbol{\psi}_i)]_m=[\mathbf{g}(\boldsymbol{\psi}_i)]_{2M+1-m}\quad(m=1,2,\ldots,2M)$$ * MERGEFORMAT (15)

Using the symmetric structure of $\mathbf{g}(\boldsymbol{\psi}_i)$, the generalized manifold vector $\mathbf{b}(\boldsymbol{\psi}_i)$ can be rewritten as

$$\begin{aligned}\mathbf{b}(\boldsymbol{\psi}_i) &= \begin{bmatrix}[\mathbf{a}(\theta_i)]_1\\ [\mathbf{a}(\theta_i)]_2\\ \vdots\\ [\mathbf{a}(\theta_i)]_{2M}\end{bmatrix}\otimes\begin{bmatrix}[\mathbf{g}(\boldsymbol{\psi}_i)]_1\\ [\mathbf{g}(\boldsymbol{\psi}_i)]_2\\ \vdots\\ [\mathbf{g}(\boldsymbol{\psi}_i)]_{2M}\end{bmatrix}\\ &=\left[\begin{array}{ccc}[\mathbf{a}(\theta_i)]_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & [\mathbf{a}(\theta_i)]_M\\ \hdashline 0 & \cdots & [\mathbf{a}(\theta_i)]_{M+1}\\ \vdots & \iddots & \vdots\\ [\mathbf{a}(\theta_i)]_{2M} & \cdots & 0\end{array}\right]\begin{bmatrix}[\mathbf{g}(\boldsymbol{\psi}_i)]_1\\ [\mathbf{g}(\boldsymbol{\psi}_i)]_2\\ \vdots\\ [\mathbf{g}(\boldsymbol{\psi}_i)]_M\end{bmatrix}\end{aligned}$$ * MERGEFORMAT (16)

Define a $2M \times M$ matrix $\mathbf{C}(\theta_i)$ and a $M \times 1$ matrix $\mathbf{g}'(\boldsymbol{\psi}_i)$ as

$$\mathbf{C}(\theta_i) = \begin{bmatrix} [\mathbf{a}(\theta_i)]_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & [\mathbf{a}(\theta_i)]_M \\ \hdashline 0 & \cdots & [\mathbf{a}(\theta_i)]_{M+1} \\ \vdots & \iddots & \vdots \\ [\mathbf{a}(\theta_i)]_{2M} & \cdots & 0 \end{bmatrix} \quad \text{* MERGEFORMAT (17)}$$

$$\mathbf{g}'(\boldsymbol{\psi}_i) = \begin{bmatrix} [\mathbf{g}(\boldsymbol{\psi}_i)]_1 \\ [\mathbf{g}(\boldsymbol{\psi}_i)]_2 \\ \vdots \\ [\mathbf{g}(\boldsymbol{\psi}_i)]_M \end{bmatrix} \quad \text{* MERGEFORMAT (18)}$$

Therefore, the generalized manifold vector can be expressed as

$$\mathbf{b}(\boldsymbol{\psi}_i) = \mathbf{C}(\theta_i)\mathbf{g}'(\boldsymbol{\psi}_i) \quad \text{* MERGEFORMAT (19)}$$

Let $\mathbf{U}_n$ be the noise subspace of $\mathbf{R}_x = E\{\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)\}$, thus, we have

$$\mathbf{b}^{\mathrm{H}}(\boldsymbol{\Psi}_i)\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{b}(\boldsymbol{\Psi}_i) = 0 \quad \text{* MERGEFORMAT (20)}$$

Inserting Eq. (19) into Eq. (20),

$$[\mathbf{g}'(\boldsymbol{\psi}_i)]^{\mathrm{H}}\mathbf{C}^{\mathrm{H}}(\theta_i)\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{C}(\theta_i)\mathbf{g}'(\boldsymbol{\psi}_i) = 0 \quad \text{* MERGEFORMAT (21)}$$

Define a $M \times M$ matrix $\mathbf{Q}(\theta_i)$ as

$$\mathbf{Q}(\theta_i) = \mathbf{C}^{\mathrm{H}}(\theta_i)\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{C}(\theta_i) \quad \text{* MERGEFORMAT (22)}$$

Then Eq. (21) can be written as

$$[\mathbf{g}'(\boldsymbol{\psi}_i)]^{\mathrm{H}}\mathbf{Q}(\theta_i)\mathbf{g}'(\boldsymbol{\psi}_i) = 0 \quad \text{* MERGEFORMAT (23)}$$

In Eq. (23), the vector $\mathbf{g}'(\boldsymbol{\psi}_i)$ is of size $M \times 1$, the matrix $\mathbf{Q}(\theta_i)$ is of size $M \times M$ and the elements of $\mathbf{g}'(\boldsymbol{\psi}_i)$ is not equal to zero, based on the RARE theory [14], Eq. (23) is satisfied only when the matrix $\mathbf{Q}(\theta_i)$ reduce rank,

$$\det\{\mathbf{Q}(\theta_i)\} = 0 \;\Rightarrow\; [\mathbf{g}'(\boldsymbol{\psi}_i)]^{\mathrm{H}}\mathbf{Q}(\theta_i)\mathbf{g}'(\boldsymbol{\psi}_i) = 0 \quad \text{* MERGEFORMAT (24)}$$

where $\det\{\cdot\}$ denotes the determinant of a matrix. Therefore,

$$P(\theta) = \frac{1}{\det\{\mathbf{Q}(\theta)\}} = \frac{1}{\det\{\mathbf{C}^{\mathrm{H}}(\theta)\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{C}(\theta)\}} \quad \text{* MERGEFORMAT (25)}$$

The spectrum peak search can be replaced by a polynomial, define $z = e^{-j\pi(d/\lambda)\sin\theta}$, the array manifold vector $\mathbf{a}(\theta)$ in Eq. (7) can be written as

$$\mathbf{a}(z) = \left[z^{-(2M-1)}, z^{-(2M-3)}, \ldots, z^{2M-1}\right]^{\mathrm{T}} \quad \text{* MERGEFORMAT (26)}$$

The matrix $\mathbf{C}(\theta)$ in Eq. (17) can be rewritten as

$$\mathbf{C}(z) = \begin{bmatrix} z^{-(2M-1)} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & z^{-1} \\ \hdashline 0 & \cdots & z \\ \vdots & \iddots & \vdots \\ z^{2M-1} & \cdots & 0 \end{bmatrix} \quad \text{* MERGEFORMAT (27)}$$

The matrix $\mathbf{Q}(\theta)$ in Eq. (22) can be expressed as

$$\mathbf{Q}(z) = \mathbf{C}^{\mathrm{H}}(z)\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{C}(z) \quad \text{* MERGEFORMAT (28)}$$

which results in

$$\frac{1}{\det\{\mathbf{Q}(z)\}} = \frac{1}{\det\left\{\mathbf{C}^{\mathrm{H}}(z)\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{C}(z)\right\}} = 0$$ * MERGEFORMAT (29)

The estimates of the central DOA are obtained as

$$\hat{\theta}_i = \arcsin\left(\frac{\lambda}{\pi d} angle\{\hat{z}_i\}\right)$$ * MERGEFORMAT (30)

4. Summary of the proposed algorithm

The method consists of the following steps:

Step 1: Calculate the covariance matrix $\mathbf{R}_x = E\left\{\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)\right\}$.

Step 2: Perform eigen-decomposition to covariance matrix $\mathbf{R}_x$.

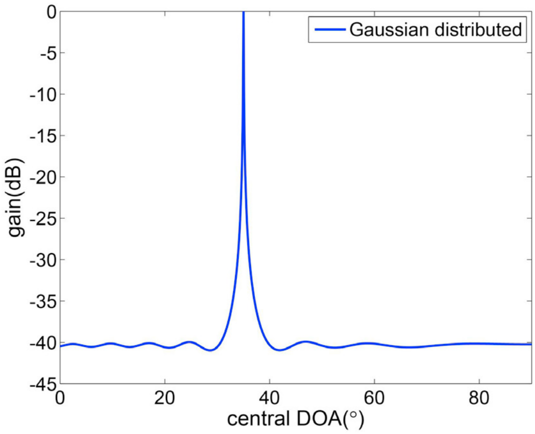
Step 3: According to the eigenvalues of $\mathbf{R}_x$, determine the number of signal sources.

Step 4: Obtain the noise subspace matrix $\mathbf{U}_\mathrm{n}$.

Step5: Construct the matrix $\mathbf{Q}(\theta)$ in Eq. (22), and estimate the cetral DOA of CD sources.

5. Simulation results

Effect of different DADFs



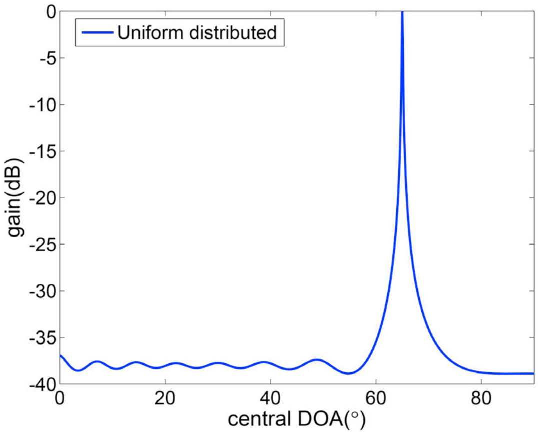


Fig. 2. (a) The spatial spectrum; (b) The spatial spectrum.

In this experiment, we compare the proposed algorithm with the DSPE algorithm [5] and the D-ESPRIT algorithm [11] when the signal-to-noise ratio (SNR) changes. We consider an array consisting of 6 elements, the distance between the adjacent sensor is $d = \lambda/2$. The searching step and number of snapshots is 0.01° and 200, respectively. $\boldsymbol{\psi}_1 = (35°, 4°)$, $\boldsymbol{\psi}_2 = (60°, 5°)$. In Fig. 3, the RMSE curves of DOA estimates for the two CD sources are given.

It can be seen that the central DOAs are estimated accurately independently of the DADFs.

Effect of SNR

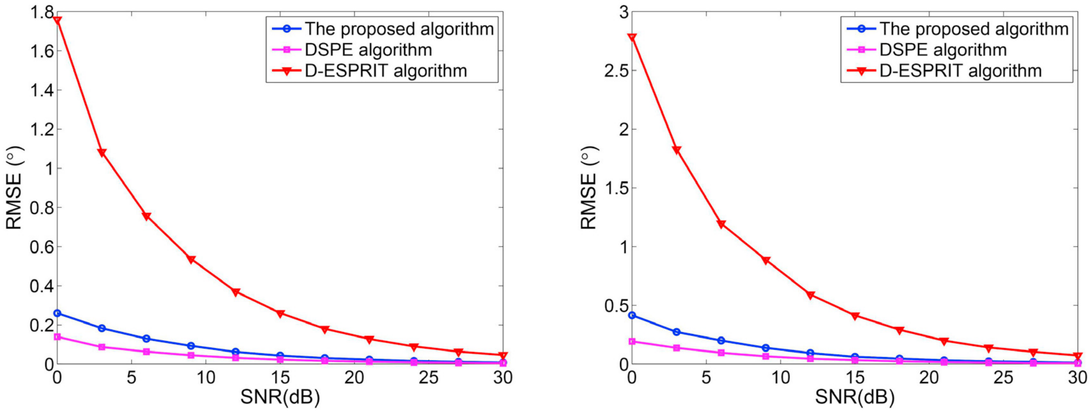


Fig. 3. (a) RMSE of DOA estimates; (b) RMSE of DOA estimates.

In this experiment, we compare the proposed algorithm with the DSPE algorithm [5] and the D-ESPRIT algorithm [11] when the signal-to-noise ratio (SNR) changes. We consider an array consisting of 6 elements, the distance between the adjacent sensor is $d = \lambda / 2$. The searching step and number of snapshots is $0.01°$ and 200, respectively. $\boldsymbol{\psi}_1 = (35°, 4°)$, $\boldsymbol{\psi}_2 = (60°, 5°)$. In Fig. 3, the RMSE curves are given.

It can be seen that the estimation accuracy of the proposed algorithm is higher than the D-ESPRIT algorithm, lower than the DSPE algorithm.

6. Conclusions

In this paper, the central DOA estimation of CD source is considered. We prove that the ASDW vector of a ULA has the symmetric structure. The RARE theory is applied to estimate the central DOA. Low computational complexity and high estimation accuracy are both obtained in the proposed algorithm. Moreover, the proposed algorithm performs independently of DADF.

Acknowledgements

This work was supported by the National Key Research and Development Program of China (2016YFB0502405).

References

1. Schmidt,R.O. 1986. Multiple emitter location and signal parameter estimation. IEEE Trans. Antennas Propag. 34, 3 (Mar. 1986), 276-280.
2. Roy, R., Paulraj, A., and Kailath, T. 1986. ESPRIT-A subspace rotation approach to estimation of parameters of cissoids in noise. IEEE Trans. Acoust., Speech, Signal Process. 34, 5 (Oct. 1986), 1340-1342.
3. Ziskind, I., and Wax, M. 1988. Maximum likelood localization of multiple sources by alternating projection. IEEE Trans. Acoust., Speech, Signal Process. 36, 10 (Oct. 1988), 1553-1560.
4. Jantti, T.P. 1992. The influence of the extended sources on theoretical performance of MUSIC and ESPRIT method: Narrow-band sources. Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. 2 (Mar. 1992), 429-432.
5. Valaee, S., Champagne, B., and Kabal, P. 1995. Parametric localization of distributed sources. IEEE Trans. Signal Process. 43, 9 (Sept. 1995), 2144-2153.
6. Astely, D., Ottersten, B., and Swindlehurst, A.L. 1998. Generalised array manifold model for wireless communication channels with local scattering. IEE Proc. Radar Sonar Navigation. 145, 1 (Feb. 1998), 51-57.
7. Astely, D., and Ottersten, B. 1999. The effects of local scattering on direction of arrival estimation with MUSIC. IEEE Trans. Signal Process. 47, 12 (Dec. 1999), 3220-3234.
8. Raich, R., Goldberg, J., and Messer, H. 2000. Bearing estimation for a distributed source: Modeling, inherent accuracy limitations and algorithms. IEEE Trans. Signal Process. 48, 2 (Feb. 2000), 429-441.

9. Bengtsson, M., and Ottersten, B. 2000. Low-complexity estimators for distributed sources. IEEE Trans. Signal Process. 48, 8 (Aug. 2000), 2185-2194.
10. Meng, Y., Stoica, P., and Wong, K.M. 1996. Estimation of the directions of arrival of spatially dispersed signals in array processing. in IEE Proc. Radar Sonar Navigation. 143, 1 (Feb. 1996), 1-9.
11. Shahbazpanahi, S., Valaee, S., and Bastani, M.H. 2001. Distributed source localization using ESPRIT algorithm. in IEEE Trans. Signal Process. 49, 10 (Oct. 2001), 2169-2178.
12. Dai, Z., Su, W., and Gu, H. 2017. Direction finding of noncircular coherently distributed sources for centrosymmetric array. in IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences. E100-A, 2 (Feb. 2017), 722-725.
13. Dai, Z., Su, W., and Gu, H. 2017. 2D central DOA estimation of coherently distributed sources using a pair of uniform circular arrays. in IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences. E100-A, 5 (May. 2017), 1179-1187.
14. Pesavento, M., and Bohme, J.F. 2002. Direction of arrival estimation in uniform circular arrays composed of directional elements. in IEEE Sensor array, Multichannel Signal Process. Workshop Proc. (2002), 503-507.