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Short communication

# Orthogonal projection method for DOA estimation in low-altitude environment based on signal subspace

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ABSTRACT

The rank-deficiency and subspace leakage caused by multipath effect are the main factors that lead to performance breakdown of direction of arrival (DOA) estimation in low-altitude environment. In this paper, we propose an orthogonal projection method based on signal subspace to overcome the negative effects of multipath. First, the signal covariance matrix is recovered to full-rank by forward and backward spatial smoothing (FBSS). Then, based on the least square technique, the signal subspace is used to establish the orthogonal projection matrix. Thereby the cross covariance matrices of signal and noise parts can be estimated and eliminated to modify the sample covariance matrix. Compared with the conventional methods that only dispose rank-deficiency, the proposed method has better performances in low-altitude environment. Besides, compared with the former orthogonal projection method based on steering matrix, this method reduces the computational complexity without iterative scheme. These conclusions are verified by simulations.

## 1. Introduction

The multipath effect (including specular reflection and diffuse reflection) in low-altitude environment will seriously degrade the performance of conventional direction of arrival (DOA) estimation algorithms such as multiple signal classification (MUSIC) [1] and estimation signal parameters via rotational invariance technique (ESPRIT) [2]. The specular reflection will lead to rank-deficiency and some directions may lose [3]. The diffuse reflection signals will be received as noise, thus the noise will be correlated with target signals to some extent. This will lead to the so-called subspace leakage problem, which means part of the true signal subspace resides in the sample noise subspace (and vice versa) [4]. Many methods have been proposed to solve the rank-deficiency problem from different aspects. There are methods such as forward and backward spatial smoothing (FBSS) [5], oblique projection [6] to recover the covariance matrix to full-rank. The [7] and [8] both adopt improved ML method to avoid the rank-deficiency problem. The [9] uses the spatial diversity of MIMO radar to avoid the targets glint caused by multipath. Based on sparse signal reconstruction method, [10] and [11] overcome the rank-deficiency problem without estimating the covariance matrixes. But the study of subspace leakage comparatively attracts less attention. To solve this problem, Steinwandt directly models leakage of noise subspace into signal subspace and then estimates the corresponding perturbation matrix [12]. [13] proposes an orthogonal projection method based on array steering matrix, however the DOA estimation procedure has to be repeated and the computational complexity is large.

In this paper, we propose a novel orthogonal projection method to improve the DOA estimation performance in low-altitude environment. The signal subspace, instead of the steering matrix, is used to establish the orthogonal projection matrix. Compared with the method of [13], the proposed method can reach the same accuracy with smaller computational complexity. The notations of $(\cdot)^{\mathrm{T}}$,$(\cdot)^*$ and $(\cdot)^{\mathrm{H}}$ denote the transpose, conjugation and the conjugation-transpose of the matrixes, respectively.

## 2. Signal model in low-altitude environment

The array geometry model in low-altitude environment is shown in Fig. 1.

Consider an uniform linear array (ULA) with $M$ sensors, whose inter-element space is half-wavelength. Far field narrowband signals impinge on this array from the directions $\theta_k$ $(k = 1,2,\cdots,K)$. $\boldsymbol{s}(t) = [s_1(t),s_2(t),\cdots,s_K(t)]^{\mathrm{T}}$ denotes the uncorrelated narrowband signal vector. The ULA receives both the direct and reflected signals. The steering matrix of the array is $\boldsymbol{A} = [\boldsymbol{a}_{d1} + \boldsymbol{a}_{r1},\cdots,\boldsymbol{a}_{dK} + \boldsymbol{a}_{rK}]$, and:

$$\boldsymbol{a}_{dk}(\theta_{dk}) = \left[1,\ \mathrm{e}^{-\mathrm{j}\,\pi\sin\theta_{dk}},\cdots,\mathrm{e}^{-\mathrm{j}\,\pi(M-1)\sin\theta_{dk}}\right]^{\mathrm{T}} \tag{1}$$

$$\boldsymbol{a}_{rk}(\theta_{rk}) = \delta_k \boldsymbol{a}_{dk}(\theta_{rk}) \tag{2}$$

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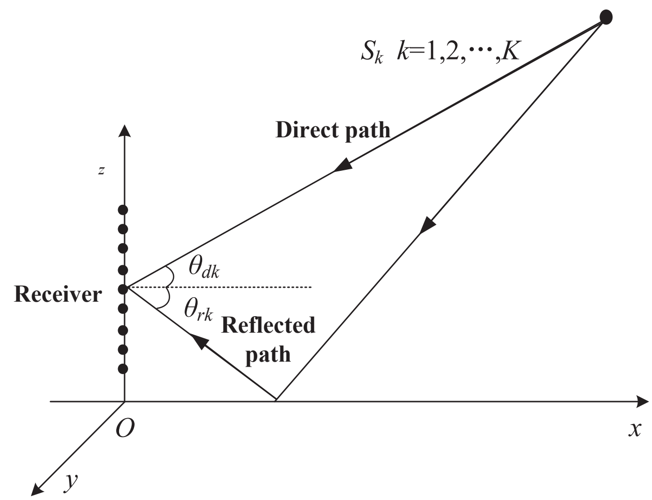


**Fig. 1.** The array geometry model in low-altitude environment.

where $\theta_{dk} = -\theta_{rk} = \theta_k$, $\delta_k$ is the multipath reflection coefficient. Thus:

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1) + \delta_1 \boldsymbol{a}(-\theta_1),\ \cdots,\ \boldsymbol{a}(\theta_K) + \delta_K \boldsymbol{a}(-\theta_K)] \tag{3}$$

The output of receiver can be represented as:

$$\boldsymbol{y}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) = \boldsymbol{B}\boldsymbol{x}(t) + \boldsymbol{n}(t) \tag{4}$$

where $\boldsymbol{n}(t)$ is complex colored noise, $\boldsymbol{B} = [\boldsymbol{a}(\theta_1),\boldsymbol{a}(-\theta_1),\cdots,\boldsymbol{a}(\theta_K),\boldsymbol{a}(-\theta_K)]_{N\times 2K}$, and $\boldsymbol{x}(t) = [s_1(t),\delta_1 s_1(t),\cdots,s_K(t),\delta_K s_K(t)]^{\mathrm{T}}_{2K\times 1}$. The covariance of $\boldsymbol{y}(t)$ is:

$$\boldsymbol{R} = E[\boldsymbol{y}(t)\boldsymbol{y}^{\mathrm{H}}(t)] = \boldsymbol{B}\boldsymbol{R}_x\boldsymbol{B}^{\mathrm{H}} + \mathrm{E}[\boldsymbol{n}(t)\boldsymbol{n}^{\mathrm{H}}(t)] \tag{5}$$

Due to the multipath effect, $\boldsymbol{x}(t)$ contains coherent signals and the $\boldsymbol{R}_x$ in (5) is a rank-deficient matrix. If $\boldsymbol{R}$ is directly used for eigen decomposition, some angles will lose. Therefore, the methods such as FBSS-MUSIC, oblique projection can be used to solve the rank-deficiency problem. Taking FBSS as an example, we can divide $\boldsymbol{y}(t)$ into $L$ forward overlapping vectors $\boldsymbol{y}_{fl}(t)$ and $L$ backward overlapping vectors $\boldsymbol{y}_{bl}(t)$, and each of them contains $m$ antennas:

$$\boldsymbol{y}_{fl}(t) = \boldsymbol{F}_l\boldsymbol{y}(t) = \boldsymbol{B}_m\boldsymbol{D}^{l-1}\boldsymbol{x}(t) + \boldsymbol{n}_l(t) \tag{6}$$

$$\boldsymbol{y}_{bl}(t) = \boldsymbol{F}_l\boldsymbol{J}\boldsymbol{y}^*(t) \tag{7}$$

where $\boldsymbol{F}_l = [0_{m\times(l-1)}\ \boldsymbol{I}_m\ 0_{m\times(M-m-l+1)}]$, $l = 1,2,\cdots,L$. $\boldsymbol{B}_m$ is the first $m$ rows of $\boldsymbol{B}$ and is an $m \times m$ exchange matrix with 1 on its anti-diagonal and 0 elsewhere. Besides:

$$\boldsymbol{D} = \mathrm{diag}[\mathrm{e}^{-\mathrm{j}\,\pi\sin\theta_1},\mathrm{e}^{\mathrm{j}\pi\sin\theta_1},\cdots,\mathrm{e}^{-\mathrm{j}\,\pi\sin\theta_K},\mathrm{e}^{\mathrm{j}\,\pi\sin\theta_K}] \tag{8}$$

The $(L-l+1)$ th backward vector can be expressed by the $l$ th forward vector as:

$$\boldsymbol{y}_{b(L-l+1)}(t) = \boldsymbol{J}\boldsymbol{y}^*_{fl}(t) \tag{9}$$

Suppose the number of snapshots is $\boldsymbol{N}$. The covariance of $\boldsymbol{y}_{fl}(t)$ and $\boldsymbol{y}_{b(L-l+1)}(t)$ can be estimated respectively:

$$\widehat{\boldsymbol{R}}_{fl} = \frac{1}{N}\sum_{t=1}^{N}[\boldsymbol{y}_{fl}(t)\boldsymbol{y}^{\mathrm{H}}_{fl}(t)] = \boldsymbol{B}_m\boldsymbol{D}^{l-1}\widehat{\boldsymbol{R}}_x(\boldsymbol{B}_m\boldsymbol{D}^{l-1})^{\mathrm{H}} + \widehat{\boldsymbol{R}}^{fl}_{\mathrm{n}} + \widehat{\boldsymbol{R}}^{fl}_{\mathrm{sn}} + \widehat{\boldsymbol{R}}^{fl}_{\mathrm{ns}} \tag{10}$$

$$\begin{aligned}\widehat{\boldsymbol{R}}_{b(L-l+1)} &= \frac{1}{N}\sum_{t=1}^{N}[\boldsymbol{y}_{b(L-l+1)}(t)\boldsymbol{y}^{\mathrm{H}}_{b(L-l+1)}(t)]\\ &= \boldsymbol{J}[\boldsymbol{B}_m\boldsymbol{D}^{l-1}\widehat{\boldsymbol{R}}_x(\boldsymbol{B}_m\boldsymbol{D}^{l-1})^{\mathrm{H}} + \widehat{\boldsymbol{R}}^{fl}_{\mathrm{n}} + \widehat{\boldsymbol{R}}^{fl}_{\mathrm{sn}} + \widehat{\boldsymbol{R}}^{fl}_{\mathrm{ns}}]^*\boldsymbol{J}\end{aligned} \tag{11}$$

Hence, the total smoothing matrix can be expressed as:

$$\overline{\boldsymbol{R}} = \frac{1}{L}\sum_{l=1}^{L}\overline{\boldsymbol{R}}_l = \frac{1}{2L}\sum_{l=1}^{L}[\widehat{\boldsymbol{R}}_{fl} + \boldsymbol{J}(\widehat{\boldsymbol{R}}_{fl})^*\boldsymbol{J}] \tag{12}$$

The amount of coherent sources that can be identified depends on the method that is used to solve the rank-deficiency problem. In this paper, we use FBSS to solve the problem. For an array with $M$ elements, the maximum coherent sources that the proposed algorithm can identify is $2M/3$, which has been proved in literature [5].

## 3. Orthogonal projection methods to solve subspace leakage

### 3.1. Orthogonal projection method based on steering matrix

The signal and noise are partially correlated, and the actual number of snapshots is finite. Thus the $\widehat{\boldsymbol{R}}^{fl}_{\mathrm{sn}}$ and $\widehat{\boldsymbol{R}}^{fl}_{\mathrm{ns}}$ $(l = 1,2,\cdots,L)$ may have significant values, which may decrease the DOA estimation performance largely. In order to remove these terms, we employ the least square technique with (6) to estimate the source signal $\boldsymbol{x}$ as:

$$\hat{\boldsymbol{x}} = [(\widehat{\boldsymbol{B}}_m\boldsymbol{D}^{l-1})^{\mathrm{H}}\widehat{\boldsymbol{B}}_m\boldsymbol{D}^{l-1}]^{-1}(\widehat{\boldsymbol{B}}_m\boldsymbol{D}^{l-1})^{\mathrm{H}}\boldsymbol{y}_{fl}(t) \tag{13}$$

where $\widehat{\boldsymbol{B}}_m$ is the estimator of $\boldsymbol{B}_m$ and the $l$ th forward noise vector can be estimated as:

$$\hat{\boldsymbol{n}}_{fl}(t) = \boldsymbol{y}_{fl}(t) - \widehat{\boldsymbol{B}}_m\boldsymbol{D}^{l-1}\hat{\boldsymbol{x}} = \boldsymbol{y}_{fl}(t) - \widehat{\boldsymbol{B}}_m(\widehat{\boldsymbol{B}}^{\mathrm{H}}_m\widehat{\boldsymbol{B}}_m)^{-1}\widehat{\boldsymbol{B}}^{\mathrm{H}}_m\boldsymbol{y}_{fl}(t) = \widehat{\boldsymbol{P}}^{\perp}_{\boldsymbol{B}_m}\boldsymbol{y}_{fl}(t) \tag{14}$$

where $\widehat{\boldsymbol{P}}^{\perp}_{\boldsymbol{B}_m} = \boldsymbol{I}_m - \widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}$, and $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m} = \widehat{\boldsymbol{B}}_m(\widehat{\boldsymbol{B}}^{\mathrm{H}}_m\widehat{\boldsymbol{B}}_m)^{-1}\widehat{\boldsymbol{B}}^{\mathrm{H}}_m$. Therefore $\widehat{\boldsymbol{R}}^{fl}_{\mathrm{sn}}$ and $\widehat{\boldsymbol{R}}^{fl}_{\mathrm{ns}}$ can be estimated as:

$$\begin{cases}\widehat{\boldsymbol{R}}^{fl}_{\mathrm{sn}} = \widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}\widehat{\boldsymbol{R}}_{fl}\widehat{\boldsymbol{P}}^{\perp}_{\boldsymbol{B}_m}\\ \widehat{\boldsymbol{R}}^{fl}_{\mathrm{ns}} = \widehat{\boldsymbol{P}}^{\perp}_{\boldsymbol{B}_m}\widehat{\boldsymbol{R}}_{fl}\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}\end{cases} \tag{15}$$

In reality, $\widehat{\boldsymbol{P}}^{\perp}_{\boldsymbol{B}_m}$ and $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}$ cannot be known at first, so [13] adopts a two-step scheme to estimate $\widehat{\boldsymbol{P}}^{\perp}_{\boldsymbol{B}_m}$ and $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}$. In the first step, the DOAs are estimated without eliminating $\widehat{\boldsymbol{R}}^{fl}_{\mathrm{sn}}$ and $\widehat{\boldsymbol{R}}^{fl}_{\mathrm{ns}}$. In the second step, the steering matrix $\widehat{\boldsymbol{B}}_m$, together with $\widehat{\boldsymbol{P}}^{\perp}_{\boldsymbol{B}_m}$ and $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}$, is obtained using the estimated DOAs, so that the $\widehat{\boldsymbol{R}}^{fl}_{\mathrm{sn}}$ and $\widehat{\boldsymbol{R}}^{fl}_{\mathrm{ns}}$ can be estimated and eliminated. Therefore, the DOAs can be estimated more precisely.

### 3.2. Improved orthogonal projection method based on signal subspace

The method based on steering matrix has to estimate the DOAs in ahead, which brings extra computational complexity to establish space spectrum and search peaks. To reduce the computational complexity, we propose a novel orthogonal projection method based on signal subspace. The eigen decomposition of $\overline{\boldsymbol{R}}$ is:

$$\overline{\boldsymbol{R}} = \widehat{\boldsymbol{U}}_{\mathrm{s}}\widehat{\boldsymbol{\Sigma}}_{\mathrm{s}}\widehat{\boldsymbol{U}}^{\mathrm{H}}_{\mathrm{s}} + \widehat{\boldsymbol{U}}_{\mathrm{n}}\widehat{\boldsymbol{\Sigma}}_{\mathrm{n}}\widehat{\boldsymbol{U}}^{\mathrm{H}}_{\mathrm{n}} \tag{16}$$

The diagonal matrix $\widehat{\boldsymbol{\Sigma}}_{\mathrm{s}}$ and $\widehat{\boldsymbol{\Sigma}}_{\mathrm{n}}$ are consisted of the largest $K$ eigenvalues and the rest $m$-$K$ eigenvalues respectively. $\widehat{\boldsymbol{U}}_{\mathrm{s}}$ and $\widehat{\boldsymbol{U}}_{\mathrm{n}}$ are the signal subspace and noise subspace respectively. The space spanned by the vectors of $\widehat{\boldsymbol{U}}_{\mathrm{s}}$ and the space spanned by the vectors of $\boldsymbol{B}_m$ are the same, namely:

$$\mathrm{span}\{\boldsymbol{u}_1,\boldsymbol{u}_2,\cdots,\boldsymbol{u}_K\} = \mathrm{span}\{\boldsymbol{b}_1,\boldsymbol{b}_2,\cdots,\boldsymbol{b}_K\} \tag{17}$$

There exists a $K \times K$ invertible matrix $\boldsymbol{V}$ which satisfies the equation of $\widehat{\boldsymbol{U}}_{\mathrm{s}}\boldsymbol{V} = \boldsymbol{B}_m$, then the following equations can be obtained:

$$\begin{aligned}\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m} &= \widehat{\boldsymbol{B}}_m(\widehat{\boldsymbol{B}}^{\mathrm{H}}_m\widehat{\boldsymbol{B}}_m)^{-1}\widehat{\boldsymbol{B}}^{\mathrm{H}}_m = \widehat{\boldsymbol{U}}_{\mathrm{s}}\boldsymbol{V}[(\widehat{\boldsymbol{U}}_{\mathrm{s}}\boldsymbol{V})^{\mathrm{H}}\widehat{\boldsymbol{U}}_{\mathrm{s}}\boldsymbol{V}]^{-1}(\widehat{\boldsymbol{U}}_{\mathrm{s}}\boldsymbol{V})^{\mathrm{H}} = \widehat{\boldsymbol{U}}_{\mathrm{s}}(\widehat{\boldsymbol{U}}^{\mathrm{H}}_{\mathrm{s}}\widehat{\boldsymbol{U}}_{\mathrm{s}})^{-1}\widehat{\boldsymbol{U}}^{\mathrm{H}}_{\mathrm{s}}\\ &= \widehat{\boldsymbol{P}}_{\mathrm{s}}\end{aligned} \tag{18}$$

So $\widehat{\boldsymbol{P}}_{\boldsymbol{B}_m}$ can be replaced with $\widehat{\boldsymbol{P}}_{\mathrm{s}}$ to perform the orthogonal projection in (15):

$$\begin{cases}\widehat{\boldsymbol{R}}^{fl}_{\mathrm{sn}} = \widehat{\boldsymbol{P}}_{\mathrm{s}}\widehat{\boldsymbol{R}}_{fl}\widehat{\boldsymbol{P}}^{\perp}_{\mathrm{s}}\\ \widehat{\boldsymbol{R}}^{fl}_{\mathrm{ns}} = \widehat{\boldsymbol{P}}^{\perp}_{\mathrm{s}}\widehat{\boldsymbol{R}}_{fl}\widehat{\boldsymbol{P}}_{\mathrm{s}}\end{cases} \tag{19}$$

The adjusted $\bar{\boldsymbol{R}}'_{fl}$ and $\bar{\boldsymbol{R}}'$ should be expressed as:

$$\hat{\boldsymbol{R}}'_{fl} = [\hat{\boldsymbol{R}}_{fl} - \gamma(\hat{\boldsymbol{R}}^{fl}_{\mathrm{sn}} + \hat{\boldsymbol{R}}^{fl}_{\mathrm{ns}})][\hat{\boldsymbol{R}}_{fl} - \gamma(\hat{\boldsymbol{R}}^{fl}_{\mathrm{sn}} + \hat{\boldsymbol{R}}^{fl}_{\mathrm{ns}})]^{\mathrm{H}} \tag{20}$$

$$\bar{\boldsymbol{R}}' = \frac{1}{L}\sum_{l=1}^{L}\bar{\boldsymbol{R}}'_l = \frac{1}{2L}\sum_{l=1}^{L}[\hat{\boldsymbol{R}}'_{fl} + \boldsymbol{J}(\hat{\boldsymbol{R}}'_{fl})^*\boldsymbol{J}] \tag{21}$$

where coefficient $\gamma \in [0,1]$ is used to revise the estimation errors. If the estimation is perfect, $\gamma$ should be equal to 1. However, estimation errors are inevitable and unknown. Based on the maximum likelihood criterion, the best $\gamma$ can be obtained by calculating (22) under different values of $\gamma$ (e.g. $\gamma = 0,0.1,0.2,\cdots,1$) [13], namely:

$$f(\gamma) = \min_{\gamma} \ln\det\left(\hat{\boldsymbol{P}}_{\mathrm{s}}^{(2)}\bar{\boldsymbol{R}}\hat{\boldsymbol{P}}_{\mathrm{s}}^{(2)} + \frac{\mathrm{Tr}\{\hat{\boldsymbol{P}}_{\mathrm{s}}^{\perp(2)}\bar{\boldsymbol{R}}\}}{m-K}\hat{\boldsymbol{P}}_{\mathrm{s}}^{\perp(2)}\right) \tag{22}$$

where $\mathrm{Tr}\{\cdot\}$ is the trace operator. $\hat{\boldsymbol{P}}_{\mathrm{s}}^{(2)}$ is the projection matrix obtained from the adjusted $\bar{\boldsymbol{R}}'$ and $\hat{\boldsymbol{P}}_{\mathrm{s}}^{\perp(2)} = \boldsymbol{I}_m - \hat{\boldsymbol{P}}_{\mathrm{s}}^{(2)}$. This kind of method avoids the calculations of space spectrum and the search of peaks, especially in the procedure of finding the optimal $\gamma$. Otherwise, once a $\gamma$ needs to be tested, an extra DOA estimation has to be conducted.

To obtain the noise subspace $\hat{\boldsymbol{U}}'_{\mathrm{n}}$, we conduct eigen decomposition of $\bar{\boldsymbol{R}}'$. Then the DOA can be estimated by searching the maximum peaks of MUSIC spatial spectrum.

### 3.3. Analysis of the proposed algorithm

The main steps of the proposed algorithm are listed as follows:

**Inputs**: $M,m,d,\lambda,N,K$ and received vectors $\boldsymbol{y}(1),\boldsymbol{y}(2),\cdots\boldsymbol{y}(N)$
**Outputs**: $\hat{\theta}_1,\hat{\theta}_2,\cdots,\hat{\theta}_K$

**Steps:**
(1) Conduct FBSS to revise the covariance matrix $\boldsymbol{R}$ to $\bar{\boldsymbol{R}}$, and conduct eigenvalue decomposition of $\bar{\boldsymbol{R}}$ to obtain estimated signal subspace $\hat{\boldsymbol{U}}_{\mathrm{s}}$
(2) Calculate $\hat{\boldsymbol{P}}_{\mathrm{s}}$ and $\hat{\boldsymbol{P}}_{\mathrm{s}}^{\perp}$, and estimate $\hat{\boldsymbol{R}}^{fl}_{\mathrm{sn}}$ and $\hat{\boldsymbol{R}}^{fl}_{\mathrm{ns}}$ by (19) $(l = 1,2,\cdots,L)$
(3) Adjust the $\bar{\boldsymbol{R}}$ according to (20) and (21), and determine the best $\gamma$ by solving (22)
(4) Estimate the DOAs based on the optimal adjusted $\bar{\boldsymbol{R}}'$

For the steering matrix based orthogonal projection MUSIC (SMOP-MUSIC) method, the DOA estimation has to be conducted once in step (2) and $G$ times in step (3), $G$ is the number of $\gamma$. Thus its computational complexity of space spectrum calculating and peaks searching is $O((G+1)\Theta m(m-K)/\Delta\theta)$, where $\Theta$ is the span of angle, and $\Delta\theta$ is the step of peaks searching. The signal subspace based orthogonal projection method reduces the computational complexity of space spectrum calculating and peaks searching to $O(\Theta m(m-K)/\Delta\theta)$, which will save much time. For the proposed method, the computation complexity of step (1)-(3) is $O((G+L+1)m^3 + m^2(GL+NL+K) + mK^2 + K^3)$. Thus the total computation complexity of the proposed method is $O((G+L+1)m^3 + m^2(GL+NL+K) + mK^2 + K^3 + \Theta m(m-K)/\Delta\theta)$.

## 4. Simulation and discussion

Simulations are conducted to verify the performance of the proposed method in low-altitude environment. The ULA has a number of $M = 35$ antennas and the number of sub-arrays is set as $L = 6$, and each sub-array has 30 antennas. Three non-coherent narrowband signal sources coming from the directions of $\theta_1 = 2^{\circ}$, $\theta_2 = 5^{\circ}$, $\theta_3 = 8^{\circ}$ are considered.

Fig. 2 shows the DOA estimate curve of different estimation algorithms, including the directions of direct and reflected signals. SNR = 0 dB, and the number of snapshots is $N = 200$.

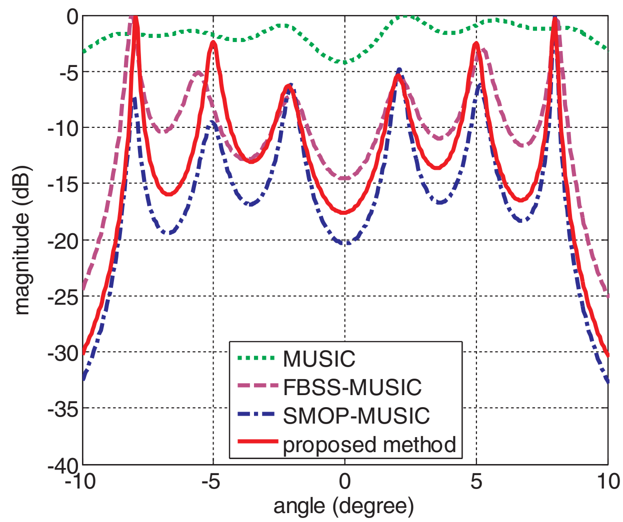


**Fig. 2.** Comparison of DOA estimation performances for different methods.

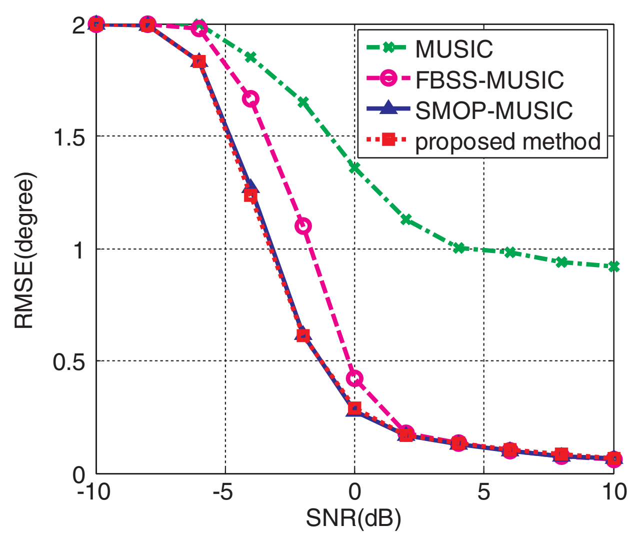


**Fig. 3.** RMSE versus SNR ($N$ = 200).

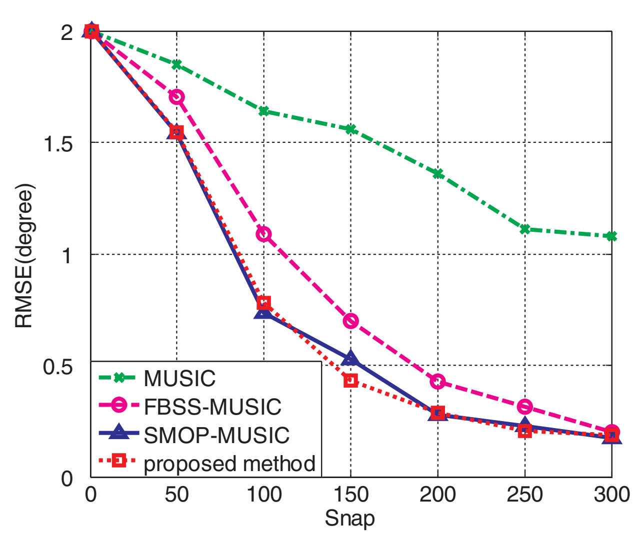


**Fig. 4.** RMSE versus snapshot (SNR = 0 dB).

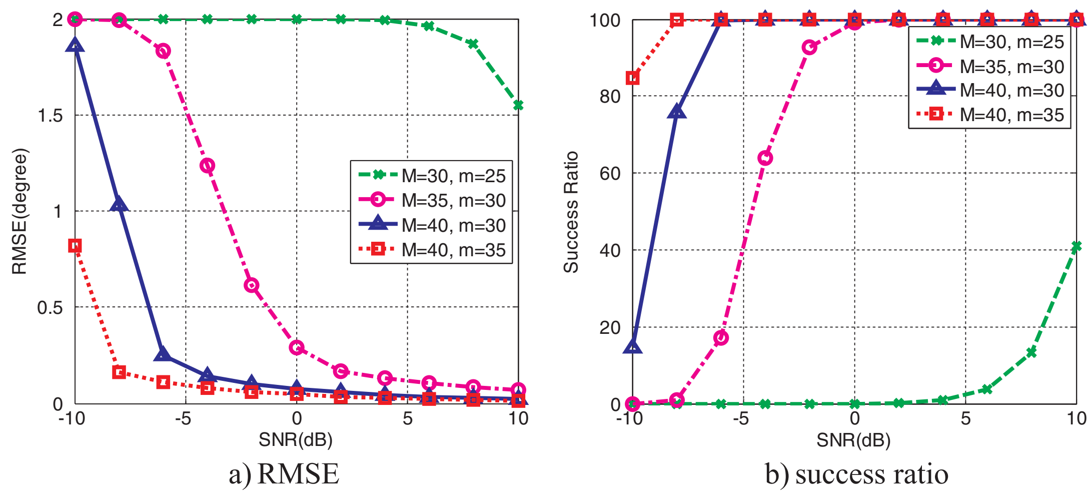


**Fig. 5.** DOA estimation performance of the proposed method under different number of sensors ($N$ = 200).

**Table 1**
Time consumption of 500 Monte Carlo simulations (units: second).

| Searching step | Proposed method | SMOP-MUSIC |
|---|---|---|
| 1° | 14.673 | 19.140 |
| 0.1° | 15.379 | 36.703 |
| 0.01° | 32.697 | 207.752 |
| 0.001° | 182.584 | 1765.243 |

It can be seen that the MUSIC fails to estimate the accurate angles of the real targets as well as the specular targets. For FBSS-MUSIC algorithm, it occurs an error in the direction of $-5^{\circ}$. Compared with FBSS-MUSIC, the SMOP-MUSIC and the proposed method, which eliminate the negative effect of subspace leakage, can estimate all the six angles precisely.

Figs. 3 and 4 show the root mean square error (RMSE) of the DOA estimation versus SNR and number of snapshots. Here we assume that if the error between the estimated angle and real target angle exceeds 2°, the estimation is failed, and the error is set as 2°.

We can find that with the increase of SNR and snapshots, the RMSE gets smaller. Apparently, the MUSIC has obvious error when estimating the three targets and their specular images. By eliminating the subspace leakage parts, the DOA estimation performance of the proposed method and the SMOP-MUSIC gets improved, and both of them perform better than FBSS-MUSIC. Besides, when SNR is low, the powers of the signal and the noise are close, so the $\hat{\boldsymbol{R}}_{\text{sn}}^{fl}$ and $\hat{\boldsymbol{R}}_{\text{ns}}^{fl}$ are close to the signal covariance matrix; when the number of snapshots is small, $\hat{\boldsymbol{R}}_{\text{sn}}^{fl}$ and $\hat{\boldsymbol{R}}_{\text{ns}}^{fl}$ may have significant values. Both situations will cause obvious interference on DOA estimation. By eliminating the $\hat{\boldsymbol{R}}_{\text{sn}}^{fl}$ and $\hat{\boldsymbol{R}}_{\text{ns}}^{fl}$, the orthogonality between the signal and the noise subspace is guaranteed, so the performances get improved.

Fig. 5 further discusses RMSE and the success ratio of the proposed method under different number of sensors. Here $N = 200$. It can be found that with the increase of $M$ or $m$, the DOA estimation performances get improved obviously.

The signal subspace based orthogonal projection method and the steering matrix based method have almost the same accuracy in the same condition. To compare the computational complexity of these two kinds of methods, we record the total time consumption of 500 Monte Carlo simulations under different peaks searching steps and list them in Table 1. SNR = 10 dB, $N = 200$, $M = 35$, and $m = 30$. It can be found that the proposed method has smaller computational complexity and it will save much time as the searching step gets smaller.

## 5. Conclusion

In this letter, a novel DOA estimation algorithm for low-altitude environment is proposed, which can eliminate the negative effects of multipath. Simulation results verify that the proposed method has better accuracy than the conventional algorithms such as MUSIC, FBSS-MUSIC, etc, especially in the conditions with low SNR or small sample size. Besides, compared with the former steering matrix based orthogonal projection method to solve subspace leakage, this method has smaller computational complexity with the same accuracy.

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## References

[1] Liu Y, Sun X, Zhao S. A covariance matrix shrinkage method with Toeplitz rectified target for DOA estimation under the uniform linear array. AEU-Int J Electron Commun 2017;81:50–5.
[2] Cui K, Wu W, Huang J, Chen X, Yuan N. DOA estimation of LFM signals based on STFT and multiple invariance ESPRIT. AEU-Int J Electron Commun 2017;77:10–7.
[3] Dai J, Ye Z. Spatial smoothing for direction of arrival estimation of coherent signals in the presence of unknown mutual coupling. IET Signal Process 2011; 5(4): 418–25.
[4] Liu J, Liu X, Ma X. First-order perturbation analysis of singular vectors in singular value decomposition. IEEE Trans Signal Process 2008; 56(7): 3044–9.
[5] Pillai SU, Kwon BH. Forward/backward spatial smoothing techniques for coherent signal identification. IEEE Trans Acoust Speech, Signal Process 1989;37(1):8–15.
[6] Tao H, Xin J, Wang J. Two-dimensional direction estimation for a mixture of noncoherent and coherent signals. IEEE Trans Signal Process 2015;63(2):318–33.
[7] Akhavan S, Bahabadi MA, Norouzi Y, Lolaee H. Direction of arrival estimation using array of antennas for low-altitude targets in multi-path environment. IET Radar Sonar Navig 2016;10(8):1439–45.
[8] Bonacci D, Vincent F, Gigleux B. Robust DoA estimation in case of multipath environment for a sense and avoid airborne radar. IET Radar Sonar Navig 2017;11(5):797–801.
[9] Shi J, Hu G, Zong B, Chen M. DOA estimation using multipath echo power for MIMO radar in low-grazing angle. IEEE Sensors J 2016;16(15):6087–94.
[10] Shi J, Hu G, Zhang X, Sun F, Zhou H. Sparsity-based 2-D DOA estimation for co-prime array: from sum-difference co-array viewpoint. IEEE Trans Signal Process 2017;65(21):5591–604.
[11] Tian Y, Xu H. DOA, power and polarization angle estimation using sparse signal reconstruction with a COLD array. AEU-Int J Electron Commun 2015;69:1606–12.

[12] Steinwandt J, Roemer F, Haardt M, Galdo GD. R-dimensional ESPRIT-type algorithms for strictly second-order non-circular sources and their performance analysis. IEEE Trans Signal Process 2014;62(18):4824–38.
[13] Shaghaghi M, Vorobyov SA. Subspace leakage analysis and improved DOA estimation with small sample size. IEEE Trans Signal Process 2015;63(12):3251–65.

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