$Z_R = R$, $Z_C = \frac{1}{j\omega C}$

12/12/24

PHYS 605 HW3

1. Low pass:

$$V_{out} = V_{in}\left(\frac{Z_C}{Z_R + Z_C}\right)$$

$$= V_{in}\left(\frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}\right) = V_{in}\left(\frac{1}{Rj\omega C + 1}\right)$$

$$\boxed{G_L(\omega) = \frac{1}{jRwC+1}}$$

High Pass:

$$V_{out} = V_{in}\left(\frac{Z_R}{Z_R + Z_C}\right)$$

$$= V_{in}\left(\frac{R}{R + \frac{1}{j\omega C}}\right) = V_{in}\left(\frac{Rj\omega C}{Rj\omega C + 1}\right)$$

$$\boxed{G_H(\omega) = \frac{jRwC}{jRwC+1}}$$

DC Behavior:

→ Low Pass - 0Hz for DC source returns input voltage value

→ High Pass - 0Hz will return 0V for Vout

AC Behavior (High freq. sin):

→ Low Pass - High frequency Vin returns 0V for Vout

→ High Pass - High frequency Vin returns ~Vin for Vout

2.

$R_1 \rightarrow Z_{R_1} = R_1$, $C_1 \rightarrow Z_{C_1} = \frac{1}{j\omega C_1}$

$$\rightarrow Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{R_1 j\omega C_1 + 1}{j\omega C_1}$$

$$Z_2 = R_2 + \frac{1}{j\omega C_2} = \frac{R_2 j\omega C_2 + 1}{j\omega C_2}$$

$$V_{out} = V_{in}\left(\frac{Z_2}{Z_1 + Z_2}\right) \Rightarrow G(\omega) = \frac{R_2 + \frac{1}{j\omega C_2}}{R_1 + \frac{1}{j\omega C_1} + R_2 + \frac{1}{j\omega C_2}} = \frac{R_2\left(1 + \frac{1}{j\omega C_1 R_1}\right)}{R_1 + \frac{R_2}{j\omega C_1 R_1} + R_2 + \frac{R_1}{j\omega C_2 R_2}}$$

a)

$C_1R_1 = C_2R_2$

$C_1 = C_2\frac{R_2}{R_1}$

$C_2 = C_1\frac{R_1}{R_2}$

$$\Rightarrow G(\omega) = \frac{R_2\left(1 + \frac{1}{j\omega C_1 R_1}\right)}{(R_1 + R_2)\left(1 + \frac{1}{j\omega C_1 R_1}\right)} = \boxed{\frac{R_2}{R_1 + R_2}}$$

a)

$C_2R_2 = C_1R_1$

b)

$R_1 = \frac{C_2R_2}{C_1}$

$R_2 = \frac{C_1R_1}{C_2}$

$$\Rightarrow \frac{R_2}{R_1 + R_2} = \frac{\frac{C_1R_1}{C_2}}{\frac{C_2R_2}{C_1} + \frac{C_1R_1}{C_2}} \overset{C_1R_1 = C_2R_2}{=} \frac{C_1R_1}{C_2R_1 + C_1R_1} = \boxed{\frac{C_1}{C_1 + C_2}}$$

b)