Parameterization Packet 2

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This packet contains a small library of explicit example parametric curves, these will be employed multiple times in example problems and exercises to give you practice with the various advanced integration. In this packet the parameterizations have been notated K_1, K_2 etc. I will refer back to this notation with the examples later in the class. In addition graphs of the curves are provided.

1. Curve K_1: the generic directed line segment connecting the point $\mathbf{a}$ to the point $\mathbf{b}$ in the plane.

$$\mathbf{r}_{K_1} = \begin{pmatrix} (b_1 - a_1)t + a_1 \\ (b_2 - a_2)t + a_2 \end{pmatrix}, \qquad t \in [0, 1]$$

$$\frac{d\mathbf{r}_{K_1}}{dt} = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \end{pmatrix} \qquad \left\| \frac{d\mathbf{r}_{K_1}}{dt} \right\| = \|\mathbf{b} - \mathbf{a}\|$$

2. Curve K_2: the unit circle traversed counterclockwise starting at the positive x axis.

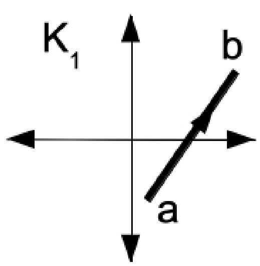
$$\mathbf{r}_{K_2} = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}, \qquad t \in [0, 2\pi]$$

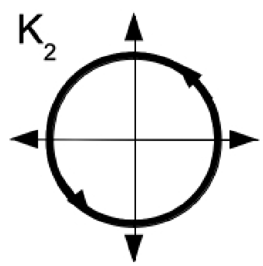
$$\frac{d\mathbf{r}_{K_2}}{dt} = \begin{pmatrix} -\sin(t) \\ \cos(t) \end{pmatrix} \qquad \left\| \frac{d\mathbf{r}_{K_2}}{dt} \right\| = 1$$

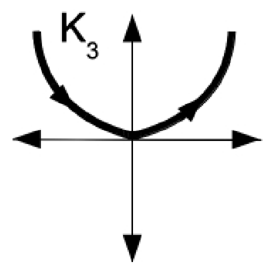
3. Curve K_3: The bottom of the standard parabola.

$$\mathbf{r}_{K_3} = \begin{pmatrix} t \\ t^2 \end{pmatrix}, \qquad t \in [-1, 1]$$

$$\frac{d\mathbf{r}_{K_3}}{dt} = \begin{pmatrix} -1 \\ 2t \end{pmatrix} \qquad \left\| \frac{d\mathbf{r}_{K_3}}{dt} \right\| = \sqrt{1 + 4t^2}$$







4. Curve K_4: a functional parameterization of the diagonal line $y = x$.

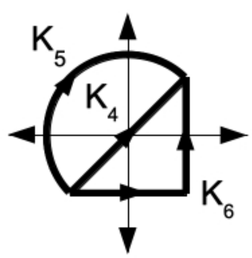
$$\mathbf{r}_{K_4} = \begin{pmatrix} t \\ t \end{pmatrix}, \qquad t \in [-1, 1]$$
$$\frac{d\mathbf{r}_{K_4}}{dt} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \left\| \frac{d\mathbf{r}_{K_4}}{dt} \right\| = \sqrt{2}$$

5. Curve K_5: A half circle which connects $(-1, -1)$ to the point $(1, 1)$.

$$\mathbf{r}_{K_5} = \begin{pmatrix} \sqrt{2}\cos(t) \\ \sqrt{2}\sin(-t) \end{pmatrix}, \qquad t \in \left[\frac{3\pi}{4}, \frac{7\pi}{4}\right]$$
$$\frac{d\mathbf{r}_{K_5}}{dt} = \begin{pmatrix} -\sqrt{2}\sin(t) \\ \sqrt{2}\cos(t) \end{pmatrix} \qquad \left\| \frac{d\mathbf{r}_{K_5}}{dt} \right\| = 2$$

6. Curve K_6: A piecewise linear curve built from one horizontal segment and one vertical segment.

$$\mathbf{r}_{K_6} = \begin{pmatrix} t \\ -1 \end{pmatrix}, \text{ for } \qquad t \in [-1, 1), \text{ and } \begin{pmatrix} 1 \\ t-2 \end{pmatrix}, \text{ for } t \in [1, 3]$$
$$\frac{d\mathbf{r}_{K_6}}{dt} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \left\| \frac{d\mathbf{r}_{K_6}}{dt} \right\| = 1$$



This set of curves has been specifically chosen so that all three curves start and end at the same points. These will help us to investigate the fundamental theorem of line integrals.