



$$3. \omega R_1 C_1 = 1, C_2 = \frac{C_1}{\alpha}, R_2 = \alpha R_1, Z_{eq} = Z_{2+3+4} = Z_2 \parallel Z_{3+4}$$

$$V_1 = V_{in} \left(\frac{Z_{eq}}{Z_1 + Z_{eq}} \right)$$

$$Z_{eq} = \left(\frac{jC_2 R_2 \omega + 1}{-C_1 C_2 R_2 \omega^2 + j\omega(C_1 + C_2)} \right)$$

$$= V_{in} \left[\frac{jC_2 R_2 \omega + 1}{C_1 C_2 \alpha R_1 R_2 \omega^2 + C_1 j R_1 \omega + C_2 j R_2 \omega + 1} \right]$$

$$V_{out} = V_1 \left(\frac{R_2}{R_2 + Z_3} \right) = V_{in} \left[\frac{jC_2 R_2 \omega}{-C_1 C_2 R_1 R_2 \omega^2 + jC_1 R_1 \omega + jC_2 R_1 \omega + jC_2 R_2 \omega + 1} \right]$$

$$\text{Using } R_2 = \alpha R_1 \text{ and } C_2 = \frac{C_1}{\alpha}$$

$$\Rightarrow V_{out} = V_{in} \left[\frac{\alpha j C_1 R_1 \omega}{-\alpha C_1^2 R_1^2 \omega^2 + 2j\alpha C_1 R_1 \omega + \alpha + jC_1 R_1 \omega} \right]$$

$$\text{Using } \omega = \frac{1}{R_1 C_1}$$

$$V_{out} = V_{in} \left[\frac{\alpha j}{-\alpha + 2\alpha j + \alpha + j} \right] = V_{in} \left[\frac{\alpha j}{j(2\alpha + 1)} \right]$$

$$\Rightarrow V_{out} = V_{in} \left[\frac{\alpha}{2\alpha + 1} \right] \Rightarrow G(\alpha) = \frac{V_{out}}{V_{in}} = \left[\frac{\alpha}{2\alpha + 1} \right] a)$$

b) input impedance: Z_{eq} from V_{in} 's perspective

$$Z_{eq} = Z_{2+3+4} \quad \omega = \frac{1}{R_1 C_1}$$

$$\Rightarrow Z_{2+3+4} = \left(\frac{jC_2 R_2 \omega + 1}{-C_1 C_2 R_2 \omega^2 + j\omega(C_1 + C_2)} \right) = \left(\frac{j+1}{-\frac{1}{R_1} + j\left(\frac{1}{R_1} + \frac{\alpha}{R_1}\right)} \right)$$

$$= \left(\frac{\alpha R_1 (j+1)}{a(j+1) + j} \right) [b] [\Omega]$$

c) output impedance: Z_{eq} from V_{out} 's perspective

$$Z_{eq} = Z_{1+2+3} = Z_{1+2} \parallel Z_3 = \left(\frac{R_1 + \frac{1}{j\omega C_1}}{\left(R_1 + \frac{1}{j\omega \left(\frac{1}{C_1} + \frac{1}{C_2} \right)} \right)} \right) = \left(\frac{\omega(C_1 + C_2)(j\omega C_1 R_1 + 1)}{C_1 C_2 j\omega^2 (jR_1 \omega(C_1 + C_2) + 1)} \right)$$

$$\text{Using: } C_2 = \frac{C_1}{\alpha}, \omega = \frac{1}{R_1 C_1}$$

$$\Rightarrow \left(\frac{\alpha(j+1)R_1}{j(\alpha+1)-1} \right) [c] [\Omega]$$