Homework : Chapter 8

1. (5 points) Do out in detail the mathematics in Section 8.2 that shows how the central force problem splits into a center of mass problem and a relative motion problem. This is an iconic derivation and worth knowing.

2. (5 points) Apply what we have learned to the earth-sun system. What is the reduced mass of this system? Where is the center of mass of this system? Is it inside or outside of the sun? Based on these values, is it reasonable to think of the sun as fixed and the earth moving around it in the center of mass frame?

3. (10 points) Problem 8.12 in Taylor, in which you find the stable fixed point for the effective potential, so you can use all of what we have been learning to do this. (Note that this problem is stated in terms of the potential instead of the force, so the derivative of the potential is zero at the fixed point).

4. (10 points) Phase space practice.

   a. Take the potential for the problem you wrote code for last week

$$U_{pendulum} = m\ g\ l\ (1 - \cos\theta)$$

OR

$$U_{beadonwire} = -\frac{1}{2} mR^2\omega^2 \sin^2\theta + \ m\ g\ R(1-\cos\theta) =$$
$$= mR^2\omega^2 \left(-\frac{1}{2}\sin^2\theta\ + \gamma^2(1-\cos\theta)\right) \quad where \qquad \gamma = \sqrt{\frac{g}{\omega^2 R}}$$

and plot the potential using whatever computational tool you want (take all constants =1).

   b. Then use that plot of U to sketch several phase space trajectories in the following steps (note that each energy is uniquely identified by its energy). For the bead on wire, pick a value of $\gamma < 1$, since that gives you a more interesting potential.
      i. Identify stable and unstable points and put those in phase space first and label as “s” or “u”.
      ii. Sketch the separatrix trajectories (if there are any). These are the trajectories that have the same energy as the unstable fixed point, but don’t start at the unstable trajectory.
      iii. Choose 4 or more energies (some above the unstable fixed point energy and some below) and sketch those trajectories by noting the location of the turning points (where the velocity is zero) and the location of the max velocity. Put those points on the phase space plot and then join smoothly. Knowing that trajectories cannot cross helps you draw these.
   c. Use the phase space plot to plot several $\theta(t)$ plots. Can the motion for your system be unpredictable/chaotic?

5. (15 points) Getting a conceptual overview of the possible central force orbits using an effective potential and phase space diagrams.
We can write the energy of this one-dimensional problem as follows

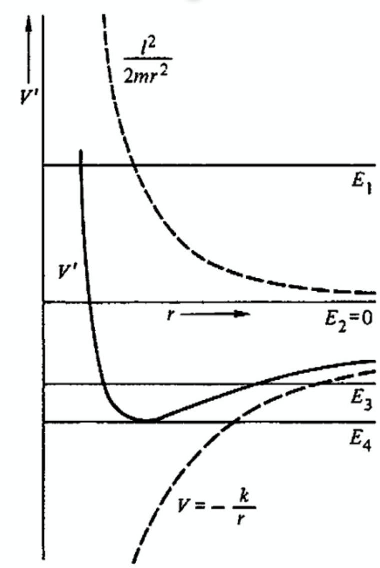
$$E = T + U = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}\mu r^2 \dot{\phi}^2 + U(r) = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}\frac{l^2}{\mu r^2} + U(r)$$

We commonly then define the *effective potential:*

$$U_{effective}(r) \equiv \frac{1}{2}\frac{l^2}{\mu r^2} + U(r)$$

$$E = T + U_{effective} = \frac{1}{2}\mu \dot{r}^2 + U_{effective}$$

What does this definition get us? My claim is that it gets us a helpful overview of the possible motions for this system.
We'll work this out in the steps below and take $U(r) = -\frac{k}{r}$ for specificity.



i. Look at the sketch given here. V' is the same as our $U_{effective}$. Explain why $U_{effective}$ is positive for small r and negative for large r. Where (if anywhere) is $U_{effective} = 0$?

ii. Using the same ideas that we used in Lecture 4.2 to describe the motion of a planar pendulum, describe the motion of the particle with total energy E4. Describe in words and sketch the trajectory on a phase space plot of r *vs.* $\dot{r}$.

iii. Similarly, describe in words and sketch the phase space trajectory (on the same plot for all the energies) for particles with energy E3, E2, and E1. In particular, which of these orbits are confined to a finite range of *r*, and which (if any) can get to $r = \infty$?

iv. It is easy to forget that *r* is not the only motion: our particle is moving in ϕ direction as well (this is hidden in the effective potential). Expand your descriptions in the previous two parts to include this motion as well (qualitatively only). (See pages 301-305 in the text.)

v. IMHO – this is the one draw back of the effective potential – we focus so much on the radial motion that we forget there is angular motion as well. Do you see other drawbacks to using the effective potential?

6. (10 points) Problem 8.16
7. (10 points) Problem 8.23