Problems 1.2 & 1.22, 1.6, 1.18, 1.43, 1.48, 1.24 do 1.32 for fun

PHYS 615 HW1 1/22/24

1. Case, N=3. (1.25 - 1.29)

1.25 $F_1^{net} = F_1 = F_{12} + F_1^{ext} + F_{13}$

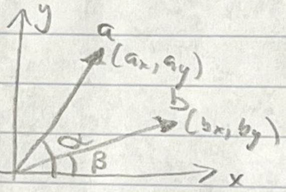
1.26 $\dot{p}_1 = F_{12} + F_{13} + F_1^{ext}$

$P = p_1 + p_2 + p_3 \Rightarrow \dot{P} = \dot{p}_1 + \dot{p}_2 + \dot{p}_3$

1.27 $\dot{P} = F_{12} + F_{13} + F_{23} + F_1^{ext} + F_2^{ext} + F_3^{ext} + F_{21} + F_{31} + F_{32}$

1.28 $\sum_{\alpha}\sum_{\alpha \neq \beta} F_{\alpha\beta} = F_{12} + F_{21} + F_{23} + F_{32} + F_{13} + F_{31} \therefore F_{\alpha\beta} + F_{\beta\alpha} = 0, \exists \alpha, \beta \in \{(\alpha,\beta) \mid \alpha \neq \beta\}$

1.29 $\boxed{\dot{P} = F_1^{ext} + F_2^{ext} + F_3^{ext}}$ 1.



2. a) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos(\alpha - \beta)$

$\vec{a} \cdot \vec{b} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} \cdot \begin{pmatrix} b_x \\ b_y \end{pmatrix} = a_x b_x + a_y b_y = ab\cos\alpha\cos\beta + ab\sin\alpha\sin\beta$

$\Rightarrow |\vec{a}||\vec{b}|\cos(\alpha - \beta) = ab(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$\Rightarrow \boxed{\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$ 2a.

b) $\vec{a} \times \vec{b} = \vec{v} = \begin{cases} v_x = \cancel{a_y b_z - a_z b_y} \to 0 \\ v_y = \cancel{a_x b_z - a_z b_x} \to 0 \\ v_z = a_x b_y - a_y b_x \end{cases}$ $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ a_x & a_y & 0 \\ b_x & b_y & 0 \end{vmatrix} = \hat{x} \cdot 0 - \hat{y} \cdot 0 + \hat{z}(a_x b_y - a_y b_x)$

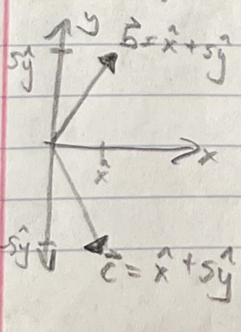
$|\vec{a}||\vec{b}|\sin(\alpha - \beta) = ab(\sin\alpha\cos\beta - \sin\beta\cos\alpha)\hat{z}$

$\Rightarrow \boxed{\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha}$ 2b.

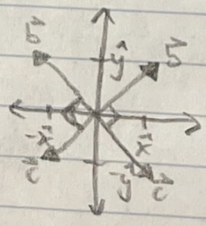
3. $\vec{b} = \hat{x} + s\hat{y} = \begin{pmatrix} 1 \\ s \end{pmatrix}, \vec{c} = \hat{x} - s\hat{y} = \begin{pmatrix} 1 \\ -s \end{pmatrix}$

$\vec{b}$ and $\vec{c}$ are orthogonal $\therefore \vec{b} \cdot \vec{c} = 0 \Rightarrow \begin{pmatrix} 1 \\ s \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -s \end{pmatrix} = 0 \Rightarrow 1 - s^2 = 0 \Rightarrow s^2 = 1$

$\Rightarrow s = 1, -1$

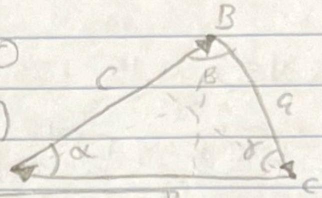


$\Rightarrow$ if $s = \pm 1$, then we have two right angles, which prove orthogonality



1.18 1.43 1.48 1.24

4.a) Area $= \frac{1}{2} b \cdot h \Rightarrow b = |\vec{b}|,\ h = |\vec{a}| \sin\gamma$



$|\vec{a} \times \vec{b}| = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = |\hat{x}(a_y b_z - a_z b_y) - \hat{y}(a_x b_z - a_z b_x) + \hat{z}(a_x b_y - a_y b_x)|$

$= ab(\sin\gamma \cos(0) - \sin(0)\cos\gamma) = \boxed{\frac{|\vec{a}||\vec{b}|\sin\gamma}{2} = \frac{b \cdot h}{2}}$ ai)

$|\vec{b} \times \vec{c}| = |\hat{z}(b_x c_y - b_y c_x)| = bc \sin\alpha$

$b \cdot h \Rightarrow b = |\vec{b}|,\ h = |\vec{c}| \sin\alpha \therefore \boxed{\frac{b \cdot h}{2} = \frac{bc \sin\alpha}{2}}$ aii)

$|\vec{c} \times \vec{a}| = |\hat{z}(c_x a_y - c_y a_x)| = ca \sin\beta,\ b \cdot h \Rightarrow b = |\vec{a}|,\ h = |\vec{c}| \sin\beta$

$\therefore \boxed{\frac{b \cdot h}{2} = \frac{ca \sin\beta}{2}}$ aiii) $\boxed{\frac{ab \sin\gamma}{2} = \frac{bc \sin\alpha}{2} = \frac{ca \sin\beta}{2} = \frac{bh}{2}}$

b) $bh = ab \sin\gamma = bc \sin\alpha = ca \sin\beta$

$\Rightarrow \frac{b \sin\gamma}{bc} = \frac{c \sin\alpha}{ac} = \frac{a \sin\beta}{ab}$

$\Rightarrow \frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \Rightarrow \boxed{\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}}$ b)

5.a) $\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{x} + y\hat{y}}{\sqrt{x^2+y^2}} = \frac{x\hat{x}}{\sqrt{x^2+y^2}} + \frac{y\hat{y}}{\sqrt{x^2+y^2}} = \overset{\cos\phi}{\frac{x}{r}}\hat{x} + \overset{\sin\phi}{\frac{y}{r}}\hat{y} = \boxed{\cos\phi\,\hat{x} + \sin\phi\,\hat{y}}$

$x = r\cos\phi \quad r = \sqrt{x^2+y^2}$

$y = r\sin\phi \quad \phi = \tan^{-1}(y/x) \quad \hat{\phi} = \hat{z} \times \hat{r} = \hat{z} \times (\cos\phi\,\hat{x} + \sin\phi\,\hat{y})$

$= \cos\phi(\hat{z} \times \hat{x}) + \sin\phi(\hat{z} \times \hat{y}) = \boxed{\cos\phi\,\hat{y} - \sin\phi\,\hat{x}}$

b) $\dot{\hat{r}} = \dot{\phi}(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}) = \dot{\phi}\hat{\phi}$

$\dot{\hat{\phi}} = -\dot{\phi}(\sin\phi\,\hat{x} + \cos\phi\,\hat{y}) = -\dot{\phi}\hat{r}$

6. $\hat{\rho} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y},\ \dot{\hat{\rho}} = \dot{\phi}(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}) = \dot{\phi}\hat{\phi}$

$\hat{\phi} = \cos\phi\,\hat{y} - \sin\phi\,\hat{x},\ \dot{\hat{\phi}} = -\dot{\phi}(\sin\phi\,\hat{x} + \cos\phi\,\hat{y}) = -\dot{\phi}\hat{r}$

$\hat{z} = \hat{z}$

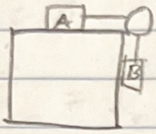
1.48 1.24

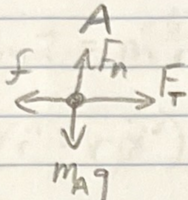
7. $\frac{df}{dt} = f \Rightarrow f' - f = 0 \quad f = Ae^{\lambda t}$

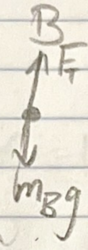
$\lambda = 1$

$f = c_1 e^{t}$ 1st order D.E. has 1 constant

8.







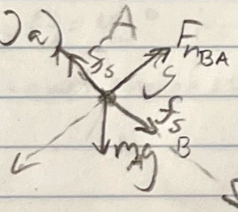
$\mu_s m_A g - m_B g = 0 \quad F_T = m_B g$

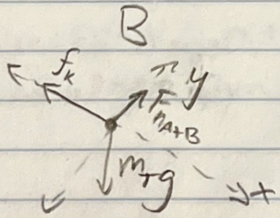
$$\boxed{m_B = \mu_s m_A}$$

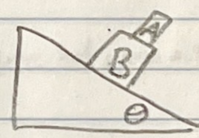
9. $\mu_s m_A g - m_B g = a(m_A + m_B)$

$$\boxed{a = \frac{(\mu_s m_A - m_B) g}{m_A + m_B}}$$

10 a)







$F_x^{net} = m_T g \sin\theta - \mu_k m_T g \cos\theta = m_T a$

$\Rightarrow a = \boxed{g(\sin\theta - \mu_k \cos\theta)\hat{x}}$

$$\boxed{m_T = m_A + m_B}$$

b) $\Sigma F_{x_A} = 0 \Rightarrow m_B g \sin\theta - \mu_k m_B g \cos\theta = \mu_s m_A g \cos\theta$

$\mu_s = \frac{m_B(\sin\theta - \cos\theta)}{m_A \cos\theta}$

c) $F_{nBA} = m_A g \cos\theta$

$F_{nA+B} = m_T g \cos\theta$