PHYS 615 HW3

1. a) $\ddot{x} = -Bx$, $x = e^{At}$

$A^2 e^{At} = -Be^{At}$

$A^2 = -B$ ✓ const. a) ✓

b) $\ddot{x} = Cx - D\dot{x}$

$A^2 e^{At} = Ce^{At} - ADe^{At}$

$A^2 + AD = C$ ✓ constant

$A = \frac{-D \pm \sqrt{D^2+4AC}}{2A}$ b) ✓

c) $\frac{d^5x}{dt^5} = -Fx$

$A^5 e^{At} = -Fe^{At}$

$A^5 = -F$ ✓ const. c) ✓

d) $\ddot{x} = Gtx$

$A^2 e^{At} = Gte^{At}$ ✓

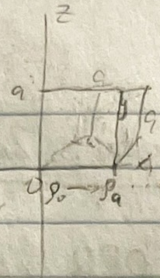
$A^2 = Gt$ X depen on time d)

e) $\ddot{x} = -kx^2$

$A^2 e^{At} = -ke^{2At}$; $e^{2At} e^{-At} = e^{At}$ ✓

$A^2 = -ke^{At}$ X dep. on time e)

the guess $x = e^{At}$ doesn't work on non-linear differential equations



Assuming cube is in I st quadrant

3. a) $\rho = \rho_0 (1 + \frac{x}{a})$

CM: $(M_x \geq \frac{a}{2}, \frac{a}{2}, \frac{a}{2})$

b) $\int \rho \, dV = \int_0^a\int_0^a\int_0^a \rho \, dx\,dy\,dz = \int_0^a\int_0^a \left[\int_0^a \rho_0 + \frac{\rho_0 x}{a} dx\right] dy\,dz$

$= \int_0^a\int_0^a dy\,dz \left[\frac{3\rho_0 a}{2}\right] = \Sigma M = \boxed{\frac{3\rho_0 a^3}{2}}$

$x = \frac{1}{M}\int_0^a\int_0^a\int_0^a \rho x \, dx\,dy\,dz = \frac{1}{M}\int_0^a\int_0^a\int_0^a \rho_0 x + \frac{\rho_0 x^2}{a} dx\,dy\,dz = \frac{\rho_0 a^2}{M}\left[\frac{x^2}{2} + \frac{x^3}{3a}\right]_0^a$ $\frac{5a^2}{6}$

$= \frac{5}{6}\frac{\rho_0 a^4}{M} = \frac{5}{6}\rho_0 \frac{a^4}{a^3} = \boxed{5/6 \cdot \rho_0 a}\, \hat{x}$

$y = \frac{1}{M}\int_0^a\int\int \rho y \, dx\,dy\,dz = \frac{\rho}{M}\cdot\frac{a^4}{2} = \frac{1}{V}\cdot\frac{a^4}{2} = \frac{a^4}{2a^3} = \boxed{a/2}\,\hat{y}$

$z = \frac{1}{M}\int_0^a\int\int \rho z \, dx\,dy\,dz = \frac{\rho}{M}\cdot\frac{a^4}{2} = \boxed{\frac{a}{2}}\,\hat{z}$

$\boxed{\vec{R} = \left(\frac{5}{6}\rho_0 a, \frac{a}{2}, \frac{a}{2}\right)}$

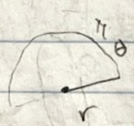
Very close, forgot factor of $\frac{2}{3}$ from $\frac{1}{M}$.

2. Find both sphere and cube center of mass then treat them like point particles to find combined C.O.M.

$\vec{R}_{cube} \Rightarrow$ symmetry $\begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases} \Rightarrow \vec{R}_c = (0,0,0)$

I used $\vec{R}_s = (0,0,3R)$ instead of $2R$.

$\vec{R}_{sphere} \Rightarrow \begin{cases} x = 0 \\ y = 0 \\ z = \frac{1}{M}\int \rho z \, dV \end{cases}$



$z = 3R$ X sol'n page 2

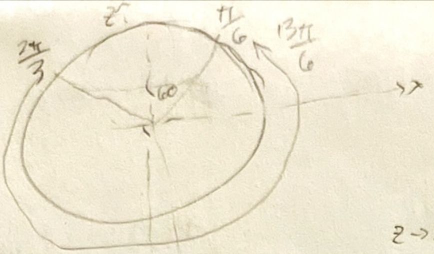
$\vec{z} = \frac{\rho}{M}\int_0^{2\pi}\int_0^{\pi}\int_0^{R} z r^2 \sin\theta \, dr\,d\theta\,d\phi \to \frac{\rho}{M}\int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta\,d\phi \cdot \frac{R^3}{3} \Rightarrow \frac{\rho R^3}{3M}\int_0^{2\pi} d\phi \cdot -\cos\theta \Big|_0^{\pi}$

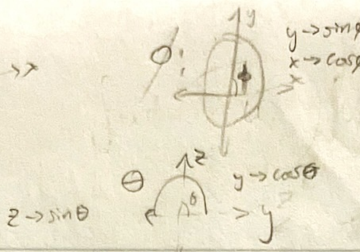
Otherwise, process was right

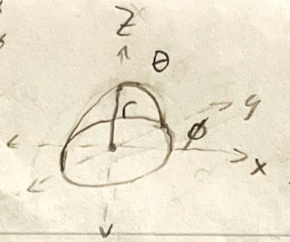
$\Rightarrow \frac{2\rho R^3}{3M} z \left[2\pi\right] = \frac{4\pi\rho R^3}{3M} z$, $\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} \Rightarrow \frac{4\pi R^3 3M}{4\pi R^3 3M} z$, $\vec{R}_s = (0,0,z) = (0,0,3R)$

$\vec{R}_{sys} = \frac{1}{M_T}(\vec{R}_c + \vec{R}_s) = \frac{1}{M_T}(3R \cdot M_s \hat{z}) \Rightarrow \frac{3RM_s \hat{z}}{M_s + M_c} = \frac{24R^4\rho\hat{z}}{\rho R^3(8 + \frac{4}{3}\pi)} = \frac{6R\hat{z}}{\frac{7}{3}\pi} = \boxed{\frac{18}{7\pi}R\hat{z}}$

$M_s = 8\rho R^3$, $M_c = \frac{4}{3}\pi\rho R^3$







4. a) Assuming ρ is const.

x, y - symmetric, $\sum_\alpha r_{x\alpha} m_\alpha + \sum_\alpha r_{y\alpha} m_\alpha = 0$

$$\vec{R} \le \begin{bmatrix} 0 \\ 0 \\ R\sin 30^\circ \end{bmatrix} \qquad dV \to \begin{cases} r = dr\,[0, R] \\ \theta = r\,d\theta\,[0, \pi] \\ \phi = r\sin\theta\,d\phi\,\left[\frac{5\pi}{6}, \frac{13\pi}{6}\right] \end{cases}$$

Misread the question, it says 60° from the z axis, which isn't true, what you did, which was correct, would be 30° from z.

b) $$M = \int \rho\, dV = \int_0^{4\pi/3}\int_0^{\pi}\int_0^R \rho r^2 \sin\theta\, dr\, d\theta\, d\phi = \frac{\rho R^3}{3}\int_0^{4\pi/3}\int_0^{\pi} \sin\theta\, d\theta\, d\phi = \frac{\rho R^3}{3}\left[-\cos(\pi) - (-\cos(0))\right]\int_0^{4\pi/3} d\phi$$

$$\Rightarrow \frac{2\rho R^3}{3}\left[\frac{4\pi}{3}\right] = \boxed{\frac{8}{9}\pi\rho R^3}$$

$x = r\sin\theta\cos\phi$ ← defined θ wrong, it should be

θ is from horizontal so is ϕ $\begin{cases} x = r\cos\phi\cos\theta, \; y = r\sin\phi\cos\theta \\ z = r\sin\theta \end{cases}$

$$M_x = \rho\int_0^{4\pi/3}\int_0^{\pi}\int_0^R r^3 \sin^2\theta\, dr\, d\theta\, d\phi$$

switched bounds to fit cartesian ~~so that rcos makes sense~~

$M_y = 0$ due to symmetry; $x, y = 0$, $z = r\sin\theta\sin\phi$

$$M_z = \rho\int_{5\pi/6}^{13\pi/6}\int_0^{\pi}\int_0^R r^3 \sin^2\theta \sin\phi\, dr\, d\theta\, d\phi = \frac{\rho R^4}{4}\int_{5\pi/6}^{13\pi/6}\int_0^{\pi}\left[\frac{1-\cos(2\theta)}{2}\right] d\theta\, d\phi = \frac{\rho R^4 \pi}{4 \cdot 2}\int_{5\pi/6}^{13\pi/6} \sin\phi\, d\phi$$

$$= \frac{\pi\rho R^4}{8}\left[-\cos\theta\right]_{5\pi/6}^{13\pi/6} = \boxed{\frac{\sqrt{3}\pi\rho R^4}{8}} \to M_z, \quad z = \frac{-\sqrt{3}\pi R^4 \cdot 9}{8 R^3 \pi \cdot 8} = \boxed{\frac{-9\sqrt{3}R}{64}\hat{z}}$$ ← too large

$\rho = \frac{M}{V}$

$V = \frac{\pi \cdot 8R^3}{9}$

$$\boxed{\vec{R} = \left(0, 0, \frac{-9\sqrt{3}R}{64}\right)}$$

c) It makes sense, as long units match.

5. $m_{initial} = 2\times10^6$ kg, $m_{final} = 1\times10^6$ kg, $V_{ex} = 3000$ m/s

$v = v_0 + V_{ex}\ln(m_0/m) \to \boxed{V = 2079.44\text{ m/s}}$ ✓

Thrust $= -\dot{m}v_{ex}$

$dm = -1\times10^6$ kg

$dt = 120$ s, thrust $= \frac{1.0\times10^6}{120}\cdot 3000 = 2.5\times10^7\text{ N} = \boxed{25\text{ MN}}$ ✓

$m_{initial} = 2\times10^6 \Rightarrow 8\%$ of thrust ← compared weight and thrust, got it backwards

6. $-\dot{m} = -\left(\frac{-1\times10^6}{120}\right)\cdot v_{ex} \ge 2\times10^6 \Rightarrow 8333.33\, V_{ex} \ge 2\times10^6$

$\boxed{V_{ex} \ge 240\text{ m/s}}$ ✗ off by a factor of 10? missed a zero :(

$P(t)$

2.a) $P(t+dt) = (m+dm)(v+dv) - dm(v - v_{ex}) = mv + mdv + dm v_{ex}$

$dP = P(t+dt) - P(t) = m dv + dm v_{ex}$

$F^{ext} = m dv + dm v_{ex}$

$\boxed{m\dot{v} = F^{ext} - \dot{m} v_{ex}}$ ✓

b) $(m_0 - kt)\dot{v} = k v_{ex} - (m_0 - kt)$

$\dot{v} = \frac{k v_{ex}}{m_0 - kt} - 1$

$v = \int \frac{k v_{ex}}{m_0 - kt} dt - t + v_0$

$v = -v_{ex} \ln(m_0 - kt) - t + v_0, \; t=0: v_0 = v_{ex}\ln(m_0)$

$v = \boxed{v_{ex}\ln(m_0 - kt) - t + v_{ex}\ln(m_0)}$ Very close, forgot about g in this, should be $v_{ex}\ln\left(\frac{m_0}{m_0 - kt}\right) - gt$

$\hookrightarrow \ln\left(\frac{m_0}{m_0 - kt}\right)$

c) $v = -3000\ln(2\times10^6 - 8.3\times10^3 \cdot 120) - 120 + 3000\ln(2\times10^6)$

$v = 1959.44 \, m/s$ ✓ correct steps, Error carry it forward

d) The rocket wouldn't be able to accelerate upwards - $\dot{v} < 0$! ✓

Includes assumption of how mass changes

3. Used: $v = v_0 + v_{ex}\left(\ln\left(\frac{m_0}{m}\right) + \ln\left(\frac{m_{02}}{m_2}\right)\right)$

Optimal masses: $m_1 = m_2$

did a simulation with this model in desmos, will either email or submit (y-intercept = v_{max})

Felt right :)