PHYS 615 HW 2

2/9/24

1. $\frac{dv}{dt} + bv = a$, $\mu = e^{bt}$

$e^{bt} \cdot v = a\int e^{bt}\,dt \Rightarrow e^{bt} \cdot v = \frac{a}{b}e^{bt} + C \Rightarrow v = \frac{a}{b} + Ce^{-bt}$, $v(0) = v_0$

$\Rightarrow v_0 = \frac{a}{b} + C \Rightarrow C = v_0 - \frac{a}{b} \therefore \boxed{v(t) = \frac{a}{b} + \left(v_0 - \frac{a}{b}\right)e^{-bt}}$

This is the stuff we talked about in class

$\int_0^t v(t)\,dt = y(t) - y_0 = \frac{a}{b}t + \left(\frac{a}{bt} - \frac{v_0}{b}\right)e^{-bt} = \boxed{\frac{a}{b}t + \left(\frac{a}{b^2} - \frac{v_0}{b}\right)e^{-bt} + y_0}$

2. "$\dot{v} = av^2 + t^3$" - not seperable

$\hookrightarrow \dot{v} = b + t^3$ - sceperable - can integrate in this form $\rightarrow \int_0^t \frac{dv}{dt}\,dt = \int_0^t b + t^3\,dt$

$\hookrightarrow \dot{v} = (b+v^3)^{-1}$ - sceperable $\Rightarrow$ integrate in this form: $\int_0^t (b+v^3)\frac{dv}{dt}\,dt = \int_0^t dt$

$\hookrightarrow \dot{v} = c\sqrt{vt}$ - seperable $\rightarrow \int_0^t \frac{1}{\sqrt{v}}\frac{dv}{dt}\,dt = \int_0^t c\sqrt{t}\,dt$

3. a) $\lim\limits_{x\to\infty} e^x = \infty$, $\lim e^{-x} = 0$, $\lim \cosh(x) = \infty$, $\lim \sinh(x) = \infty$

$\lim\limits_{x\to\infty}\left[\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - \cancel{e^{-x}}^{\,0}}{e^x + \cancel{e^{-x}}} = \frac{e^x}{e^x}\right] = 1$

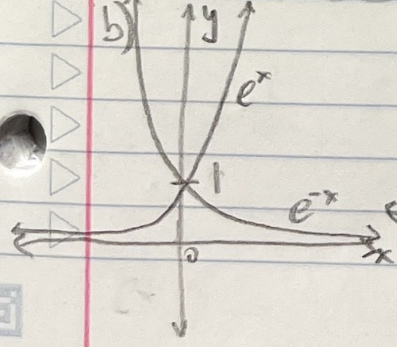
$\lim\limits_{x\to-\infty} e^x = 0$, $\lim e^{-x} = \infty$, $\lim \cosh(x) = \infty$, $\lim \sinh(x) = -\infty$

$\lim\limits_{x\to-\infty}\left[\tanh(x) = \frac{\cancel{e^x}^{\,0} - e^{-x}}{\cancel{e^x} + e^{-x}} = -\frac{\cancel{e^{-x}}}{\cancel{e^{-x}}}\right] = -1$

$\lim\limits_{x\to 0} e^x = 1$, $\lim e^{-x} = 1$, $\lim \cosh(x) = 1$, $\lim \sinh(x) = 0$,

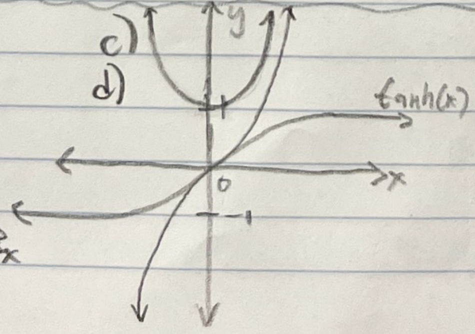
$\lim\limits_{x\to 0}\left[\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1-1}{1+1}\right] = 0$

b)



c)

d)



3.d) $\cosh^2(x)-\sinh^2(x)=\left(\frac{e^x+e^{-x}}{2}\right)^2-\left(\frac{e^x-e^{-x}}{2}\right)^2$

$=\frac{1}{4}\left(\left(e^{2x}+e^{-2x}+2\right)-\left(e^{2x}+e^{-2x}-2\right)\right)=\frac{1}{4}\cdot 4=\boxed{1}$

4. $e^{\theta}=1+\theta+\frac{\theta^2}{2}+\frac{\theta^3}{3!}+\dots,\ e^{i\theta}=1+i\theta-\frac{\theta^2}{2}-\frac{i\theta^3}{3!}+\frac{\theta^4}{4!}+\frac{i\theta^5}{5!}-\frac{\theta^6}{6!}+\dots$

$e^{i\theta}=\left[1-\frac{\theta^2}{2}+\frac{\theta^4}{4!}-\frac{\theta^6}{6!}+\dots\right]+i\left[\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-\frac{\theta^7}{7!}+\dots\right]=\boxed{\cos\theta+i\sin\theta}$

5.a) $v_y(t)=v_{ter}\left(1-e^{-t/\tau}\right),\ \tau=\frac{m}{b},\ e^{-\frac{b}{m}t}\approx 1-\frac{b}{m}t$

$\Rightarrow v_y(t)=v_{ter}\left(\frac{b}{m}t\right),\ v_{ter}=g\tau\ \therefore\ v_y=g\tau\left(\frac{t}{\tau}\right)=\boxed{gt}$ ✓

b) $\int_0^t v_y(t)\,dt=y(t)-\cancelto{0}{y_0}=\int_0^t gt\,dt=\boxed{\frac{1}{2}gt^2}$

6. $v(0)=v_0\quad f_D=-cv^{3/2}\quad t=m\int_{v_0}^{v}\frac{1}{-cv'^{3/2}}dv'=-\frac{m}{c}\int_{v_0}^{v}v'^{-3/2}dv'$

$t=-\frac{m}{c}\left(-2v'^{-1/2}\right)\Big|_{v_0}^{v}=-\frac{m}{c}\left(-\frac{2}{\sqrt{v}}+\frac{2}{\sqrt{v_0}}\right)\Rightarrow t=-\frac{2m}{c}\left(\frac{1}{\sqrt{v_0}}-\frac{1}{\sqrt{v}}\right)=\frac{2m}{c\sqrt{v}}-\frac{2m}{c\sqrt{v_0}}$

$\Rightarrow\sqrt{v}\left(t+\frac{2m}{c\sqrt{v_0}}\right)=\frac{2m}{c}\Rightarrow\boxed{v=\left(\frac{2m}{ct+\frac{2m}{c\sqrt{v_0}}}\right)^2}$ $\boxed{\text{v will never come to rest}}$