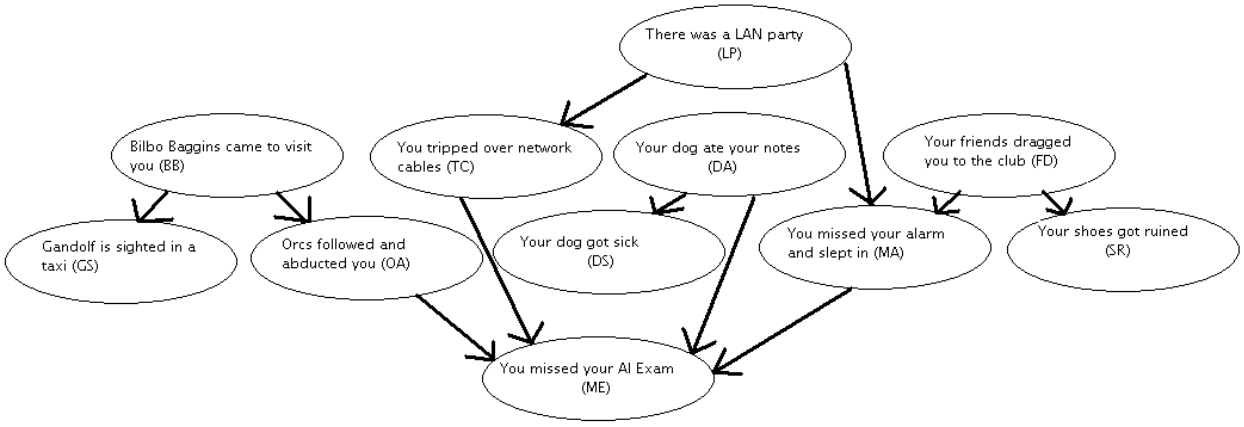
1 D-Separation

You missed the Artificial Intelligence Exam! When you try to explain to your professor your excuses for missing the exam, he stops you. He offers to give you a grade based on your performance of mapping your excuses and determining the D-separation of the excuses and the events associated with them. You draw out the events as a Bayes net seen above. You must now answer the following questions about the net to see what your grade for the exam you missed!



Consider the following pairs of variables to answer the questions below:

(a) GS and OA ; (b) DA and MA ; (c) LP and FD ; (d) TC and MA ; (e) BB and ME
Given the stated evidence, list the variables that are independent or conditionally independent.

1. (4 points) No information (even the evidence that you missed the exam).

   (b), (c)

2. (4 points) MA is observed.

   (b), (d)

3. (4 points) BB is observed.

   (a), (b), (c), (e)

4. (4 points) LP is observed.

   (b), (c), (d)

5. (4 points) TC is observed.

   (b), (c), (d)

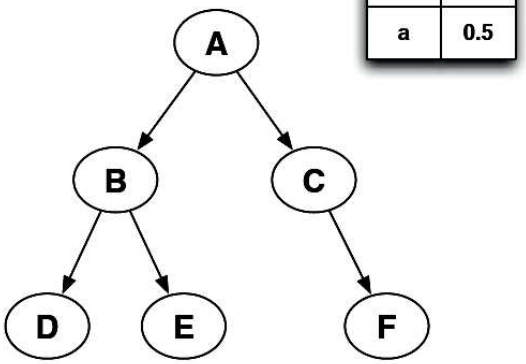
6. (4 points) LP and OA are observed.

   (a), (b), (c), (d), (e)

7. (4 points) ME is observed.

   (e)

2 Inference by Enumeration



A	P(A)
a	0.5

F	C	P(FIC)
f	c	0.5
f	~c	0.8

C	A	P(CIA)
c	a	0.9
c	~a	0.7

D	B	P(DIB)
d	b	0.7
d	~b	0.2

E	B	P(EIB)
e	b	0.6
e	~b	0.2

B	A	P(BIA)
b	a	0.8
b	~a	0.5

1. What is the expression for $P(A, b, C, \sim d, E, f)$ given the structure of this Bayes' net and conditional probability tables?

$$P(A, b, C, \sim d, E, f) = P(A)P(b|A)P(C|A)P(\sim d|b)P(E|b)P(f|C)$$

2. Form the joint distribution $P(A, b, C, \sim d, E, f)$ using factors. Please give the details of your derivation.

   First, extract entries from the tables consistent with the evidence.

A	$P(b\|A)$
a	0.8
$\sim a$	0.5

E	$P(E\|b)$
e	0.6
$\sim e$	0.4

C	$P(f\|C)$
c	0.5
$\sim c$	0.8

   Rewrite the joint distribution in terms of factors:

$$P(A, b, C, \sim d, E, f) = 0.3 f_1(A) \times f_2(A, b) \times f_3(A, C) \times f_4(b, E) \times f_5(C, f)$$

   where

$$f_1(A) = P(A),\ f_2(A, b) = P(b|A),\ f_3(A, C) = P(C|A),\ f_4(b, E) = P(E|b),\ f_5(C, f) = P(f|C)$$

   First, let's combine factors in C: $f_6(A, C, f) = f_3(A, C) \times f_5(C, f)$.

A	C	f	$f_6(A, C, f)$
a	c	f	$.9 * .5 = .45$
a	$\sim c$	f	$.1 * .8 = .08$
$\sim a$	c	f	$.7 * .5 = .35$
$\sim a$	$\sim c$	f	$.3 * .8 = .24$

Next, combine factors in A: $f_7(A, b, C, f) = f_1(A) \times f_2(A, b) \times f_6(A, C, f)$.

A	B	C	F	$f_7(A, b, C, f)$
a	b	c	f	$.5 * .8 * .45 = .18$
a	b	$\sim c$	f	$.5 * .8 * .08 = .032$
$\sim a$	b	c	f	$.5 * .5 * .35 = .0875$
$\sim a$	b	$\sim c$	f	$.5 * .5 * .24 = .06$

Finally, combine the factor in E, which does not depend on A and C.

A	B	C	D	E	F	$P(A, b, C, \sim d, E, f)$
a	b	c	$\sim d$	e	f	$.3 * .18 * 0.6 = .0324$
a	b	c	$\sim d$	$\sim e$	f	$.3 * .18 * 0.4 = .0216$
a	b	$\sim c$	$\sim d$	e	f	$.3 * .032 * 0.6 = .00576$
a	b	$\sim c$	$\sim d$	$\sim e$	f	$.3 * .032 * 0.4 = .00384$
$\sim a$	b	c	$\sim d$	e	f	$.3 * .0875 * 0.6 = .01575$
$\sim a$	b	c	$\sim d$	$\sim e$	f	$.3 * .0875 * 0.4 = .0105$
$\sim a$	b	$\sim c$	$\sim d$	e	f	$.3 * .06 * 0.6 = .0108$
$\sim a$	b	$\sim c$	$\sim d$	$\sim e$	f	$.3 * .06 * 0.4 = .0072$

3. Solve for the query $P(C|b, \sim d, f)$.

From the product rule,

$$P(C|b, \sim d, f) = \alpha P(A, C, \sim d, f) = \alpha \sum_a \sum_e P(A, b, C, \sim d, E, f)$$

First, marginalize on E.

A	B	C	D	F	$P(A, b, C, \sim d, f)$
a	b	c	$\sim d$	f	.054
a	b	$\sim c$	$\sim d$	f	.0096
$\sim a$	b	c	$\sim d$	f	.02625
$\sim a$	b	$\sim c$	$\sim d$	f	.018

Next, marginalize on A.

B	C	D	F	$P(b, C, \sim d, f)$
b	c	$\sim d$	f	$.054 + .02625 = .08025$
b	$\sim c$	$\sim d$	f	$.0096 + .018 = .0276$

Finally, $1/\alpha = .0825 + .0276 = .10786$.

$$P(C|b, \sim d, f) = \frac{1}{.10786} P(b, C, \sim d, f)$$