CS 6300 **HW08: Bayes Nets II** **April 14, 2022**

Please use the LaTeX template to produce your writeups. Hand in using gradescope.

1 Variable Elimination

As an anthropologist, you are studying ancient human races. You are also a performing artist, and have an excellent model of the way humans invent music and dance. The key variables are:

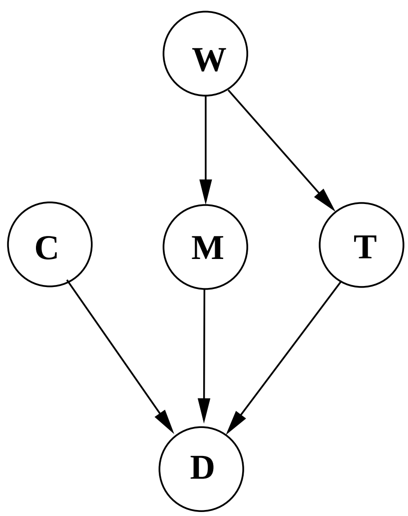
Writing (W): Whether or not a race invented some form of writing

Cold climate (C): Whether or not the race lived in a cold weather climate

Music (M): Whether or not a race invented music

Tools (T): Whether or a race had metal tools

You model the relationships between these variables and **dance (D)** using the Bayes net below.



The conditional probabilities of the Bayes net are listed below.

C	M	T	D	$P(D\|C,M,T)$
$+c$	$+m$	$+t$	$+d$	0.9
$+c$	$+m$	$+t$	$-d$	0.1
$+c$	$+m$	$-t$	$+d$	0.8
$+c$	$+m$	$-t$	$-d$	0.2
$+c$	$-m$	$+t$	$+d$	0.8
$+c$	$-m$	$+t$	$-d$	0.2
$+c$	$-m$	$-t$	$+d$	0.2
$+c$	$-m$	$-t$	$-d$	0.8
$-c$	$+m$	$+t$	$+d$	0.8
$-c$	$+m$	$+t$	$-d$	0.2
$-c$	$+m$	$-t$	$+d$	0.5
$-c$	$+m$	$-t$	$-d$	0.5
$-c$	$-m$	$+t$	$+d$	0.6
$-c$	$-m$	$+t$	$-d$	0.4
$-c$	$-m$	$-t$	$+d$	0.1
$-c$	$-m$	$-t$	$-d$	0.9

W	M	$P(M\|W)$
$+w$	$+m$	0.8
$+w$	$-m$	0.2
$-w$	$+m$	0.1
$-w$	$-m$	0.9

W	$P(W)$
$+w$	0.9
$-w$	0.1

W	T	$P(T\|W)$
$+w$	$+t$	0.7
$+w$	$-t$	0.3
$-w$	$+t$	0.9
$-w$	$-t$	0.1

C	$P(C)$
$+c$	0.5
$-c$	0.5

You want to know how likely it is for a dancing, writing race to invent music.

1. Write the formula to compute $P(+m|+d,+w)$ using variable elimination.

$$\begin{aligned}
P(+m|+d,+w)P(+d,+w) &= P(+m,+d,+w) \\
&= \sum_c \sum_t P(+m,+d,+w,c,t) \\
&= \sum_c \sum_t P(+w)P(+m|+w)P(C)P(T|+w)P(+d|C,+m,T) \\
&= P(+w)P(+m|+w)\sum_c P(C)\sum_t P(t|+w)P(+d|c,+m,t)
\end{aligned}$$

2. Now compute $P(+m|+d,+w)$.

   Write in terms of factors.

$$P(+m|+d,+w)P(+d,+w) = 0.9 * 0.8 * \sum_c f_1(C) \times \sum_t f_2(T) \times f_3(C,T)$$

   where factor $f_4(C)$ is defined and computed as:

$$\begin{aligned}
f_4(c) &= \sum_t f_2(t) \times f_3(c,t) \\
&= f_2(+t) f_3(C,+t) + f_2(-t) f_3(C,-t) \\
&= 0.7 * \begin{bmatrix} 0.9 \\ 0.8 \end{bmatrix} + 0.3 * \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix} \\
&= \begin{bmatrix} 0.87 \\ 0.71 \end{bmatrix}
\end{aligned}$$

Now combine $f_1(C)$ and $f_4(C)$ to yield the factor f_5.

$$\begin{aligned}
f_5 &= \sum_c f_1(C) * f_4(C) \\
&= f_1(+c) * f_4(+c) + f_1(-c) * f_4(-c) \\
&= 0.5 * 0.87 + 0.5 * 0.71 \\
&= 0.79
\end{aligned}$$

Finally,

$$P(+m| + d, +w) P(+d, +w) = 0.72 * 0.79 = 0.5688$$

To avoid computing the probability of the evidence, we repeat the above derivation for $P(-m| + d, +w)$ and then normalize.

$$P(-m|+d,+w)P(+d,+w) = P(+w)P(-m|+w) \sum_c P(c) \sum_t P(t|+w) P(+d|c,-m,t)$$

Write in terms of factors.

$$P(-m| + d, +w) P(+d, +w) = 0.9 * 0.2 * \sum_c f_1(C) \times \sum_t f_2(T) \times f_3(C,T)$$

where factor $f_4(C)$ is defined and computed as:

$$
\begin{aligned}
f_4(C) &= \sum_t f_2(T) * f_3(C,T) \\
&= f_2(+t)f_3(C,+t) + f_2(-t)f_3(C,-t) \\
&= 0.7 * \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix} + 0.3 * \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix} \\
&= \begin{bmatrix} 0.62 \\ 0.45 \end{bmatrix}
\end{aligned}
$$

Now combine $f_1(C)$ and $f_4(C)$ to yield the factor f_5.

$$
\begin{aligned}
f_5 &= \sum_c f_1(C) * f_4(C) \\
&= f_1(+c) * f_4(+c) + f_1(-c) * f_4(-c) \\
&= 0.5 * 0.62 + 0.5 * 0.45 \\
&= 0.535
\end{aligned}
$$

Finally,

$$P(-m| + d, +w)P(+d, +w) = 0.18 * 0.535 = 0.09958$$

To normalize, $1/\alpha = 0.5688 + 0.09958 = 0.66838$. Hence

$$P(+m| + d, +w) = 0.5688/0.66838 = 0.851$$

2 Sampling

You decide to use sampling instead. You use prior sampling to draw the samples below:

W	C	M	T	D
$-w$	$+c$	$+m$	$-t$	$+d$
$+w$	$+c$	$-m$	$-t$	$-d$
$+w$	$-c$	$-m$	$+t$	$-d$
$+w$	$+c$	$+m$	$-t$	$+d$
$+w$	$+c$	$-m$	$+t$	$+d$
$+w$	$-c$	$-m$	$+t$	$-d$
$+w$	$-c$	$-m$	$-t$	$-d$
$+w$	$+c$	$+m$	$+t$	$+d$
$+w$	$+c$	$-m$	$+t$	$-d$
$-w$	$-c$	$-m$	$-t$	$-d$

1. Based on rejection sampling using the samples above, what is the answer to your query $P(+m|+d,+w)$?

   2/3

2. While your sampling method has worked fairly well in many cases, for rare cases (like a race that doesn't write) your results are less accurate as rejection sampling rejects almost all of the data. You decide to use likelihood weighting instead.

   You now wish to compute the probability that a race that has no writing ($-w$) or dancing ($-d$) nonetheless has music ($+m$), using likelihood weighting. I.e., you want $P(+m|-w,-d)$.

   (a) You draw four samples, using likelihood weighting. The following random numbers are generated, used in the order CT.

   0.45 0.85 0.12 0.95 0.66 0.23 0.07 0.46

   Complete the following tables with the four samples.

W	C	M	T	D	weight
$-w$	$+c$	$+m$	$+t$	$-d$	0.01
$-w$	$+c$	$-m$	$-t$	$-d$	0.08
$-w$	$-c$	$-m$	$+t$	$-d$	0.04
$-w$	$+c$	$+m$	$+t$	$-d$	0.01

   (b) For each of these samples, indicate its weight in the same table.

   (c) Compute the answer to your query, $P(+m|-w,-d)$, using likelihood weighting with these samples.

   1/7