CS 6300 **HW02: CSPs and Minimax** **Ryan Dalby** **Due February 1, 2022**

Please use LaTeX to produce your writeups. See the Homework Assignments page on the class website for details.

1 CSPs

You are in charge of scheduling arrivals for Pohonk Internation Airport. The airport has two runways (R and L), and four landing time slots (T1, T2, T3, T4). Today, there will be 4 arrivals, each of which you must assign a runway and a time slot. Here are the requirements for the schedule:

1. Air Force One (AF1) must land on runway R due to motorcade and secret service logistics.
2. The airport closes (no landings) for one timeslot before, one during, and one after the arrival of AF1.
3. The Blue Angels (BA) will land in formation, which requires both runways at the same time.
4. The Blue Angels are exempt from AF1 related airport closures.
5. The new Boeing 777 Dreamliner (B777) must land before the Blue Angels.
6. The B777 must land on runway L.
7. The Cessna (C) must not land beside or one timestep after the B777, due to turbulence considerations.
8. No two planes may land on the same runway at the same time.

	T1	**T2**	**T3**	**T4**
R				
L				

1. Represent the problem with 4 variables: AF1, B777, C, and BA. The domain of BA is a time between 1 and 4 that the Blue Angels will arrive; the domain of the other three variables is a time between 1 and 4 plus ‘R’ or ‘L’ to specify the runway being landed on. Enumerate separately the unary and binary constraints in this problem. For the binary constraints, you may use pseudocode for implicit functions, like *beside(?,?)*.

   Unary Constraints:

   AF1 $\in \{$R1, R2, R3, R4$\}$
   B777 $\in \{$L1, L2, L3, L4$\}$

   Binary Constraints: Note: $before$, $justBefore$, $during$, $justAfter$, and $after$ implies a landing time slot value being relative to another. $beside$ implies same landing time slot but different runway.

   $\neg$($justBefore$(AF1,X), $during$(AF1,X), $justAfter$(AF1,X)) for X $\in \{$B777, C$\}$
   $before$(B777, BA)
   $\neg$($justAfter$(C,B777) or $beside$(C,B777))
   $alldiff$(X,Y) which means if X$\neq$Y for X,Y $\in \{$AF1, B777, C, BA$\}$

2. Write constraint 5 in explicit form.

   (B777, BA) $\in$ {(R1,R2&L2), (L1,R2&L2), (R2,R3&L3), (L2,R3&L3), (R3,R4&L4), (L3,R4&L4), (R1,R3&L3), (L1,R3&L3), (R1,R4&L4), (L1,R4&L4), (R2,R4&L4), (L2,R4&L4)}

3. Enforce all *unary* constraints by deleting values in the table below.

AF1	R1	R2	R3	R4				
B777					L1	L2	L3	L4
C	R1	R2	R3	R4	L1	L2	L3	L4
BA	R1&L1	R2&L2	R3&L3	R4&L4				

4. Transfer your answer from part 3 to the table below. Then run arc-consistency. Show your sequence of steps; i.e., the arc you picked and the resulting domain change.

   (a) Arc $B777->BA$: B777 loses value L4 due to constraint 5.

AF1	R1	R2	R3	R4				
B777					L1	L2	L3	
C	R1	R2	R3	R4	L1	L2	L3	L4
BA	R1&L1	R2&L2	R3&L3	R4&L4				

   (b) Arc $BA->B777$: BA loses value R1&L1 due to constraint 5.

AF1	R1	R2	R3	R4				
B777					L1	L2	L3	
C	R1	R2	R3	R4	L1	L2	L3	L4
BA		R2&L2	R3&L3	R4&L4				

   (c) Arc $AF1->B777$: AF1 loses value R2 due to constraint 2.

AF1	R1		R3	R4				
B777					L1	L2	L3	
C	R1	R2	R3	R4	L1	L2	L3	L4
BA		R2&L2	R3&L3	R4&L4				

5. Assuming you have not yet found a unique solution, perform backtracking search, and maintain arc-consistency after each variable assignment. Use the Minimum Remaining Values (MRV) heuristic to choose which variable to assign first, breaking ties in the order AF1, B777, C, BA. After each variable assignment, reassign the domains in the grid.

   (a) Variable assignment: AF1=R4

AF1				R4				
B777						L2		
C	R1				L1			
BA			R3&L3					

   (b) Variable assignment: B777=L2

AF1			R4		
B777					L2
C	R1			L1	
BA		R3&L3			

(c) Variable assignment: BA=R3&L3

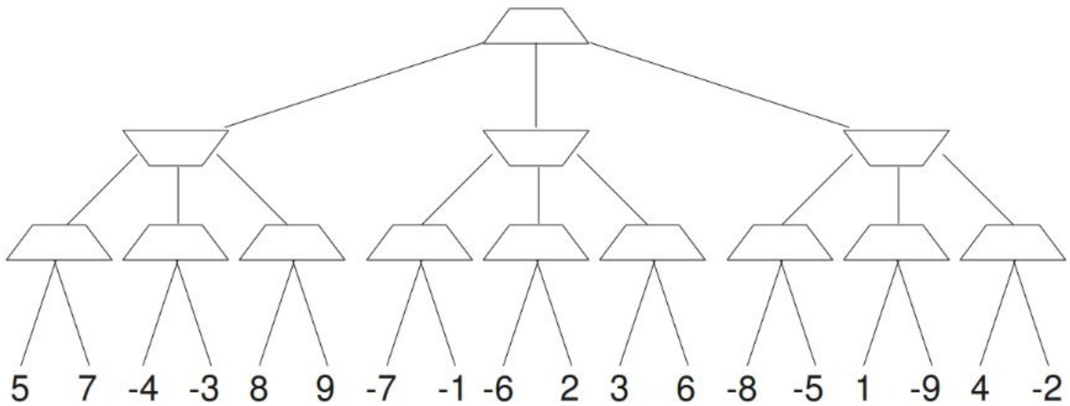
AF1			R4		
B777					L2
C	R1			L1	
BA		R3&L3			

(d) Variable assignment: C=R1

AF1			R4	
B777				L2
C	R1			
BA		R3&L3		

2 Minimax

Consider the two-player minimax game tree below. Suppose the top node is labeled A, the nodes at the next level A_1, A_2, A_3 from left to right, the nodes at the next level under A_1 as A_{11}, A_{12}, A_{13} from left to right, the nodes under A_2 as A_{21}, A_{22}, A_{23}, etc. The terminal nodes have 3 indexes A_{ijk}.



1. Carry out minimax search. Give the values for each node.

   $A = -1$

   $A_1 = -3, A_2 = -1, A_3 = -5$

   $A_{11} = 7, A_{12} = -3, A_{13} = 9$

   $A_{21} = -1, A_{22} = 2, A_{23} = 6$

   $A_{31} = -5, A_{32} = 1, A_{33} = 4$

2. Now use $\alpha - \beta$ pruning. Let ab_i be the $\alpha - \beta$ values passed down an edge to node i, etc., for all the nodes with appropriate change of index or indices. Similarly, v_i is the value passed up edge i, etc.. Show the sequence of steps, by giving the ab values on the way down, and the v values on the way up.

   (Note ab_i implies (α_i, β_i). All v_{xyz} (leaf/terminal nodes, depth 3) are implicitly passed up after an ab_{xyz} unless a pruning was specified)

   - $ab = (-\infty, \infty)$
   - $ab_1 = (-\infty, \infty)$
   - $ab_{11} = (-\infty, \infty)$
   - $ab_{111} = (-\infty, \infty)$
   - $ab_{112} = (5, \infty)$
   - $v_{11} = 7$
   - $ab_{12} = (-\infty, 7)$
   - $ab_{121} = (-\infty, 7)$
   - $ab_{122} = (-4, 7)$
   - $v_{12} = -3$
   - $ab_{13} = (-\infty, -3)$

- $ab_{131} = (-\infty, -3)$
- If going to the next step $ab_{132} = (8, -3)$ thus ab_{132} is pruned
- $v_{13} = 8$
- $v_1 = -3$
- $ab_2 = (-3, \infty)$
- $ab_{21} = (-3, \infty)$
- $ab_{211} = (-3, \infty)$
- $ab_{212} = (-3, \infty)$
- $v_{21} = -1$
- $ab_{22} = (-3, -1)$
- $ab_{221} = (-3, -1)$
- $ab_{222} = (-3, -1)$
- $v_{22} = 2$
- $ab_{23} = (-3, -1)$
- $ab_{231} = (-3, -1)$
- If going to the next step $ab_{232} = (3, -1)$ thus ab_{232} is pruned
- $v_{23} = 3$
- $v_2 = -1$
- $ab_3 = (-1, \infty)$
- $ab_{31} = (-1, \infty)$
- $ab_{311} = (-1, \infty)$
- $ab_{312} = (-1, \infty)$
- $v_{31} = -5$
- If going to the next step $ab_{32} = (-1, -5)$ thus the rest of $ab_{3...}$ (the unvisited child nodes of node 3) are pruned
- $v_3 = -5$
- $v = -1$