CS 6300 **HW03: Expectimax and Probability** **Ryan Dalby** **Due February 8, 2022**

Please use LaTeX to produce your writeups. See the Homework Assignments page on the class website for details.

1 Expectimax

Bob has found an unfair coin and an unfair 4-sided die. The coin comes up heads twice as frequently as it comes up tails. The die on the other hand comes up even twice as often as it comes up odd. I.e. $P(H) = \frac{2}{3}$, $P(T) = \frac{1}{3}$, $P(\text{x even}) = \frac{1}{3}$, and $P(\text{x odd}) = \frac{1}{6}$.

After a little thought Bob decides that he can devise a game which he can never loose in hopes that he can trick his friend Tom out of soda. Bob tells Tom that Tom can win $10 by playing. The game proceeds as follows.

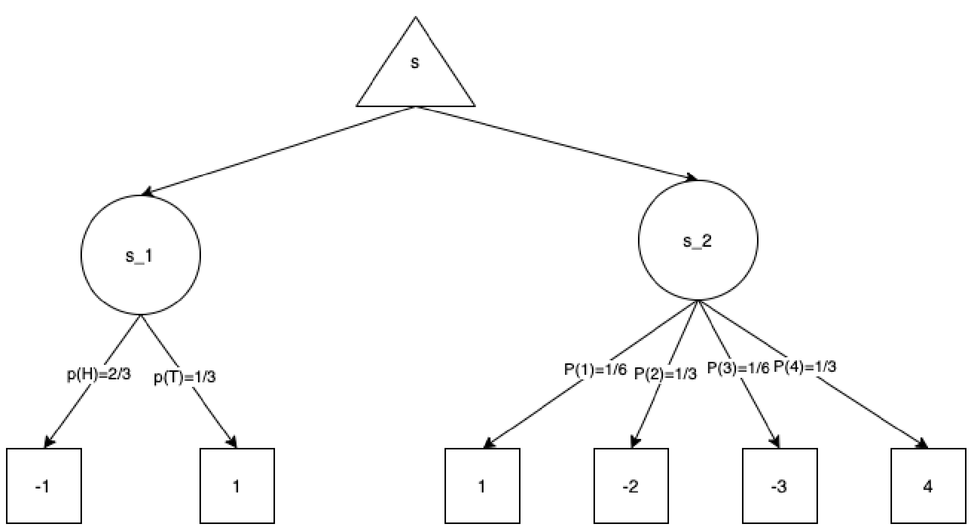
Tom makes the first move. He can either toss the coin or permit Bob to roll the die. The outcome of the game is -1 if the coin toss results in heads and 1 if it comes up tails. Otherwise it's the value of the die, except that it's -2 and -3 for those values.

In this game Tom wins if the outcome of the game is a positive number and looses if it's a negative number. **Note:** The outcome should never be 0.

Assumptions (problem wording is a bit ambiguous):

- The dice outcome is -2 if 2 is rolled and -3 if 3 is rolled.
- Since there really is just one choice, made by Tom, in this game everything is "relative" to Tom's perspective (Tom's goal would be to try to get a positive number outcome).

1. Draw the game tree for this game. Don't skip any layers (i.e. include chance nodes even where the outcome is guaranteed to happen).



2. Determine the value for each node in the tree and give the probability for each edge out of a chance node. Show your work.

   For probabilities see diagram above.

   $s_1 = (\frac{2}{3})(-1) + (\frac{1}{3})(1) = -\frac{1}{3}$

   $s_2 = (\frac{1}{6})(1) + (\frac{1}{3})(-2) + (\frac{1}{6})(-3) + (\frac{1}{3})(4) = \frac{1}{3}$

   $s = \frac{1}{3}$ (assuming the top node representing Tom's choice is a max node where he will rationally choose the maximizing option to win him the game (since there is a positive expectation from the dice game he can win over time))

3. Clearly Bob did not succeed at making a game he could not loose. Suggest a non-trivial change to the outcomes that would correct this issue and explain why it works. The solution will be simple but changing all values to positive numbers would be trivial.

   Played over a long-run Bob could simply assign -1 to if a 1 is rolled, 2 if a 2 is rolled, 3 if a 3 is rolled, and -4 if a 4 is rolled. This would result in $s_2 = (\frac{1}{6})(-1)+(\frac{1}{3})(2)+(\frac{1}{6})(3)+(\frac{1}{3})(-4) = -\frac{1}{3}$ meaning that Tom's maximizing choice would result in Bob winning no matter if Tom chooses the coin flip or Bob rolling the dice over time.

2 Probability

Sometimes, there is traffic (cars) on the freeway (C=+c). This could either be because of a ball game (B=+b) or because of an accident (A=+a). Consider the following joint probability table P(A,B,C) for the domain.

A	B	C	P(A,B,C)
+a	+b	+c	0.018
+a	+b	-c	0.002
+a	-b	+c	0.126
+a	-b	-c	0.054
-a	+b	+c	0.064
-a	+b	-c	0.016
-a	-b	+c	0.072
-a	-b	-c	0.648

1. What is the distribution $P(A, B)$?

A	B	P(A,B)
+a	+b	0.020
+a	-b	0.180
-a	+b	0.080
-a	-b	0.720

2. Are A and B independent in this model given no evidence?

A	P(A)
+a	0.200
-a	0.800

B	P(B)
+b	0.100
-b	0.900

By defintion of independence $P(X,Y) = P(X)P(Y) \quad \forall x, y; P(x,y) = P(x)P(y)$:

$$P(+a,+b) = 0.020 = P(+a)P(+b) = (0.200)(0.100) = 0.020$$

$$P(+a,-b) = 0.180 = P(+a)P(-b) = (0.200)(0.900) = 0.180$$

$$P(-a,+b) = 0.080 = P(-a)P(+b) = (0.800)(0.100) = 0.0.080$$

$$P(-a,-b) = 0.720 = P(-a)P(-b) = (0.800)(0.900) = 0.720$$

Thus A and B are independent given no evidence.

3. What is $P(A| + c)$?

A	C	P(A,C)
+a	+c	0.144
+a	-c	0.056
-a	+c	0.136
-a	-c	0.664

$$P(A|+c) = \alpha P(A,+c) \quad \text{where} \quad \alpha = \frac{1}{(0.144 + 0.136)}$$

A	$P(A,+c)$
+a	0.514
-a	0.486

4. What is $P(A|+b,+c)$?

$$P(A|+b,+c) = \alpha P(A,+b,+c) \quad \text{where} \quad \alpha = \frac{1}{(0.018 + 0.064)}$$

A	$P(A|+b,+c)$
+a	0.220
-a	0.780