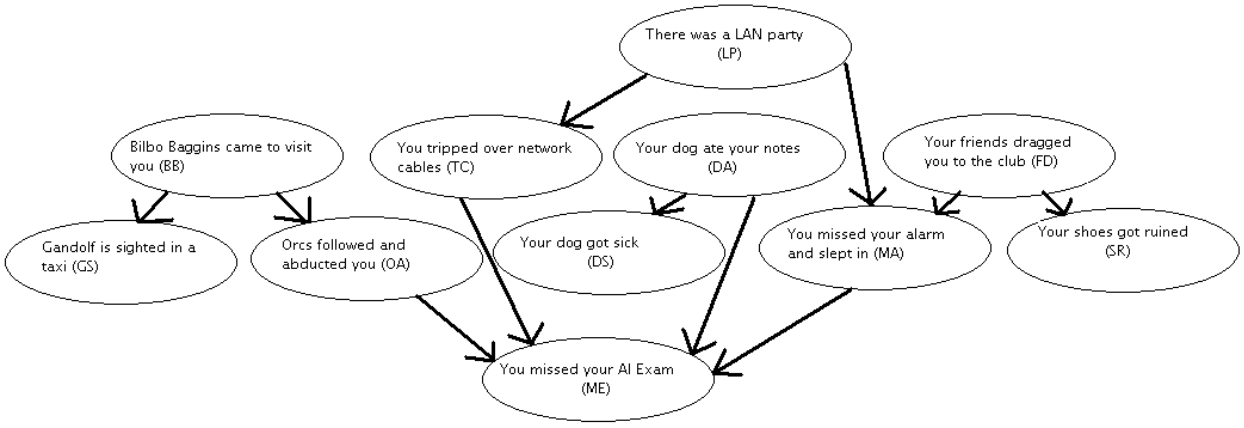
CS 6300 **HW07: Bayes Nets I** **Ryan Dalby** **April 5, 2022**

1 D-Separation

You missed the Artificial Intelligence Exam! When you try to explain to your professor your excuses for missing the exam, he stops you. He offers to give you a grade based on your performance of mapping your excuses and determining the D-separation of the excuses and the events associated with them. You draw out the events as a Bayes net seen above. You must now answer the following questions about the net to see what your grade for the exam you missed!



Consider the following pairs of variables to answer the questions below:

(a) GS and OA ; (b) DA and MA ; (c) LP and FD ; (d) TC and MA ; (e) BB and ME
Given the stated evidence, list the variables that are independent or conditionally independent.

1. (4 points) No information (even the evidence that you missed the exam).
   (b) and (c) are independent.

   a. Active:

Path	Triple	Inactive Triple?
GS,BB,OA	BB,GS,OA	No

   b. Inactive, independent:

Path	Triple	Inactive Triple?
DA,ME,MA	DA,MA,ME	Yes
DA,ME,TC,LP,MA	DA,TC,ME	Yes

   c. Inactive, independent:

Path	Triple	Inactive Triple?
LP,MA,FD	LP,FD,MA	Yes
LP,TC,ME,MA,FD	LP,TC,ME	No
	TC,MA,ME	Yes

d. Active:

Path	Triple	Inactive Triple?
TC,LP,MA	LP,TC,MA	No

e. Active:

Path	Triple	Inactive Triple?
BB,OA,ME	BB,OA,ME	No

2. (4 points) MA is observed.

(b) is independent given the evidence.

a. Active:

Path	Triple	Inactive Triple?
GS,BB,OA	BB,GS,OA	No

b. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
DA,ME,MA	DA,MA,ME	Yes
DA,ME,TC,LP,MA	DA,TC,ME	Yes

c. Active:

Path	Triple	Inactive Triple?
LP,MA,FD	LP,FD,MA	No

d. Active:

Path	Triple	Inactive Triple?
TC,LP,MA	LP,TC,MA	No

e. Active:

Path	Triple	Inactive Triple?
BB,OA,ME	BB,OA,ME	No

3. (4 points) BB is observed.

(a), (b), and (c) are independent given the evidence.

a. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
GS,BB,OA	BB,GS,OA	Yes

b. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
DA,ME,MA	DA,MA,ME	Yes
DA,ME,TC,LP,MA	DA,TC,ME	Yes

c. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
LP,MA,FD	LP,FD,MA	Yes
LP,TC,ME,MA,FD	LP,TC,ME	No
	TC,MA,ME	Yes

d. Active:

Path	Triple	Inactive Triple?
TC,LP,MA	LP,TC,MA	No

e. Active:

Path	Triple	Inactive Triple?
BB,OA,ME	BB,OA,ME	No

4. (4 points) LP is observed.

   (b), (c), and (d) are independent given the evidence.

   a. Active:

Path	Triple	Inactive Triple?
GS,BB,OA	BB,GS,OA	No

   b. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
DA,ME,MA	DA,MA,ME	Yes
DA,ME,TC,LP,MA	DA,TC,ME	Yes

   c. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
LP,MA,FD	LP,FD,MA	Yes
LP,TC,ME,MA,FD	LP,TC,ME	No
	TC,MA,ME	Yes

   d. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
TC,LP,MA	LP,TC,MA	Yes
TC,ME,MA	TC,MA,ME	Yes

   e. Active:

Path	Triple	Inactive Triple?
BB,OA,ME	BB,OA,ME	No

5. (4 points) TC is observed.

   (b) and (c) are independent given the evidence.

a. Active:

Path	Triple	Inactive Triple?
GS,BB,OA	BB,GS,OA	No

b. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
DA,ME,MA	DA,MA,ME	Yes
DA,ME,TC,LP,MA	DA,TC,ME	Yes

c. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
LP,MA,FD	LP,FD,MA	Yes
LP,TC,ME,MA,FD	LP,TC,ME	Yes

d. Active:

Path	Triple	Inactive Triple?
TC,LP,MA	LP,TC,MA	No

e. Active:

Path	Triple	Inactive Triple?
BB,OA,ME	BB,OA,ME	No

6. (4 points) LP and OA are observed.

(b), (c), (d), and (e) are independent given the evidence.

a. Active:

Path	Triple	Inactive Triple?
GS,BB,OA	BB,GS,OA	No

b. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
DA,ME,MA	DA,MA,ME	Yes
DA,ME,TC,LP,MA	DA,TC,ME	Yes

c. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
LP,MA,FD	LP,FD,MA	Yes
LP,TC,ME,MA,FD	LP,TC,ME	No
	TC,MA,ME	Yes

d. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
TC,LP,MA	LP,TC,MA	Yes
TC,ME,MA	TC,MA,ME	Yes

e. Inactive, independent given the evidence:

Path	Triple	Inactive Triple?
BB,OA,ME	BB,OA,ME	Yes

7. (4 points) ME is observed.

   None can be said to be independent given the evidence.

   a. Active:

Path	Triple	Inactive Triple?
GS,BB,OA	BB,GS,OA	No

   b. Active:

Path	Triple	Inactive Triple?
DA,ME,MA	DA,MA,ME	No

   c. Active:

Path	Triple	Inactive Triple?
LP,MA,FD	LP,FD,MA (...ME)	No

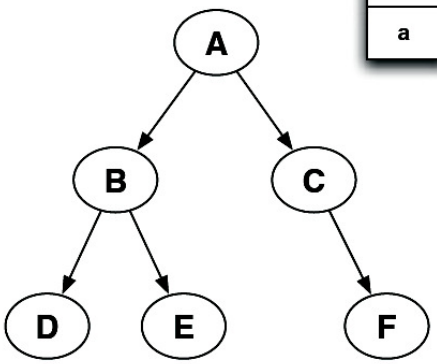
   d. Active:

Path	Triple	Inactive Triple?
TC,LP,MA	LP,TC,MA	No

   e. Active:

Path	Triple	Inactive Triple?
BB,OA,ME	BB,OA,ME	No

2 Inference by Enumeration



A	P(A)
a	0.5

F	C	P(FIC)
f	c	0.5
f	~c	0.8

C	A	P(CIA)
c	a	0.9
c	~a	0.7

D	B	P(DIB)
d	b	0.7
d	~b	0.2

E	B	P(EIB)
e	b	0.6
e	~b	0.2

B	A	P(BIA)
b	a	0.8
b	~a	0.5

Note: for this problem I am assuming all variables X can take on only two values of x or $\sim x$.

1. What is the expression for $P(A, b, C, \sim d, E, f)$ given the structure of this Bayes' net and conditional probability tables?

   $P(A, B, C, D, E, F) = P(A)P(B|A)P(C|A)P(D|B)P(E|B)P(F|C)$

   $P(A, b, C, \sim d, E, f) = P(A)P(b|A)P(C|A)P(\sim d|b)P(E|b)P(f|C)$

2. Form the joint distribution $P(A, b, C, \sim d, E, f)$ using factors. Please give the details of your derivation.

   $f_1(A) = P(A)$

   $f_2(A, B) = f_2(A) = P(b|A)$

   $f_3(A, C) = P(C|A)$

   $f_4(B, D) = f_4 = P(\sim d|b) = 0.3$

   $f_5(B, E) = f_5(E) = P(E|b)$

   $f_6(C, F) = f_6(C) = P(f|C)$

   $P(A, b, C, \sim d, E, f) = f_1(A) \times f_2(A) \times f_3(A, C) \times f_4 \times f_5(E) \times f_6(C)$

   Joining on A:

   $f_7(A, C) = f_1(A) \times f_2(A) \times f_3(A, C)$

A	C	$f_7(A, C)$
a	c	$(0.5)(0.8)(0.9) = 0.36$
a	$\sim c$	$(0.5)(0.8)(0.1) = 0.04$
$\sim a$	c	$(0.5)(0.5)(0.7) = 0.175$
$\sim a$	$\sim c$	$(0.5)(0.5)(0.3) = 0.075$

Joining on C:

$f_8(A, C) = f_7(A, C) \times f_6(C)$

A	C	$f_8(A, C)$
a	c	$(0.36)(0.5) = 0.18$
a	$\sim c$	$(0.04)(0.8) = 0.032$
$\sim a$	c	$(0.175)(0.5) = 0.0875$
$\sim a$	$\sim c$	$(0.075)(0.8) = 0.06$

Then finally forming the factor that is equivalent to the desired expression:

$P(A, b, C, \sim d, E, f) = f_9(A, C, E) = f_8(A, C) \times f_4 \times f_5(E) = f_8(A, C) \times (0.3) \times f_5(E)$

A	C	E	$P(A, b, C, \sim d, E, f)$
a	c	e	$(0.18)(0.3)(0.6) = 0.0324$
a	c	$\sim e$	$(0.18)(0.3)(0.4) = 0.0216$
a	$\sim c$	e	$(0.032)(0.3)(0.6) = 0.00576$
a	$\sim c$	$\sim e$	$(0.032)(0.3)(0.4) = 0.00384$
$\sim a$	c	e	$(0.0875)(0.3)(0.6) = 0.01575$
$\sim a$	c	$\sim e$	$(0.0875)(0.3)(0.4) = 0.0105$
$\sim a$	$\sim c$	e	$(0.06)(0.3)(0.6) = 0.0072$
$\sim a$	$\sim c$	$\sim e$	$(0.06)(0.3)(0.4) = 0.0108$

3. Solve for the query $P(C|b, \sim d, f)$.

$$P(C|b, \sim d, f) = \frac{P(C, b, \sim d, f)}{P(b, \sim d, f)} = \alpha P(C, b, \sim d, f)$$

where α is a constant

$$P(C, b, \sim d, f) = \sum_A \sum_E P(A, b, C, \sim d, E, f)$$

A	C	$P(A, b, C, \sim d, E, f)$
a	c	0.054
a	$\sim c$	0.0096
$\sim a$	c	0.02625
$\sim a$	$\sim c$	0.018

$$P(C, b, \sim d, f) = \sum_{A} P(A, b, C, \sim d, f)$$

C	$P(C, b, \sim d, f)$
c	0.08025
$\sim c$	0.0276

Now using the laws of probability, $\alpha = \frac{1}{0.08025+0.0276} = 9.272$

C	$P(C\|b, \sim d, f)$
c	0.744
$\sim c$	0.256