

Exercise 1

$$v(t) = \sqrt{\frac{gm}{cd}} \tanh\left(\sqrt{\frac{gcm}{m}} t\right)$$

$$x(10s) - x(0s) = \int_0^{10} \sqrt{\frac{gm}{cd}} \tanh\left(\sqrt{\frac{gcm}{m}} t\right) dt$$

$$g = 9.81 \text{ m/s}^2$$

$$m = 68.1 \text{ kg}$$

$$cd = 0.25$$

$$\sqrt{\frac{gm}{cd}} \cdot \frac{1}{\sqrt{\frac{gcm}{m}}} \cdot \ln\left[\cosh\left(\sqrt{\frac{gcm}{m}} t\right)\right] \Big|_0^{10}$$

$$51.694 \cdot 5.269 \cdot \ln[\cosh(1.18986)] \Big|_0^{10}$$

$$272.4 [\ln[\cosh(1.1898)] - \ln[\cosh[0]]]$$

$$272.4 [\ln[3.41627214] - \ln[1]]$$

$$\boxed{334.178 \text{ m}}$$

Python 3.6.5 |Anaconda, Inc.| (default, Mar 29 2018, 13:32:41) [MSC v.1900 64 bit (AMD64)]
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IPython 6.4.0 -- An enhanced Interactive Python.

```
In [1]: runfile('C:/Users/hoops/OneDrive/Documents/School/ME EN 2450 Numerical Methods/  
HW6/HW6b.py', wdir='C:/Users/hoops/OneDrive/Documents/School/ME EN 2450 Numerical Methods/  
HW6')
```

Part 1:

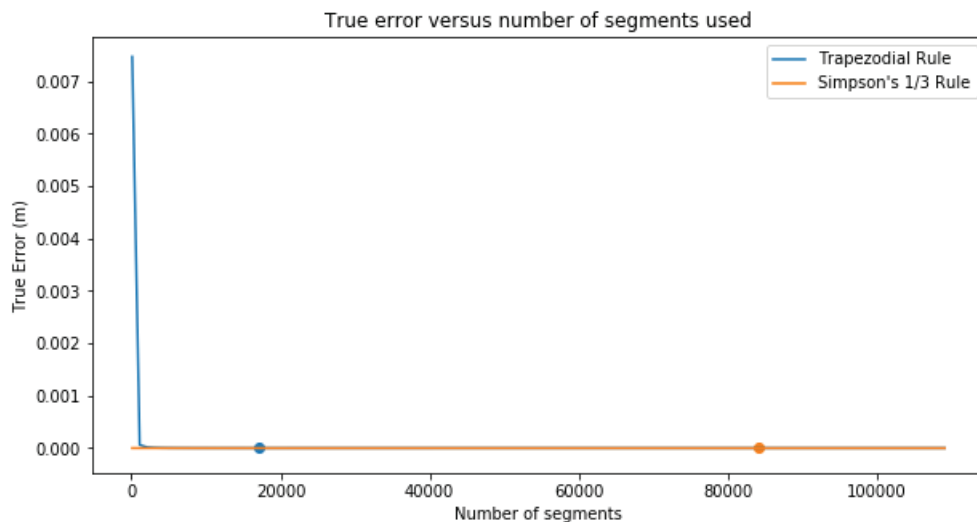
Using the Trapezoidal Rule Integration with 10 segments the distance fallen after 10s = 333.42982m

Using the Simpson's 1/3 Rule with 10 segments distance the fallen after 10s = 334.18282m

Part 2:

The optimum number of segments using the Trapezoidal Rule for this problem is 17100

The optimum number of segments using Simpson's 1/3 Rule for this problem is 84100



```
In [2]:
```

```

# -*- coding: utf-8 -*-
"""
Created on Sun Apr 7 22:43:06 2019
HW6b
@author: Ryan Dalby
"""
import numpy as np
import matplotlib.pyplot as plt

def trapezoidalRuleIntegration(func, xi, xf, numSegments):
    """
    Given a function f(x) and starting integration value (xi) and ending integration value (xf)
    along with the number of segments to apply the trapezoidal rule will take the definite integral between the
    """
    h = (xf-xi)/numSegments
    x = xi #current x
    runningSum = func(x) #first segment
    x += h
    #now loop over middle segments
    for i in range(numSegments-1):
        runningSum += 2 * func(x)
        x += h
    runningSum += func(x) #final segment
    I = (h/2.0) * runningSum #integral value
    return I

def simpsonsOneThirdRuleIntegration(func, xi, xf, numSegments):
    """
    Given a function f(x) and starting integration value (xi) and ending integration value (xf)
    along with the number of segments (must be even number) to apply simpson's 1/3 rule to take
    the definite integral between the designated values.
    """
    if(np.mod(numSegments, 2) == 1): #if total number of segments is odd then can't use method
        raise Exception("Number of segments must be even")
    h = (xf-xi)/numSegments
    x = xi #current x
    runningSum = func(x) #first segment
    x += h
    #now loop over middle segments
    for i in range(numSegments-1):
        if (np.mod((i+1), 2) == 1 : #Segment is odd
            runningSum += 4 * func(x)
        else: #segment is even
            runningSum += 2 * func(x)
        x += h
    runningSum += func(x) #final segment
    I = (h/3.0) * runningSum #integral value
    return I

g = 9.81 #m/s^2
m = 68.1 #kg
cd = 0.25 #kg/m
vFunc = lambda t: np.sqrt((g*m)/cd) * np.tanh(np.sqrt((g * cd)/m) * t)

#Part 1
print("Part 1:")
print("Using the Trapezoidal Rule Integration with 10 segments the distance fallen after 10s = {:.5f}m".format

```

```
print("Using the Simpson's 1/3 Rule with 10 segments distance the fallen after 10s = {:.5f}m".format(simpsonsOneThirdRuleIntegration(vFunc,0,10,10)))
print()
```

```
#Part 2
```

```
print("Part 2:")
```

```
iterVals = np.arange(100, 110000, 1000)
```

```
trapError = []
```

```
simpError = []
```

```
trueVal = 334.178167 #analytical solution
```

```
for i in iterVals: #get true error for each segment size
```

```
    trapError.append(np.abs(trueVal - trapezoidalRuleIntegration(vFunc,0,10, i)))
```

```
    simpError.append(np.abs(trueVal - simpsonsOneThirdRuleIntegration(vFunc,0,10, i)))
```

```
trapMinIndex = iterVals[np.argmin(trapError)]
```

```
trapMin = np.min(trapError)
```

```
simpMinIndex = iterVals[np.argmin(simpError)]
```

```
simpMin = np.min(simpError)
```

```
print("The optimum number of segments using the Trapezodial Rule for this problem is {}".format(trapMinIndex))
```

```
print("The optimum number of segments using Simpson's 1/3 Rule for this problem is {}".format(simpMinIndex))
```

```
fig, ax1 = plt.subplots(figsize = (10,5))
```

```
ax1.plot(iterVals, trapError, label = "Trapezodial Rule")
```

```
ax1.plot(iterVals, simpError, label = "Simpson's 1/3 Rule")
```

```
ax1.scatter(trapMinIndex, trapMin)
```

```
ax1.scatter(simpMinIndex, simpMin)
```

```
ax1.set_title("True error versus number of segments used")
```

```
ax1.set_xlabel("Number of segments")
```

```
ax1.set_ylabel("True Error (m)")
```

```
ax1.legend()
```