ME 2450 Assignment 05

Name: ____________________

Due: March 28, 2019 before midnight

Collaborators: ____________________

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I declare that the assignment here submitted is original except for source material explicitly acknowledged.

I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the University website.

Name	Date
Signature	Student ID

Score

Exercise Graded:	
Presentation:	/2
Technical Content:	/8
Total:	/10

Background

Heat Transfer in a Thin Rod

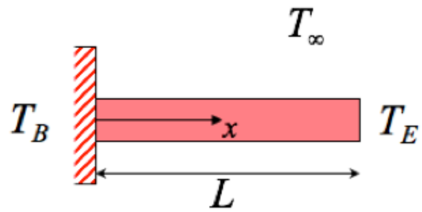

Figure 1: Schematic of heat transfer in a thin rod

In class, we have discussed a model of heat transfer in a thin rod (Figure **??**). This might represent the cooling effect of a heatsink in a computer, or the cooling of a rod in a nuclear reactor. In the case shown, the representative differential equation is

$$\frac{\partial^2 T}{\partial x^2} = \frac{hP}{\kappa A}(T - T_\infty) \tag{1}$$

where the parameters are defined as follows:

- h is the heat transfer coefficient (in W/m˙K)
- κ is the thermal conductivity of the fin material (in W/m $\cdot$ K)
- P is the perimeter of the cross-sectional area of the fin (in m)
- A is the cross-sectional area of the fin (in m^2)
- T_∞ is the ambient temperature (in K)

In the situation where one end of the fin is open to the air, we would not have a constraint holding the temperature of the end of the fin (T_E in Figure **??**) at a constant value. However, we might have a goal temperature that we want it to achieve.

Numerical Methods

Shooting Method

The Shooting Method (Section 27.1.1 of the Textbook) is a way to solve boundary value problems, which are systems of ordinary differential equations (ODE) where we do not have an initial condition for every dependent variable. Instead, we have some initial conditions and some other (usually final) conditions. The shooting method may be summarized in a few steps:

- Choose an initial condition for each dependent variable that is missing one.
- Run one of our ODE solvers (i.e., Euler, Heun, Runge Kutta 4) on the system to find a set of values at the other boundary.
- Choose a second, different, initial condition for each dependent variable that is missing one.

- Run the ODE solver on the system again to find a new set of values at the other boundary.
- Interpolate between the first and second initial condition(s) to find a new estimate for the initial condition(s).
- Run the ODE solver on the system again to determine whether this new estimate is close enough. If it is not, return to step 1 and repeat until satisfied.

Finite Difference Method

As discussed in lecture, the finite difference method involves replacing derivatives in a differential equation with finite difference approximations. For this assignment, you will use this method and one of your linear algebra methods to solve for temperatures in a cooling fin.

Exercises

In this assignment, you will perform the shooting method in two different ways:

- by hand to solve for the temperature profile in a cooling fin with a very coarse grid using a few iterations of the fourth-order Runge-Kutta method; and
- with your own Runge-Kutta function to perform the shooting method on a model of a cooling fin with a much finer grid.

First, you will apply the method to a coarse grid and solve the resulting system of equations by hand. Then, you will apply the method to a model of a cooling fin with a much finer grid and use one of your MATLAB or Python functions to solve the resulting system of equations. Use the parameters from the shooting method section for these problems.

For all exercises, use the following parameters

$h = 20\text{W/m}^2 \cdot \text{K}$	$\kappa = 200\text{W/m} \cdot \text{K}$	
$L = 2.0\text{m}$	$D = 0.1\text{m}$	
$T_\infty = 300\text{K}$	$T_B = 600\text{K}$	$T_E = 350\text{K}$

Exercise 1

Decompose the second-order differential in (**??**) into a system of first-order ODE.

Exercise 2: Shooting Method

Solve the ODE in Exercise 1 by hand. Divide the rod into a grid with 4 nodes, where one node lies at the left edge and one lies at the right edge. Determine the temperature profile of the cooling fin at these points using the shooting method and the fourth-order Runge-Kutta method. Follow the steps outlined in the shooting method section above. Lecture notes as well as the textbook examples 25.10 and 27.1 may also be helpful guides.

This exercise can be time consuming.

Exercise 3: Shooting Method

Divide the rod into a grid with 51 points. Write a script called `find_temperature_profile_shooting` that determines the temperature profile of the cooling fin at these points using the shooting method with a `runge_kutta_four` function that you write. A function called `cooling_fin` is available on Canvas which implements the system of equations derived in Exercise 1. This function can be used in your script as the `ode_fcn` input parameter to `runge_kutta_four`. Since (**??**) is linear, this script will not need to loop. Plot the temperature distribution (i.e., T vs x).

Exercise 4: Finite Difference Method

Similar to Exercise 2, divide the rod into a grid but this time use 5 nodes. Derive the system of equations to solve for the temperatures at these nodes using the finite difference method. Refer to the lecture notes for help. Solve these equations by hand using Naïve Gauss Elimination.

Exercise 5: Finite Difference Method

Now, divide the rod into a grid with 51 nodes, as in Exercise 3. Derive the system of equations to solve for the temperatures at these nodes using the finite difference method (you should be able to show what this system would look like without writing out the entire system of equations by hand). Write a script called `find_temperature_profile_finite` that determines the temperature profile of the cooling fin at these nodes using the finite difference method. In other words, it should set up the system linear algebraic equations and solve it. Your script should use one of the three methods we have developed in assignments and labs (Naïve Gauss elimination, LU decomposition, or Gauss-Seidel). Plot the temperature distribution (i.e., T vs x).