ME 2450 Assignment 1

Name: _____

Due: January 17th, 2019

Collaborators: _____

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I declare that the assignment here submitted is original except for source material explicitly acknowledged.

I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the University website.

Name	Date
Signature	Student ID

Score

Exercise Graded:	
Presentation:	/2
Technical Content:	/8
Total:	/10

Exercise 1

A cylindrical storage tank, with cross-sectional area A, contains liquid at a depth y, defined such that $y = 0$ when the tank is half full. Liquid is withdrawn from the tank at a volume flow rate of $Q_{\text{out}} = \alpha (1+y)^{3/2}$, which depends on the depth of water in the tank. At the same time, liquid is replenished at a volume flow rate of $Q_{\text{in}} = 2Q \sin^2(t)$. The differential equation describing the rate of change of the depth of the water in the tank is

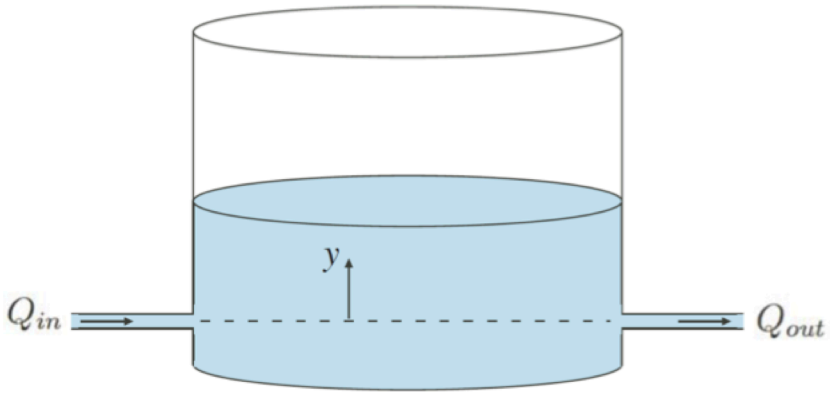
$$\frac{\mathrm{d}y}{\mathrm{d}t} = 2\frac{Q}{A}\sin^2(t) - \frac{\alpha (1+y)^{3/2}}{A}.$$

Consider parameter values of $A = 850\ m^2$, $Q = 325\ m^3/s$, and $\alpha = 200\ m^{3/2}/s$. Assume that the initial condition is $y(t=0) = 2\ m$.

It is difficult to obtain an analytical solution in this case. Instead, we can obtain an *approximate* solution using Euler's method as follows

$$y_{i+1} = y_i + \left(2\frac{Q}{A}\sin^2(t_i) - \frac{\alpha (1+y_i)^{3/2}}{A} \right) h,$$

where y_i is the water level at the current time t_i and y_{i+1} is the predicted water level at the next time step.



a) Write out by hand an estimate of the water level y at time $t = 1.5$ sec using Euler's method with a step size of $h = 0.5$ sec. (2 pts)

b) Implement Euler's method in a *fully commented* computer code to solve for the depth of water as a function of time from $t = 0$ to 10 sec with a step size of 0.5 sec. (Print out your code and attach it to your assignment.) (2 pts)

c) Plot the solution (y versus t) for $0 \leq t \leq 10$. Be sure to correctly label the axes. (2 pts)

d) Please explain, in your own words and reasoning, why you think your numerical approximation is accurate. For this, you can try verifying against an analytical solution, for example, or simply providing a reasoning of the general form of the result, i.e. increasing/decreasing, etc. (2 pts)