Flow around a Circular Cylinder

Thermal, Fluids, and Energy
Systems Lab

(ME EN 4650)

Prof. M Metzger
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University of Utah

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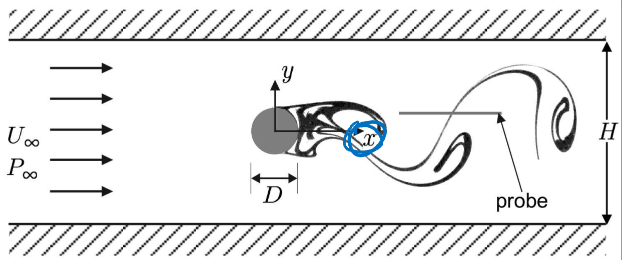
Flow around Circular Cylinder at High Re

What we want to know:

- drag coefficient versus Reynolds number
- static pressure around object

$Re_D \approx 1 \times 10^4$



$D = 0.75\,\mathrm{in},\ L = 12\,\mathrm{in}$

What we can measure:

- Streamwise velocity (Pitot-static probe & pressure transducer)
- Static pressure on cylinder (tap & pressure transducer)

ΔP

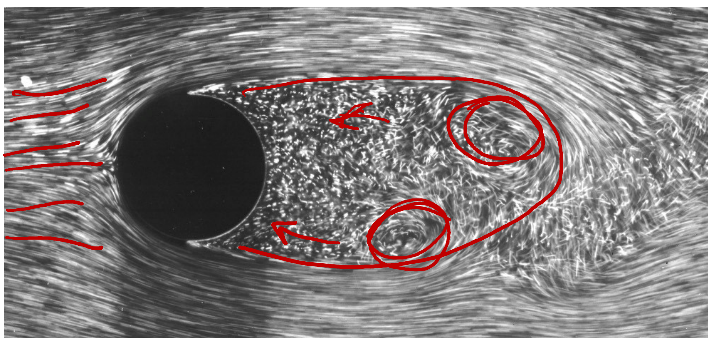
Lecture Outline

- Background (Flow Visualization) $Re_D \approx 10,000$
- Experimental Setup
- Measurements and Instrumentation
- Required Figures
- Data Analysis (Uncertainty Analysis)

Flow around Circular Cylinder at High Re

steady

flow

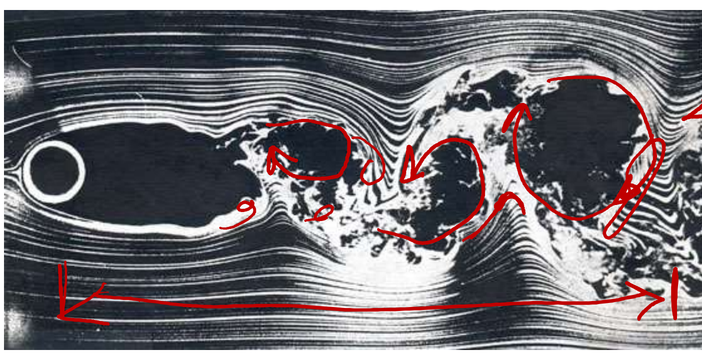


vortices

$Re_D = 2,000$

unsteady

flow



$Re_D = 10,000$

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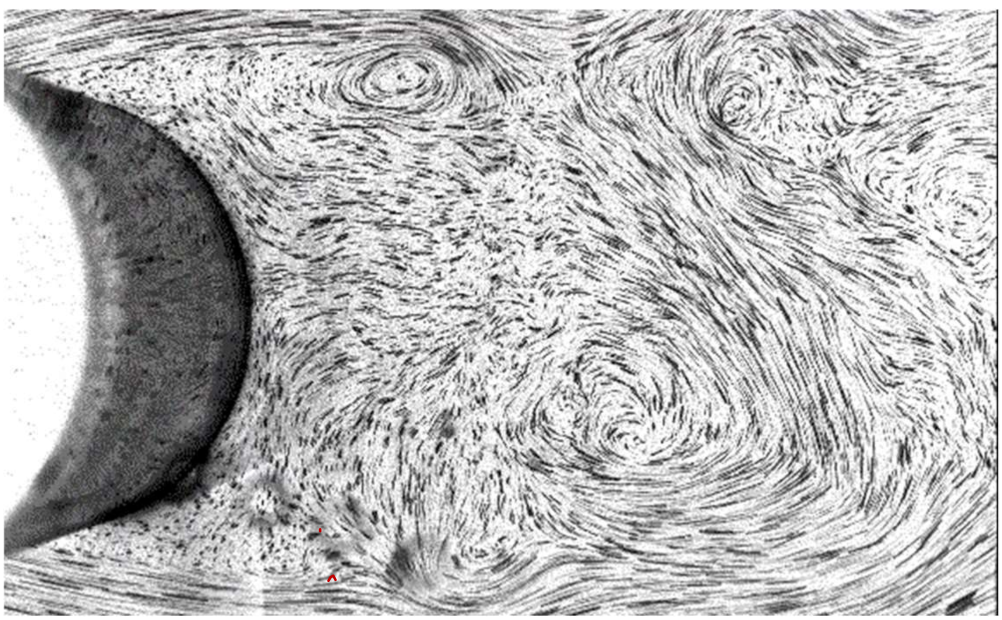
Experimental Measurement of Turbulent Flow in the Wake

Particle Image Velocimetry (PIV)



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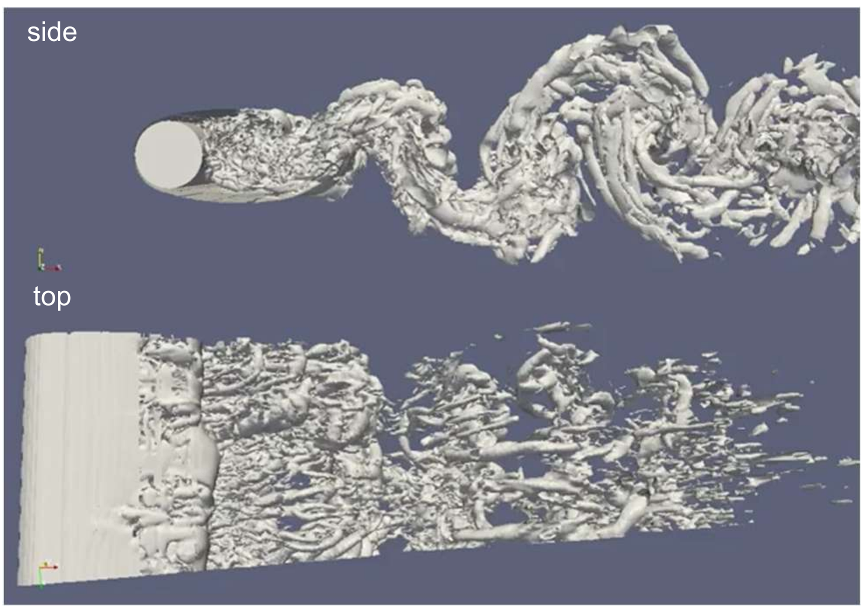
Direct Numerical Simulation of Flow around Circular Cylinder

flow



flow

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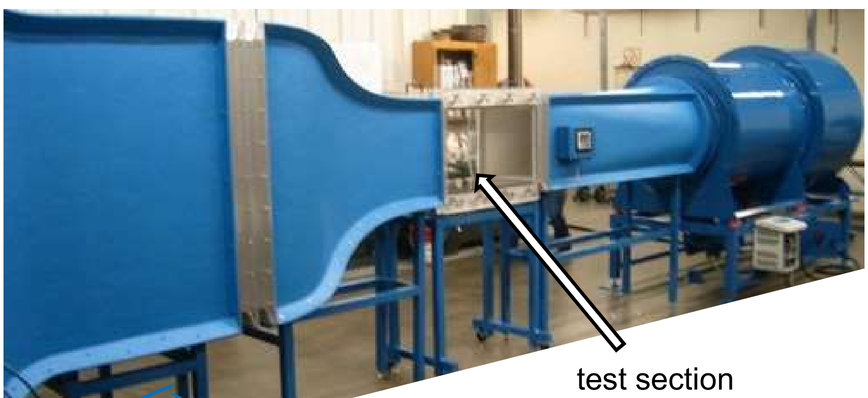
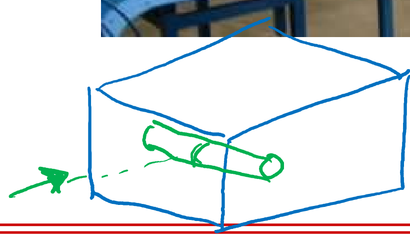
Experimental Setup: Wind Tunnel



test section

$12'' \times 12'' \times 24''$



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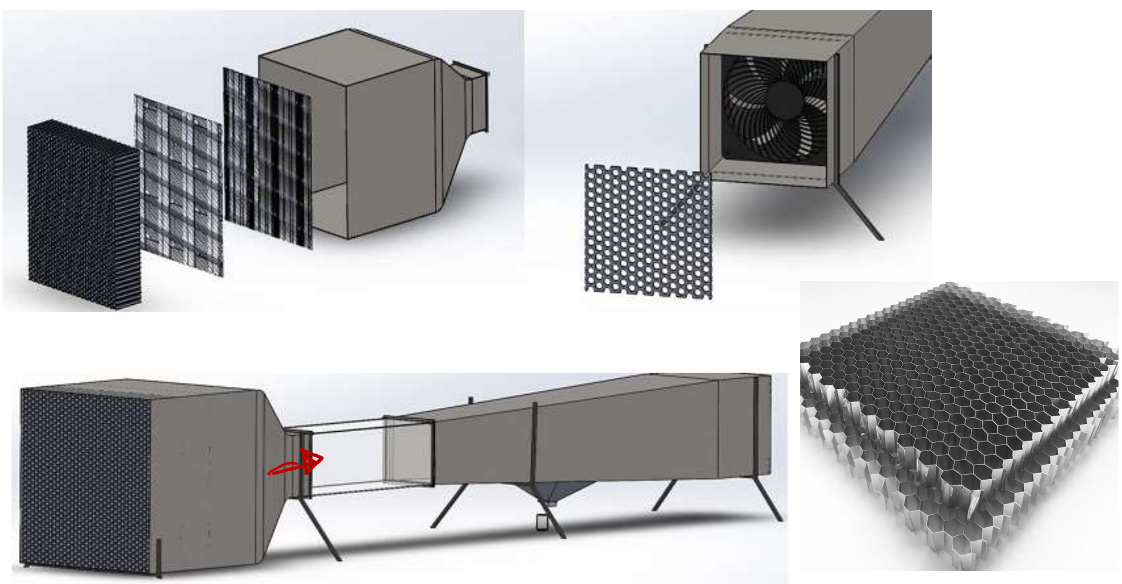
Experimental Setup: Wind Tunnel



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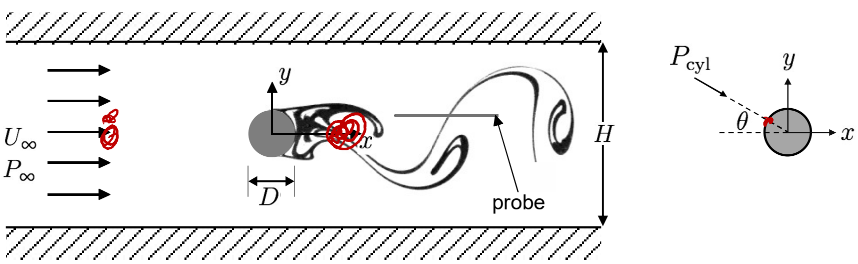
Measurements



Quantity	Symbol	Units	Instrument
Static *vacuum* pressure around cylinder	$P_{\mathrm{atm}} - P_{\mathrm{cyl}}$	mm Hg	tap on cylinder surface
Dynamic pressure in wake & freestream	ΔP	mm Hg	Pitot-static probe
Static *vacuum* pressure in wake & freestream	$P_{\mathrm{atm}} - P_\infty$	mm Hg	Pitot-static probe (static holes only)

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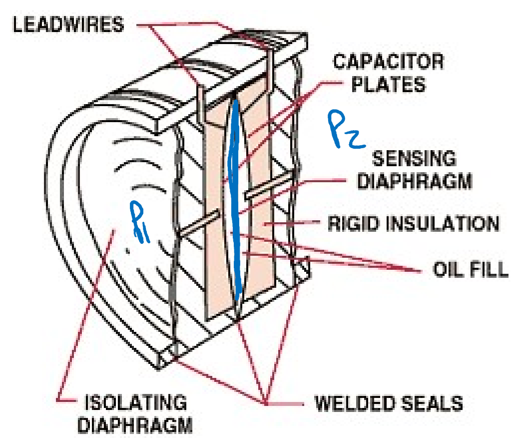
Instrumentation: Pressure Transducer

Differential Pressure: Capacitive Pressure Sensor



~ 100 Hz

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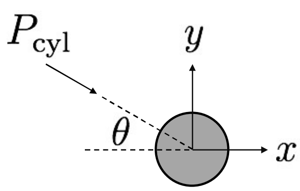
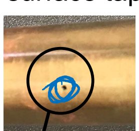
Instrumentation: Pressure Tap

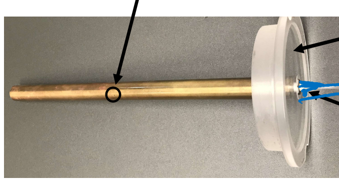


Quantity	Symbol	Units	Instrument
Static *vacuum* pressure around cylinder	$P_{\text{atm}} - P_{\text{cyl}}$	mm Hg	tap on cylinder surface

output = $P_{atm} - P_{cyl}$ (mmHg)

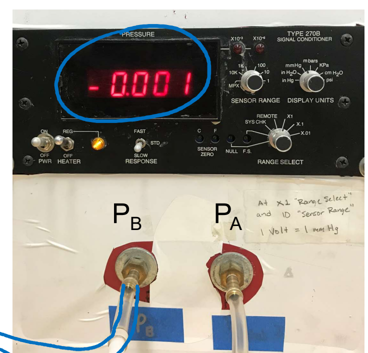
surface tap





mounting plug

hose barb fitting



P_{cyl} open (P_{atm})

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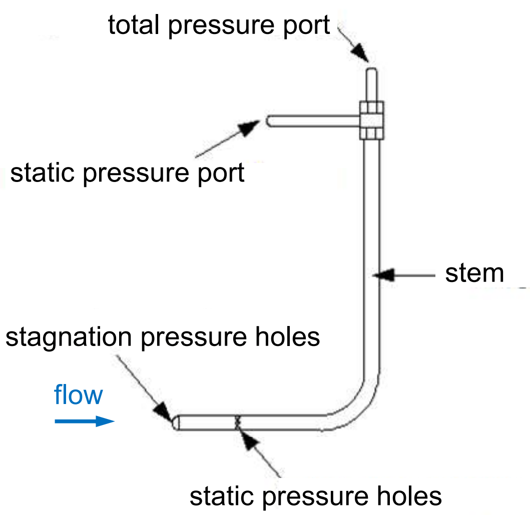
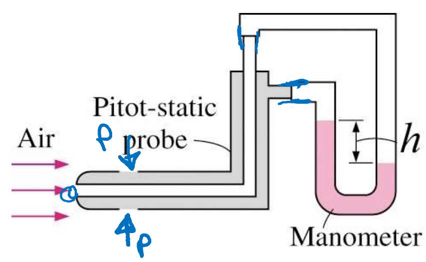
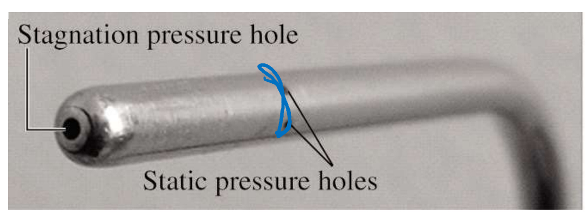
Instrumentation: Pitot-Static Probe







Pitot-Static Probe: Bernoulli's Equation

Quantity	Symbol	Units	Instrument
Dynamic pressure in wake & freestream	ΔP	mm Hg	Pitot-static probe

upper: $\frac{1}{2}\rho u^2 + P = \frac{1}{2}\rho u^2 + P_2$

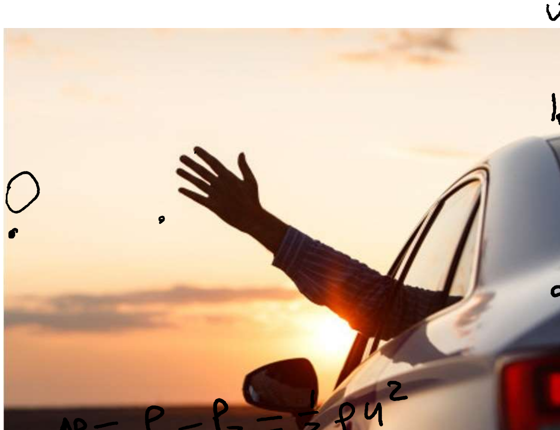
$P = P_2$

lower: $\frac{1}{2}\rho u^2 + P = \frac{1}{2}\rho \cancel{u_1^2} + P_1$

$\cancel{u_1^2} = 0$

$$\underbrace{\underbrace{\frac{1}{2}\rho u^2}_{\text{dynamic pressure}} + \underbrace{P}_{\text{static pressure}}}_{\text{total pressure}} = P_1$$

(stagnation pressure)



$\Delta P = P_1 - P_2 = \frac{1}{2}\rho u^2$

$$\boxed{u = \sqrt{2\Delta P/\rho}}$$

ΔP : Pa ← mmHg → Pa

ρ : kg/m^3

u : m/s

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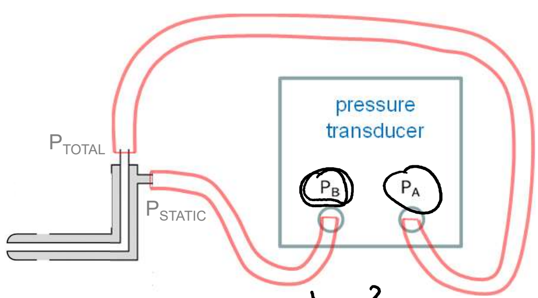
Pitot-Static Probe: Connection to Pressure Transducer

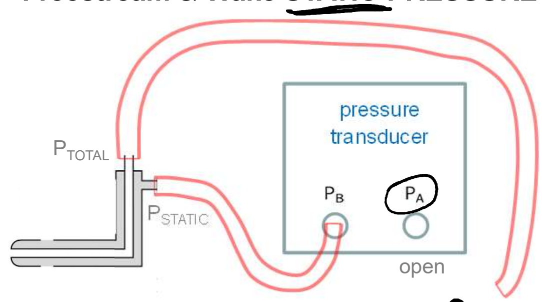
Quantity	Symbol	Units	Instrument
Dynamic pressure in wake & freestream	ΔP	mm Hg	Pitot-static probe
Static *vacuum* pressure in wake & freestream	$P_{\text{atm}} - p$	mm Hg	Pitot-static probe (static holes only)

Freestream & Wake VELOCITY



$$\text{output} = \Delta P = \frac{1}{2}\rho u^2$$

Freestream & Wake STATIC PRESSURE



$$\text{output} = P_{atm} - P$$

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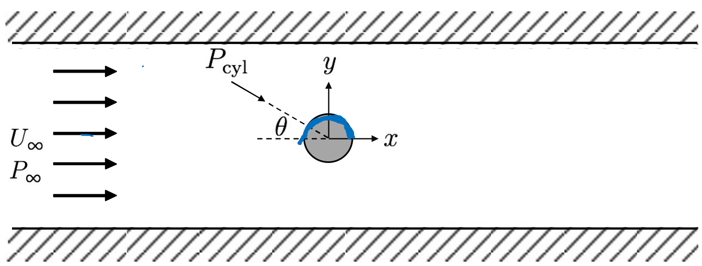
Experiments



pitot-static probe

Experiment 1: V_∞ & P_∞ (5 y-locations)
15 sec @ 100 Hz

Experiment 2: u & p (~20 y-locations)

cylinder tap

Experiment 3: P_{cyl} (~20 angles)

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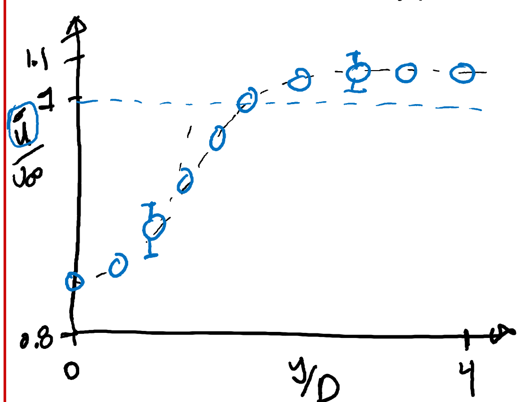
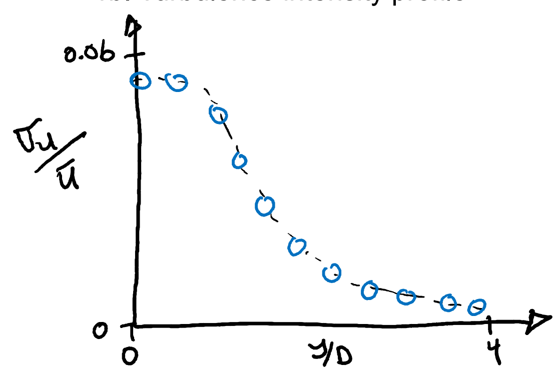
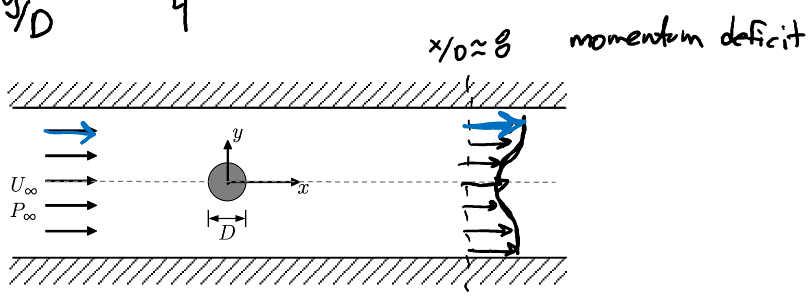
Figures 1a & 1b

1a. Mean streamwise velocity profile



1b. Turbulence intensity profile





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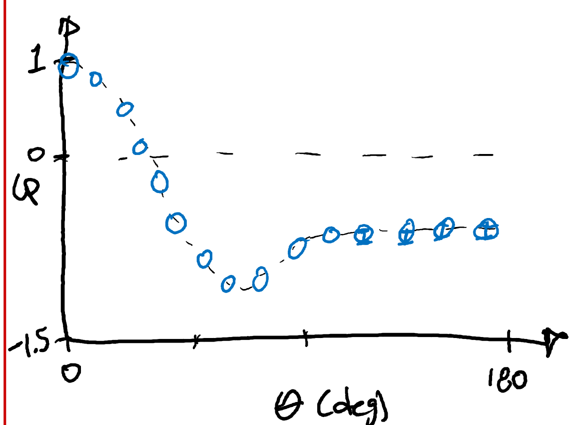
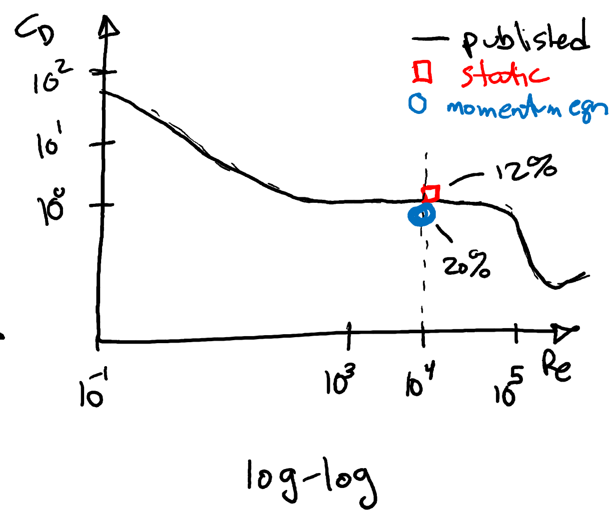
Figures 1c & 1d

1c. Coefficient of pressure versus angle



1d. Drag coefficient versus Reynolds number



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Flow around a Circular Cylinder: Part II

Thermal, Fluids, and Energy Systems Lab

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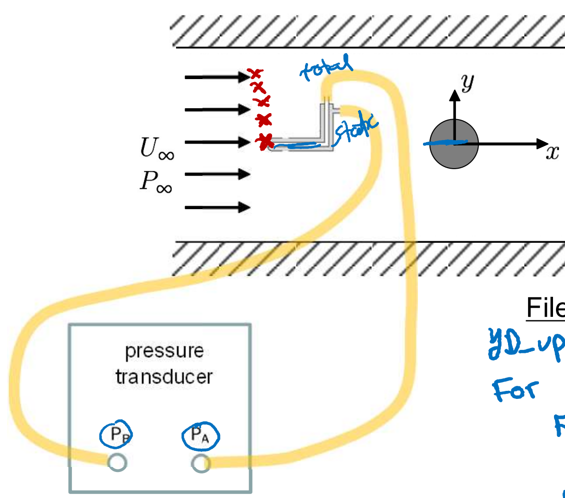
Quantities of Interest (Required Figures)

- $\overline{u}/U_\infty$ vs y/D (with errorbars) → wake profile
- $\sigma_u/\overline{u}$ vs y/D → turbulence intensity
- C_p vs θ (with errorbars) → coefficient of pressure
- C_D vs Re_D → drag coefficient
  - published results over range $10^{-1} \leq Re_D \leq 10^5$
  - conservation of momentum method at $Re_D \approx 10^4$
  - integration of static pressure around cylinder at $Re_D \approx 10^4$

Post-Processing the Data

Freestream Velocity



air density: $\rho = \frac{P_{atm}}{RT}$ (ideal gas law)

Pconvert=133.3224;

output: $\Delta P = P_A - P_B$

$$\underbrace{\Delta P = \frac{1}{2}\rho U_\infty^2}_{\text{dynamic pressure}}$$

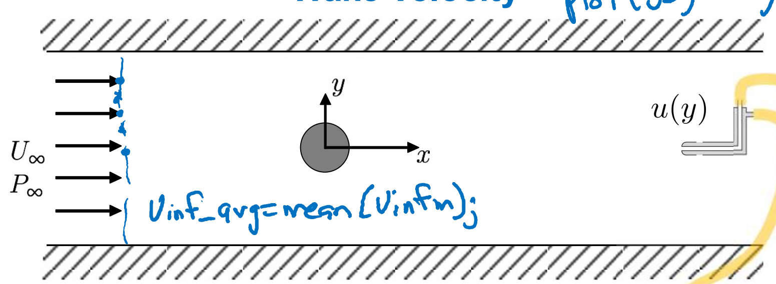
Filename: `Uinf_yD0.csv` (contains ΔP) mmHg

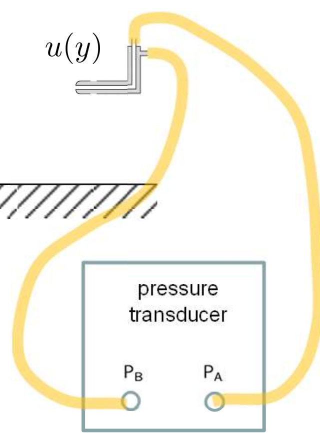
```
yD_up=[0,1,2,3,4];
For (j=1 : length(yD_up))
  Filename=['Uinf_yD', num2str(yD_up(j)),
           '.csv'];
  Delta_P= readmatrix(Filename);
  Delta_P= Delta_P * Pconvert;
  Uinf= sqrt(2*Delta_P/rho);
  Uinfm(j) = mean(Uinf);
end
```

Post-Processing the Data

Wake Velocity

plot(yD, ustd./umean, 'bo')





n=50;

Filename: `Uwake_yD0.csv` (contains ΔP) mmHg

```
yD=[0,0.1, . . . . , 4];
For (j=1 : length(yD))
    :
    u=sqrt(2* Delta P / rho);
  → umean(j) = mean(u);
  → ustd(j) = std(u);
    uerr(j)= 2* ustd(j) / √n ;
end
errorbar(yD, u/Vinf_avg, uerr/Vinf_avg, 'bo')
```

output: $\Delta P = P_A - P_B$

$$\Delta P = \frac{1}{2}\rho u^2$$

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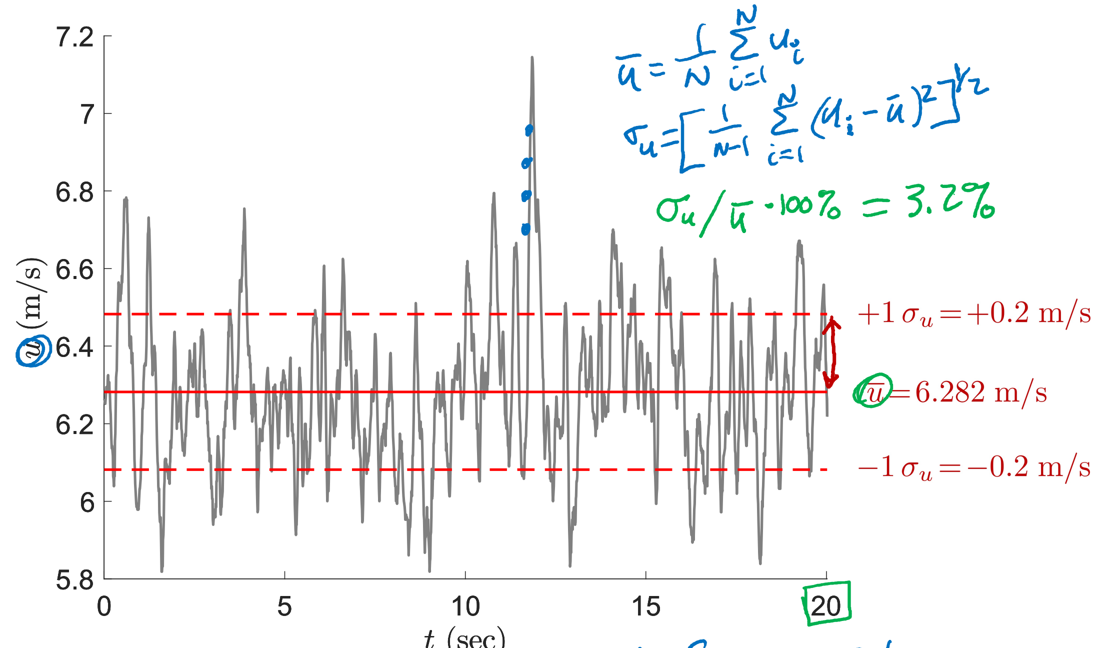
Velocity Time Series in Wake



$\bar{u} = \frac{1}{N}\sum_{i=1}^{N} u_i$

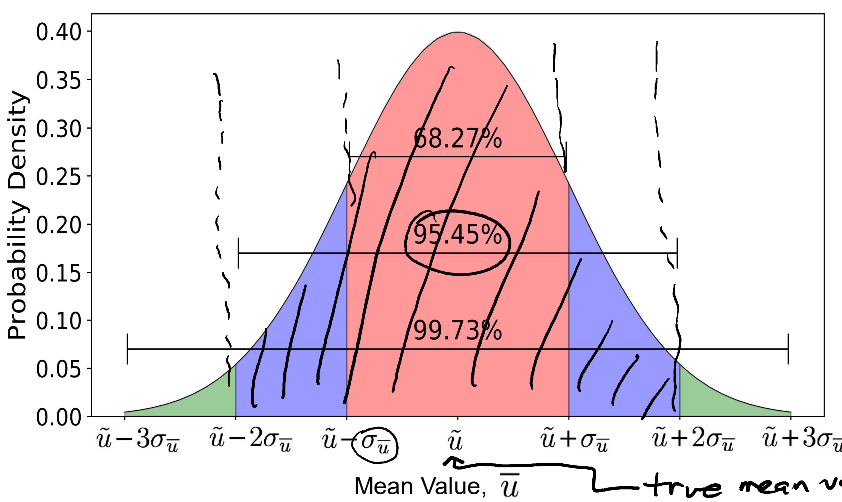
$\sigma_u = \left[\frac{1}{N-1}\sum_{i=1}^{N}(u_i - \bar{u})^2\right]^{1/2}$

$\sigma_u / \bar{u} \cdot 100\% = 3.2\%$

$f = 100$ Hz (100 samples/s), $N = f \cdot 20 = 2000$ pts

Central Limit Theorem

Probability of the mean value of any random process follows a Gaussian Distribution



$\overline{u} \pm 2\sigma_{\overline{u}}$

95% confidence interval

true mean value

Standard error: $\sigma_{\overline{u}} = \frac{\sigma_u}{\sqrt{n}}$

# of independent samples

$= 50$

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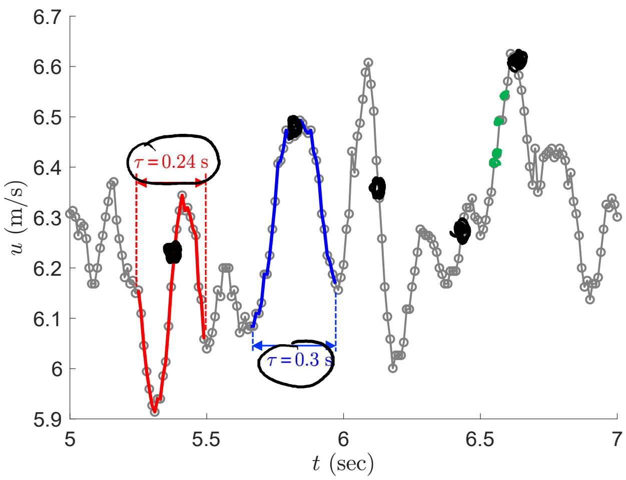
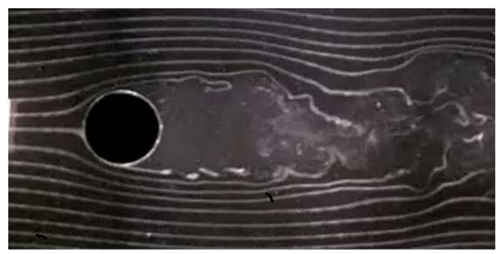
Behavior of Turbulence





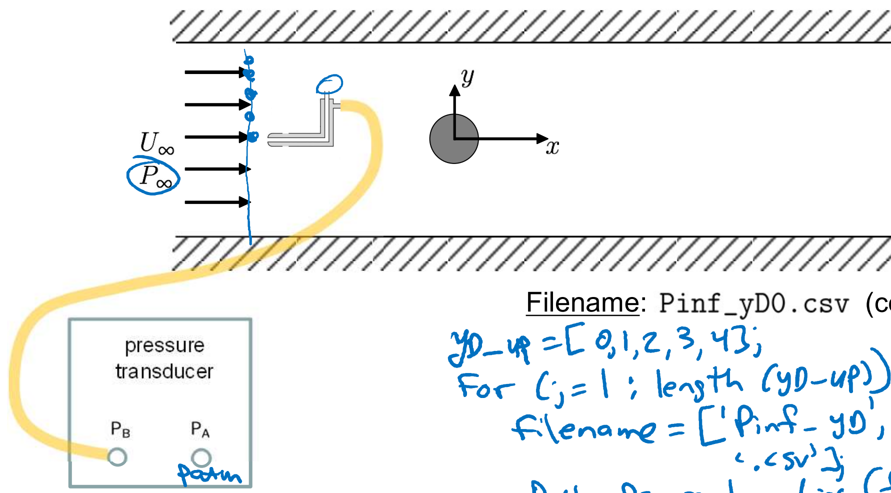
$f_c = \frac{1}{\tau}$

correlation timescale $\approx 0.3s$

$15s/0.3s = 50$ independent samples

Post-Processing the Data

Freestream Static Pressure



Filename: `Pinf_yD0.csv` (contains ΔP) mmHg

output: $\Delta P = P_A - P_B$

$\Delta P = P_{atm} - P_\infty$

```
yD_up = [0,1,2,3,4];
For (j=1 : length (yD_up))
    filename = ['Pinf_yD', num2str(yD_up(j)),
                '.csv'];
    Delta_P = readmatrix (filename);
    Pinf(j) = mean((Patm - Delta_P)*Pconvert);
end
Pinf_avg = mean(Pinf);
```

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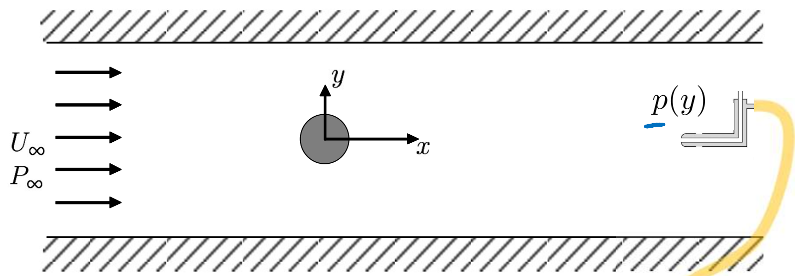
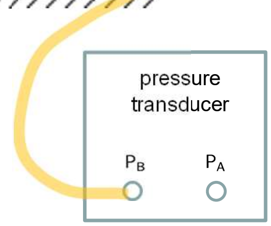
Post-Processing the Data

Wake Static Pressure



Filename: `Pwake_yD0.csv` (contains ΔP)



FOR
;
p = (Patm - Delta_P) * Pconvert
pmean(j) = mean(p);
pstd(j) = std(p);
end
perr = 2 * pstd / sqrt(n)
=50

output: $\Delta P = P_A - P_B$

$\Delta P = P_{\mathrm{atm}} - p$

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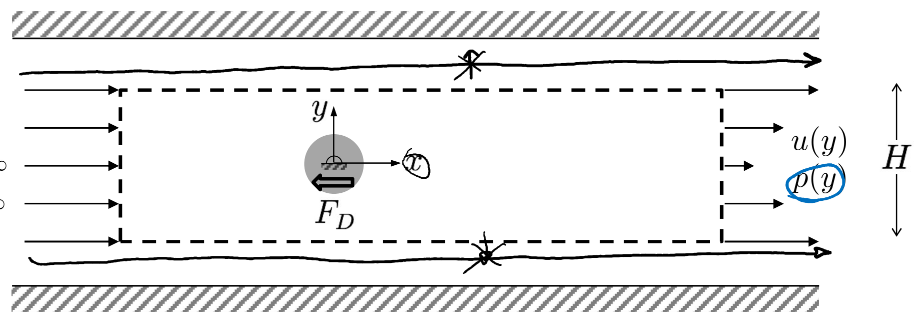
Drag Force: Control Volume Analysis

Conservation of x-momentum



$$F_D = 2\underbrace{\int_{y=0}^{H/2} P_\infty(y)\,dy}_{\mathrm{I}} - 2\underbrace{\int_{y=0}^{H/2} p(y)\,dy}_{\mathrm{II}} + 2\rho\underbrace{\int_{y=0}^{H/2} U_\infty^2(y)\,dy}_{\mathrm{III}} - 2\rho\underbrace{\int_{y=0}^{H/2} u(y)^2\,dy}_{\mathrm{IV}}$$

```
T1 = 2*trapz(yup, Pinf);    yup = yD-vp*D
T2 = 2*trapz(y, pmean);
T3 = 2*rho*trapz(yup, Uinf.^2);
T4 = 2*rho*trapz(y, umean.^2);
```

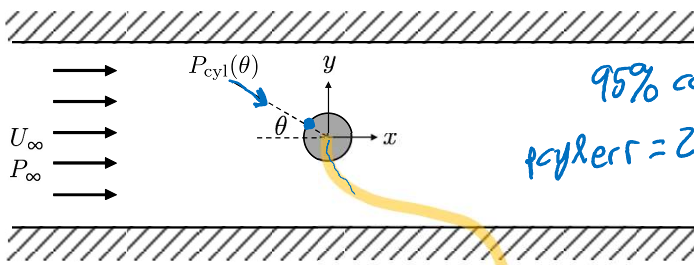
$F_D = T1 - T2 + T3 - T4$

$C_D = F_D / (\frac{1}{2}\rho U_\infty^2 D L)$

Uinf_avg

Post-Processing the Data

Cylinder Static Pressure



95% confidence

pcylerr = 2*Pcylstd/sqrt(n)

9

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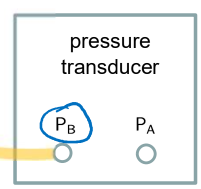
<u>Filename</u>: `Pcyl_deg0.csv` (contains ΔP) mmHg




```
theta=[0,5,....,180];
For (j=1:length(theta)).
  Filename=['Pcyl_deg', num2str(theta(j)),
            '.csv'];
  Delta_P= readmatrix(Filename);
  Pcyl= (Patm- Delta_P)* Pconvert;
  Pcylmean(j)= mean(Pcyl);
  Pcylstd(j)= std(Pcyl);
end
```

$$\text{output}:\ \Delta P = P_A - P_B$$

$$\Delta P = P_{\rm atm} - P_{\rm cyl}$$

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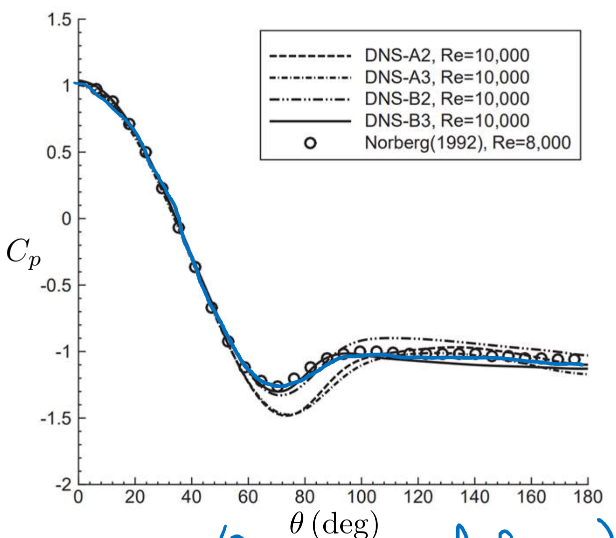
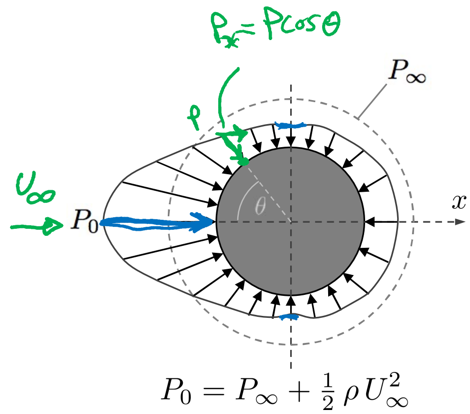
Pressure Distribution





C_p = (Pcyl mean - Pinf_avg)/(0.5*rho*Vinf_avg^2)

$$C_D = \int_{\theta=0}^{\pi} C_p(\theta)\cos\theta\, d\theta \Rightarrow \text{trapz(theta, Cp.*cos(theta))}$$

radians!

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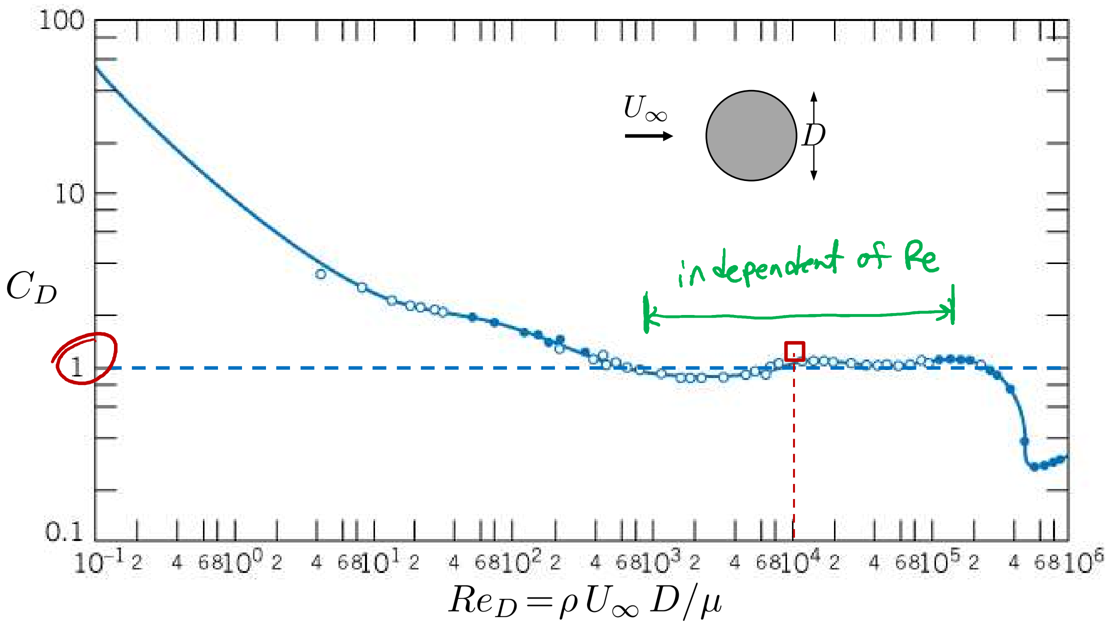
Drag Coefficient vs. Reynolds Number



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Questions??

Thank you for your attention!

Let me or the TAs know if you have questions

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