TFES Lab (ME EN 4650)
Shell and Tube Heat Exchanger

Textbook Reference: Sections 11.1 – 11.4, 9.1 – 9.3, 9.6.3, from Incropera, DeWitt, Bergman, and Lavine, 6th ed., John Wiley & Sons

Objectives

(i) examine the performance of a shell and tube heat exchanger for a range of flow rates,

(ii) compare effectiveness values obtained from the experimental measurements to those predicted by the Effectiveness-NTU theory, and

(iii) estimate the heat lost from the shell casing to the surroundings via convection and radiation.

Background

Heat exchangers are devices that facilitate the exchange of heat between two fluids at different temperatures that are separated by a solid surface, and not permitted to mix. Shell and tube heat exchangers are the most widely used type of heat exchanger. They can handle a wide range of allowable design pressures and temperatures, from full vacuum to 41 MPa (6000 psig) and from cryogenic temperatures near absolute zero to 1100°C (2000°F). Thus, this class of heat exchanger is the most versatile and finds application in the process, power, petrochemical, and refrigeration industries.

Figure 1 illustrates one of the simplest types of shell and tube heat exchangers, namely the single-tube-pass, counterflow type. In this configuration, several straight tubes are bundled together and placed within a larger shell. Baffles inside the shell are used to support the tube bundle and direct the flow over the tubes, thereby creating turbulence and inducing a crossflow velocity component. This, in turn, promotes higher heat transfer rates between the

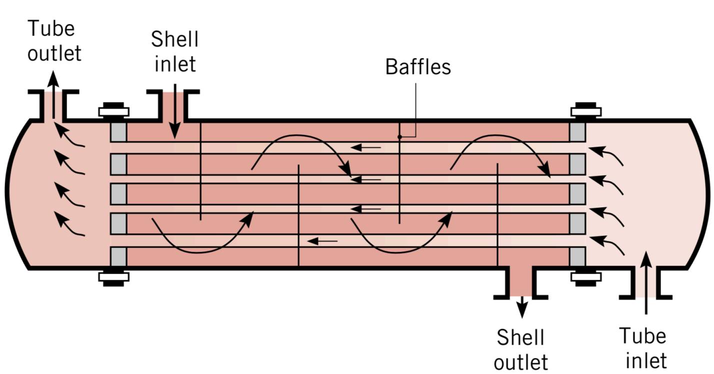

Figure 1. Single-tube pass, counterflow, shell and tube heat exchanger showing the direction of flow through the tubes and shell.

shell-side fluid and tube-side fluid that serves to increase the overall heat transfer coefficient. In the counterflow configuration, the tube-side fluid enters the heat exchanger at one end; while, the shell-side fluid enters at the opposite end. There is usually one inlet and one outlet for both the shell-side and tube-side fluids. A plenum is typically utilized on the tube-side to facilitate the equal distribution of flow through the tubes at the inlet, and through the fitting at the outlet. A variety of other types of arrangements are possible, such as multiple shell passes and even multiple (two, four, etc.) tube passes. However, the present experiment will focus on the single-tube-pass counterflow type.

The temperature distributions expected in a counterflow heat exchanger are shown in Figure 2. The nomenclature used is such that T_h and T_c denote the temperatures of the hot-side and cold-side fluid, respectively. Furthermore, we use the subscripts o and i to indicate the inlet and outlet. For example, $T_{h,o}$ represents the temperature of the hot-side fluid at the outlet. As illustrated in Figure 2, the temperatures of the two fluids change continuously with x distance along the heat exchanger. At any x location, we can write the energy balance on a differential element as

$$dq_h = -\dot{m}_h\, c_{p,h}\, dT_h \qquad \text{and} \qquad dq_c = +\dot{m}_c\, c_{p,c}\, dT_c\,, \tag{1}$$

where $\dot{m}$ is the mass flow rate and c_p is the specific heat at constant pressure. The negative sign in (1) for dq_h is because heat is leaving the control volume on the hot side; whereas dq_c

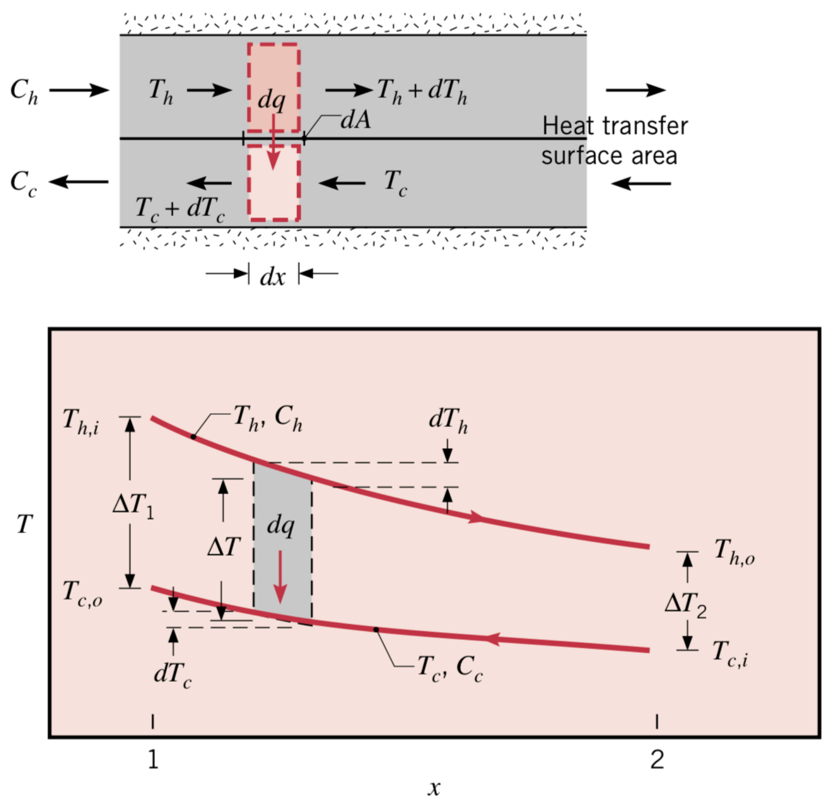

Figure 2. Temperature distributions in a counterflow heat exchanger. In this scenario, the cold-side fluid enters at $x = 2$; while, the hot-side fluid enters at $x = 1$.

Table 1. List of measurements acquired in the experiment with their native units.

Quantity	Symbol	Units	Instrument
Temperature of hot-side	$T_{h,i}$, $T_{h,o}$	°F	thermocouple
Temperature of cold-side	$T_{c,i}$, $T_{c,o}$	°F	thermocouple
Temperature of shell casing	T_{shell}	°F	thermocouple
Temperature of ambient air	T_{air}	°F	thermocouple
Volume flow rate	$\dot{V}_c$, $\dot{V}_h$	gpm	rotameter

is positive, because heat is entering the control volume on the cold side. The above equations may be integrated along x from the inlet to the outlet, in order to obtain the overall energy balances on both sides. Assuming constant mass flow rate and constant specific heat along the length of the heat exchanger, (1) can be written as

$$q_h = -\dot{m}_h \, c_{p,h} \, (T_{h,o} - T_{h,i}) = C_h \, (T_{h,i} - T_{h,o}) \tag{2}$$

and

$$q_c = \dot{m}_c \, c_{p,c} \, (T_{c,o} - T_{c,i}) = C_c \, (T_{c,o} - T_{c,i}) \tag{3}$$

where C $(= \dot{m} \, c_p)$ denotes the *heat capacity* of the fluid. Theoretically, we expect $q_h = q_c$. However, in reality, this will not be the case, because some heat is transfered to the surroundings through the shell casing.

Experimental Setup

In the lab, we are interested in examining how the mass flow rate of the hot-side and cold-side fluid ($\dot{m}_h$ and $\dot{m}_c$, respectively) impacts the overall heat transfer rate (q) and how effective the heat exchanger is in transferring heat from the hot-side to the cold-side. Figure 3 shows the shell and tube heat exchanger used in the experiment along with its relevant specifications. The heat exchanger is instrumented with T-type thermocouples, providing temperatures of both the hot-side and cold-side fluids at the inlets and outlets, as well as the shell casing temperature and ambient air temperature. The thermocouples are connected to a digital thermometer, which converts the thermocouple analog signal to a digital reading.

Cold water from the building domestic water supply is connected to the tube-side inlet. Cold water output from the tube-side is routed to the building water drain system. On the shell-side, hot water is supplied via a recirculating loop consisting of a hot water tank with electrical resistive heaters, a temperature controller, pump, and several valves. Inline rotameters provide a measure of the volumetric flow rate in gallons per minute (gpm) on both the hot-side and cold-side. Table 1 lists the measurements to be acquired during the experiment along with their native units. A schematic of the test apparatus is shown in Figure 4.

Specification	Value	Units
shell outside diameter	2.12	in
shell (and tube) length	9.0	in
baffle spacing	1.125	in
tube outside diameter	0.25	in
tube wall thickness	0.028	in
number of tubes	31	—
number tube passes	1	—
tube material	copper	—
shell material	brass	—

Figure 3. Photograph of the Young Radiator Company (model HF–201–HY–1P) shell and tube heat exchanger examined in the experiment along with its relevant specifications.

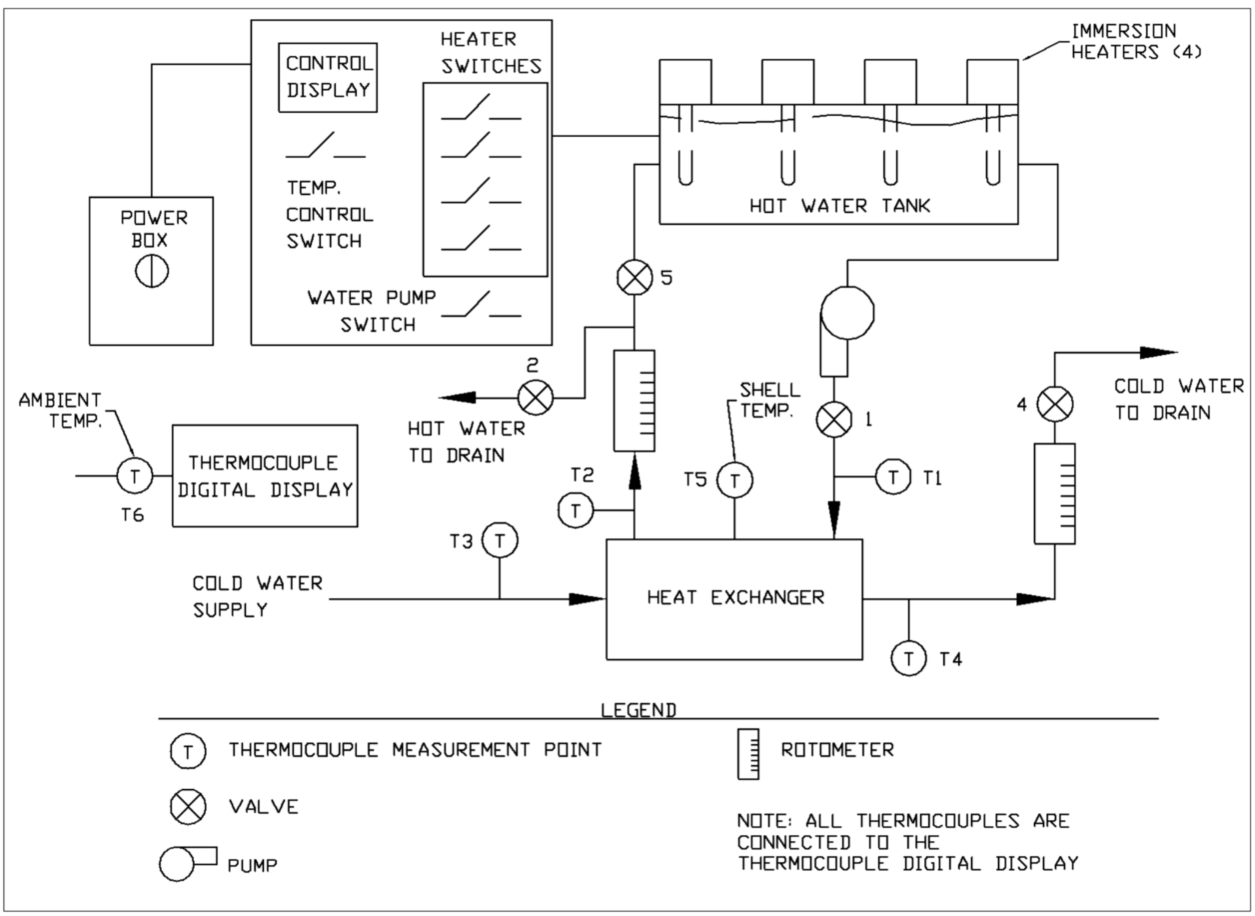

Figure 4. Schematic of the experimental apparatus showing the measurement locations.

Laboratory Procedure

1. Determine the flow direction of each fluid and select the fluid type (hot or cold) to route through the tube-side and shell-side.

2. Examine the water level in the vertical pipe connected to the hot-water tank. If instructed by your TA, add water to the tank until the water free surface is visible a few inches below the top of the pipe. Use valve V2 and the faucet for the hot-water supply line near the sink in the laboratory.

3. Your TA will direct you to open the faucet for the cold water supply line near the sink in the laboratory. Set the cold water control valve on the outlet side of the heat exchanger using valve V4 to about 5–6 gpm, as indicated by the rotameter (measure with respect to the top of the float). Always use the cold-water control valve to set and control the flow rate. Always maintain the cold-water flow rate at greater than 3 gpm. It is recommended that the combined flow rates for the hot and cold fluids be less than 10 gpm.

4. When instructed by your TA, complete the following:
    a. Open the hot-water control valve on the inlet side of the heat exchanger (valve V1).
    b. Open the hot-water control valve on the outlet side of the heat exchanger (near the hot- water tank inlet, valve V5).
    c. Turn on power to all equipment using the large red switch on the control panel.
    d. Turn on the power to the pump using the marked toggle switch.

5. Use the hot-water control valve (valve V5) downstream of the rotameter to control the flow rate. Begin with this valve fully open, producing the maximum possible flow rate.

6. When instructed by your TA, turn on the Temperature Controller. Set the desired temperature (indicated by SV — Set Value) as follows: Press the SV button. Press the up arrow key ($\wedge$) under the digit to be changed — digit should then be in a flashing mode. Use the up arrow ($\wedge$) and down arrow ($\vee$) keys to change the digit to the desired value. Press the Enter key — digit will stop flashing. Repeat this sequence for each digit to be changed. The hot water temperature should be set in the range from 90 – 130°F (32 – 54°C).

7. Turn on all 4 heaters using the individual toggle switches.

8. Monitor the outlet temperature from the hot-water tank using the Present Value (PV) display. Wait until steady state is reached (i.e., until PV has reached the set value SV on the display).

9. Use the digital thermometer with accompanying thermocouples to measure all system temperatures. Record all fluid temperatures, shell casing temperature, ambient air temperature, and flow rates for the heat exchanger.

10. Reduce the hot-water flow rate ($\dot{V}_h$) slightly using valve V5. Wait for a steady state condition. Record all temperatures and flow rates.
11. Make a note of the uncertainties in the flow rates. This will be based on the observed fluctuations of the float in the rotameter and the spacing between tick marks on the rotameter scale.
12. Keep the cold flow rate constant ($\dot{V}_c$) and reduce the hot flow rate ($\dot{V}_h$) by closing the hot-water control valve to at least its half-open position or less. Wait for a steady-state condition. Record all temperatures and flow rates.
13. Reduce the cold flow rate ($\dot{V}_c$) by approximately half, by partially closing the cold-water valve V4. Set the hot-water flow rate ($\dot{V}_h$) back to its maximum value by fully opening the hot-water valve. Wait for a steady-state condition. Record all temperatures and flow rates.
14. Keep the cold flow rate constant ($\dot{V}_c$) at the same flow rate used above, and reduce the hot flow rate ($\dot{V}_h$) by closing the hot-water control valve to at least its half-open position or less. Wait for a steady-state condition. Record all temperatures and flow rates.
15. When instructed by your TA, shut down the heat exchanger apparatus using the following steps:
    a. Turn off all 4 heaters using the toggle switches.
    b. Turn off the Temperature Controller toggle switch.
    c. Allow the heaters to cool 3–5 minutes with the cold and hot water flowing. Then turn off the pump using the toggle switch.
    d. Shut off the cold water supply at valve V4. Leave the control valves in their present positions.
    e. Turn off the digital thermometer if you are in the last lab of the day.

Heat Exchanger Theory

There are two main types of methods for analyzing heat exchanger performance: the Log Mean Temperature Difference (LMTD) method and the Effectiveness-NTU method. When both the inlet and outlet temperatures are known, the LMTD method provides the simplest means of evaluating performance. However, if only the inlet temperatures are known, the Effectiveness-NTU method is preferable. In this lab exercise, we will utilize both methods.

Log Mean Temperature Difference (LMTD) Method

The LMTD method is based on the idea that the interfluid heat transfer rate can be written as

$$q = U_i \, A_i \, F \, \Delta T_{\mathrm{lm}} \tag{4}$$

where U_i denotes the overall heat transfer coefficient (based on the total inside tube surface area), A_i is the total tube inside surface area, F denotes a correction factor, and ΔT_{lm} is the log mean temperature difference. For a counterflow, single-tube-pass, shell and tube heat exchanger, $F = 1$. Note, (4) may also be defined using $U_o\, A_o$, instead of $U_i\, A_i$, where the subscript o indicates that those quantities are based on the total *outside* surface area of the tubes. The log mean temperature difference is defined to be consistent with Newton's Law of Cooling, such that

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}, \tag{5}$$

where, for a counterflow heat exchanger,

$$\Delta T_1 = T_{h_i} - T_{c_o} \qquad \text{and} \qquad \Delta T_2 = T_{h,o} - T_{c,i}\,. \tag{6}$$

The overall heat transfer coefficient U_i appearing in (4) is the sum of the thermal resistances due to forced convection through the tubes and shell, plus conduction through the tube wall, plus an additional thermal resistance due to fouling (i.e., the deposition of a film/scale or the formation of rust on the inside surfaces of the tubes and shell due to impurities in the fluids from longtime use without cleaning). Because of the complexities of the geometry, especially through the shell, U_i (or U_o) is generally determined experimentally for shell and tube heat exchangers.

Effectiveness-NTU Method

The effectiveness, ε, provides a measure of the efficiency of the heat exchanger, and is defined as the ratio of the actual heat transfer rate over the maximum possible heat transfer rate,

$$\varepsilon = \frac{q}{C_{\text{min}}\,(T_{h,i} - T_{c,i})}, \tag{7}$$

where $C_{\text{min}} = \min(C_c,\, C_h)$. Rearranging (7) and solving for q yields another equation for the heat transfer rate,

$$q = \varepsilon\, C_{\text{min}}\,(T_{h_i} - T_{c,i}). \tag{8}$$

The above equation is attractive, because we only need to have knowledge of the inlet temperatures. By equating (8) and (2), we can obtain an equation for the hot-side outlet temperature,

$$T_{h,o} = T_{h,i}\left(1 - \frac{\varepsilon\, C_{\text{min}}}{C_h}\right) + T_{c,i}\left(\frac{\varepsilon\, C_{\text{min}}}{C_h}\right). \tag{9}$$

The above equation is particularly useful in heat exchanger design, because it allows one to predict the outlet temperature of the hot fluid based solely on knowledge of inlet fluid temperatures (and flow rates). Typically in the design of a heat exchanger, a target outlet temperature of the hot fluid is specified (e.g., the hot fluid needs to be cooled by 10°C). Equation (9) provides a means for determining the minimum effectiveness, ε, required to achieve the target outlet fluid temperature. Therefore, it is important to have a relatively accurate characterization of ε for your heat exchanger.

For every heat exchanger, we can find a function $\mathcal{F}$ such that

$$\varepsilon = \mathcal{F}(NTU,\, C_r), \tag{10}$$

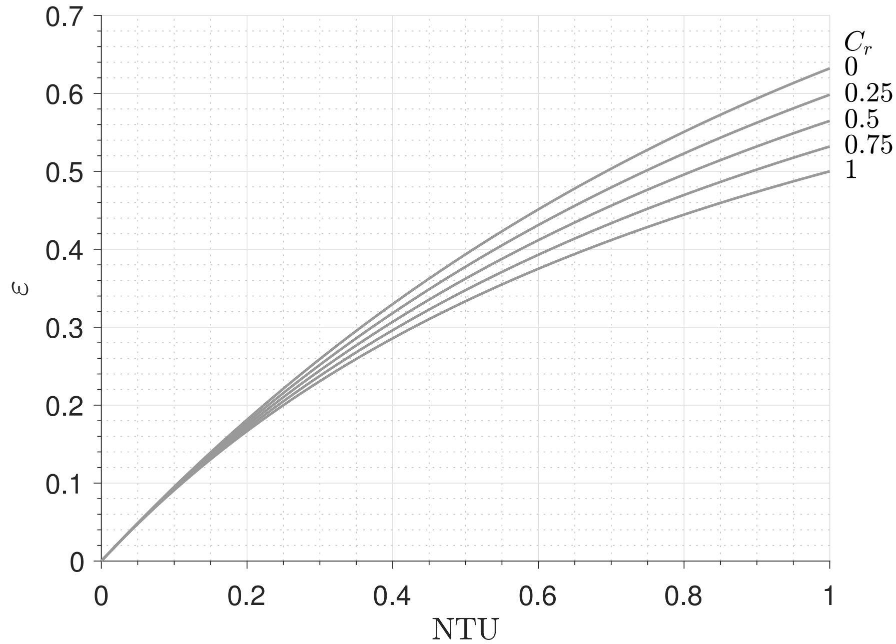

Figure 5. Theoretical effectiveness (ε) versus number of transfer units (NTU) for five different heat capacity ratios (C_r) according to the relationship in (12).

where C_r ($= C_{\min}/C_{\max}$) denotes the ratio of heat capacities, and NTU refers to the "number of transfer units", defined mathematically as

$$NTU = \frac{U_i\,A_i}{C_{\min}} = \frac{U_o\,A_o}{C_{\min}}\,. \tag{11}$$

The value of NTU represents the nondimensional size of the heat exchanger and provides some measure of how much heat can be transfered between the two fluids. The actual functional form of $\mathcal{F}(NTU, C_r)$ depends on the type of heat exchanger. The theoretical relationship for a counterflow, single-tube-pass, shell and tube heat exchanger is

$$\varepsilon_{\text{theory}} = \begin{cases} \dfrac{1-\exp[-NTU\,(1-C_r)]}{1-C_r\,\exp[-NTU\,(1-Cr)]}, & C_r < 1, \\[2ex] \dfrac{NTU}{1+NTU}, & C_r = 1, \end{cases} \tag{12}$$

Figure 5 shows a plot of the relationship in (12). For a given NTU value, the effectiveness, ε, increases with decreasing C_r. We can understand why this is the case by substituting (2) or (3) into (7). For example, if $C_{\min} = C_c$, then $C_{\max} = C_h$ and we would substitute (2) into (7) giving

$$\varepsilon = \frac{C_{\max}\,(T_{h,i} - T_{h,o})}{C_{\min}\,(T_{h,i} - T_{c,i})} = \frac{1}{C_r}\frac{(T_{h,i} - T_{h,o})}{(T_{h,i} - T_{c,i})} \tag{13}$$

From the above equation, we see that ε is inversely proportional to C_r.

Data Analysis

The following list outlines the data analysis steps necessary for creating the required figures/tables, and responding to the short-answer questions.

1. Convert all quantities into their proper SI units.
2. Calculate the average temperatures on both the cold and hot sides

$$\overline{T}_c = \tfrac{1}{2}(T_{c,i} + T_{c,o}) \qquad \text{and} \qquad \overline{T}_h = \tfrac{1}{2}(T_{h,i} + T_{h,o}) \tag{14}$$

3. Use the average temperatures to compute the fluid properties on the cold and hot sides, specifically the fluid densities (ρ_c, ρ_h) and specific heats ($c_{p,c}$, $c_{p,h}$).
4. Calculate the mass flow rates

$$\dot{m}_c = \rho_c \dot{V}_c \qquad \text{and} \qquad \dot{m}_h = \rho_h \dot{V}_h \tag{15}$$

5. Calculate the heat capacities: C_c, C_h, $C_{\min}$, $C_{\max}$, C_r.
6. Calculate the cold and hot-side heat transfer rates, q_c and q_h, using (3) and (2), respectively. These should be identical. However due to measurement uncertainty and fouling, you will notice a discrepancy.

   The main source of measurement uncertainty here is due to the fact that we have a single-point measurement of temperature at the inlets and outlet. We then assume that the temperatures are uniform across the inlet/outlet, such that the point measurement provides an accurate representation of the temperature distribution across the entire inlet/outlet planes. This is likely a good assumption at the inlet since the flow is well mixed by the turbulent flow through the delivery pipes. However, due to fouling inside the heat exchanger (and the nonuniform deposit of minerals on the tube side walls), the temperature may vary significantly across the outlet on both sides of the heat exchanger.
7. Calculate the effectiveness, ε, based on the measurements using (7). Note, q_h should be utilized in (7), rather than q_c, as it yields superior results, and provides a better estimate of how much heat was removed from the hot fluid.
8. Calculate the log mean temperature difference, ΔT_{lm}, using (5).
9. Calculate the overall heat transfer coefficient based on the *inside* tube surface area, U_i, using (4). Note, q_h should be utilized in (4), rather than q_c, for consistency with previous steps.
10. Calculate NTU based on (11).
11. Calculate the theoretical effectiveness, $\varepsilon_{\text{theory}}$, using (12).

12. Estimate the rate of heat transfer from the shell casing to the surroundings due to natural convection (q_{conv}) and thermal radiation (q_{rad}). In order to determine q_{rad}, we need to estimate the emissivity of the shell material (listed as brass by the manufacturer). If you consult engineeringtoolbox, for example, you will see that the emissivity for brass can vary from 0.03-0.6 depending on the surface finish and the oxidation properties. However, since the shell is painted gray, the emissivity increases to approximately 0.95. See the APPENDIX for further details on estimating q_{conv} and q_{rad}.

13. Estimate the percent uncertainty in the measured heat transfer rates. To do this, we perform a Taylor's series expansion of the relationships in (2) and (3). After simplification, it can be shown that

$$\frac{\sigma_{q_c}}{q_c} = \left[\left(\frac{\sigma_{\dot{V}_c}}{\dot{V}_c}\right)^2 + \left(\frac{\sigma_{\Delta T_c}}{\Delta T_c}\right)^2\right]^{1/2} \cdot 100\%\,, \tag{16}$$

$$\frac{\sigma_{q_h}}{q_h} = \left[\left(\frac{\sigma_{\dot{V}_h}}{\dot{V}_h}\right)^2 + \left(\frac{\sigma_{\Delta T_h}}{\Delta T_h}\right)^2\right]^{1/2} \cdot 100\%\,. \tag{17}$$

In the above expressions, $\Delta T_c = T_{c,o} - T_{c,i}$ and $\Delta T_h = T_{h,i} - T_{h,o}$. Furthermore, $\sigma_{\dot{V}}$ represents the uncertainty in the measured volume flow rate and $\sigma_{\Delta T}$ represents the uncertainty in the measured temperature difference. Note, in writing (16) and (17), we have ignored the uncertainty in the fluid properties (ρ and c_p). Because the temperature change between the inlets and outlets is relatively small, assuming constant fluid properties across the length of the heat exchanger does not significantly increase the overall uncertainty in the calculations.

The standard uncertainty in T-type thermocouples, which are the ones being used in this experiment, is ± 0.1°C. Therefore, $\sigma_{\Delta T} = \pm 0.1$°C. The uncertainty in the volume flow rate is based on the uncertainty of reading the water level using the scale on the rotameter. In terms of visual inspection of a measurement scale, the standard uncertainty is taken to be half the difference between markings on the scale. For the case of the rotameter in the experiment, this translates into $\sigma_{\dot{V}} = \pm 0.2$ gpm.

Required Figures and Tables

Captions

A meaningful and comprehensive caption must accompany all figures and tables. For the two tables, the caption is placed *above* the table and includes the label Figure 1x., where x denotes the letter a–b according to the plot order listed below. For the figure, the caption is placed *below* the figure and includes the label Table 1c.

Plots and Tables

1a. Create a table that looks like the one below, inserting the values from your analysis in each of the shaded boxes. Note, for cases 1a & 1b, the cold-side flow rate $\dot{m}_c$ is relatively fast. While, for cases 2a & 2b, the cold-side flow rate is relatively slow, approximately half that of cases 1a & 1b. The temperature differences between the inlet and outlet are $\Delta T_c = T_{c,o} - T_{c,i}$ and $\Delta T_h = T_{h,i} - T_{h,o}$. The heat transfer rates q_c and q_h are calculated based on (3) and (2), respectively. Finally, the percent difference in the hot-side and cold-side heat transfer rates (Δq) is quantified using the following expression

$$\Delta q = \frac{|q_h - q_c|}{\frac{1}{2}(q_h + q_c)} \cdot 100\% \,. \tag{18}$$

Case	Flow Rate (kg/s)		Temperature (°C)		U_i	Heat Transfer Rate (kW)		
	$\dot{m}_c$	$\dot{m}_h$	ΔT_h	ΔT_c	(W/K m^2)	q_c	q_h	Δq (%)
1a	(fast)	(fast)						
1b	(fast)	(slow)						
2a	(slow)	(fast)						
2b	(slow)	(slow)						

1b. Create a table that looks like the one below, inserting the values from your analysis in each of the shaded boxes. Values of the measured effectiveness, ε, are calculated from (7); while, values of the theoretical effectiveness, $\varepsilon_{\text{theory}}$, are calculated from (12). The percent relative difference between the measured and theoretical effectiveness is calculated using the following expression

$$\Delta\varepsilon = \frac{|\varepsilon - \varepsilon_{\text{theory}}|}{\varepsilon_{\text{theory}}} \cdot 100\% \,. \tag{19}$$

Case	Flow Rate (kg/s)		C_r	NTU	ε		
	$\dot{m}_c$	$\dot{m}_h$			measured	theory	Δε (%)
1a	(fast)	(fast)					
1b	(fast)	(slow)					
2a	(slow)	(fast)					
2b	(slow)	(slow)					

1c. Download the Matlab file for Figure 5 (EffectivenessNTUFigure.fig) that shows theoretical curves of ε versus NTU over a range of C_r values for a single tube-pass, single-shell pass counterflow heat exchanger. On this same figure, plot ε calculated from (7) based on your measurements using ○ markers. Also on this same figure, plot $\varepsilon_{\text{theory}}$ calculated from (12) using + markers.

Create two subplots placed side-by-side. The left subplot should be the same plot as generated above and should contain a legend. The right subplot should be zoomed-in near the cluster of data to highlight the discrepancy between the measurements and theory. Use the following axis limits:

(i) Left subplot: $0 \leq \varepsilon \leq 0.7$ and $0 \leq NTU \leq 1$.

(ii) Right subplot: $0.16 \leq \varepsilon \leq 0.22$ and $0.18 \leq NTU \leq 0.26$

Short-Answer Questions

2a. Theoretically, we expect $q_h = q_c$. However, your measurements will not likely support this. In your analysis, you calculated the percent relative difference, Δq, between q_c and q_h. State the range of Δq calculated from your measurements. In addition, state the range of percent relative uncertainty in the calculated heat transfer rates: $(\sigma_{q_c}/q_c) \cdot 100\%$ and $(\sigma_{q_h}/q_h) \cdot 100\%$. State the extent to which the uncertainty in the measurements helps explain the observed difference between q_c and q_h? [3–4 sentences]

2b. State the percent difference in effectiveness values obtained from the measurements compared to theory ($\Delta\varepsilon$ in %). Based on your engineering judgment, does the theory adequately describe the observations? For example, is it possible to use the theory to predict the effectiveness of the present heat exchanger for the following case: $\dot{m}_c = 0.3$ kg/s and $\dot{m}_h = 0.4$ kg/s? If yes, explain how. If no, explain why not. [3–5 sentences]

2c. Estimate the rate of heat transfer from the shell casing to the surroundings due to natural convection (q_{conv}) and radiation (q_{rad}). State the q_{conv} and q_{rad} values in kW averaged over all four test cases. Based on your engineering judgment, are these losses important and would you recommend insulating/covering the shell casing to mitigate these losses? Explain why or why not. [3–4 sentences]

*Note, be sure to include your calculations for q_{conv} and q_{rad} in your computer code.

APPENDIX I: Heat Loss from Shell Casing

Natural Convection Cooling

Heating/cooling of the shell casing by the hot-side/cold-side fluid will create buoyancy effects in the air neighboring the non-insulated heat exchanger. We can estimate the amount of heat transfer to the surroundings by modeling the shell as a long, horizontal, circular cylinder of length L and diameter D_s. Figure 6 illustrates the flow field in such a scenario. The heat transfer rate due to natural convective cooling of the shell casing is given by

$$q_{\text{conv}} = \overline{h}\, \pi\, D_s\, L\, (T_s - T_\infty)\,, \tag{20}$$

where T_s is the temperature of the shell casing, T_∞ is the ambient air temperature far from the shell casing, and $\overline{h}$ is the average heat transfer coefficient, defined as

$$\overline{h} = \frac{\kappa\, \overline{Nu}_D}{D_s}\,. \tag{21}$$

In the above expression, κ is the thermal conductivity of the air and $\overline{Nu}_D$ is the average Nusselt number. Note, all air properties should be evaluated at the film temperature,

$$T_f = \tfrac{1}{2}(T_s + T_\infty)\,. \tag{22}$$

Because the flow is driven by buoyancy, (and not inertial effects), the important nondimensional parameter governing the Nusselt number is the Raleigh number (not the Reynolds number). The Raleigh number is defined as

$$Ra_D = \frac{g\, \beta\, (T_s - T_\infty)\, D_s^3}{\nu\, \alpha}\,, \tag{23}$$

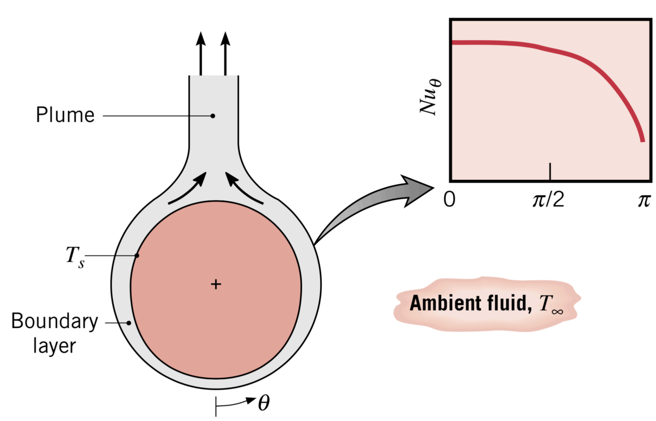

Figure 6. Schematic of the flow over a long, horizontal, circular cylinder driven purely by buoyancy effects. The Nusselt number is nearly independent of angle θ, except near the top of the cylinder at $\theta \approx \pi$.

where g is the gravitational constant, ν is the kinematic viscosity of the air, α is the thermal diffusivity of the air, and β is the volumetric thermal expansion coefficient. The kinematic viscosity and thermal diffusivity are fluid properties defined as

$$\nu = \frac{\mu}{\rho} \qquad \text{and} \qquad \alpha = \frac{\kappa}{\rho\, c_p}, \tag{24}$$

where μ is the dynamic viscosity of the air, ρ is the air density, and c_p is the specific heat of the air. Again, all air properties should be evaluated at the film temperature. For an ideal gas such as air, it can be shown that

$$\beta = \frac{1}{T_f}\,. \tag{25}$$

We will use the Nusselt number correlation developed by Morgan† that is appropriate for the range $Ra_D = 10^4 - 10^7$,

$$\overline{Nu}_D = 0.48\, Ra_D^{1/4}\,. \tag{26}$$

Note, this analysis does not consider end effects, i.e., natural convection around the ends of the heat exchanger; but, these should be relatively small compared to the heat transfer along the main body of the heat exchanger.

Thermal Radiation

The rate of heat transfer from the shell casing to the surroundings due to thermal radiation can be estimated from

$$q_{\text{rad}} = \epsilon\, \sigma\, (T_s^4 - T_\infty^4)\, \pi\, D_s\, L\,, \tag{27}$$

where ϵ denotes the emissivity of the shell material, and σ (= 5.6703×10^{-8} W/m^{2}K^4) is the Stefan-Boltzmann constant. Here again, we are ignoring end effects, which should be relatively insignificant compared to radiation from the main body of the heat exchanger.

†Morgan, *Heat Transfer Engineering*, **18(1)**, 25–33, 1997.

APPENDIX II: Calculating Air Fluid Properties

Students can utilize a custom Matlab function entitled `AirProperties`, provided by the instructor, in order to calculate the air fluid properties for a given absolute temperature (in degrees K) and absolute pressure (in Pa). The "help" information for the function is printed below.

```
%------------------------------------------------------------------------
% function [rho,mu,k,Cp] = AirProperties(T,P)
%
% Properties of DRY air based on the ideal gas law. Note, for ideal
% gases, Cp, k, and mu are independent of pressure.
%
% INPUTS:
%    T      Temperature in Kelvin
%    P      Atmospheric pressure in Pascal
%
% OUTPUTS:
%    rho    density in kg/m^3
%    mu     absolute viscosity in kg/(m*s)
%    k      thermal conductivity in W/(m*K)
%    Cp     specific heat in J/(kg*K)
%
% M Metzger
%------------------------------------------------------------------------
```

Note, you can pass either a single temperature and pressure value as input arguments, or an array of temperature and pressure values. In the latter case, the arrays must be the same size. To call this function in Matlab, consider the following example where the temperature is 30°C and the atmospheric pressure is 655 mmHg:

```
T=30+273.15;                          %temperature in K
P=655*133.3224;                       %pressure in Pa

[rho,nu,k,Cp]=AirProperties(T,P);  %call function
```

The output arguments, in the order returned, are: density in kg/m^3, dynamic (or absolute) viscosity in kg/(m·s), thermal conductivity in W/(m·K), and specific heat in J/(kg·K).

APPENDIX III: Calculating Water Fluid Properties

Students can utilize a custom Matlab function entitled `WaterProperties`, provided by the instructor, in order to calculate the water fluid properties for a given temperature (in °C). The "help" information for the function is printed below.

```
%----------------------------------------------------------------------
% function [rho,Cp] = WaterProperties(T)
%
% Properties of saturated liquid water at atmospheric pressure. Useful
% in analyzing heat exchanger lab data.
%
% INPUTS:
%    T      Temperature in oC
%
% OUTPUTS:
%    rho    density in kg/m^3
%    Cp     specific heat in J/(kg*K)
%
% M Metzger
%----------------------------------------------------------------------
```

Note, you can pass either a single temperature value as the input argument or an array of temperature values. The temperature must be in units of °C. The output density and specific heat are in the standard SI units.

To call this function in Matlab, consider the following example where the average temperature between the inlet and outlet temperature on the hot-side of the heat exchanger is 40°C:

```
T=40;                          %temperature in oC
[rho,Cp]=WaterProperties(T);  %call function
```

The output arguments, in the order returned, are: density in kg/m^3 and specific heat in J/(kg·K).