

1.  $w=1$  rad/s  $G(s) = \frac{1}{s+10} \Rightarrow G(j) = \frac{1}{j+10} \cdot \frac{-j+10}{-j+10} = \frac{1}{j+10} \cdot \frac{-j+10}{1+100}$   
 $G(j) = \frac{10}{101} - \frac{j}{101} \Rightarrow \angle G(j) = -5.71^\circ \quad |G(j)| = 0.10$

$w=2$  rad/s  $G(2j) = \frac{1}{2j+10} \cdot \frac{10-2j}{10-2j} = \frac{10-2j}{4+100} = \frac{10-2j}{104}$   
 $\frac{10}{104} - \frac{2}{104}j \Rightarrow \angle G(2j) = -11.31^\circ \quad |G(2j)| = 0.098$

$w=5$  rad/s  $G(5j) = \frac{1}{5j+10} \cdot \frac{-5j+10}{-5j+10} = \frac{-5j+10}{25+100}$   
 $\frac{10}{125} - \frac{5}{125}j \Rightarrow \angle G(5j) = -26.57^\circ \quad |G(5j)| = 0.089$

$w=10$  rad/s  $G(10j) = \frac{1}{10j+10} \cdot \frac{-10j+10}{-10j+10} = \frac{-10j+10}{100+100}$   
 $\frac{10}{200} - \frac{10}{200}j \Rightarrow \angle G(10j) = -45^\circ \quad |G(10j)| = 0.0707$

$w=20$  rad/s  $G(20j) = \frac{1}{20j+10} \cdot \frac{-20j+10}{-20j+10} = \frac{-20j+10}{400+100}$   
 $\frac{10}{500} - \frac{20}{500}j \Rightarrow \angle G(20j) = 63.43^\circ \quad |G(20j)| = 0.0447$

$w=50$  rad/s  $G(50j) = \frac{1}{50j+10} \cdot \frac{-50j+10}{-50j+10} = \frac{-50j+10}{2500+100}$   
 $\frac{10}{2600} - \frac{50}{2600}j \Rightarrow \angle G(50j) = -78.7^\circ \quad |G(50j)| = 0.0196$

$w=100$  rad/s  $G(100j) = \frac{1}{100j+10} \cdot \frac{-100j+10}{-100j+10} = \frac{-100j+10}{10000+100}$   
 $\frac{10}{10100} - \frac{100}{10100}j \Rightarrow \angle G(100j) = -84.29^\circ \quad |G(100j)| = 0.00995$

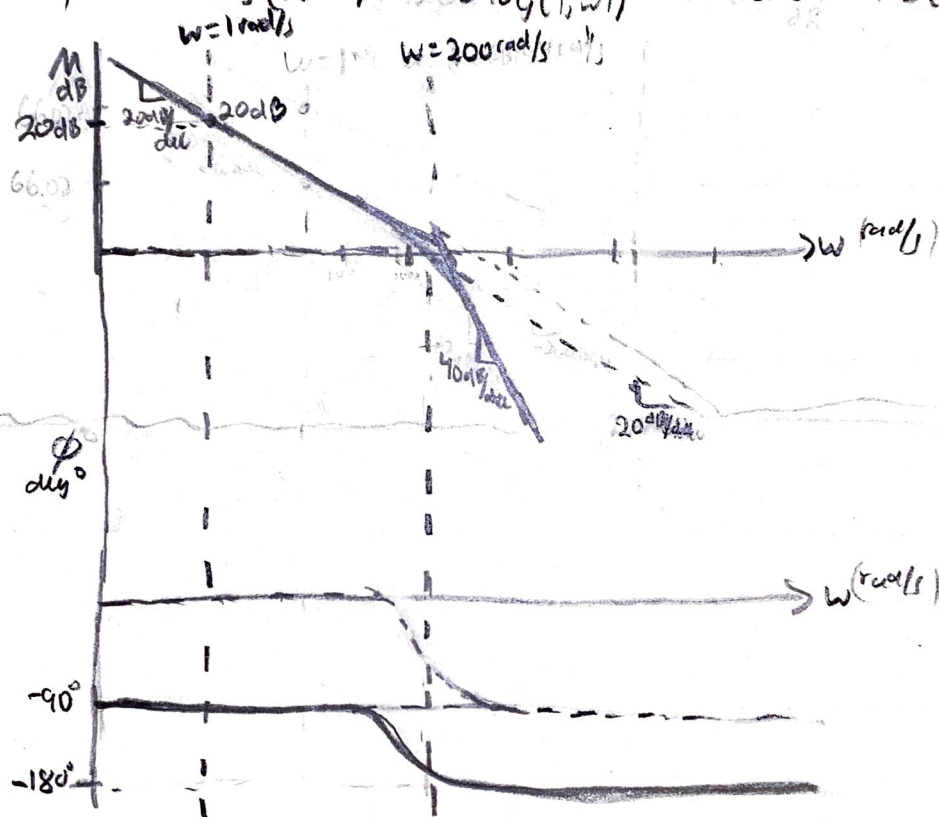
2.9)

$$L(s) = \frac{2000}{s(s+200)} = \frac{2000}{s^2+200s} \Rightarrow \frac{10}{s(1/200)s+1}$$

$$\Rightarrow \frac{10}{s} \Rightarrow K_c(j\omega)^n \quad \text{where}$$

$$n = -1 \quad K_c = 1000$$

$$\text{dB} \rightarrow 20 \log(1000) - 20 \log(1/j\omega) = 60 \text{ dB} - 20(1/j\omega)$$



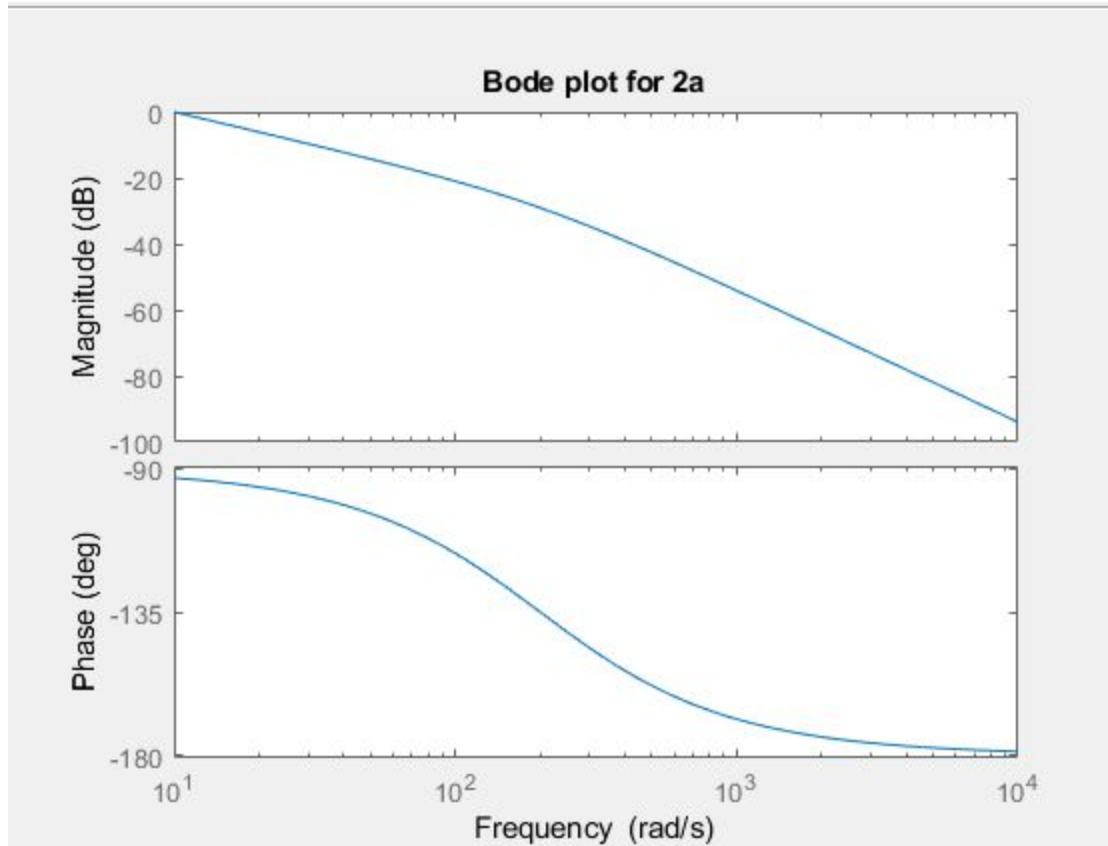
$$(s+200)^{-1} = (1/200)s + 1)^{-1} \quad \tau = 1/200 \text{ s}$$

$$\omega_B = 200 \text{ rad/s}$$

$$n = -1$$

**Problem 2**

a)



This plot is very similar to my sketch except there are smoother transitions in the actual bode plot. The phase plots look identical.

$$2. b) L(s) = \frac{100}{s(0.1s+1)(0.5s+1)} = \frac{100}{s(0.05s^2 + 0.1s + 0.5s + 1)} = \frac{100}{0.05s^3 + 0.6s^2 + s}$$

$$\frac{100}{s} \Rightarrow K_G = 100 \quad 20 \log(K_G) = 40 \text{ dB}$$

$$n = -1 \quad 20n \log(1/s) = -20 \log(1/s)$$

$$\phi = -90^\circ$$

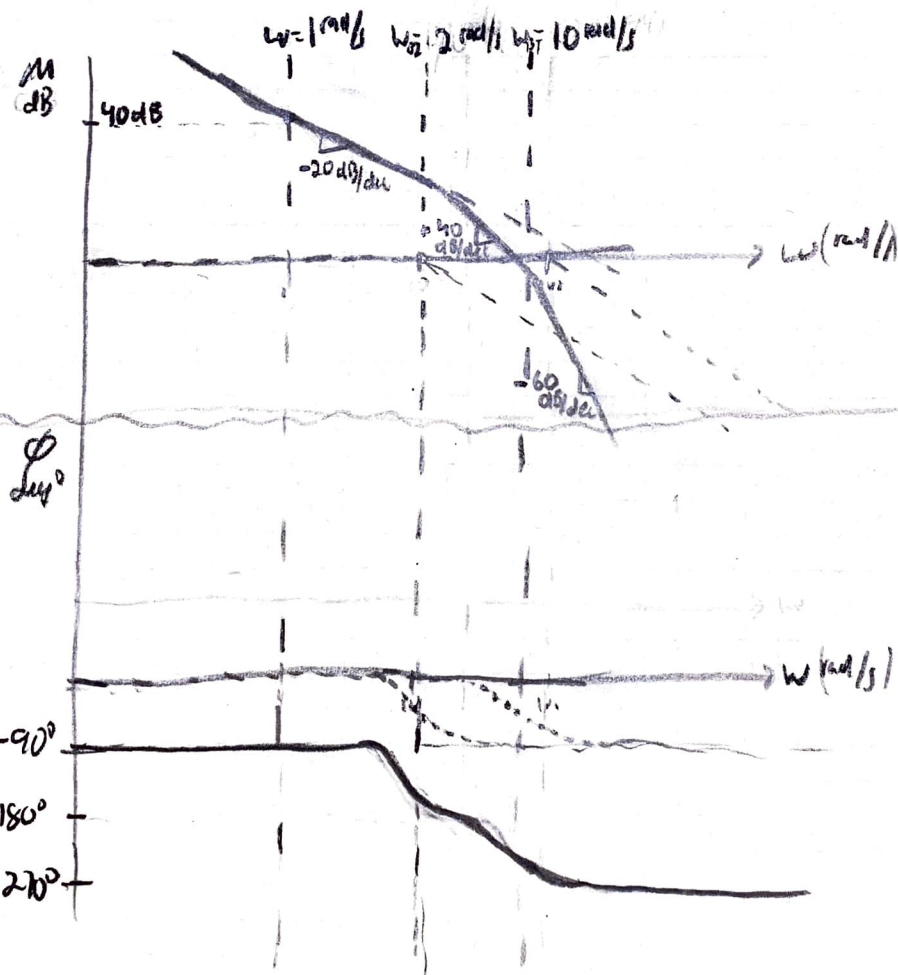
$$(0.1s+1)^{-1} (0.5s+1)^{-1} \Rightarrow ((1/10)s+1)^{-1} ((1/2)s+1)^{-1}$$

$$n = -1 \quad 0^\circ \rightarrow -90^\circ \quad \tau_1 = 1/10$$

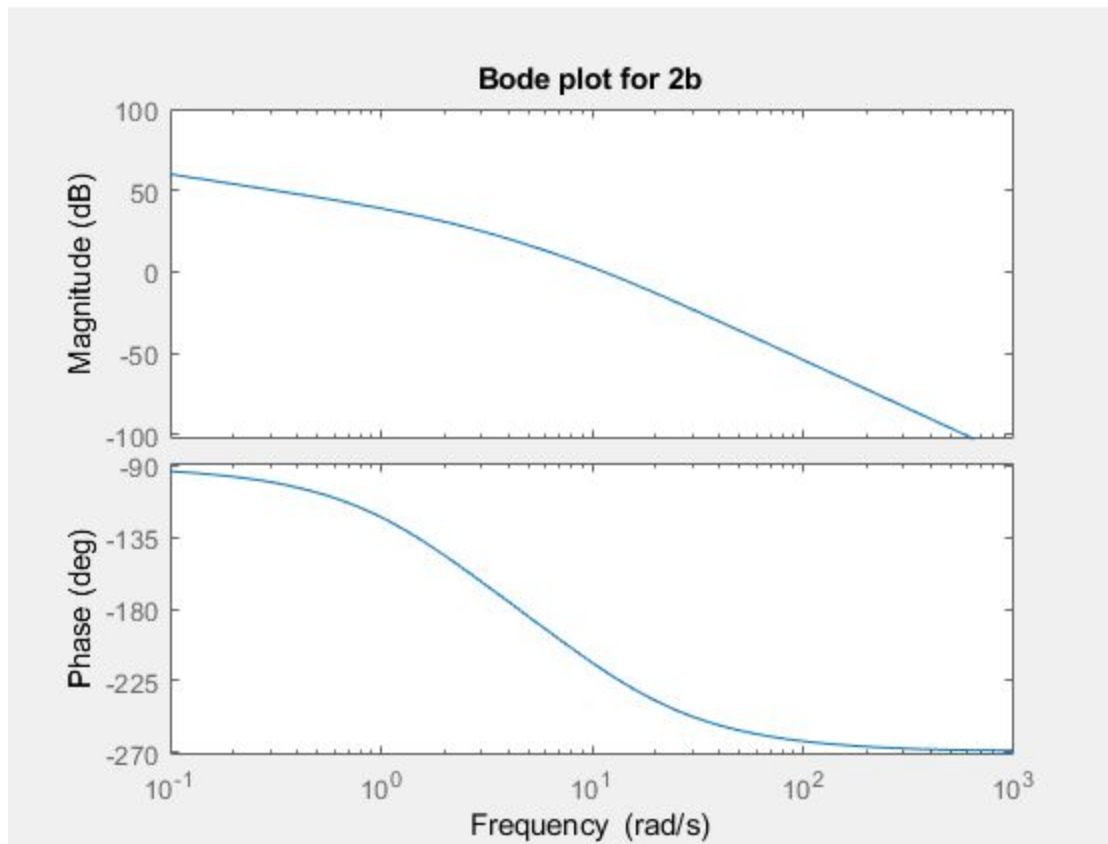
$$\omega_{B1} = 10 \text{ rad/s} \quad 62.83 \text{ rad/s} \quad n = -1 \quad -20 \text{ dB/dec}$$

$$n = -1 \quad 0 \rightarrow -90^\circ \quad \tau_2 = 1/2$$

$$\omega_{B2} = 2 \text{ rad/s} \quad 12.57 \text{ rad/s} \quad n = -1 \quad -20 \text{ dB/dec}$$



b)



This plot is very similar to my sketch except there are smoother transitions in the actual bode plot. Between 2 rad/s and 10 rad/s it is harder to discern the transitions on the bode plot because of the scale.



$$2. c) L(s) = \frac{1}{s(s+1)(0.02s+1)} = \frac{1}{s(0.02s^2 + 1.02s + 1)} = \frac{1}{0.02s^3 + 1.02s^2 + s}$$

$$\frac{1}{s} \Rightarrow K_G = 1 \quad 20 \log(K_G) = 0 \text{ dB}$$

$$n = -1 \quad 20n \log(1/1) = -20 \log(1/1)$$

$$\phi = -90^\circ$$

$$(j\omega + 1)^{-1} (0.02j\omega + 1)^{-1}$$

$$n = -1 \quad 0^\circ \rightarrow -90^\circ$$

$$\tau_1 = 1$$

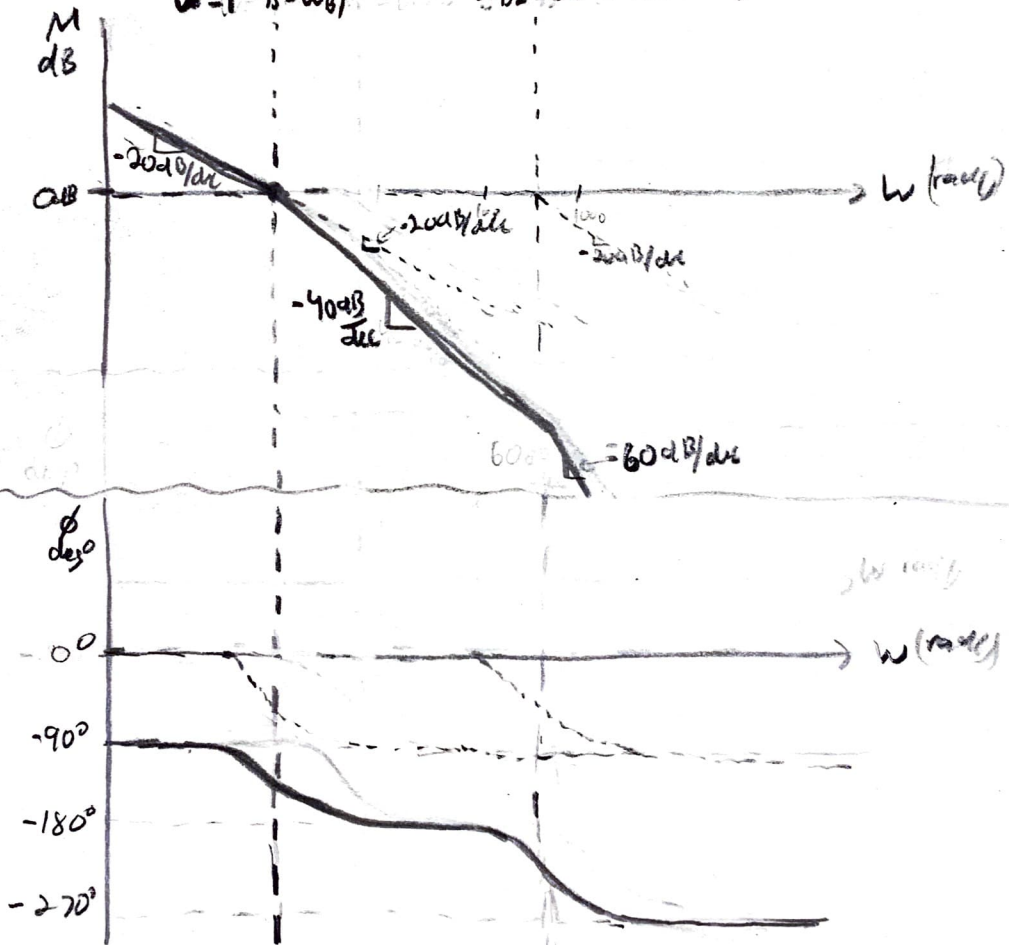
$$\omega_{B1} = 1 \text{ rad/s} \quad 20 \log(1) = 0 \text{ dB} \quad n = -1 \quad -20 \text{ dB/dec}$$

$$n = -1 \quad 0^\circ \rightarrow -90^\circ$$

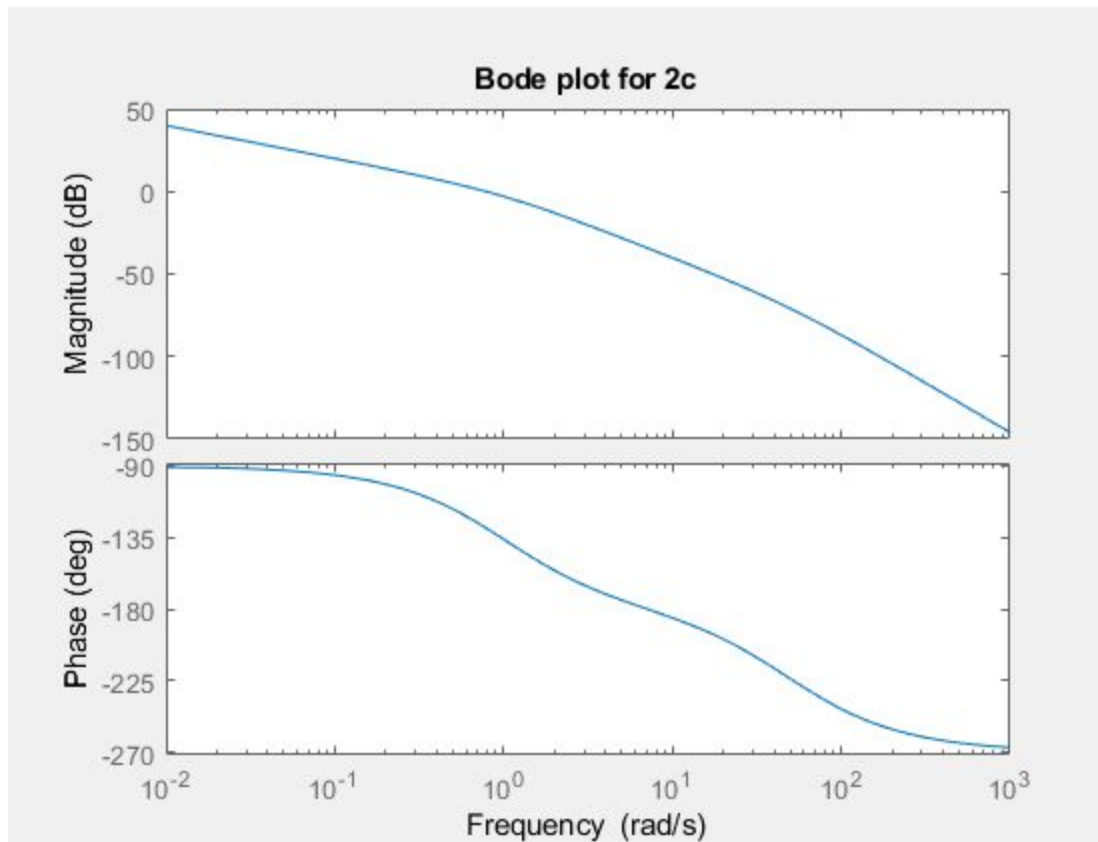
$$\tau_2 = 2/100$$

$$\omega_{B2} = 50 \text{ rad/s} \quad 20 \log(50) = 34.159 \text{ dB} \quad n = -1 \quad -20 \text{ dB/dec}$$

$$\omega = 1 \text{ rad/s} = \omega_{B1} \quad \omega_{B2} = 50 \text{ rad/s}$$



c)



This plot is very similar to my sketch except there are smoother transitions in the actual bode plot. Once again because of the bode plot scale the two “bumps” of the phase plot are not as pronounced as compared to the sketch, but the plots appear the same.

3. a)

$$L(s) = \frac{(s+2)}{s(s+10)(s^2+2s+2)} \Rightarrow \frac{(s+2)}{s(s^3+12s^2+22s+20)}$$

$$\Rightarrow \frac{(1/10)(1/2)(2)(1/2)(s+1)}{s(1/10)(s+1)(1/2)(s^2+s+1)}$$

$$(10s)^{-1} \Rightarrow t_0 = 1/10 \Rightarrow 20 \log(1/10) = -20 \text{ dB}$$

$$\phi: -90^\circ \quad n: -1 \quad 20 \log(|j\omega|) \Rightarrow -20 \text{ dB/dec}$$

$$((1/2)s+1) \Rightarrow \tau_1 = 1/2 \text{ s} \quad \omega_{B1} = 2 \text{ rad/s} \rightarrow 20 \text{ dB/dec} \quad n=1$$

$$((1/10)s+1)^{-1} \Rightarrow \tau_2 = 1/10 \text{ s} \quad \omega_{B2} = 10 \text{ rad/s} \rightarrow -20 \text{ dB/dec} \quad n=-1$$

$$\phi_1: 0^\circ \rightarrow 90^\circ \text{ at } \omega_{B1} \quad \phi_2: 0^\circ \rightarrow -90^\circ \text{ at } \omega_{B2}$$

$$((1/2)s^2 + s + 1)^{-1} \Rightarrow \left[ \left( \frac{j\omega}{\omega_n} \right)^2 + 2\zeta \left( \frac{j\omega}{\omega_n} \right) + 1 \right]^{-1}$$

$$1/2 \text{ sec} = (1/\omega_n)$$

$$\sqrt{2} = \omega_n$$

$$\omega_{B3} = \omega_n = \sqrt{2} \text{ rad/s}$$

$$-40 \text{ dB/dec} \text{ since } n=-1$$

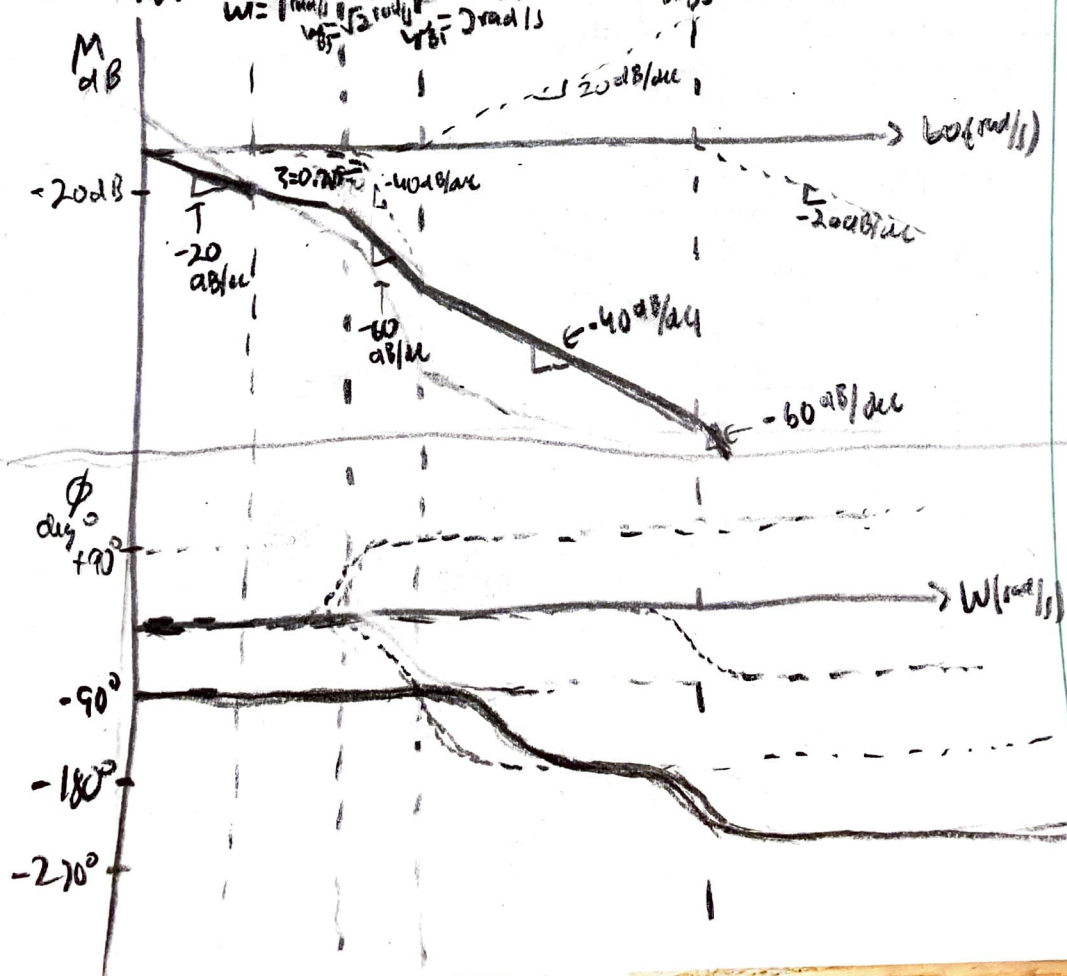
$$\zeta = \sqrt{2}/2 = 0.7071$$

How big the "peak" at  $\omega_n$  is

$$0 \rightarrow -180^\circ$$

$$\text{and } -90^\circ \text{ at } \omega_{B3}$$

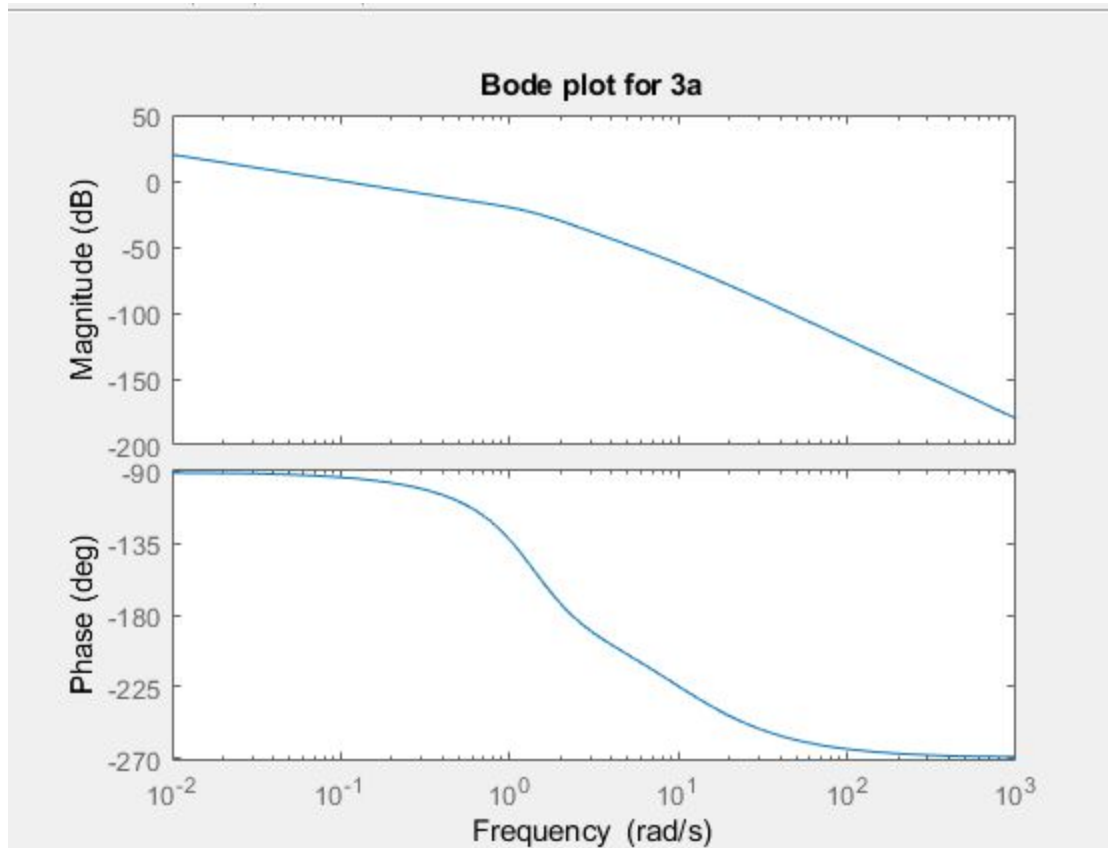
$$\omega_{B3} = 10 \text{ rad/s}$$





### Problem 3

a)



This plot is very close to the sketch I created, although the phase plot and magnitude plot don't show as detailed transitions between 1 rad/s and 2 rad/s when compared to my sketch because of the scale.

$$3. b) L(s) = \frac{(s+2)}{s^2(s+10)(s^2+6s+25)} = \frac{(s+2)}{s^2(s^3+16s^2+85s+250)}$$

$$\frac{1}{125} \Rightarrow \frac{(2)(1/10)(1/25)(1/2)j\omega+1}{(j\omega)^2((1/10)j\omega+1)((1/25)(j\omega)^2+6/25j\omega+1)}$$

$$(125(j\omega)^{-2}) \Rightarrow K_G = 1/125 \Rightarrow 20 \log(1/125) = -41.94 \text{ dB}$$

$$\phi = -180^\circ \quad n = -2 \quad 20n \log(1/\omega) = -40 \text{ dB/dec}$$

$$((1/2)j\omega+1) \Rightarrow T_1 = 1/2 \quad \omega_{B1} = 2 \rightarrow 20 \text{ dB/dec } n=1 \quad \phi: 0^\circ \rightarrow -90^\circ @ \omega_{B1}$$

$$((1/10)j\omega+1) \Rightarrow T_2 = 1/10 \quad \omega_{B2} = 10 \rightarrow -20 \text{ dB/dec } n=-1 \quad \phi: 0^\circ \rightarrow +90^\circ @ \omega_{B2}$$

$$[(1/25)(j\omega)^2 + 6/25j\omega + 1]^{-1} \Rightarrow \left[ \left( \frac{j\omega}{\omega_n} \right)^2 + 2\zeta \left( \frac{j\omega}{\omega_n} \right) + 1 \right]^{-1}$$

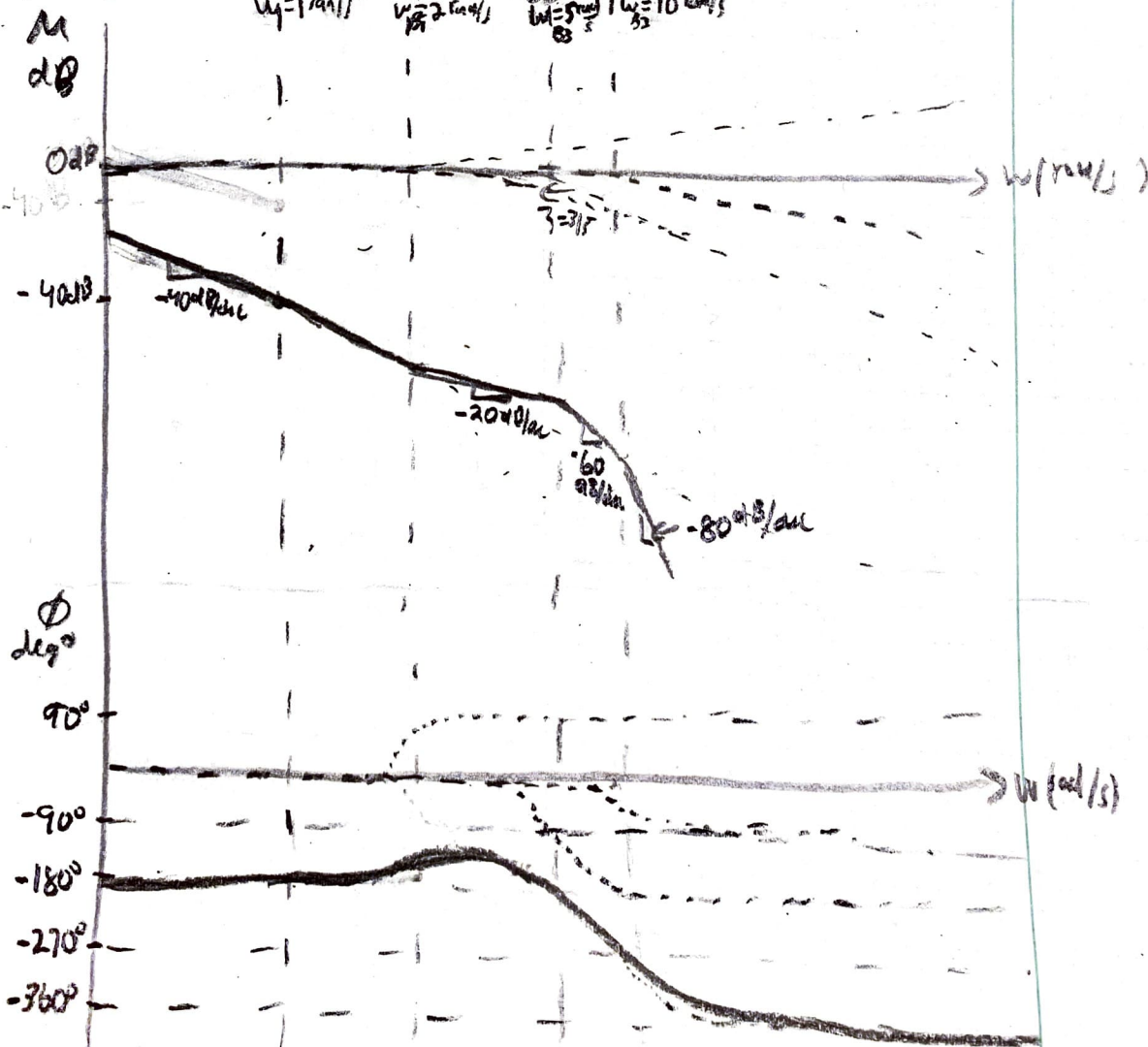
$$\omega_{B3} = \omega_n = \sqrt{25} = 5 \text{ rad/s} \quad \zeta = 3/5 \quad \frac{\phi}{2\zeta} = \frac{23}{5}$$

$$-40 \text{ dB/dec since } n = -1$$

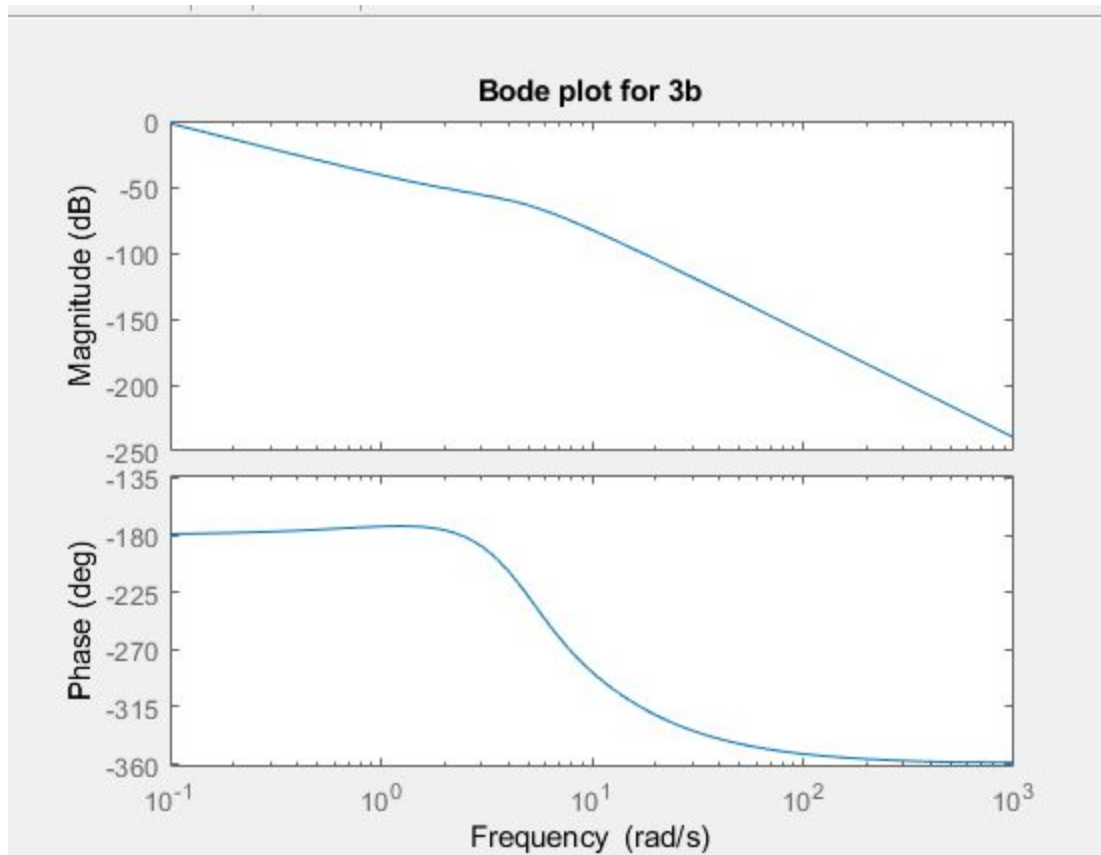
1/25 big "peak" at  $\omega_n = 5$

$$0 \rightarrow -180^\circ \quad \text{and } \odot = -90^\circ @ \omega_{B2}$$

$$\omega_{B1} = 1 \text{ rad/s} \quad \omega_{B2} = 2 \text{ rad/s} \quad \omega_{B3} = 5 \text{ rad/s} \quad \omega_{B4} = 10 \text{ rad/s}$$



b)



This plot is close to my sketch but between .1 rad/s and 1 rad/s the phase plot appears to be increasing on the actual bode plot and this doesn't appear as much on my sketch. This is likely because of the influence of the breakpoint at 2 rad/s beginning to have some influence before the plot reaches that breakpoint.