MB EN 6200 | Homework 9 | Ryan Dalby | Pascal

1. a) $G(s) = \frac{1}{s^2}$ $D_c(s) = K \cdot \frac{(s+z)}{(s+p)}$ s.t. $z < p \Rightarrow$ Lead Compensator

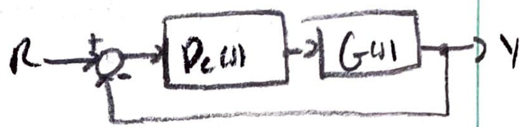
Goal: Design $D_c(s)$ so dominant closed-loop poles are $s = -2 \pm 2j$

$(s+2+2j)(s+2-2j)$ ↳ Need to find K for this

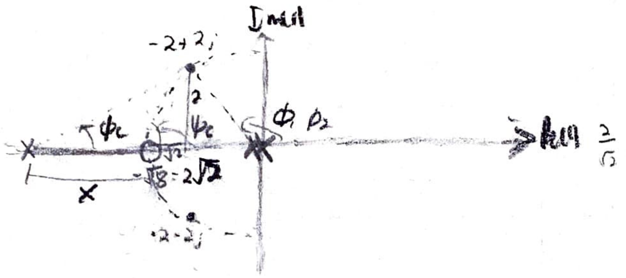
$s^2 + 2s - 2js + 2s + 4 - 4j + 2js + 4j + 4$

$s^2 + 4s + 8$

$s^2 + 2\zeta\omega_n s + \omega_n^2$



$\omega_n^2 = 8$ $\omega_n = \sqrt{8}$



Poles of $G(s)$: 0, 0
$D_c(s)$: $-z$, $-p$

take $z = \sqrt{8}$

$\phi_1 = \phi_2 = 135°$ $\psi = 54.74° = \arctan(2/\sqrt{2})$

$\psi_c - (\phi_c + \phi_1 + \phi_2) = 180 + 360°(l-1)$

$54.74 - (\phi_c + 135 + 135) = 180 + 360°(l-1)$

$\phi_c = 395.26 = 35.26°$

$\tan(\phi_c) = 2/(x+\sqrt{2})$ $x = 1.4147$

$p = x + \sqrt{2} + 2 = 4.83$

$D_c(s) = K \cdot \frac{(s+\sqrt{8})}{(s+4.83)}$ $1 + K\left(\frac{(s+\sqrt{8})}{(s+4.83)}\right) G(s) = 0$

$K = \left| -\frac{1}{\left(\frac{(s+\sqrt{8})}{(s+4.83)s^2}\right)} \right|_{s=-2\pm 2j} = \left| -\frac{(-2+2j)^2(-2+2j+4.83)}{(-2+2j+\sqrt{8})} \right|$

$K = \left| -\frac{(-8j)(-2+2j+4.83)}{(-2+2j+\sqrt{8})} \right| = \left| \frac{16j + 16 - 38.64j}{-2+2j+\sqrt{8}} \right|$

$K = \left| \frac{-16 + 22.64j}{0.828 + 2j} \right| = \frac{\sqrt{16^2 + 22.64^2}}{\sqrt{0.828^2 + 2^2}} = \frac{27.72}{2.164} = 12.8$

$$\boxed{D_c(s) = 12.8 \cdot \frac{(s+\sqrt{8})}{(s+4.83)}}$$

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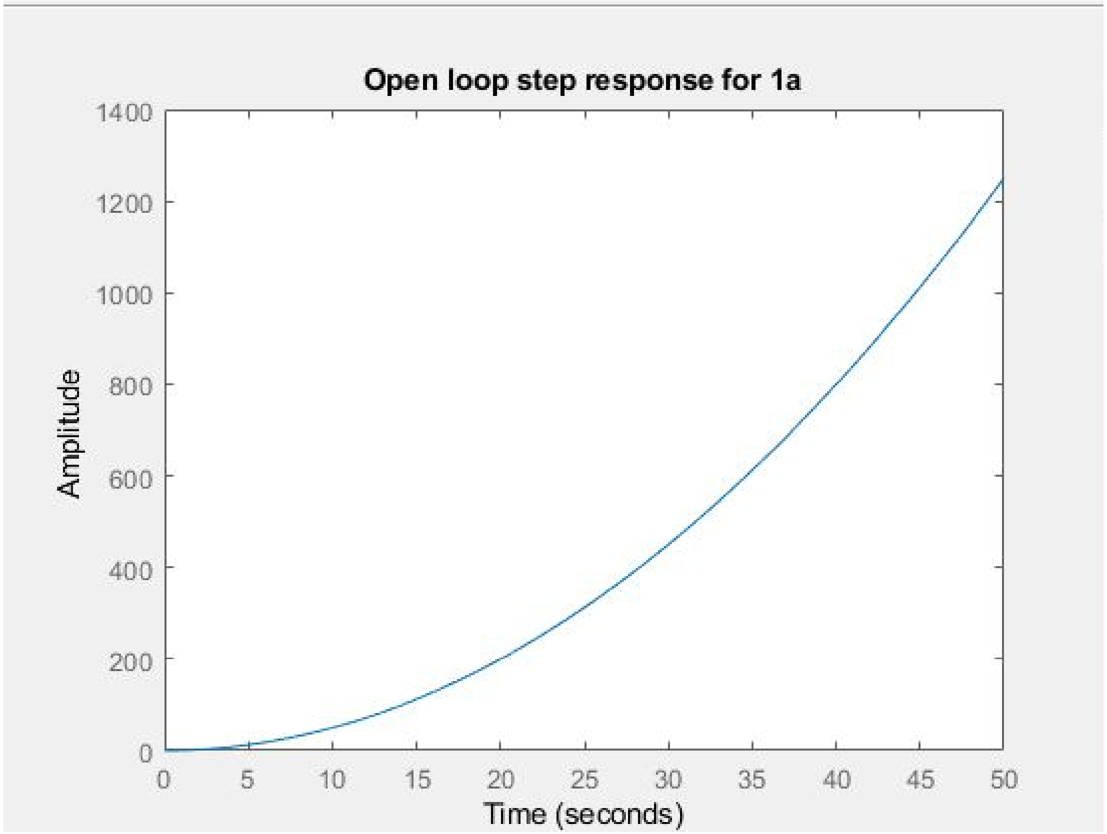
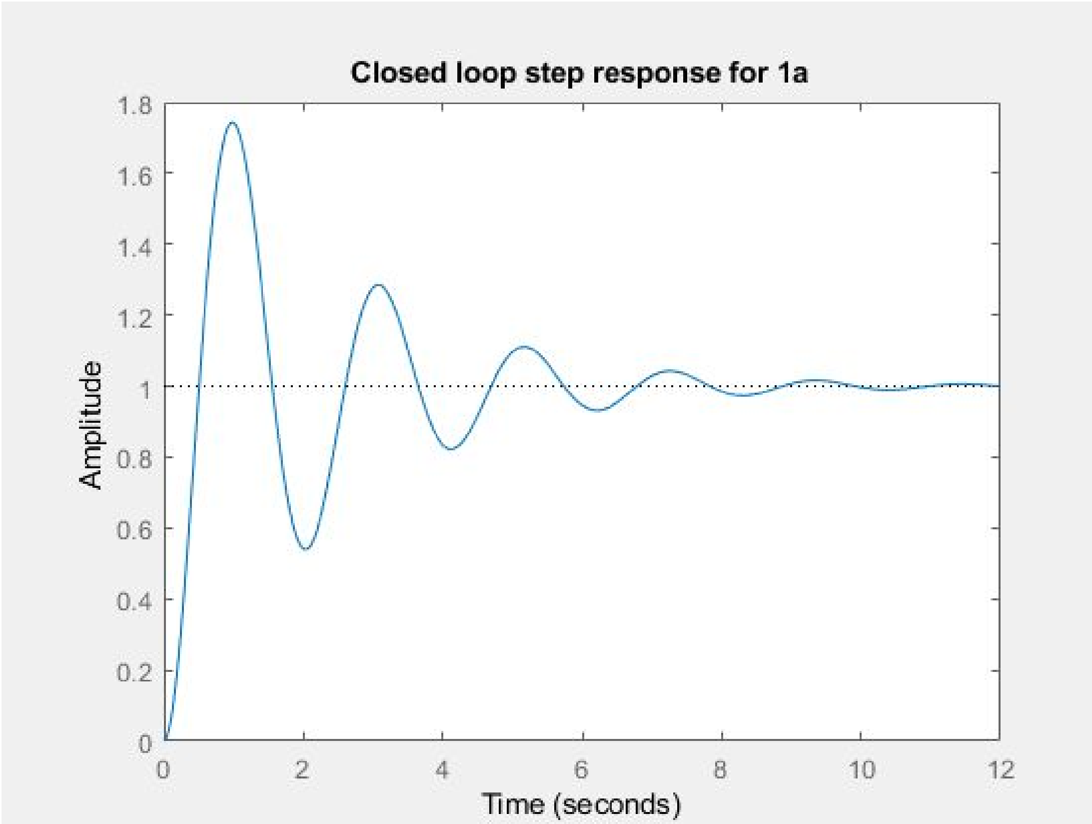
Problem 1

a)



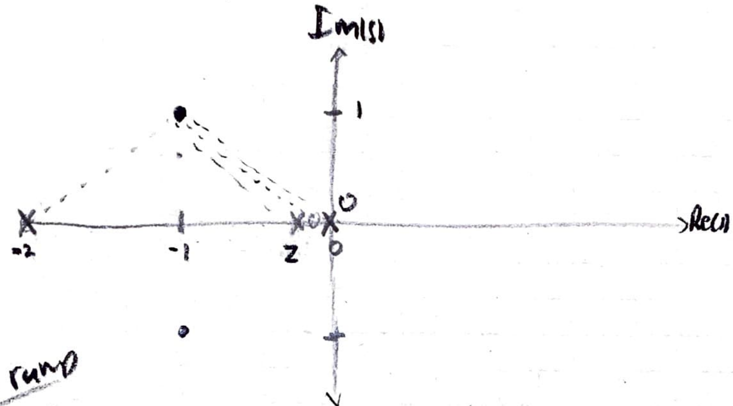


As can be seen above the open loop increases towards infinity as time increases thus indicating that the open loop system is unstable for the input. The closed loop step response with the lead compensator results in a response that shows the system is stable for the input. As can be seen adding the controller resulted in a stable system which has the desired step response properties of a system with dominant -2 +- 2j poles. (natural frequency of sqrt(8))

```
%% 1
% a
% Open loop response
G = tf(1, [1 0 0]);
figure;
step(G);
title('Open loop step response for 1a');
% Closed loop response
z = sqrt(8);
p = 4.83;
k = 12.8;
Dc = tf([1 z], [1 p]);
cltf = feedback(k*Dc*G, 1);
figure;
step(cltf);
title('Closed loop step response for 1a');
```

2 b) $G(s) = \frac{1}{s(s+2)}$ $D_c(s) = K \cdot \frac{(s+z)}{(s+p)}$ s.t. $z > p \Rightarrow$ Lag Compensator

Goal: Design $D_c(s)$ so dominant closed loop poles are at $s = -1 \pm j$
and $e_{ss} = 0.2$ ↳ Need to find K for this
↳ Due to a ramp input

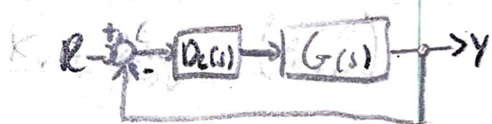


Poles of $G(s)$: $s = 0, -2$
$D_c(s)$: $s = -z, -p$

Take z = small number
↳ $z = 0.1$

Assume unity ramp

For a unity ramp w/ unity feedback: $e_{ss} = \frac{1}{K_v} \leq 0.2$



$K_v = \lim_{s \to 0} s\, G(s) D_c(s) = K \frac{(s+z)}{(s+2)(s+p)}$

$K_v = \frac{zK}{2p}$

$e_{ss} = \frac{2p}{zK} \leq 0.2$

$1 + D_c(s) G(s) = 0 \Rightarrow 1 + K \frac{(s+z)}{(s+p)} \frac{1}{s(s+2)} \Big|_{s=-1+j} = 0$ $= \frac{(s+z)}{s(s+p)(s+2)}$

$K = - \frac{s(s+p)(s+2)}{(s+z)} \Big|_{s=-1+j}$ $\Rightarrow 2p \leq (0.2)(zK)$

$\frac{10p}{z} \leq K$

$|100p| = \left| - \frac{(-1+j)(-1+j+p)(-1+j+2)}{(-1+j+0.1)} \right|$ ⟵ taking $z = 0.1$

$100p \leq K$
take $K = 100p$

$|100p| = \frac{|-1+j|\,|-1+p+j|\,|1+j|}{|-0.9+j|}$

$100p = \frac{(\sqrt{2})(\sqrt{(-1+p)^2+1})(\sqrt{2})}{\sqrt{0.9^2+1^2}} = \frac{2\sqrt{p^2-2p+2}}{1.345}$

$4525p^2 = p^2 - 2p + 2 \Rightarrow 4524p^2 + 2p - 2 = 0$

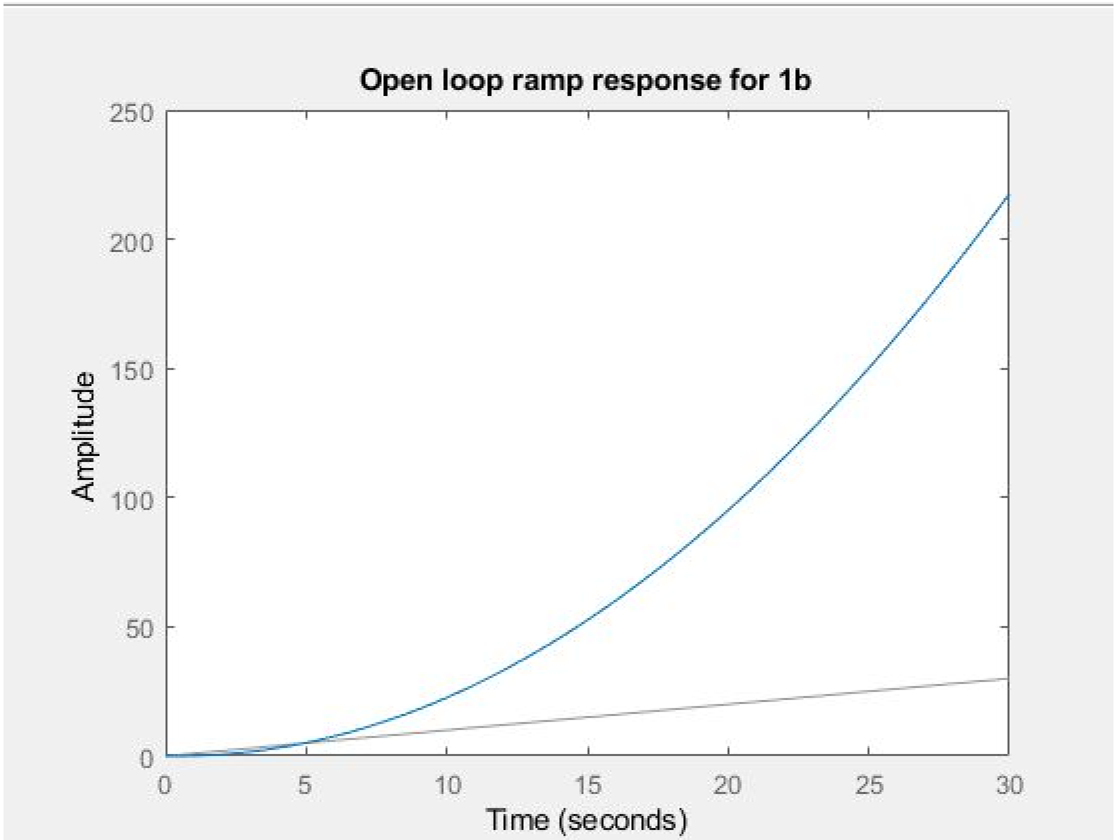
$p = 0.0208, -0.0212$

b) $P = 0.0208$ $\qquad$ $z = 0.1$

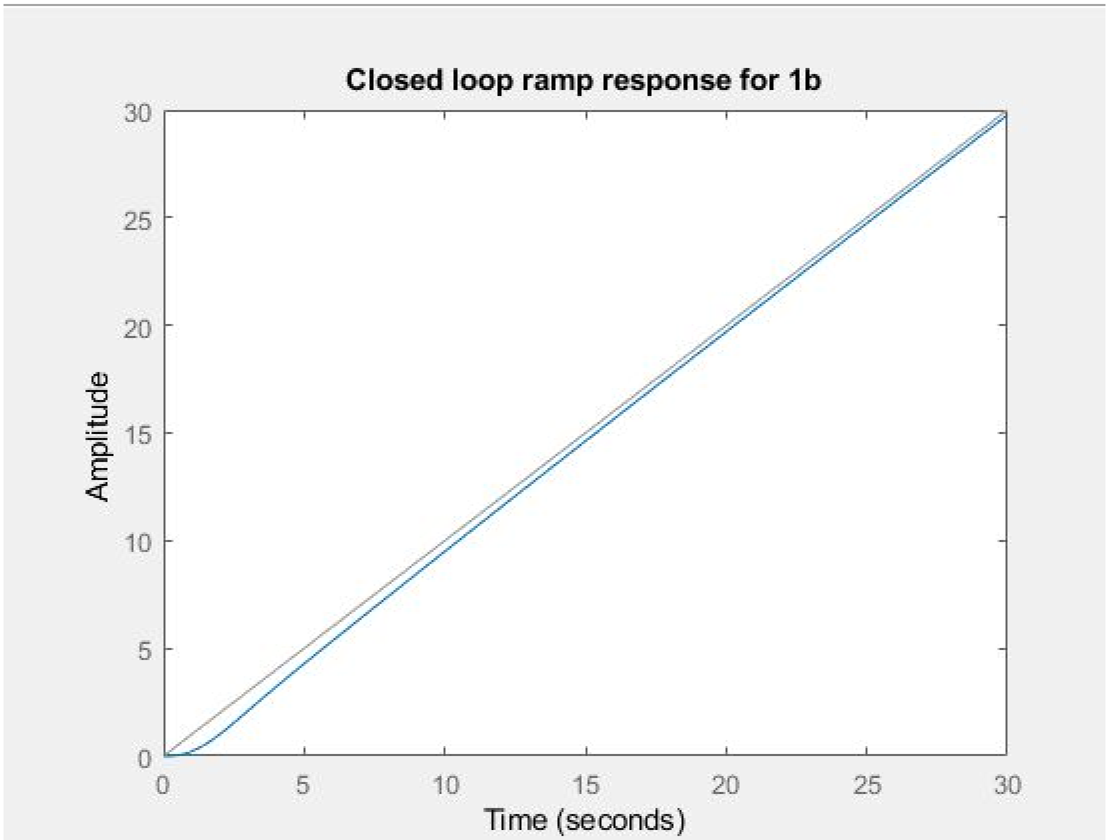
$K = 100(0.0208) = 2.08$

$$\boxed{D_c(s) = 2.08\frac{(s+0.1)}{(s+0.0208)}}$$

b)



Open loop ramp response for 1b
Amplitude
Time (seconds)
250
200
150
100
50
0
0
5
10
15
20
25
30




```
The steady-state error of the closed loop ramp response for 1b is 0.20
```

The open loop ramp response shows an exponentially growing output as time increases.This indicates that the open loop system is not stable for a ramp input. The closed loop ramp response shows a response that tracks the input with a steady-state error of 0.20. This indicates that the closed loop system is stable for the input. Thus the closed loop system achieves the desired steady-state behavior as indicated in the design specifications for this problem.

```
% b
% Open loop response
G = tf(1, [1 2 0]);
figure;
t_vals = 0:0.1:30;
ramp_vals = t_vals;
lsim(G, ramp_vals, t_vals);
title('Open loop ramp response for 1b');

% Closed loop response
z = 0.1;
p = 0.0208;
k = 2.08;
```

```
Dc = tf([1 z], [1 p]);
cltf = feedback(k*Dc*G, 1);
figure;
lsim(cltf, ramp_vals, t_vals);
title('Closed loop ramp response for 1b');

% Determine steady-state error
t_vals = 0:0.1:10000;
ramp_vals = t_vals;
[y,t] = lsim(cltf, ramp_vals, t_vals);
ess = abs(ramp_vals(end) - y(end));
fprintf('The steady-state error of the closed loop ramp response for 1b is
%.2f\n', ess);
```