1. a)

$$LC\ddot{V}_2 + RC\dot{V}_2 + V_2 = LC\ddot{V}_1 + RC\dot{V}_1 + V_3$$

voltages are implicitly relative to the same ground, taking that ground to be V_3, thus $V_3 = 0$, $\dot{V}_3 = 0$, $\ddot{V}_3 = 0$

$$LC(\ddot{V}_2 - \cancel{\ddot{V}_3}) + RC(\dot{V}_2 - \cancel{\dot{V}_3}) + V_2 - \cancel{V_3} = LC(\ddot{V}_1 - \cancel{\ddot{V}_3}) + RC(\dot{V}_1 - \cancel{\dot{V}_3}) + \cancel{V_3 - V_3}$$

thus relative to V_3:

$$\mathcal{L}\{LC\ddot{V}_2 + RC\dot{V}_2 + V_2\} = \mathcal{L}\{LC\ddot{V}_1 + RC\dot{V}_1\}$$

↳ Assuming $\dot{V}_2(0) = 0$ $V_2(0) = 0$ $\dot{V}_1(0) = 0$ $V_1(0) = 0$

$$s^2 LCV_2(s) + sRCV_2(s) + V_2(s) = s^2 LCV_1(s) + sRCV_1(s)$$

$$V_2(s)[s^2LC + sRC + 1] = V_1(s)[s^2LC + sRC]$$

$$\boxed{\frac{V_2(s)}{V_1(s)} = \frac{LCs^2 + RCs}{LCs^2 + RCs + 1}}$$

output: V_2 relative to V_3
input: V_1 relative to V_3
add V_3 for relative to ground if looking at V_2 for time domain

b) Taking ground to be V_1, $V_1 = 0$ $\dot{V}_1 = 0$ $\ddot{V}_1 = 0$

relative to V_1 $$LC(\ddot{V}_2 - \cancel{\ddot{V}_1}) + RC(\dot{V}_2 - \cancel{\dot{V}_1}) + V_2 - \cancel{V_1} = LC(\cancel{\ddot{V}_1 - \ddot{V}_1}) + RC(\cancel{\dot{V}_1 - \dot{V}_1}) + V_3 - \cancel{V_1}$$

$$\mathcal{L}\{LC\ddot{V}_2 + RC\dot{V}_2 + V_2\} = \mathcal{L}\{V_3\}$$

↳ Assuming $\dot{V}_2(0) = 0$ $V_2(0) = 0$

$$s^2 LCV_2(s) + sRCV_2(s) + V_2(s) = V_3(s)$$

$$V_2(s)[LCs^2 + RCs + 1] = V_3(s)$$

$$\boxed{\frac{V_2(s)}{V_3(s)} = \frac{1}{LCs^2 + RCs + 1}}$$

output: V_2 relative to V_1
input: V_3 relative to V_1
add V_1 for relative to ground if looking at V_2 in time domain

2. a)
$$CR_5\dot{V}_{out} + \frac{R_5}{R_6}V_{out} = \frac{R_3}{R_1}V_1 - \left(\frac{R_1+R_3}{R_1}\right)\left(\frac{R_4}{R_2+R_4}\right)V_2$$

Inputs are V_1 and V_2, relative to ground, taking all voltages to be relative to V_2 we set $V_2 = 0$

$$CR_5\dot{V}_{out} + \frac{R_5}{R_6}V_{out} = \frac{R_3}{R_1}V_1$$

$$\mathcal{L}\left\{CR_5\dot{V}_{out} + \frac{R_5}{R_6}V_{out}\right\} = \mathcal{L}\left\{\frac{R_3}{R_1}V_1\right\}$$

↳ Assume $V_{out}(0) = 0$

$$sCR_5V_{out}(s) + \frac{R_5}{R_6}V_{out}(s) = \frac{R_3}{R_1}V_1(s)$$

$$V_{out}(s)\left[CR_5 s + \frac{R_5}{R_6}\right] = \frac{R_3}{R_1}V_1(s)$$

$$\boxed{\frac{V_{out}(s)}{V_1(s)} = \frac{R_3/R_1}{CR_5 s + R_5/R_6} = \frac{R_3}{CR_5R_1 s + R_5R_1/R_6}}$$

b) taking all voltage to be relative to V_1, $V_1 = 0$

$$\mathcal{L}\left\{CR_5\dot{V}_{out} + \frac{R_5}{R_6}V_{out}\right\} = \mathcal{L}\left\{-\left(\frac{R_1+R_3}{R_1}\right)\left(\frac{R_4}{R_2+R_4}\right)V_2\right\}$$

↳ Assuming $V_{out}(0) = 0$

$$sCR_5V_{out}(s) + \frac{R_5}{R_6}V_{out}(s) = -\left(\frac{R_1+R_3}{R_1}\right)\left(\frac{R_4}{R_2+R_4}\right)V_2(s)$$

$$V_{out}(s)\left[CR_5 s + \frac{R_5}{R_6}\right] = -\left(\frac{R_1+R_3}{R_1}\right)\left(\frac{R_4}{R_2+R_4}\right)V_2(s)$$

$$\boxed{\frac{V_{out}(s)}{V_2(s)} = \frac{-\left(\frac{R_1+R_3}{R_1}\right)\left(\frac{R_4}{R_2+R_4}\right)}{CR_5 s + R_5/R_6}} = \frac{\frac{R_1R_2 + 2R_1R_4 + R_2R_3 + R_3R_4}{R_1(R_2+R_4)}}{CRs + R5/R_6}$$

"simplified"

$$\frac{V_{out}(s)}{V_2(s)} = \frac{R_1R_2 + 2R_1R_4 + R_2R_3 + R_3R_4}{(CRs + R_5/R_6)(\underbrace{R_1(R_2+R_4)}_{R_1R_2+R_1R_4})} = \boxed{\frac{R_1R_2 + 2R_1R_4 + R_2R_3 + R_3R_4}{CRs(R_1R_2 + R_1R_4) + \frac{R_5}{R_6}(R_1R_2 + R_1R_4)}}$$

3. a) $F(s) = \frac{1}{s(s+1)} = \frac{K_1}{s+1} + \frac{K_2}{s}$

$\frac{s}{s(s+1)} = \frac{sK_1}{s+1} + \frac{sK_2}{s}$

$\frac{1}{s+1} = \frac{sK_1}{s+1} + K_2$ $\quad s \to 0$

$\frac{1}{1} = K_2$

$\frac{s+1}{s(s+1)} = \frac{(s+1)K_1}{s+1} + \frac{(s+1)K_2}{s}$

$\frac{1}{s} = K_1 + \frac{(s+1)K_2}{s}$

$K_1 = \frac{1}{-1} = -1$

$K_1 = \frac{1(s+1)}{s(s+1)}\Big|_{s\to -1} = -1$

$K_2 = \frac{1(s)}{s(s+1)}\Big|_{s\to 0} = 1$

$F(s) = -\frac{1}{s+1} + \frac{1}{s}$

$a=-1 \quad \frac{1}{s-a} = e^{at}$

$$\boxed{f(t) = 1 - e^{-t}}$$

b) $F(s) = \frac{5}{s(s+1)(s+5)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+5}$

$K_1 = \frac{5(s)}{s(s+1)(s+5)}\Big|_{s\to 0} = 1$

$K_3 = \frac{5(s+5)}{(s+5)\,s(s+1)}\Big|_{s\to -5} = -1/4$

$K_2 = \frac{5(s+1)}{(s+1)s(s+5)}\Big|_{s\to -1} = -5/4$

$F(s) = \frac{1}{s} - \frac{5}{4(s+1)} - \frac{1}{4(s+5)}$

$$\boxed{f(t) = 1 - \frac{5}{4}e^{-t} - \frac{1}{4}e^{-5t}}$$

c) $F(s) = \frac{3s+2}{s^2+2s+10} = \frac{3s+2}{(s+1)^2+9} = \frac{3s+2}{(s+1)^2+3^2}$

Aside: $s = \frac{-2 \pm \sqrt{4-40}}{2} = \frac{-2 \pm 6i}{2} = -1 \pm 3i$

complex

Aside from table: $e^{at}\cos(bt) = \frac{s-a}{(s-a)^2+b^2}$

By inspection

From textbook pg 42

$\frac{3s+2}{(s+1)^2+3^2} = \frac{A(s+a)+B\omega}{(s+a)^2+\omega^2} = \frac{A(s+1)+3B}{(s+1)^2+3^2}$ $\quad \omega = 3, \; a = 1$

A, B are constants for any s

$A(s+1)+3B = 3s+2$ $\quad$ Say $s=-1$ $\quad 3B = -3+2$ $\quad B = -1/3$ $\quad A=3$

$As + A + 3(-1/3) = 3s+2$ $\quad As + A = 3s+3$ $\quad \underline{A=3}$

$\mathcal{L}^{-1}\left\{\frac{A(s+a)+B\omega}{(s+a)^2+\omega^2}\right\} = Ae^{-at}\cos(\omega t) + Be^{-at}\sin(\omega t)$

$$\mathcal{L}^{-1}\left\{\frac{3s+2}{s^2+2s+10}\right\} = \boxed{3e^{-t}\cos(3t) - 1/3\, e^{-t}\sin(3t)}$$

3. d)

$$F(s) = \frac{3s^2+6s+6}{(s+1)(s^2+6s+10)}$$

$$s = \frac{-6 \pm \sqrt{36-40}}{2} = \frac{-6 \pm 2i}{2} = -3 \pm i$$

↳ complex

$-1 \pm i$

$$\frac{3s^2+6s+6}{(s+1)(s^2+6s+10)} = \frac{K_1}{s+1} + \frac{K_2 s + K_3}{s^2+6s+10}$$ ← Assume form from pg. 42

$$K_1 = \frac{(3s^2+6s+6)(s+1)}{(s^2+6s+10)(s+1)}\Big|_{s\to -1} = \frac{3-6+6}{1-6+10} = \frac{3}{5} = K_1$$

$$K_1(s^2+6s+10) + (K_2 s + K_3)(s+1) = 3s^2+6s+6$$

$$K_1 s^2 + K_1 6s + 10K_1 + K_2 s^2 + K_2 s + K_3 s + K_3$$

$$(3/5 + K_2)s^2 + (6(3/5) + K_2 + K_3)s + (10(3/5) + K_3) = 3s^2+6s+6$$

$\underline{K_2 = 12/5}$ $\quad 18/5 + 12/5 + K_3 = 6 \quad$ $\underline{K_3 = 0}$

$$\frac{3s^2+6s+6}{(s+1)(s^2+6s+10)} = \frac{3/5}{s+1} + \left(\frac{12}{5}\right)\frac{s}{s^2+6s+10}$$

$s(s+6)+10$

Assume form from Pg. 42: $\dfrac{A(s+a)+B\omega}{(s+a)^2+\omega^2} \pm \left(\dfrac{12}{5}\right)\dfrac{(s+3)-3}{(s+3)^2+1^2}$

$A = 12/5 \quad B = -36/5$

$a = 3 \quad \omega = 1$

$Ae^{-at}\cos\omega t + Be^{-at}\sin\omega t$

$$\boxed{f(t) = \frac{3}{5}e^{-t} + \frac{12}{5}e^{-3t}\cos(t) - \frac{36}{5}e^{-3t}\sin(t)}$$

e) $F(s) = \dfrac{1}{s^2+16}$ $\quad s = \dfrac{0 \pm \sqrt{0-64}}{2} = \dfrac{8i}{2} = 4i$

From Laplace table on "Paul's Online Notes"

$$\sin(at) = \frac{a}{s^2+a^2}$$

$a = 4$

$$F(s) = \frac{1}{4}\frac{4}{s^2+16}$$

$$\boxed{f(t) = \frac{1}{4}\sin(4t)}$$

4. a) $\mathcal{L}\{\ddot{y}(t)+\dot{y}(t)+3y(t)\}=0 \qquad y(0)=1 \quad \dot{y}(0)=2$

$(s^2Y(s)-sy(0)-\dot{y}(0))+(sY(s)-y(0))+3Y(s)=0$

$s^2Y(s)-s-2+sY(s)-1+3Y(s)=0$

$s^2Y(s)+sY(s)+3Y(s)=s+3 \qquad Y(s)=\dfrac{s+3}{s^2+s+3} \qquad s=\dfrac{-1\pm\sqrt{1-12}}{2}=-\dfrac{1}{2}\pm\dfrac{\sqrt{11}}{2}$

$b/2a$ complete the square $1/2$

$Y(s)=\dfrac{(s+\frac{1}{2})+\frac{5}{2}}{(s+\frac{1}{2})^2+\frac{11}{4}}=\dfrac{A(s+a)+B\omega}{(s+a)^2+\omega^2} \qquad A=1 \quad a=\frac{1}{2} \quad \omega=\sqrt{\frac{11}{4}}=\frac{\sqrt{11}}{2}$

$=Ae^{-at}\cos\omega t+Be^{-at}\sin\omega t \qquad B\sqrt{\frac{11}{4}}=\frac{5}{2}\Rightarrow 5/\sqrt{11}=B$

$\mathcal{L}^{-1}(Y(s))=\boxed{y(t)=e^{-t/2}\cos\left(\frac{\sqrt{11}}{2}t\right)+\frac{5}{\sqrt{11}}e^{-t/2}\sin\left(\frac{\sqrt{11}}{2}t\right)}$

b) $\mathcal{L}\{\ddot{y}(t)-2\dot{y}(t)+4y(t)=0\} \qquad y(0)=1 \quad \dot{y}(0)=2$

$[s^2Y(s)-sy(0)-\dot{y}(0)]-2[sY(s)-y(0)]+4Y(s)=0$

$s^2Y(s)-s-2-2sY(s)+2+4Y(s)=0$

$Y(s)[s^2-2s+4]=s+2-2 \qquad Y(s)=\dfrac{s}{s^2-2s+4} \qquad \dfrac{-2}{2}=-1$

$Y(s)=\dfrac{s-1+1}{(s-1)^2+3}=\dfrac{A(s+a)+B\omega}{(s+a)^2+\omega^2} \qquad A=1 \quad a=-1 \quad B=1/\sqrt{3} \quad \omega=\sqrt{3}$

$\mathcal{L}\{Y(s)\}=\boxed{y(t)=e^{t}\cos(\sqrt{3}t)+1/\sqrt{3}\,e^{t}\sin(\sqrt{3}t)}$

c) $\mathcal{L}\{\ddot{y}(t)+\dot{y}(t)\}=\mathcal{L}\{\sin(t)\} \qquad y(0)=1 \quad \dot{y}(0)=2$

$[s^2Y(s)-sy(0)-\dot{y}(0)]+[sY(s)-y(0)]=\dfrac{1}{s^2+1} \qquad 1+s^3+s+3s^2+3$

$s^2Y(s)-s-2+sY(s)-1=\dfrac{1}{s^2+1} \rightarrow \dfrac{1+s(s^2+1)+3(s^2+1)}{s^2+1}$

$Y(s)[s^2+s]=\dfrac{1}{s^2+1}+s+3$

$\dfrac{A(s+a)+B\omega}{(s+a)^2+\omega^2} \quad \dfrac{As+B}{s^2+1} \rightarrow \omega=1$

$Y(s)=\dfrac{1+s(s^2+1)+3(s^2+1)}{(s^2+s)(s^2+1)}=\dfrac{s^3+3s^2+s+4}{s(s+1)(s^2+1)}=\dfrac{k_1}{s}+\dfrac{k_2}{s+1}+\dfrac{As+B}{s^2+1}$

$k_1=\dfrac{0+0+0+4}{1}=4 \qquad k_2=\dfrac{-1+3-1+4}{-2}=-5/2$

$4(s+1)(s^2+1)+(-\frac{5}{2})(s)(s^2+1)+(As+B)(s)(s+1)=s^3+3s^2+s+4$

try $-1/2$ so we solve for A and B — Evaluate with two s

$A=-1/2 \qquad B=-1/2$

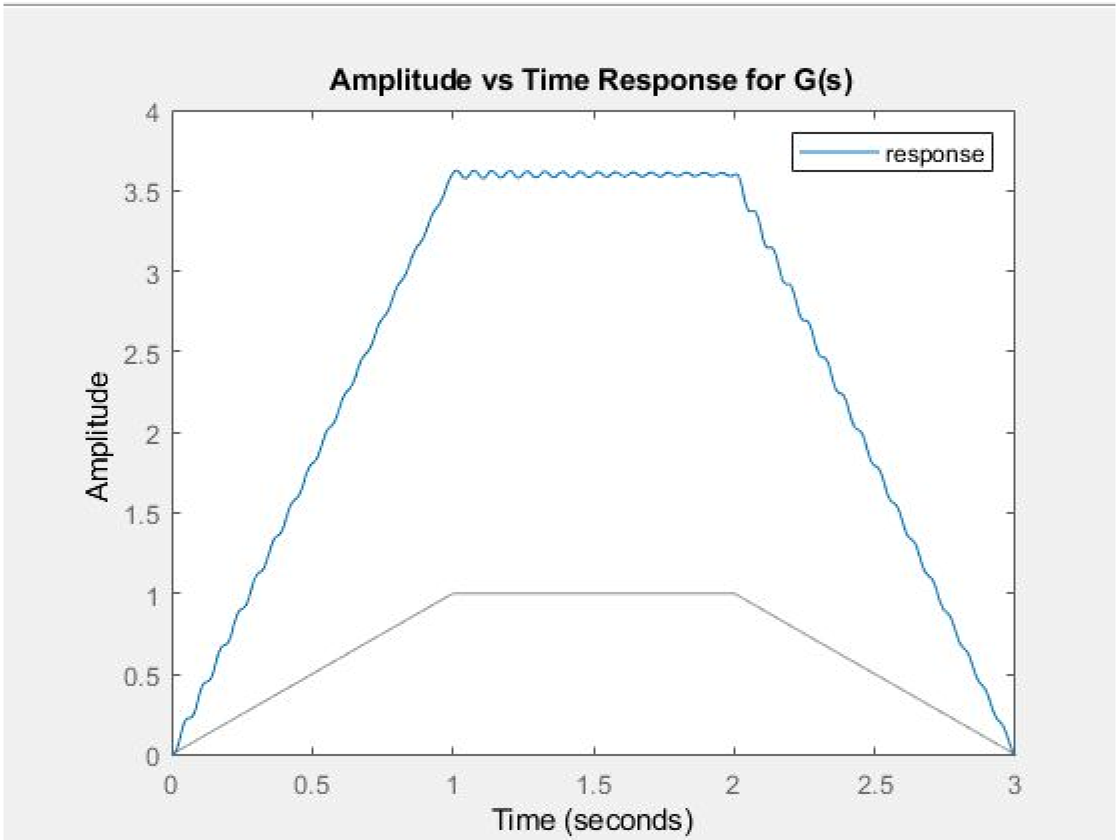
$\dfrac{4}{s}-\dfrac{5}{2}\dfrac{1}{s+1}+\dfrac{-1/2\,s-1/2}{s^2+1}=-\dfrac{1}{2}\dfrac{s}{s^2+1}-\dfrac{1}{2}\dfrac{1}{s^2+1}-5/2\dfrac{1}{s+1}+4/s$

$\boxed{y(t)=-\frac{1}{2}\cos(t)-\frac{1}{2}\sin(t)-5/2e^{-t}+4}$

ME EN 6200
Homework 2
Ryan Dalby

Problem 5

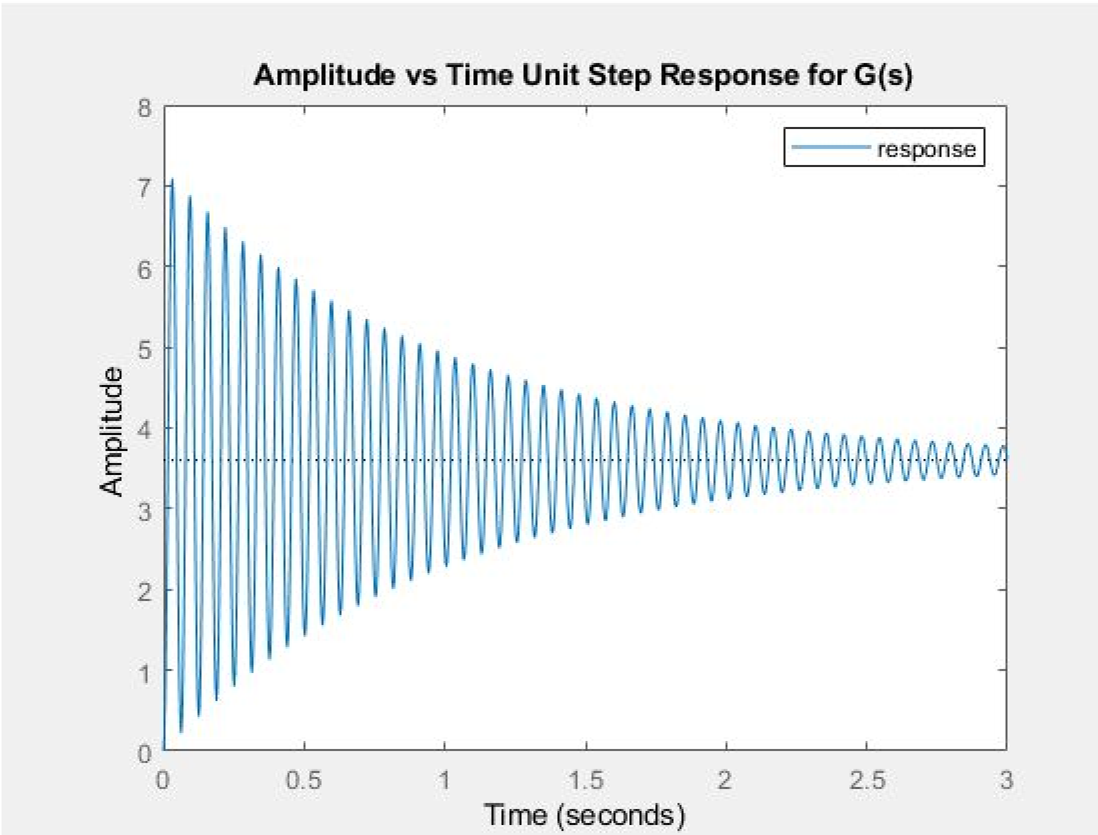
(a) Show a plot of output vs. time, label all axes.



(b) Briefly describe the response based on the input from Part (a) -- what's happening?

The response ramps up to approximately 3.5 from t=0 to t=1 then stays at approximately 3.5 from t=1 to t=2. The response finally goes back down to 0 from t=2 to t=3.The response is essentially the same signal as the input signal but amplified by a gain of around 3.5.

(c) Now suppose the input is a unit step instead of the input u(t) shown above. Simulate the response and provide a plot of output vs. time. Label all axes appropriately.



(d) For Part (c), what is the final value?

```
The final value of the unit step response is 3.61
```

(e) Rather than get the final value from the plot in Part (c), how else could you have done it?

Since the final value at t=3 is close to the settled value I could have used the final value theorem for u(s)*G(s). I could use this because all the poles for s*u(s)*G(s) = G(s) are in the open left half plane which is because the roots of the denominator of G(s), which is s^2 + 2s + 10000, have a negative real part.

(f) Provide print out of your Matlab code (m-file, Simulink model, etc.) to justify how you created your plots.

```
%% ME EN 6200 Homework 2 Ryan Dalby
%%
clear;
close all;
%% Problem 5
omega_n = 100; % Hz natural frequency
zeta = 0.01; % Damping constant
K = 3.6;

G = tf((K*omega_n^2),[1, (2*zeta*omega_n), (omega_n^2)]);
u = @(t) t.*heaviside(t) - t.*heaviside(t-1) + heaviside(t-1) -
heaviside(t-2) - (t-3).*heaviside(t-2) + (t-3).*heaviside(t-3);
dt = 0.001;
t_vals = 0:dt:3;

% a
figure;
lsim(G,u(t_vals),t_vals);
title('Amplitude vs Time Response for G(s)');
legend('response');

% c
figure;
step(G,3);
title('Amplitude vs Time Unit Step Response for G(s)');
legend('response');

% d
step_response = step(G,3);
fprintf('The final value of the unit step response is
%.2f\n',step_response(end));
```