MEEN 6200 | Homework 4 | Ryan Dalby | Page 1

1. a) $\frac{Y(s)}{U(s)} = G(s) = \frac{s+2}{s^2+3s+36}$ $Y(s) = \frac{s+2}{s(s^2+3s+36)}$ → Poles: $s=0$, $s = -\frac{3}{2} \pm \sqrt{15}\, i$

$U(s) = \frac{1}{s}$ ← step input

$$Y(s) = \frac{s+2}{s(s^2+3s+36)} = \frac{k_1}{s} + \frac{k_2 s + k_3}{s^2+3s+36}$$

$$k_1 = \frac{(s+2)\cancel{s}}{\cancel{s}(s^2+3s+36)}\Big|_{s\to 0} = \frac{2}{36} = \frac{1}{18}$$

$s+2 = k_1(s^2+3s+36) + (k_2 s + k_3)(s)$

$s+2 = k_1 s^2 + k_1 3s + k_1 36 + k_2 s^2 + k_3 s$

$-k_1 s^2 = k_2 s^2$ $k_2 = -k_1$

$s = (k_1 3s + k_3)s$

$\frac{3}{18} + k_3 = 1$ $k_3 = 15/18$

$k_2 = -\frac{1}{18}$ $k_3 =$ signs

$$\boxed{Y(s) = \frac{1}{18s} + \frac{-1/18\, s + 15/18}{s^2+3s+36}}$$

$(s+3/2)^2 + \frac{135}{4} = (s+3/2)^2 + \left(\frac{\sqrt{135}}{2}\right)^2$

b) $\frac{Y(s)}{U(s)} = G(s) = \frac{s-2}{s^2+3s+36}$ $Y(s) = \frac{s-2}{s(s^2+3s+36)} = \frac{k_1}{s} + \frac{k_2 s + k_3}{s^2+3s+36}$

$$k_1 = \frac{(s-2)\cancel{s}}{\cancel{s}(s^2+3s+36)}\Big|_{s\to 0} = -\frac{1}{18}$$

$s-2 = k_1(s^2+3s+36) + (k_2 s + k_3)s$

$k_2 = -k_1$

$s = (k_1 3s + k_3)s$

$-3/18 + k_3 = 1$

$k_2 = \frac{1}{18}$ $k_3 = \frac{21}{18}$ signs

$$\boxed{Y(s) = -\frac{1}{18s} + \frac{1/18\, s + 21/18}{s^2+3s+36}}$$

↳ $(s+3/2)^2 + \left(\frac{\sqrt{135}}{2}\right)^2$

$-\frac{27}{\sqrt{135}}$

$\frac{\sqrt{135}}{2} \cdot B = \frac{27}{36} - 3/2 + \ldots = -15$

a → $Y(s) = \frac{1}{18s} - \frac{1}{18}\frac{s - 15}{(s+3/2)^2 + \left(\sqrt{\frac{135}{2}}\right)^2} = \frac{A(s+a) + B\omega}{(s+a)^2 + \omega^2} = Ae^{-at}\cos(\omega t) + Be^{-at}\sin(\omega t)$

$Y(s) = \frac{1}{18s} - \frac{1/18(s+3/2) - \left(\frac{2}{\sqrt{135}}\right)\left(\frac{\sqrt{135}}{2}\right)}{(s+3/2)^2 + \left(\frac{\sqrt{135}}{2}\right)^2}$

b) → $Y(s) = -\frac{1}{18s} + \frac{1}{18}\frac{s + 21}{(s+3/2)^2 + \left(\frac{\sqrt{135}}{2}\right)^2} = \frac{A(s+a) + B\omega}{(s+a)^2 + \omega^2} = Ae^{-at}\cos(\omega t) + Be^{-at}\sin(\omega t)$

Explanation to follow

Note: The signs of the boxed answers.

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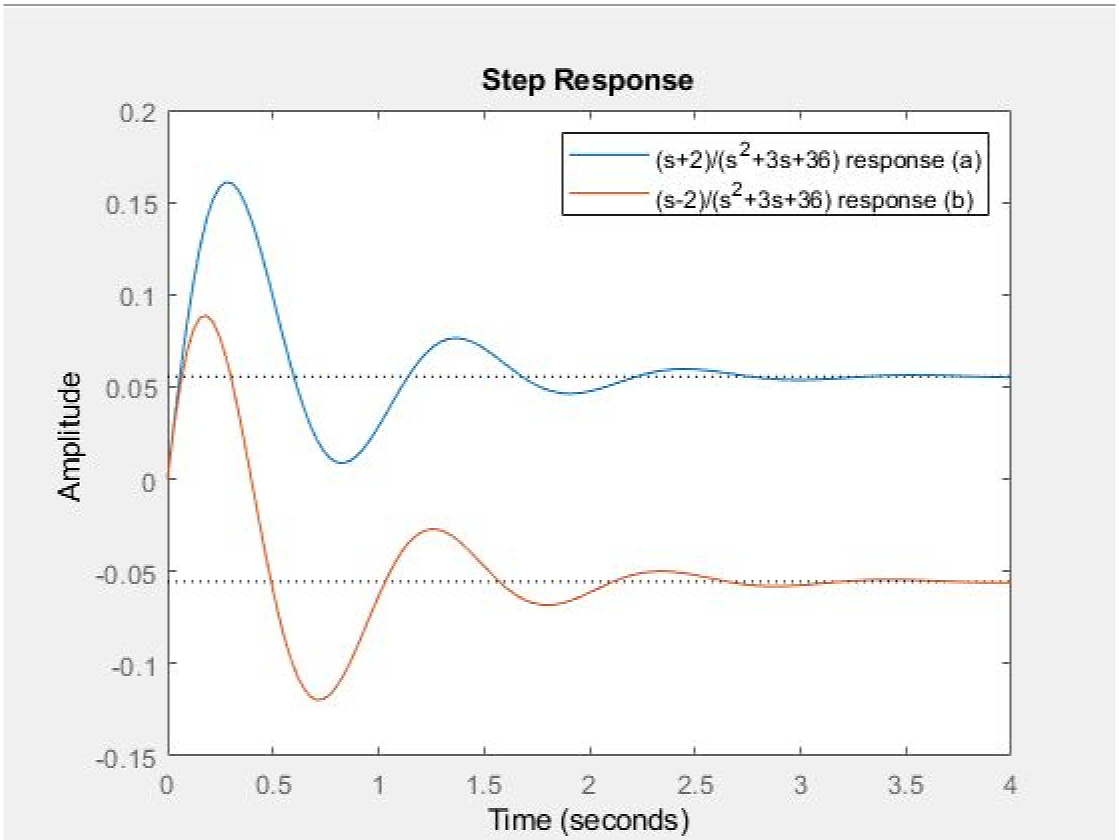
Problem 1



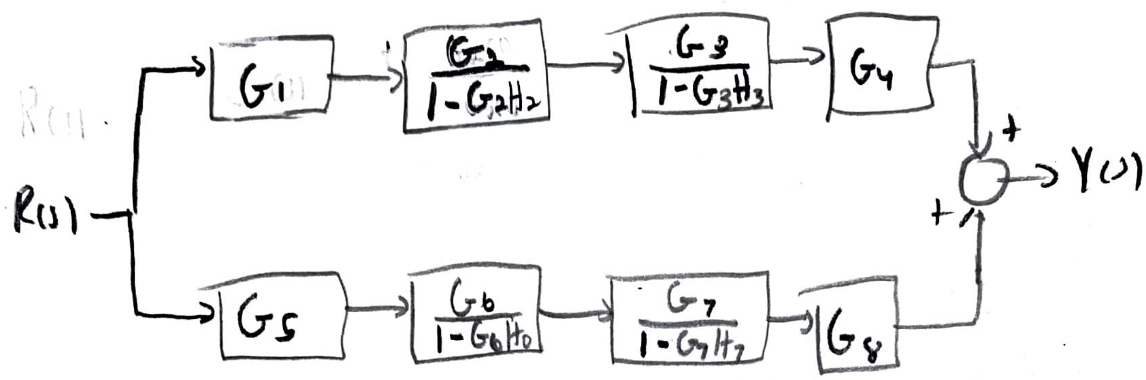

```
%% 1

G1 = tf([1,2],[1,3,36]); % a
G2 = tf([1,-2],[1,3,36]); % b

figure;
step(G1);
hold on;
step(G2);
legend('(s+2)/(s^2+3s+36) response (a)' , '(s-2)/(s^2+3s+36) response (b)');
```

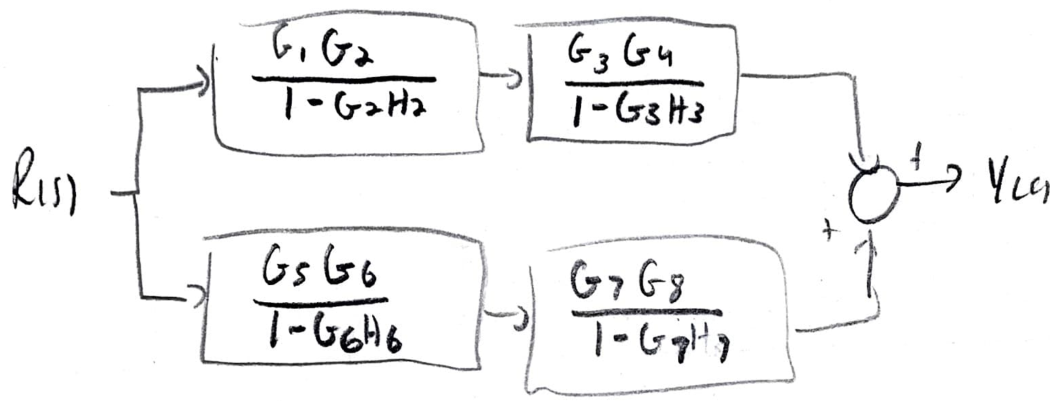
Comparing and contrasting response a and b we see that both a and b have similar shapes (settling times and time of each oscillation) but different amplitudes for each oscillation (notice the different final values). Response a peaks higher but settles to a positive value while response b peaks lower (still positive peak) and yet settles to a negative value.

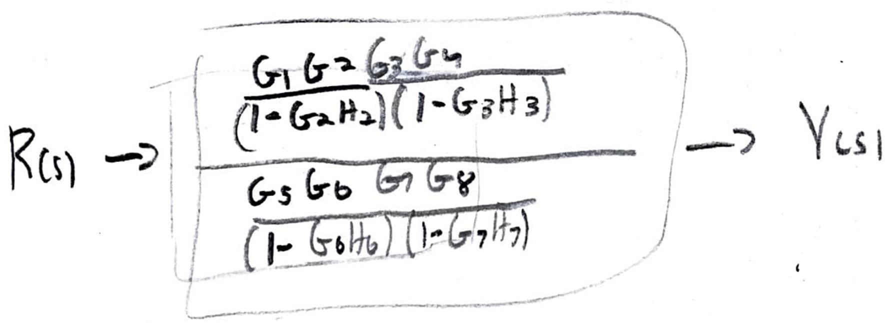
Response a has a zero at s = -2 while response b has a zero at s = 2. The poles for both are equivalent with s=0, -3/2 (+/-) i * sqrt(15). From the text, page 191, we note that we can represent the transfer functions for a and b as (s+a)/((s+b)(s+c)) where for transfer function a the value a is equal to 2 while for transfer function b the value a is equal to -2. Thus we note that system b is a non minimum-phase system. The zero is in the right half plane and the response for b "turns" toward the negative direction even though it initially increased.

This behavior can also be realized for these specific responses by looking at the partial fraction expansion for the step response of both polynomials as shown above. We notice for response b the sin and cos terms will both have the same signs but the final constant term is negative, this is why we get the initial positive behavior then settling behavior at a negative constant term.

2. Combining all feedback loops







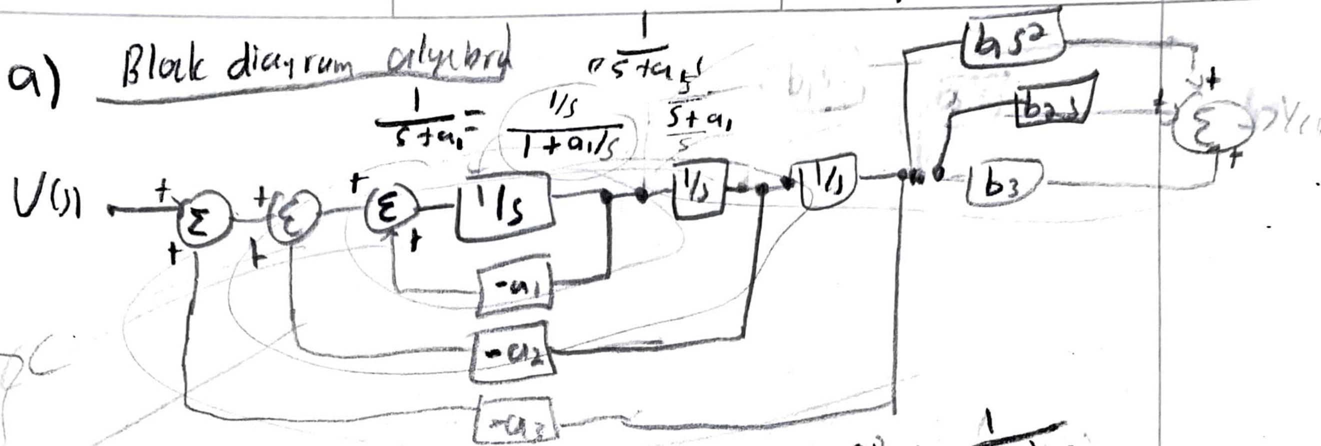
$$\boxed{\frac{Y(s)}{R(s)} = \frac{G_1(s)G_2(s)G_3(s)G_4(s)(1-G_6(s)H_6(s))(1-G_7(s)H_7(s))}{G_5(s)G_6(s)G_7(s)G_8(s)(1-G_2(s)H_2(s))(1-G_3(s)H_3(s))}}$$

Consider this simplified because if the G's or H's are fractions (represent transfer functions themselves) the 1 - GH can be simplified by taking a common denominator which will easily combine

If needed, expanding can be carried out much more effectively by a computer than me.

3. a) Block diagram algebra

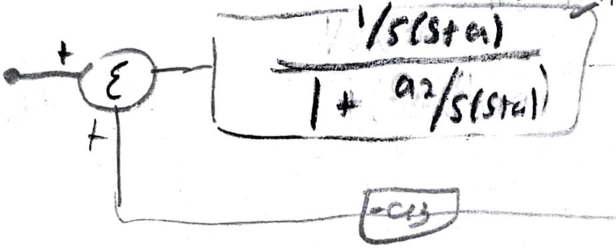
$\frac{1}{s+a_1} = \frac{1/s}{1+a_1/s}$ $\qquad \frac{1}{s^2+a_1 s} = \frac{\frac{1}{s}}{\frac{s+a_1}{s}}$

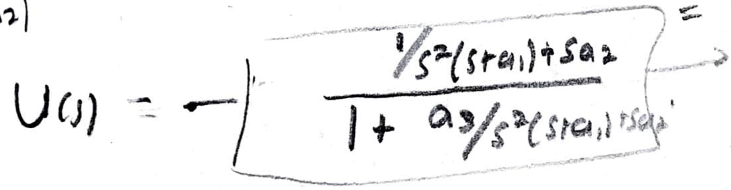


$\frac{1}{s(s+a_1)}$

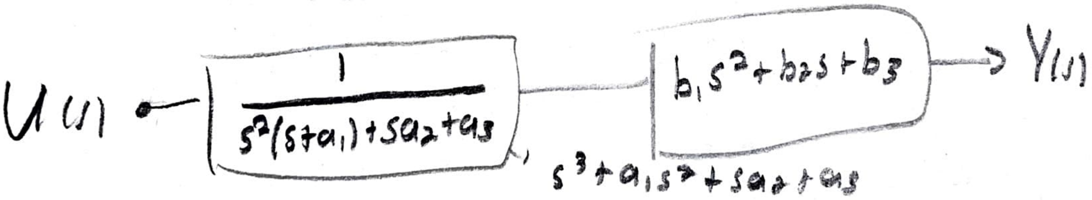
$\frac{1/s(s+a_1)}{\frac{s(s+a_1)+a_2}{s(s+a_1)}} = \frac{1}{s(s+a_1)+a_2}$

$\frac{1}{s^2(s+a_1)+sa_2} = \frac{1}{s(s(s+a_1)+a_2)}$





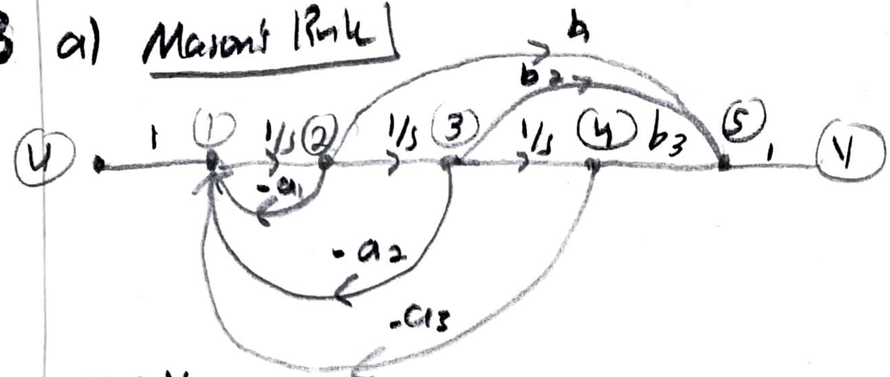
$= \frac{1}{s^2(s+a_1)+sa_2+a_3}$



$s^3+a_1 s^2+sa_2+a_3$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{b_1 s^2+b_2 s+b_3}{s^3+a_1 s^2+s a_2+a_3}}$$

3 a) Mason's Rule



U → Y

$F_1 = (1)(1/s)(1/s)(1/s)(b_3) = b_3/s^3$

$F_2 = (1)(1/s)(b_1) = b_1/s$

$F_3 = (1)(1/s)(1/s)(b_2) = b_2/s^2$

$L_1 = (1/s)(-a_1) = -a_1/s$

$L_2 = (1/s)(1/s)(-a_2) = -a_2/s^2$

$L_3 = (1/s)(1/s)(1/s)(-a_3) = -a_3/s^3$

$\Delta = 1 - \left[-a_1/s - a_2/s^2 - a_3/s^3\right] + \left[\cancel{\quad}\ \right]$ all loops touch at 1

$\Delta = 1 + a_1/s + a_2/s + a_3/s^3$

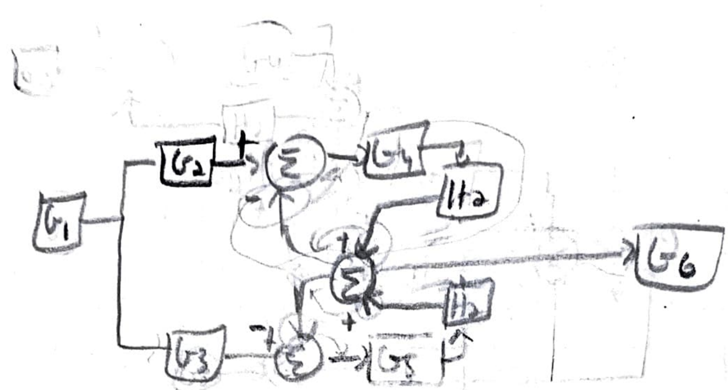
$\Delta_1 = 1$
$\Delta_2 = 1$
$\Delta_3 = 1$

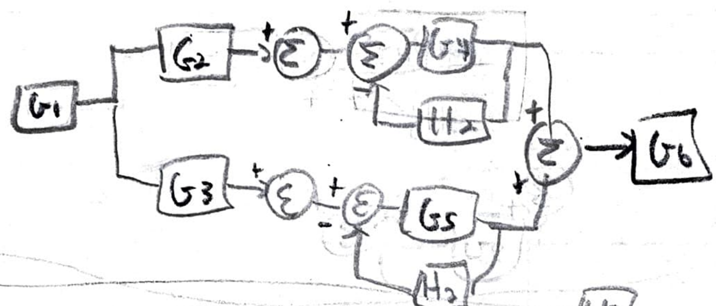
Each F_i touches 1 which all a_i are connected to through 1

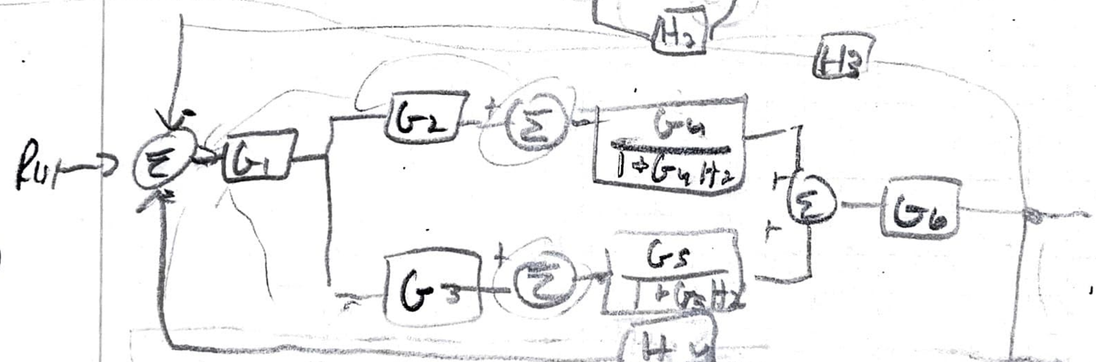
$$\frac{Y(s)}{U(s)} = \frac{1}{\Delta}\left[F_1\Delta_1 + F_2\Delta_2 + F_3\Delta_3\right] = \frac{b_3/s^3 + b_1/s + b_2/s^2}{1 + a_1/s + a_2/s^2 + a_3/s^3}\cdot\frac{s^3}{s^3}$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}}$$ ✓

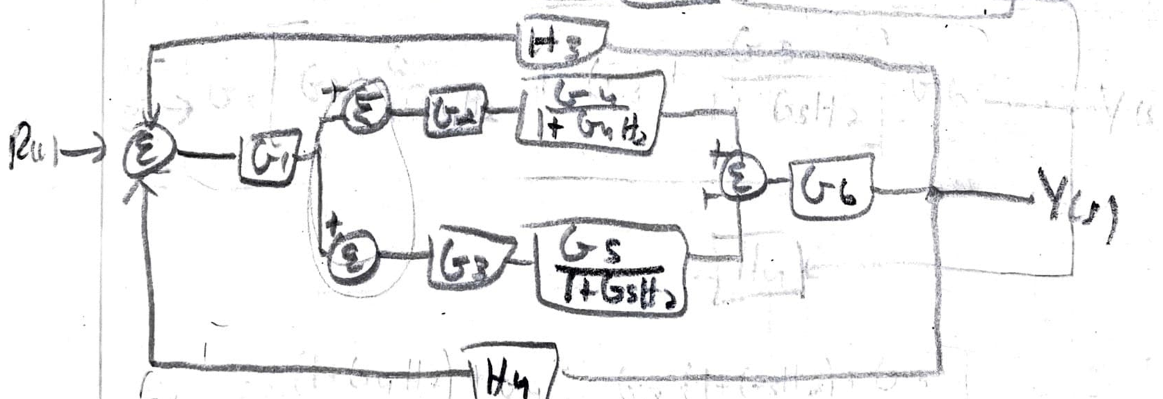
3.3 b)

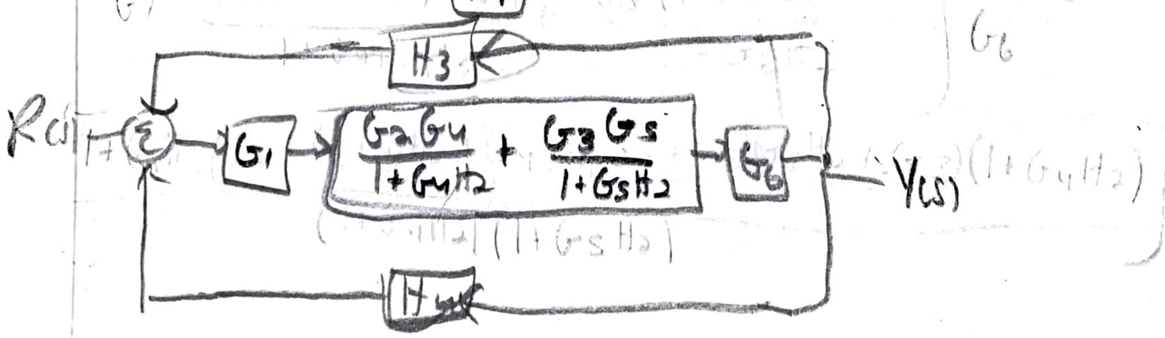

G1
G2
G3
G4
G5
G6
H2
Σ
+
-


G1
G2
G3
G4
G5
G6
H2
Σ
+
-

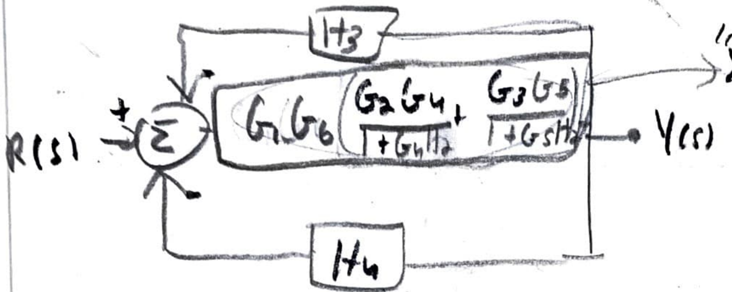

R(s)
G1
G2
G3
$\frac{G_4}{1+G_4H_2}$
$\frac{G_5}{1+G_5H_2}$
G6
H3
H4
Σ
Y(s)

X + Y +
Z + W

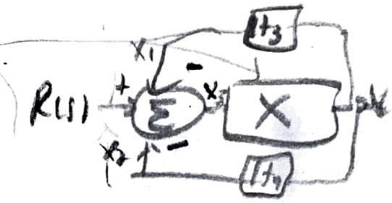

R(s)
G1
G2
G3
$\frac{G_4}{1+G_4H_2}$
$\frac{G_5}{1+G_5H_2}$
G6
H3
H4
Σ
Y(s)


R(s)
G1
$\frac{G_2G_4}{1+G_4H_2} + \frac{G_3G_5}{1+G_5H_2}$
G6
H3
H4
Σ
Y(s)

3. b) continued



"X"



$$(R + X_1 - X_2)X = Y$$

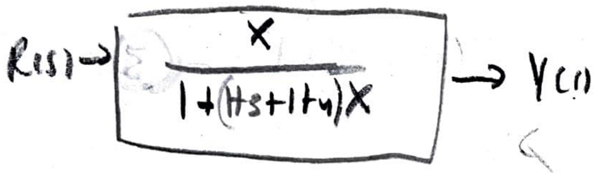
$$X_1 = YH_3$$

$$X_2 = YH_4$$

$$(R - YH_3 - YH_4)X = Y$$

$$RX = Y(H_3X + H_4X + 1)$$

$$\frac{Y}{R} = \frac{X}{1+(H_3+H_4)X}$$



$$G_1G_6\left[\frac{G_2G_4(1+G_5H_2)+G_3G_5(1+G_4H_2)}{(1+G_4H_2)(1+G_5H_2)}\right]$$

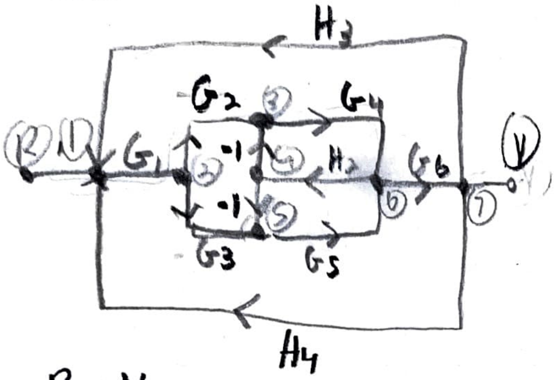
$$\frac{\frac{G_1G_6G_2G_4(1+G_5H_2)+G_1G_6G_3G_5(1+G_4H_2)}{(1+G_4H_2)(1+G_5H_2)}}{1+(H_3+H_4)\left[\frac{G_1G_6G_2G_4(1+G_5H_2)+G_1G_6G_3G_5(1+G_4H_2)}{(1+G_4H_2)(1+G_5H_2)}\right]}$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{G_1G_6G_2G_4(1+G_5H_2)+G_1G_6G_3G_5(1+G_4H_2)}{(1+G_4H_2)(1+G_5H_2)+(H_3+H_4)\left[G_1G_6G_2G_4(1+G_5H_2)+G_1G_6G_3G_5(1+G_4H_2)\right]}}$$

↳ Assuming this is equivalent to

$$\boxed{\frac{Y(s)}{R(s)} = \frac{G_1G_2G_4G_6+G_1G_3G_5G_6}{1-G_1G_2G_4G_6H_3-G_1G_3G_5G_6H_4-G_1G_3G_5G_6H_3-G_1G_2G_4G_6H_4+G_4H_2+G_5H_2}}$$

3. b) Mason's Rule



$R \to Y$

$F_1 = G_1 G_2 G_4 G_6$

$F_2 = G_1 G_3 G_5 G_6$

$L_1 = G_1 G_2 G_4 G_6 H_3$

$L_2 = G_1 G_3 G_5 G_6 H_4$

$L_3 = G_1 G_3 G_5 G_6 H_3$

$L_4 = G_1 G_2 G_4 G_6 H_4$

$L_5 = -G_4 H_2 \; (-1)$

$L_6 = -H_2 G_5 \; (-1)$

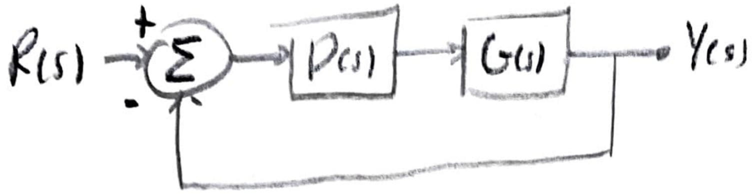
$\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5 + L_6]$

$\Delta_1 = 1 \qquad \Delta_2 = 1$

$$\frac{Y(s)}{R(s)} = \frac{1}{\Delta} \cdot [F_1 \Delta_1 + F_2 \Delta_2]$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_4 G_6 + G_1 G_3 G_5 G_6}{1 - G_1 G_2 G_4 G_6 H_3 - G_1 G_3 G_5 G_6 H_4 - G_1 G_3 G_5 G_6 H_3 - G_1 G_2 G_4 G_6 H_4 + G_4 H_2 + G_5 H_2}}$$

4. a)



$G(s) = \frac{1}{s(s+3)}$ $\qquad D = \frac{K(s+z)}{s+p}$

Open loop system $\quad R(s) \to \boxed{G(s)} \to Y(s)$ $\qquad G(s) = \frac{1}{s^2+3s}$

$\boxed{\text{Order 2}}$

b) closed loop $\quad R(s) \to \boxed{\frac{D(s)G(s)}{1+D(s)G(s)}} \to Y(s)$ $\qquad \frac{\frac{K(s+z)}{s(s+3)(s+p)}}{1+\frac{K(s+z)}{s(s+3)(s+p)}}$

$H(s)=1$

$$\frac{Y(s)}{R(s)} = \frac{K(s+z)}{s(s+3)(s+p)+K(s+z)} = \frac{K(s+z)}{s^3+s^2p+3s^2+3ps+Ks+Kz}$$

$\boxed{\text{Order 3}}$ $\qquad \frac{Y(s)}{R(s)} = \frac{Ks+Kz}{s^3+(p+3)s^2+(3p+K)s+Kz}$

c) We will represent this order 3 system as the product of an order 1 and order 2 general system. Assuming standard forms parameterized by a, ζ, ω_n

Taking $R(s) = 1/s$

$$Y(s) = \frac{\bar{K}}{s}\left[\frac{a}{s+a} \cdot \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\right]$$

Approach

Poles will dictate system behavior

↳ will fit denominator, assuming dominant second order system and try to fit first order pole $(-a)$ over 10 times the fitted 2nd order part

Equate denominators ($1/s$ cancel out)

$$(s+a)(s^2+2\zeta\omega_n s+\omega_n^2) = s^3+(p+3)s^2+(3p+K)s+Kz$$

$$s^3+(2\zeta\omega_n+a)s^2+(\omega_n^2+2\zeta\omega_n a)s+\omega_n^2 a = s^3+(p+3)s^2+(3p+K)s+Kz$$

subject to $\%OS = 10 \quad t_s = 1.5$

$$\zeta = \frac{-\ln(0.10)}{\sqrt{\pi^2+\ln(0.10)^2}} = 0.591 \qquad t_s \approx \frac{4}{\zeta\omega_n} \approx 1.5 \qquad \omega_n \approx 4.51$$

Look at poles of 2nd order part

$$s^2+2(0.5)(4.51)s+(4.51)^2 = s^2+4.51s+20.35$$

$$s = \frac{-4.51 \pm \sqrt{20.35-4(20.35)}}{2} = -2.26 \pm \frac{\sqrt{61}\,i}{2}$$

want a to be over 10 times this to have little effect

say $a = 25$ thus $s = -25 \gg -2.26$

4.c) Continued

$\zeta = 0.591$ $\quad w_n = 4.51$ $\quad a = 25$

Equate s^2 coefficients:

$2\zeta w_n + a = p + 3 \rightarrow \boxed{p = 27.33}$

Equate s coefficients:

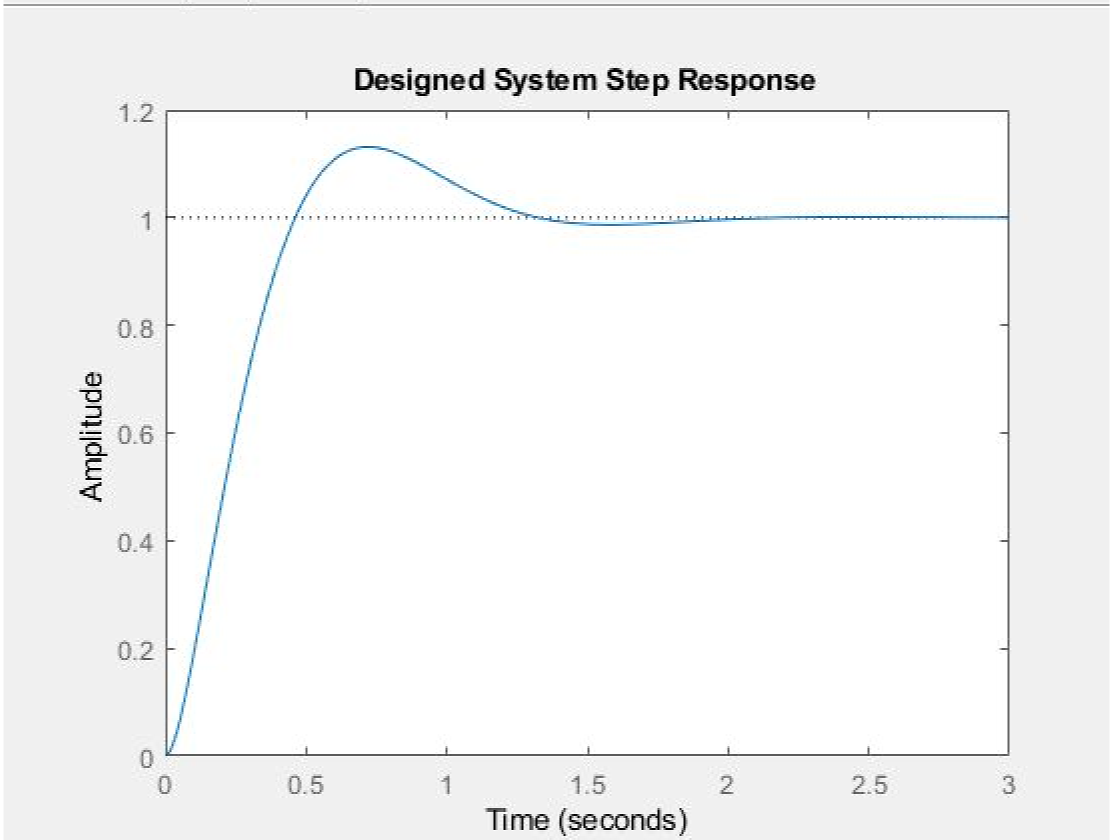
$w_n^2 + 2\zeta w_n a = 3p + k \rightarrow \boxed{k = 71.68}$

$= 82 + k$

Equate constant coefficient

$w_n^2 a = kz \rightarrow \boxed{z = 7.097}$

Problem 4




```
       RiseTime: 0.3240
   SettlingTime: 1.1987
    SettlingMin: 0.9118
    SettlingMax: 1.1312
      Overshoot: 13.1199
     Undershoot: 0
           Peak: 1.1312
       PeakTime: 0.7253
```

```
%% 4
k = 71.68;
p = 27.33;
z = 7.097;

G = tf(1,[1, 3, 0]);
D = tf([k,k*z],[1,p]);
sys = feedback(G*D, 1);

figure;
step(sys);
title('Designed System Step Response');
```

```
% Expected: 10% OS and 1.5 second(+/-2%) settling time
sys_inf = stepinfo(sys);
disp(sys_inf);
```