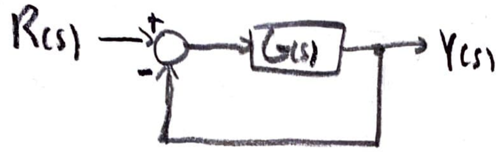
ME EN 6200 | Homework 6 | Ryan Dalby | Page 1

1. a)



$$G(s) = \frac{75(s+1)}{s(s+5)(s+25)} = \frac{75s+75}{s^3+30s^2+12\ldots}$$

Unity feedback so

$$\frac{e(s)}{R(s)} = \left[\frac{1}{1+\frac{75(s+1)}{s(s+5)(s+25)}}\right] = \frac{s(s+5)(s+25)}{s(s+5)(s+25)+75(s+1)}$$

$$(s^2+5s)(s+25)+75s+75 = s^3+30s^2+125s+75s+75$$

$$d(s) = s^3+30s^2+200s+75$$

Routh Table

1	200	0
30	75	0
197.5	0	
75		

No sign change

$\Rightarrow$ All poles of $se(s)$ are in OLHP, we have stable system (error stable $\rightarrow$ system stable). Thus we can look at e_{ss}.

$$R(s) = \frac{A}{s^2}$$

$$e_{ss} = \frac{A}{\lim_{s\to 0} sG(s)} = \frac{A}{\lim_{s\to 0}\frac{75s(s+1)}{s(s+5)(s+25)}} = \frac{125A}{75} = \boxed{e_{ss} = \frac{5A}{3}}$$

$\Rightarrow$ Also see $G(s)$ is type 1 system w/ unity feedback thus $e_{ss} = \frac{A}{K_v}$

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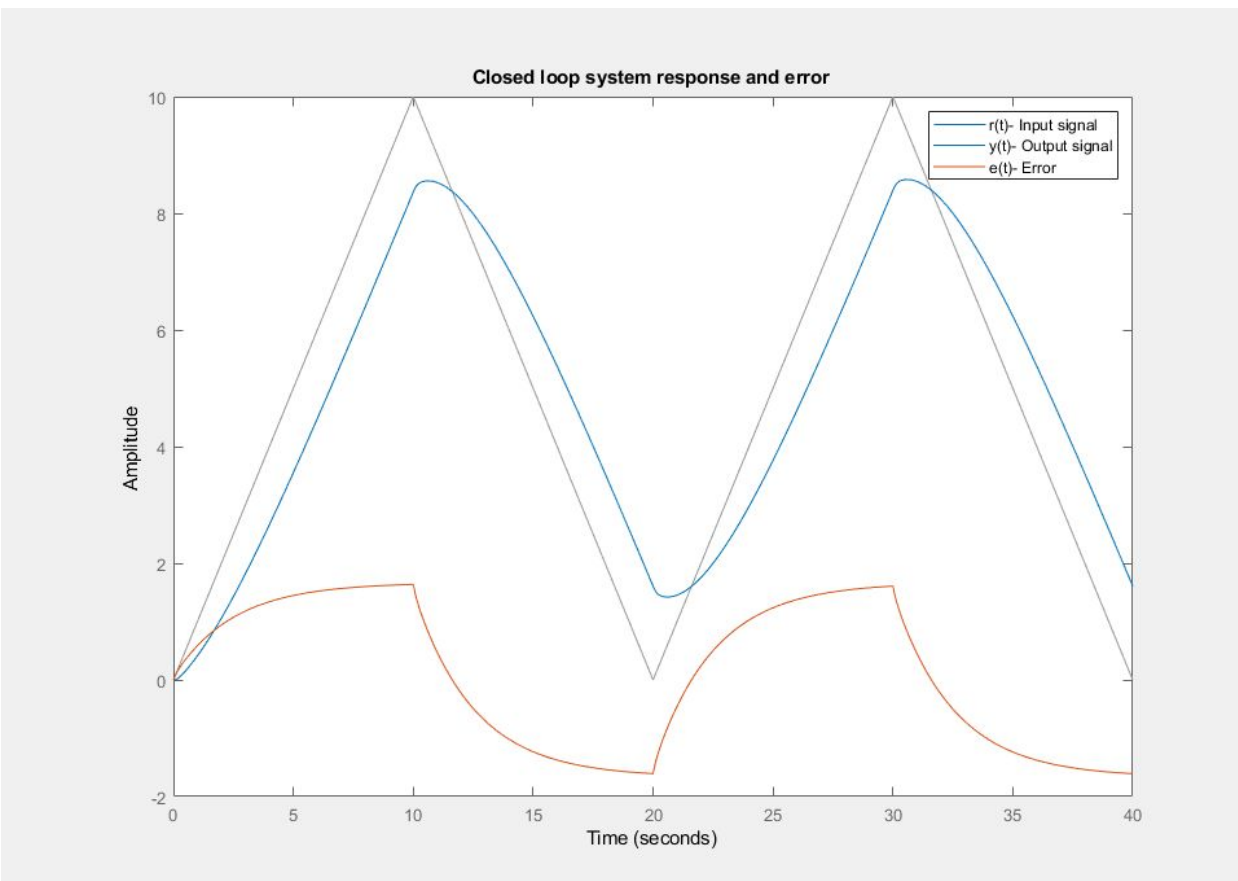
Problem 1

b)

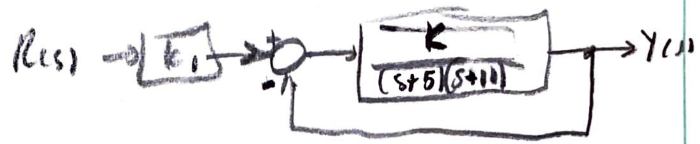
As we can see in the plot, the first ramp the error increases but slows down as it begins to approach its steady state error value. This steady state error value appears to follow what I got in part (a) which was a steady state error of 5/3 (approximately 1.6) for an A = 1. For the rest of the ramps we see similar behavior but our starting value for these ramps do not begin at the input signal value as it did for the first ramp. The general behavior of these ramps is identical to the behavior of the first ramp but offset or flipped (Like for the A=-1 ramps we approach -5/3 error). This makes sense since we have a linear system and we are essentially superimposing different constituent signal componentes to make the input signal, and thus the output and error behaves in a similarly superimposed way.



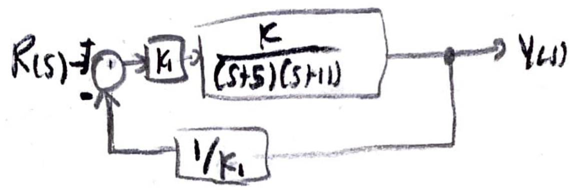
```
% b
t_step = .01; % s
t_vals = 0:t_step:40; % s
r_vals = [0:t_step:10, (10-t_step):-t_step:0, t_step:t_step:10,
(10-t_step):-t_step:0];
G = tf([75,75],[1,30 125,0]);
Y_R = feedback(G, 1);
E_R = tf([1,30,125,0],[1,30,200,75]);

figure;
plot(t_vals, r_vals);
hold on;
lsim(Y_R,r_vals,t_vals);
hold on;
lsim(E_R,r_vals,t_vals);
title('Closed loop system response and error');
legend('r(t)- Input signal','y(t)- Output signal', 'e(t)- Error');
```

2. a) $e(s) = R(s) - Y(s)$



R(s) = unit step



$$e(s) = R(s)\left[1 - T(s)\right]$$

$$T(s) = \frac{\frac{K_1 K}{(s+5)(s+11)}}{1 + \frac{K}{(s+5)(s+11)}} = \frac{K_1 K}{(s+5)(s+11)+K}$$

$$\frac{e(s)}{R(s)} = \frac{(s+5)(s+11)+K-K_1K}{(s+5)(s+11)+K}$$

$$1 - \frac{K_1 K}{(s+5)(s+11)+K} = \frac{(s+5)(s+11) - K_1 K + K}{(s+5)(s+11)+K}$$

Check stability: $d(s) = (s+5)(s+11)+K = s^2+16s+(55+K)$

$$s = \frac{-16 \pm \sqrt{16^2 - 4(55+K)}}{2} = \frac{-16 \pm \sqrt{36-4K}}{2} = -8 \pm \sqrt{9-K}$$

Assume $K > 0$ then stable ✓

$$e_{ss} = \lim_{s\to 0} \frac{s\left((s+5)(s+11)+K-K_1K\right)}{(s+5)(s+11)+K}\underbrace{\left(\frac{1}{s}\right)}_{R(s)\text{ unit step}} = \frac{55+K-K_1K}{55+K} = 1 - \frac{K_1K}{55+K}$$

$$\boxed{e_{ss} = 1 - \frac{K_1 K}{55+K}}$$

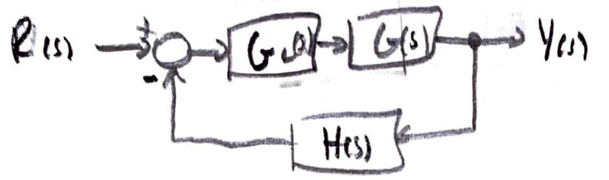
b) K_1 s.t. $e_{ss} = 0$

$$1 - \frac{K_1K}{55+K} = 0 \qquad \frac{K_1K}{55+K} = 1$$

$$K_1 K = 55 + K \qquad K_1 = 55/K + 1 = \frac{55+K}{K}$$

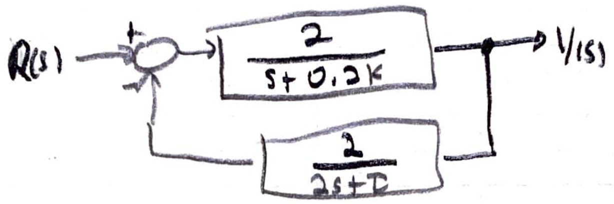
$$\boxed{K_1 = \frac{55+K}{K} = 1 + \frac{55}{K}}$$

3. a)



$G_c(s)G(s) = \frac{2}{s+0.2K}$

$H(s) = \frac{2}{2s+\tau}$



b) $e(s) = R - Y = R - T(s)R(s) = [1-T(s)]R(s)$

$$T(s) = \frac{\frac{2}{s+0.2K}}{1+\frac{4}{(s+0.2K)(2s+\tau)}} = \frac{\frac{2}{s+0.2K}}{\frac{4+(s+0.2K)(2s+\tau)}{(s+0.2K)(2s+\tau)}} = \frac{2(2s+\tau)}{4+(s+0.2K)(2s+\tau)}$$

$$\frac{e(s)}{R(s)} = \frac{(4+(s+0.2K)(2s+\tau)) - \underbrace{2(2s+\tau)}_{-4s-2\tau}}{4+(s+0.2K)(2s+\tau)} = \frac{2s^2+(\tau+0.4K-4)s+(0.2K\tau+4-2\tau)}{4+2s^2+\tau s+0.4Ks+0.2K\tau}$$

$\hookrightarrow 2s^2+(\tau+0.4K)s+(0.2K\tau+4)$

$$\boxed{\frac{e(s)}{R(s)} = \frac{2s^2+(\tau+0.4K-4)s+(0.2K\tau-2\tau+4)}{2s^2+(\tau+0.4K)s+(0.2K\tau+4)}}$$

c) $e_{ss} = \lim_{s\to 0} s\,e(s) = \lim_{s\to 0} R(s)\left[\frac{s[2s^2+(\tau+0.4K-4)s+(0.2K\tau-2\tau+4)]}{2s^2+(\tau+0.4K)s+(0.2\tau+4)}\right]$

d) $R(s) = 1/s$ $\quad \tau = 2.4 \quad$ determine K s.t. $e_{ss} = 0$

$$\lim_{s\to 0}\frac{\cancel{s}}{\cancel{s}}\left[\frac{2s^2+(2.4+0.4K-4)s+(0.2K\tau-2\tau+4)}{2s^2+\tau(0.4K)s+(0.2\tau+4)}\right] = \frac{0.2K(2.4)-2(2.4)+4}{0.2(2.4)+4}$$

$$e_{ss} = 0 = \frac{0.48K-4.8+4}{0.48+4} \Rightarrow 0.48K-0.8 = 0$$

$K = 0.8/0.48$

$\boxed{K = 5/3} = 1.\bar{6}$

3. e) For $K = 5/3$ determine %OS, t_s, t_p

$$T(s) = \frac{2(2s+\tau)}{4+(s+0.2K)(2s+\tau)} \qquad \tau = 2.4 \quad K = 5/3 \qquad R(s) = 1/s$$

$$T(s) = \frac{4s+4.8}{4+(s+1/3)(2s+2.4)} = \frac{4s+4.8}{\underbrace{2s^2+2.4s+2/3s}_{3.06s}+\underbrace{0.8+4}_{4.8}}$$

$$T(s) = \frac{2s+2.4}{s^2+1.53s+2.4}$$

2nd order system form/parameterization w/ unit step

$$d(s) = s^2+1.53s+2.4 = s^2+2\zeta\omega_n s+\omega_n^2$$

$\omega_n^2 = 2.4 \qquad \omega_n = 1.55$

$1.53 = 2\zeta(1.55) \qquad \zeta = 0.49$ underdamped

$$\%OS = \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)\times 100 = \boxed{16.8\%}$$

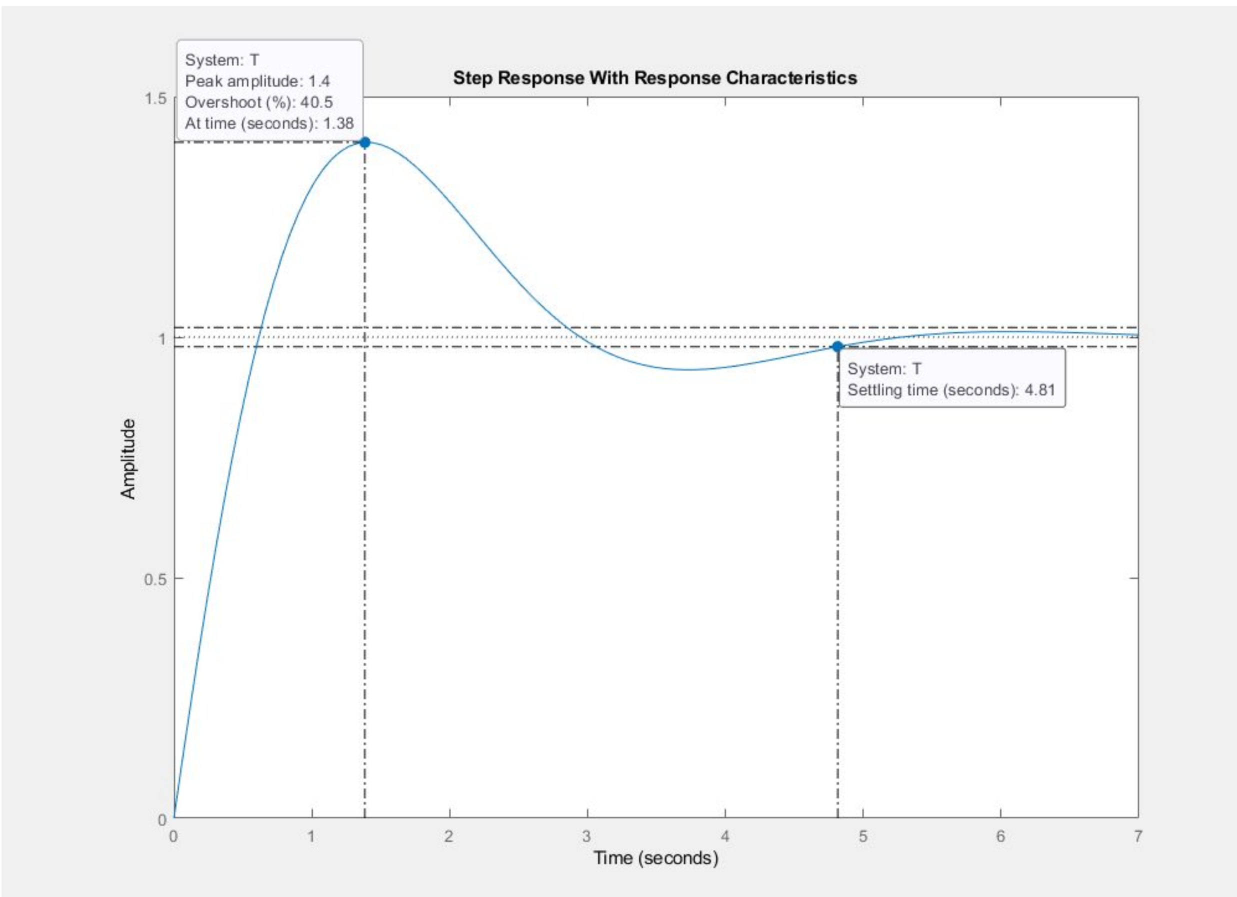
$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{(0.49)(1.55)} = \boxed{5.2 \text{ seconds}}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{(1.55)\sqrt{1-(0.49)^2}} = \boxed{2.3 \text{ seconds}}$$

Problem 3

f)

As we can see in the step response, the estimate for the transfer function characteristics in part (e) was not close. For (e) the response was predicted to have approximately 17%OS, a 5.2 second settling time, and a 2.3 second time to peak. The plotted response has a 40.5%OS, a 4.8 second settling time, and a 1.4 second time to peak. Thus it is clear that the response characteristics as estimated by the simple second order system parameterization with no zeroes is an ineffective estimate for this system. This is likely because the system contains a zero in the numerator that affects the speed of the response of the system. In this case the zero makes the system respond faster as indicated by a quicker time to peak and a higher percent overshoot. Both systems have similar settling times.




```
     RiseTime: 0.4825
 SettlingTime: 4.8144
  SettlingMin: 0.9115
  SettlingMax: 1.4048
    Overshoot: 40.4808
   Undershoot: 0
         Peak: 1.4048
     PeakTime: 1.3846
```

```
%% 3
% f
K = 5/3;
tau = 2.4;
T = tf([2,2.4],[1,1.53,2.4]);

figure;
step(T);
title('Step Response With Response Characteristics');
disp(stepinfo(T));
```

4. a) $G(s) = \frac{A}{s(s+a)}$ Sensitivity for A = Assuming $T(s)$ is unity feedback

$$S_A^{T(s)} = \lim_{\Delta A \to 0} \frac{\Delta T / T}{\Delta R / R} = \frac{A}{T(s)} \cdot \frac{\partial T(s)}{\partial A}$$

$$T(s) = \frac{\frac{A}{s(s+a)}}{1+\frac{A}{s(s+a)}} = \frac{A}{s(s+a)} \frac{s(s+a)}{s(s+a)+A} = \frac{A}{s(s+a)+A} = A\left(s(s+a)+A\right)^{-1}$$

$$S_A^{T(s)} = \frac{A}{\frac{A}{s(s+a)+A}} \cdot \frac{\partial T(s)}{\partial A}$$

$$\frac{\partial T(s)}{\partial A} = (s(s+a)+A)^{-1} + A \cdot -A(s(s+a)+A)^{-2}$$

$$S_A^{T(s)} = s(s+a)+A\left[\frac{s(s+a)}{(s(s+a)+A)^{\cancel{2}}}\right]$$

$$\frac{\partial T(s)}{\partial A} = \frac{1}{(s(s+a)+A)} - \frac{A}{(s(s+a)+A)^2}$$

$$\frac{\partial T(s)}{\partial A} = \frac{s(s+a)}{(s(s+a)+A)^2}$$

$$\boxed{S_A^{T(s)} = \frac{s(s+a)}{s(s+a)+A}}$$

b) $$S_a^{T(s)} = \frac{a}{\frac{A}{s(s+a)+A}} \cdot \frac{\partial T(s)}{\partial a}$$

$$\frac{\partial T(s)}{\partial a} = \frac{\partial}{\partial a}\left[\frac{A}{s^2+as+A}\right] = A(s^2+As)^{-1}$$

$$\frac{\partial T(s)}{\partial a} = -sA(s^2+as+A)^{-2}$$

$$S_a^{T(s)} = \frac{a[\cancel{s(s+a)+A}]}{A} \cdot \frac{-s\cancel{A}}{(s(s+a)+A)^{\cancel{2}}}$$

$$\boxed{S_a^{T(s)} = \frac{-as}{s(s+a)+A}}$$

c) $T(s)$ with β feedback

$$T(s) = \frac{\frac{A}{s(s+a)}}{1+\frac{\beta A}{s(s+a)}} = \frac{A}{s(s+a)} \frac{s(s+a)}{s(s+a)+\beta A} = \frac{A}{s(s+a)+\beta A}$$

$$S_\beta^{T(s)} = \frac{\beta}{\frac{A}{s(s+a)+\beta A}} \frac{\partial T(s)}{\partial \beta}$$

$$\frac{\partial T(s)}{\partial \beta} = A\left(s(s+a)+\beta A\right)^{-1} = -A^2\left(s(s+a)+\beta A\right)^{-2}$$

$$S_\beta^{T(s)} = \frac{\beta[\cancel{s(s+a)+\beta A}]}{\cancel{A}} \cdot \frac{-A^{\cancel{2}}}{(s(s+a)+\beta A)^{\cancel{2}}} = \boxed{S_\beta^{T(s)} = \frac{-\beta A}{(s(s+a)+\beta A)}}$$

Problem 4

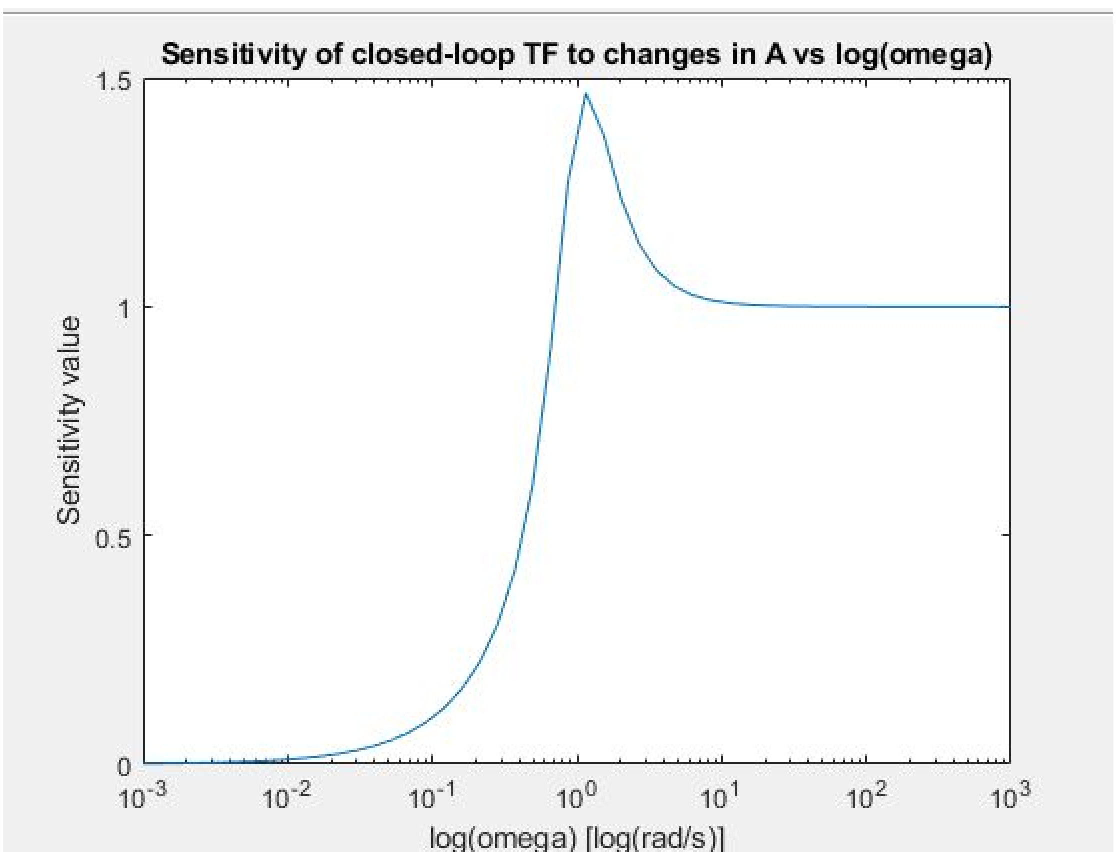
d)

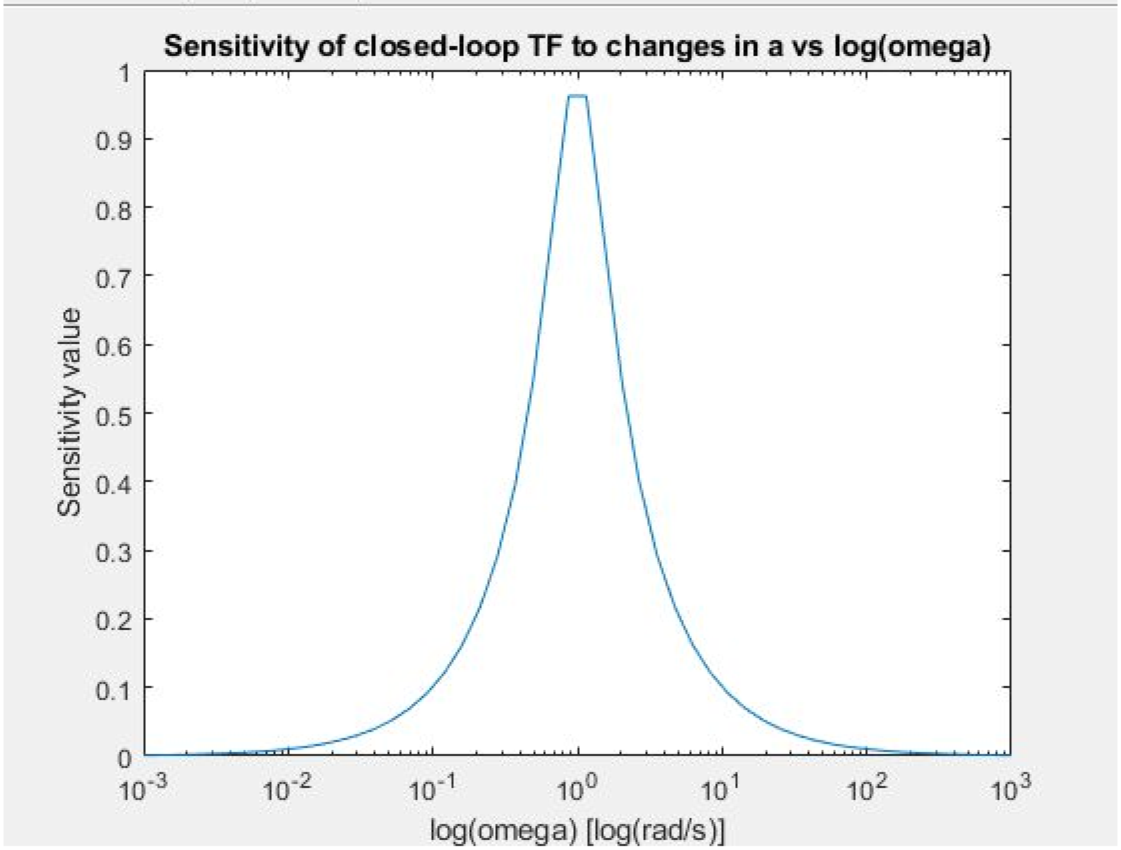
(Note as indicated in the problem A was taken to be 1 and a was taken to be 1. Thus beta was given a few values to investigate how beta itself effects its own sensitivity equation)

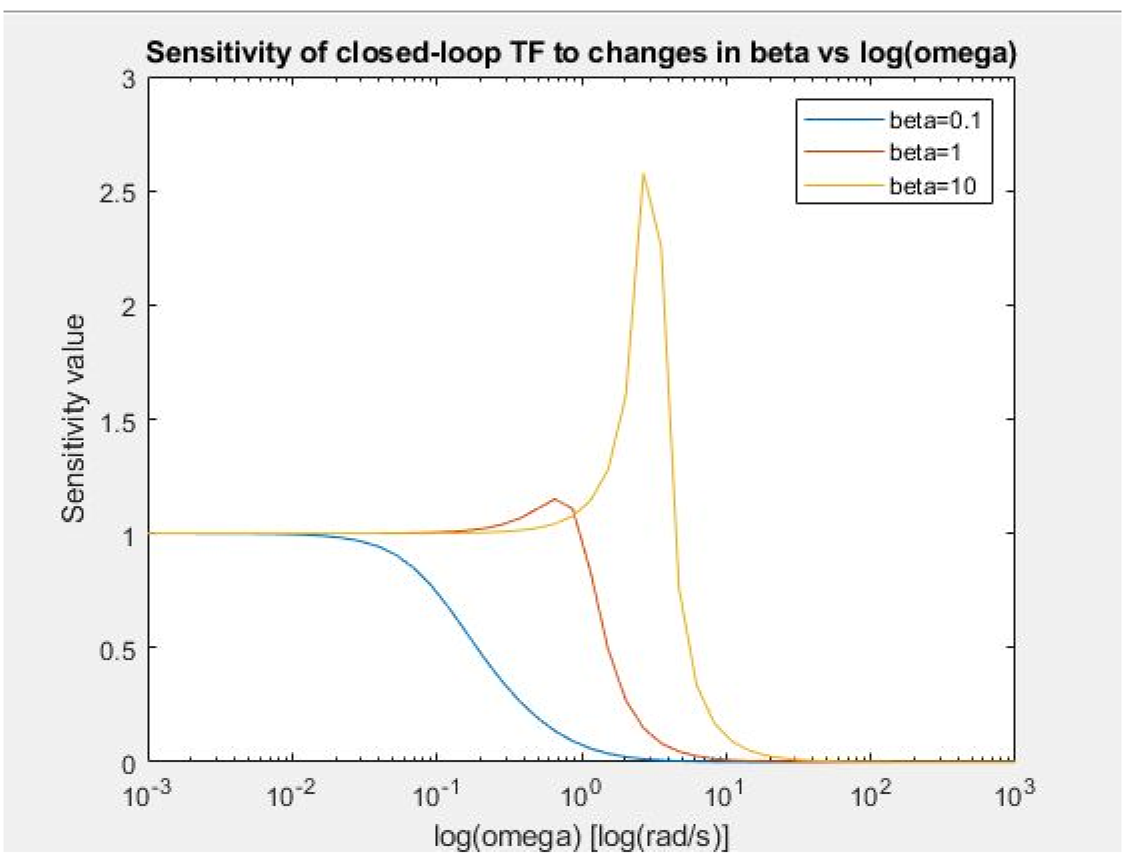
We can see that varying A has a very prominent effect on the system when at frequencies near 1 rad/s or greater. At a frequency of 0 rad/s A has little to no effect on the system. At frequencies above 1 rad/s it appears that the system settles to a sensitivity value of 1 which means varying A does have a prominent effect on the system.

We can see that varying “a” has a very prominent effect on the system when at frequencies near 1 rad/s. At a frequency of 0 rad/s or a frequency much greater than 1 rad/s “a” has little to no effect on the system.

We can see that varying beta has different effects depending on what beta is. The general pattern of beta is that near a frequency of 0 rad/s it has a sensitivity of 1 which means that changing beta at frequencies near 0 rad/s has a large effect on the system. At much higher frequencies (much greater than 1 rad/s) we see that beta has little to no effect on the system. In the middle (near a frequency of 1 rad/s) the effect on the system varies based on the value of beta itself, with higher beta values giving a very high sensitivity value just after a frequency of 1 rad/s while lower frequencies do not peak the sensitivity value just past 1 rad/s.




Sensitivity of closed-loop TF to changes in a vs log(omega)
Sensitivity value
log(omega) [log(rad/s)]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}


Sensitivity of closed-loop TF to changes in beta vs log(omega)
beta=0.1
beta=1
beta=10
Sensitivity value
log(omega) [log(rad/s)]
3
2.5
2
1.5
1
0.5
0
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

```
%% 4
% d
omega_vals = logspace(-3,3);
a = 1;
A = 1;
beta = 0.1;
s = 1i .* omega_vals;

S_A = (s.*(s+a)) ./ (s.*(s+a) + A);
S_a = (-a.*s) ./ (s.*(s+a) + A);
S_beta = (-beta .* A) ./ (s.*(s+a) + beta*A);

figure;
semilogx(omega_vals, abs(S_A));
title('Sensitivity of closed-loop TF to changes in A vs log(omega)');
xlabel('log(omega) [log(rad/s)]');
ylabel('Sensitivity value');

figure;
semilogx(omega_vals, abs(S_a));
title('Sensitivity of closed-loop TF to changes in a vs log(omega)');
xlabel('log(omega) [log(rad/s)]');
ylabel('Sensitivity value');

figure;
semilogx(omega_vals, abs(S_beta));
hold on;
beta = 1;
S_beta = (-beta .* A) ./ (s.*(s+a) + beta*A);
semilogx(omega_vals, abs(S_beta));
hold on;
beta = 10;
S_beta = (-beta .* A) ./ (s.*(s+a) + beta*A);
semilogx(omega_vals, abs(S_beta));
title('Sensitivity of closed-loop TF to changes in beta vs log(omega)');
legend('beta=0.1', 'beta=1', 'beta=10');
xlabel('log(omega) [log(rad/s)]');
ylabel('Sensitivity value');
```