

Do the following problems and show all your work for full credit. Note: not all problems will be graded, but you must complete all problems to get full credit.

Problem 1 [40 points]

Consider the following polynomials that represent the characteristic equation of a system. Using Routh-Hurwitz stability criterion to determine the system's stability:

- (a) $s^3 + 4s^2 + 8s + 4$
- (b) $s^3 + 2s^2 - 6s + 20$
- (c) $s^4 + s^2 + 2s^2 + 12s + 10$
- (d) $s^6 + 2s^5 + 12s^4 + 4s^3 + 21s^2 + 2s + 10$

Check your results with Matlab's 'roots' function.

Problem 2 [25 points]

Consider a standard feedback control system where the controller $C(s) = K$, where K is a constant and the system transfer function is given by $G(s) = \frac{s+40}{s(s+10)}$ and the feedback transfer function is given by $F(s) = \frac{1}{s+20}$.

- (a) [5 pts] Draw the block diagram for this scenario and label all signals and blocks.
- (b) [10 pts] Determine the range of K for stability of the closed-loop system.
- (c) [10 pts] For the gain that makes the system marginally stable (i.e., poles on the $j\omega$ -axis), determine the value of ω . In other words, determine the frequency of oscillation.

Problem 3 [30 points]

Consider a standard feedback control system where the closed-loop system has the following characteristic equation:

$$s^3 + (1+K)s^2 + 10s + (5+15K) = 0$$

Assume K is positive. (a) What is the maximum value of K before the system becomes unstable? (b) For this value of K, what is the frequency of oscillation? Note, this is asking the same as Problem 2(c).

Problem 4 [30 points]

A closed-loop feedback system has the transfer function given by:

$$T(s) = \frac{1}{s^3 + 5s^2 + 20s + 6}$$

- (a) Determine whether the system is stable
- (b) Plot the response for a unit step response and comment on the behavior and how it relates to the result in (a).