1. a) s^3+4s^2+8s+4

s^3	1	8	0
s^2	4	4	0
s^1	7	0	0
s^0	4	0	0

$\frac{-(4-32)}{4} = 7$ $\quad \frac{0-0}{4}$

$\frac{-(0-28)}{7} = 4$

No sign changes → System is stable (~~Possibly marginally~~) No rows of zeroes

0 poles in ORHP $\boxed{\text{It is not unstable}}$ (0 poles in ORHP)

b) $s^3+2s^2-6s+20$

s^3	1	-6	0
s^2	2	20	0
s^1	-16	0	0
s^0	20	0	0

$\frac{-(20-(-12))}{2} = -16$

$\frac{-(0-(-16\cdot 20))}{-16} = 20$

2 sign changes → 2 poles in ORHP → System is Unstable

c) $s^4+s^3+2s^2+12s+10$

s^4	1	2	10
s^3	1	12	0
s^2	-10	10	0
s^1	13	0	0
s^0	10	0	0

$\frac{-(12-2)}{1} = -10$ $\quad \frac{-(0-10)}{1} = 10$

$-\frac{(10+(10)(12))}{-10} = 13$

$\frac{-(0-130)}{13} = 10$

2 sign changes → 2 poles in ORHP → System is unstable

1. d) $s^6+2s^5+12s^4+4s^3+21s^2+2s+10$

s^6	1	12	21	10
s^5	2	4	2	0
s^4	10	20	10	0
s^3	$\cancel{0}$ 40	$\cancel{0}$ 40	$\cancel{0}$ 0	0
s^2	10	10	0	0
s^1	$\cancel{0}$ 20	0	0	0
s^0	10	0	0	0

$\frac{-(4-24)}{2}=10$ $\quad \frac{-(0-20)}{2}=10$

$\frac{-(2-42)}{2}=20$

$\frac{-(40-40)}{40/10}=0$

$\frac{-(20-20)}{10}=0$

Auxillary equation: $10s^4+20s^2+10=A(s)$

$$\frac{dA(s)}{ds}=40s^3+40s$$

$\frac{-(400-800)}{40}=10$ $\quad \frac{-(0-400)}{40}=10$

$\frac{-(400-400)}{10}=0$

Auxillary equation: $10s^2+10=A(s)$

$$\frac{dA(s)}{ds}=20s$$

$$\frac{-(0-200)}{20}=10$$

Analysis: No sign changes → 0 poles in ORHP

↳ System is not unstable

System is stable (possibly marginally)

↳ likely "Marginally stable since we had a row of zeros implying $10s^4+20s^2+10$ is a factor of $d(s)$ thus there are likely poles on the $j\omega$ axis.

ME EN 6200
Homework 5
Ryan Dalby

Problem 1

```
(stable) a roots:
  -1.6478 + 1.7214i
  -1.6478 - 1.7214i
  -0.7044 + 0.0000i

(unstable) b roots:
  -4.3981 + 0.0000i
   1.1991 + 1.7634i
   1.1991 - 1.7634i

(unstable) c roots:
   0.9237 + 2.1353i
   0.9237 - 2.1353i
  -1.8474 + 0.0000i
  -1.0000 + 0.0000i

(marginally stable) d roots:
  -1.0000 + 3.0000i
  -1.0000 - 3.0000i
   0.0000 + 1.0000i
   0.0000 - 1.0000i
  -0.0000 + 1.0000i
  -0.0000 - 1.0000i
```

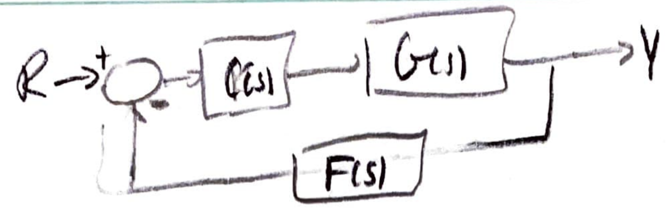
```
%% 1
% a
disp('(stable) a roots:');
disp(roots([1 4 8 4]));

% b
disp('(unstable) b roots:');
disp(roots([1 2 -6 20]));

% c
disp('(unstable) c roots:');
disp(roots([1 1 2 12 10]));

% d
disp('(marginally stable) d roots:');
disp(roots([1 2 12 4 21 2 10]));
```

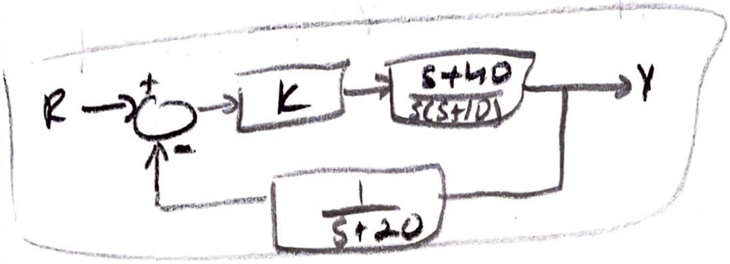
2. a)



$C(s) = K$

$G(s) = \dfrac{s+40}{s(s+10)}$

$F(s) = \dfrac{1}{s+20}$



b)

$$\frac{\dfrac{K(s+40)}{s(s+10)}}{1 + \dfrac{K(s+40)}{s(s+10)}\dfrac{1}{(s+20)}} = \frac{K(s+40)}{\cancel{s(s+10)}} \cdot \frac{\cancel{s(s+10)}(s+20)}{s(s+10)(s+20)+K(s+40)}$$

Ks

$$R \rightarrow \boxed{\frac{K(s+40)(s+20)}{s(s+10)(s+20)+K(s+40)}} \rightarrow Y$$

$d(s) = (s^2+10s)(s+20) + Ks + 40K$ ← Monic polynomial

$d(s) = s^3 + 20s^2 + 10s^2 + 200s + Ks + 40K$

$d(s) = s^3 + 30s^2 + (200+K)s + 40K$

s^3	1	(200+K)	0
s^2	30	40K	0
s^1	$200 - K/3$	0	0
s^0	40K	0	0

>0 for stability

$$-\frac{\left[40K - (6000+30K)\right]}{30} = \frac{-6000+10K}{-30}$$

$200 - 1/3K$

200

600

for stability these must be true

$200 - K/3 > 0$

$40K > 0$

$$\frac{\left[0 - 40K(200-K/3)\right]}{-(200-K/3)} = 40K$$

K<600

40K>0

$K < 600$

$K > 0 \qquad K < 600$

$$\boxed{0 < K < 600}$$

2. c) If poles are on $j\omega$ axis then we have a row of zeros on the Routh table. (an even polynomial) ($d(s)$ has a constant term so no pole at $s=0$)
↳ will look at these scenarios and make sure they lie on $j\omega$ axis (If (with a row of zeros) its a factor)

Suppose $k=600$

s^3	1	800	0
s^2	30	24000	0
s^1	0	0	0
s^0			

$A(s) = 30s^2 + 24000$ ← Even factor of $d(s)$

$30s^2 + 24000 = 0$ $\qquad s^2 = -800$

$s = \pm\sqrt{800}\,i$

$\boxed{\omega_n = \sqrt{800}\ \text{rad/s}}$

↳ 28.28 rad/s

3. a) $s^3 + (1+K)s^2 + 10s + (5+15K) = 0$

Assume $K>0$ $\qquad$ Max K before we get poles in ORHP?

s^3	1	10	0
s^2	$1+K$	$5+15K$	0
s^1	$\frac{5(1-K)}{1+K}$	0	0
s^0	$\frac{5(-3K^2+2K+1)}{1+K}$	0	0

$\frac{-(5+15K-10-10K)}{1+K} = \frac{+5-5K}{1+K} = \frac{5(1-K)}{1+K}$

$\frac{-\left(0-\left[\frac{5-5K}{1+K}(5+15K)\right]\right)}{\frac{5-5K}{1+K}} = \frac{-(25+75K-25K-75K^2 \ldots)}{1+K} = \frac{\ldots}{1+K}$

$\frac{-15(K+1/3)(K-1)}{1-K} = \frac{-15K^2+10K+5}{1-K} = \frac{-75K^2+50K+25}{5-5K}$

$\frac{-10\pm\sqrt{100+300}}{-30}$ $\quad \frac{-10\pm 20}{-30}$ $\quad -1/3,\ 1$

For no poles in ORHP ($K>0$)

$1+K > 0$ ✓ $K>0$

$\frac{5(1-K)}{1+K} \geq 0 \Rightarrow 0<K<1$

$\frac{5(-3K^2+2K+1)}{1-K} > 0$

$K<1$

$0<K\leq 1$

$\boxed{K=1}$ → will check this edge case but $0<K<1$ will have no poles in ORHP (thus NOT unstable)

3. b) If $K=1$

s^3	1	10	0
s^2	2	20	0
s^1	~~0~~ 4	0	0
s^0	20		

$A(s) = 2s^2 + 20$ ← This is a auxiliary polynomial

$A'(s) = 4s$

$A(s) = 2s^2+20 = 0$ ← even factor of $d(s)$

$s^2 = -10$ $\quad s = \pm\sqrt{-10}$ $\quad s = \pm\sqrt{10}\, i$

$\boxed{\omega_n = \sqrt{10} \text{ rad/s}}$

4. a) $T(s) = \dfrac{1}{s^3+5s^2+20s+6}$

will look at poles: (Routh table)
$d(s) = s^3+5s^2+20s+6=0$

s^3	1	20	0
s^2	5	6	0
s^1	94/5	0	
s^0	6		

$\dfrac{-(6 \cdot 1 - 100)}{5} = \dfrac{94}{5}$

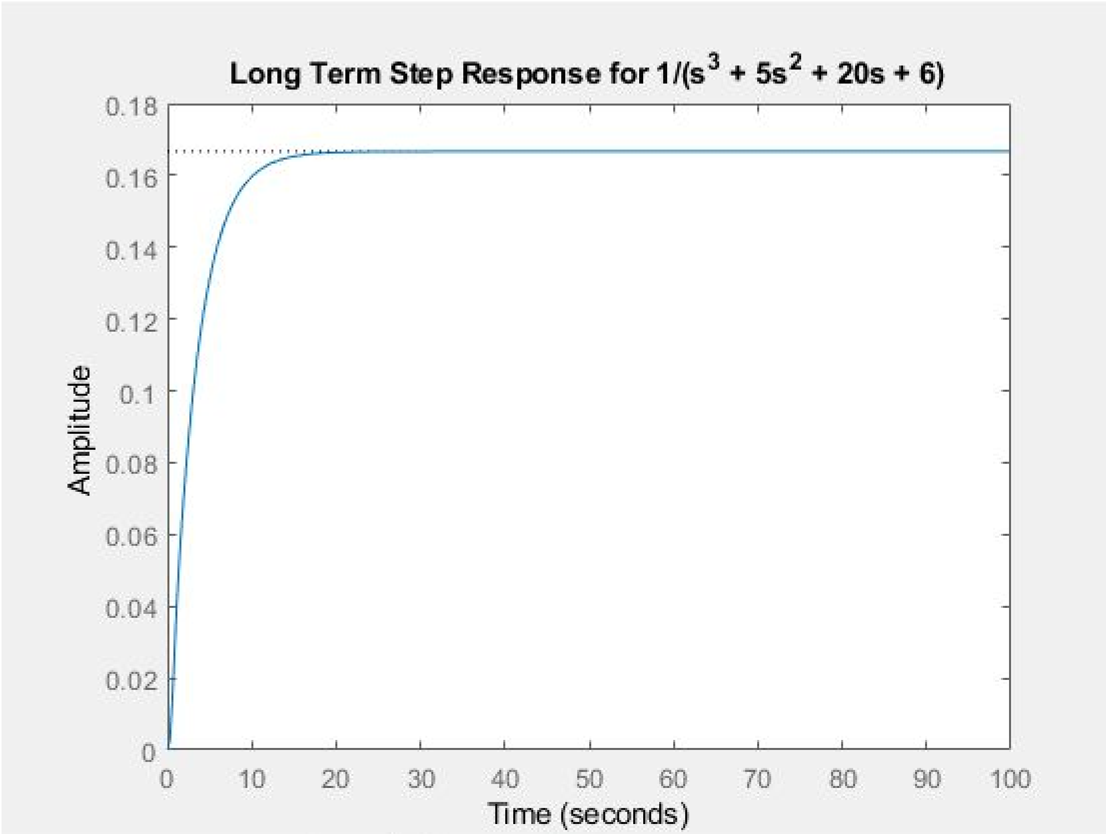
$\dfrac{-(0 - 94/5)6)}{94/5} = 6$

We see no sign changes thus NO poles on ORHP

↳ system is $\boxed{\text{not unstable}}$

↳ $\boxed{\text{System is Stable}}$ since we have no 0 rows (thus no even divisors of the polynomial) so no poles on $j\omega$ axis

Problem 4




```
%% 4
T = tf(1,[1 5 20 6]);
tFinal = 100; %s Will see long term step response
figure;
step(T, tFinal);
title('Long Term Step Response for 1/(s^3 + 5s^2 + 20s + 6)');
```

b) Looking at the step response of this closed loop feedback transfer function we can see a stable response where as time increases the amplitude of the response approaches a steady value. This is the behavior of a response which is stable and does not grow unbounded as time approaches infinity. This is directly in line with the results from the Routh–Hurwitz stability criterion which showed there were no sign changes in the first column of the completed Routh table. This implies that the system has no poles in the open right hand plane thus it is at least marginally stable. Thus this system is a stable system.