1. $w = 1$ rad/s: $G(s) = \frac{1}{s+10} \Rightarrow G(j) = \frac{1}{j+10} \cdot \frac{-j+10}{-j+10} = \frac{-j+10}{1+100}$

$G(j) = 10/101 - 1/101 j$ $\boxed{\angle G(j) = -5.71°}$ $\boxed{|G(j)| = 0.10}$

$w = 2$ rad/s $G(2j) = \frac{1}{2j+10} \cdot \frac{10-2j}{10-2j} = \frac{10-2j}{4+100} = \frac{10-2j}{104}$

$10/104 - 2/104 j \Rightarrow$ $\boxed{\angle G(2j) = -11.31°}$ $\boxed{|G(2j)| = 0.098}$

$w = 5$ rad/s $G(5j) = \frac{1}{5j+10} \cdot \frac{-5j+10}{-5j+10} = \frac{-5j+10}{25+100}$

$\frac{10}{125} - \frac{5}{125} j \Rightarrow$ $\boxed{\angle G(5j) = -26.57°}$ $\boxed{|G(5j)| = 0.089}$

$w = 10$ rad/s $G(10j) = \frac{1}{10j+10} \cdot \frac{-10j+10}{-10j+10} = \frac{-10j+10}{100+100}$

$\frac{10}{200} - \frac{10}{200} j \Rightarrow$ $\boxed{\angle G(10j) = -45°}$ $\boxed{|G(10j)| = 0.0707}$

$w = 20$ rad/s $G(20j)$ $\frac{1}{20j+10} \cdot \frac{-20j+10}{-20j+10} = \frac{-20j+10}{400+100}$

$\frac{10}{500} - \frac{20}{500} j \Rightarrow$ $\boxed{\angle G(20j) = 63.43°}$ $\boxed{|G(20j)| = 0.0447}$

$w = 50$ rad/s $\frac{1}{50j+10} \cdot \frac{-50j+10}{-50j+10} = \frac{-50j+10}{2500+100}$

$10/2600 - 50/2600 j \Rightarrow$ $\boxed{\angle G(50j) = -78.7°}$ $\boxed{|G(50j)| = 0.0196}$

$w = 100$ rad/s $\frac{1}{100j+10} \cdot \frac{-100j+10}{-100j+10} = \frac{-100j+10}{10000+100}$

$10/10100 - 100/10100 j \Rightarrow$ $\boxed{\angle G(100j) = -84.29°}$ $\boxed{|G(100j)| = 0.00995}$

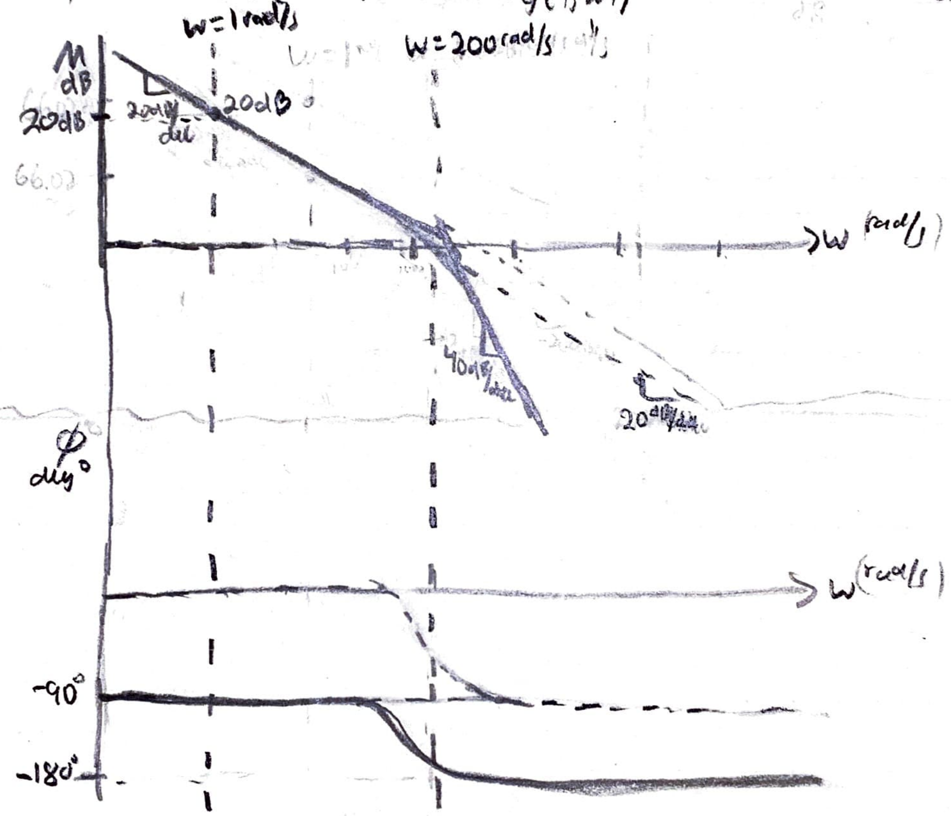
2.a)

$$L(s) = \frac{2000}{s(s+200)} = \frac{2000}{s^2+200s} \Rightarrow \frac{10}{s((1/200)j\omega+1)}$$

$$\rightarrow \frac{10}{s} \Rightarrow K_v(j\omega)^n \quad \text{where}$$

$$n = -1 \qquad K_v = 10$$

$$dB) \rightarrow 20\log(10) - 20\log(1, j\omega 1) = 20 - 20(1, j\omega 1)$$



$$(s+200)^{-1} = ((1/200)j\omega+1)^{-1} \qquad \tau = 1/200\,s$$

$$\underline{\omega_B = 200\ \text{rad/s}} \qquad n = -1$$

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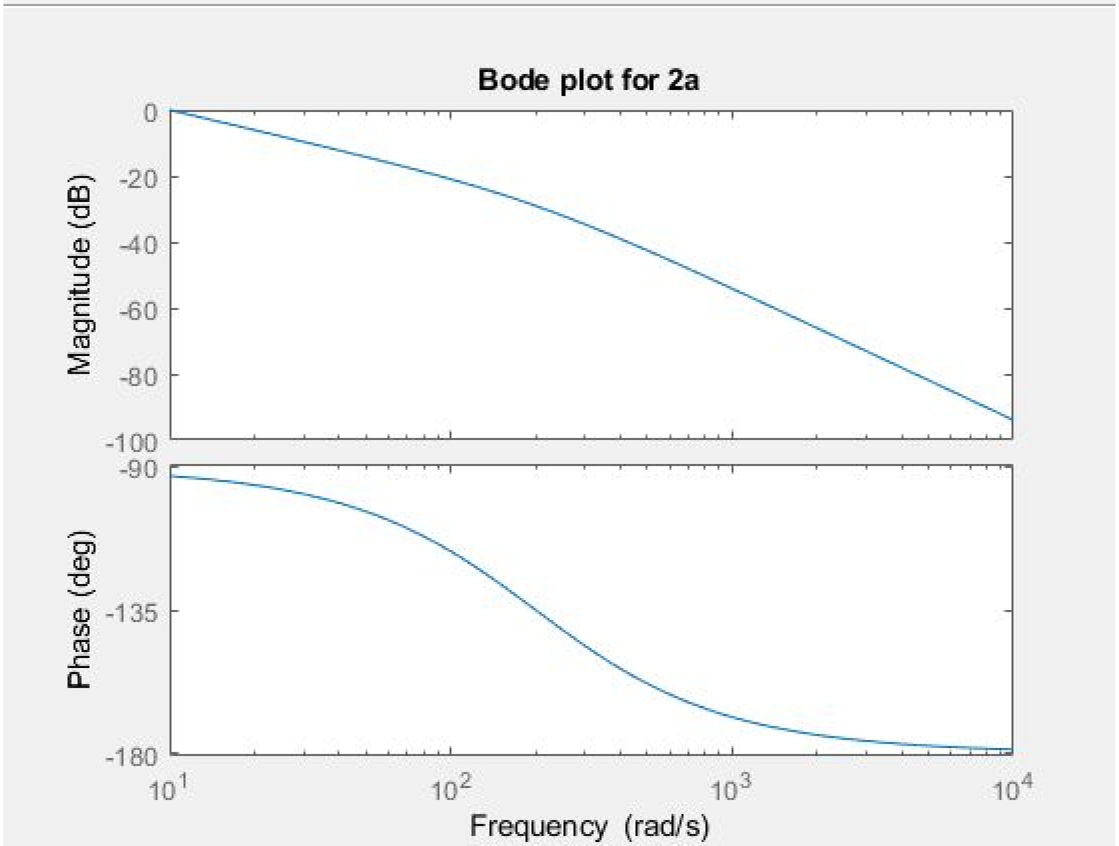
Homework 10
Ryan Dalby

<u>Problem 2</u>
a)



This plot is very similar to my sketch except there are smoother transitions in the actual bode plot. The phase plots look identical.

2. b) $L(s) = \frac{100}{s(0.1s+1)(0.5s+1)} = \frac{100}{s(0.05s^2+0.1s+0.5s+1)} = \frac{100}{0.05s^3+0.6s^2+s}$

$\frac{100}{s} \Rightarrow \quad K_0 = 100 \quad 20\log(K_0) = 40\,dB$

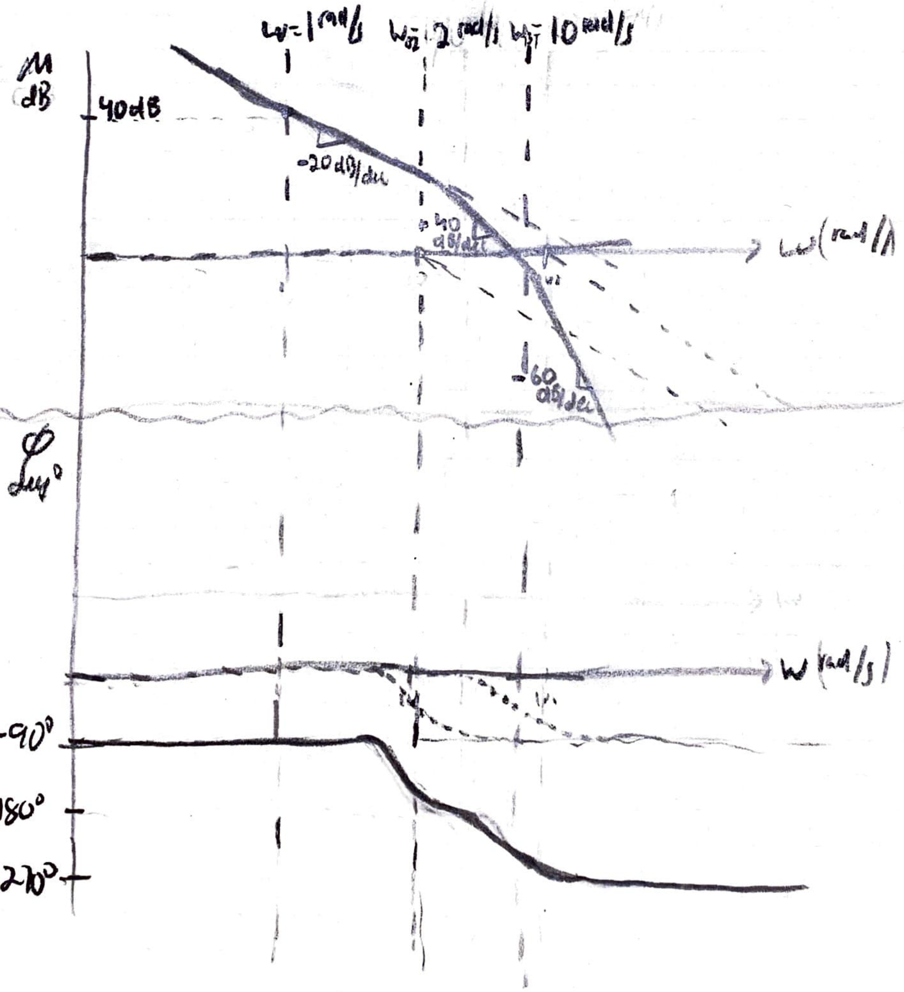
$n = -1 \quad 20n\log(|j\omega|) = -20\log(|j\omega|)$

$\phi = -90°$

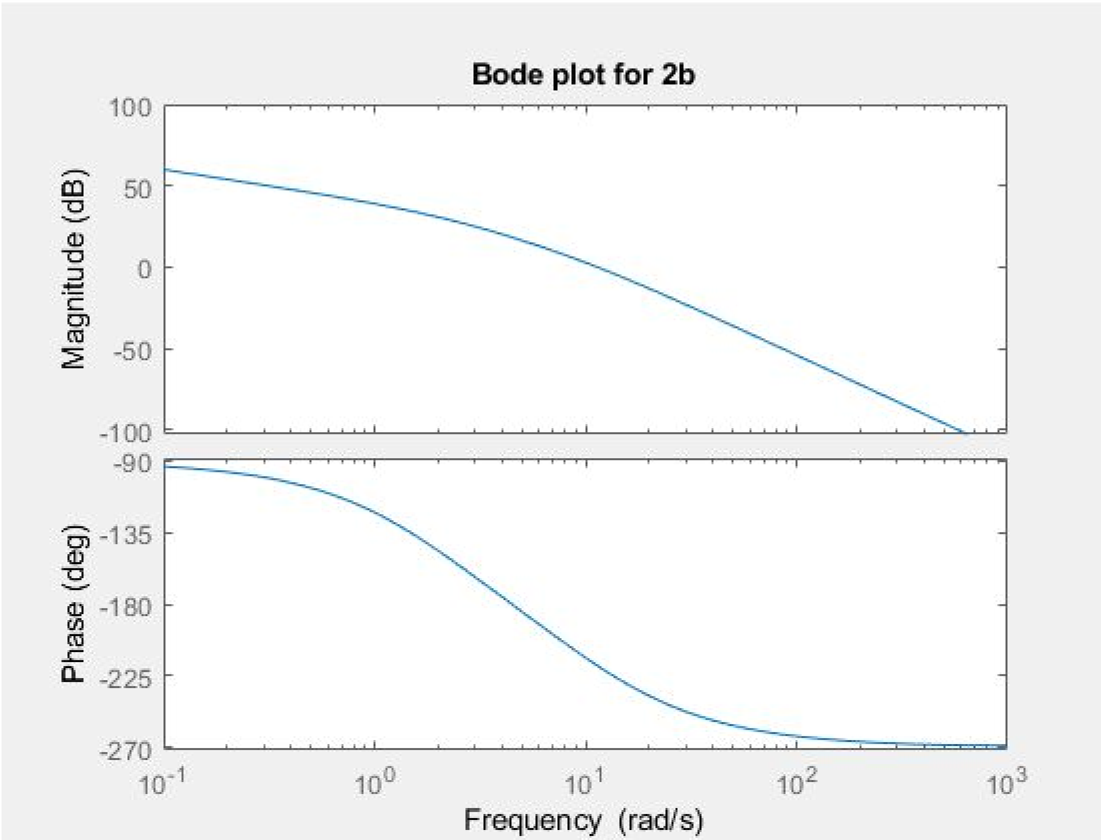
$(0.1s+1)^{-1}(0.5s+1)^{-1} \Rightarrow ((1/10)j\omega+1)^{-1}((1/2)j\omega+1)^{-1}$

$n=-1 \quad 0° \to -90° \quad \tau_1 = 1/10 \quad \omega_{B1} = 10\,rad/s \quad n=-1 \quad -20\,dB/dec$

$n=-1 \quad 0 \to -90° \quad \tau_2 = 1/2 \quad \omega_{B2} = 2\,rad/s \quad n=-1 \quad -20\,dB/dec$



b)



This plot is very similar to my sketch except there are smoother transitions in the actual bode plot. Between 2 rad/s and 10 rad/s it is harder to discern the transitions on the bode plot because of the scale.

2. c) $L(s) = \dfrac{1}{s(s+1)(0.02s+1)} = \dfrac{1}{s(0.02s^2+1.02s+1)} = \dfrac{1}{0.02s^3+1.02s^2+s}$

$\frac{1}{s} \Rightarrow K_G = 1 \qquad 20\log(K_G) = 0\text{ dB}$

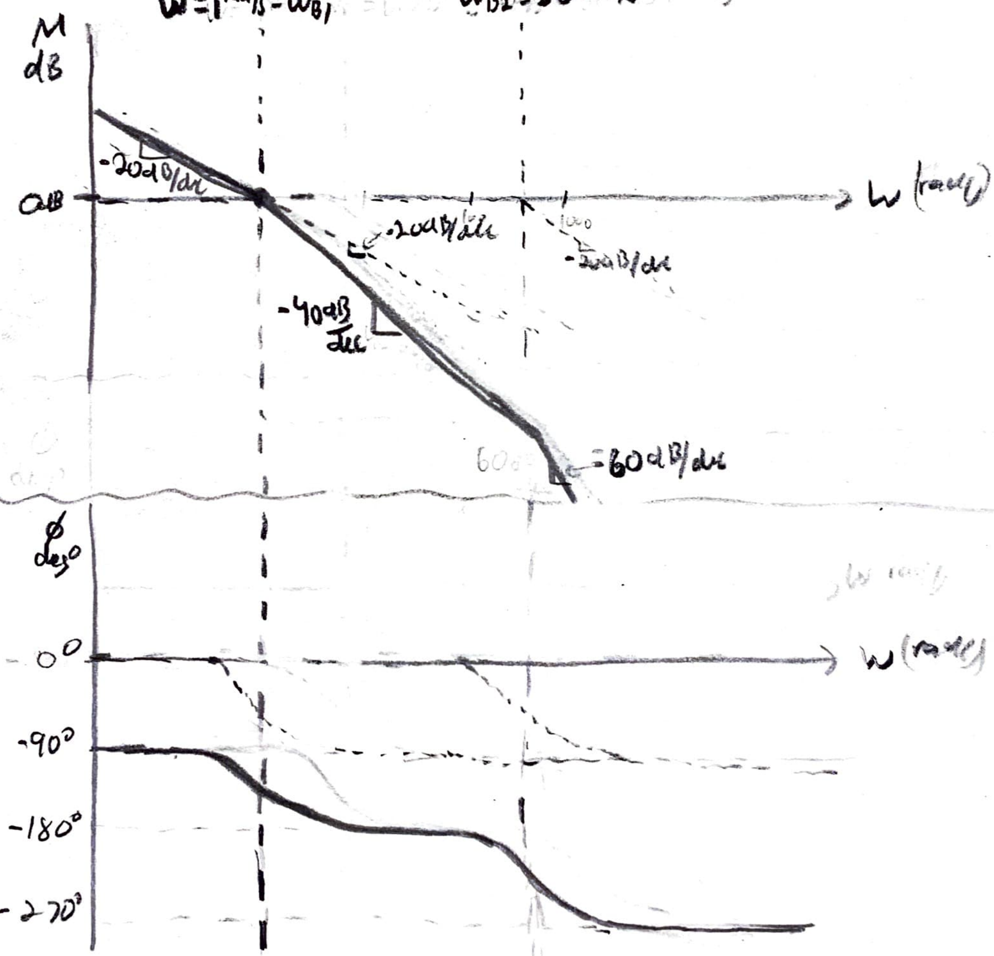
$n = -1 \qquad 20n\log(|j\omega|) = -20\log(|j\omega|)$

$\phi = -90°$

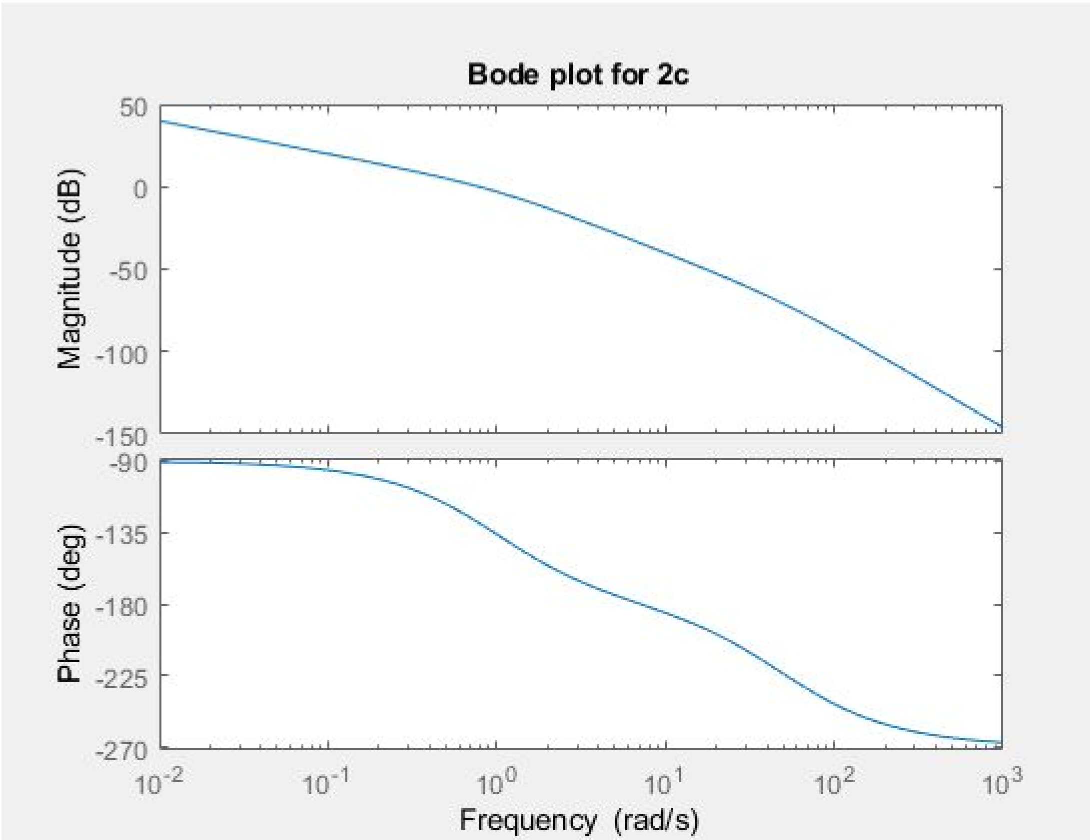
$(j\omega+1)^{-1}((2/100)j\omega+1)^{-1}$

$n=-1 \quad 0° \to -90° \qquad \tau_1 = 1 \qquad \omega_{B1} = 1\text{ rad/s} \qquad n=-1 \quad -20\text{ dB/dec}$

$n=-1 \quad 0° \to -90° \qquad \tau_2 = 2/100 \qquad \omega_{B2} = 50\text{ rad/s} \qquad n=-1 \quad -20\text{ dB/dec}$



c)



This plot is very similar to my sketch except there are smoother transitions in the actual bode plot. Once again because of the bode plot scale the two “bumps” of the phase plot are not as pronounced as compared to the sketch, but the plots appear the same.

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3. a) $L(s) = \frac{(s+2)}{s(s+10)(s^2+2s+2)} \Rightarrow \frac{(s+2)}{s(s^3+12s^2+22s+20)}$

$\Rightarrow \frac{(1/10)(1/2)(2)\ ((1/2)s+1)}{s((1/10)s+1)((1/2)s^2+s+1)}$ $(10s)^{-1}$

$(10s)^{-1} \Rightarrow K_0 = 1/10 \Rightarrow 20\log(1/10) = -20\,dB$

$\phi: -90°$ $n = -1$ $20n\log(|j\omega|) \Rightarrow -20\,dB/dec$

$((1/2)j\omega+1) \Rightarrow \tau_1 = 1/2\ s$ $\omega_{B1} = 2\ rad/s$ $20\,dB/dec$ $n = 1$

$((1/10)j\omega+1)^{-1} \Rightarrow \tau_2 = 1/10\ s$ $\omega_{B2} = 10\ rad/s$ $-20\,dB/dec$ $n = -1$

$\phi_1: 0° \to 90°$ at ω_{B1} $\phi_2: 0° \to -90°$ at ω_{B2}

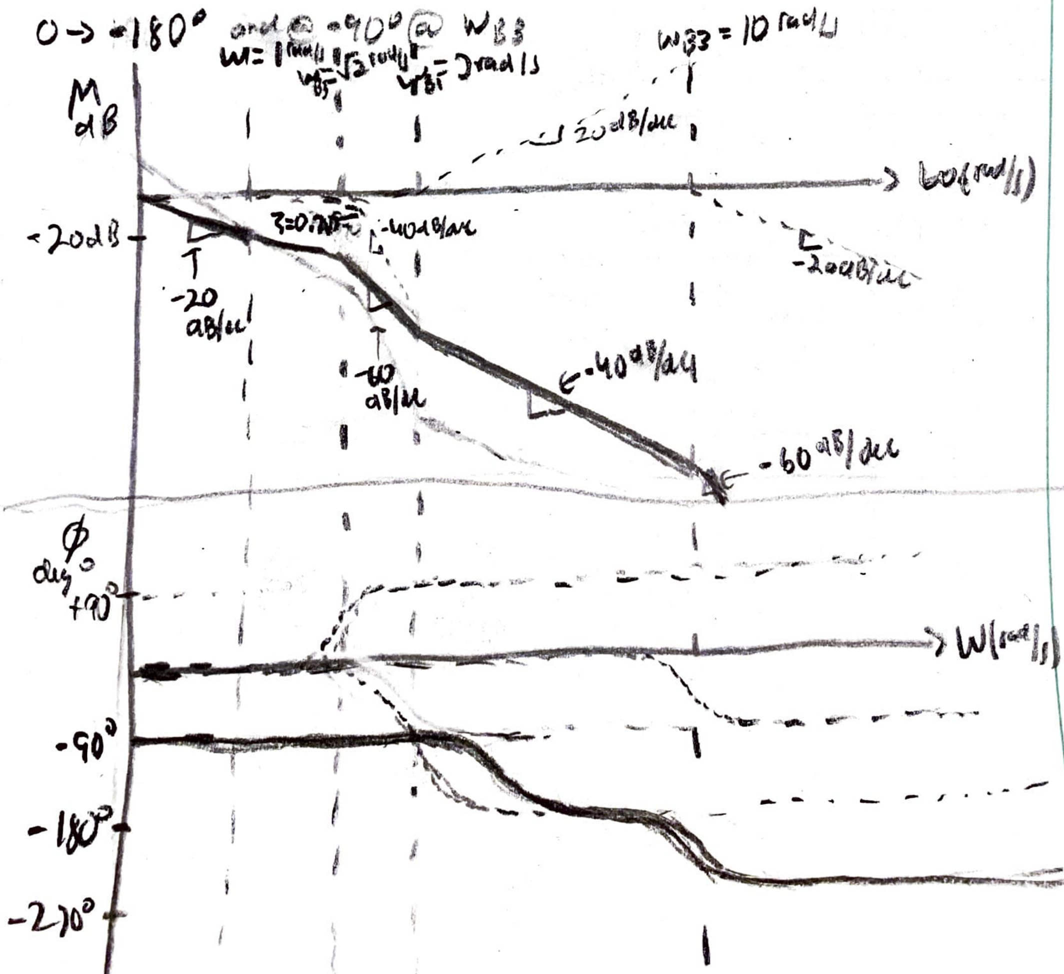
$[(1/2)s^2+s+1]^{-1} \Rightarrow [(\frac{j\omega}{\omega_n})^2 + 2\zeta(\frac{j\omega}{\omega_n})+1]^{-1}$

$1/2\ sec = (\frac{1}{\omega_n})$ $\omega_{B3} = \omega_n = \sqrt{2}\ rad/s$ $\zeta = \sqrt{2}/2 = 0.7071$

$\sqrt{2} = \omega_n$

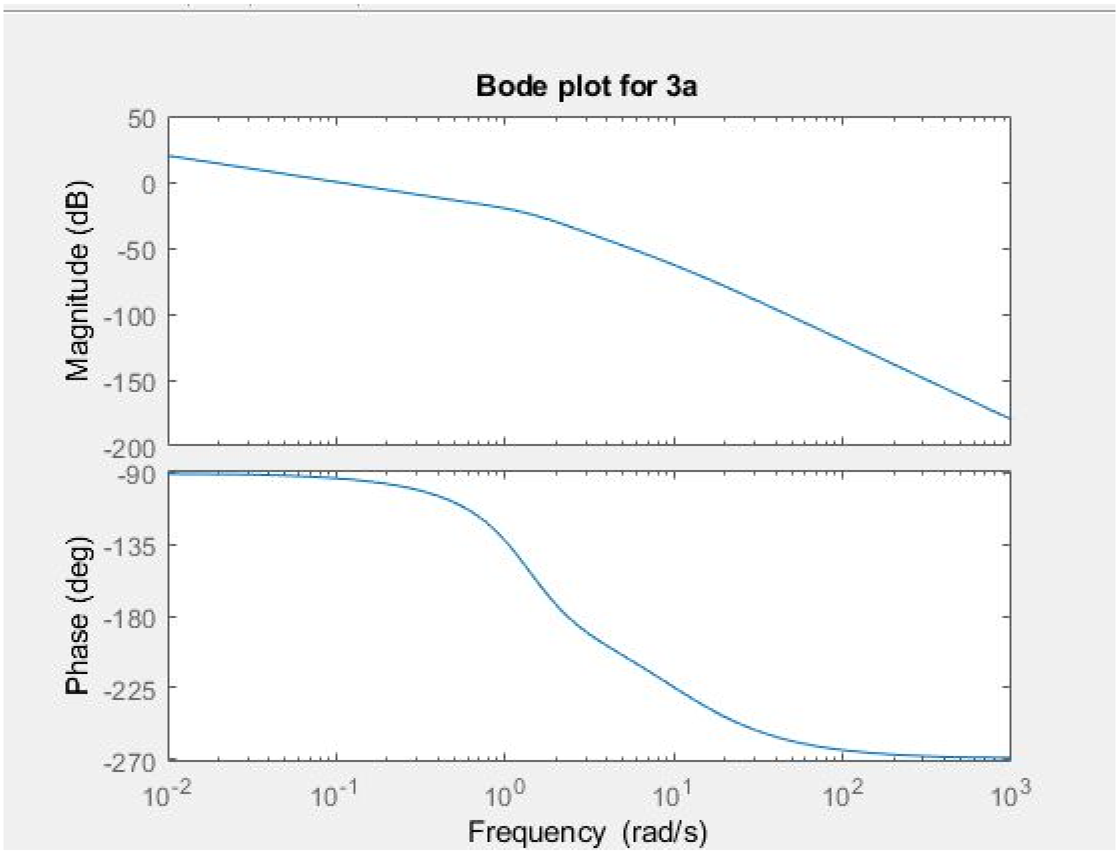
$-40\,dB/dec$ since $n = -1$ How big the "peak" at ω_n is

$0 \to -180°$ and @ $-90°$ @ ω_{B3}



Problem 3

a)



This plot is very close to the sketch I created, although the phase plot and magnitude plot don't show as detailed transitions between 1 rad/s and 2 rad/s when compared to my sketch because of the scale.

3. b) $L(s) = \dfrac{(s+2)}{s^2(s+10)(s^2+6s+25)} = \dfrac{(s+2)}{s^2(s^3+16s^2+85s+250)}$

$\Rightarrow \dfrac{\underbrace{(2)(1/10)(1/25)}_{1/125}((1/2)j\omega+1)}{(j\omega)^2((1/10)j\omega+1)((1/25)(j\omega)^2+6/25j\omega+1)}$

$(125^{-1})(j\omega)^{-2} \Rightarrow K_G = 1/125 \Rightarrow 20\log(1/125) = -41.94\ dB$

$\phi = -180°$ $\quad n=-2 \quad 20n\log(|j\omega|) = -40\ dB/dec$

$((1/2)j\omega+1) \Rightarrow \tau_1 = 1/2 \quad \omega_{B1} = 2 \quad \rightarrow 20\ dB/dec \quad n=1 \quad \phi_1: 0° \rightarrow 90°\ @\omega_{B1}$

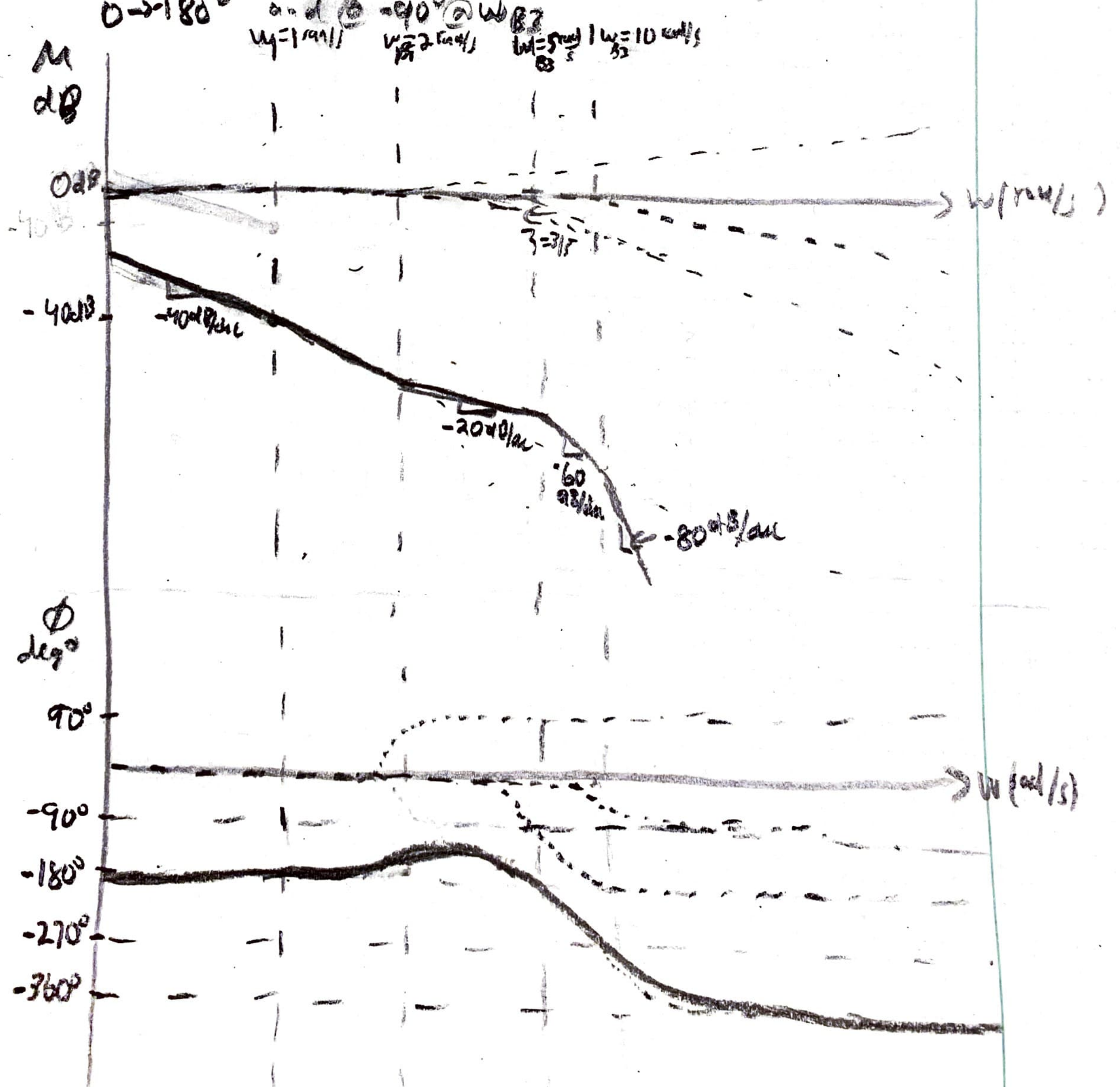
$((1/10)j\omega+1)^{-1} \Rightarrow \tau_2 = 1/10 \quad \omega_{B2} = 10 \quad \rightarrow -20\ dB/dec \quad n=-1 \quad \phi_2: 0° \rightarrow -90°\ @\omega_{B2}$

$[(1/25)(j\omega)^2 + 6/25 j\omega + 1]^{-1} \Rightarrow [(\frac{j\omega}{\omega_n})^2 + 2\zeta(\frac{j\omega}{\omega_n}) + 1]^{-1}$ $\quad \frac{6}{25} = \frac{2\zeta}{5}$

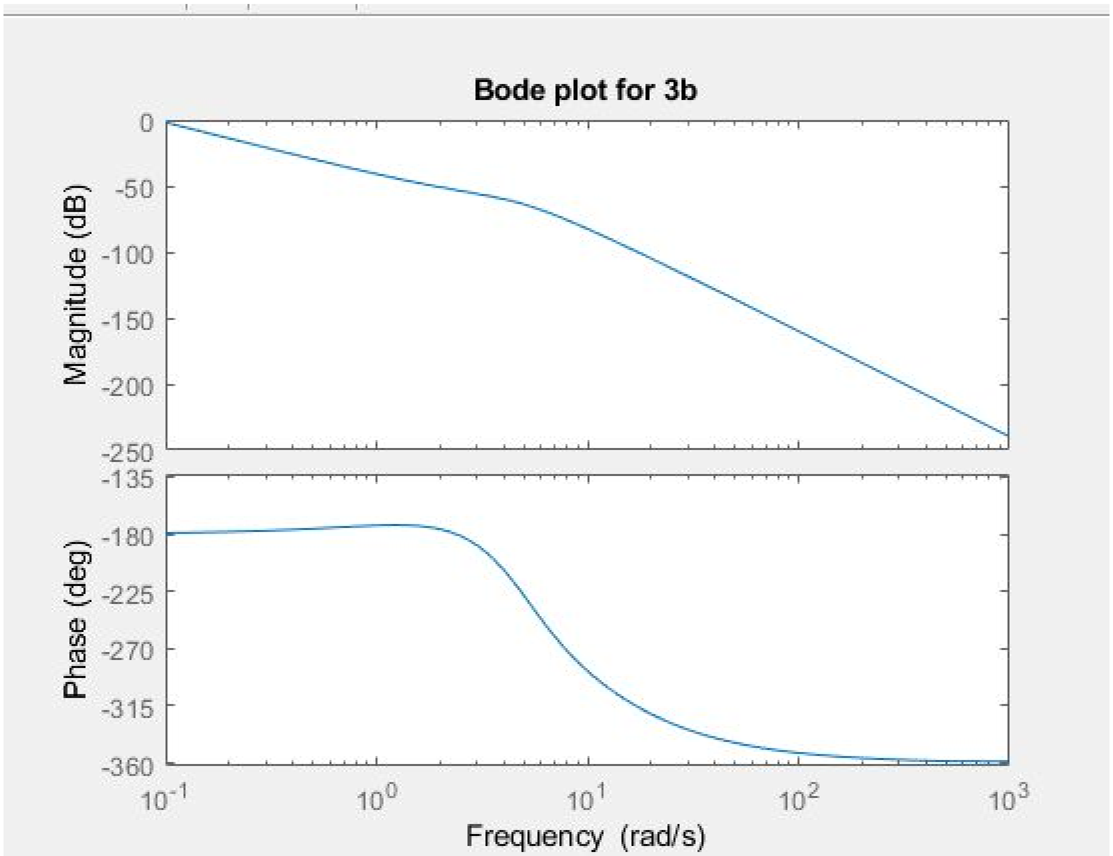
$\omega_{B3} = \omega_n = \sqrt{25} = 5\ rad/s \quad \zeta = 3/5$

$-40\ dB/dec$ since $n=-1$ $\quad$ ← "how big peak" at ω_n is

$0 \rightarrow -180°$ and $@ -90° @ \omega_{B3}$



b)



This plot is close to my sketch but between .1 rad/s and 1 rad/s the phase plot appears to be increasing on the actual bode plot and this doesn't appear as much on my sketch. This is likely because of the influence of the breakpoint at 2 rad/s beginning to have some influence before the plot reaches that breakpoint.

```matlab
%% ME EN 6200 Homework 10 Ryan Dalby
%%
clear;
close all;

%% 2
% a
La = tf(2000,[1 200 0]);
figure;
bode(La);
title('Bode plot for 2a');
% b
Lb = tf(100,[0.05 0.6 1 0]);
figure;
bode(Lb);
title('Bode plot for 2b');
% c
Lc = tf(1,[0.02 1.02 1 0]);
figure;
bode(Lc);
title('Bode plot for 2c');
%% 3
% a
La = tf([1 2],[1 12 22 20 0]);
figure;
bode(La);
title('Bode plot for 3a');
% b
Lb = tf([1 2],[1 16 85 250 0 0]);
figure;
bode(Lb);
title('Bode plot for 3b');
```