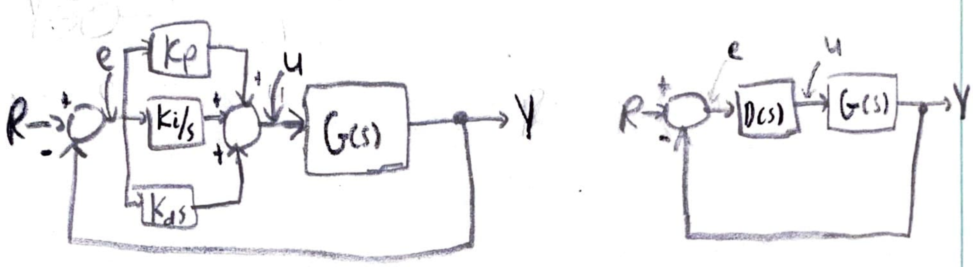
ME EN 6200 Homework 8 Ryan [illegible]

1. $G(s) = \frac{a}{(s+b)}$ $a = 0.1$ $b = 10$ $\boxed{G(s) = \frac{0.1}{s+10}}$

a)



$\boxed{u(s) = \left(K_p + K_i/s + K_d s\right) e(s)}$ $\boxed{e = R - Y}$ $D(s) = K_p + K_i/s + K_d s$

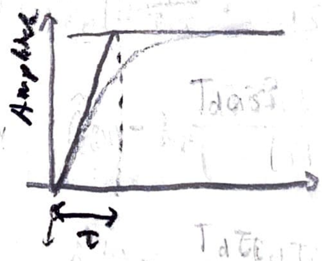
b)

$$R \rightarrow \boxed{\frac{D(s)G(s)}{1 + D(s)G(s)}} \rightarrow Y$$

$$\frac{(K_p + K_i/s + K_d s)\,G(s)}{1 + (K_p + K_i/s + K_d s)\,G(s)} = \frac{a(K_p + K_i/s + K_d s)}{s + b + (K_p + K_i/s + K_d s)a} = \frac{aK_p s + K_i a + aK_d s^2}{s^2 + bs + K_p a s + K_i a + K_d a s^2}$$

$$R \rightarrow \boxed{\frac{a(K_d s^2 + K_p s + K_i)}{(K_d a + 1)s^2 + (K_p a + b)s + K_i a}} \rightarrow Y$$

d) 25% decay approach
↳ Assume $L \approx 0$ but $L \neq 0$
so take $L = 0.01$



$\tau = 0.1$ $A = 0.01$

$R = 0.100$

$\boxed{K_p = 1200 \quad K_i = 60000 \quad K_d = 6}$

$\boxed{\begin{array}{l} \%OS = 11.13 \\ t_s = 0.0802s \\ e_{ss} \approx 0 \end{array}}$

c) See attached
SSvalue $= 0.010$
$t_s = 0.391s$

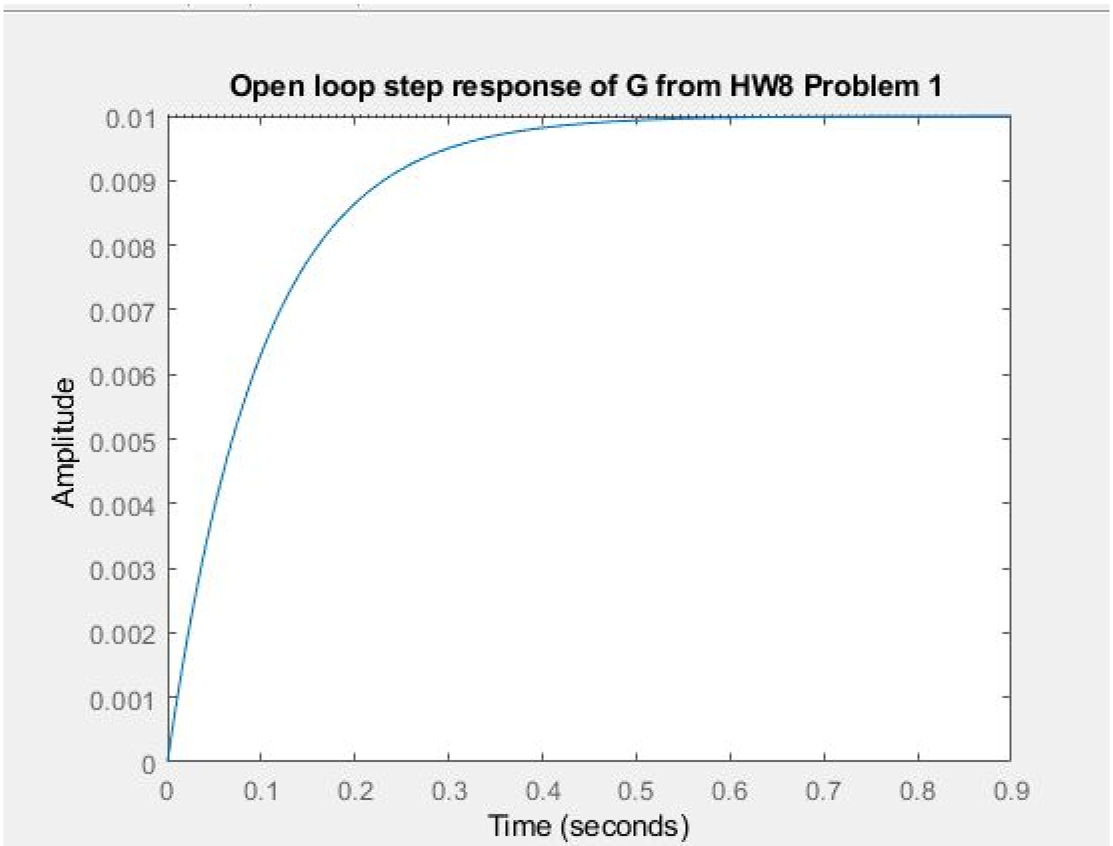
f) Improving settling time and overshoot
$\boxed{K_p = 3000 \quad K_i = 80000 \quad K_d = 8}$

$\boxed{\begin{array}{l} \%OS = 3.287 \\ t_s = 0.07735s \\ e_{ss} \approx 0 \end{array}}$

ME EN 6200
Homework 8
Ryan Dalby

Problem 1

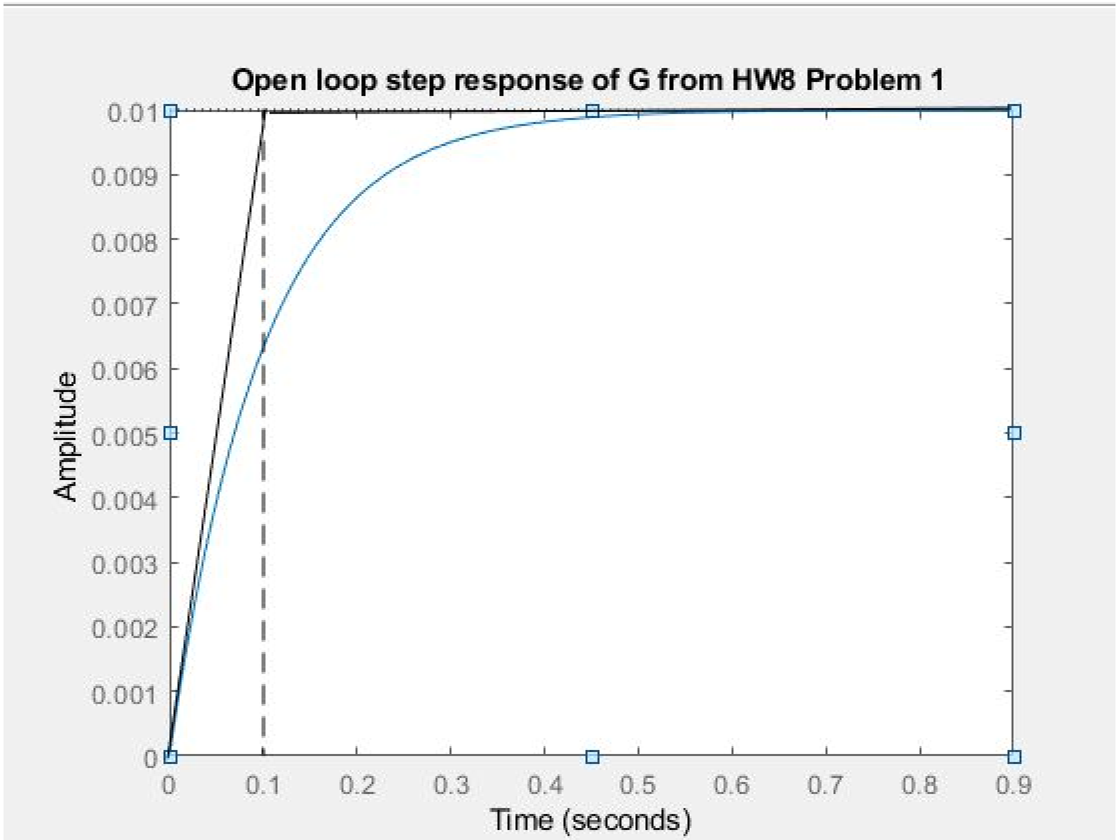
c)




```
OLTF: Steady state value is 0.010000 and the settling time is 0.391207s
```

```
% c
a = 0.1;
b = 10;

% Open loop step response
G = tf(a,[1 b]);
figure;
step(G);
title('Open loop step response of G from HW8 Problem 1');
G_step_info = stepinfo(G);
fprintf('OLTF: Steady state value is %f and the settling time is %fs\n\n',
G_step_info.SettlingMax, G_step_info.SettlingTime);
```

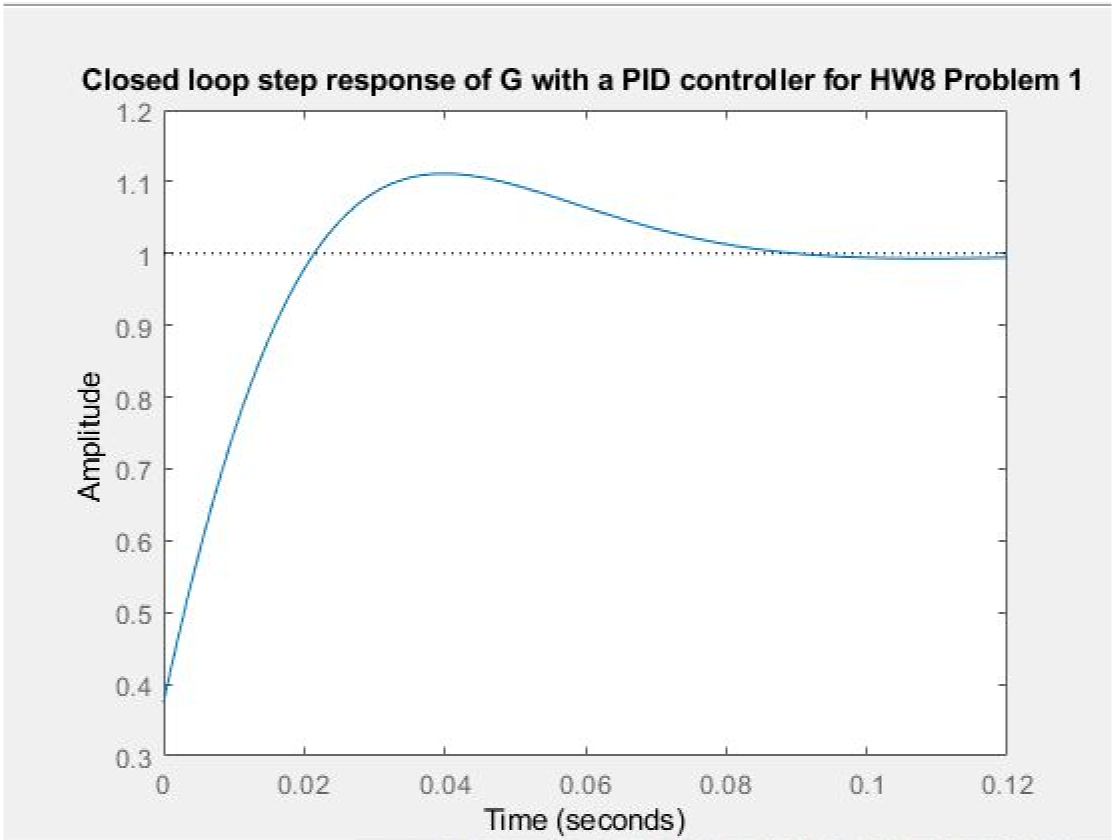
d)




```
25 Percent Method Gains: Kp=1200.000000, Ki=60000.000000, and Kd=6.000000
```

```
% d
% 25% method
L = 0.01;
A = 0.01;
tau = 0.1;
R = A/tau;

kp = 1.2/(R*L);
Ti = 2*L;
Td = 0.5*L;
ki = kp/Ti;
kd = kp*Td;

fprintf('25 Percent Method Gains: Kp=%f, Ki=%f, and Kd=%f\n\n', kp,ki,kd);
```

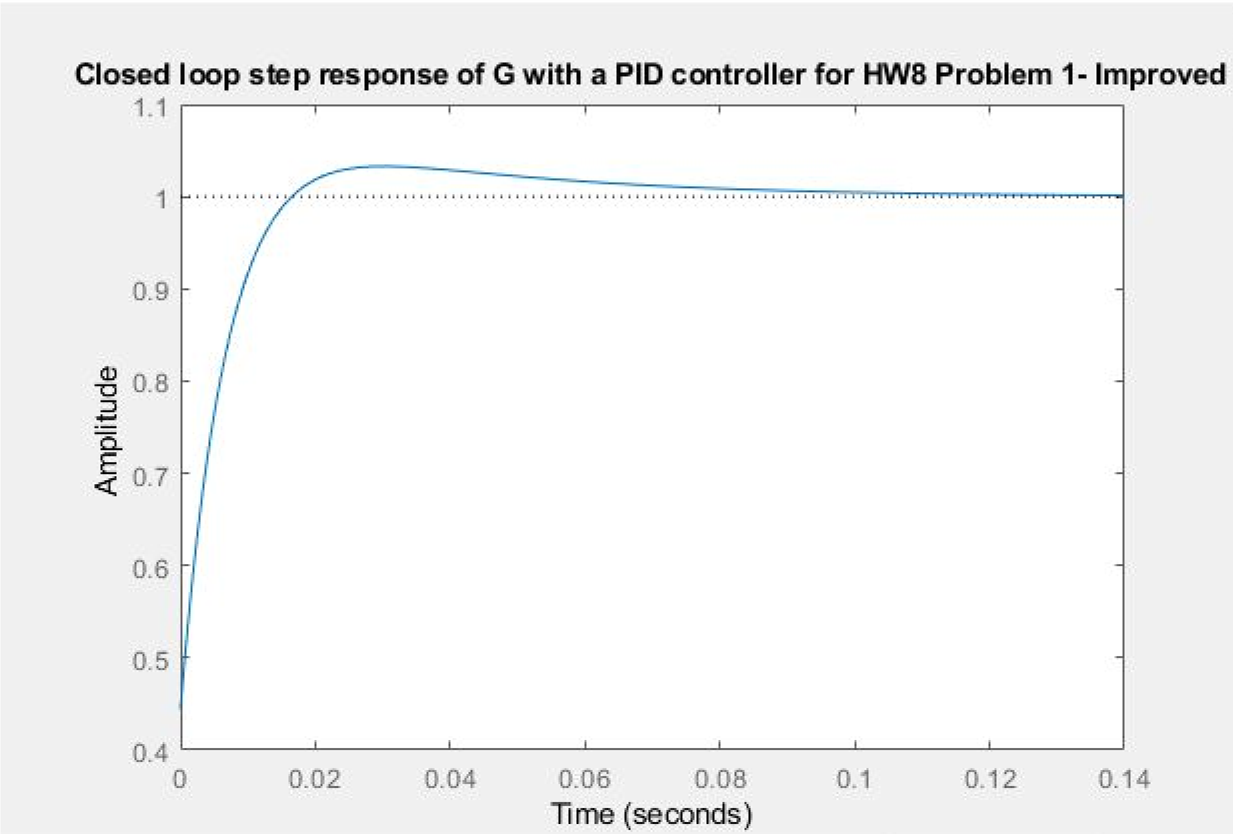
e)




```
CLTF: Percent overshoot is 11.132516, the settling time is 0.080225s, and the steady state error is 0.006623
```

```
% e
% Closed loop step response for optimum gains
cltf = tf([a*kd a*kp a*ki],[(kd*a+1) (kp*a+b) (ki*a)]);
figure;
step(cltf);
[cltf_response,~] = step(cltf);
title('Closed loop step response of G with a PID controller for HW8 Problem
1');
cltf_ss_error = 1 - cltf_response(end);
cltf_step_info = stepinfo(cltf);
fprintf('CLTF: Percent overshoot is %f, the settling time is %fs, and the
steady state error is %f\n\n', cltf_step_info.Overshoot,
cltf_step_info.SettlingTime, cltf_ss_error);
```

f)




```
CLTF Improved: Percent overshoot is 3.286701, the settling time is 0.072497s, and the steady state error is 0.006623
```

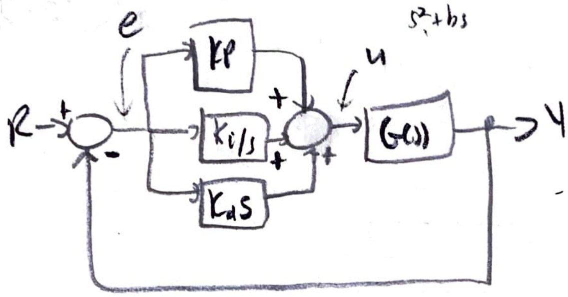
```
% f
% Improving tuning values
kp=3000;
ki=80000;
kd=8;
cltf = tf([a*kd a*kp a*ki],[(kd*a+1) (kp*a+b) (ki*a)]);

figure;
step(cltf);
title('Closed loop step response of G with a PID controller for HW8 Problem 1- Improved');
cltf_ss_error = 1 - cltf_response(end);
cltf_step_info = stepinfo(cltf);
fprintf('CLTF Improved: Percent overshoot is %f, the settling time is %fs, and the steady state error is %f\n\n\n', cltf_step_info.Overshoot, cltf_step_info.SettlingTime, cltf_ss_error);
```

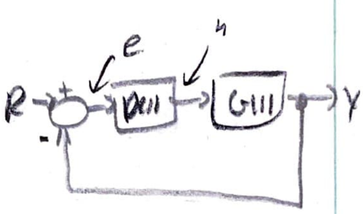
2. a)

$$G(s) = \frac{1}{s(s+b)(s^2+2\zeta\omega_n s+\omega_n^2)}$$

$b = 10$
$\zeta = 0.707$
$\omega_n = 4\ \text{rad/s}$

$$G(s) = \frac{1}{s^4+(2\zeta\omega_n+b)s^3+(\omega_n^2+2\zeta\omega_n b)s^2+b\omega_n^2 s^1}$$





$u(s) = (k_p + k_i/s + k_d s)e(s)$ $\quad e = R - Y \quad$ $D(s) = k_p + k_i/s + k_d s$

$$G(s) = \frac{1}{s(s+10)(s^2+5.66s+16)}$$

b) $R \rightarrow \boxed{\frac{D(s)G(s)}{1+D(s)G(s)}} \rightarrow Y$

$$\frac{\frac{(k_p + k_i/s + k_d s)}{s(s+b)(s^2+2\zeta\omega_n s+\omega_n^2)}}{1+\frac{(k_p+k_i/s+k_d s)}{s(s+b)(s^2+2\zeta\omega_n s+\omega_n^2)}} = \frac{k_p + k_i/s + k_d s}{s(s+b)(s^2+2\zeta\omega_n s+\omega_n^2)+k_p+k_i/s+k_d s}$$

$$= \frac{k_p s + k_i + k_d s^2}{\underbrace{s(s^2+bs)}_{s^3+bs^2}(s^2+2\zeta\omega_n s+\omega_n^2)+k_p s+k_i+k_d s^2} = \frac{k_p s + k_i + k_d s^2}{s^5+2\zeta\omega_n s^4+\omega_n^2 s^3+bs^4+2\zeta\omega_n b s^3+\omega_n^2 b s^2+k_p s+k_i+k_d s^2}$$

$$R \rightarrow \boxed{\frac{k_d s^2 + k_p s + k_i}{s^5+(2\zeta\omega_n+b)s^4+(\omega_n^2+2\zeta\omega_n b)s^3+(\omega_n^2 b+k_d)s^2+k_p s+k_d}} \rightarrow Y$$

c) See attached

$$\boxed{\begin{array}{l} \text{SS value} = \underline{\text{None}} \\ t_s = \underline{\text{None}} \end{array}}$$

→ Open loop system is not stable

2. d) $D(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$ $\quad T_i = \frac{K_p}{K_i}$ $\quad T_d = \frac{K_d}{K_p}$

Taking $K_i = 0$ and $K_d = 0$

$$R \rightarrow \boxed{\frac{K_p}{s^4 + (2\zeta\omega_n + b)s^3 + (\omega_n^2 + 2\zeta\omega_n b)s^2 + b\omega_n^2 s + K_p}} \rightarrow Y$$

↳ Can determine P_u and K_p^* from CLTF response

then

$K_p = 1.6 K_p^*$ $\quad T_i = 0.5 P_u$ $\quad T_d = 0.125 P_u$

$K_i = K_p / T_i$ $\quad K_d = K_p \cdot T_d$

From MATLAB: $K_p^* = 637.1004$ $\quad P_u = 2$ seconds

$\boxed{K_p = 1019.4 \quad K_i = 1019.4 \quad K_d = 254.87}$

$\boxed{\begin{array}{l} \%OS = 89.53 \\ t_s = 18.5\ s \\ e_{ss} \approx 0 \end{array}}$

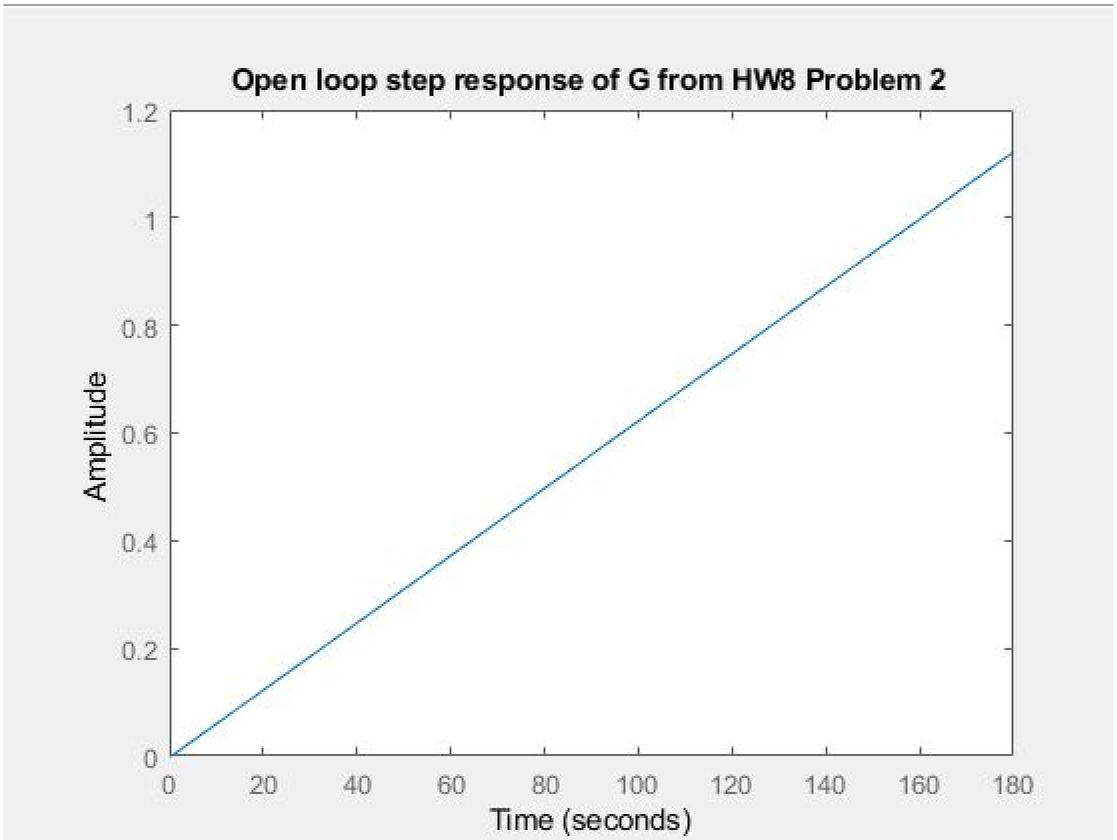
f) Improving response:

$\boxed{K_p = 750 \quad K_i = 750 \quad K_d = 330}$

$\boxed{\begin{array}{l} \%OS = 60.32 \\ t_s = 5.387\ s \\ e_{ss} \approx 0 \end{array}}$

Problem 2

c)

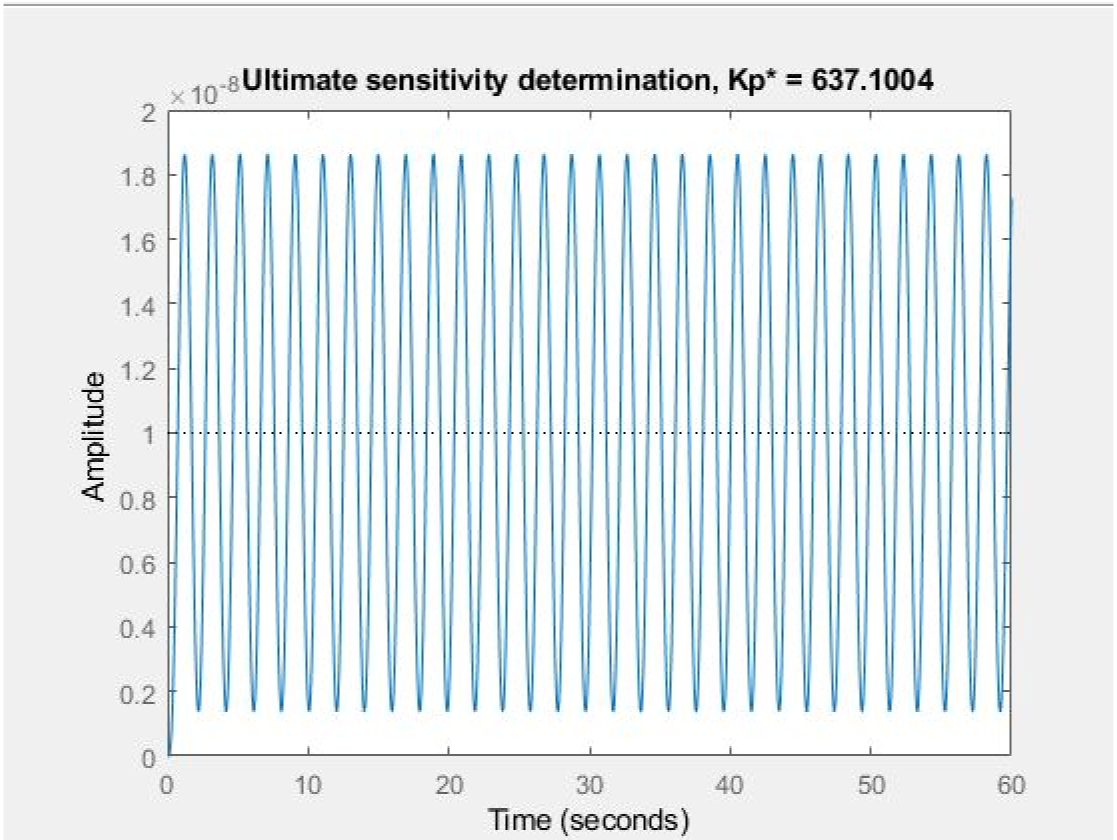



```
OLTF: Steady state value is NaN and the settling time is NaNs
```

\

```
% c
b = 10;
zeta = 0.707;
wn = 4; % rad/s

% Open loop step response
G = tf(1,[1 (2*zeta*wn+b) (wn^2+2*zeta*wn*b) (b*wn^2) 0]);
figure;
step(G);
title('Open loop step response of G from HW8 Problem 2');
G_step_info = stepinfo(G);
fprintf('OLTF: Steady state value is %f and the settling time is %fs\n\n',
G_step_info.SettlingMax, G_step_info.SettlingTime);
```

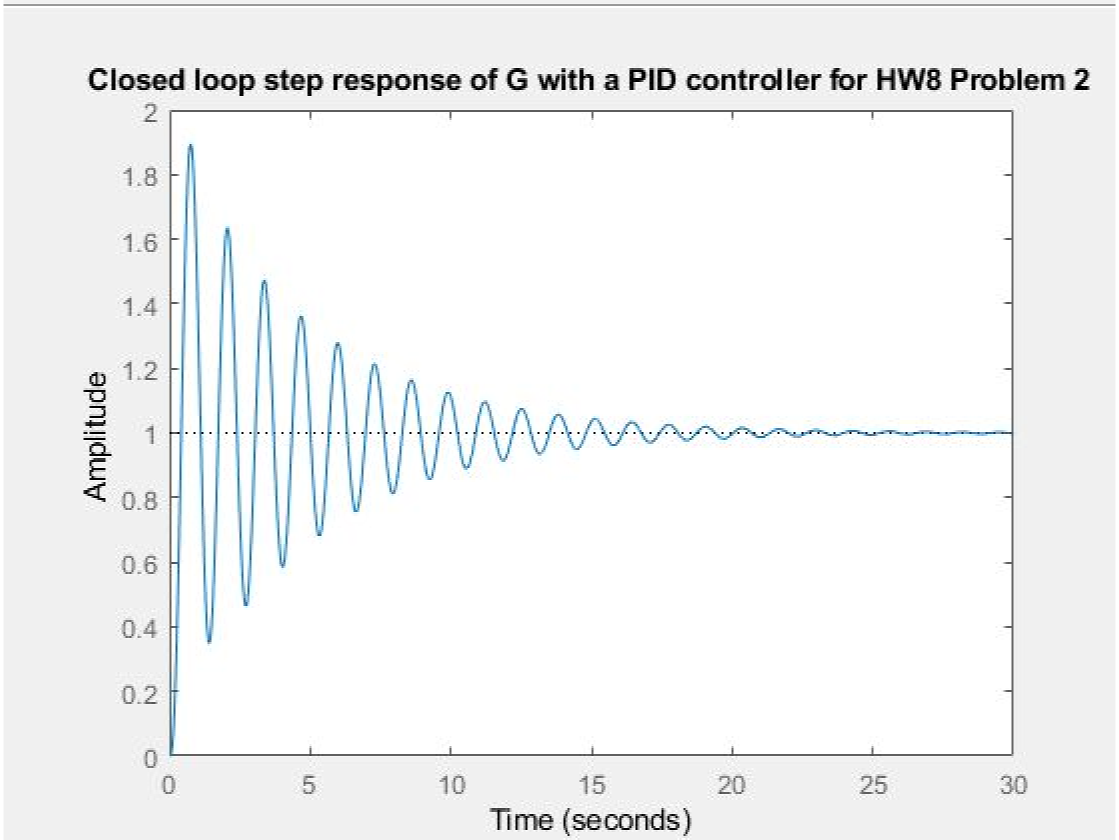
d)




```
Ultimate Sensitivity Gains: Kp=1019.360640, Ki=1019.360640, and Kd=254.840160
```

```
% d
% Ultimate sensitivity
kp_star = 637.1004;
opt = stepDataOptions('StepAmplitude', 0.00000001);
G_kp = tf(kp_star,[1 (2*zeta*wn+b) (wn^2+2*zeta*wn*b) (b*wn^2) kp_star]);
figure;
step(G_kp, opt);
title('Ultimate sensitivity determination, Kp* = 637.1004');
Pu = 2; % s % From inspection

% Optimum gains from Zieglaer-Nichols
kp = 1.6*kp_star;
Ti = 0.5*Pu;
Td = 0.125*Pu;
ki = kp/Ti;
kd = kp*Td;
fprintf('Ultimate Sensitivity Gains: Kp=%f, Ki=%f, and Kd=%f\n\n',
kp,ki,kd);
```

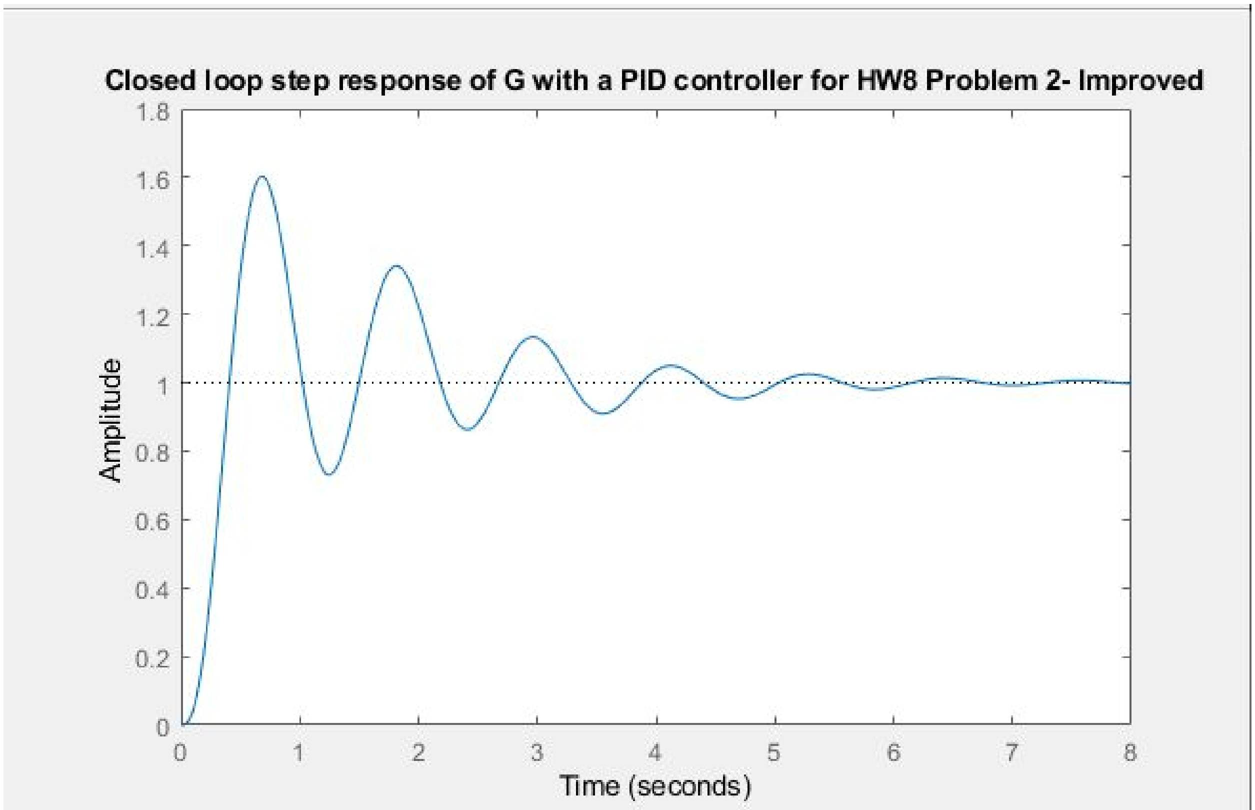
e)




```
CLTF: Percent overshoot is 89.528380, the settling time is 18.497767s, and the steady state error is 0.002816
```

```
% e
% Closed loop step response for optimum gains
cltf = tf([kd kp ki],[1 (2*zeta*wn+b) (wn^2+2*zeta*wn*b) (b*wn^2 + kd) kp
ki]);
figure;
step(cltf);
[cltf_response,~] = step(cltf);
title('Closed loop step response of G with a PID controller for HW8 Problem
2');

cltf_step_info = stepinfo(cltf);
cltf_ss_error = 1 - cltf_response(end);
fprintf('CLTF: Percent overshoot is %f, the settling time is %fs, and the
steady state error is %f\n\n', cltf_step_info.Overshoot,
cltf_step_info.SettlingTime, cltf_ss_error);
```

f)




```
CLTF Improved: Percent overshoot is 60.318252, the settling time is 5.386918s, and the steady state error is 0.002816
```

```
% f
% Improving tuning values
kp=750;
ki=750;
kd=330;
cltf = tf([kd kp ki],[1 (2*zeta*wn+b) (wn^2+2*zeta*wn*b) (b*wn^2 + kd) kp
ki]);

figure;
step(cltf);
title('Closed loop step response of G with a PID controller for HW8 Problem
2- Improved');
cltf_ss_error = 1 - cltf_response(end);
cltf_step_info = stepinfo(cltf);
fprintf('CLTF Improved: Percent overshoot is %f, the settling time is %fs,
and the steady state error is %f\n\n\n', cltf_step_info.Overshoot,
cltf_step_info.SettlingTime, cltf_ss_error);
```