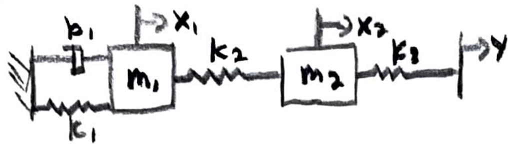
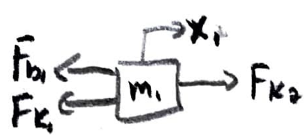
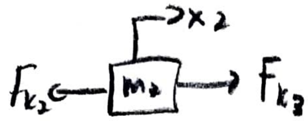
1.

a)



Assume $y > x_2 > x_1$





$$\Sigma F = m_1 \ddot{x}_1$$

$$-F_{b1} - F_{k1} + F_{k2} = m_1\ddot{x}_1$$

$$-b_1(\dot{x}_1 - 0) + k_1(x_1 - 0) + k_2(x_2 - x_1) = m_1\ddot{x}_1$$

$$\boxed{m_1\ddot{x}_1 + b_1\dot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = 0}$$

$$\Sigma F = m_2\ddot{x}_2$$

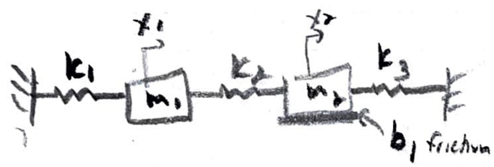
$$-F_{k2} + F_{k3} = m_2\ddot{x}_2$$

input

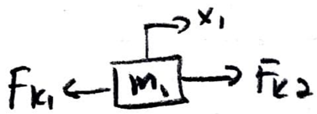
$$-k_2(x_2 - x_1) + k_3(y - x_2) = m_2\ddot{x}_2$$

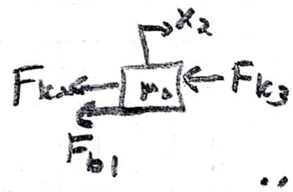
$$\boxed{m_2\ddot{x}_2 + (k_2+k_3)x_2 - k_2x_1 = k_3 y}$$

b)



Assume $x_2 > x_1$





$$\Sigma F = m_1\ddot{x}_1$$

$$-F_{k1} + F_{k2} = m_1\ddot{x}_1$$

$$-k_1(x_1 - 0) + k_2(x_2 - x_1) = m_1\ddot{x}_1$$

$$\boxed{m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = 0}$$

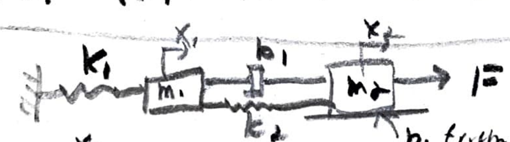
$$\Sigma F = m_2\ddot{x}_2$$

$$-F_{k2} - F_{b1} - F_{k3} = m_2\ddot{x}_2$$

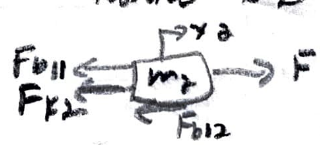
$$-k_2(x_2 - x_1) - b_1\dot{x}_2 - k_3(x_2 - 0) = m_2\ddot{x}_2$$

$$\boxed{m_2\ddot{x}_2 + b_1\dot{x}_2 + (k_2+k_3)x_2 - k_2x_1 = 0}$$

c)



Assume $x_2 > x_1$



$$-F_{k1} + F_{b11} + F_{k2} = m_1\ddot{x}_1$$

$$-k_1(x_1 - 0) + b_1(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = m_1\ddot{x}_1$$

$$\boxed{m_1\ddot{x}_1 + b_1\dot{x}_1 + (k_1+k_2)x_1 - b_1\dot{x}_2 - k_2x_2 = 0}$$

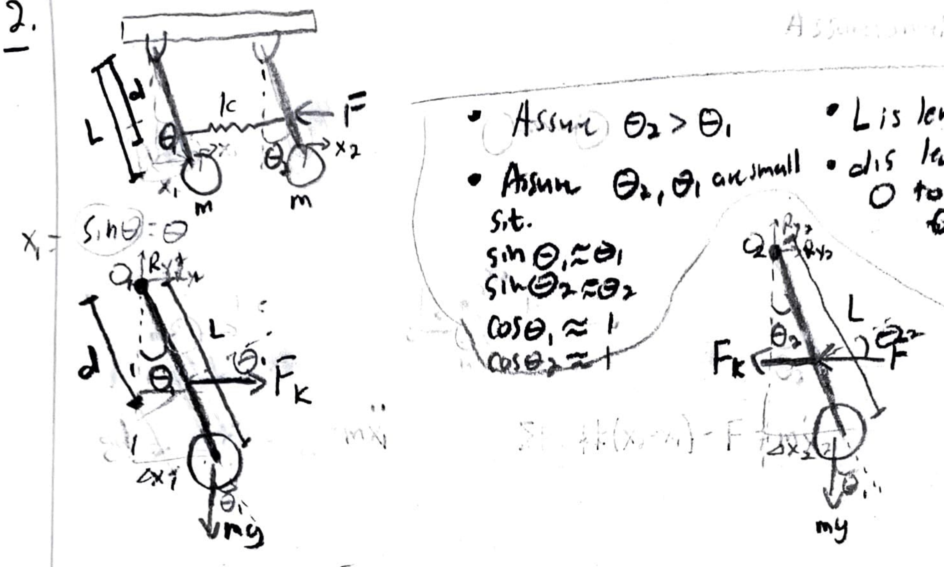
$$-F_{b11} - F_{k2} - F_{b12} + F = m_2\ddot{x}_2$$

input

$$-b_1(\dot{x}_2 - \dot{x}_1) - k_2(x_2 - x_1) - b_1\dot{x}_2 + F = m_2\ddot{x}_2$$

$$\boxed{m_2\ddot{x}_2 + 2b_1\dot{x}_2 + k_2x_2 - b_1\dot{x}_1 - k_2x_1 = F}$$

2.




- Assume $\theta_2 > \theta_1$
- Assume $\theta_2, \theta_1$ are small s.t.
  $\sin\theta_1 \approx \theta_1$
  $\sin\theta_2 \approx \theta_2$
  $\cos\theta_1 \approx 1$
  $\cos\theta_2 \approx 1$
- L is length of rod from O to C not mass
- d is length from O to spring / force application point

$$\circlearrowleft + \;\; \sum \tau_{O_1} = I\ddot{\theta}_1$$

$$F_k d\cos\theta_1 - mgL\sin\theta_1 = I\ddot{\theta}_1$$

$$F_k = (\underbrace{d\sin\theta_2}_{\Delta x_2} - \underbrace{d\sin\theta_1}_{\Delta x_1})k$$

by assumptions

$$F_k = dk(\theta_2 - \theta_1)$$

$$F_k d - mgL\theta_1 = I\ddot{\theta}_1$$

$$dk(\theta_2 - \theta_1)d - mgL\theta_1 = I\ddot{\theta}_1$$

$$\boxed{I\ddot{\theta}_1 + (mgL + d^2k)\theta_1 - d^2k\theta_2 = 0}$$

$$\circlearrowleft + \;\; \sum \tau_{O_2} = I\ddot{\theta}_2$$

$$-F_k d\cos\theta_2 - mgL\sin\theta_2 - Fd\cos\theta_2 = I\ddot{\theta}_2$$

$$F_k = dk(\theta_2 - \theta_1)$$

by assumptions

$$-F_k d - mgL\theta_2 - Fd = I\ddot{\theta}_2$$

$$-dk(\theta_2 - \theta_1)d - mgL\theta_2 - Fd = I\ddot{\theta}_2$$

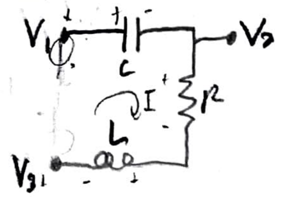
$$\boxed{I\ddot{\theta}_2 + (mgL + d^2k)\theta_2 - d^2k\theta_1 = -Fd}$$

$\boxed{\text{when } I = mL^2}$

(Assume m is close to point mass)

Could relate d to L now like $d = \frac{1}{2}L$ if d is $\frac{1}{2}L$

**3.**




$V_1$ and $V_3$ are inputs $\quad$ $V_2$ is output

KVL: $-V_1 + V_C + V_R + V_L + V_3 = 0$

$$-V_1 + \frac{1}{C}\int I\,dt + IR + L\frac{dI}{dt} + V_3 = 0$$

$$V_1 - V_3 = \frac{1}{C}\int I\,dt + IR + \frac{L\,dI}{dt}$$

$$V_1 - V_2 = \frac{1}{C}\int I\,dt$$

$$I = C(\dot{V}_1 - \dot{V}_2)$$

$$V_1 - V_3 = V_1 - V_2 + RC(\dot{V}_1 - \dot{V}_2) + LC(\ddot{V}_1 - \ddot{V}_2)$$

$$V_3 = V_2 + RC(\dot{V}_2 - \dot{V}_1) + LC(\ddot{V}_2 - \ddot{V}_1)$$

$$\boxed{LC\ddot{V}_1 + RC\dot{V}_1 + V_3 = LC\ddot{V}_2 + RC\dot{V}_2 + V_2}$$

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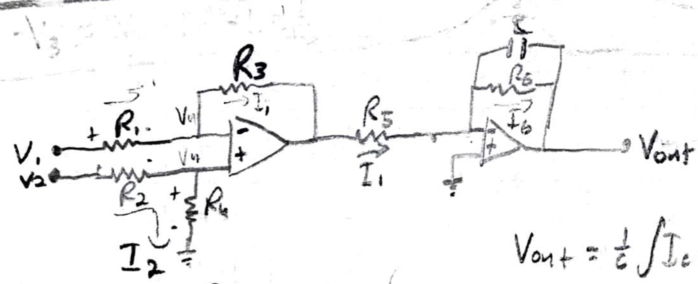
**4.**




$V_1$ and $V_2$ are inputs

$$I_c = C\frac{dV_{out}}{dt} = C\dot{V}_{out}$$

$$I_6 = \frac{V_{out}}{R_6}$$

$$V_{out} = \frac{1}{C}\int I_c\,dt = I_6 R_6$$


$$I_1 = \frac{V_1 - V_4}{R_1}$$

Ohm's Law

$$V_1 - V_4 = I_1 R_1$$

voltage divider

$$V_4 = \frac{R_4}{R_2 + R_4} V_2$$

$$V_1 - \frac{R_4}{R_2 + R_4} V_2 = I_1 R_1$$

$$I_1 = \frac{V_1}{R_1} - \frac{R_4}{R_1(R_2 + R_4)} V_2$$

$I_1$ in terms of $V_1$ and $V_2$

in terms of $V_1$ and $V_2$

KCL: $I_1 = I_c + I_6$ ← in terms of $V_{out}$

$$\frac{V_1}{R_1} - \frac{R_4}{R_1(R_2 + R_4)} V_2 = C\dot{V}_{out} + \frac{V_{out}}{R_6}$$

$$\boxed{\frac{1}{R_1} V_1 - \frac{R_4}{R_1(R_2 + R_4)} V_2 = C\dot{V}_{out} + \frac{V_{out}}{R_6}}$$

↳ $V_1$ and $V_2$ related to $V_{out}$