MECHEN 6220 | Lab 1 | Ryan Dalby

$X(0) = [V_1]$ $\quad x_1(t_1) = x_2(t_1) = [V_2]$ $\quad X(t_2) = [V_3]$

$x_1(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ → 4 coeffs

$x_2(t) = b_0 + b_1 (t-t_1) + b_2 (t-t_1)^2 + b_3 (t-t_1)^3$ → 4 coeffs

8 coefficients

2 binary
6 unary
constraints

$\dot{x}_1(0) = 0$ $\quad \dot{x}_2(t_2) = 0$

$\dot{x}_1(t_1) = \dot{x}_2(t_1)$ $\quad \ddot{x}_1(t_1) = \ddot{x}_2(t_1)$

$\dot{x}_1(t) = a_1 + 2a_2 t + 3a_3 t^2$

$\dot{x}_2(t) = b_1 + 2b_2 (t-t_1) + 3b_3 (t-t_1)^2$

$\ddot{x}_1(t) = 2a_2 + 6a_3 t$

$\ddot{x}_2(t) = 2b_2 + 6b_3 (t-t_1)$

$a_0 = V_1$ → $a_0 + a_1 t_1 + a_2 t_1^2 + a_3 t_1^3 = V_2$

$b_0 = V_2$ → $b_0 + b_1 (t_2-t_1) + b_2 (t_2-t_1)^2 + b_3 (t_2-t_1)^3 = V_3$

$a_1 = 0$ → $b_1 + 2b_2 (t_2-t_1) + 3b_3 (t_2-t_1)^2 = 0$

→ $a_1 + 2a_2 t_1 + 3a_3 t_1^2 = b_1$

→ $2a_2 + 6a_3 t_1 = 2b_2$

$a_0 = V_1$

$b_0 = V_2$

$a_1 = 0$

Eqn (1) $V_1 + a_2 t_1^2 + a_3 t_1^3 = V_2$

Eqn (2) $V_2 + b_1 (t_2-t_1) + b_2 (t_2-t_1)^2 + b_3 (t_2-t_1)^3 = V_3$

Eqn (3) $b_1 + 2b_2 (t_2-t_1) + 3b_3 (t_2-t_1)^2 = 0$

Eqn (4) $2a_2 t_1 + 3a_3 t_1^2 = b_1$

Eqn (5) $2a_2 + 6a_3 t_1 = 2b_2$

↑ Numerical solution

V_1 is start position
V_2 is mid position
V_3 is end position