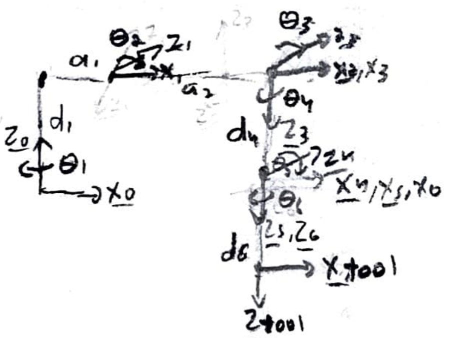
ME EN 6220 Lab 3 — Ryan Nelly — Paul

i	a_i	d_i	α_i	θ_i
1	a_1	d_1	$-\pi/2$	*
2	a_2	0	0	*
3	0	0	$-\pi/2$	*
4	0	d_4	$\pi/2$	*
5	0	0	$-\pi/2$	*
6	0	d_6	0	*

From HW 6

$\Rightarrow$

i	a_i	d_i	α_i	θ_i
1	a_1	d_1	$-\pi/2$	*
2	a_2	0	0	*
3	0	0	$-\pi/2$	*
4	0	d_4	$\pi/2$	*
5	0	0	$-\pi/2$	*
6	0	0	0	*
tool	0	d_6	0	0



Program

Given $\quad {}^wT_{tool} = \begin{bmatrix} {}^wR_{tool} & {}^wd_{w,tool} \\ \underline{0} & 1 \end{bmatrix}$ $\qquad {}^wT_0 = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & L \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & h \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^wR_0 & {}^wd_{w,0} \\ \underline{0} & 1 \end{bmatrix}$

$${}^6R_{tool} = R_z(\theta_{tool})\,R_x(\alpha_{tool}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

From HW 6 b) $\rightarrow$ $\boxed{{}^0R_6 = {}^wR_0^T\,{}^wR_{tool}\,{}^6R_{tool}^T \Rightarrow {}^0R_6 = {}^wR_0^T\,{}^wR_{tool}}$

$${}^6d_{4,tool} = d_6\,{}^6z_6 = [0 \;\; 0 \;\; d_6]^T$$

$${}^0d_{4,tool} = {}^0R_6\,{}^6d_{4,tool} = {}^0R_6\,[0 \;\; 0 \;\; d_6]^T$$

$${}^0d_{04} = {}^wR_0^T\,{}^w\underline{d}_{w,tool} - {}^wR_0^T\,{}^wd_{w,0} - {}^0\underline{d}_{4,tool} = {}^wR_0^T\left({}^wd_{0,tool}\right) - {}^0d_{4,tool}$$

${}^wd_{w,tool} - {}^wd_{w,0}$

$= {}^0R_w\left({}^wd_{0,tool}\right) - {}^0\underline{d}_{4,tool}$

$= {}^0d_{0,tool} - {}^0d_{4,tool}$

$\hookrightarrow$ Now we can use previous solutions

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Program: Extract information from ${}^{w}T_{tool} = \begin{bmatrix} {}^{w}R_{tool} & {}^{w}d_{w,tool} \\ 0 & 1 \end{bmatrix}$

${}^{0}R_{6} = {}^{w}R_{0}^{T}\, {}^{w}R_{tool}$

${}^{0}d_{0,tool} = {}^{w}R_{0}^{T}({}^{w}d_{0,tool})$

${}^{0}d_{04} = {}^{0}d_{0,tool} - {}^{0}d_{4,tool}$ where ${}^{0}d_{4,tool} = {}^{0}R_{6}[0\ 0\ d_{6}]^{T}$

↳ Check if ${}^{0}d_{04}$ ($= {}^{0}d_{04}$) is in workspace. Assuming $d_1 = 64$ mm, $d_2 = 370.62$ m, $d_4 = 374.24$ m

Primary workspace diameter: $[a_1 + a_2 - d_4,\ a_2 + d_4 - a_1]$

Now ${}^{0}d_{04} = [X, Y, Z]$ left is "default"

$\theta_1 = \arctan\left(\frac{Y}{X}\right)$ → Left and Right (choose)

$\theta_3 = \alpha - \pi/2$ where $\alpha = \pm 2\left(\arctan\sqrt{\frac{(a_2+d_4)^2 - ({}^{1}d_{14x}^2 + {}^{1}d_{14y}^2)}{({}^{1}d_{14x}^2 + {}^{1}d_{14y}^2) - (a_2 - d_4)^2}}\right)$ → Elbow up, Elbow Down (choose) "default" flip w/ (-) and elbow up

$\theta_2 = \Phi - \Psi$ where $\Psi = \text{atan2}(d_4 \sin(\alpha),\ a_2 + d_4\cos(\alpha))$

$\Phi = \text{atan2}({}^{1}d_{14y},\ {}^{1}d_{14x})$

Find $\theta_4, \theta_5, \theta_6$ using ${}^{3}R_{6} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

$\text{atan2}\left(\frac{-r_{23}}{-r_{13}}\right)$ usually in quad 3 (NF=1)

$\theta_4 = \text{atan2}\left(\frac{r_{23}}{r_{13}}\right)$

Choose: NoFlip, Flip

NF=0 [illegible] π

$\theta_5 = \text{atan2}(-s_{13}, s_{23})$ ${}^{4}R_{6} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}$

$\theta_6 = \text{atan2}(-s_{31}, -s_{32})$

Code for ikinelbow:

```
% ikinelbow computes the inverse kinematics for the 7 DoF Baxter robot
% constrained to be 6 DoF with theta3=0
%
%     theta = ikinelbow(a, d, Tw_tool, LR, UD, NF) Calculates a vector of
%     theta (joint angle) values given specification of the robot geometry
%     and a desired end position and orientation
%
%     a = non zero a DH parameters in sequential order starting with a1
%     i.e. (a1, a2)
%   d = non zero d DH parameters in sequential order starting with d1
%   i.e. (d1, d4, d6)
%   Tw_tool  = Homogeneous transformation matrix which specifies a desired
%   location and orientation of the end effector from the tool frame with
%   respect to the world frame
%   LR = 1 if specifying the "lefty" solution and 0 if specifying the
%   "righty" solution
%   UD = 1 if specifying the "elbow up" solution and 0 if specifying the
%   "elbow down" solution
%   NF = 1 if specifying the "flip" solution and 0 if specifying the
%   "no flip" solution
%
%     Ryan Dalby
%     ME EN 6220
%     11/17/2020
function [theta] = ikinelbow(a,d,Tw_tool,LR,UD,NF)
theta = zeros(1,6);
R_w_tool = Tw_tool(1:3,1:3);
d_w_wtool = Tw_tool(1:3,4);
a1 = a(1);
a2 = a(2);
d1 = d(1);
d4 = d(2);
d6 = d(3);

%
% Information for transformation between world frame and frame 0 and also
%
R_w_0 = [sqrt(2)/2 sqrt(2)/2 0;...
         -sqrt(2)/2 sqrt(2)/2 0;...
         0 0 1];
L = 221; % mm
h = 22; % mm
```

```
H = 1104; % mm
d_w_w0 = [L; h; H];

%
% Information for transformation between frame 6 and tool frame
%
R_6_tool = [1 0 0;...
         0 1 0;...
         0 0 1];
d_6_4tool = [0; 0; d6];

%
% Extract necessary information to use solution used in ikinebaxter
%
R_0_6 = transpose(R_w_0)*R_w_tool*transpose(R_6_tool);
d_0_0tool = transpose(R_w_0)*(d_w_wtool-d_w_w0);
d_0_4tool = R_0_6 * d_6_4tool;
d_0_04 = d_0_0tool - d_0_4tool;

%
% Workspace determination
%
% Assume that d4=374.29mm, a2=370.82mm, and a1=69mm
d_0_06 = d_0_04;
d_0_06_mag = norm(d_0_06);
inner_primary_workspace_diameter = a1+a2-d4;
outer_primary_workspace_diameter = a2+d4-a1;
% Make sure we are in primary workspace or exit function
if (d_0_06_mag < inner_primary_workspace_diameter) || (d_0_06_mag >
outer_primary_workspace_diameter)
    disp('Outside of primary workspace');
    return;
end



%
% Regional structure inverse kinematics
%

% theta 1 determination
theta(1) = atan2(d_0_04(2), d_0_04(1));
if (~LR)
    theta(1) = theta(1) + pi;
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```
end

% theta 3 and theta 2 determination
d_0_01 = [a1*cos(theta(1)); a1*sin(theta(1)); d1];
R_1_0 = [cos(theta(1)) sin(theta(1)) 0;...
         0 0 -1;...
         -sin(theta(1)) cos(theta(1)) 0];
d_1_14 = R_1_0 * (d_0_04-d_0_01);

r2 = d_1_14(1)^2 + d_1_14(2)^2;

alpha = 2*atan2(sqrt((a2+d4)^2 - (r2)) , sqrt((r2) - (a2-d4)^2));
if(~UD)
    alpha = alpha * -1;
end
theta(3) = alpha - pi/2;

psi = atan2((d4*sin(alpha)),(a2+d4*cos(alpha)));
phi = atan2(d_1_14(2),d_1_14(1));
theta(2) = phi - psi;

%
% Orientation structure inverse kinematics
%
R_0_1 = transpose(R_1_0);
R_1_2 = [cos(theta(2)) -sin(theta(2)) 0;...
         sin(theta(2)) cos(theta(2)) 0;...
         0 0 1];
R_2_3 = [cos(theta(3)) 0 -sin(theta(3));...
         sin(theta(3)) 0 cos(theta(3));...
         0 -1 0];
R_0_3 = R_0_1*R_1_2*R_2_3;

R_3_6 = transpose(R_0_3) * R_0_6;

% theta 4 determination
theta(4) = atan2(-R_3_6(2,3),-R_3_6(1,3));
if(NF)
    theta(4) = theta(4) + pi;
end

% theta 5 and theta 6 determination
R_3_4 = [cos(theta(4)) 0 sin(theta(4));...
```

```
          sin(theta(4)) 0 -cos(theta(4));...
          0 1 0];

R_4_6 = transpose(R_3_4) * R_3_6;

theta(5) = atan2(-R_4_6(1,3),R_4_6(2,3));

theta(6) = atan2(-R_4_6(3,1),-R_4_6(3,2));
end
```