Problem Set #4: Open Loop Response

Suppose we have a single DOF robot, namely the Quanser SRV-02, which has been turned on its side with a pendulum attached to the output shaft, as shown in Figure 1.

Figure 1. Single DOF Robot

Depending on whether the SRV-02 is controlled by a current or voltage amplifier, the linearized dynamics are described by the following differential equations:

A. Voltage amp:

$$\left(\frac{Nk_t}{R_a}\right) V(t) = (I_1 + N^2 J_m)\,\ddot{\theta} \;+ N^2\left(b + \frac{k_t^2}{R_a}\right)\dot{\theta} + m_1 g\, r_{01}\theta$$

B. Current amp:

$$(Nk_t)\, i(t) = (I_1 + N^2 J_m)\,\ddot{\theta} \;+ N^2 b\dot{\theta} + m_1 g\, r_{01}\theta$$

where θ is the angle of the output shaft, $V(t)$ is the motor voltage, and $i(t)$ is the motor current. Notice that with the voltage amp, there is an extra damping term due to the effect of the back EMF.

Assume that the parameters are known to be:

Motor torque constant: k_t	0.0077 N·m/A
Armature resistance: R_a	2.6 Ω
Gear ratio: N	70
Moment of inertia of link: I_1	0.83×10^{-3} kg·m^2
Motor inertia: J_m	0.65×10^{-6} kg·m^2
Mechanical damping: b	3.1×10^{-6} N·m·s/rad
Gravitational torque constant: $m_1 g\, r_{01}$	0.067 N·m/rad

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1. Imagine that you as the robot designer could have chosen a set of gears with a different gear ratio *N*. For the SRV-02 parameters shown in the table above, use impedance matching to find the ideal gear ratio (to the nearest integer) that would maximize the ability of the robot to accelerate from rest.

2. Now assume that the gear ratio is *N*=70 as shown in the table. For each of the two amplifier scenarios above (A & B) with the given parameters, sketch by hand the open-loop response $\theta(t)$ of the system, assuming that a unit step input $V(t)\ or\ i(t)$ is applied at $t = 0$ and all initial conditions are zero. In each case, first find the open loop poles and decide what kind of a step response to expect (overdamped, underdamped, or critically damped). If the response looks like a first-order response, then compute the time constant and final steady-state value. If the response looks like a second-order response, then compute peak time, 2% settling time, percent overshoot, and final value. Use these to sketch the response of the system.