ME EN 5230/6230, CS 6330

Intro to Robot Control – Spring 2021

Problem Set #2: Manipulator Dynamics

For this problem, we consider the dynamics of the planar 2-DOF Quanser robot in the lab, shown in Figure 1. Be sure to include the masses and moments of inertia of both links, but do not worry about the numerical values. We will only be concerned with the algebraic solution at this time. You can assume that the centers of mass of the links are located at distances r_{01} and r_{12} along the x_1 and x_2 axis, respectively.

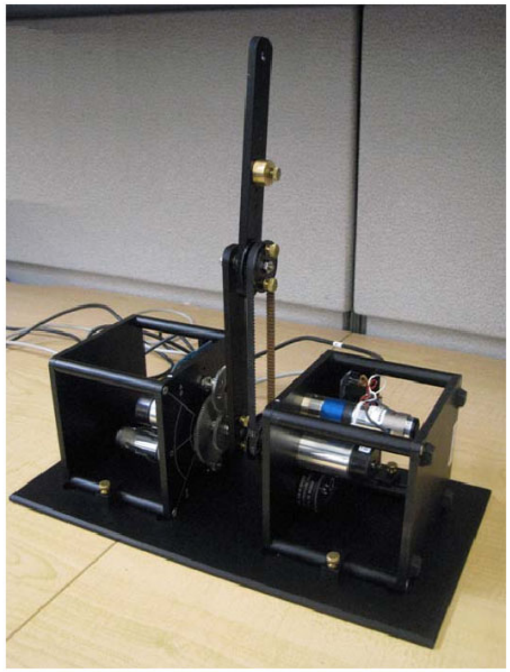

Figure 1. Planar 2-DOF Quanser Robot

In class we learned that you can put the inverse dynamic solution into a closed form:

$$\boldsymbol{\tau} = \boldsymbol{H}(\boldsymbol{\Theta})\ddot{\boldsymbol{\Theta}} + \boldsymbol{V}\left(\boldsymbol{\Theta}, \dot{\boldsymbol{\Theta}}\right) + \boldsymbol{G}(\boldsymbol{\Theta})$$

where **H** is the inertia matrix, **V** is a matrix of centripetal and coriolis terms, and **G** is a matrix of body or gravitational torques. For the 2-DOF robot example in class, this matrix equation took the form of:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 & -2h & -h \\ h & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_2^2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$$

Use the recursive Newton-Euler algorithm to derive the algebraic solutions for the terms in the above equations.

Hint: You can follow the derivations in Ch. 10 of Hollerbach's notes, EXCEPT for the part where he assumes the centers of mass of the links are exactly halfway along each link, and you'll need to include gravitational effects.