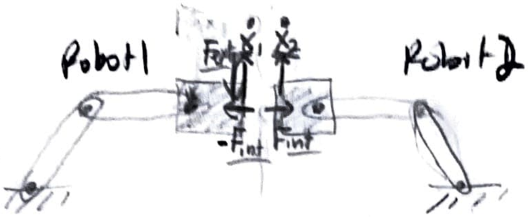
1. Setting up relative task frame



$\underline{\dot{\Theta}}_1$ = Robot 1 joint velocities $\dot{\theta}_{11}$ and $\dot{\theta}_{12}$

$\underline{\dot{\Theta}}_2$ = Robot 2 joint velocities $\dot{\theta}_{21}$ and $\dot{\theta}_{22}$

$\underline{\tau_1}$ = Robot 1 joint torques τ_{11}, τ_{12}

$\underline{\tau_2}$ = Robot 2 joint torques τ_{21}, τ_{22}

$$\underline{\dot{x}_1} = \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} \qquad \underline{\dot{x}_2} = \begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} \qquad \underline{\dot{x}_{rel}} = \underline{\dot{x}_2} - \underline{\dot{x}_1} = \begin{bmatrix} \dot{x}_2 - \dot{x}_1 \\ \dot{y}_2 - \dot{y}_1 \end{bmatrix} = \begin{bmatrix} \dot{x}_{rel} \\ \dot{y}_{rel} \end{bmatrix}$$

$$\underline{F_{ext}} = \begin{bmatrix} f_x \\ f_y \end{bmatrix} \qquad \underline{F_{int}} = \begin{bmatrix} f_{x,rel} \\ f_{y,rel} \end{bmatrix}$$

New task space:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = J_1 \underline{\dot{\Theta}}_1 \qquad \begin{bmatrix} \dot{x}_{rel} \\ \dot{y}_{rel} \end{bmatrix} = J_2 \underline{\dot{\Theta}}_2 - J_1 \underline{\dot{\Theta}}_1$$

$$\underline{\tau}_1 = J_1^T \begin{bmatrix} f_{x,ext} \\ f_{y,ext} \end{bmatrix} - J_1^T \begin{bmatrix} f_{x,int} \\ f_{y,int} \end{bmatrix} \qquad \underline{\tau}_2 = J_2^T \begin{bmatrix} f_{x,int} \\ f_{y,int} \end{bmatrix}$$

↳ Matrix form:

$$\boxed{\begin{bmatrix} \underline{\dot{x}_1} \\ \underline{\dot{x}_{rel}} \end{bmatrix} = \underbrace{\begin{bmatrix} J_1 & 0 \\ -J_1 & J_2 \end{bmatrix}}_{J_{rel}} \begin{bmatrix} \underline{\dot{\Theta}}_1 \\ \underline{\dot{\Theta}}_2 \end{bmatrix}} \qquad \begin{bmatrix} \underline{\tau}_1 \\ \underline{\tau}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} J_1^T & -J_1^T \\ 0 & J_2^T \end{bmatrix}}_{J_{rel}^T} \begin{bmatrix} \underline{F_{ext}} \\ \underline{F_{int}} \end{bmatrix}$$

Note: J_1 is Jacobian for Robot 1 mapping from $\underline{\Theta}_1$ to $\underline{x}_1$
J_2 is Jacobian for Robot 2 mapping from $\underline{\Theta}_2$ to $\underline{x}_2$

Constraints	Kinematic	Static
Natural Constraints	$\dot{x}_{rel} = 0$ $\dot{y}_{rel} = 0$	$f_{x,ext} = 0$ $f_{y,ext} = 0$
Artificial Constraints	$\dot{x}_1 = \dot{x}_{des}$ $\dot{y}_1 = \dot{y}_{des}$	$f_{x,int} = f_{x,des}$ $f_{y,int} = f_{y,des}$
	Position Control	Force Control

See Model (1.1) for implementation of a Multi-Arm Position/Force controller w/ selection matrix of

$$S = \begin{array}{c|c} & \dot{x}_1 \quad \dot{y}_1 \\ \hline & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ f_{x,int} \\ \\ f_{y,int} \end{array}$$

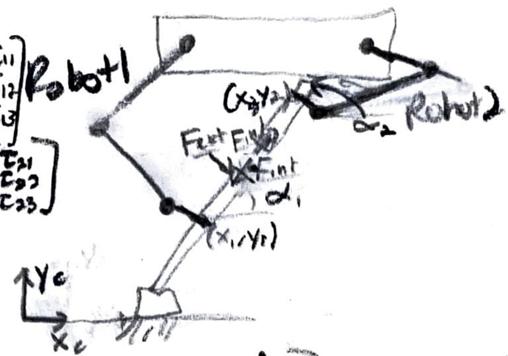
1 → Position control

0 → Force control

2. 2.1)

τ_1 = Robot 1 joint torques $= \begin{bmatrix}\tau_{11}\\ \tau_{12}\\ \tau_{13}\end{bmatrix}$

τ_2 = Robot 2 joint torques $= \begin{bmatrix}\tau_{21}\\ \tau_{22}\\ \tau_{23}\end{bmatrix}$



$\underline{\dot{\theta}_1}$ = Robot 1 joint angle velocities $\dot{\theta}_{11}, \dot{\theta}_{12}, \dot{\theta}_{13}$

$\underline{\dot{\theta}_2}$ = Robot 2 joint angle velocities $\dot{\theta}_{21}, \dot{\theta}_{22}, \dot{\theta}_{23}$

Joint Space

$$\underline{\dot{X}_1} = \begin{bmatrix}\dot{x}_1\\ \dot{y}_1\\ \dot{\alpha}_1\end{bmatrix} \qquad \underline{\dot{X}_2} = \begin{bmatrix}\dot{x}_2\\ \dot{y}_2\\ \dot{\alpha}_2\end{bmatrix}$$

$$\underline{\dot{X}_1} = J_1 \underline{\dot{\theta}_1} \qquad \underline{\dot{X}_2} = J_2 \underline{\dot{\theta}_2}$$

Relative Task space:

$$\underline{\dot{X}_{rel}} = \begin{bmatrix}\dot{x}_{rel}\\ \dot{y}_{rel}\\ \dot{\alpha}_{rel}\end{bmatrix} = \begin{bmatrix}\dot{x}_2-\dot{x}_1\\ \dot{y}_2-\dot{y}_1\\ \dot{\alpha}_2-\dot{\alpha}_1\end{bmatrix}$$

$$\Rightarrow \quad \underline{\dot{X}_{rel}} = J_2\underline{\dot{\theta}_2} - J_1\underline{\dot{\theta}_1}$$

Note:

$$\underline{F_{int}} = \begin{bmatrix}f_{x,int}\\ f_{y,int}\\ \tau_{int}\end{bmatrix} \qquad \underline{F_{ext}} = \begin{bmatrix}f_{x,ext}\\ f_{y,ext}\\ \tau_{ext}\end{bmatrix}$$

To map between joint space and relative task spaces

$$\begin{bmatrix}\underline{\dot{X}_1}\\ \underline{\dot{X}_{rel}}\end{bmatrix} = \begin{bmatrix}J_1 & 0\\ -J_1 & J_2\end{bmatrix}\begin{bmatrix}\underline{\dot{\theta}_1}\\ \underline{\dot{\theta}_2}\end{bmatrix} = J_{rel}\,\underline{\dot{\theta}} = \underline{\dot{X}}$$

Also Note:

$$\begin{bmatrix}\underline{\tau_1}\\ \underline{\tau_2}\end{bmatrix} = J_{rel}^T\begin{bmatrix}\underline{F_{ext}}\\ \underline{F_{int}}\end{bmatrix}$$

$$\underline{\tau} = J_{rel}^T\,\underline{F}$$

2.2)

Will assume broom begins contacting surface thus $\dot{y}_{des}$ can be 0.

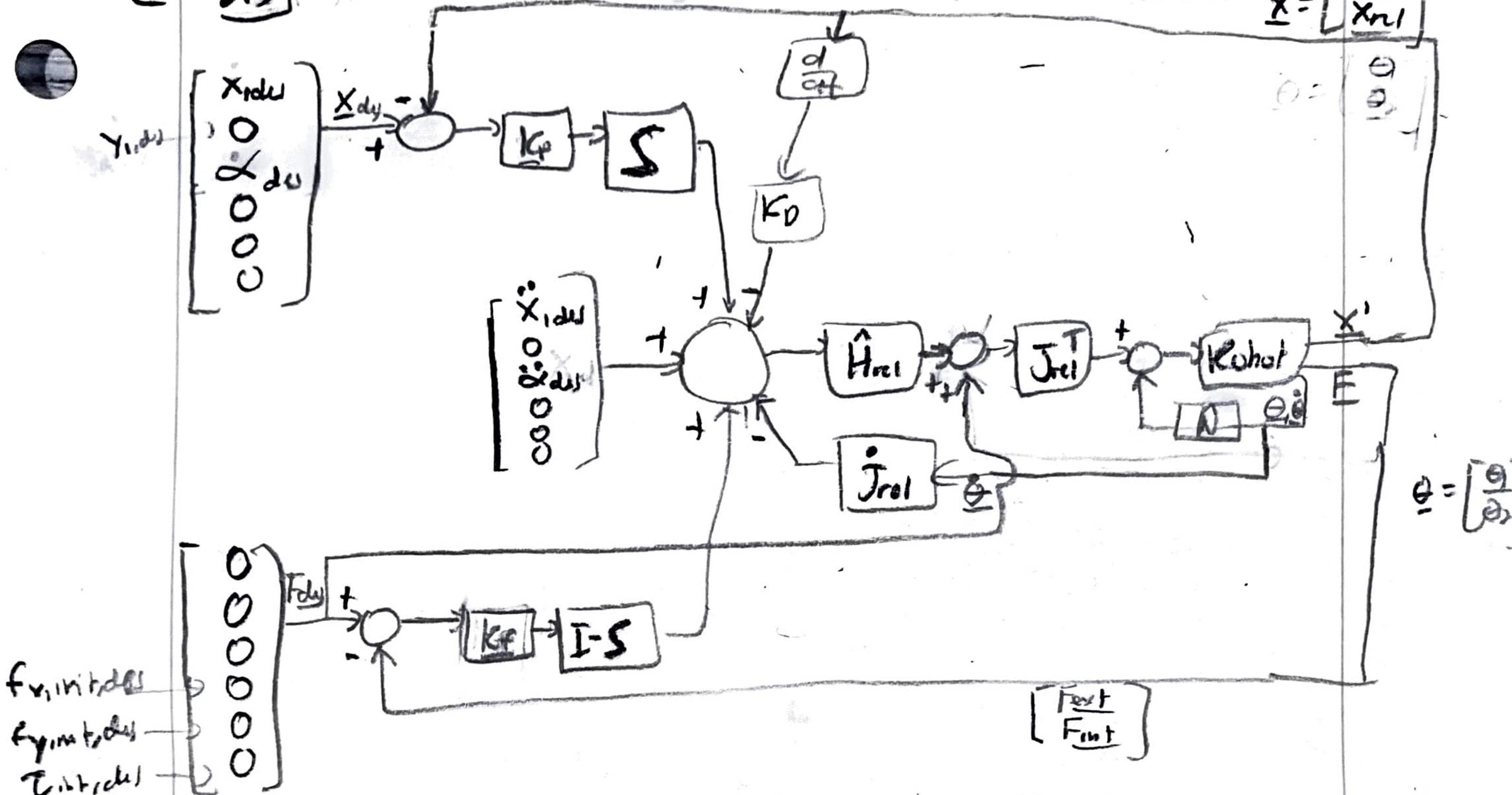
Constraints	Kinematic	Static
Natural constraints	$\dot{x}_{rel}=0$ $\dot{y}_{rel}=0$ $\dot{\alpha}_{rel}=0$	$f_{x,ext}=0$ $f_{y,ext}=0$ $\tau_{ext}=0$ corresponds to $\dot{\alpha}_1$
Artificial constraints	$\dot{x}_1=\dot{x}_{des}$ $\dot{y}_1=\dot{y}_{des}=0$ $\dot{\alpha}_1=\dot{\alpha}_{des}$	$f_{x,int}=f_{x,int,des}=0$ $f_{y,int}=f_{y,int,des}=0$ ← Force Control $\tau_{int}=\tau_{int,des}=0$ corresponds to $\dot{\alpha}_{rel}$

Position Control

Assuming all values are in end-effector frame, although for $\dot{x}$, $\dot{y}$, and $\dot{\alpha}$ may make more sense to put in constraint frame, (would define as "broom frame") will not worry about it here.

$$S = diag\left(\left[\begin{array}{cccccc} \dot{x}_1 & \dot{y}_1 & \dot{\alpha}_1 & \dot{x}_{rel} & \dot{y}_{rel} & \dot{\alpha}_{rel}\\ 1 & 1 & 1 & 0 & 0 & 0\end{array}\right]\right) = \begin{bmatrix}1 & & & & & \\ & 1 & & & 0 & \\ & & 1 & & & \\ & & & 0 & & \\ & 0 & & & 0 & \\ & & & & & 0\end{bmatrix}$$

2. 2.3



$$\hat{H}_{rel} = J_{rel}^{-T} \begin{bmatrix} H_1(\underline{\theta}_1) & 0 \\ 0 & H_2(\underline{\theta}_2) \end{bmatrix} J_{rel}^{-1}$$