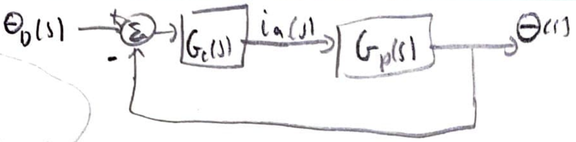
MEEN 6230 | Problem Set 5 | Ryan Dalby | Page 1

1.



$K_t = 0.007 N\cdot m/A$
$R_a = 2.6 \Omega$
$N = 70$
$I_1 = 0.83 \times 10^{-3} kg\cdot m^2$
$J_m = 0.65 \times 10^{-6} kg\cdot m^2$
$b = 3.1 \times 10^{-6} N\cdot m\cdot s/rad$
$m_1 g r_{c1} = 0.067 N\cdot m/rad$

$(NK_t) i(t) = (I_1 + N^2 J_m)\ddot{\theta} + N^2 b\dot{\theta} + m_1 g r_{c1}\theta$

$\Downarrow$

$G_c(s) = K_p + K_D s$

$$G_p(s) = \frac{NK_t}{(I_1 + N^2 J_m)s^2 + (N^2 b)s + m_1 g r_{c1}}$$

$PS\#4 \rightarrow G_p(s) = \frac{0.534}{0.004015 s^2 + 0.01519 s + 0.067}$

Design for $\%OS = 20\%$ $T_s = 0.2s$

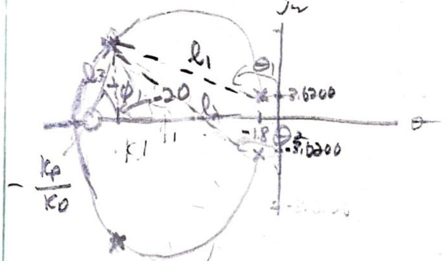
$$G_c(s) G_p(s) = \frac{NK_t(K_p + K_D s)}{(I_1 + N^2 J_m)s^2 + (N^2 b)s + m_1 g r_{c1}} = \frac{NK_t K_D(s + K_P/K_D)}{(I_1 + N^2 J_m)s^2 + (N^2 b)s + m_1 g r_{c1}}$$

plugging in parameter values ↳ (same as PS #4)

$$G_c(s) G_p(s) = \frac{0.539 K_D(s + ~~K_D~~ K_P/K_d)}{0.004015 s^2 + 0.01519 s + 0.067} = \frac{134.247 K_d(s + K_P/K_D)}{s^2 + 3.783s + 16.687}$$

O.L. zeros: $s = -K_P/K_D$

O.L. poles: $s = -1.8917 \pm 3.6206j$



$$\zeta = \frac{-\ln(20/100)}{\sqrt{\pi^2 + \ln^2(20/100)}} = 0.456$$

$T_s = \frac{4}{\zeta\omega_n} = 0.2$

$\omega_n = 43.86$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$\omega_d = 39.04$

Desired closed loop poles: $s_{1,2} = -\zeta\omega_n \pm \omega_d j$

$\boxed{s_{1,2} = -20 \pm 39.04j}$

Angle condition

$\phi - \theta_1 - \theta_2 = \pm 180°$

$\phi = \pm 180° + \tan^{-1}\left(\frac{\overset{39.04-3.621}{35.42}}{-18.2}\right) + \tan^{-1}\left(\frac{\overset{39.04+3.62}{42.66}}{-18.2}\right)$

$\phi = \pm 180° + 117.20 + 113.10 \Rightarrow 50.3°$

$\tan(50.3°) = \left(\frac{39.04}{-20-z}\right)$

$z = -52.41 = -K_P/K_D$

Loop Gain, Magnitude Condition

$$134.247 K_D = \frac{l_1 l_2}{l_3} = \frac{\sqrt{35.42^2 + 18.2^2}\sqrt{42.60^2 + 18.2^2}}{\sqrt{32.41^2 + 39.04^2}} = 36.40$$

$K_D = 36.4/134.247 = 0.271 \Rightarrow K_P = K_D(52.41) = 14.210$

$\boxed{K_P = 14.21 \quad K_D = 0.27}$

↳ The closed loop zero may interfere

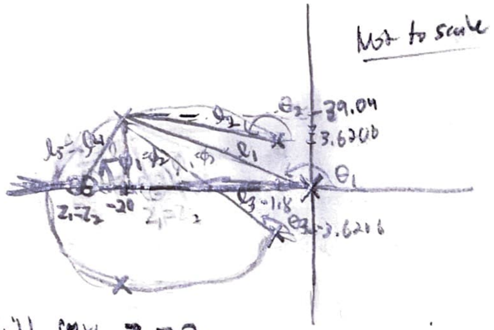
2. $G_c(s) = K_P + K_D s + \frac{K_I}{s}$ $\qquad G_p(s) = \frac{N K_t}{(J_l + N^2 J_m)s^2 + (N^2 b)s + m_l g r_{lc}}$

$G_c(s) = \frac{K_D s^2 + K_P s + K_I}{s}$

$^{K_P}/_{K_D}$ $^{K_I}/_{K_D}$

$$G_c G_p = \frac{134.247(K_D s^2 + K_P s + K_I)}{s(s^2 + 3.783s + 16.687)} = \frac{134.247 K_D (s+z_1)(s+z_2)}{s(s^2+3.783s+16.687)}$$

↳ $s = -1.8917 \pm 3.6206j$



From 1.:
Desired dominant closed loop poles should be (for %OS = 20%, $T_s = 0.2s$)
$s = -20 \pm 39.04j$

Will say $z_1 = z_2$

Angle Condition: $2\phi_1 - \theta_1 - \theta_2 - \theta_3 = \pm 180°$

$$\phi_1 = \frac{\theta_1 + \theta_2 + \theta_3 \pm 180°}{2} = \frac{\tan^{-1}\left(\frac{39.04}{-20}\right) + \tan^{-1}\left(\frac{35.42}{-18.1}\right) + \tan^{-1}\left(\frac{42.66}{-18.1}\right) \pm 180°}{2}$$

$$\phi_1 = \frac{117.1258 + 117.20 + 113.10 \pm 180}{2} = \frac{167.42}{2} = 83.713°$$

$\tan(83.713°) = \left(\frac{39.04}{-20-z_1}\right)$ $\qquad z_1 = -24.3$

Magnitude Condition:

$$134.247 K_D = \frac{l_1 l_2 l_3}{2 l_4} = \frac{\sqrt{(39.04)^2+(20)^2}\sqrt{(35.42)^2+(18.1)^2}\sqrt{(42.66)^2+(18.2)^2}}{2\sqrt{(4.3)^2+(39.04)^2}} = 1030.15$$

↑ $-20-(-24.3)$

$K_D = {}^{1030.15}/_{134.247} = 7.67$

$K_D(s+z_1)(s+z_2) = K_D s^2 + K_D(z_1+z_2)s + K_D(z_1 z_2) = K_D s^2 + K_P s + K_I$

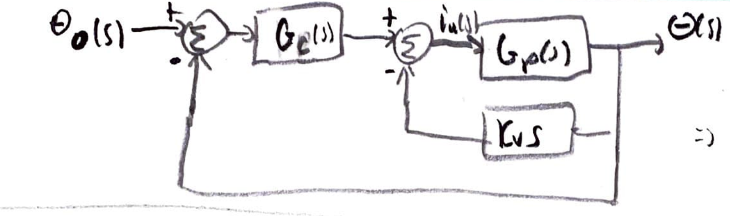
$K_D(z_1+z_2) = 2K_D(z_1) = K_P = 372.76$ $\quad$ (z_1: -24.3)

$K_D(z_1 z_2) = K_D(z_1)^2 = K_I = 4529.06$

$\boxed{K_P = 372.76}$ $\boxed{K_D = 7.67}$ $\boxed{K_I = 4529.06}$

↳ The closed loop zeros may (likely) interfere
↳ We also assumed dominant second order behavior but all 3 poles may dominate.

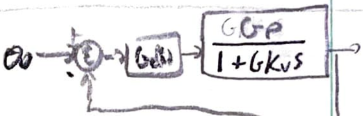
4.



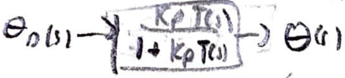
$$G_p(s) = \frac{134.247}{s^2 + 3.7833s + 16.687} \Rightarrow = \frac{134.2}{s^2 + 3.8s + 16.7}$$

PV Control:

$G_c(s) = K_p$



$$\frac{G_p}{1+G_pK_vs} = \frac{\frac{134.2}{s^2+3.8s+16.7}}{1+\frac{134.2(K_v)s}{s^2+3.8s+16.7}} = \frac{134.2}{s^2+(3.8+134.2K_v)s+16.7} = T(s)$$



$$\frac{\Theta(s)}{\Theta_D(s)} = \frac{K_pT(s)}{1+K_pT(s)}$$

PV CLTF

$K_p = 14.21$
$K_D = K_v = 0.27$

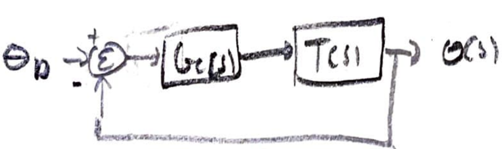
$$\frac{\Theta(s)}{\Theta_D(s)} = \frac{K_p(134.2)}{s^2+(3.8+134.2K_v)s+(16.7+134.2K_p)} = \frac{1906.962}{s^2+40.034s+1928.682}$$

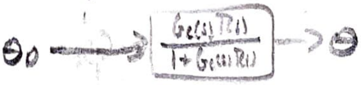
↳ C.L. Poles: $s = -20.17 \pm 38.95j \approx -20 \pm 39.04j$ ✓ PD CL Poles:

There was 1 CL zero for PD ← (No closed loop zeroes!)

PIV Control:

$$G_c(s) = K_p + \frac{K_I}{s} = \frac{K_ps+K_I}{s}$$





$$G_c(s)T(s) = \frac{(K_ps+K_I)134.2}{s(s^2+(3.8+134.2K_v)s+16.7)}$$

$K_p = 372.76$
K_v: $K_D = 7.67$
$K_I = 4529.06$

$$\frac{\Theta(s)}{\Theta_D(s)} = \frac{(K_ps+K_I)134.2}{s^3+(3.8+134.2K_v)s^2+(16.7+134.2K_p)s+134.2K_I} = \frac{50024s+607800}{s^3+1046s^2+50041s+607800}$$

C.L. Poles: $s = -996.39, -27.10, -22.51 \approx s = -988.2, -28.85, -21.43$

↳ Approximately equal to closed loop zeroes found using matlab: (see attached code)

↳ We have 1 closed loop zero instead of 2 with PID