MEEN 6230 | Problem Set 4 | Ryan Dalby | Page 1

1. SRV-02 Linearized Dynamics:

A: Voltage Amp: $\left(\frac{N K_t}{R_a}\right) V(t) = (I_1 + N^2 J_m)\ddot{\theta} + N^2\left(b + \frac{K_t^2}{R_a}\right)\dot{\theta} + m_1 g r_{01}\theta$

B: Current Amp:

$$(N K_t) i(t) = (I_1 + N^2 J_m)\ddot{\theta} + N^2 b\dot{\theta} + m_1 g r_{01}\theta$$

θ: angle of output shaft $V(t)$ is motor voltage $i(t)$ is motor current

$K_t = 0.0077$ N·m/A
$R_a = 2.6\ \Omega$
$N = 70$
$I_1 = 0.83\times10^{-3}$ kg·m²

$J_m = 0.65\times10^{-6}$ kg·m²
$b = 3.1\times10^{-6}$ N·m·s/rad
$m_1 g r_{01} = 0.067$ N·m/rad

1. Use Impedance Matching to find N to maximize ability of robot to accelerate from rest

Assumptions: Neglect Gravity and Friction and assume $\dot{\theta} = 0$ (from rest)

$$N \tau_m = (I_1 + N^2 J_m)\ddot{\theta}$$

$$\ddot{\theta} = \frac{N\tau_m}{(I_1 + N^2 J_m)}$$ ⇒ want to find N that maximizes $\ddot{\theta}$

$$\frac{d\ddot{\theta}}{dN} = \frac{\tau_m(I_1 + N^2 J_m) - N\tau_m(2 N J_m)}{(I_1 + N^2 J_m)^2} = 0$$

$$I_1 + N^2 J_m - 2N^2 J_m = 0$$
$$I_1 - N^2 J_m = 0$$

$$N = \sqrt{\frac{I_1}{J_m}}$$

Thus as a designer would choose $N = \sqrt{\frac{0.83\times10^{-3}\,[\text{kg}\cdot\text{m}^2]}{0.65\times10^{-6}\,[\text{kg}\cdot\text{m}^2]}}$

$\therefore N = 35.73$

$$\boxed{N = 36}$$

2. N=70, Sketch OLR $\theta(t)$ for unit step at $t=0$ w/ ICs = 0
↳ Determine OLP → Determine step response type we expect (over/under/critically damped)
→ Determine if 1st or 2nd order-like → Determine characteristics of the system (τ, ζ, ω_n etc.)
↳ Then sketch

A: $$\left(\frac{NK_t}{R_a}\right)V(s) = (I_1+N^2J_m)s^2\Theta(s) + N^2\left(b+\frac{K_t^2}{R_a}\right)s\Theta(s) + m_1 g r_{01}\Theta(s)$$

$$\left(\frac{NK_t}{R_a}\right)V(s) = \left[(I_1+N^2J_m)s^2 + N^2\left(b+\frac{K_t^2}{R_a}\right)s + m_1 g r_{01}\right]\Theta(s)$$

$$\frac{\Theta(s)}{V(s)} = \frac{NK_t}{R_a(I_1+N^2J_m)s^2 + R_a N^2\left(b+\frac{K_t^2}{R_a}\right)s + R_a m_1 g r_{01}}$$

$$\frac{\Theta(s)}{V(s)} = \frac{0.539}{0.010439s^2 + 0.330015s + 0.1742}$$

$$\frac{\Theta(s)}{V(s)} = \frac{0.539}{0.010439(s+0.536976)(s+31.076685)}$$

We have 2 real poles : expected response: $\boxed{\text{over damped}}$

Openloop Poles: $\boxed{s_1 = -0.536976 \quad s_2 = -31.076685}$

Since $s_1 \ll 5s_2$ we can neglect s_2 contribution, $\Rightarrow$ $\boxed{\text{1st order-like response}}$

Will determine τ and $\theta(\infty)$

↳ $$\frac{\Theta(s)}{V(s)} \approx \frac{0.539}{(s+0.536976)}$$ where $a=0.536976$

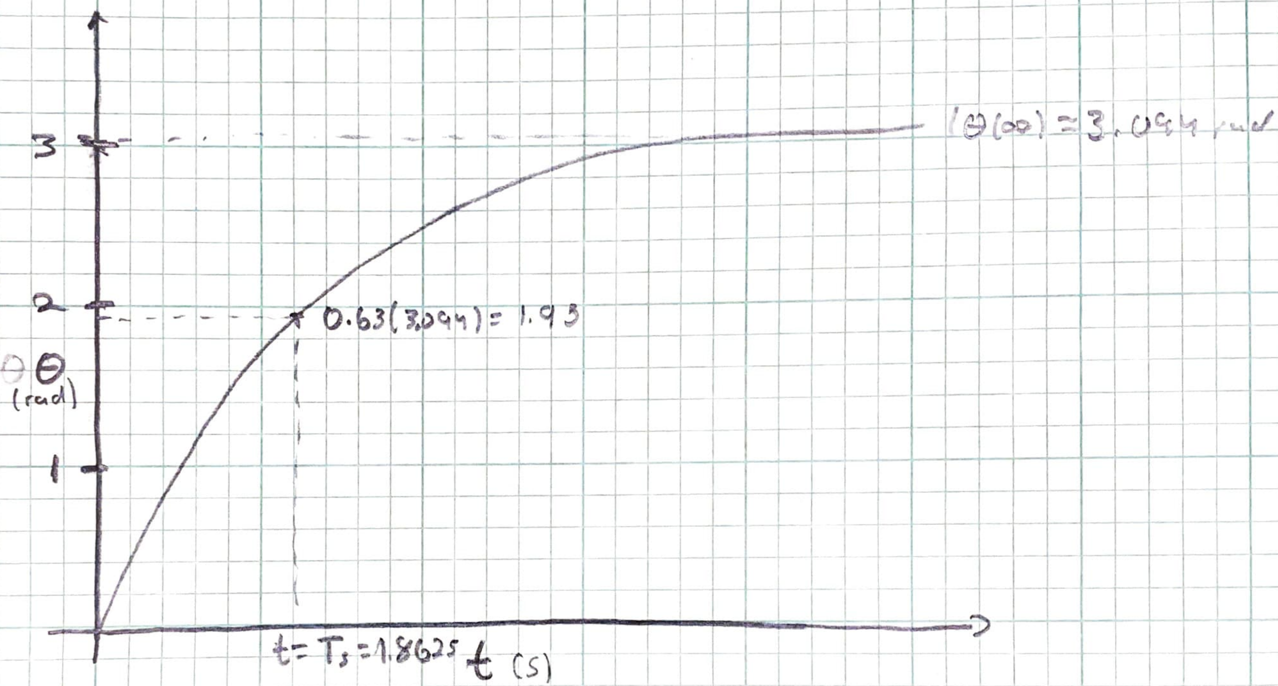
Neglect other pole just for characteristics of plot (not for values)

$\boxed{\tau = 1/a = 1.862 \text{ seconds}}$

FVT: $$\lim_{s\to 0} s\cdot\frac{1}{s}\,\frac{0.539}{0.010439s^2+0.330015s+0.1742} = 3.094$$

$\boxed{\theta(\infty) \approx 3.094}$ rad

Given a unit step voltage the Θ response looks like:



Page 2: Plot

2: B: $(NK_t)\,i(s) = \left[(I_1 + N^2 J_m)s^2 + N^2 b s + m_1 g r_{01}\right]\Theta(s)$

$$\frac{\Theta(s)}{i(s)} = \frac{N K_t}{(I_1 + N^2 J_m)s^2 + N^2 b s + m_1 g r_{01}}$$

$$\frac{\Theta(s)}{i(s)} = \frac{0.539}{0.004015 s^2 + 0.01519 s + 0.067} = \frac{0.539}{0.004015(s^2 + 3.783 s + 16.687)}$$

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0.01519 \pm \sqrt{0.01519^2 - 4(.004015)(0.067)}}{2(0.004015)} = -1.8917 \pm 3.6206j$$

We have complex conjugate poles: expected response: $\boxed{\text{underdamped}}$

Open loop Poles: $\boxed{S_1 = -1.8917 + 3.6206j \quad S_2 = -1.8917 - 3.6206j}$

(2nd order-like response)

↳ will determine T_p, $T_s^{2\%}$, $\%OS$, $\theta(\infty)$

$\omega_n^2 = 16.687 \Rightarrow \omega_n = 4.085$

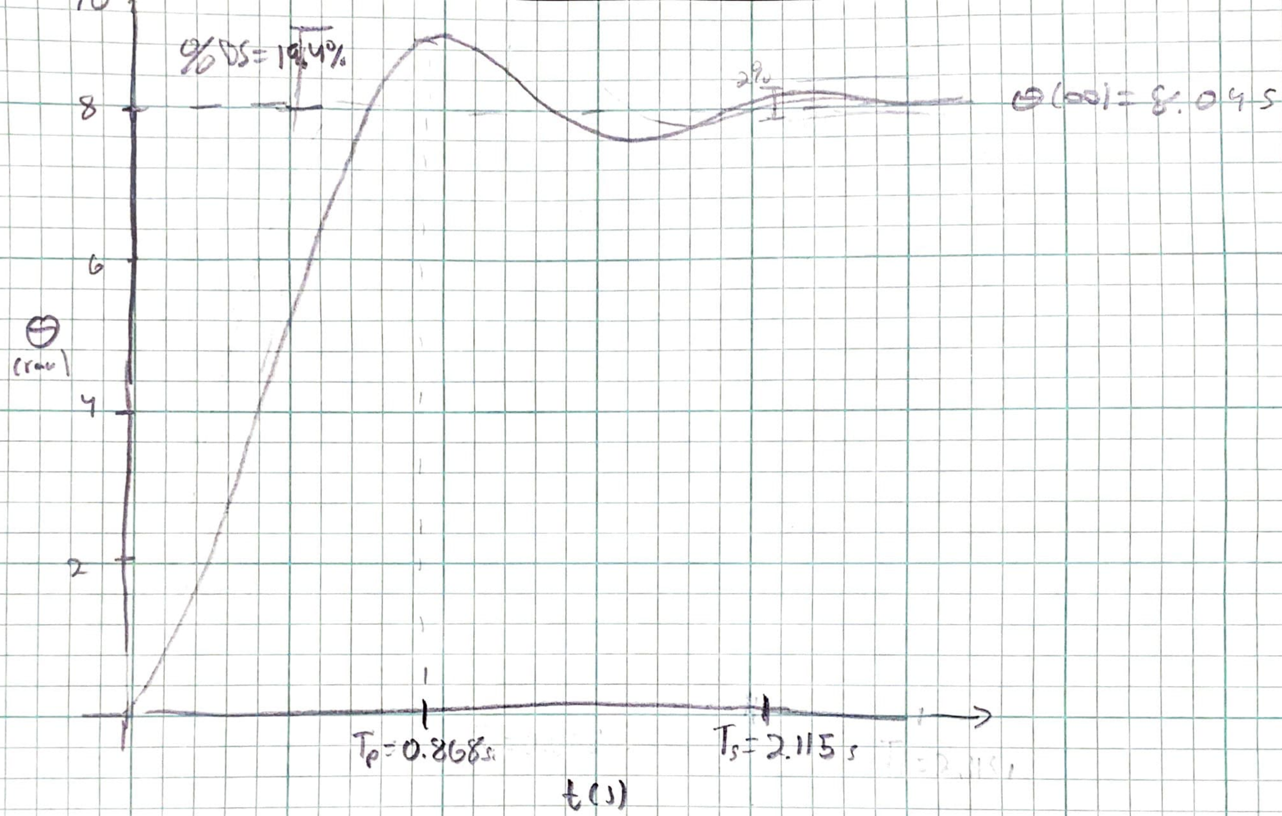
$2\zeta\omega_n = 3.783 \Rightarrow \zeta = 0.4631$

$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \Rightarrow \boxed{T_p = 0.868\,s}$

$T_s \approx \frac{4}{\zeta\omega_n} \Rightarrow \boxed{T_s = 2.115\,s}$

$\%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100 = \boxed{19.4\%}$

$\theta(\infty) = \lim_{s\to 0} s \cdot \frac{1}{s} \cdot \frac{0.539}{0.004015 s^2 + 0.01519 s + 0.067} = \boxed{8.045\text{ rad}}$



Part 3 - Plot