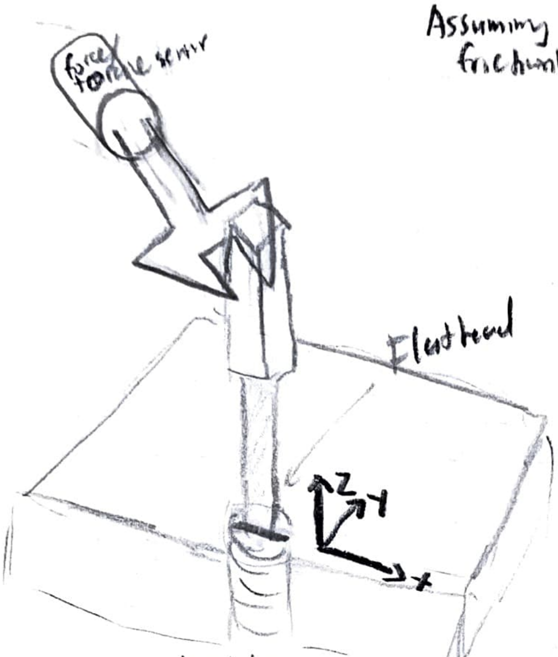
2. 2.1

6DoF:
V_x, V_y, V_z
w_x, w_y, w_z



Assuming frictionless:

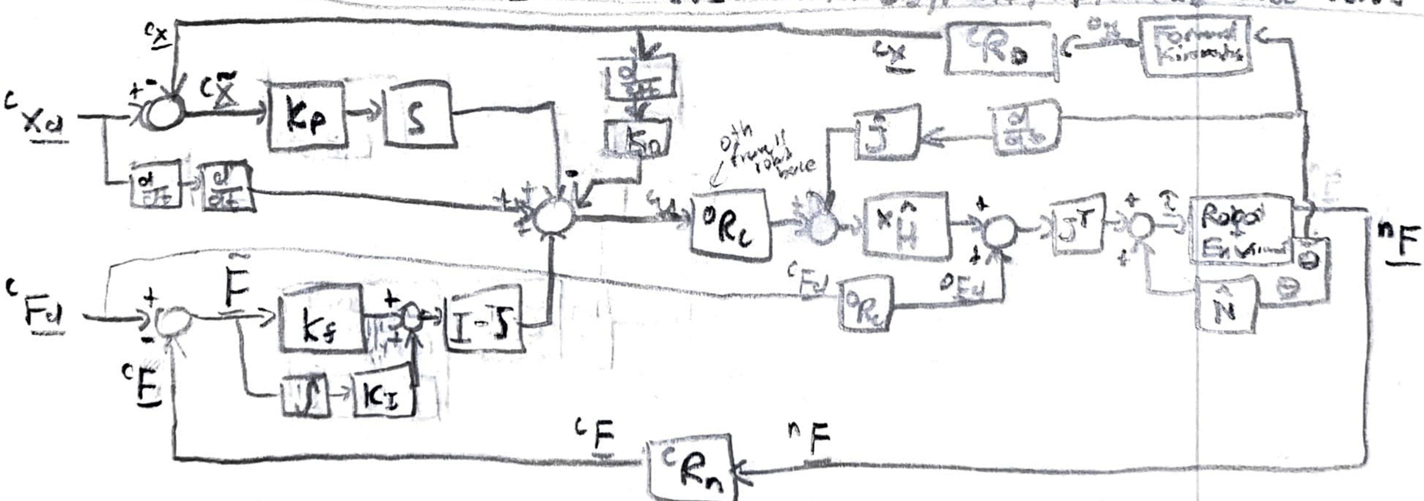
	(Velocity) Kinematic	(Force) Static
Natural Constraints (Geometry)	$V_y = 0$ $V_z = 0$ $w_x = 0$ $w_y = 0$	$f_x = 0$ $\tau_z = 0$
Artificial Constraints (Control)	$V_x = V_{xd} = 0$ $w_z = w_{zd}$	$f_y = f_{yd} = 0$ $f_z = f_{zd}$ $\tau_x = \tau_{xd} = 0$ $\tau_y = \tau_{yd} = 0$
	Position Control	Force Control

$$\sigma = \overbrace{[\,\overset{x}{1}\ \ \overset{y}{0}\ \ \overset{z}{0}}^{\text{translation}}\ \ \overbrace{\overset{x}{0}\ \ \overset{y}{0}\ \ \overset{z}{1}\,]}^{\text{rotation}}$$

$$\text{diag}(\sigma) = \boxed{S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}$$

2.2

(Note:) ${}^c\underline{X_d} = [{}^cx_d\ {}^cy_d\ {}^cz_d\ {}^c\theta_d\ {}^c\phi_d\ {}^c\psi_d]$ (Euler angles); ${}^c\underline{F_d} = [{}^cF_{xd}\ {}^cF_{yd}\ {}^cF_{zd}\ {}^c\tau_{xd}\ {}^c\tau_{yd}\ {}^c\tau_{zd}]$;
$I - S = \text{diag}([0\ 1\ 1\ 1\ 1\ 0])$; K_f, K_p, and K_D can be vectors



for moments of $\underline{F}$ would have to use ${}^c\tau = {}^cR_n{}^n\underline{\tau} + {}^c d_{n}\times {}^cR_n{}^n\underline{F}$

2. 2.2 continued

Control law: $\underline{\tau} = \hat{N} + J^T({}^{0}\underline{F_d} + {}^{x}\hat{H}({}^{0}\underline{u} - \dot{J}\dot{\underline{\theta}}))$

where:

$${}^{0}\underline{u} = {}^{0}R_c\left[{}^{c}\ddot{x}_d + K_p S\,{}^{c}\underline{\epsilon} + (I-S)(K_s\hat{E} + K_I\int \hat{E}) - K_D\,{}^{c}\dot{\underline{x}}\right]$$

2.3

↳ The constraint frame should move with the tool assuming the tool moves with the screw, this would keep the natural and thus artificial constraints the same even as motion occurs.

↳ Assuming the wrist is the n^{th} frame of this robot the measured force and torque will be in the n^{th} frame not the constraint frame. Thus to be compared to the force in the constraint frame it will be necessary to perform a coordinate transformation.

(Rigid manipulator, so could place final frame (n) in wrist out of convenience)

As shown in 2.2 coordinate transforms would be used to go from the n^{th} frame to the constraint frame, mathematically:

$$\boxed{{}^{c}\underline{F} = {}^{c}R_n\,{}^{n}\underline{F}}$$

Do note the torques would use ${}^{c}\underline{\tau} = {}^{c}R_n\,{}^{n}\underline{\tau} + {}^{c}d_{cn} \times {}^{c}R_n\,{}^{n}\underline{f}$ when ${}^{c}d_{cn}$ is the interframe between the constraint and n^{th} frame.

$${}^{n}\underline{F} = [{}^{n}F_x\ {}^{n}F_y\ {}^{n}F_z\ {}^{n}\tau_x\ {}^{n}\tau_y\ {}^{n}\tau_z]$$

$${}^{c}\underline{F} = [{}^{c}F_x\ {}^{c}F_y\ {}^{c}F_z\ {}^{c}\tau_x\ {}^{c}\tau_y\ {}^{c}\tau_z]$$