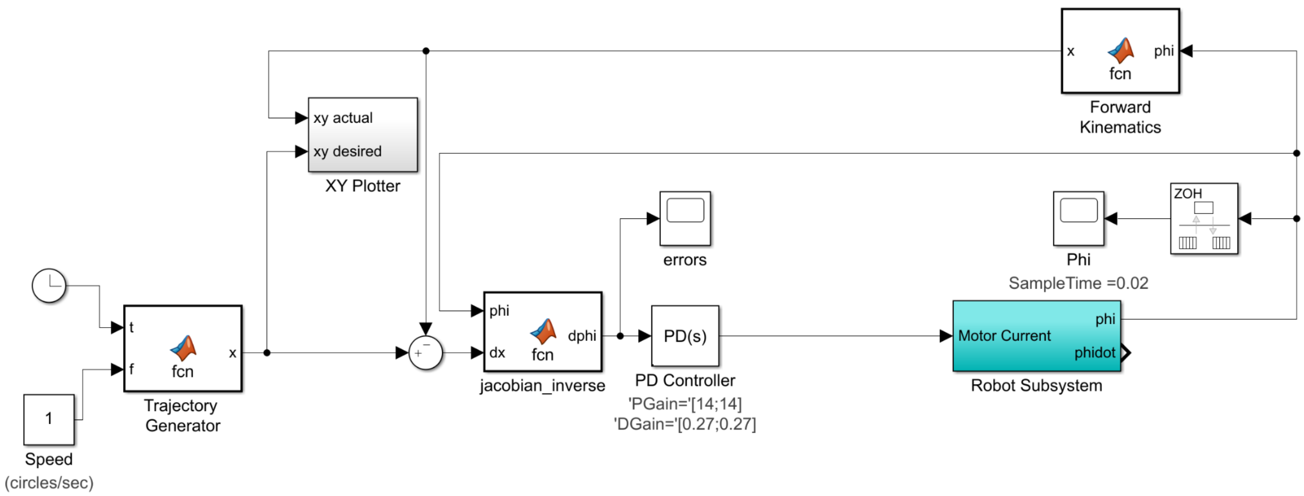
ME EN 6230
Lab 3
Ryan Dalby

Note that lab station 2 was used for this lab. The amp bias used was [-0.042; 0.065] and the current sense bias used was [0.02; 0.037].

1. Jacobian Inverse Control

PD gains are Kp = 14 and Kd = 0.27 for jacobian inverse control.

1.1. Model



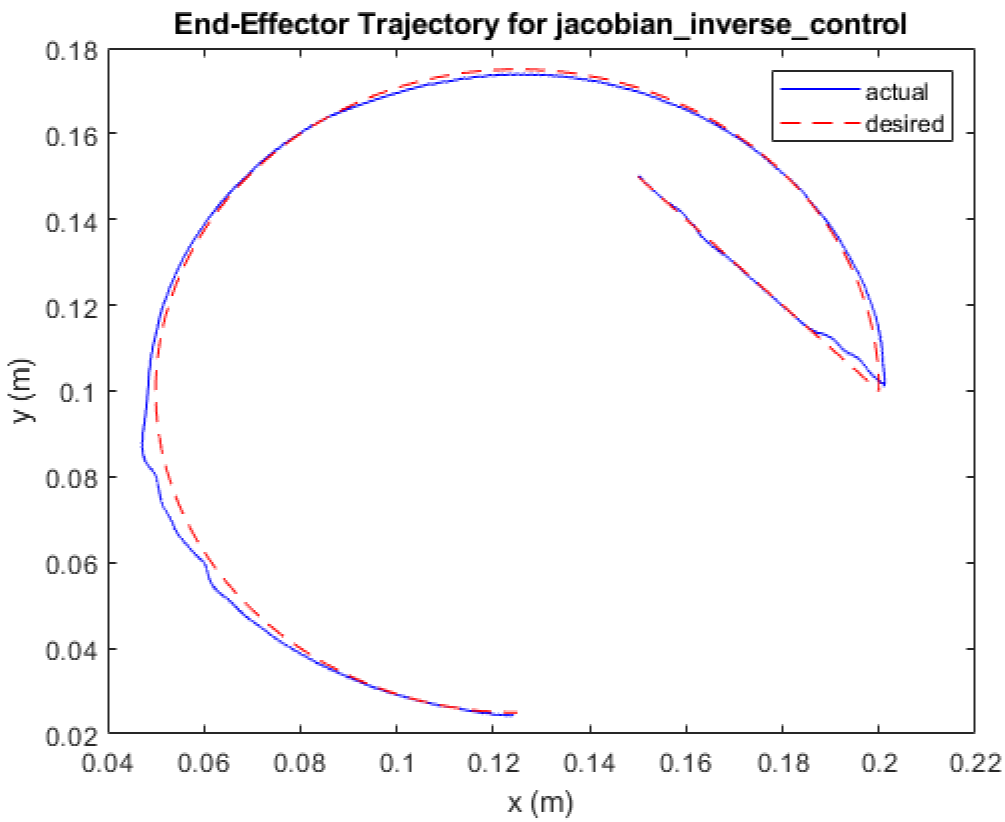
1.2. Code

Code for jacobian_inverse block:

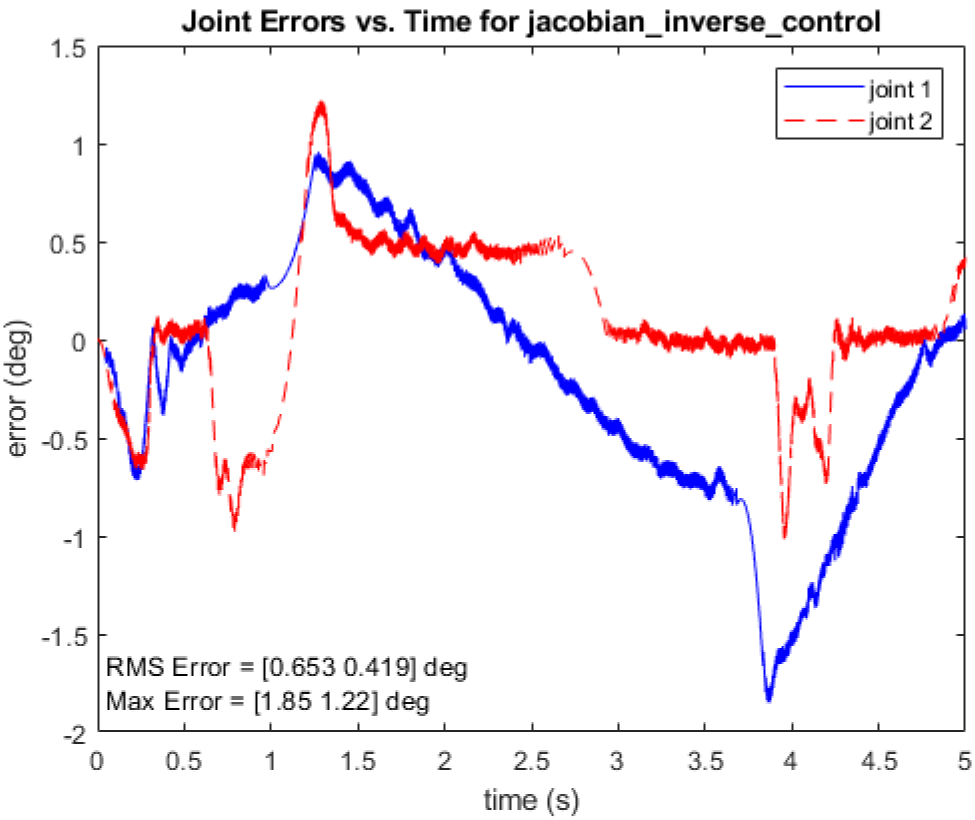
```
function dphi = fcn(phi, dx)
% This block supports an embeddable subset of the MATLAB language.
% See the help menu for details.

a1 = 0.15;  % link 1 length
a2 = 0.15;  % link 2 length

% Describe combined relationship of jacobian from phi to x using jacobian
% (Combines transmission jacobian from phi to theta and manipulator
% jacobian from theta to x)
J11 = -a1*sin(phi(1));
J12 = -a2*sin(phi(2));
J21 = a1*cos(phi(1));
J22 = a2*cos(phi(2));
J = [J11 J12; J21 J22];

dphi = inv(J)*dx;
```

1.3. Low Speed Simulation (f=0.2 circles/s)

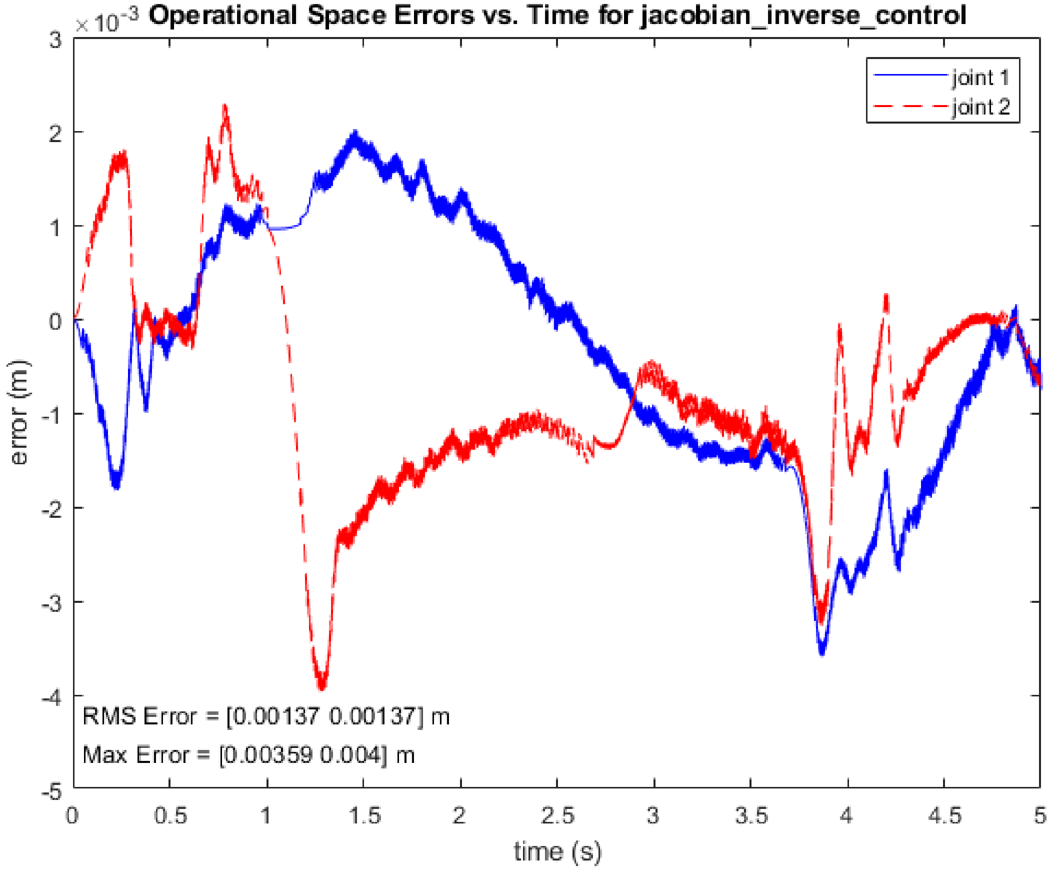
1.3.1. X-Y Trajectory



1.3.2. Joint Angle Errors

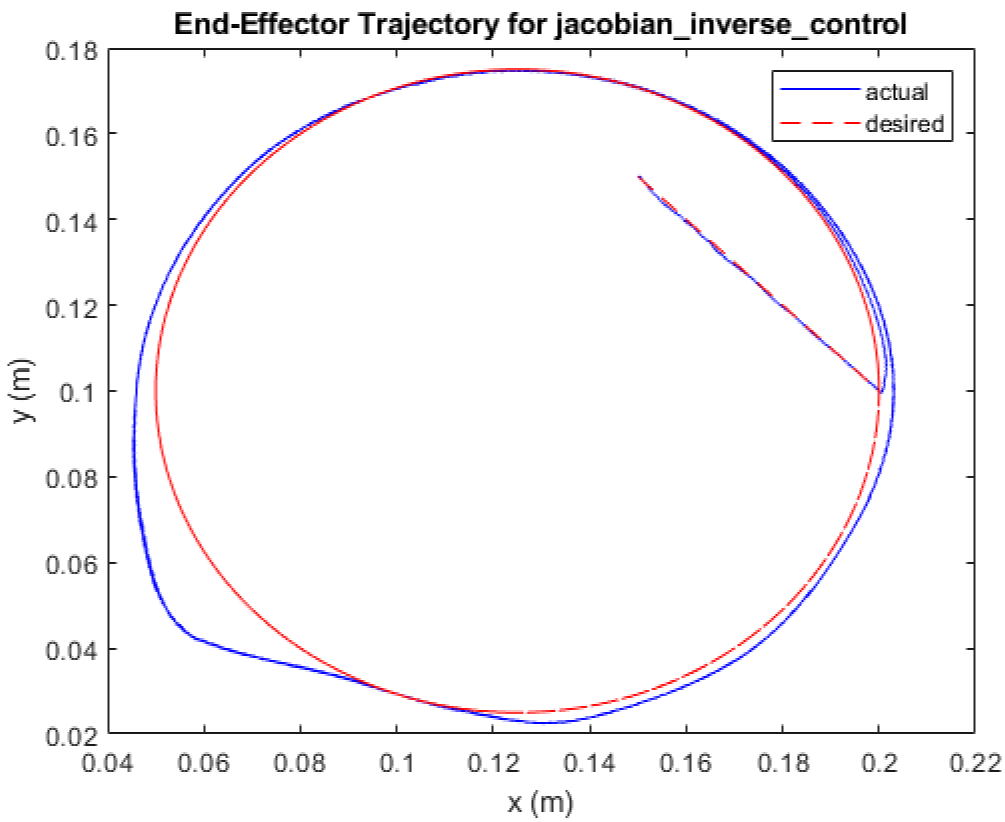


1.3.3. Operational Space Errors

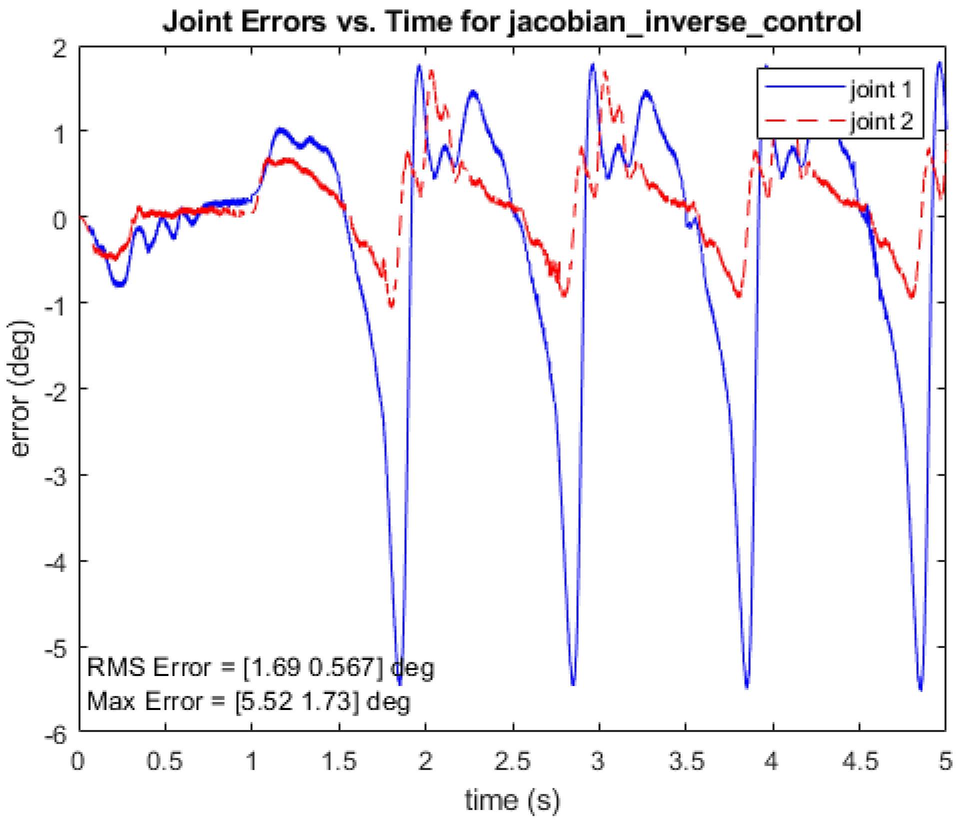


1.4. High Speed Simulation (f=1 circles/s)

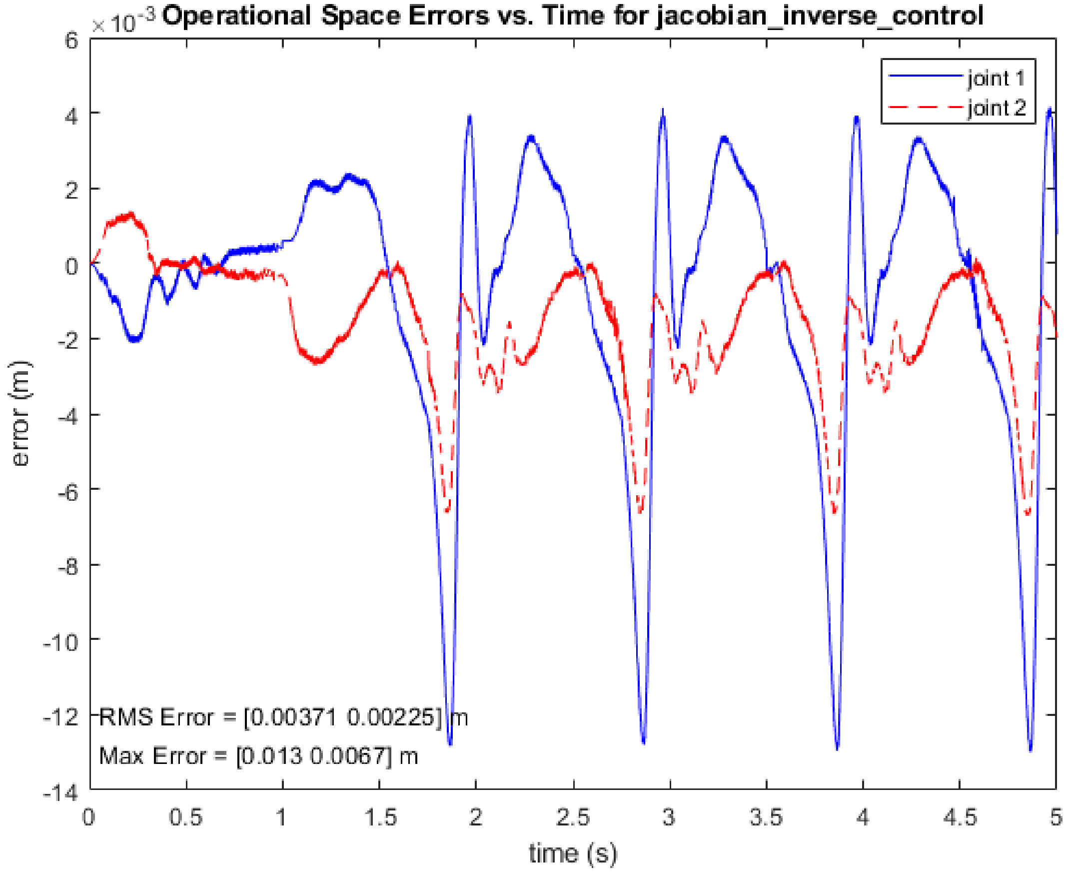
1.4.1. X-Y Trajectory



1.4.2. Joint Angle Errors

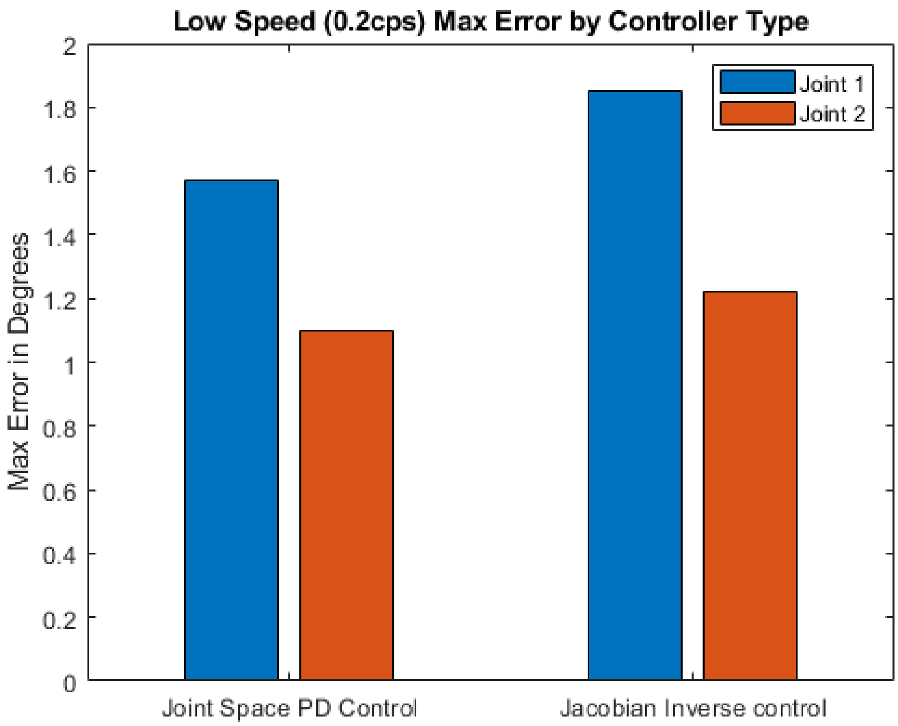


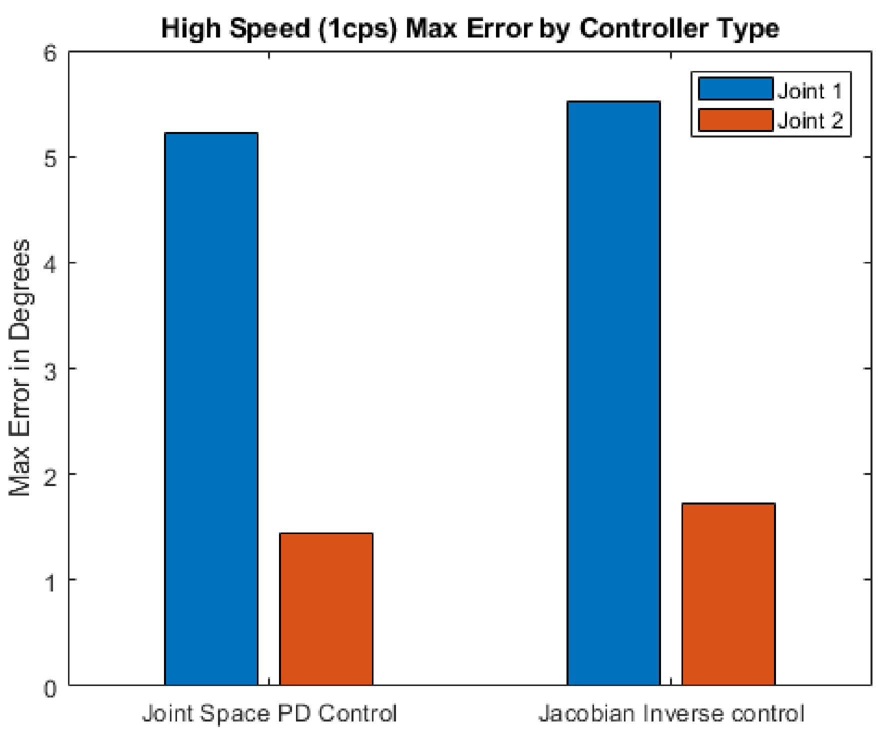
1.4.3. Operational Space Errors



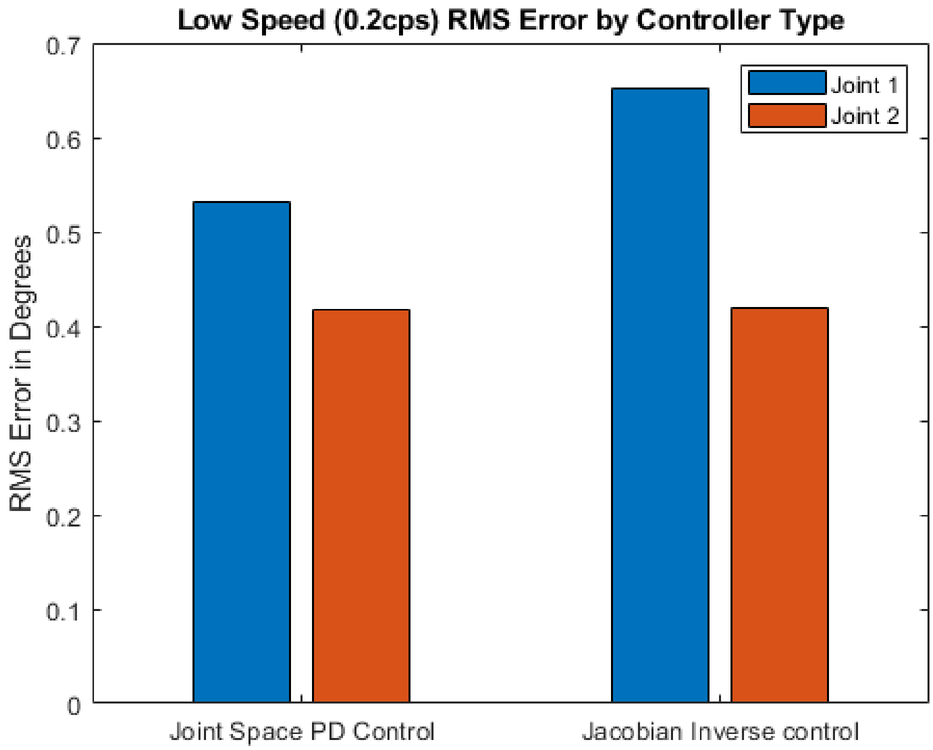
1.5. Controller Comparison

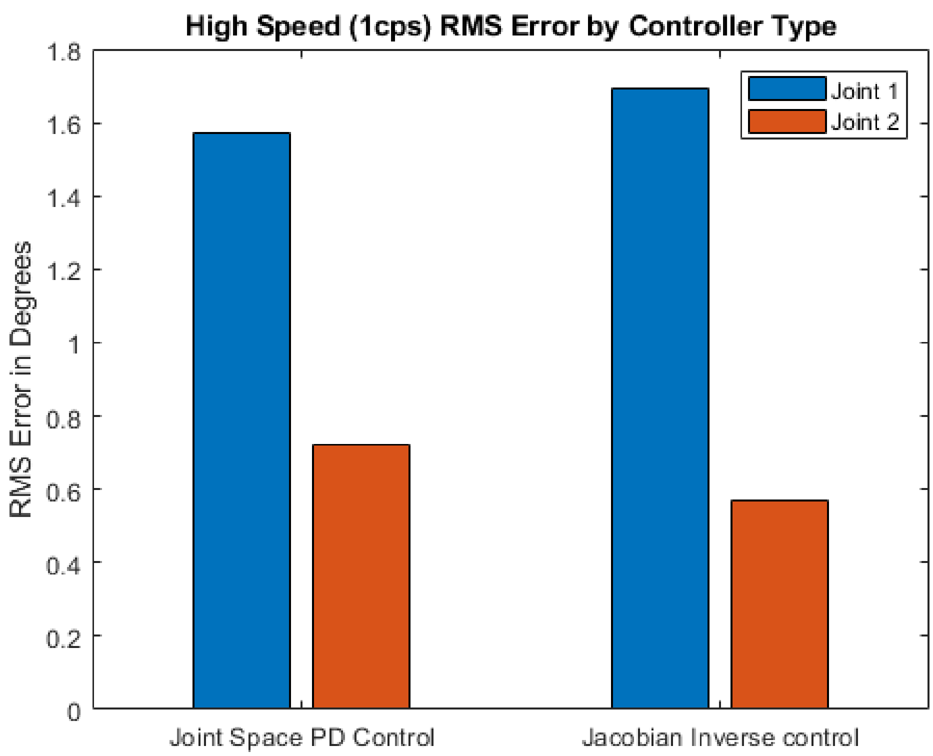
1.5.1. Peak Joint Errors





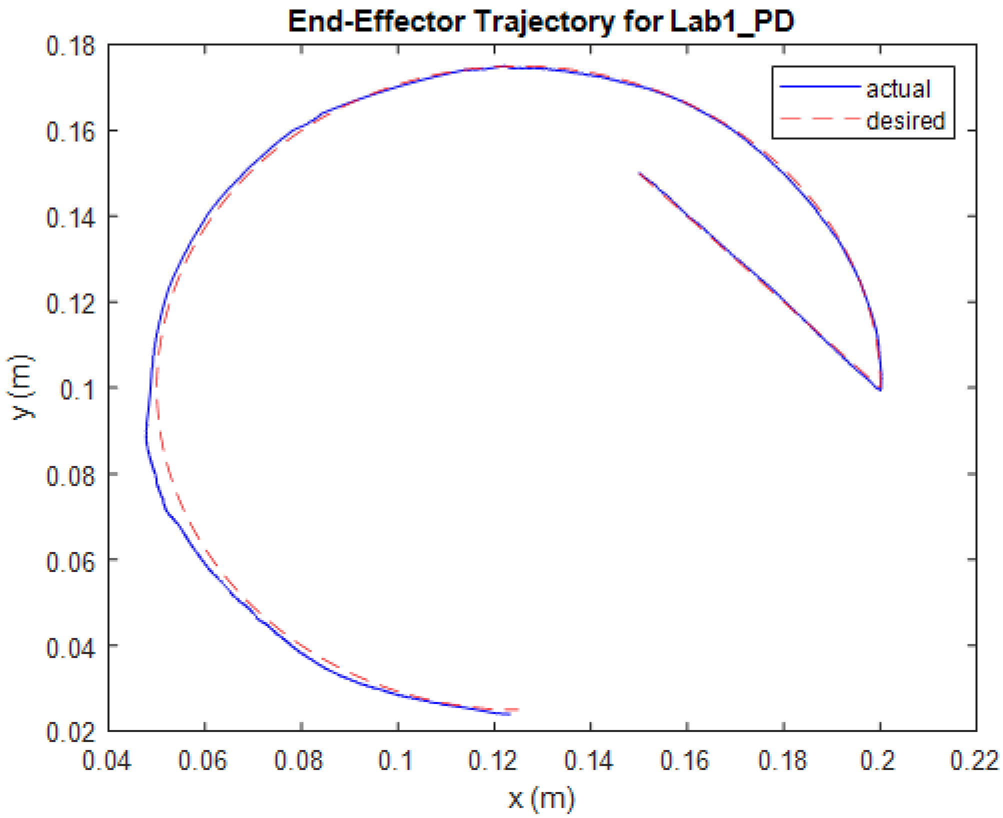
1.5.2. Root-Mean-Square Joint Errors





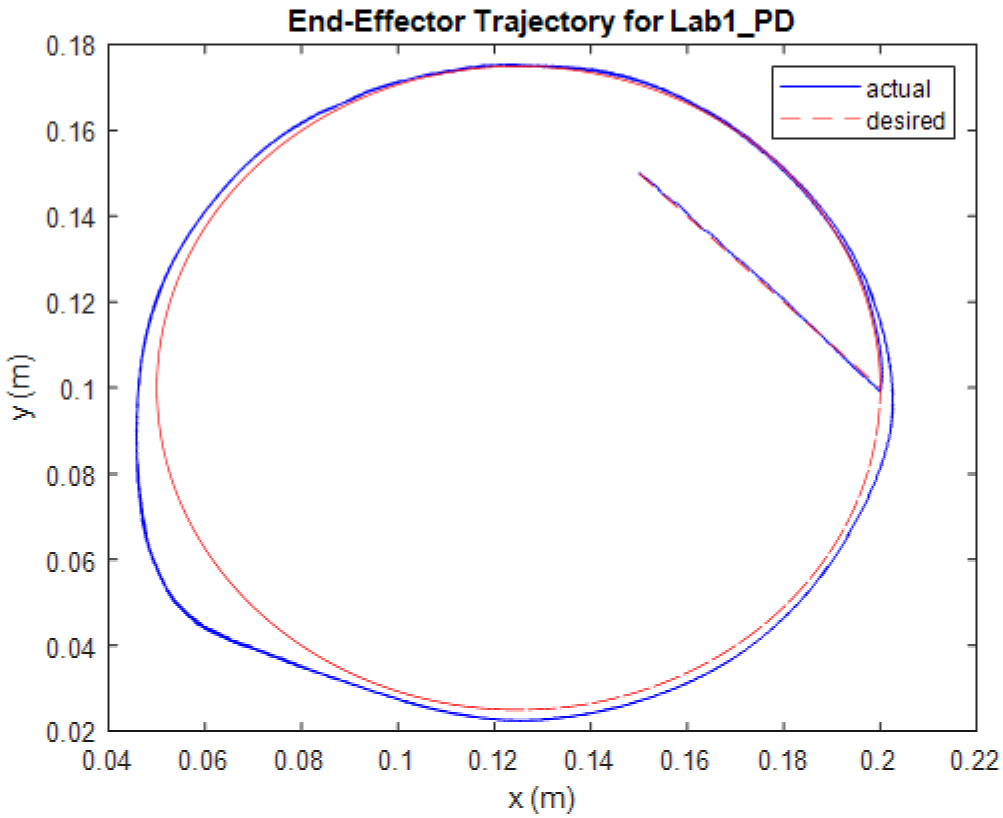
1.5.3. Trajectories

1.5.3.1. PD Control Low Speed Simulation (f=0.2 circles/s) (Compare to 1.3.1.)



1.5.3.2. PD Control High Speed Simulation (f=1 circles/s)

(Compare to 1.4.1.)



1.5.4. Comparison

Comparing Jacobian inverse control to joint space PD control with the same PD gains the actual experimental results are close to identical with the joint space PD control having slightly better tracking overall. As can be seen in the trajectory plots in 1.5.3 the trajectories appear to be very similar. As can be seen in 1.5.1 and 1.5.2, both the RMS and max errors are comparable with the joint space PD control being better most of the time. In the end, the differences can likely be attributed to the robot used which was not the same between this lab and lab 1 so these controllers can be considered roughly equivalent.

1.5.5. Comparison Code

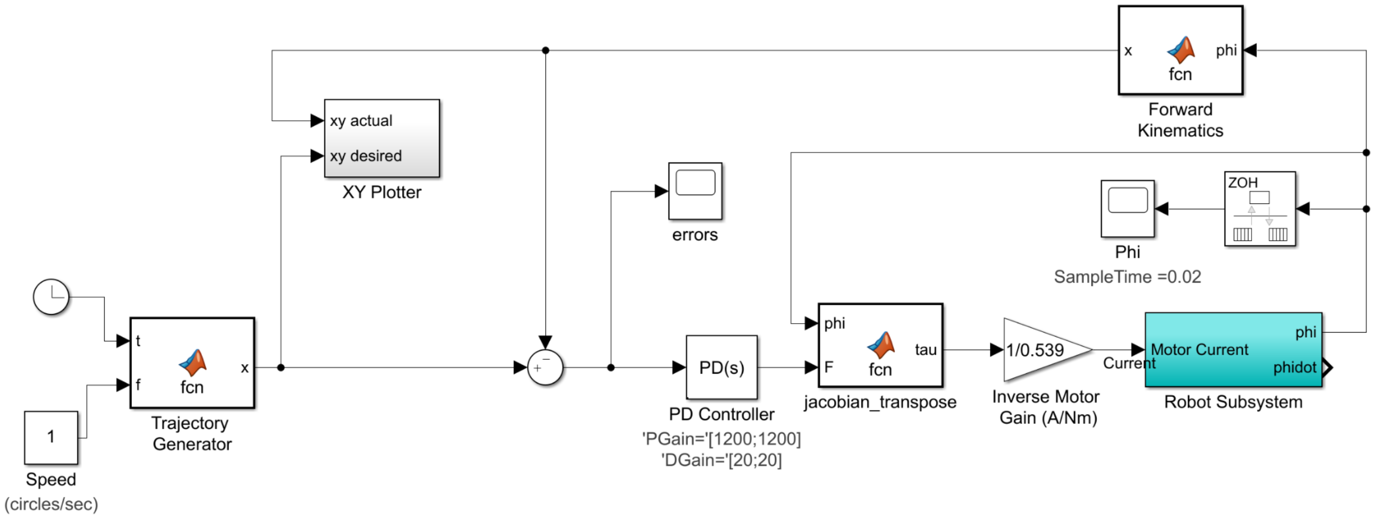
```
%% ME EN 6230 Lab 3 Ryan Dalby Jacobian Inverse and Joint Space PD
Controller Comparison
set(groot, 'DefaultTextInterpreter', 'none') % Stops underscore = subscript
% Joint Space PD Controller errors in degrees
pd_slow_rms = [0.532 0.417];
pd_slow_max = [1.57 1.1];
pd_fast_rms = [1.57 0.722];
pd_fast_max = [5.23 1.44];
% Jacobian Inverse Controller errors in degrees
feedbackcomp_slow_rms = [0.653 0.419];
feedbackcomp_slow_max = [1.85 1.22];
feedbackcomp_fast_rms = [1.69 0.567];
feedbackcomp_fast_max = [5.52 1.73];
slow_rms = [pd_slow_rms; feedbackcomp_slow_rms];
fast_rms = [pd_fast_rms; feedbackcomp_fast_rms];
slow_max = [pd_slow_max; feedbackcomp_slow_max];
fast_max = [pd_fast_max; feedbackcomp_fast_max];
controller_labels = {'Joint Space PD Control', 'Jacobian Inverse control'};
controller_labels_cat = categorical(controller_labels);
controller_labels_cat = reordercats(controller_labels_cat,
string(controller_labels_cat));
figure;
bar(controller_labels_cat,slow_rms);
title('Low Speed (0.2cps) RMS Error by Controller Type');
legend('Joint 1', 'Joint 2');
ylabel('RMS Error in Degrees');
figure;
bar(controller_labels_cat,fast_rms);
title('High Speed (1cps) RMS Error by Controller Type');
legend('Joint 1', 'Joint 2');
ylabel('RMS Error in Degrees');
figure;
bar(controller_labels_cat,slow_max);
title('Low Speed (0.2cps) Max Error by Controller Type');
legend('Joint 1', 'Joint 2');
ylabel('Max Error in Degrees');
figure;
bar(controller_labels_cat,fast_max);
title('High Speed (1cps) Max Error by Controller Type');
legend('Joint 1', 'Joint 2');
ylabel('Max Error in Degrees');
```

2. Jacobian Transpose Related Control

PD gains are Kp = 1200 and Kd = 20 for all jacobian transpose related control.

2.1. Jacobian Transpose Control

2.1.1. Model



2.1.2. Code

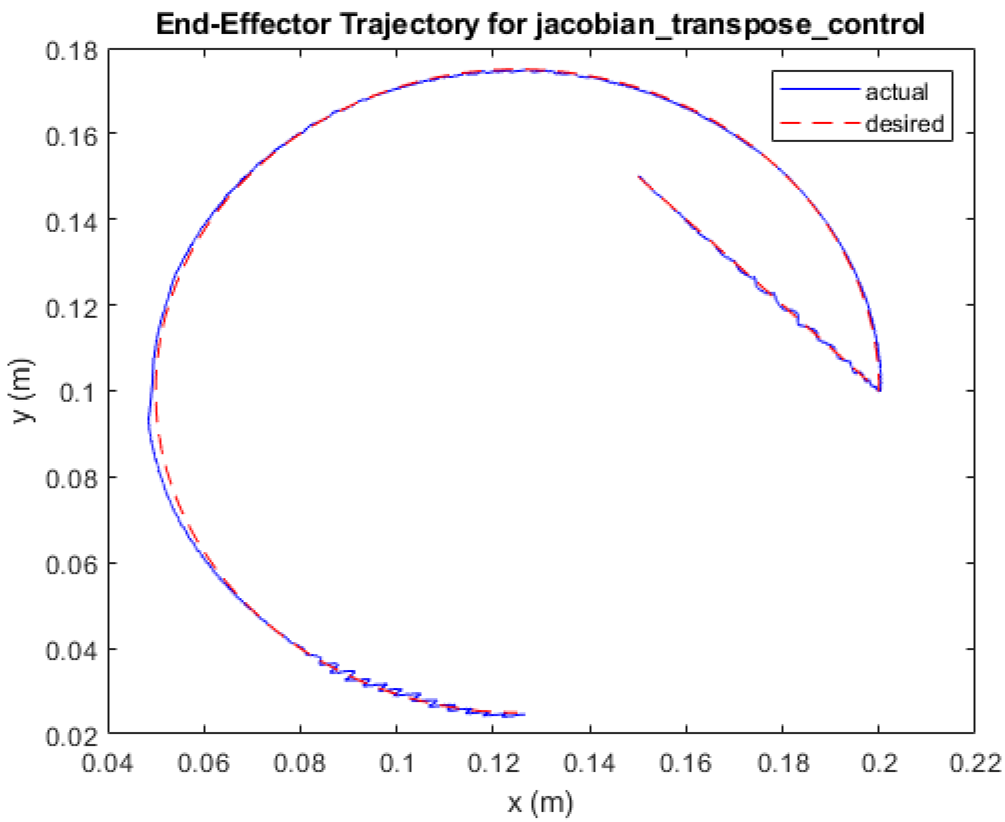
Code for the jacobian_transpose block (also used for inverse dynamics control in operational space and robust control in operational space):

```
function tau = fcn(phi, F)
% This block supports an embeddable subset of the MATLAB language.
% See the help menu for details.

a1 = 0.15;  % link 1 length
a2 = 0.15;  % link 2 length

% Describe combined relationship of jacobian from phi to x using jacobian
% (Combines transmission jacobian from phi to theta and manipulator
% jacobian from theta to x)
J11 = -a1*sin(phi(1));
J12 = -a2*sin(phi(2));
J21 = a1*cos(phi(1));
J22 = a2*cos(phi(2));
J = [J11 J12; J21 J22];

tau = transpose(J)*F;
```

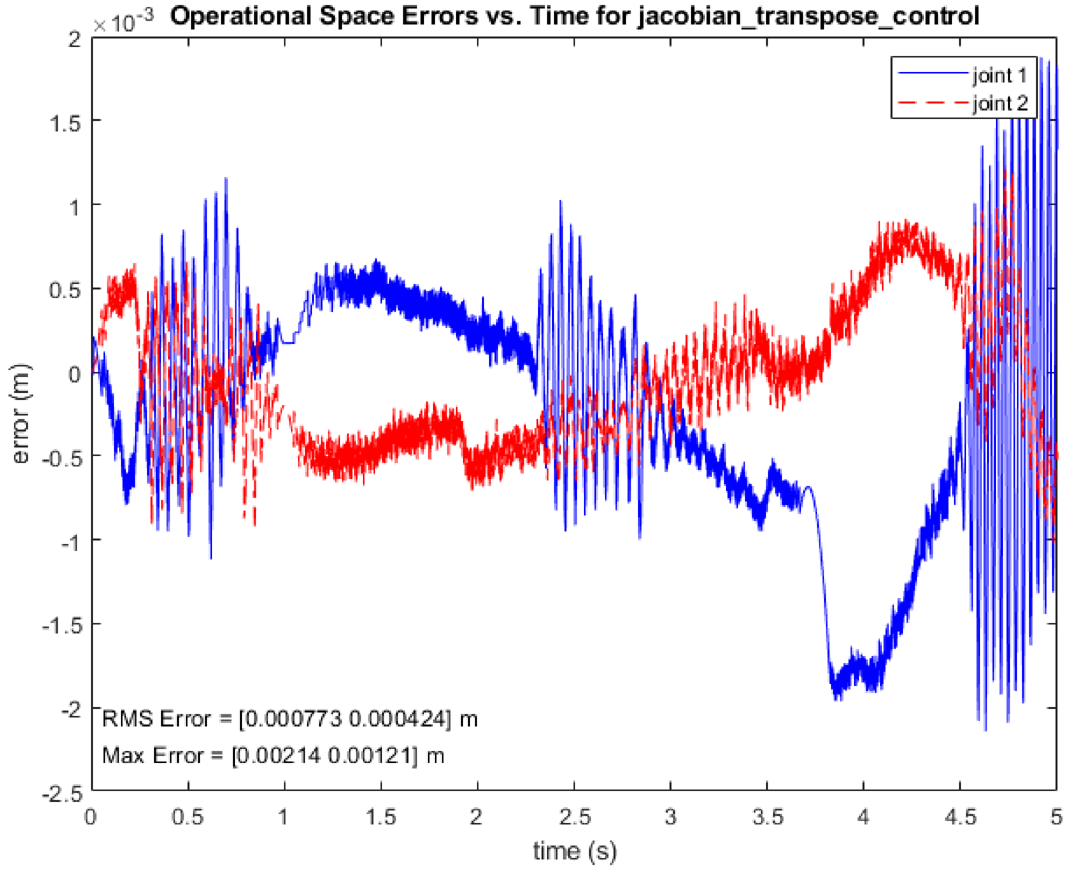
]

2.1.3. Low Speed Simulation (f=0.2 circles/s)

2.1.3.1. X-Y Trajectory

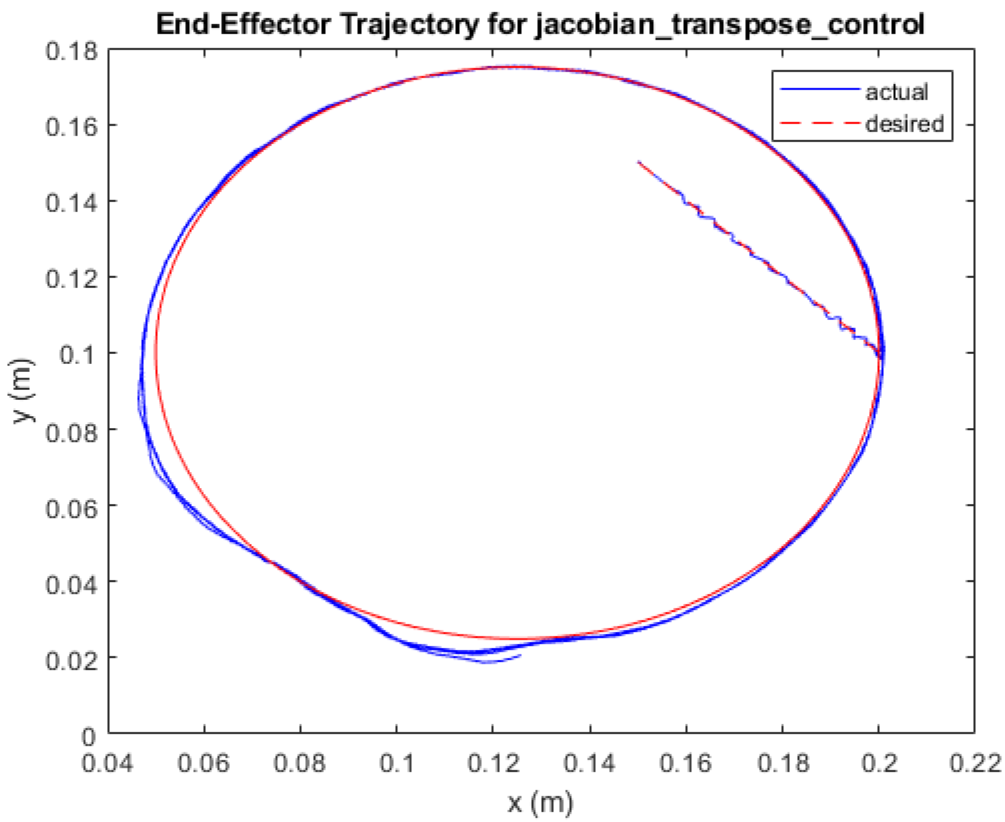


2.1.3.2. Operational Space Errors

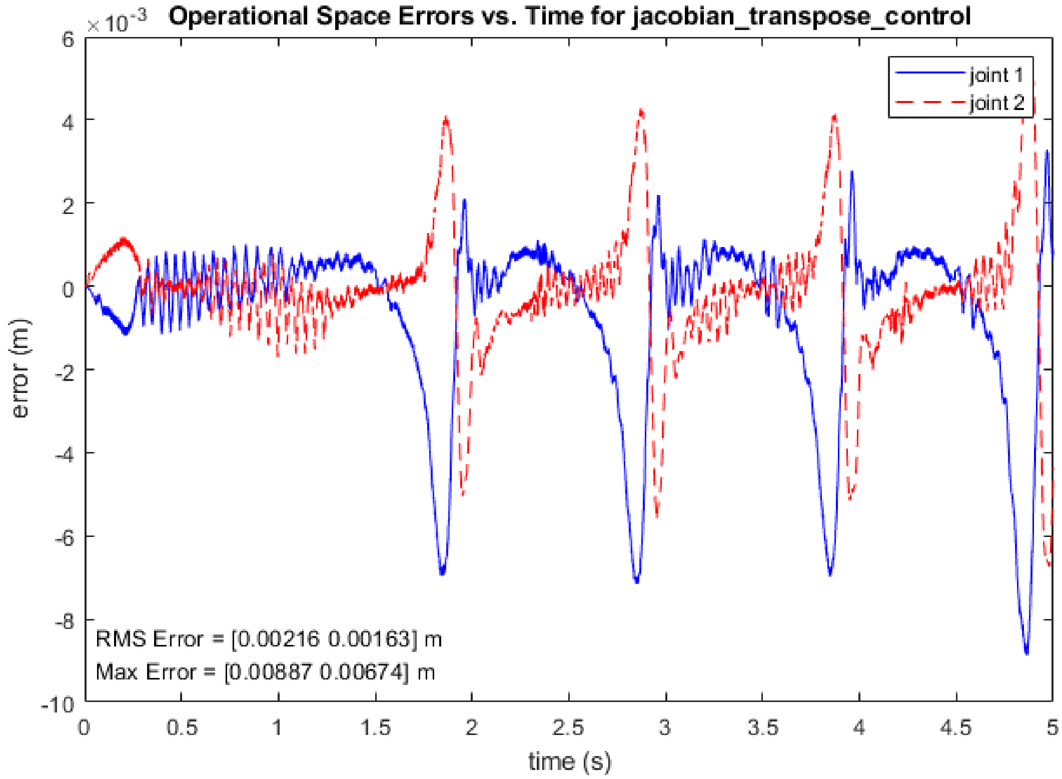


2.1.4. High Speed Simulation (f=1 circles/s)

2.1.4.1. X-Y Trajectory

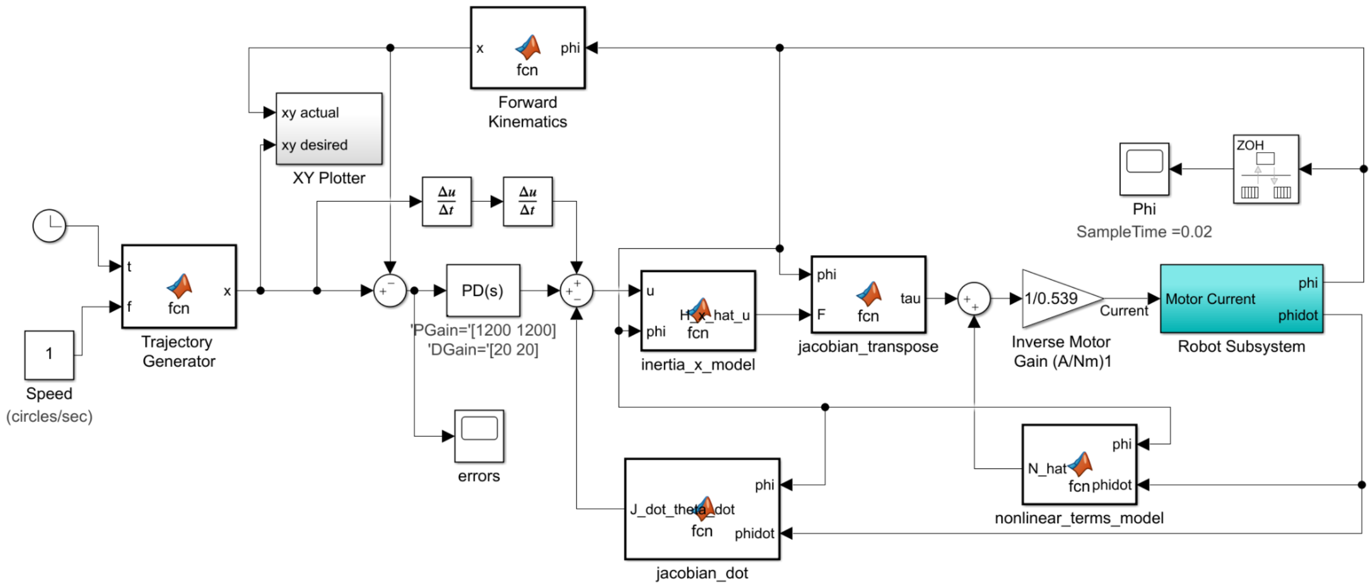


2.1.4.2. Operational Space Errors



2.2. Inverse Dynamics Control in Operational Space

2.2.1. Model



2.2.2. Code

Code for the jacobian_dot block:

```
function J_dot_theta_dot = fcn(phi, phidot)
% This block supports an embeddable subset of the MATLAB language.
% See the help menu for details.

a1 = 0.15;  % link 1 length
a2 = 0.15;  % link 2 length

J11_dot = -a1*cos(phi(1))*phidot(1);
J12_dot = -a2*cos(phi(2))*phidot(2);
J21_dot = -a1*sin(phi(1))*phidot(1);
J22_dot = -a2*sin(phi(2))*phidot(2);

J_dot = [J11_dot J12_dot; J21_dot J22_dot];

J_dot_theta_dot = J_dot * phidot;
```

Code for the inertia_x_model block:

```
function H_x_hat_u = fcn(u,phi)
% This block supports an embeddable subset of the MATLAB language.
% See the help menu for details.

a1 = 0.15;  % link 1 length
a2 = 0.15;  % link 2 length
m1 = 0.092;  % link 1 mass
m2 = 0.077; % link 2 mas
r01 = 0.062; % link 1 center of mass
r12 = 0.036; % link 2 COM
I1 = 0.64e-3;  % link 1 inertia
I2 = 0.30e-3;  % link 2 inertia
Jm1 = 0.65e-6; % motor inertias
Jm2 = 0.65e-6;
b1 = 3.1e-6; % viscous damping constants
b2 = 3.1e-6;
c1 = 0.0001; % coulomb friction constants
c2 = 0.0001;
g = 9.8; % gravitational constant
N1 = 70; % gear ratios
N2 = 70;

a1 = 0.15;  % link 1 length
```

```
a2 = 0.15;  % link 2 length
% Describe combined relationship of jacobian from phi to x using jacobian
% (Combines transmission jacobian from phi to theta and manipulator
% jacobian from theta to x)
J11 = -a1*sin(phi(1));
J12 = -a2*sin(phi(2));
J21 = a1*cos(phi(1));
J22 = a2*cos(phi(2));
J = [J11 J12; J21 J22];

H11 = N1^2*Jm1 + I1 + m2*a1^2;
H12 = a1*r12*m2*cos(phi(2)-phi(1));
H21 = H12;
H22 = N2^2*Jm2 + I2;

H_hat = [H11 H12; H21 H22]; % inertia matrix

H_x_hat = inv(transpose(J)) * H_hat * inv(J);

H_x_hat_u = H_x_hat * u;
```

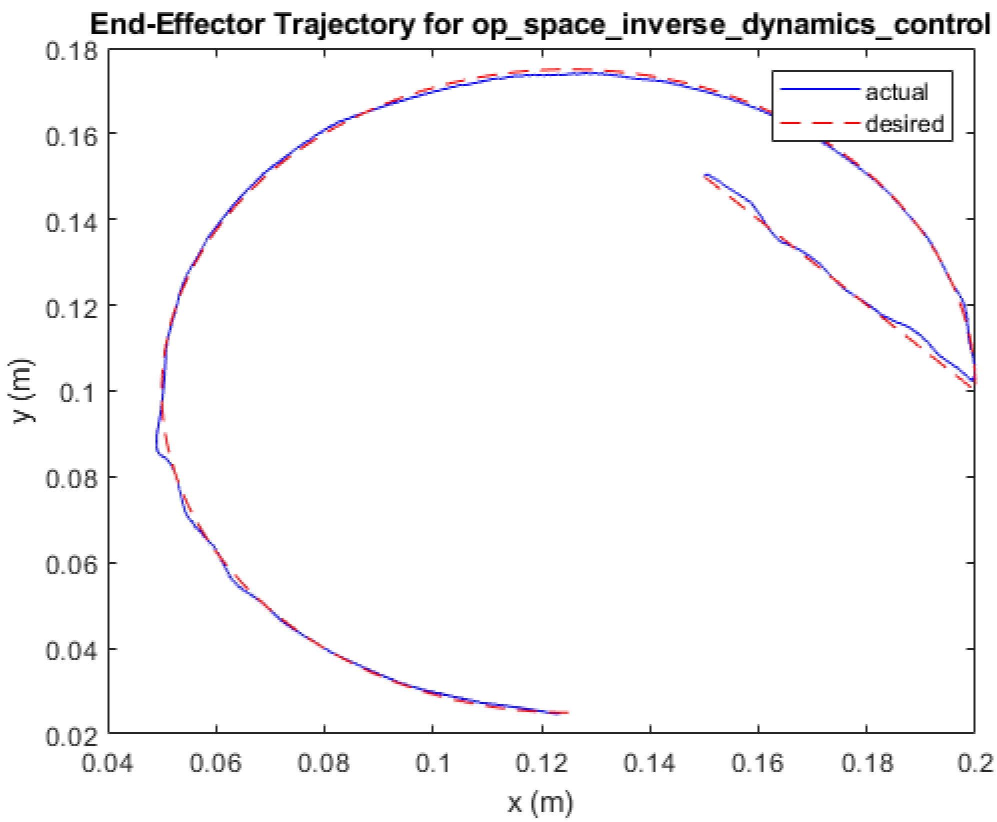
Code for nonlinear_terms_model block:

```
function N_hat = fcn(phi,phidot)
% This block supports an embeddable subset of the MATLAB language.
% See the help menu for details.

a1 = 0.15;  % link 1 length
a2 = 0.15;  % link 2 length
m1 = 0.092;  % link 1 mass
m2 = 0.077; % link 2 mas
r01 = 0.062; % link 1 center of mass
r12 = 0.036; % link 2 COM
I1 = 0.64e-3;  % link 1 inertia
I2 = 0.30e-3;  % link 2 inertia
Jm1 = 0.65e-6; % motor inertias
Jm2 = 0.65e-6;
b1 = 3.1e-6; % viscous damping constants
b2 = 3.1e-6;
c1 = 0.0001; % coulomb friction constants
c2 = 0.0001;
g = 9.8; % gravitational constant
N1 = 70; % gear ratios
```

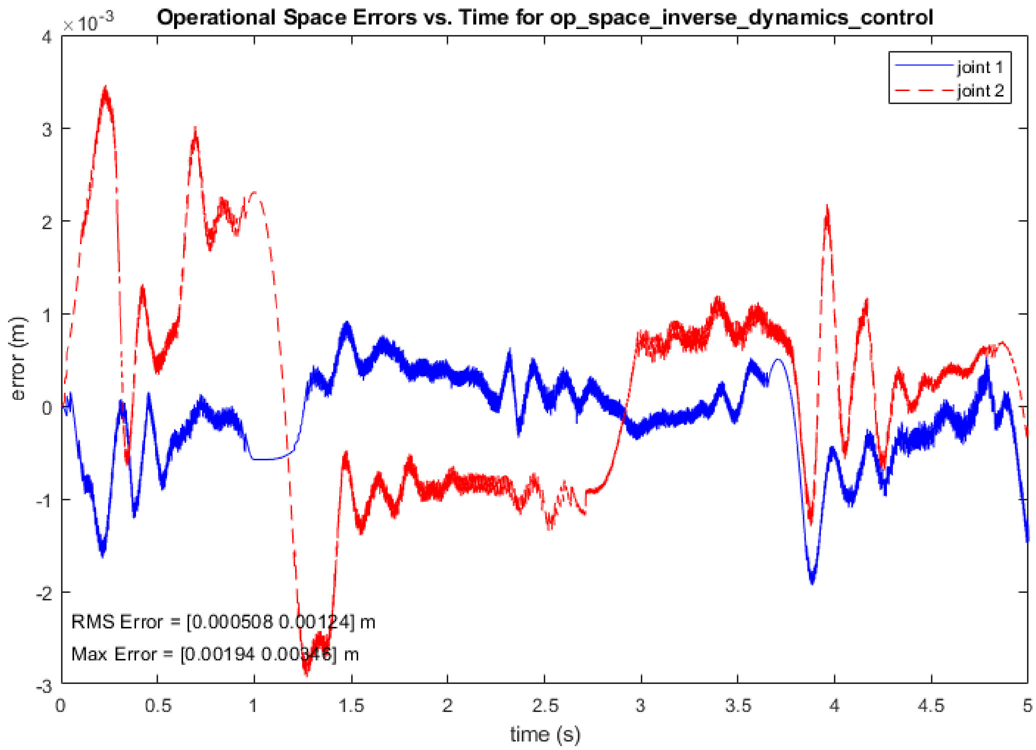
```
N2 = 70;

h = a1*r12*m2*sin(phi(2)-phi(1));
G1 = (r01*m1+a1*m2)*g*cos(phi(1));
G2 = r12*m2*g*cos(phi(2));
F1 = N1^2*b1*phidot(1) + N1*c1*sign(phidot(1));
F2 = N2^2*b2*phidot(2) + N2*c2*sign(phidot(2));

V_hat = [0 -h ;h 0]*[phidot(1)^2;phidot(2)^2]; % centripetal torques
G_hat = [G1;G2]; % gravity torques
F_hat = [F1;F2]; % frictional torques

N_hat = V_hat + G_hat + F_hat;
```

2.2.3. Low Speed Simulation (f=0.2 circles/s)

2.2.3.1. X-Y Trajectory

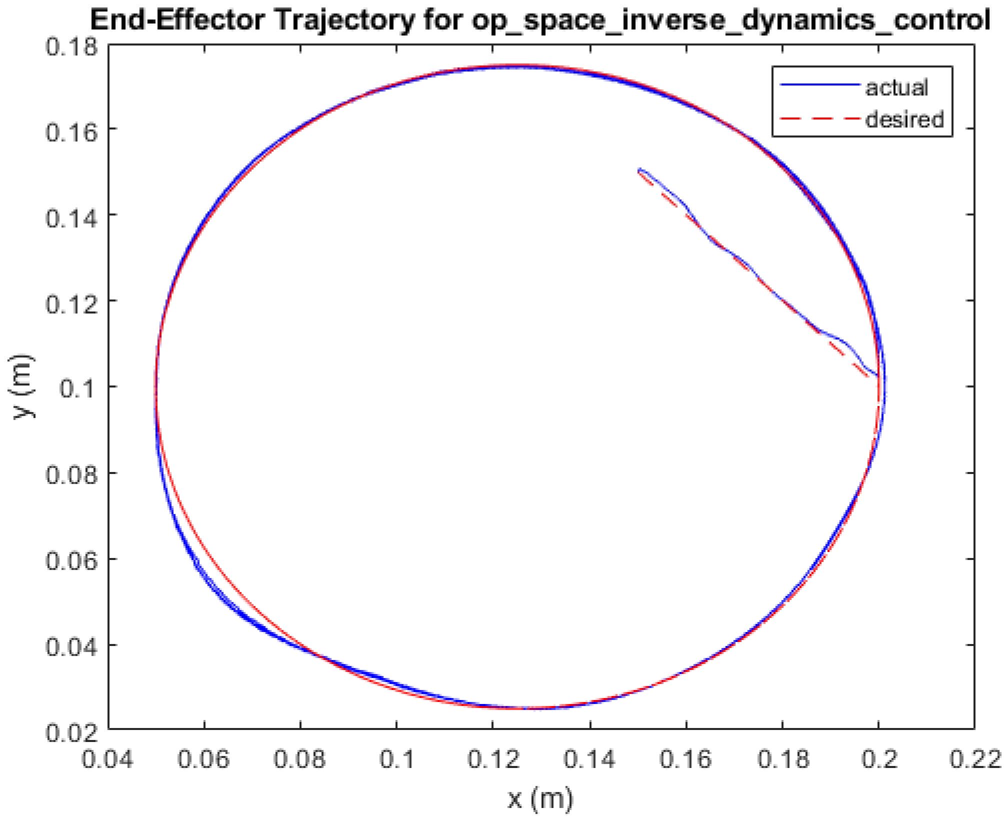


2.2.3.2. Operational Space Errors

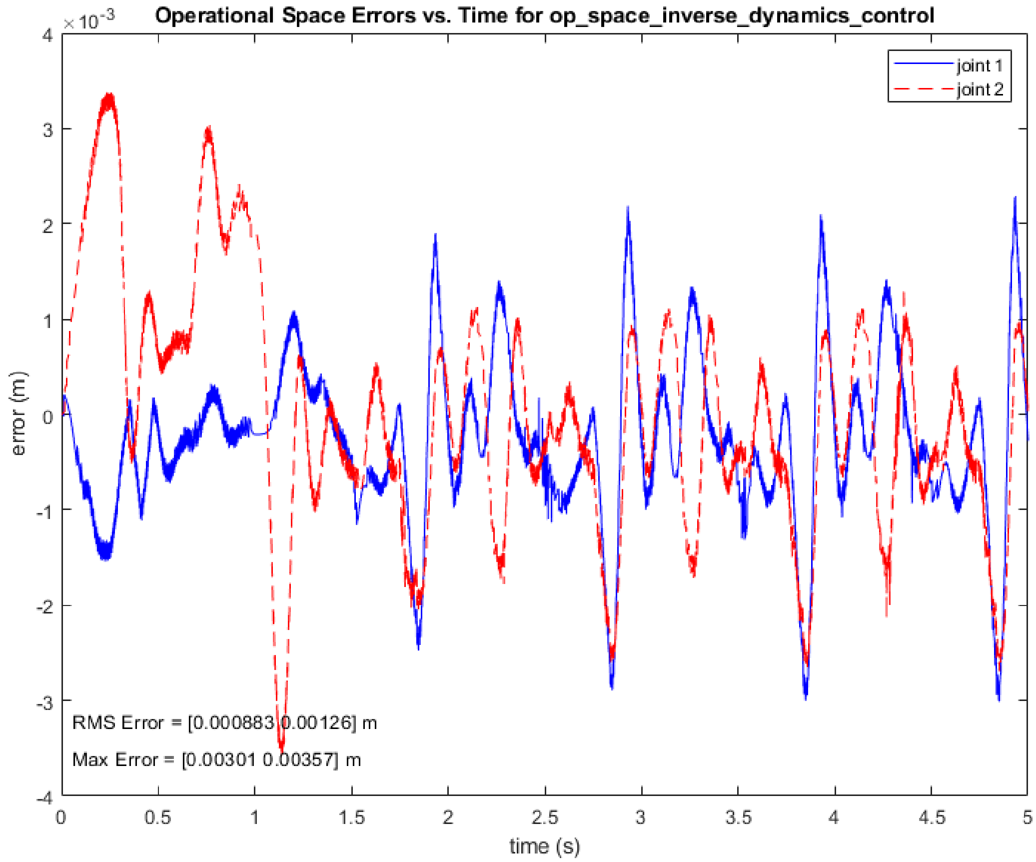


2.2.4. High Speed Simulation (f=1 circles/s)

2.2.4.1. X-Y Trajectory



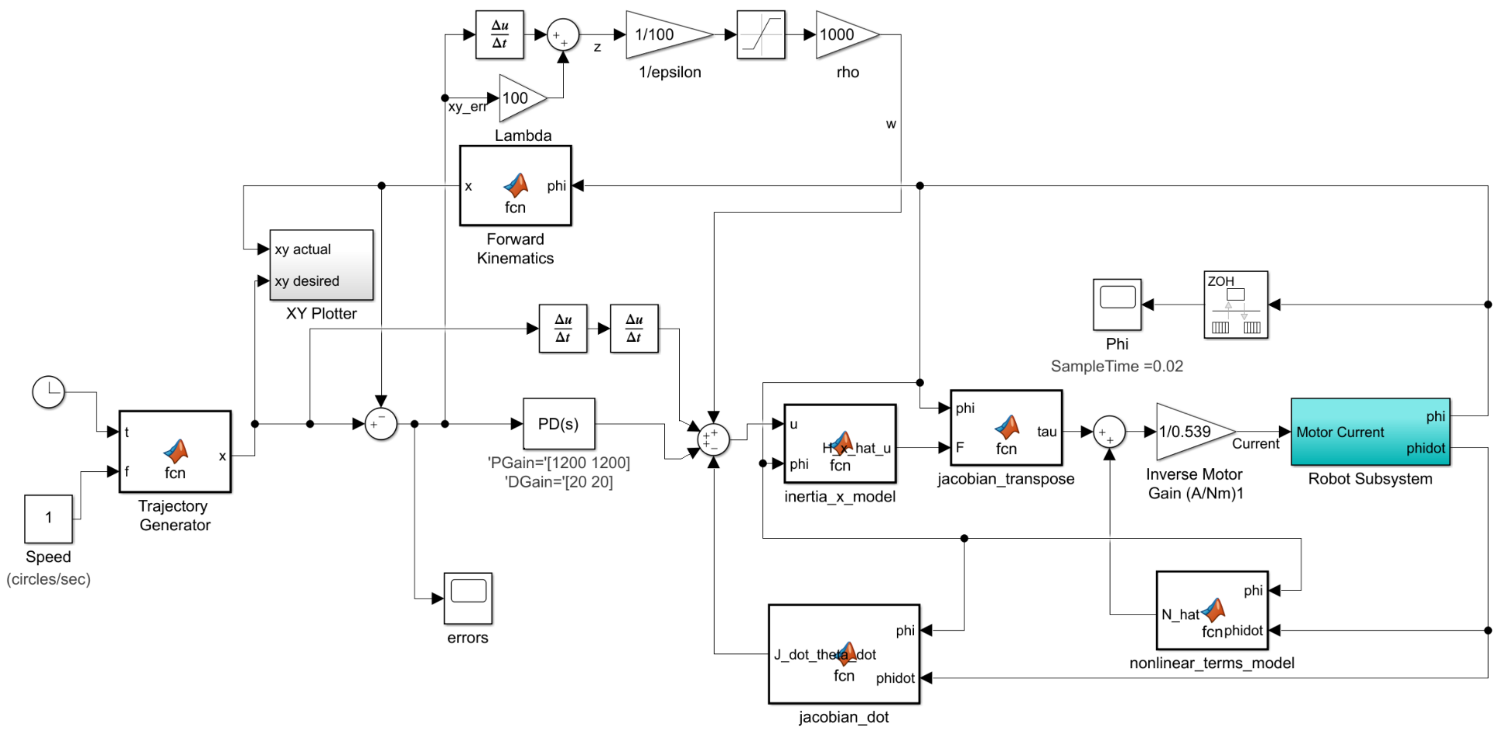
2.2.4.2. Operational Space Errors



2.3. Robust Control in Operational Space

Note: The sliding mode control parameters were modified from simulation to function well on the real-world robot. Epsilon was set to 100 to increase the boundary layer thickness and prevent chattering and saturation of the amplifier. Gamma was also increased to 1000 to compensate for the change in epsilon and to get closer to using the full range of the amplifier.

2.3.1. Model

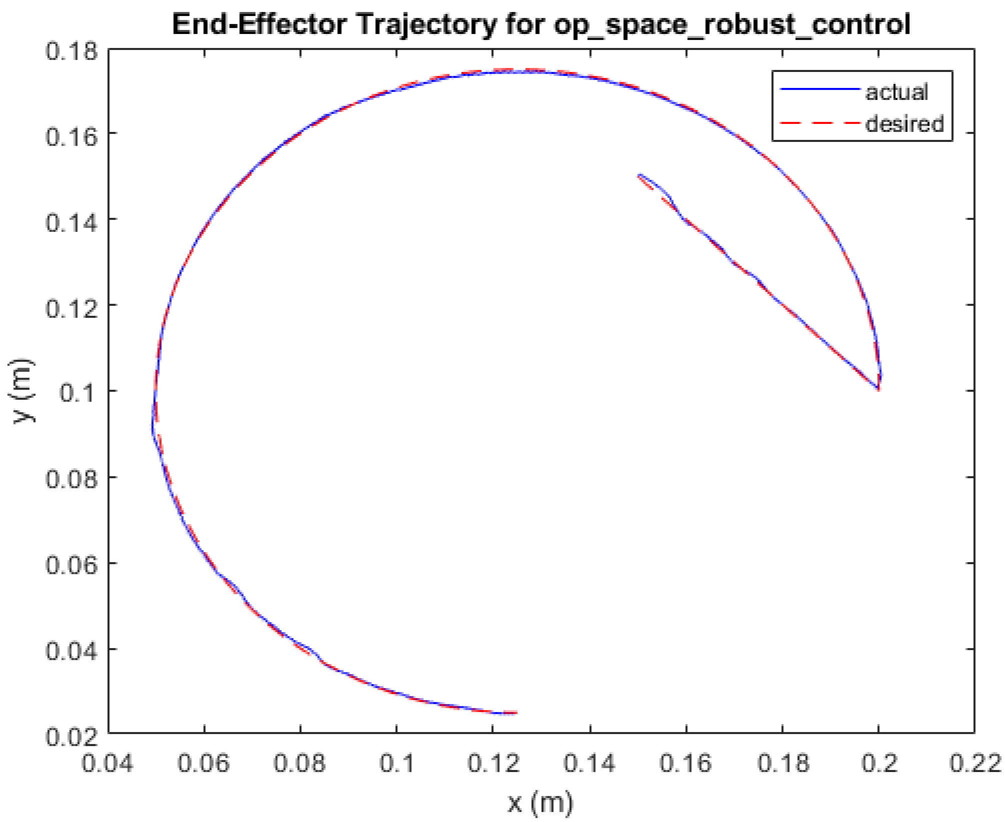


2.3.2. Code

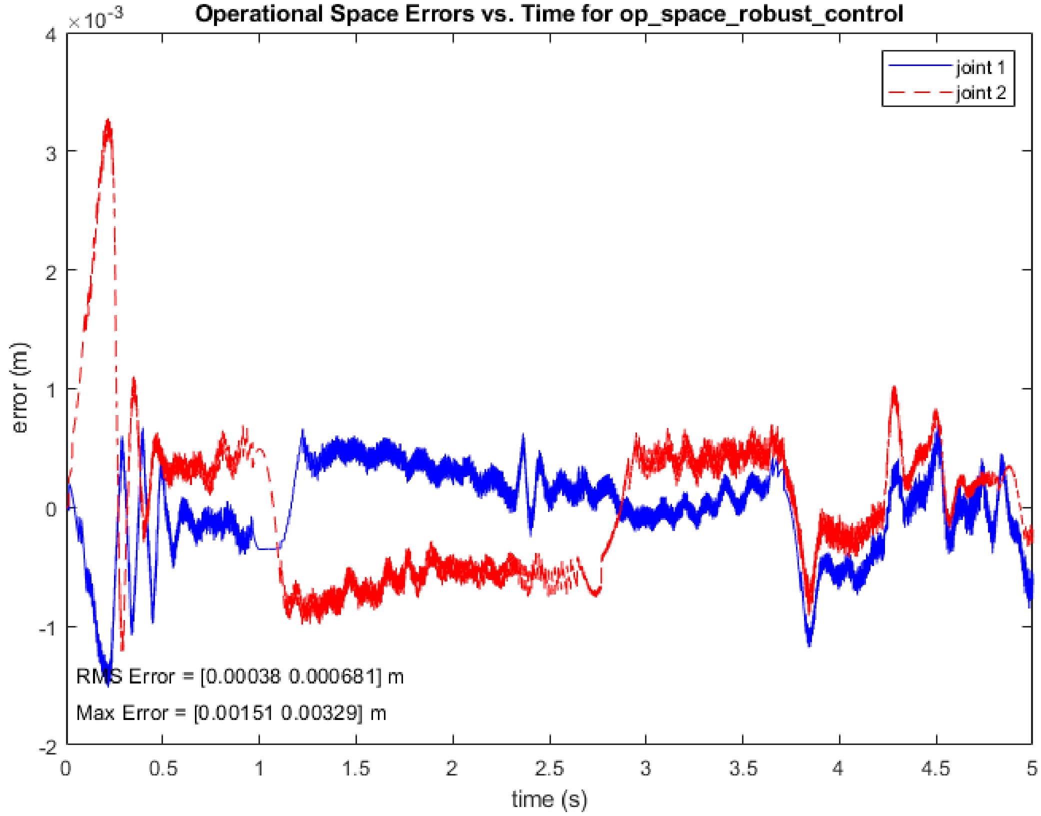
There are no new MATLAB function blocks that have not been described previously..

2.3.3. Low Speed Simulation (f=0.2 circles/s)

2.3.3.1. X-Y Trajectory

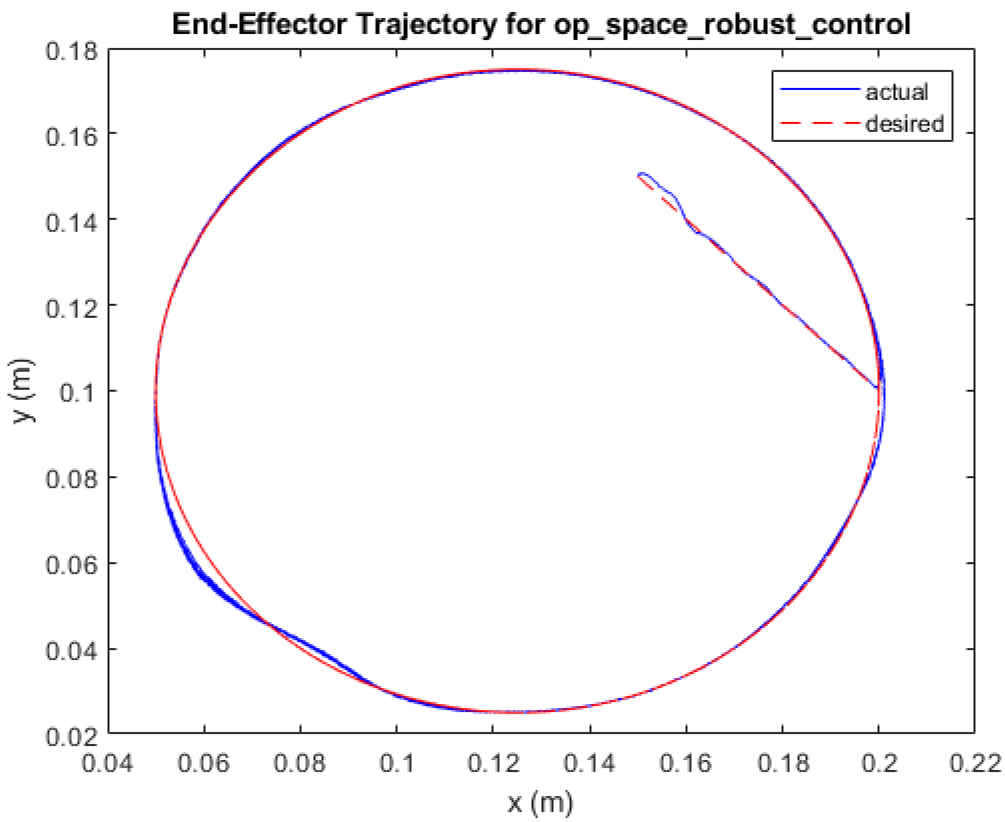


2.3.3.2. Operational Space Errors

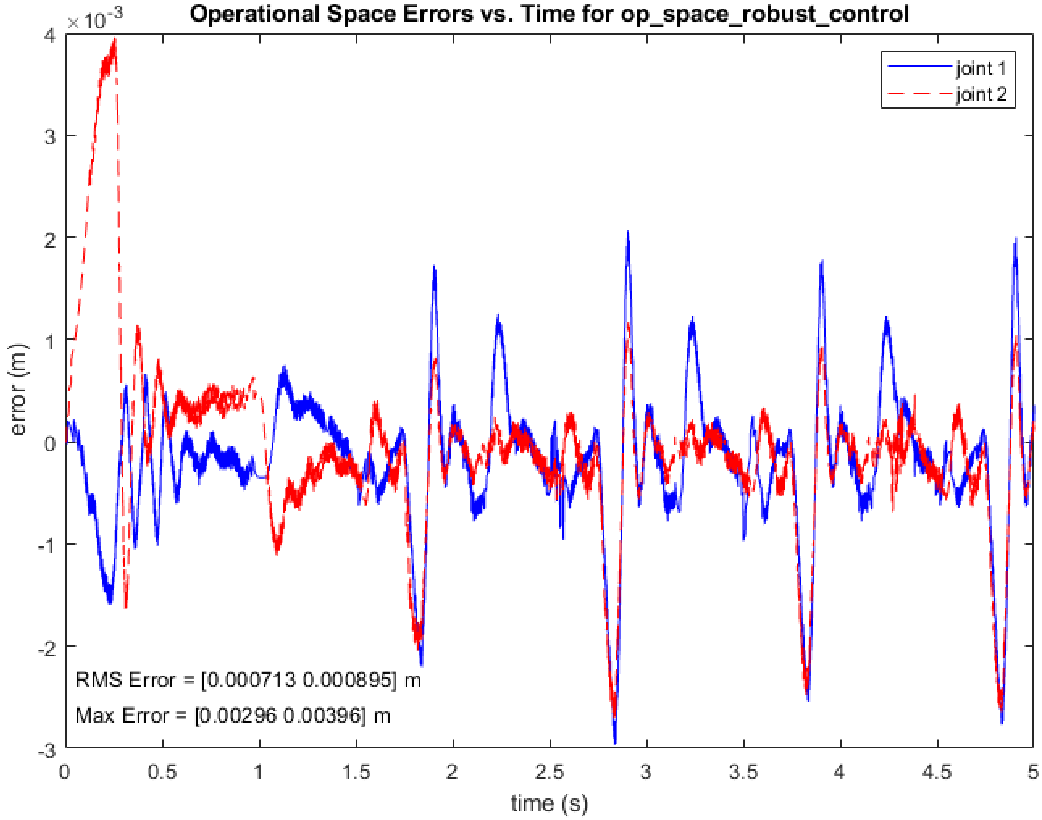


2.3.4. High Speed Simulation (f=1 circles/s)

2.3.4.1. X-Y Trajectory

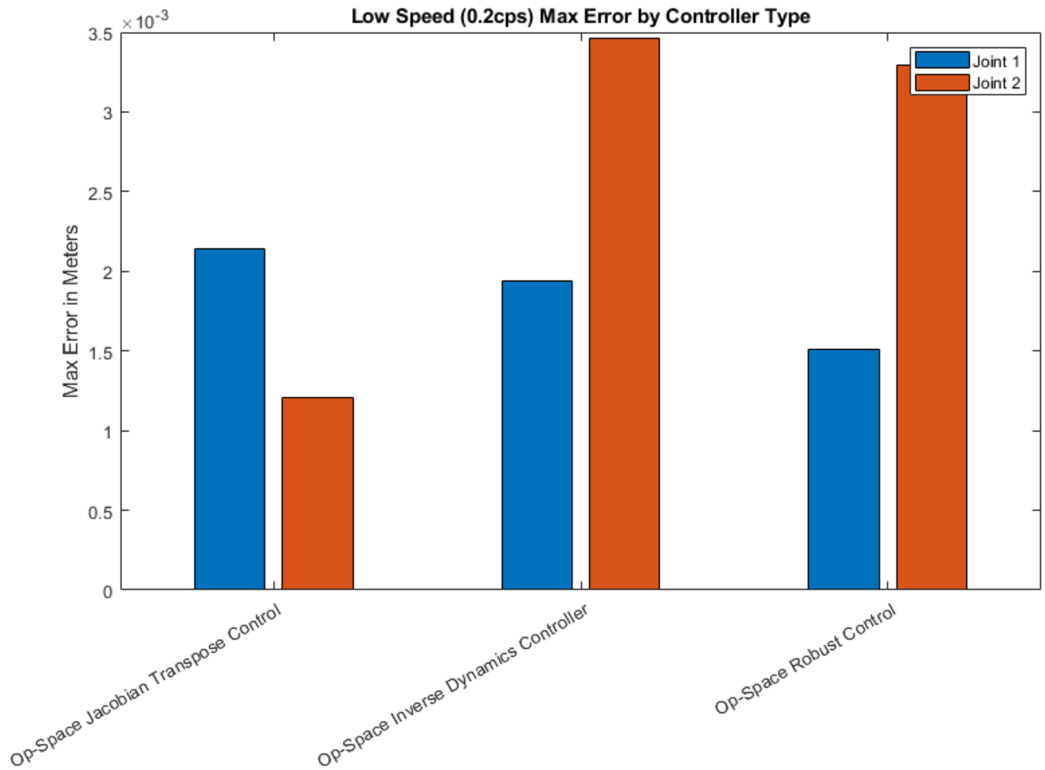


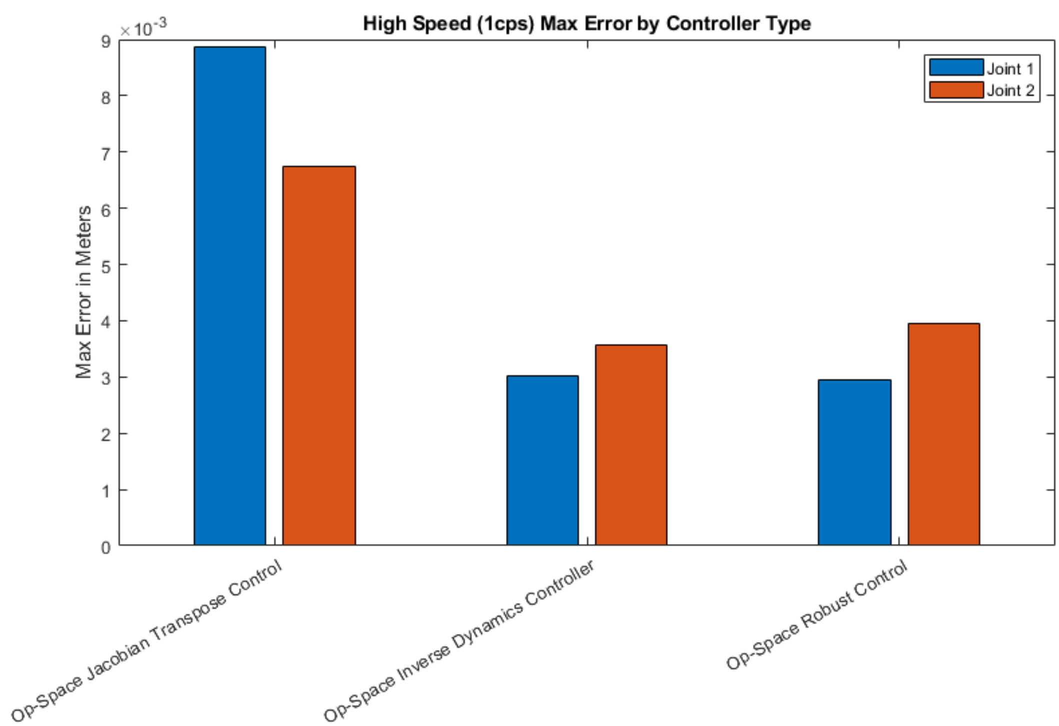
2.3.4.2. Operational Space Errors



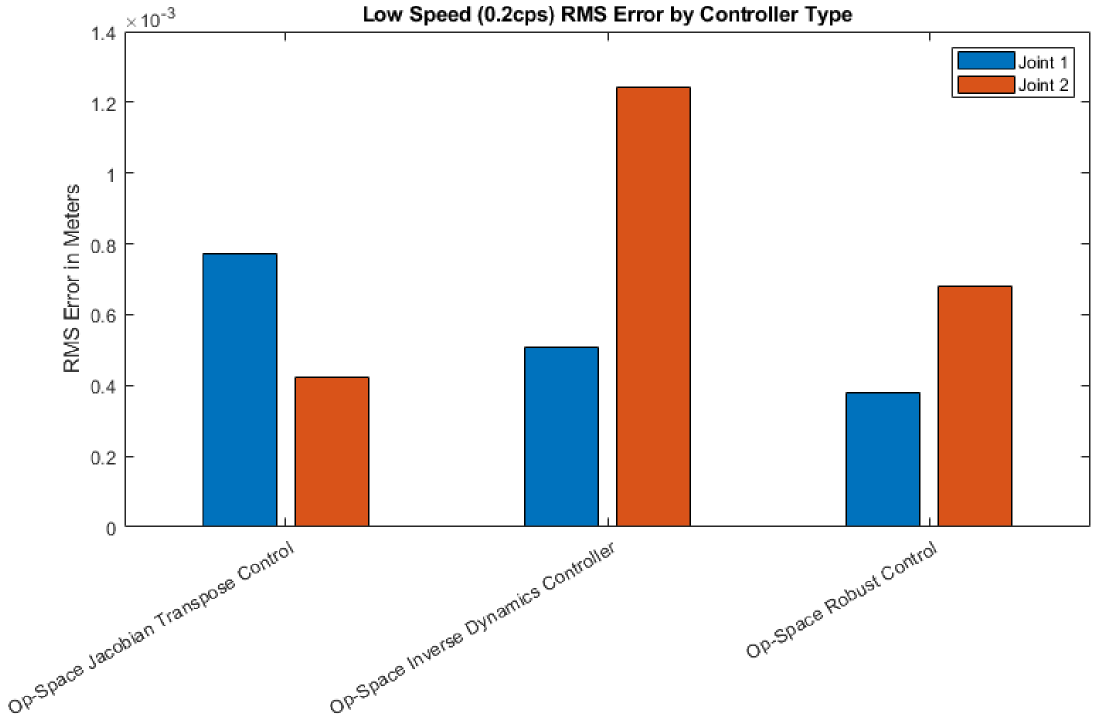
2.4. Controller Comparison

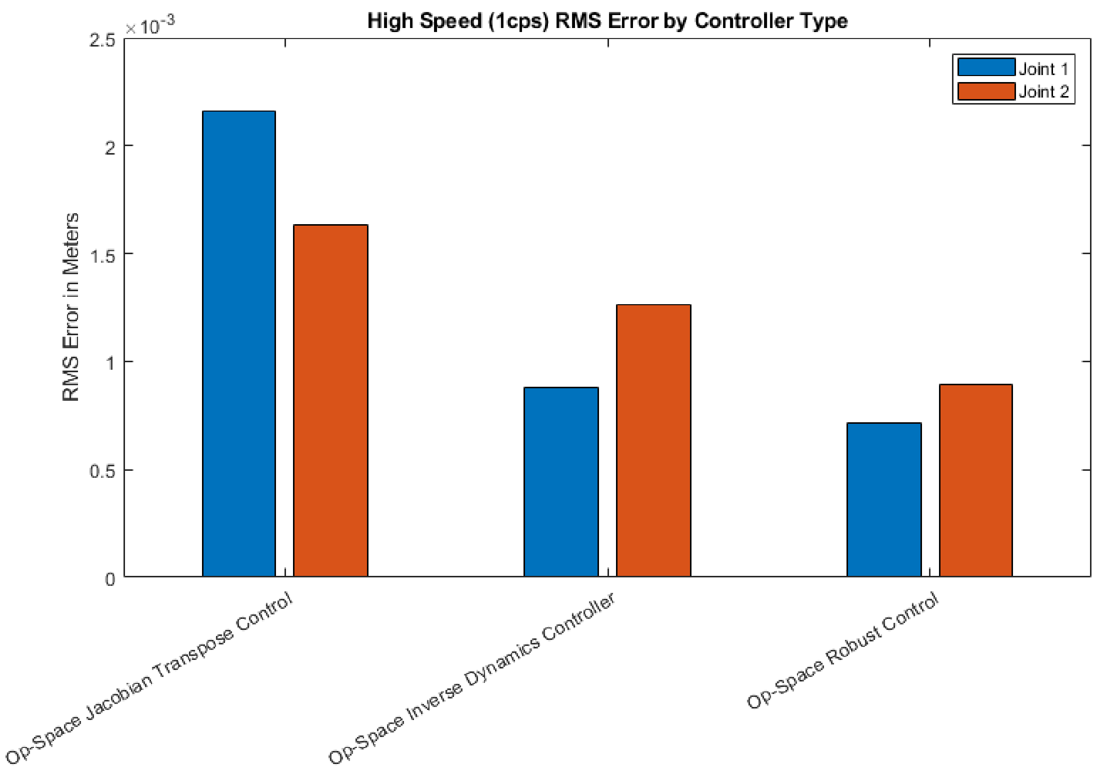
2.4.1. Peak Operational Space Errors





2.4.2. Root-Mean-Square Operational Space Errors





2.4.3. Comparison

Looking at the comparison plots above it can be seen that for high speed trajectories for both joints the overall rms error is lowest for op-space robust control and then op-space inverse dynamics control, then with the highest for op-space jacobian transpose control. Max error for high speed trajectories follows the same trend as high speed trajectories rms error with the exception of joint 2. There were some interesting results for low speed trajectories with op-space jacobian transpose control performing better in terms of lower max error and rms error for joint 2 but the same results as for high speed trajectories held for joint 1. Joint 2 seemed to be harder for the inverse dynamics controller and robust controller to track but this is likely because of experimental variation from tightening some screws on the robot. I also noticed that the inverse dynamics controller and robust controller were better at handling high speed trajectories but actually struggled at lower speed trajectoires without higher PD gains, this can likely be attributed to experimental differences between each run and that the controllers could not be run with higher PD gains. Some accompanying interesting observations were that I had to lower the PD gains specifically to keep jacobian transpose control stable. The other controllers did just fine with higher PD gains and tracked the trajectories even better, although for a fair comparison I had to lower the gains to match jacobian transpose control.

2.4.4. Comparison Code

```
%% ME EN 6230 Problem Set 8 Ryan Dalby
% Jacobian Transpose Related Controller Comparison
% close all;
set(groot, 'DefaultTextInterpreter', 'none') % Prevents underscore from
becoming subscript

% errors are in meters
% Operational space jacobian transpose controller
jacobian_transpose_slow_rms = [0.000773 0.000424];
jacobian_transpose_slow_max = [0.00214 0.00121];
jacobian_transpose_fast_rms = [0.00216 0.00163];
jacobian_transpose_fast_max = [0.00887 0.00674];

% Operational space inverse dynamics controller
op_space_idc_slow_rms = [0.000508 0.00124];
op_space_idc_slow_max = [0.00194 0.00346];
op_space_idc_fast_rms = [0.000883 0.00126];
op_space_idc_fast_max = [0.00301 0.00357];

% Operational space robust inverse dynamics control
op_space_robust_slow_rms = [0.00038 0.000681];
op_space_robust_slow_max = [0.00151 0.00329];
op_space_robust_fast_rms = [0.000713 0.000895];
op_space_robust_fast_max = [0.00296 0.00396];


slow_rms = [jacobian_transpose_slow_rms; op_space_idc_slow_rms;
op_space_robust_slow_rms];
fast_rms = [jacobian_transpose_fast_rms; op_space_idc_fast_rms;
op_space_robust_fast_rms];
slow_max = [jacobian_transpose_slow_max; op_space_idc_slow_max;
op_space_robust_slow_max];
fast_max = [jacobian_transpose_fast_max; op_space_idc_fast_max;
op_space_robust_fast_max];

controller_labels = {'Op-Space Jacobian Transpose Control', 'Op-Space
Inverse Dynamics Controller', 'Op-Space Robust Control'};
controller_labels_cat = categorical(controller_labels);
controller_labels_cat = reordercats(controller_labels_cat,
string(controller_labels_cat));

figure;
```

```
bar(controller_labels_cat,slow_rms);
title('Low Speed (0.2cps) RMS Error by Controller Type');
legend('Joint 1', 'Joint 2');
ylabel('RMS Error in Meters');

figure;
bar(controller_labels_cat,fast_rms);
title('High Speed (1cps) RMS Error by Controller Type');
legend('Joint 1', 'Joint 2');
ylabel('RMS Error in Meters');

figure;
bar(controller_labels_cat,slow_max);
title('Low Speed (0.2cps) Max Error by Controller Type');
legend('Joint 1', 'Joint 2');
ylabel('Max Error in Meters');

figure;
bar(controller_labels_cat,fast_max);
title('High Speed (1cps) Max Error by Controller Type');
legend('Joint 1', 'Joint 2');
ylabel('Max Error in Meters');
```

2.4.5. Appendix- Plotting Code (Used to make simulation plots)

```
%% ME EN 6230 Lab 3 Ryan Dalby
% close all;
set(groot, 'DefaultTextInterpreter', 'none') % Prevents underscore from
becoming subscript

% Extract necessary data, will error if the data does not exist
time = errors.time; % s
model_title = extractBefore(errors.blockName, "/");
joint_errors = rad2deg(errors.signals.values); % deg
op_errors = xy - xy_d; % m
actual_trajectory = xy; % m
desired_trajectory = xy_d; % m

% RMS Joint Errors
RMS_joint_error = rms(joint_errors); % deg
% Max Joint Errors
max_joint_error = max(abs(joint_errors)); % deg

% RMS Operational Space Errors
RMS_op_error = rms(op_errors); % deg
% Max Operational Space Errors
max_op_error = max(abs(op_errors)); % deg

% Plot Joint Errors vs Time
if strcmp(model_title, "jacobian_inverse_control")
    figure;
    plot(time, joint_errors(:,1), 'b-');
    hold on;
    plot(time, joint_errors(:,2), 'r--');
    hold on;
    text(0.01,0.10,strcat("RMS Error = ", mat2str(RMS_joint_error,3),"
deg"), 'Units', 'normalized');
    hold on;
    text(0.01,0.05,strcat("Max Error = ", mat2str(max_joint_error,3),"
deg"), 'Units', 'normalized');
    title(strcat("Joint Errors vs. Time for ", model_title));
    xlabel("time (s)");
    ylabel("error (deg)");
    legend("joint 1", "joint 2");
end

% Plot Operational Space Errors vs Time
```

```matlab
figure;
plot(time, op_errors(:,1), 'b-');
hold on;
plot(time, op_errors(:,2), 'r--');
hold on;
text(0.01,0.10,strcat("RMS Error = ", mat2str(RMS_op_error,3)," m"),
'Units', 'normalized');
hold on;
text(0.01,0.05,strcat("Max Error = ", mat2str(max_op_error,3)," m"),
'Units', 'normalized');
title(strcat("Operational Space Errors vs. Time for ", model_title));
xlabel("time (s)");
ylabel("error (m)");
legend("joint 1", "joint 2");

% Plot End-Effector Trajectory
figure;
plot(xy(:,1), xy(:,2), 'b-');
hold on;
plot(xy_d(:,1), xy_d(:,2), 'r--');
title(strcat("End-Effector Trajectory for ", model_title));
xlabel("x (m)");
ylabel("y (m)");
legend("actual", "desired");
```