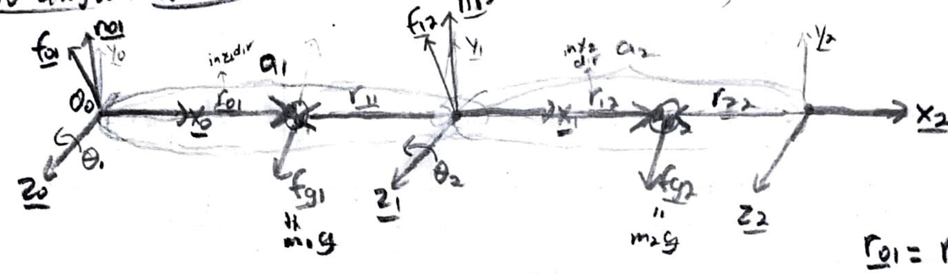
2 DOF Quanser robot Diagrams and Robot Description

i	a_i	d_i	α_i	θ_i
1	a_1	0	0	*
2	a_2	0	0	*

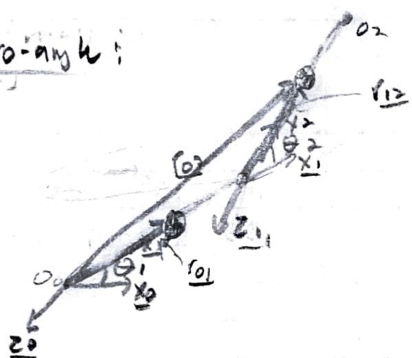
Zero angle and FBD



$\underline{r_{01}} = r_{01}\underline{x_1}$

$\underline{r_{12}} = r_{12}\underline{x_2}$

Non-zero-angle:



Define: $\underline{r_{11}} = -a_1\underline{x_1} + \underline{r_{01}} = -a_1\underline{x_1} + r_{01}\underline{x_1} = (r_{01}-a_1)\underline{x_1}$

$\underline{r_{22}} = -a_2\underline{x_2} + \underline{r_{12}}$

$$\underline{f_1} = \underline{f_{01}} + \underline{f_{21}} + m_1\underline{g} = m_1\ddot{\underline{r_{01}}}$$

about O_1: $$\underline{n_1} = \underline{n_{01}} + \underline{n_{21}} - \underline{r_{01}} \times \underline{f_{01}} + \underline{r_{11}} \times \underline{f_{21}} = I_1\dot{\underline{w_{01}}} + \underline{w_{01}} \times I_1\underline{w_{01}}$$

$$\underline{f_2} = \underline{f_{12}} + m_2\underline{g} = m_2\ddot{\underline{r_{02}}}$$

about O_2: $$\underline{n_2} = \underline{n_{12}} - \underline{r_{12}} \times \underline{f_{12}} = I_2\dot{\underline{w_{02}}} + \underline{w_{02}} \times I_2\underline{w_{02}}$$

↳ Now Recursive Newton-Euler Algorithm

Will compute velocities and accelerations starting at $i=1$, $i=2$

$\boxed{\underline{w_{01}} = \dot{\theta}_1\underline{z_0}}$ $\boxed{\dot{\underline{w_{01}}} = \ddot{\theta}_1\underline{z_0}}$

Note $\underline{x_1} = \underline{x_0}c\theta_1 + \underline{y_0}s\theta_1$

thus $\underline{z_0} \times a_1(\underline{x_0}c\theta_1 + \underline{y_0}s\theta_1) = a_1c\theta_1\underline{y_0} - a_1s\theta_1\underline{x_0}$

$$\ddot{\underline{d_{01}}} = \dot{\underline{w_{01}}} \times \underline{d_{01}} + \underline{w_{01}} \times (\underline{w_{01}} \times \underline{d_{01}}) = \ddot{\theta}_1\underline{z_0} \times a_1\underline{x_1} + \dot{\theta}_1\underline{z_0} \times (\dot{\theta}_1\underline{z_0} \times a_1\underline{x_1})$$

$$\ddot{\underline{d_{01}}} = a_1\ddot{\theta}_1(-s\theta_1\underline{x_0} + c\theta_1\underline{y_0}) + a_1\dot{\theta}_1^2\underline{z_0} \times (-s\theta_1\underline{x_0} + c\theta_1\underline{y_0})$$

$$\boxed{\ddot{\underline{d_{01}}} = a_1\ddot{\theta}_1(-s\theta_1\underline{x_0} + c\theta_1\underline{y_0}) + a_1\dot{\theta}_1^2(-c\theta_1\underline{x_0} - s\theta_1\underline{y_0})}$$

When computing the dynamics I will use this form not this (also for $\ddot{r}_{01}$) (it was more complicated in 0 frame)

$\underline{\ddot{r}_{01}} = \underline{\dot{w}_{01}} \times \underline{r_{01}} + \underline{w_{01}} \times (\underline{w_{01}} \times \underline{r_{01}}) = \ddot{\theta}_1 \underline{z_0} \times (r_{01} \underline{x_1}) + \dot{\theta}_1 \underline{z_0} \times (\dot{\theta}_1 \underline{z_0} \times r_{01} \underline{x_1})$

$\boxed{\underline{\ddot{r}_{01}} = r_{01}\ddot{\theta}_1(-s\theta_1 \underline{x_0} + c\theta_1 \underline{y_0}) + r_{01}\dot{\theta}_1^2(-c\theta_1 \underline{x_0} - s\theta_1 \underline{y_0})}$

For $i=2$:

$\underline{w_{02}} = \underline{w_{01}} + \dot{\theta}_2 \underline{z_1} = \underline{w_{02}} = \dot{\theta}_1 \underline{z_0} + \dot{\theta}_2 \underline{z_1}$ Note $\underline{z_0} = \underline{z_1}$

$\boxed{\underline{w_{02}} = (\dot{\theta}_1 + \dot{\theta}_2)\underline{z_0}}$

$\underline{\dot{w}_{02}} = \underline{\dot{w}_{01}} + \ddot{\theta}_2 \underline{z_1} + \dot{\theta}_2 \underline{w_{01}} \times \underline{z_1} = \ddot{\theta}_1 \underline{z_0} + \ddot{\theta}_2 \overset{=z_0}{\underline{z_1}} + \cancel{\dot{\theta}_2 (\dot{\theta}_1 z) \times z_1}^{=z_0}$

$\boxed{\underline{\dot{w}_{02}} = (\ddot{\theta}_1 + \ddot{\theta}_2)\underline{z_0}}$

$\underline{\ddot{d}_{02}} = \underline{\ddot{d}_{01}} + \underline{\dot{w}_{02}} \times \underline{d_{12}} + \underline{w_{02}} \times (\underline{w_{02}} \times \underline{d_{12}})$

$\underline{\ddot{d}_{02}} = \underline{\ddot{d}_{01}} + (\ddot{\theta}_1 + \ddot{\theta}_2)\underline{z_0} \times (a_2 \underline{x_2}) + (\dot{\theta}_1 + \dot{\theta}_2)\underline{z_0} \times ((\dot{\theta}_1 + \dot{\theta}_2)\underline{z_0} \times (a_2 \underline{x_2}))$

$\underline{z_0} \times \underline{x_2} = z_0 \times (c\theta_2 \underline{x_1} + s\theta_2 \underline{y_1}) = z_0 \times (c\theta_2(c\theta_1 \underline{x_0} + s\theta_1 \underline{y_0}) + s\theta_2(c\theta_1 \underline{y_0} - s\theta_1 \underline{x_0}))$

$= z_0 \times ((c\theta_1 c\theta_2 - s\theta_1 s\theta_2)\underline{x_0} + (s\theta_1 c\theta_2 + c\theta_1 s\theta_2)\underline{y_0})$

$\underline{z_0} \times \underline{x_2} = (-s\theta_1 c\theta_2 - c\theta_1 s\theta_2)\underline{x_0} + (c\theta_1 c\theta_2 - s\theta_1 s\theta_2)\underline{y_0}$

$\underline{z_0} \times \underline{z_0} \times x_2 = (-c\theta_1 c\theta_2 + s\theta_1 s\theta_2)\underline{x_0} + (-s\theta_1 c\theta_2 - c\theta_1 s\theta_2)\underline{y_0}$

$\underline{\ddot{d}_{02}} = \underline{\ddot{d}_{01}} + (\ddot{\theta}_1 + \ddot{\theta}_2)a_2(\underline{z_0} \times \underline{x_2}) + (\dot{\theta}_1 + \dot{\theta}_2)\underline{z_0} \times a_2(\dot{\theta}_1 + \dot{\theta}_2)(\underline{z_0} \times \underline{x_2})$

$\underline{\ddot{d}_{02}} = \underline{\ddot{d}_{01}} + (\ddot{\theta}_1 + \ddot{\theta}_2)a_2(z_0 \times x_2) + (\dot{\theta}_1 + \dot{\theta}_2)^2 a_2(z_0 \times z_0 \times x_2)$

$$\boxed{\begin{aligned}\underline{\ddot{d}_{02}} = \ddot{d}_{01} &+ a_2(\ddot{\theta}_1 + \ddot{\theta}_2)\left[(-s\theta_1 c\theta_2 - c\theta_1 s\theta_2)\underline{x_0} + (c\theta_1 c\theta_2 - s\theta_1 s\theta_2)\underline{y_0}\right] \\ &+ a_2(\dot{\theta}_1 + \dot{\theta}_2)^2\left[(-c\theta_1 c\theta_2 + s\theta_1 s\theta_2)\underline{x_0} + (-s\theta_1 c\theta_2 - c\theta_1 s\theta_2)\underline{y_0}\right]\end{aligned}}$$

$\underline{\ddot{r}_{02}} = \underline{\ddot{d}_{01}} + (\ddot{\theta}_1 + \ddot{\theta}_2)\underline{z_0} \times (r_{12} \underline{x_2}) + (\dot{\theta}_1 + \dot{\theta}_2)\underline{z_0} \times ((\dot{\theta}_1 + \dot{\theta}_2)\underline{z_0} \times (r_{12} \underline{x_2}))$

$\Downarrow$ similarly to $\underline{\ddot{d}_{02}}$

$$\boxed{\begin{aligned}\underline{\ddot{r}_{02}} = \underline{\ddot{d}_{01}} &+ r_{12}(\ddot{\theta}_1 + \ddot{\theta}_2)\left[(-s\theta_1 c\theta_2 - c\theta_1 s\theta_2)\underline{x_0} + (c\theta_1 c\theta_2 - s\theta_1 s\theta_2)\underline{y_0}\right] \\ &+ r_{12}(\dot{\theta}_1 + \dot{\theta}_2)^2\left[(-c\theta_1 c\theta_2 + s\theta_1 s\theta_2)\underline{x_0} + (-s\theta_1 c\theta_2 - c\theta_1 s\theta_2)\underline{y_0}\right]\end{aligned}}$$

When computing the dynamics I will use this instead of this since it was more complicated to use the 0 frame evaluation

Now will compute joint torques starting at $i=2$ to $i=1$:

From page 1:

$$\underline{f_{12}} = m_2 \underline{\ddot{r}_{02}} - m_2 g = m_2(\underline{\ddot{r}_{02}} - g)$$

$$\underline{n_{12}} = \underline{r_{12}} \times \underline{f_{12}} + I_2 \underline{\dot{w}_{02}} + \underline{w_{02}} \times I_2 \underline{w_{02}}$$

$$\tau_2 = \underline{z_1} \cdot \underline{n_{12}}$$

$$\tau_2 = z_1 \cdot \underbrace{\left[r_{12} \underline{x_2} \times \left(m_2 \left(\underline{\ddot{d}_{01}} + \cancel{(z_0 \times x_2)} + \cancel{z_0 \times z_0 \times x_2} \right) - g \right) \right]}_{\text{term } \boxed{1}} + z_1 \cdot \underbrace{\left[I_2 \dot{w}_{02} + w_{02} \times I_2 w_{02} \right]}_{\text{term } \boxed{2}}$$

$a \cdot b \times c = a \times b \cdot c$

$$\text{(1)}: \underline{z_1} \cdot r_{12} \underline{x_2} \times m_2 \left[\left(\ddot{\theta}_1 \underline{z_0} \times a_1 \underline{x_1} + \dot{\theta}_1 \underline{z_0} \times (\dot{\theta}_1 \underline{z_0} \times a_1 \underline{x_1}) \right) - g \right]$$

$$r_{12} m_2 \underline{y_2} \cdot \left[(\ddot{\theta}_1 a_1 \underline{y_1} - \dot{\theta}_1^2 a_1 \underline{x_1}) - g \right]$$

$$r_{12} m_2 (c\theta_2 \underline{y_1} - s\theta_2 \underline{x_1}) \cdot \left[(\ddot{\theta}_1 a_1 \underline{y_1} - \dot{\theta}_1^2 a_1 \underline{x_1}) - g \right]$$

$$r_{12} m_2 (c\theta_2 \ddot{\theta}_1 a_1 + s\theta_2 \dot{\theta}_1^2 a_1) + \left[r_{12} m_2 (c\theta_2 \underline{y_1} - s\theta_2 \underline{x_1}) \cdot -g \right]$$

$z_0 \times$
$I_{2,13}(\dot{\theta}_1+\dot{\theta}_2)$
$I_{2,23}(\dot{\theta}_1+\dot{\theta}_2)$
$\cancel{I_{2,33}(\dot{\theta}_1+\dot{\theta}_2)}$

$$\boxed{2}: \underbrace{\begin{bmatrix}0\\0\\1\end{bmatrix}}_{z_1 = z_0} \cdot \left[\begin{bmatrix} I_{2,11} & I_{2,12} & I_{2,13} \\ I_{2,21} & I_{2,22} & I_{2,23} \\ I_{2,31} & I_{2,32} & I_{2,33} \end{bmatrix} (\ddot{\theta}_1 + \ddot{\theta}_2) \underbrace{\begin{bmatrix}0\\0\\1\end{bmatrix}}_{z_0} + \cancel{(\dot{\theta}_1 + \dot{\theta}_2) \begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix} I_{2,11} & I_{2,12} & I_{2,13} \\ I_{2,21} & I_{2,22} & I_{2,23} \\ I_{2,31} & I_{2,32} & I_{2,33} \end{bmatrix} (\dot{\theta}_1 + \dot{\theta}_2) \begin{bmatrix}0\\0\\1\end{bmatrix}} \right]$$

$$\begin{bmatrix}0\\0\\1\end{bmatrix} \cdot \begin{bmatrix} I_{2,13}(\ddot{\theta}_1 + \ddot{\theta}_2) \\ I_{2,23}(\ddot{\theta}_1 + \ddot{\theta}_2) \\ I_{2,33}(\ddot{\theta}_1 + \ddot{\theta}_2) \end{bmatrix} = I_{2,33}(\ddot{\theta}_1 + \ddot{\theta}_2)$$

$$\boxed{\tau_2 = r_{12} m_2 a_1 (c\theta_2 \ddot{\theta}_1 + s\theta_2 \dot{\theta}_1^2) + I_{2,33}(\ddot{\theta}_1 + \ddot{\theta}_2) + r_{12} m_2 (c\theta_2 \underline{y_1} - s\theta_2 \underline{x_1}) \cdot -g}$$

Now collect terms: $\ddot{\theta}_1$: $\boxed{H_{21} = r_{12} m_2 a_1 c\theta_2 + I_{2,33}}$ $\ddot{\theta}_2$: $\boxed{H_{22} = I_{2,33}}$

$\dot{\theta}_1^2$: $\boxed{h = r_{12} m_2 a_1 s\theta_2}$

$\boxed{G_2 = r_{12} m_2 (c\theta_2 \underline{y_1} - s\theta_2 \underline{x_1}) \cdot -g}$ ← since g vector was not specified (direction) cannot simplify to scalar

Now for $i=1$

$$\underline{f_{01}} = m_1 \ddot{\underline{r_{01}}} + \underline{f_{12}} - m_1 \underline{g} = m_1(\ddot{\underline{r_{01}}} - \underline{g}) + m_2(\ddot{\underline{r_{02}}} - \underline{g}) = m_1 \ddot{\underline{r_{01}}} + m_2 \ddot{\underline{r_{02}}} - (m_1+m_2)\underline{g}$$

$$\underline{n_{01}} = \underline{n_{12}} + \underline{r_{01}} \times \underline{f_{01}} - \underline{r_{11}} \times \underline{f_{12}} + I_1 \dot{\underline{w_{01}}} + \underline{w_{01}} \times I_1 \underline{w_{01}}$$

$$\tau_1 = \underline{z_0} \cdot \underline{n_{01}}$$

$$\tau_1 = \underbrace{\overset{=z_1}{\underline{z_0}} \cdot \underline{n_{12}}}_{\tau_2} + \underbrace{\underline{z_0} \cdot r_{01}\underline{x_1} \times \underline{f_{01}}}_{\boxed{1}} + \underbrace{\underline{z_0} \cdot (a_1 - r_{01})\underline{x_1} \times \underline{f_{12}}}_{\boxed{2}} + \underbrace{\underline{z_0} \cdot \left[I_1 \dot{\underline{w_{01}}} + \underline{w_{01}} \times I_1 \underline{w_{01}} \right]}_{\boxed{3}}$$

$$\dot{\underline{w_{01}}} = \ddot{\theta}_1 \underline{z_0}$$

$$\boxed{1} = \overset{=z_1}{\underline{z_0}} \times r_{01}\underline{x_1} \cdot \left[m_1 \ddot{\underline{r_{01}}} + m_2 \ddot{\underline{r_{02}}} - (m_1+m_2)\underline{g} \right]$$

$$-\underline{x_2} = -c\theta_2 \underline{x_1} - s\theta_2 \underline{y_1}$$

$$\boxed{1} = r_{01}\underline{y_1} \cdot \Big[m_1\big(\ddot{\theta}_1 \overset{=z_1}{\underline{z_0}} \times (r_{01}\underline{x_1}) + \dot{\theta}_1 \underline{z_0} \times \cancel{(\dot{\theta}_1 z_1 \times r_{01}\underline{x_1})}\big) +$$

$$m_2\big(\ddot{\theta}_1 \overset{z_1}{\underline{z_0}} \times a_1\underline{x_1} + \dot{\theta}_1 \cancel{\underline{z_0} \times (\dot{\theta}_1 \underline{z_0} \times a_1 \underline{x_1})}\big) + (\ddot{\theta}_1 + \ddot{\theta}_2)\overset{=z_2}{\underline{z_0}} \times (r_{12}\underline{x_2}) + (\dot{\theta}_1 + \dot{\theta}_2)\overset{=z_2}{\underline{z_0}} \times \big((\dot{\theta}_1 + \dot{\theta}_2)\underline{z_0} \times (r_{12}\underline{x_2})\big)\Big]$$

will select $\underline{y_1}$ terms

$$- r_{01}\underline{y_1} \cdot (m_1+m_2)\underline{g}$$

$$\underline{y_2} = c\theta_2 \underline{y_1} - s\theta_2 \underline{x_1}$$

$$\boxed{1} = r_{01}^2 m_1 \ddot{\theta}_1 + r_{01} a_1 m_2 \ddot{\theta}_1 + r_{01} r_{12} c\theta_2 m_2 (\ddot{\theta}_1 + \ddot{\theta}_2) - r_{01}\underline{y_1} \cdot (m_1+m_2)\underline{g}$$
$$+ r_{01} r_{12} s\theta_2 m_2 (\dot{\theta}_1 + \dot{\theta}_2)^2$$

$$\boxed{2} = \underline{z_0} \times (a_1 - r_{01})\underline{x_1} \cdot \left[m_2 \ddot{\underline{r_{02}}} - m_2 \underline{g} \right]$$

$$\boxed{2} = (a_1 - r_{01})\underline{y_1} \cdot \Big[m_2\big(\ddot{\theta}_1 \overset{z_1}{\underline{z_0}} \times a_1 \underline{x_1} + \cancel{\dot{\theta}_1 \underline{z_0} \times (\dot{\theta}_1 \underline{z_0} \times a_1 \underline{x_1})}\big) + (\ddot{\theta}_1 + \ddot{\theta}_2)\underline{z_0} \times (r_{12}\underline{x_2}) + (\ \cdots\ )\Big]$$

$$+ (a_1 - r_{01})\underline{y_1} \cdot m_2 \underline{g}$$

$$\underline{y_2} = c\theta_2 \underline{y_1} - s\theta_2 \underline{x_1}$$

The analysis of term $\boxed{2}$ is same as $\boxed{1}$ but different coefficient

$$\boxed{2} = (a_1 - r_{01}) a_1 m_2 \ddot{\theta}_1 + (a_1 - r_{01}) r_{12} c\theta_2 m_2 (\ddot{\theta}_1 + \ddot{\theta}_2) - (a_1 - r_{01})\underline{y_1} \cdot m_2 \underline{g}$$
$$+ (a_1 - r_{01}) r_{12} s\theta_2 m_2 (\dot{\theta}_1 + \dot{\theta}_2)^2$$

$$\boxed{3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot \left(\begin{bmatrix} I_{1,11} & I_{1,12} & I_{1,13} \\ I_{1,21} & I_{1,22} & I_{1,23} \\ I_{1,31} & I_{1,32} & I_{1,33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix} + \cancel{\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \times \begin{bmatrix} I_{1,11} & I_{1,12} & I_{1,13} \\ I_{1,21} & I_{1,22} & I_{1,23} \\ I_{1,31} & I_{1,32} & I_{1,33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}} \right)$$

$$\boxed{3} = I_{1,33} \ddot{\theta}_1$$

$$\boxed{\begin{aligned} \tau_1 = \tau_2 &+ r_{01}^2 m_1 \ddot{\theta}_1 + a_1^2 m_2 \ddot{\theta}_1 + a_1 r_{12} c\theta_2 m_2 (\ddot{\theta}_1 + \ddot{\theta}_2) - a_1 r_{12} s\theta_2 m_2 (\dot{\theta}_1 + \dot{\theta}_2)^2 + I_{1,33}\ddot{\theta}_1 \\ &- r_{01}\underline{y_1} \cdot (m_1+m_2)\underline{g} - (a_1 - r_{01})\underline{y_1} \cdot m_2 \underline{g} \end{aligned}}$$

$$(-r_{01}\underline{y_1} \cdot m_1 \underline{g} - a_1 \underline{y_1} \cdot m_2 \underline{g}) = -r_{01} m_1 (\underline{y_1} \cdot \underline{g}) - a_1 m_2 (\underline{y_1} \cdot \underline{g}) = -(r_{01} m_1 - a_1 m_2)(\underline{y_1} \cdot \underline{g})$$

$$(\dot{\theta}_1 + \dot{\theta}_2)^2 = \dot{\theta}_1^2 + 2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2$$

Now collect terms: $\ddot{\theta}_1$: $H_{11} = r_{12} m_2 a_1 c\theta_2 + I_{2,33} + r_{01}^2 m_1 + a_1^2 m_2 + a_1 r_{12} c\theta_2 m_2 + I_{1,33}$

$$\boxed{H_{11} = r_{01}^2 m_1 + a_1^2 m_2 + 2 r_{12} m_2 a_1 c\theta_2 + I_{1,33} + I_{2,33}}$$

$\ddot{\theta}_2$: $\boxed{H_{12} = I_{2,33} + a_1 r_{12} c\theta_2 m_2}$

$\dot{\theta}_1^2$: $\cancel{r_{12} m_2 a_1 s\theta_2 - a_1 r_{12} m_2 s\theta_2}$

$\dot{\theta}_1 \dot{\theta}_2$: $-2(r_{12} m_2 a_1 s\theta_2)$

$\dot{\theta}_2^2$ $= -r_{12} m_2 a_1 s\theta_2$

$$\boxed{h = r_{12} m_2 a_1 s\theta_2}$$

$$\boxed{G_1 = (r_{01} m_1 + a_1 m_2)(\underline{y}_1 \cdot -\underline{g}) + (r_{12} m_2)(c\theta_2 \underline{y}_1 - s\theta_2 \underline{x}_1) \cdot -\underline{g}}$$ → can factor out $\cdot \underline{g}$

Conclusion/summary of previous results

$$\boxed{H(\underline{\theta}) = \begin{bmatrix} r_{01}^2 m_1 + a_1^2 m_2 + 2 r_{12} m_2 a_1 c\theta_2 + I_{1,33} + I_{2,33} & I_{2,33} + a_1 r_{12} c\theta_2 m_2 \\ r_{12} m_2 a_1 c\theta_2 + I_{2,33} & I_{2,33} \end{bmatrix}}$$ (2×2)

$$\boxed{V(\underline{\theta}, \dot{\underline{\theta}}) = \begin{bmatrix} 0 & -2 r_{12} m_2 a_1 s\theta_2 & -r_{12} m_2 a_1 s\theta_2 \\ r_{12} m_2 a_1 s\theta_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_2^2 \end{bmatrix}}$$ 2×3 3×1 (2×1)

$$\boxed{G(\underline{\theta}) = \begin{bmatrix} [r_{12} m_2 (c\theta_2 \underline{y}_1 - s\theta_2 \underline{x}_1) + (r_{01} m_1 + a_1 m_2) \underline{y}_1] \cdot -\underline{g} \\ (r_{12} m_2)(c\theta_2 \underline{y}_1 - s\theta_2 \underline{x}_1) \cdot -\underline{g} \end{bmatrix}}$$ (2×1)

→ still in vector form since the gravity vector $\underline{g}$ was <u>NOT</u> given

$$\boxed{\underbrace{\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}}_{(2\times1)} = \underbrace{\underline{\tau}}_{(2\times1)} = \underbrace{H(\underline{\theta})\ddot{\underline{\theta}}}_{(2\times1)} + \underbrace{V(\underline{\theta}, \dot{\underline{\theta}})}_{(2\times1)} + \underbrace{G(\underline{\theta})}_{(2\times1)}}$$

$$\ddot{\underline{\theta}} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$

2×2 2×1