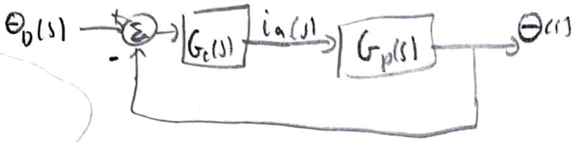
MEEN 6230 | Problem Set 5 | Ryan Dalby | Page 1

1.



$K_t = 0.0077 N\cdot m/A$
$R_a = 2.6\Omega$
$N = 70$
$I_1 = 0.83\times10^{-3} kg\cdot m^2$
$J_m = 0.65\times10^{-6} kg\cdot m^2$
$b = 3.1\times10^{-6} N\cdot m\cdot s/rad$
$m_1 g r_{01} = 0.067 N\cdot m/rad$

$(NK_t)i(t) = (I_1 + N^2 J_m)\ddot{\theta} + N^2 b\dot{\theta} + m_1 g r_{01}\theta$

$\Downarrow$

$G_c(s) = K_P + K_D s$

$$G_p(s) = \frac{NK_t}{(I_1+N^2J_m)s^2 + (N^2 b)s + m_1 g r_{01}}$$

PS#4 $\rightarrow$ $G_p(s) = \dfrac{0.539}{0.004015 s^2 + 0.01519 s + 0.067}$

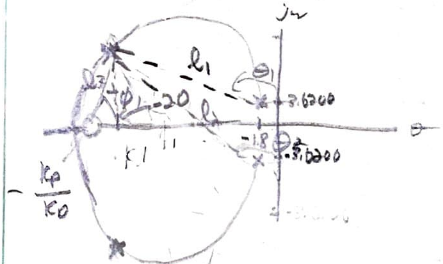
Design for %OS = 20% $T_s = 0.2s$

$$G_c(s)G_p(s) = \frac{NK_t(K_P + K_D s)}{(I_1+N^2J_m)s^2 + (N^2b)s + m_1 g r_{01}} = \frac{NK_tK_D(s + K_P/K_D)}{(I_1+N^2J_m)s^2 + (N^2b)s + m_1 g r_{01}}$$

Plugging in parameter values $\hookrightarrow$ (same as PS #4)

$$G_c(s)G_p(s) = \frac{0.539K_D(s + K_P/K_D)}{0.004015 s^2 + 0.01519 s + 0.067} = \frac{134.247K_D(s + K_P/K_D)}{s^2 + 3.783s + 16.687}$$

O.L. zeros: $s = -K_P/K_D$ O.L. poles: $s = -1.8917 \pm 3.6206j$



$$\zeta = \frac{-\ln(20/100)}{\sqrt{\pi^2 + \ln^2(20/100)}} = 0.456$$

$T_s = \dfrac{4}{\zeta\omega_n} = 0.2$ $\omega_n = 43.86$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$
$\omega_d = 39.04$

Desired closed loop poles: $s_{1,2} = -\zeta\omega_n \pm \omega_d j$
$\hookrightarrow$ $\boxed{s_{1,2} = -20 \pm 39.04j}$

Angle condition

$\phi - \theta_1 - \theta_2 = \pm 180°$

$$\phi = \pm180° + \tan^{-1}\left(\frac{\overbrace{35.42}^{39.04-3.621}}{-18.2}\right) + \tan^{-1}\left(\frac{\overbrace{42.66}^{39.04+3.62}}{-18.2}\right)$$

$\phi = \pm180° + 117.20 + 113.10 \Rightarrow 50.3°$

$\tan(50.3°) = \left(\dfrac{39.04}{-20-z}\right)$ $z = -52.41 = -K_P/K_D$

Loop Gain, Magnitude Condition:

$$134.247K_D = \frac{l_1 l_2}{l_3} = \frac{\sqrt{35.42^2+18.2^2}\sqrt{42.66^2+18.2^2}}{\sqrt{32.41^2+39.04^2}} = 36.40$$

$K_D = 36.4/134.247 = 0.271 \Rightarrow K_P = K_D(52.41) = 14.210$

$\boxed{K_P = 14.21 \quad K_D = 0.271}$

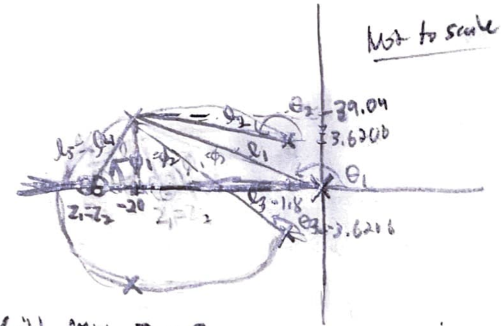
$\hookrightarrow$ The closed loop zero may interfere

2. $G_c(s) = K_P + K_D s + \frac{K_I}{s}$ $G_p(s) = \frac{NK_t}{(J_1 + N^2 J_m)s^2 + (N^2 b)s + m_1 g r_{c1}}$

$G_c(s) = \frac{K_D s^2 + K_P s + K_I}{s}$

$G_c G_p = \frac{134.247(K_D s^2 + K_P s + K_I)}{s(s^2 + 3.783s + 16.687)} = \frac{134.247 K_D (s+z_1)(s+z_2)}{s(s^2+3.783s+16.687)}$

$\hookrightarrow s = -1.8917 \pm 3.6206j$



From 1.:
Desired dominant closed loop poles should be (for %OS = 20%, $T_s = 0.2s$)
$s = -20 \pm 39.04j$

Will say $z_1 = z_2$

Angle Condition: $2\phi_1 - \theta_1 - \theta_2 - \theta_3 = \pm 180°$

$\phi_1 = \frac{\theta_1 + \theta_2 + \theta_3 \pm 180°}{2} = \frac{\tan^{-1}\left(\frac{39.04}{-20}\right) + \tan^{-1}\left(\frac{35.42}{-18.1}\right) + \tan^{-1}\left(\frac{42.66}{-18.1}\right) \pm 180°}{2}$

$\phi_1 = \frac{117.1258 + 117.20 + 113.10 \pm 180}{2} = \frac{167.42}{2} = 83.713°$

$\tan(83.713°) = \left(\frac{39.04}{-20-z_1}\right)$ $z_1 = -24.3$

Magnitude Condition: $134.247 K_D = \frac{l_1 l_2 l_3}{2 l_4} = \frac{\sqrt{(39.04)^2+(20)^2}\sqrt{(35.42)^2+(18.1)^2}\sqrt{(42.66)^2+(18.1)^2}}{2\sqrt{(4.3)^2+(39.04)^2}} = 1030.15$

$4.3 = -20-(-24.3)$

$K_D = 1030.15/134.247 = 7.67$

$K_D(s+z_1)(s+z_2) = K_D s^2 + K_D(z_1+z_2)s + K_D(z_1 z_2) = K_D s^2 + K_P s + K_I$

$K_D(z_1+z_2) = 2K_D z_1 = K_P = 372.76$ ($z_1 = 24.3$)

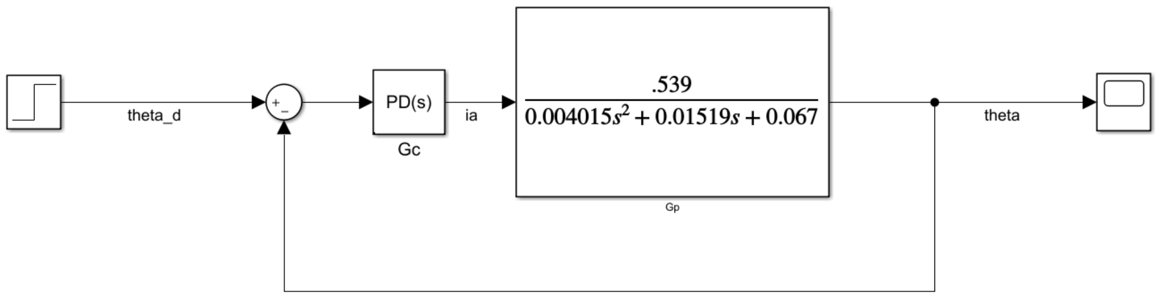
$K_D(z_1 z_2) = K_D z_1^2 = K_I = 4529.06$

$\boxed{K_P = 372.76}$ $\boxed{K_D = 7.67}$ $\boxed{K_I = 4529.06}$

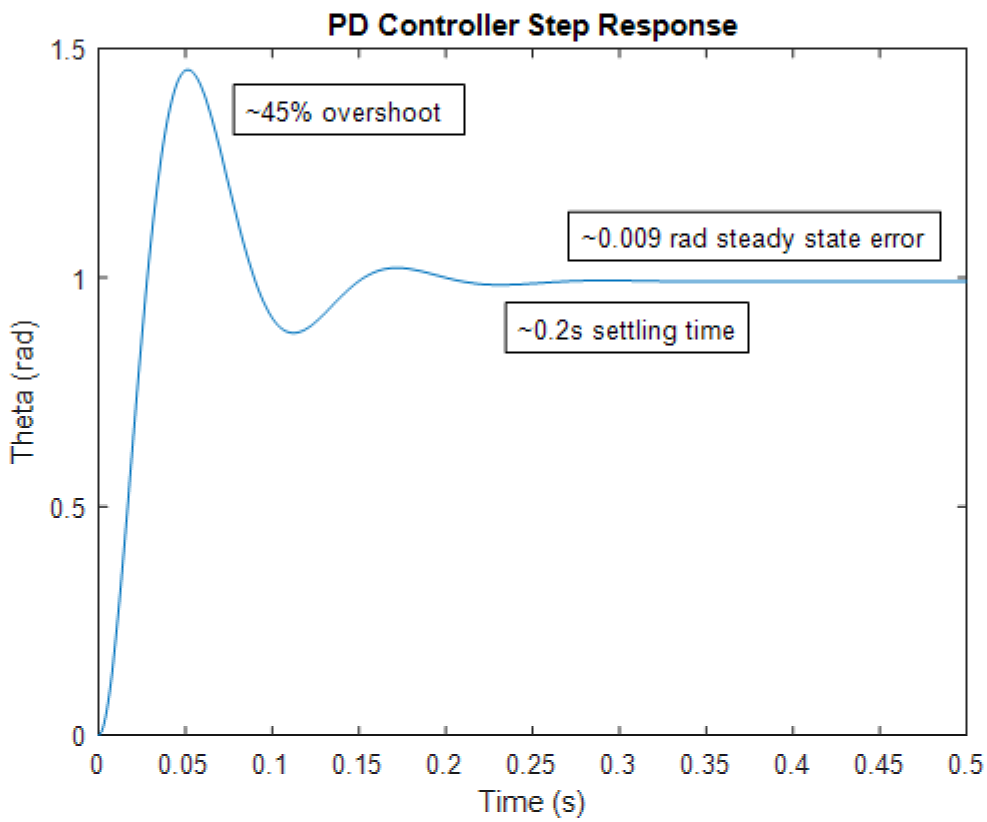
↳ The closed loop zeros may likely interfere
↳ We also assumed dominant second order behavior but all 3 poles may dominate.

3.

- PD Controller (Problem 1):
  - Model:

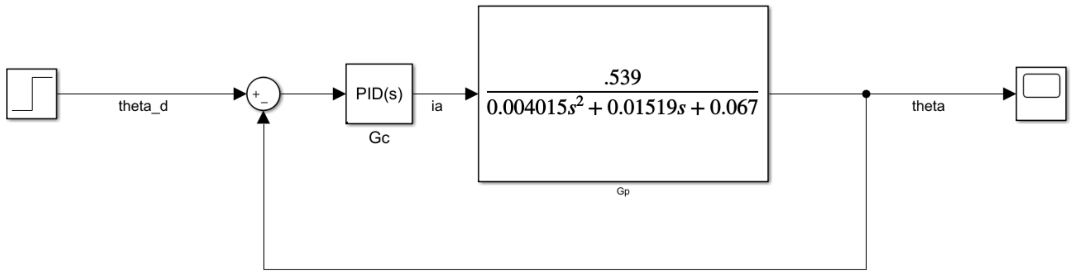


  - Step Response:

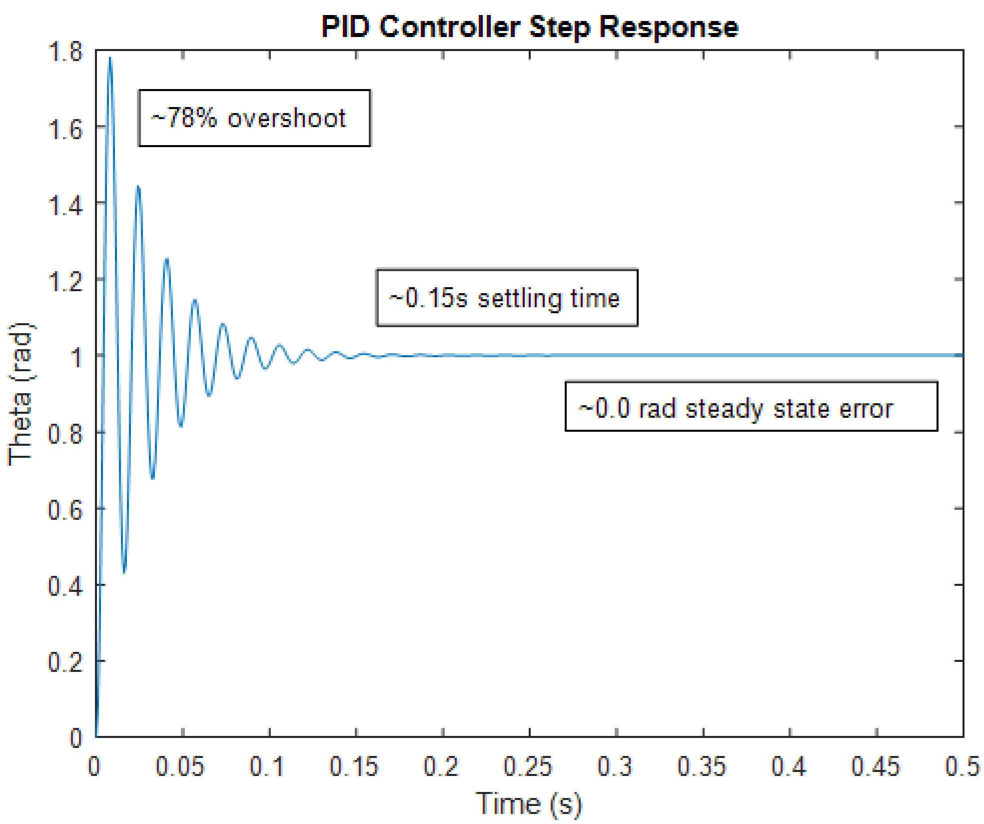


  - Performance:
    - This PD step response does not match the desired 20% overshoot. This is likely due to numerical differentiation of the step function and influence from the closed-loop zero. There is some steady-state error.

- PID Controller (Problem 2):
    - Model:



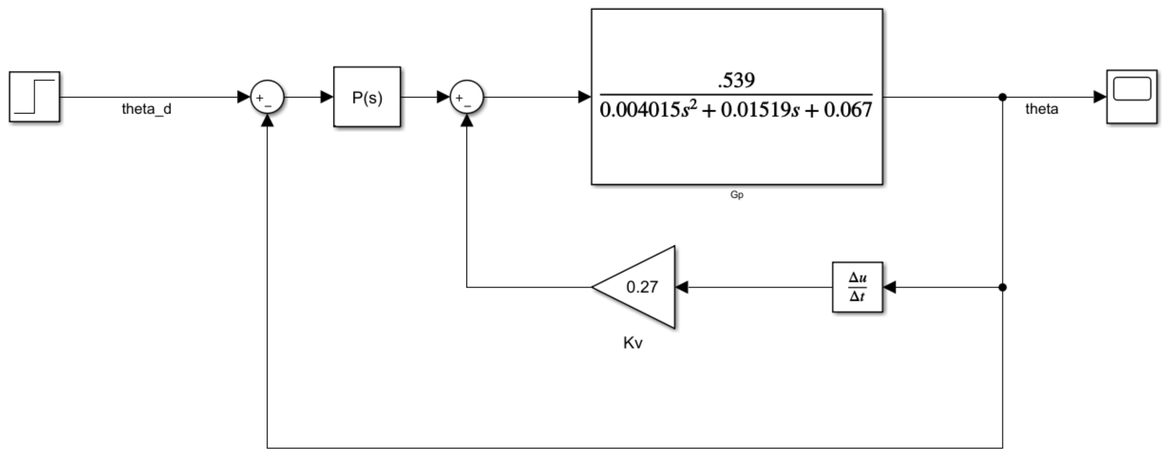
    - Step Response:



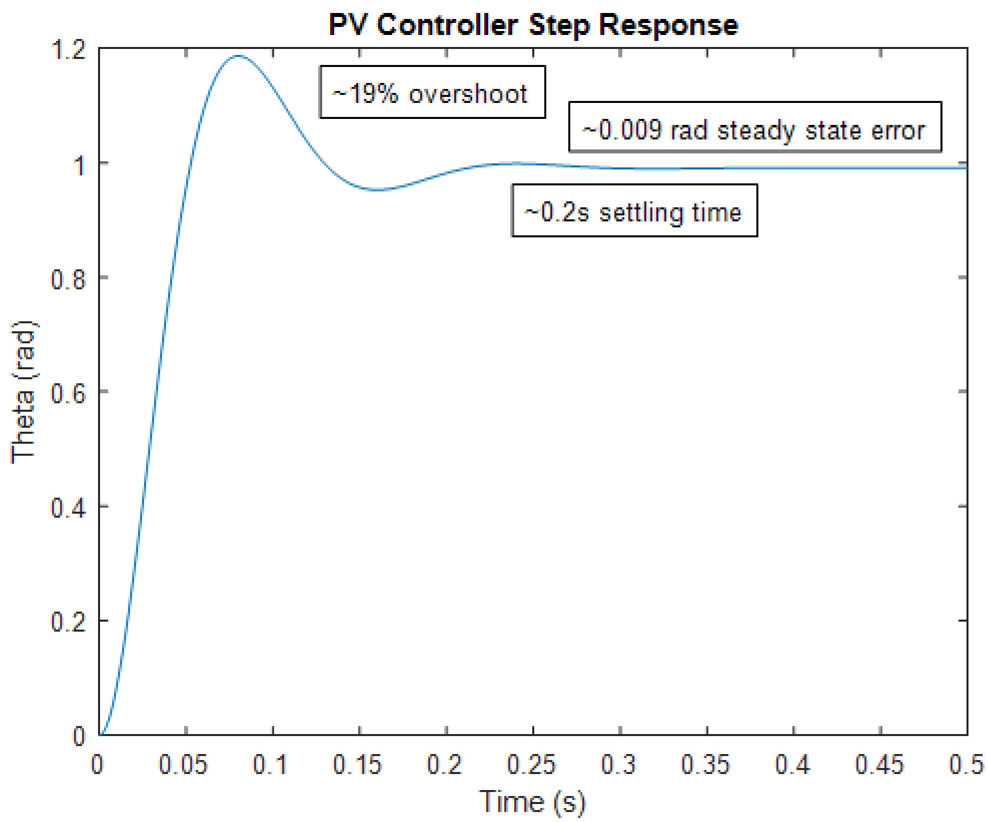
    - Performance:
        - This PID step response does not match the desired 20% overshoot or the 0.2 second settling time. This is likely due to the third closed-loop pole and the two closed-loop zeros. Numerical differentiation of the step function may also be a contributing factor as well. With more tuning this controller could theoretically achieve better performance than the PD controller. Steady-state error is zero.

4. Note: See written work for the equivalence of closed-loop poles for PD/PV and PID/PIV controllers as well as comparison between the number of closed-loop zeroes.

- PV Controller:
    - Model:

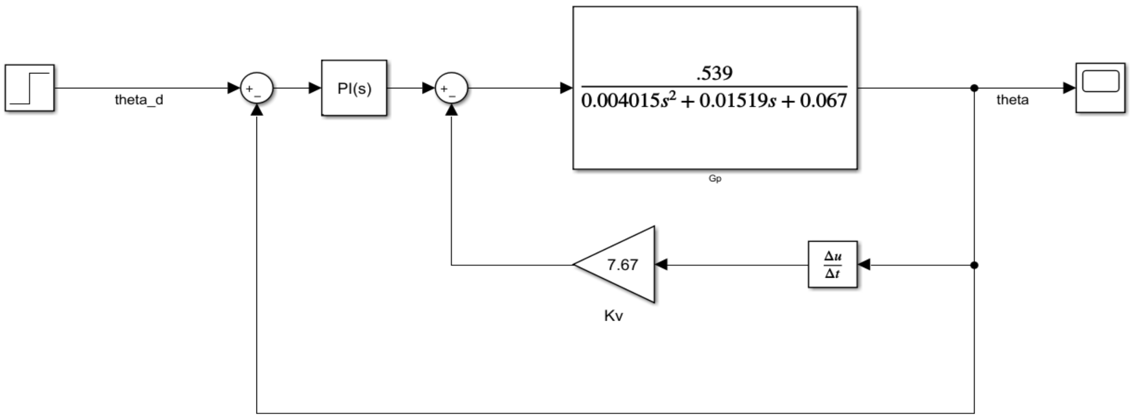


    - Step Response:

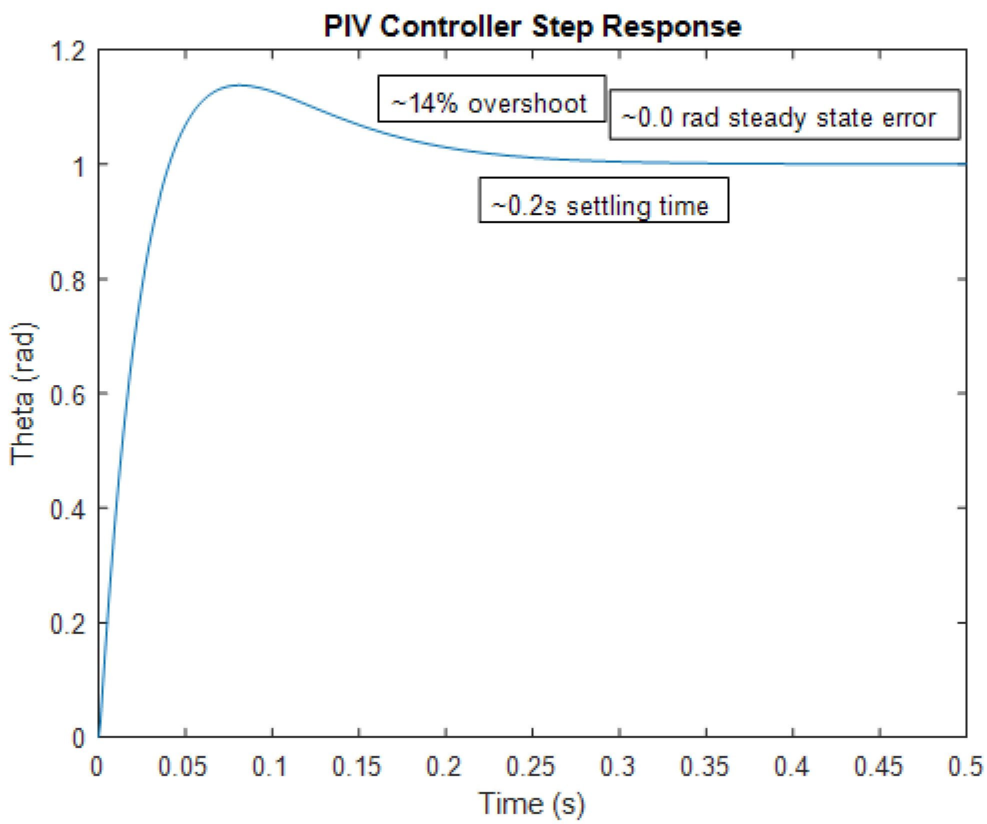


    - Performance:
        - This PV step response closely matches the desired percent overshoot and settling time. The PV controller removes the influence of the closed loop zero and numerical differentiation of the step function that was an issue with the PD controller. There is steady-state error.

- PIV Controller:
    - Model:

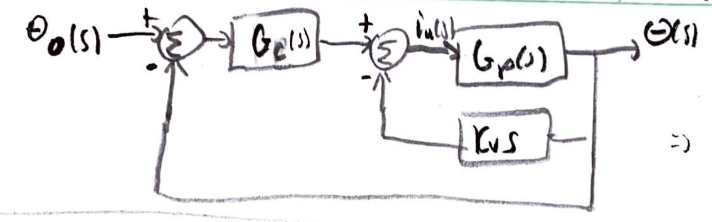


    - Step Response:



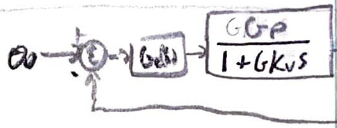
    - Performance:
        - This PIV step response does not match the desired 20% overshoot. This is likely due to the third closed-loop pole and the single closed-loop zero. The controller achieves a first order-like response and does achieve the desired settling time unlike the PID controller likely due to one less closed-loop zero. With more tuning this controller could theoretically achieve better performance Steady-state error is zero.

4.



$$G_p(s) = \frac{134.247}{s^2 + 3.7833s + 16.687} \Rightarrow = \frac{134.2}{s^2 + 3.8s + 16.7}$$

PV Control: $G_c(s) = K_p$



$$\frac{G_p}{1 + G_p K_v s} = \frac{\frac{134.2}{s^2 + 3.8s + 16.7}}{1 + \frac{134.2 (K_v) s}{s^2 + 3.8s + 16.7}} = \frac{134.2}{s^2 + (3.8 + 134.2K_v)s + 16.7} = T(s)$$

$$\Theta_D(s) \rightarrow \boxed{\frac{K_p T(s)}{1 + K_p T(s)}} \rightarrow \Theta(s) \qquad \frac{\Theta(s)}{\Theta_D(s)} = \frac{K_p T(s)}{1 + K_p T(s)}$$

PV CLTF

$K_p = 14.21$
$K_D = K_v = 0.27$

$$\frac{\Theta(s)}{\Theta_D(s)} = \frac{K_p (134.2)}{s^2 + (3.8 + 134.2K_v)s + (16.7 + 134.2K_p)} = \boxed{\frac{1906.962}{s^2 + 40.034s + 1923.682}}$$

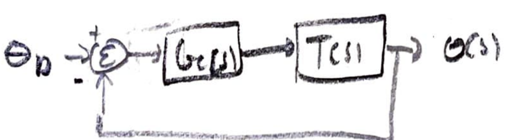
PD CL Poles:

$\boxed{\hookrightarrow \text{C.L. Poles: } s = -20.17 \pm 38.95j \approx -20 \pm 39.04j}$

There was 1 CL zero for PD

No closed loop zeros!

PIV Control: $G_c(s) = K_p + \frac{K_I}{s} = \frac{K_p s + K_I}{s}$



$$G_c(s)T(s) = \frac{(K_p s + K_I)\,134.2}{s\left(s^2 + (3.8 + 134.2K_v)s + 16.7\right)}$$

$K_p = 372.76$
$K_v = K_D = 7.67$
$K_I = 4529.06$

$$\frac{\Theta(s)}{\Theta_D(s)} = \frac{(K_p s + K_I)\,134.2}{s^3 + (3.8 + 134.2K_v)s^2 + (16.7 + 134.2K_p)s + 134.2K_I} = \frac{50024s + 607800}{s^3 + 1046s^2 + 50041s + 607800}$$

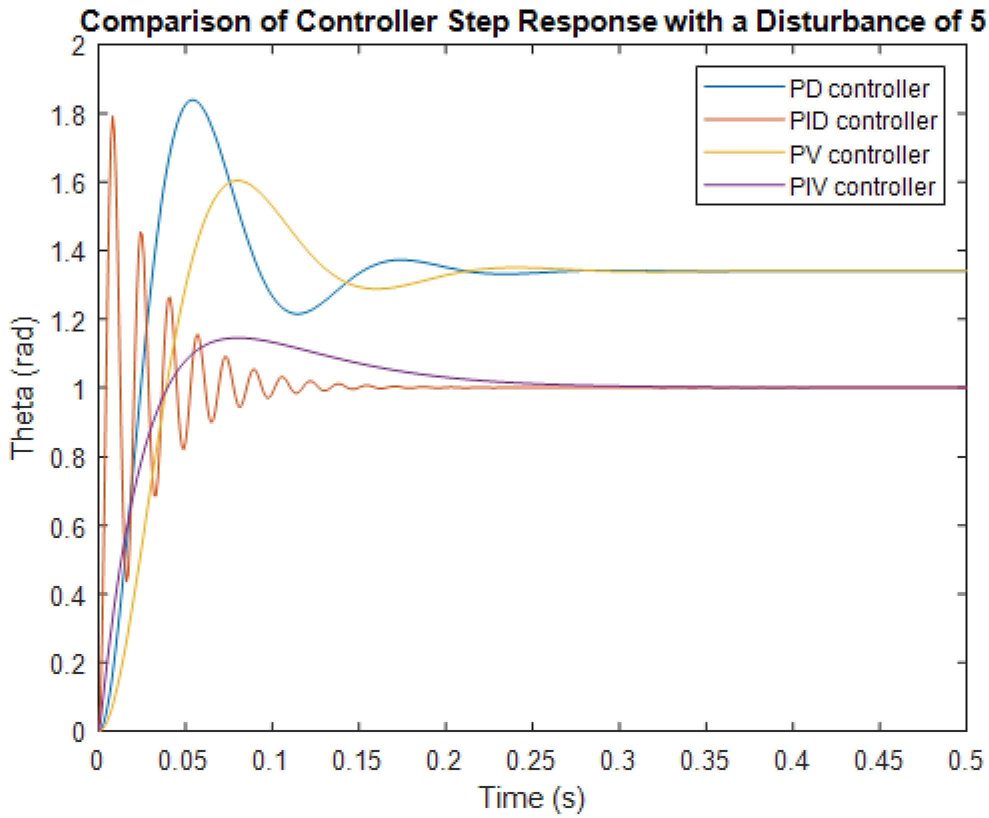
C.L. Poles: $s = -996.39,\ -27.10,\ -22.51 \approx s = -988.2,\ -28.85,\ -21.43$

$\hookrightarrow$ Approximately equal to closed loop zeroes found using matlab: (see attached code)

$\hookrightarrow$ We have 1 closed loop zero instead of 2 with PID

5.

- Disturbance influence comparison plot:



- Comparison:
    - The PD/PV controllers are strongly influenced by the disturbance in terms of steady-state error. As can be seen in the plot, the PD/PV controllers have very large steady-state error when a disturbance is introduced. From experimentation with more disturbance the steady-state error continues to grow larger for the PD/PV controllers. On the other hand, the steady-state error for both the PID/PIV controller is still zero even with a large disturbance. The integral portion of these controllers expands the capability of the PID/PIV controllers to deal with input disturbances compared to PD/PV controllers.

Appendix:

- Code to replicate plots given collected data from Simulink:

```
%% ME EN 6230 Problem Set 5 Ryan Dalby
%%
close all;
%% System Description
Gp = tf(.539, [0.004015 0.01519 0.067]);
PD = tf([0.27 14.21], 1);
PID = tf([7.67 372.76 4529.06], [1 0]);

%%
% Send data for PD controller to workspace, then execute this cell
PD_step_response_data = out.ScopeData;
%%
% Send data for PID controller to workspace, then execute this cell
PID_step_response_data = out.ScopeData;
%%
% Send data for PV controller to workspace, then execute this cell
PV_step_response_data = out.ScopeData;
%%
% Send data for PIV controller to workspace, then execute this cell
PIV_step_response_data = out.ScopeData;

%%
% Send data for PD controller with disturbance to workspace, then execute
this cell
PDdist_step_response_data = out.ScopeData;
%%
% Send data for PID controller with disturbance to workspace, then execute
this cell
PIDdist_step_response_data = out.ScopeData;
%%
% Send data for PV controller with disturbance to workspace, then execute
this cell
PVdist_step_response_data = out.ScopeData;
%%
% Send data for PIV controller with distrubance to workspace, then execute
this cell
PIVdist_step_response_data = out.ScopeData;
%% Problem 3
% PD plot
figure;
plot(PD_step_response_data(:,1), PD_step_response_data(:,2));
```

```
xlabel('Time (s)');
ylabel('Theta (rad)');
title('PD Controller Step Response');
annotation('textbox',...
    [0.251 0.823809523809524 0.206142857142857 0.0595238095238098],...
    'String',{'~45% overshoot'},...
    'FitBoxToText','off');
annotation('textbox',...
    [0.493857142857142 0.564285714285717 0.216857142857143
0.0595238095238098],...
    'String','~0.2s settling time',...
    'FitBoxToText','off');
annotation('textbox',...
    [0.55 0.673809523809525 0.332142857142857 0.0581812787420278],...
    'String','~0.009 rad steady state error',...
    'FitBoxToText','off');

% PID plot
figure;
plot(PID_step_response_data(:,1), PID_step_response_data(:,2));
xlabel('Time (s)');
ylabel('Theta (rad)');
title('PID Controller Step Response');
annotation('textbox',...
    [0.168857142857143 0.811904761904762 0.2065 0.0666666666666675],...
    'String','~78% overshoot',...
    'FitBoxToText','off');
annotation('textbox',...
    [0.381357142857142 0.598571428571431 0.232928571428572
0.0666666666666674],...
    'String','~0.15s settling time',...
    'FitBoxToText','off');
annotation('textbox',...
    [0.55 0.473809523809525 0.332142857142857 0.0581812787420278],...
    'String','~0.0 rad steady state error',...
    'FitBoxToText','off');
%% Problem 4
% Closed loop poles of PID system (these are not exactly the desired poles
% since our assumptions of a second order system are not accurately valid,
% there is interference from the third closed loop pole and two closed loop
% zeroes. Will compare this value to PIV closed loop zeroes
disp(pole(feedback(Gp*PID,1)));
```

```
% PV plot
figure;
plot(PV_step_response_data(:,1), PV_step_response_data(:,2));
xlabel('Time (s)');
ylabel('Theta (rad)');
title('PV Controller Step Response');
annotation('textbox',...
    [0.327785714285714 0.842857142857144 0.200785714285714
0.061904761904763],...
    'String',{'~19% overshoot'},...
    'FitBoxToText','off');
annotation('textbox',...
    [0.499214285714285 0.702380952380956 0.218642857142858
0.061904761904763],...
    'String','~0.2s settling time',...
    'FitBoxToText','off');
annotation('textbox',...
    [0.55 0.803809523809525 0.332142857142857 0.0581812787420278],...
    'String','~0.009 rad steady state error',...
    'FitBoxToText','off');

% PIV plot
figure;
plot(PIV_step_response_data(:,1), PIV_step_response_data(:,2));
xlabel('Time (s)');
ylabel('Theta (rad)');
title('PIV Controller Step Response');
annotation('textbox',...
    [0.380285714285714 0.840476190476191 0.202571428571429
0.0547619047619079],...
    'String',{'~14% overshoot'},...
    'FitBoxToText','off');
annotation('textbox',...
    [0.470642857142857 0.721428571428572 0.222214285714286
0.0523809523809583],...
    'String','~0.2s settling time',...
    'FitBoxToText','off');
annotation('textbox',...
    [0.587 0.82 0.312142857142857 0.0581812787420278],...
    'String','~0.0 rad steady state error',...
    'FitBoxToText','off');

%% Problem 5
```

```
figure;
plot(PDdist_step_response_data(:,1), PDdist_step_response_data(:,2));
hold on;
plot(PIDdist_step_response_data(:,1), PIDdist_step_response_data(:,2));
hold on;
plot(PVdist_step_response_data(:,1), PVdist_step_response_data(:,2));
hold on;
plot(PIVdist_step_response_data(:,1), PIVdist_step_response_data(:,2));
hold on;
legend('PD controller', 'PID controller', 'PV controller', 'PIV
controller');
xlabel('Time (s)');
ylabel('Theta (rad)');
title('Comparison of Controller Step Response with a Disturbance of 5');
```