

UNIVERSITY OF PENNSYLVANIA
The Wharton School

FNCE 100

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PROBLEM SET #2

Fall Term 2005

CAPITAL BUDGETING UNDER CERTAINTY

1. (a) Plot the NPV as a function of the interest rate for the following sequences of cash flows:

| | <u>Sequence A</u> | <u>Sequence B</u> |
|---|-------------------|-------------------|
| 1 | −100 | −100 |
| 2 | 100 | 0 |
| 3 | 200 | 100 |
| 4 | −100 | 200 |

- (b) What is the IRR for each sequence? Show this on your graphs.

2. BICCC is considering whether to replace an existing machine or to spend money on overhauling it. The replacement machine would cost \$9000 now and would require maintenance of \$1000 at the end of every year for 8 years. At the end of 8 years it would have a scrap value of \$2000 and would be sold. The existing machine requires increasing amounts of maintenance each year, and its salvage value is falling as shown below:

| <u>Year</u> | <u>Maintenance</u> | <u>Salvage</u> |
|-------------|--------------------|----------------|
| Now | 0 | 4000 |
| 1 | 1000 | 2500 |
| 2 | 2000 | 1500 |
| 3 | 3000 | 1000 |
| 4 | 2000 | 0 |

(The existing machine can be sold for \$4000 now. If it is sold in 1 year, the price will be \$2500, and \$1000 will have to be spent on maintenance. The machine will last for 4 more years before it falls apart.) If BICCC faces an opportunity cost of capital of 15 percent, when should it replace the machine?

3. Five years ago, trucking magnate Leonard Rose purchased ten trucks for \$15,000 each. He is now considering replacing those trucks with five new models. One new model can

do the work of two of the present trucks. Each new truck will cost \$25,000 and has an expected life of ten years with a \$1,000 salvage value. It is expected that the current trucks have ten years of service left at the end of which they will have no salvage value. If the five new trucks are purchased, Rose expects total operating expenses for the company to decrease by \$10,000 per year. The old trucks could currently be sold for \$3,000 each. Taxes are at a 40% rate. The Rose Company uses straight line depreciation. Rose's costs of capital is 8%. Should he replace the trucks?

4. The H. O. Company is considering purchasing a machine in order to produce a new product. The new product will generate revenues of \$50,000 per year for five years. The cost of materials and labor needed to generate these revenues will total \$30,000 per year and other cash expenses will be \$10,000 per year. Net working capital of \$5,000 will be required immediately and this amount will be freed-up at the end of the fifth year. The machine will cost \$25,000. It will be depreciated using MACRS for the three year asset class. The three year class depreciates the asset over 4 years as follows: 33.33%, 44.45%, 14.81%, and 7.81%. Finally, the firm's tax rate is 35% and its required return is 10%.

Using the net present value method, determine whether investment in the new machine is economically acceptable.

5. Mr. T. Thumb is the president of Microtechnology, Inc. a very small private company. He expects revenues in the forthcoming year of \$20 million and costs (including taxes) of \$15 million. During the subsequent 5 years (i.e., years 2 through 6) Mr. Thumb forecasts that revenues and costs will grow by 25 percent a year, but he anticipates that all profits will need to be plowed back into the business. Thereafter he forecasts that growth will drop to 5 percent a year and that the company will need to plow back only 40 percent of profits. Mr. Thumb has recently been offered \$75 million in cash for his company. Is this a fair offer if the opportunity cost of capital is 12 percent?
6. Mr. Cyrus Clops, the president of Giant Enterprises, has to make a choice between two possible investments:

| Cash Flows, Thousands of Dollars | | | | |
|----------------------------------|-------|-------|-------|----------------|
| Project | C_0 | C_1 | C_2 | IRR Percent |
| A | -400 | +241 | +293 | 21 |
| B | -200 | +131 | +172 | 31 |

The opportunity cost of capital is 9 percent. Mr. Clops is tempted to take B, which has the higher IRR.

- (a) Explain to Mr. Clops why this is not the correct procedure.
 - (b) Show him how to adapt the IRR rule to choose the best project.
 - (c) Show him that this project also has the higher NPV.
7. Brylcream Oil Company's properties are expected to yield 1000 barrels a year forever. After all expenses except taxes, Brylcream can sell this year's production for \$20 per barrel (to be received at the end of the year). This pre-tax net-income is expected to increase at an 8 percent annual rate. Brylcream faces a constant 40 percent income tax rate and does not have access to depletion allowances or other tax shields. Brylcream's opportunity rate is (quite naturally) 10 percent. How much is Brylcream Oil Company worth today? (Show calculations.)
8. Calculate the NPV of the following project given a 0 (yes, zero) percent real discount rate. Show your cash flow calculations.

| | Today ($t = 0$) | Year 1 ($t = 1$) | Year 2 ($t = 2$) |
|---------------------|----------------------|-----------------------|-----------------------|
| Investment | \$40,000 | 0 | 0 |
| Output (# widgets) | 0 | 2,000 | 2,000 |
| Labor input (hours) | 0 | 100 | 100 |
| Cash flow | | | |
| NPV | | | |

The firm faces a 40 percent tax rate and uses straight-line depreciation. The salvage value of the investment will be zero at $t=2$. Labor costs today are \$12 per hour and will increase at 10 percent per year. In real terms, Widgets will sell for 300 each today. This price will remain constant in real terms. The expected annual inflation rate equals 10%.

9. Conocococonut is considering the purchase of a new harvester. They are currently involved in deliberations with the manufacturer and as of yet the parties have not come to a settlement regarding the final purchase price. The management of Conocococonut has hired you, a high-priced consultant, to arrive at the break-even purchase price such that the NPV of the project will be zero. This will be of obvious use to Conocococonut in their negotiation with the capital equipment manufacturer.

FACTS:

- (1) The new harvester is not expected to affect revenues, but operating expenses will be reduced by \$10,000 per year for 10 years.

- (2) The old harvester is now 5 years old with 10 years of its scheduled life remaining. It was purchased for \$45,000 and depreciated on a straight-line basis to zero over 15 years. Its salvage value is expected to be \$5,000 at the end of 15 years.
 - (3) The old harvester has a current market value of \$25,000.
 - (4) The new harvester will be depreciated straight-line over its 10 year life to zero. Its salvage value is expected to be \$10,000.
 - (5) The corporate tax rate is 35%.
 - (6) The firm's cost of capital is 15%.
10. The Pratt Piston Company is considering an investment in a new plant which will entail an immediate capital expenditure of 2000 (\$1000s). The plant is to be depreciated on a straight-line basis over 10 years and there will be no salvage value. Annual operating revenues (before depreciation and taxes) are expected to be 550 for the first 5 years of the plant and to be 400 for the following 5 years. Assuming that the company has a required return of 12 percent after taxes for such an investment and that its marginal tax rate is 40 percent, calculate (a) average return on book, (b) the payback period, (c) NPV, (d) IRR, (e) the profitability index.
11. A machine costs \$3500 and has an expected life of 7 years with no salvage value. Calculate the depreciation in each year, and the present value of the depreciation tax shield at an interest rate of 12 percent under (a) straight-line depreciation & (b) MACRS (7 year class). The corporate tax rate is 35 percent. What difference does the choice of depreciation method make? What difference does it make to your calculations if the asset is depreciated over 5 years instead of 7 years? [Note: Depreciation rates for MACRS 7 year asset class are 14.29%, 24.49%, 17.49%, 12.49%, 8.93%, 8.93%, 8.93%, and 4.45% over eight years. Depreciation rates for MACRS 5 year asset class are 20.00%, 32.00%, 19.20%, 11.52%, 11.52%, and 5.76% over six years.]
12. The following example serves to illustrate the importance of taking anticipated inflation into account explicitly when estimating future cash flows. Too often, there is a tendency not to consider its effect in these estimates. Because anticipated inflation is embodied in the required rate of return, not to take account of it in the cash-flow estimates will result in a biased appraisal of the project and, in turn, the possibility of a less than optimal allocation of capital.
- To illustrate this bias, assume that a project which costs \$100,000 at time 0 was under consideration and was expected to provide cash-flow benefits over the next 5 years. Assume further straight-line depreciation and a corporate tax rate of 50%. Suppose that cash flows were estimated on the basis of price levels at time 0, with no

consideration of the effect of future inflation, and that these estimates were:

END OF YEAR:

| | 1 | 2 | 3 | 4 | 5 |
|----------------------------------|---------------|---------------|---------------|---------------|---------------|
| Expected Cash Inflow | \$30,000 | \$40,000 | \$50,000 | \$50,000 | \$30,000 |
| Expected Cash Outflow | <u>10,000</u> | <u>10,000</u> | <u>10,000</u> | <u>10,000</u> | <u>10,000</u> |
| Incremental Net Operating Income | 20,000 | 30,000 | 40,000 | 40,000 | 20,000 |
| 1-tax rate | <u>.50</u> | <u>.50</u> | <u>.50</u> | <u>.50</u> | <u>.50</u> |
| | 10,000 | 15,000 | 20,000 | 20,000 | 10,000 |
| Depreciation · tax rate | <u>10,000</u> | <u>10,000</u> | <u>10,000</u> | <u>10,000</u> | <u>10,000</u> |
| Net Cash Flow | \$20,000 | \$25,000 | \$30,000 | \$30,000 | \$20,000 |

(a) If the relevant cost of capital is 12%, what is the project's calculated net present value? Should the firm accept the project according to this analysis?

(b) Now recalculate your answer to (a), assuming a 5% inflation rate.

13. The Borstal Company has to choose between two machines which do the same job but have different lives. The two machines have the following costs:

| Year | Machine A | Machine B |
|------|------------------|-----------------|
| 0 | \$40,000 | \$50,000 |
| 1 | 10,000 | 8,000 |
| 2 | 10,000 | 8,000 |
| 3 | 10,000 + replace | 8,000 |
| 4 | ... | 8,000 + replace |

If the discount rate is 12 percent, which machine would Borstal buy?

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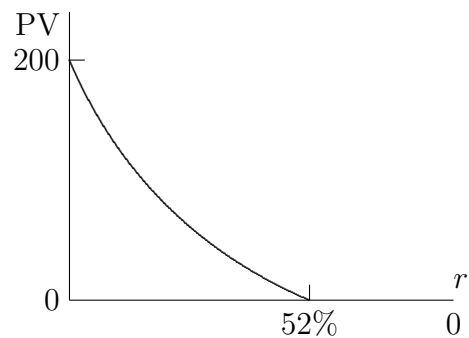
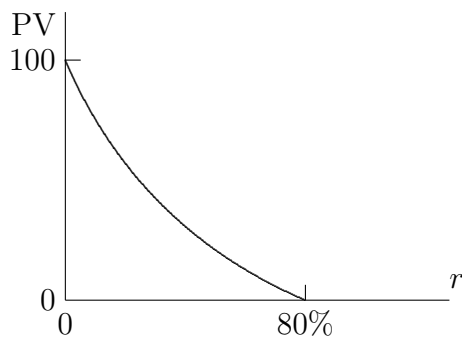
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Solutions to PROBLEM SET #2

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1. (a)



(b) A: $0 = -100 + \frac{100}{1+IRR} + \frac{200}{(1+IRR)^2} - \frac{100}{(1+IRR)^3} \rightarrow IRR = 80\%$.

B: $0 = -100 + \frac{0}{1+IRR} + \frac{100}{(1+IRR)^2} + \frac{200}{(1+IRR)^3} \rightarrow IRR = 52\%$.

2. Equivalent annual cost of replacement machine (EAC):

$$PV_{\text{costs}} = \$9000 + \sum_{t=1}^8 \frac{\$1000}{(1.15)^t} - \frac{2000}{(1.15)^8} =$$

$$\$9000 + 4.487(\$1000) - \$2000(0.3269) = \$12,833$$

$$\text{EAC of new machine} = \frac{PV}{\text{8 year annuity factor @ 15 percent}} = \frac{\$12,833}{4.487} = \$2860$$

Set up keeping machine for 1 year as an alternative and also consider keeping machine for 2, 3, and 4 years as separate alternatives.

Cost of keeping existing machine 1 year:

- The opportunity cost of not selling it now is \$4000.
- Maintenance is \$1000.
- Salvage value is \$2500.

Thus, for the present value of costs we have:

$$PV = \$4000 + \frac{\$1000}{1.15} - \frac{\$2500}{1.15} = \$2696$$

This gives an equivalent annual cost of keeping the machine for 1 year of \$3100 since at $r = 15\%$ the one year annuity factor is 0.8696. One is better off selling it now, since the EAC of a new machine is \$2860.

Cost of keeping existing machine two years:

- The opportunity cost of not selling it now is \$4000.
- Maintenance is \$1000 year 1 and \$2000 year 2.
- Salvage value is \$1500.

Thus, for the present value of costs we have:

$$PV = \$4000 + \left[\frac{\$1000}{1.15} + \frac{\$2000}{1.15^2} \right] - \frac{\$1500}{1.15^2} = \$5248$$

This gives an equivalent annual cost of keeping the machine for 2 years of \$3228 since at $r = 15\%$ the two year annuity factor is 1.626. One is better off selling it now, since the EAC of a new machine is \$2860.

Similar calculations show the equivalent annual cost of keeping the existing machine for 3 and 4 years is also in excess of the EAC of buying a new machine. Thus, *BICCC is best off buying a new machine immediately.*

3. Since the old trucks have 10 years of service left from today, they had an original useful life of 15 years and zero salvage value.

cash flow, $t = 0$:

| | | |
|---------------------|---|-----------------|
| sale of old trucks: | salvage = | 30,000 |
| | tax shelter = $[(10,000 - 3000) \cdot 4]10 =$ | <u>28,000</u> |
| | | 58,000 |
| cost of new trucks | | <u>-125,000</u> |
| | | -67,000 |

cash flows, $t = 1, 10$:

| | |
|--|--------------|
| after tax reduction in expenses $(10,000 \times .6)$ | 6,000 |
| increase in depreciation tax shield | |
| $(12500 \times .4) - (10000 \times .4) =$ | <u>1,000</u> |
| | 7,000 |

cash flow, $t = 10$:

| | |
|--|---------------|
| salvage of new trucks (5×1000) | 5,000 |
| taxes on capital gain ($\$5000 \times 0.40$) | <u>-2,000</u> |
| | 3,000 |

$$\begin{aligned} \text{NPV} &= -67,000 + 7000(10 \text{ yr annuity discount factor @ } 8\%) + \frac{3000}{(1.08)^{10}} \\ &= -67,000 + 46970 + 1390 = -\$18,640 \end{aligned}$$

→ KEEP OLD TRUCKS

4. cash flow, $t = 0$:

| | |
|-------------------------------|---------------|
| initial investment | -25,000 |
| investment in working capital | <u>-5,000</u> |
| | -30,000 |

cash flows, $t = 1, 5$:

| | |
|---|-------|
| after tax net operating cash flows all years ($50,000 - 30,000 - 10,000$).65 | 6,500 |
| Depreciation tax shield | |
| $t = 1: 0.3333 \times 25,000 \times .35 =$ | 2,916 |
| $t = 2: 0.4445 \times 25,000 \times .35 =$ | 3,889 |
| $t = 3: 0.1481 \times 25,000 \times .35 =$ | 1,296 |
| $t = 4: 0.0741 \times 25,000 \times .35 =$ | 648 |

cash flow, $t = 5$:

| | |
|--|-------|
| recovery of working capital investment = | 5,000 |
|--|-------|

$$\begin{aligned} \text{NPV} &= -30,000 + 6,500 (5 \text{ yr annuity discount factor @ } 10\%) \\ &\quad + \frac{2916}{(1.10)} + \frac{3889}{(1.10)^2} + \frac{1296}{(1.10)^3} + \frac{648}{(1.10)^4} + \frac{5000}{(1.10)^5} = \$5,027 \end{aligned}$$

→ purchase the machine

5. PV of payments to shareholders

$$\text{PV} = \frac{5m}{1.12} + \frac{\left[\frac{(5m)(1.25)^5(1.05)(.6)}{.12-.05} \right]}{(1.12)^6} = \$74.04 \text{ million}$$

\$75m is higher than the fair value so the offer is more than fair.

6. (a) This procedure is incorrect because the IRR does not take into account the relative scale of mutually exclusive projects. Because of this, a large scale project that may contribute more to firm value will be rejected due to its high initial investment. The proper way to evaluate the projects, if IRR must be used, is to calculate the value of the incremental project (i.e., $C = A - B$), to see if the additional investment necessary for A is worth undertaking. If C is worth undertaking (i.e. $IRR > 9\%$) and B is worth undertaking, then, since $A = C + B$, A is the project which should be chosen.

(b) IRR of the increment:

$$A: \begin{array}{ccc} \frac{C_0}{-200} & \frac{C_1}{110} & \frac{C_2}{121} \end{array}$$

$$\text{solve for IRR: } -200 + \frac{110}{(1+IRR)} + \frac{121}{(1+IRR)^2} = 0$$

IRR = 10% > 9%; accept A

$$\begin{aligned} \text{(c) } NPV_A &= -400 + (241)(.9174) + (293)(.8417) = 67.71 \\ NPV_B &= -200 + (131)(.9174) + (172)(.8417) = 64.9518 \end{aligned}$$

$$\underline{A > B}$$

$$7. PV = \frac{C_1}{r - g} = \frac{1000 \cdot \$20(1 - 0.4)}{0.10 - 0.08} = \frac{\$12000}{0.02} = \$600,000$$

8. The cash flow to a company is the sum of after-tax cash flows from operations plus the cash resulting from depreciation tax shield, i.e., $(\Delta S - \Delta OC)(1 - \tau) + \tau \text{ DEP}$. From the data given in the problem the first component is easily put in real terms. The second must be deflated:

$$\text{year 1: } ((2000)(300) - (100)(12))(.6) + \frac{(.4)(20,000)}{1.10} = 366,553$$

$$\text{year 2: } ((2000)(300) - (100)(12))(.6) + \frac{.4(20,000)}{(1.10)^2} = 365,892$$

$$NPV = -40,000 + 366,553 + 365,892 = \$692,445$$

9. Calculate incremental cash flows for new harvester:

Time 0 cash flows:

$$\text{(a) Payment to manufacturer} = -X$$

$$\text{(b) Sale of old} = 25,000$$

$$\text{(c) Book value old: } 45,000 - 5(3000) = 30,000$$

$$\text{Cap. Gain} = \text{Market Value} - \text{Book Value} = 25,000 - 30,000 = -5,000$$

$$\text{Tax savings} = (5,000)(.35) = 1750$$

Cash Flow Increment years 1 to 10:

$$\begin{aligned} & (\Delta S - \Delta OC)(1 - \tau) + \tau \Delta \text{DEP} \\ & = (10,000)(.65) + .35 \left[\frac{X}{10} - 3000 \right] = .035X + 5450 \end{aligned}$$

Salvage values at time 10 (Book values are 0)

| | |
|-------------------------------------|---------------|
| after tax flow from not selling old | -3,250 |
| after tax flow from selling new | +6,500 |
| incremental flow | <u>+3,250</u> |

Set NPV Equal to Zero and Find X:

$$\begin{aligned} -X + 25,000 + 1,750 + \sum_{t=1}^{10} \frac{.035X + 5450}{(1.15)^t} + \frac{3250}{(1.15)^{10}} &= 0 \\ -X + 26,750 + 0.035X \sum_{t=1}^{10} \frac{1}{(1.15)^t} + \sum_{t=1}^{10} \frac{5450}{(1.15)^t} + 803 &= 0 \\ -0.824X + 54,906 &= 0 \end{aligned}$$

$$X = \$66,605$$

10. (a) Average Return on Book:

| | |
|-----------------|-----------------|
| NI 1-5 | NI 6-10 |
| 550,000 | 400,000 |
| <u>-200,000</u> | <u>-200,000</u> |
| 350,000 | 200,000 |
| <u>-140,000</u> | <u>-80,000</u> |
| 210,000 | 120,000 |

$$\text{A.R.B.} = \frac{\frac{(5)(210,000) + 5(120,000)}{10}}{\frac{\sum_{t=0}^{10} 2,000,000 - 200,000 \cdot (t)}{11}} = 16.5\%$$

- (b) 4.88 years
- (c) \$132,501.23
- (d) 13.71%

(e) 1.066

11.

| Year | S.L.D. | | | | MACRS | | | |
|------------|-------------|------|-------------|------|-------------|------|-------------|------|
| | 7 yrs. | T.S. | 5 yrs. | T.S. | 7 yrs. | T.S. | 5 yrs. | T.S. |
| 1 | 500 | 175 | 700 | 245 | 500 | 175 | 700 | 245 |
| 2 | 500 | 175 | 700 | 245 | 857 | 300 | 1120 | 392 |
| 3 | 500 | 175 | 700 | 245 | 612 | 214 | 672 | 235 |
| 4 | 500 | 175 | 700 | 245 | 437 | 153 | 403 | 141 |
| 5 | 500 | 175 | 700 | 245 | 313 | 110 | 403 | 141 |
| 6 | 500 | 175 | | | 313 | 110 | 202 | 71 |
| 7 | 500 | 175 | | | 313 | 110 | | |
| 8 | | | | | 155 | 54 | | |
| | <u>3500</u> | | <u>3500</u> | | <u>3500</u> | | <u>3500</u> | |
| PV at 12%: | | | | | | | | |
| | | 799 | | 883 | | 834 | | 904 |

12. (a)
$$\begin{aligned} \text{NPV} &= -100,000 + \frac{20,000}{(1.12)} + \frac{25,000}{(1.12)^2} + \frac{30,000}{(1.12)^3} + \frac{30,000}{(1.12)^4} + \frac{20,000}{(1.12)^5} \\ &= -100,000 + 17,860 + 19,930 + 21,350 + 19,070 + 11,350 \\ &= -10,440 \quad \text{Reject.} \end{aligned}$$

(b)

| | 1 | 2 | 3 | 4 | 5 |
|--|-------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| (Inc.NOI)(1 - τ)(1 + π) ^{yr} | 10,500 | 16,538 | 23,153 | 24,310 | 12,763 |
| +(DEP)(τ) | <u>+10,000</u> | <u>+10,000</u> | <u>+10,000</u> | <u>+10,000</u> | <u>+10,000</u> |
| | 20,500 | 26,538 | 33,153 | 34,310 | 22,763 |
| | <u>$\div 1.12$</u> | <u>$\div (1.12)^2$</u> | <u>$\div (1.12)^3$</u> | <u>$\div (1.12)^4$</u> | <u>$\div (1.12)^5$</u> |

$$\begin{aligned} \text{NPV} &= -100,000 + 18,304 + 21,156 + 23,598 + 21,805 + 12,916 \\ &= -2,221 \end{aligned}$$

Still reject, but project looks much better than under (a).

13. Use the method of equivalent annual cost (EAC).

Find the amount that, if paid in each year (starting in year 1), would have the same present value as the actual stream of costs.

Machine A:

$$40,000 + \frac{10,000}{1+r} + \frac{10,000}{(1+r)^2} + \frac{10,000}{(1+r)^3} = \frac{EAC_A}{(1+r)} + \frac{EAC_A}{(1+r)^2} + \frac{EAC_A}{(1+r)^3}$$

$$\begin{aligned} r = .12 &\longrightarrow 64,018 = EAC_A(2.4018) \\ &\longrightarrow EAC_A = 26,654 \end{aligned}$$

Machine B:

$$50,000 + \frac{8,000}{1+r} + \frac{8,000}{(1+r)^2} + \frac{8,000}{(1+r)^3} + \frac{8,000}{(1+r)^4} = \frac{EAC_B}{1+r} + \frac{EAC_B}{(1+r)^2} + \frac{EAC_B}{(1+r)^3} + \frac{EAC_B}{(1+r)^4}$$

$$\begin{aligned} r = .12 &\longrightarrow 74,299 = EAC_B(3.0373) \\ &\longrightarrow EAC_B = 24,462 \end{aligned}$$

Borstal should buy machine B because it has a lower EAC.

Note: The PV of the infinite stream of costs is:

$$\begin{aligned} PV_A &= \frac{EAC_A}{r} = \frac{26,654}{.12} = 222,117 \\ PV_B &= \frac{EAC_B}{r} = \frac{24,462}{.12} = 203,850. \end{aligned}$$