Formulas in Longstaff & Schwartz 1995 for Risky Debt

Source:
Longstaff, Francis A., Schwartz, Eduardo S. A simple approach to Valuing Risky Fixed and Floating Rate Debt. Journal of Finance. Vol 3. July 1995.
Vasicek, O. 1977 "An Equilibrium Characterization of the term structure." Journal of Financial Economics 5: 177-188.

Firm asset value process:

$$dV = \mu V dt + \sigma V dZ_1$$

Interest rate process:

$$dr = (\zeta - \beta r)dt + \eta dZ_2$$

with parameters
V: firm asset value
μ: drift rate of asset process
r: short-term riskless interest rate
σ: instanteaneous stdev of asset process (constant)
ζ: long-term equilibrium of mean reverting process (constant)
β: "pull-back" factor - speed of adjustment (constant)
η: spot rate volatility (constant)
$dZ_{1,2}$ standard Wiener processes with correlation ρdt
K: Bankruptcy threshold - financial distress if V falls below K
X: ratio of V/K - firm asset value as % of bankruptcy threshold
α: ζ plus constant (c) to represent market price of risk
T: time to maturity
w: writedown = 1 - recovery rate

Value of risky discount bond:

$$P(X,r,T) = D(r,T) - wD(r,T)Q(X,r,T)$$

D(r,T) is the value of riskfree (no credit risk) discount bond according to Vasicek (1977).

$$D(r,T) = e^{A(T) - B(T)r}$$

with

$$A(T) = \left(\frac{\eta^2}{2\beta^2} - \frac{\alpha}{\beta}\right)T + \left(\frac{\eta^2}{\beta^3} - \frac{\alpha}{\beta^2}\right)\left(e^{-\beta T} - 1\right) - \left(\frac{\eta^2}{4\beta^3}\right)\left(e^{-2\beta T} - 1\right)$$

$$B(T) = \frac{1 - e^{-\beta T}}{\alpha}$$

Q(X,r,T) term can be interpreted as probability - under risk neutral measure - that default occurs.

$$Q(X,r,T,n) = \sum_{i=1}^{n} q_i$$

$$q_1 = N(a_1)$$
$$q_i = N(a_1) - \sum_{j=1}^{i-1} q_j N(b_{ij})$$

N(.) denotes the cumulative standard normal distribution

with

$$a_i = \frac{-\ln X - M(iT/n, T)}{\sqrt{S(iT/n)}} \qquad b_{ij} = \frac{M(jT/n, T) - M(iT/n, T)}{\sqrt{S(iT/n) - S(jT/n)}}$$

$$\begin{aligned} M(t,T) = {} & \left(\frac{\alpha - \rho\sigma\eta}{\beta} - \frac{\eta^2}{\beta^2} - \frac{\sigma^2}{2}\right)t \\ & + \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{\eta^2}{2\beta^3}\right)\exp(-\beta T)(\exp(\beta t) - 1) \\ & + \left(\frac{r}{\beta} - \frac{\alpha}{\beta^2} + \frac{\eta^2}{\beta^3}\right)(1 - \exp(-\beta t)) \\ & - \left(\frac{\eta^2}{2\beta^3}\right)\exp(-\beta T)(1 - \exp(-\beta t)) \end{aligned}$$

$$\begin{aligned} S(T) = {} & \left(\frac{\rho\sigma\eta}{\beta} + \frac{\eta^2}{\beta^2} + \sigma^2\right)t \\ & - \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{2\eta^2}{\beta^3}\right)(1 - \exp(-\beta t)) \\ & + \left(\frac{\eta^2}{2\beta^3}\right)(1 - \exp(-2\beta t)) \end{aligned}$$