Assignment 1: Answers

(March 24, 1999)

1. *Calibration to DM rates.*

   (a) Properties of DM rates:

Maturity	Mean	Std Dev
1	3.503	0.437
24	4.154	0.433
36	4.609	0.454
60	5.377	0.519
120	6.388	0.663

   (b) You might choose parameters like this: (i) $\varphi = 0.92$. In the data, the autocorrelations are a little less than this. (ii) $\theta = 3.503/1200 = 0.00292$ (the mean short rate). (iii) $\sigma = (1 - \varphi^2)^{1/2}(0.437/1200)$ (the standard deviation of the short rate). This is a lot smaller than we saw for the US. The big issue here is the short sample. With a highly persistent series, you need a lot of data to get reliable estimates, and this isn't enough. (iv) $\lambda = -1.50$ hits the 10-year rate. See graph below.

   (c) The long rates here are swap rates, which are par yields. They solve (for a seminannual swap and a monthly model)

$$\text{Swap Rate} \;=\; 200 \times \frac{b^6 + b^{12} + \cdots + b^n}{1 - b^n},$$

   where n is the maturity of the swap in months. For the model, we could compute discount factors b^j for the mean value of the state variable (namely, θ) and compute the value of the swap rate associated with it. The result is shown in the figure (o's), where we see that they're not much different from the spot rates. What we have here is two off-setting errors: semiannual compounding raises the rates and swap rates (which mix short maturities with long ones) lower them. The result would have little effect on our calibration one way or the other.

2. *Vasicek for long rates.*

   (a) This follows similar principles, but is a little harder since we can't find the parameters one at a time. (i) $\varphi = 0.978$, the autocorrelation of the 5-year rate. (ii) The variance of the 5-year rate in the data is $(2.212/1200)^2$. In the model, it's

$$(B_{60}/60)^2\sigma^2/(1-\varphi^2) \;=\; \left(\frac{1-\varphi^{60}}{1-\varphi}\right)^2 \sigma^2/(1-\varphi^2).$$

Equating the two gives us $\sigma = 0.0159$. (iii,iv) We need to pick λ and θ together. $\lambda = -0.055$ and $\theta = 0.00623$ comes close. But as before, there is too little curvature, so short rates are too high.

(b) This is a little terse, but here it goes: volatility of spot rates in this model declines with maturity – see below. With the given choices of φ, the rate of decline is greater than we see in the data. For this reason and others, many experts think a model should have a second factor that is very persistent. Even many one-factor models build in the presumption that φ is one, Ho-Lee and Black-Derman-Toy being prominent examples.

Figures follow: