

Advanced Fixed Income Analytics for Professionals

Leonard N. Stern School of Business
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What Is This Course About?

Quantitative models play an increasingly important role in the financial services industry — in valuation, trading, structuring, and risk management. In this course, you will learn how such models are built and used, put them to work yourself using proprietary software, develop insights into their strengths and weaknesses, and hear from experts about industry best practice.

Our approach to modeling introduces state-of-the-art structure with relatively modest technical requirements. Make no mistake: this is a quantitative course. But the level of mathematics rarely rises as high as high school calculus. Our approach includes discrete time and continuous states. In our view, this is both a marked improvement in accuracy over the binomial models typically used in MBA classes and a substantial gain in simplicity over the stochastic calculus of high-end financial theory.

Prerequisite: B40.3333 Debt Instruments and Markets or the equivalent expertise from other sources. A good introduction is chapter 7 of the manuscript from Debt Instruments and Markets; see <http://www.stern.nyu.edu/~dbackus/dbtclass.htm>.

Meeting Times

We meet once a week for 7 weeks: Wednesdays from 5:30 to 8:20pm, March 10 to April 28 excluding March 17 (spring break).

Grades

Grades will be based on the best 4 of 6 weekly assignments. Students who make an honest effort to do 4 or more assignments will get at least a B.

Home Page

Most of what you need for the course will be on the course home page:

<http://www.stern.nyu.edu/~dbackus/3176>

Text files are pdf format, which you can view and print with Adobe's Acrobat Reader (available free if you don't have it already).

Class Materials

The essential material for the course will be weekly lecture notes (distributed in class and posted on the Web) and a collection of readings (distributed in class). We ordered several books, but suggest that you delay buying any of them until a pressing need arises (if ever):

- Babbel and Merrill, *Valuation of Interest-Sensitive Financial Instruments*, Society of Actuaries Monograph M-FI96-1, 1996; a nice book, comprehensive and not overly technical.
- Chriss, *Black-Scholes and Beyond*, McGraw Hill, 1997; nice review of option theory by a Goldman manager who teaches at Courant, very clear logical structure, not overly technical, not particularly concerned with fixed income.
- Hull, *Options, Futures, and Other Derivative Securities (Third Edition)*, Prentice-Hall, 1997; a standard reference, popular and comprehensive but, in our view, not as clear as it might be.
- Haug, *Complete Guide to Option Pricing Formulas*, McGraw-Hill, 1998; useful reference to option pricing formulas.
- Rebonato, *Interest-Rate Option Models (Second Edition)*, Wiley, 1998; more mathematical than necessary, but covers a lot of ground if you don't mind a physicist's idea of intuition.

Office Hours

David Backus: Wednesday, 4:00-5:30pm, and by appointment, Kaufman Education Center 11-55, (212) 998-0907 and dbackus@stern.nyu.edu.

Stanley Zin: Wednesday, 4:00-5:30pm, and by appointment, Kaufman Education Center 9-58, (212) 998-0722 and szin@stern.nyu.edu.

Most Tuesdays we will be at The Apple on Waverly, between Greene and Mercer, starting about 7:30pm. Call or email to verify.

Operating Procedures

- We will start and end class on time and typically take a short intermission.
- Everyone can and will be asked questions in class.
- Assignments will be handed out each class for the next class. They can be done in groups of up to four.
- The syllabus is a forecast: we will modify it (and the assignments) to suit student interests. A new version will be posted each week.
- We are readily available by email. You'll often get a reply within an hour or two, and almost always within a business day. For best service, choose one of us at a time.
- Assignments are due at the start of class. When appropriate, answers will be distributed at the same time. Late assignments will not be accepted without prior arrangement. If necessary, you can fax them to Backus at (212) 995-4212.
- Readings are optional. The lecture notes are intended to be self-sufficient. The readings are intended to point you in the right direction should you decide to pursue any of the topics in more depth.
- If we can't answer a question in class, we will do our best to answer it in the next class.
- Material handed out in class will not be available in the next class. You can get copies of old handouts from classmates or the home page.

Schedule of Classes (subject to change)

Class 1 (March 10): What's a Model (Vasicek)?

- Uses of models
- Spot rates and their properties
- Fundamental theorem of arbitrage-free pricing
- Vasicek model: solution and properties
- Vasicek model: parameter values (“calibration”)
- Application to hedging: Vasicek v. duration
- Model assessment: where are the bodies buried?
- Assignment:
 - calibration of Vasicek model to DM interest rate data
 - calibration to 5-year spot rate
- Reading:
 - Backus, Foresi, and Telmer, “Discrete-time models of bond pricing,” Sections 2-4, reading package and home page; summary, in more technical terms than this course, of a variety of popular models expressed in a common framework.
 - Derman, “Valuing models and modeling value,” *Journal of Portfolio Management* (Spring 1996), 106-114, reading package; nice nontechnical overview of the role of models in industry.
 - Harvey, *Time Series Models (Second Edition)*, MIT Press, 1993; nice book on time series models for those who want a more advanced treatment of the methods used in the notes.

Classes 2 & 3 (March 24, April 1): Options and Volatility

- Two approaches to valuation
- The Black-Scholes formula
- The term structure of volatility
- Extended Vasicek
- Options on zeros

- Options on eurocurrency futures
- Caplets and caps
- Swap essentials
- Swaptions
- Normal and log-normal
- Delta hedging
- Assignments:
 - volatility smile for options on eurodollar futures
 - disaster strikes NatWest (1997)
- Reading:
 - Brenner and Subrahmanyam, “A simple formula to compute the implied standard deviation,” *Financial Analysts Journal*, September-October 1988, 80-83, and “A simple approach to option valuation and hedging in the Black-Scholes model,” *Financial Analysts Journal*, March-April 1994, 25-28; approximations for implied volatility, delta, etc, that aid computation and intuition both.
 - Hull, *Options, Futures, and Other Derivative Securities (Third Edition)*, Prentice-Hall, 1997, chs 10 and 11; a good overview of the basic option model.
 - Jamshidian, “An exact bond option pricing formula,” *Journal of Finance* 44 (March 1989), 205-209; nice treatment, in continuous time, of option prices for coupon bonds in a Vasicek setting.

Class 4 (April 1): Binomial Models

- Short rate trees
- Two approaches to valuation: recursive and Duffie’s formula
- Calibration to current spot rates
- Replication and hedging
- Applications
- Common variants: Ho and Lee, Black-Derman-Toy, Stapleton-Subrahmanyam
- Model assessment
- Assignment:

- calibration of Black-Derman-Toy model
- valuation of interest rate caps and implied volatilities
- Reading:
 - Backus, “Introduction to state-contingent claims,” in *Manuscript: Debt Instruments and Markets*, home page and reading package; notes for Debt Instruments.
 - Black, Derman, and Toy, “A one-factor model of interest rates and its application to treasury bond options,” *Financial Analysts Journal*, January-February 1990, 33-39, reading package; a classic that introduces a workhorse model in industry.
 - Hull, *Options, Futures, and Other Derivative Securities (Third Edition)*, Prentice-Hall, 1997, ch 15; a good reference to fixed income models, despite excessive eclecticism.

Class 5 (April 14): Comparison of Vasicek and Binomial Models

- Pricing kernels and risk-neutral probabilities
- Comparison of (extended) Vasicek and binomial models
- Where’s λ ?
- Options on zeros
- Exploiting mistakes in binomial models
- Assignment: case tba
- Reading:
 - Backus, Foresi, and Zin, “Arbitrage opportunities in arbitrage-free models of bond pricing,” *Journal of Economic and Business Statistics*, January 1998, home page and reading package; an assessment, in moderately technical language, of the role of the mean reversion parameter in option pricing (mistakes you could make with BDT).
 - Backus, Foresi, and Telmer, “Discrete-time models of bond pricing,” Sections 5-6, reading package and home page; the basis, in more technical language, of much of the lecture.

Class 6 (April 21): Legendary Mistakes

- Case tba.
- Industry visitor: models and reality

Class 7 (April 28): Hull and White

- Mean reversion in trees
- Comparison to Vasicek
- More complex lattice models
- Assignment tba
- Reading:
 - Hull and White, “Numerical procedures for implementing term structure models I: Single-factor models,” *Journal of Derivatives*, Fall 1994, reading package; introduction to their popular “trinomial” implementation of the Vasicek model.
 - Hull and White, “Using Hull-White interest rate trees,” *Journal of Derivatives*, Spring 1996, reading package; continuation with comments and extensions.

Class 7 (April 28): Bells and Whistles

- Multifactor models
- CMS swaps
- Volatility smiles and their interpretation
- “Jumps” and stochastic volatility
- Implications for options
- Delta-hedging revisited
- Volatility trades
- Reading:
 - Backus, Foresi, and Telmer, “Discrete-time models of bond pricing,” Sections 7 and 8; reading package and home page.
 - Backus, Foresi, Li, and Wu, “Accounting for biases in Black-Scholes,” Sections 1-4, home page and reading package; a moderately technical approach to volatility smiles leading to a very simple result.

Assignment 1

Due Wednesday March 24 at the start of class.

1. *Calibration to DM rates.* The goal is to choose parameters for the Vasicek model that reproduce the properties of interest rates implied by DM swaps.
 - (a) Use the DM rates supplied on the home page (the spreadsheet for Lecture 1) to compute the mean and standard deviation of DM rates for maturities of 1 month and 2, 3, 5, and 10 years. Plot the means against maturity.
 - (b) Suppose the autocorrelation of the 1-month rate is 0.92. Choose the parameters of the Vasicek model to reproduce the mean, standard deviation, and autocorrelation of the 1-month rate and the mean 10-year rate.
 - (c) (optional) In (b) we intended for you to regard the rates as continuously-compounded spot rates, just like those we used in class. In fact they were swap rates. Discuss how you might adapt your calibration to take this into account.
2. *Vasicek for long rates.* The classic approach to fixed income modeling starts with the short rate. We could with equal justification choose parameters to reproduce the properties of (say) the 5-year rate. Using the US treasury rates described in class:
 - (a) Choose the parameters of the model to reproduce the mean, standard deviation, and autocorrelation of the 5-year rate, and the mean of the 10-year rate.
 - (b) Compare the parameters to those estimated in class. For both sets of parameters, plot $\text{Var}(y^n)$ versus n . Comment.(Warning: this is harder than it looks. Give it a try but don't get bogged down if it doesn't seem to be working out.)

Assignment 2

Due Wednesday March 31 at the start of class.

Eurodollar volatility smile. We have focussed on the term structure of volatility: differences in volatility for options of different maturities. Another dimension of practical interest is “moneyness”: differences in implied volatility across strike prices for otherwise similar options.

Your mission is to explore this issue with eurodollar futures options. The following prices were reported by Bloomberg on March 16 for June options on the June contract:

Strike	Call Price	Put Price
94.50	(na)	0.0075
94.63	(na)	0.0150
94.75	0.2225	0.0225
94.88	0.1125	0.0375
95.00	0.0425	0.0925
95.13	0.0225	(na)
95.25	0.0125	(na)
95.38	0.0075	(na)
95.50	0.0075	(na)

For all options, the futures price is 94.955 and the 3-month discount factor is 0.9876.

- (a) Compute implied *yield* volatility for each of the put options. When you do this, remember to convert the put on the futures to a call on the yield.
- (b) For the put at 95.00, suppose the 3-month yield is 4%, rather than 5%, implying a discount factor of $b = 0.9900$, rather than $b = 0.9876$. How much difference does this make to your estimated volatility?
- (c) Compute implied *yield* volatility for each of the call options. Remember to use put-call parity to find the price of a call on the yield.
- (d) Compare the volatilities of at-the-money and out-of-the-money calls. If the call at 95.5 had the same volatility as the call at 95, what would its price be?
- (e) Graph implied volatility against the strike price, using different symbols to denote calls and puts. How would you interpret the evidence?

Assignment 3

Due Wednesday April 7 at the start of class.

Disaster at NatWest (adapted from Subrahmanyam and Richardson). As derivatives markets develop, we see periodic signs of strain on firms' ability to value complex instruments and manage the associated risks. NatWest's 1997 loss of £90mm on fixed income options is a case in point. The attached articles summarize the public information surrounding this debacle.

Your job is to write a 2-4 page essay touching on at least 3 of the following issues:

- (a) How did NatWest lose this much money?
- (b) How was the loss concealed for 3 years?
- (c) Who was to blame, and why?
- (d) Was fraud involved?
- (e) What specific problems related to the valuation and hedging of options are highlighted by this case?
- (f) What lessons would you draw for risk management and control?

Assignment 4

Due Wednesday April 7 at the start of class.

Interest rate caps in the BDT model. The Black-Derman-Toy model is an industry standard for fixed income valuation — not the most sophisticated model in use, but a widely-used benchmark. Your mission is to calibrate the BDT model to market conditions and use it to value interest rate caps of different maturities. Current market conditions include (this is adapted from the quote sheet we handed out):

Maturity	Spot Rate	Cap Rate	Cap Price
0.5	4.989	(na)	(na)
1.0	5.129	5.22	0.14
1.5	5.209	(na)	(na)
2.0	5.294	5.37	0.55

- (a) With a time interval of $h = 0.5$ years and a volatility term structure of (10, 12, 13, 14) (percent!), construct a 4-period BDT short rate tree consistent with the spot rates above. What are the state prices?
- (b) Compute $Y = 6$ -month LIBOR for each node in the interest rate tree.
- (c) A semi-annual interest rate cap generates cash flows of

$$(1 + Y/200)^{-1}(Y - K)^+/2$$

each period after the current one for every hundred dollars notional principal. For $K = 5.22$, calculate the cash flows generated by a one-year interest rate cap.

- (d) For $K = 5.37$, calculate the cash flows for a two-year interest rate cap.
- (e) (optional) Given the cap prices above, how would you adjust the volatility parameters?

Assignment 5

Due Wednesday April 28 at the start of class.

Cancellable swaps. Consider the version of the Black-Derman-Toy model used in class (eg, p 4-6 of Lecture 4). Your mission is to compute the value of a 1-year cancellation option on a 2.5-year swap: the ability to cancel the swap at par (namely, zero) at any date in the first year of the swap. You might recall from the lecture that the discount factors and swap rates on which the model is based are:

Period (j)	Disc Factor	Swap Rate
1	0.975365	5.052
2	0.949999	5.194
3	0.924837	5.274
4	0.899541	5.358
5	0.874550	5.426
6	0.849939	5.483

- (a) Consider the swap as (say) a long position in a fixed rate bond and a short position in a floating rate note. What are the bond's cash flows? Compute its value for every node in the tree.
- (b) If a floating rate note is worth 100, what is the value of the swap in each node of the tree?
- (c) The swap can be cancelled free at any time during the first year. What cash flows are generated by cancelling?
- (d) Apply the logic of American options to value the option. What premium should be charged for the cancellation option?