

Writing time: 9⁰⁰ – 1200

Justify all solutions chosen. Submit everything you come up with, including partial solutions.

ATTENTION! Solve each task on a separate sheet! Write legibly!

Each solution gives 0 – 7 points.

Good luck!

1. Before the HMT final last year, Erland, Rebecca and Olov guessed who the easiest and the most difficult problems would be. They listed the problems starting with what they guessed would be the easiest to the most difficult:

	Lättast 3					Svårast
Erland		2	4	1	5	6
Rebecca	2	1	5	3	6	4
Fire	4	2	5	3	6	1

It turned out that in every placement at least one of them was right, and they got them all exactly the same number of rights.

- How many correctly placed problems did each of them have?
- List the problems from easiest to hardest.

2. Let a and b be positive numbers such that $a^2 + b^2 = 1$. Show that

$$3 \leq \frac{a^2}{1+b} + \frac{b^2}{1+a} \leq 4$$

3. A square ABCD has a side of 12 cm. From B, a line is drawn to the midpoint of the side AD. From D, a line is drawn to the midpoint of side AB. These two lines intersect at the point M. Through M a line is drawn parallel to the diagonal BD. This line intersects the side AB at point E. How long is the distance AE?

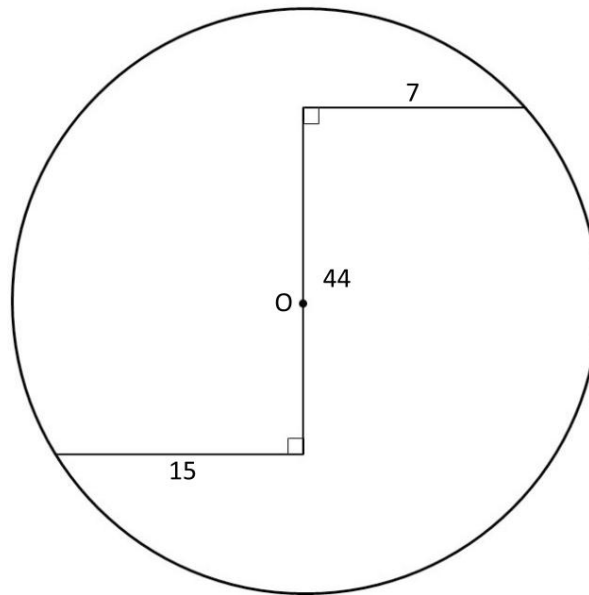
4. Aunt Ester loves parties, she almost always has guests. She has invited some guests (including four priests from Manchester). At the party, all guests play several rounds of a game. After each round, Aunt Ester hands out prizes. Whoever came last in the round gets one coin, the second last gets two coins, etc., so that each placement gives one coin more than the above.

When they have finished playing all the rounds, Aunt Ester has just run out of all her 2024 coin.

How many guests did Aunt Ester have?

Have a good time!

5. The circle below is divided into two parts by three straight sections of given lengths, where one of the sections goes through the center point O. What is the diameter of the circle?



6. A grid of 2024×2024 squares is randomly filled with integers between -2023 and 2023 . A move consists of choosing a row or a column and then multiplying all the numbers in it by ± 1 , and you can make as many moves as you like. Show that it is always possible to finally get a grid such that none of the grid's row or column sums are negative, regardless of how the grid looked from the beginning the orja.