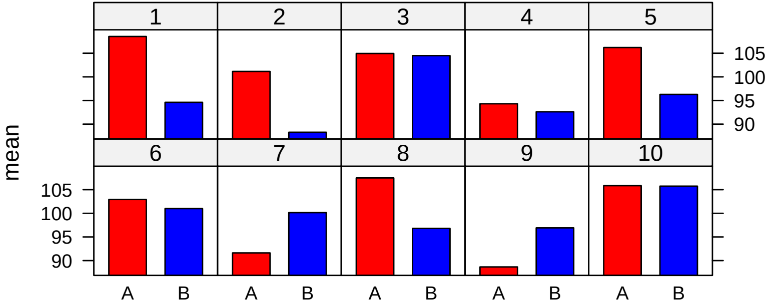
Is the effect real, or due to chance?

observation = truth + error

- Sampling/measurement introduces an element of *chance* into our observations
  - Patients who receive cognitive therapy showed 10% greater improvement than patients in a control group.
  - For each 5 additional hrs spent playing video games per week, 1.3 more aggressive incidents toward peers.
  - In a psych experiment, people remembered twice as many words when the words appeared in a sentence as when they were presented in isolation.

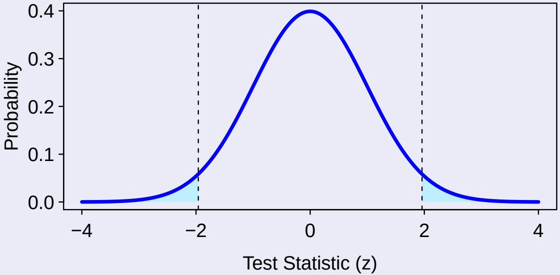
Simulated data, population known

- $\mu = 100$, $\sigma = 20$
- 10 samples, each with 20 participants
  - arbitrarily assign 10 to *A*, 10 to *B*



Null hypotheses and p-values

Sampling distribution



Definition (p-value)

assuming H_0 to be true, the probability of obtaining a test statistic at least as extreme as the one obtained

Basic probability

The probability of an event X is denoted by:
$P(X) = p$, where $0 \leq p \leq 1$.
$P(\sim X) = 1 - p$.

Another way to think about probability is in terms of the ratio:

$$\frac{\text{frequency of an event}}{\text{total number of possible outcomes}}$$

- Probability of rolling a 1 on a single six-side die: $\frac{1}{6}$ or .167.
- Probability that the number rolled is larger than 1: $\frac{5}{6}$ or .833.
- Probability that a card drawn from a deck of 52 is a 10: $\frac{4}{52}$ or .077.
- Probability that the card is NOT a 10 is 1-.077 or .923.

Joint probability

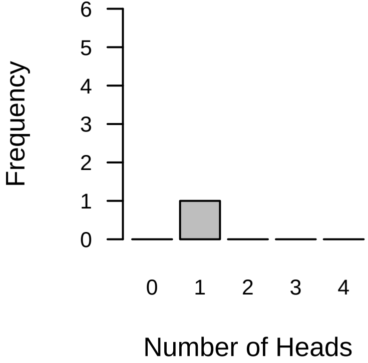
The probability of getting heads on a single coin flip is $\frac{1}{2}$ or .5. But what about getting X heads on four flips? (the criterion)

Empirical Strategy: Repeat the game N_R times, count no. of heads, and construct a distribution over all replications. Get *approximate* probability by dividing the number of games meeting the criterion by the total number of replications.

Analytical Strategy: Enumerate all of the possible outcomes of four coin flips, and then count the number of them that meet the criterion. The ratio of these two numbers is the *exact* probability.

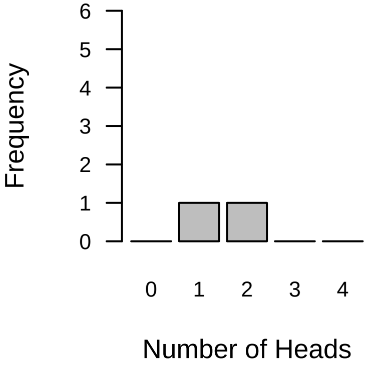
Empirical strategy: Flip, count, replicate

Game	Outcome	NHeads
1	TTHT	1



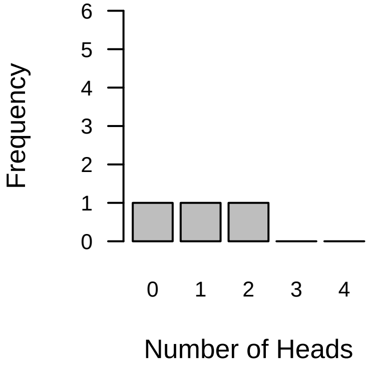
Empirical strategy: Flip, count, replicate

Game	Outcome	NHeads
1	TTHT	1
2	HHTT	2



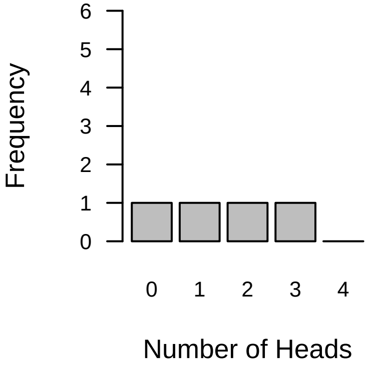
Empirical strategy: Flip, count, replicate

Game	Outcome	NHeads
1	TTHT	1
2	HHTT	2
3	TTTT	0



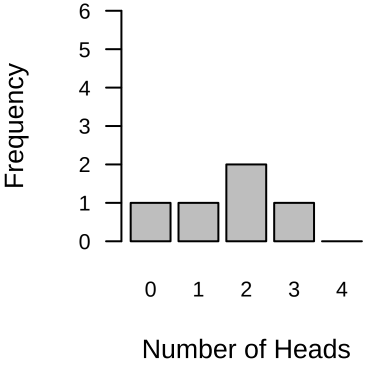
Empirical strategy: Flip, count, replicate

Game	Outcome	NHeads
1	TTHT	1
2	HHTT	2
3	TTTT	0
4	THHH	3



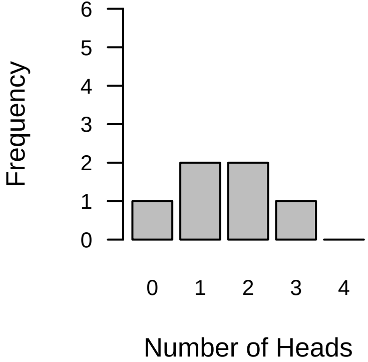
Empirical strategy: Flip, count, replicate

Game	Outcome	NHeads
1	TTHT	1
2	HHTT	2
3	TTTT	0
4	THHH	3
5	TTHH	2



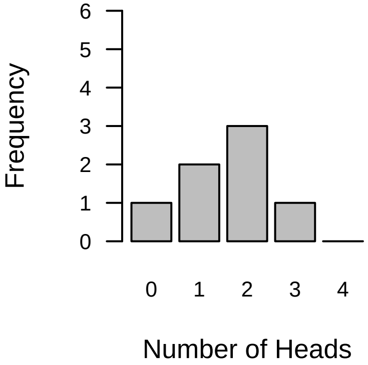
Empirical strategy: Flip, count, replicate

Game	Outcome	NHeads
1	TTHT	1
2	HHTT	2
3	TTTT	0
4	THHH	3
5	TTHH	2
6	THTT	1



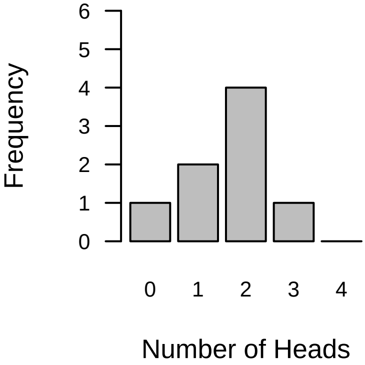
Empirical strategy: Flip, count, replicate

Game	Outcome	NHeads
1	TTHT	1
2	HHTT	2
3	TTTT	0
4	THHH	3
5	TTHH	2
6	THTT	1
7	HTTH	2



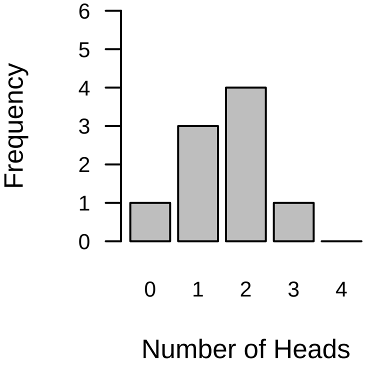
Empirical strategy: Flip, count, replicate

Game	Outcome	NHeads
1	TTHT	1
2	HHTT	2
3	TTTT	0
4	THHH	3
5	TTHH	2
6	THTT	1
7	HTTH	2
8	THTH	2



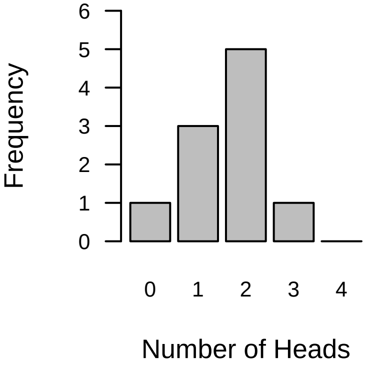
Empirical strategy: Flip, count, replicate

Game	Outcome	NHeads
1	TTHT	1
2	HHTT	2
3	TTTT	0
4	THHH	3
5	TTHH	2
6	THTT	1
7	HTTH	2
8	THTH	2
9	TTTH	1

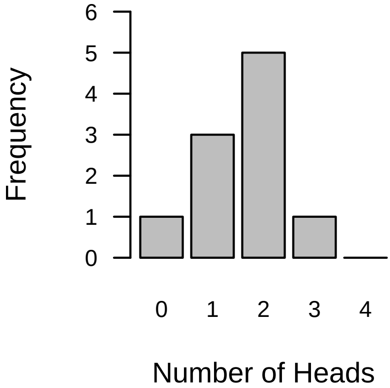


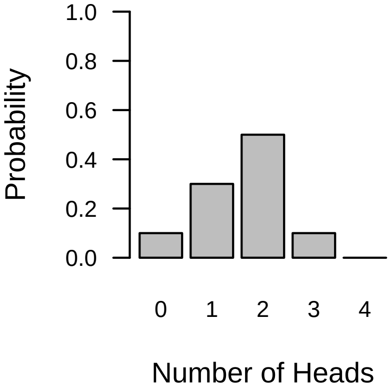
Empirical strategy: Flip, count, replicate

Game	Outcome	NHeads
1	TTHT	1
2	HHTT	2
3	TTTT	0
4	THHH	3
5	TTHH	2
6	THTT	1
7	HTTH	2
8	THTH	2
9	TTTH	1
10	TTHH	2



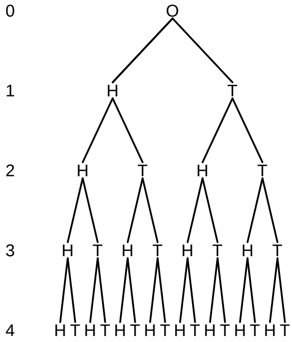
Empirical strategy

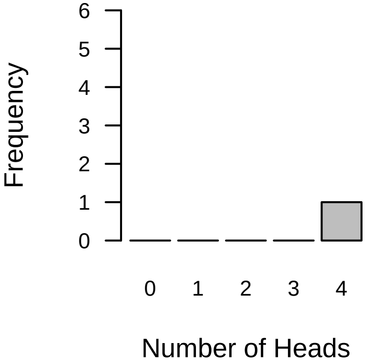




$$P(X) = \frac{N_X}{N_R}$$
$$P(X \vee Y) = P(X) + P(Y) - P(X \wedge Y)$$

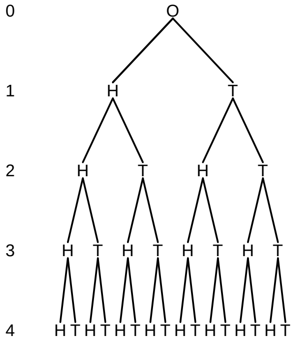
Analytical strategy

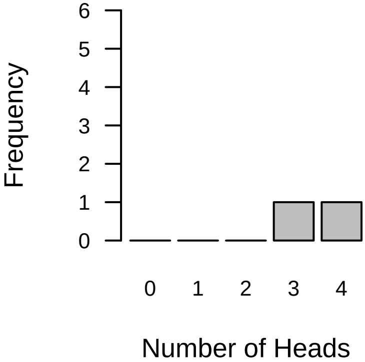




HHHH (4)

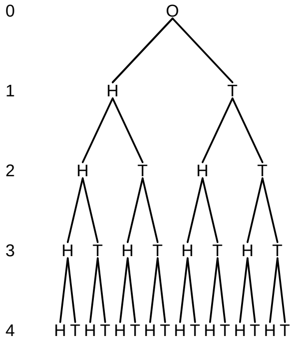
Analytical strategy

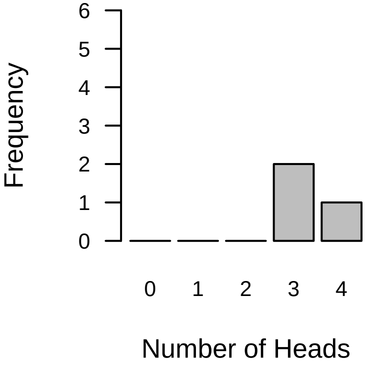




HHHH (4)   HHHT (3)

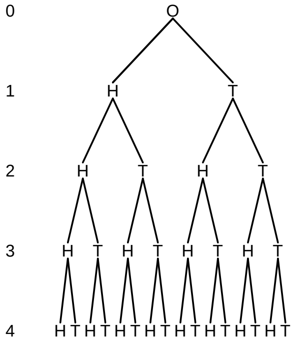
Analytical strategy



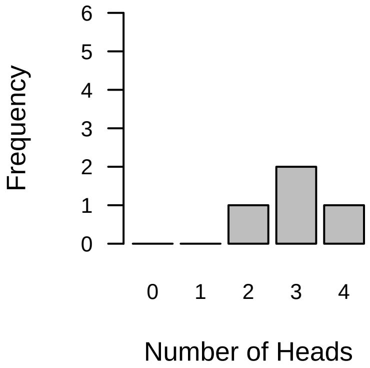


HHHH (4) HHHT (3) HHTH (3)

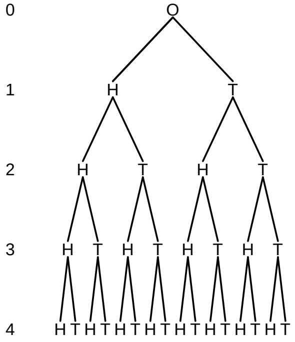
Analytical strategy



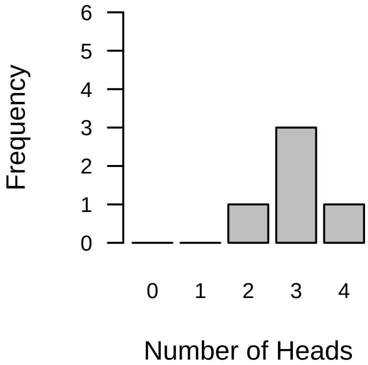
HHHH (4)   HHHT (3)   HHTH (3)   HHTT (2)



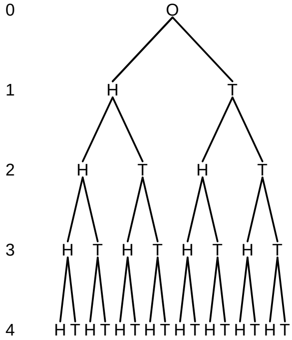
Analytical strategy

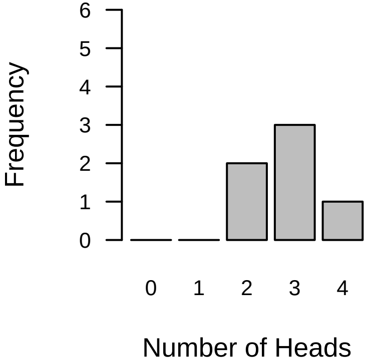


HHHH (4) HHHT (3) HHTH (3) HHTT (2) HTHH (3)



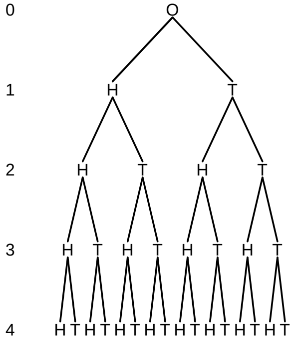
Analytical strategy

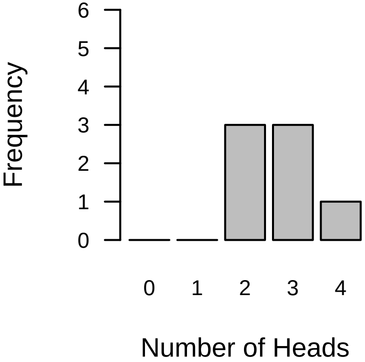




HHHH (4) HHHT (3) HHTH (3) HHTT (2) HTHH (3) HTHT (2)

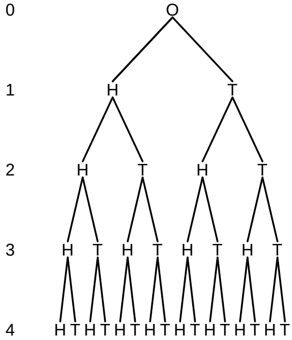
Analytical strategy

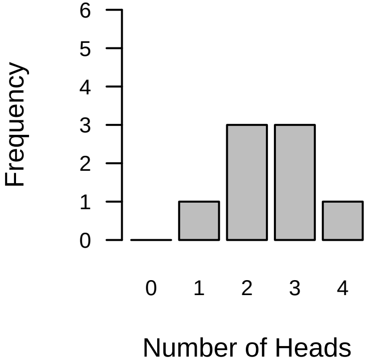




HHHH (4) HHHT (3) HHTH (3) HHTT (2) HTHH (3) HTHT (2) HTTH (2)

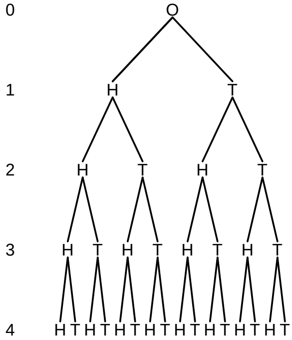
Analytical strategy

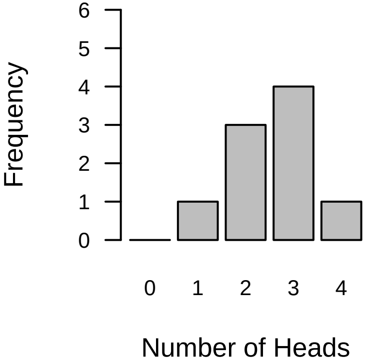




HHHH (4) HHHT (3) HHTH (3) HHTT (2) HTHH (3) HTHT (2) HTTH (2) HTTT (1)

Analytical strategy





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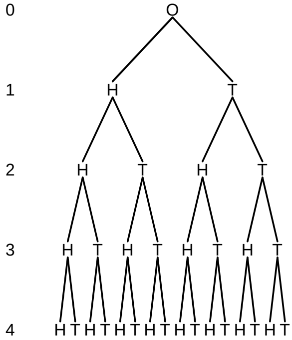
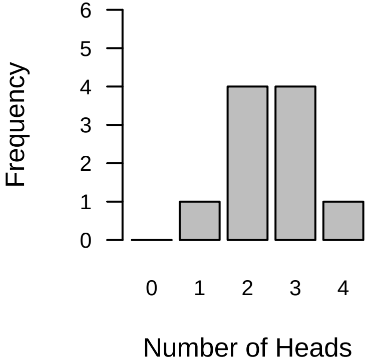
THHH (3)

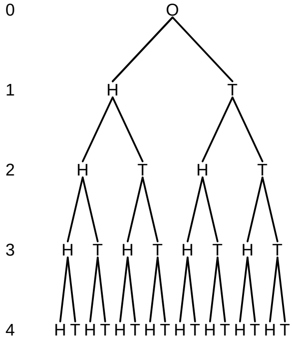
Analytical strategy

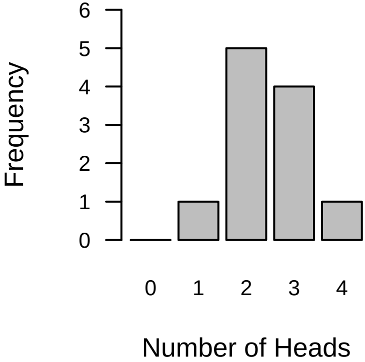




HHHH (4) HHHT (3) HHTH (3) HHTT (2) HTHH (3) HTHT (2) HTTH (2) HTTT (1)
THHH (3) THHT (2)

Analytical strategy





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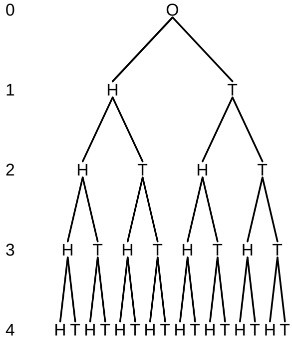
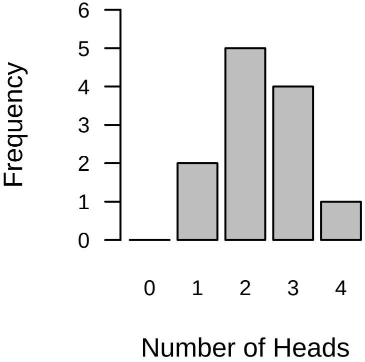
Analytical strategy





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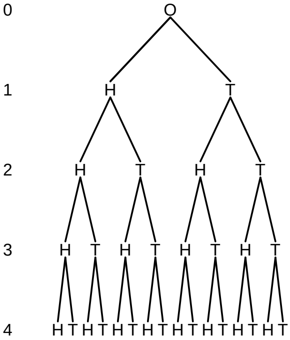
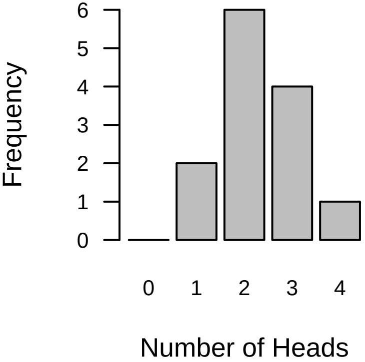
THHH (3) THHT (2) THTH (2) THTT (1)

Analytical strategy





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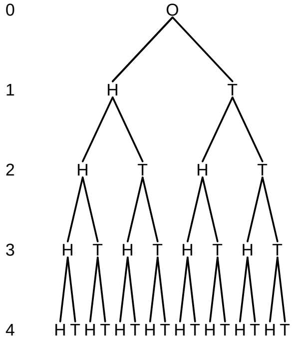
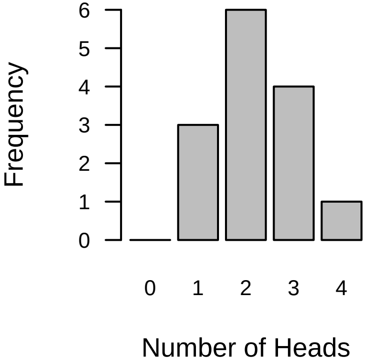
THHH (3) THHT (2) THTH (2) THTT (1) TTHH (2)

Analytical strategy





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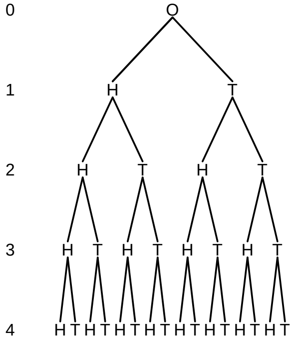
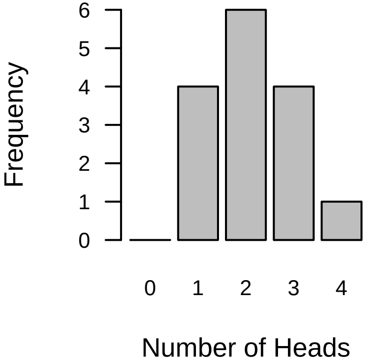
THHH (3) THHT (2) THTH (2) THTT (1) TTHH (2) TTHT (1)

Analytical strategy





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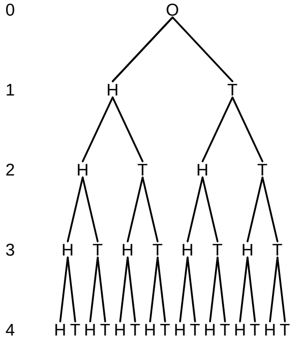
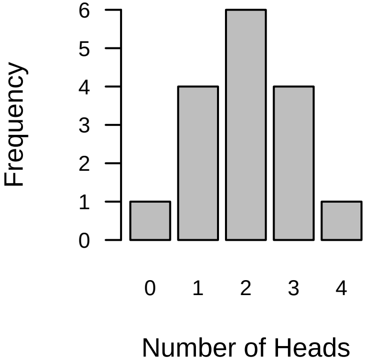
THHH (3) THHT (2) THTH (2) THTT (1) TTHH (2) TTHT (1) TTTH (1)

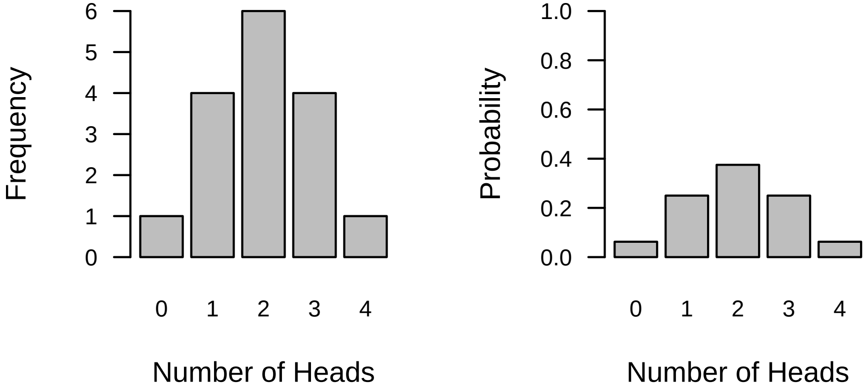
Analytical strategy





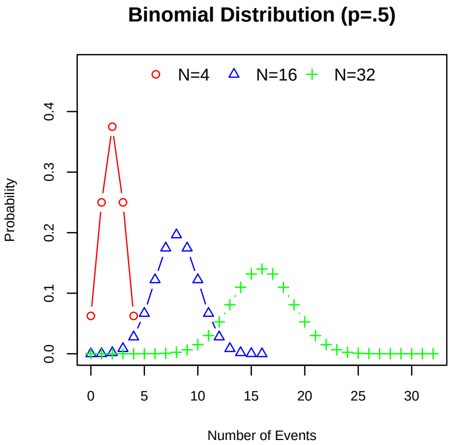
HHHH (4)	HHHT (3)	HHTH (3)	HHTT (2)	HTHH (3)	HTHT (2)	HTTH (2)	HTTT (1)
THHH (3)	THHT (2)	THTH (2)	THTT (1)	TTHH (2)	TTHT (1)	TTTH (1)	TTTT (0)

Analytical strategy



$$P(X) = \frac{N_X}{N_{tot}}; P(X \vee Y) = P(X) + P(Y) - P(X \wedge Y)$$

The binomial distribution



A theoretical distribution for binary events that gives the probability of k "successes" in n trials, with the probability of "success" equal to p

$Y \sim B(n, p)$

$$P(K = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

Note: $n! = 1 \times 2 \times \ldots \times n$

Using the binomial distribution

$$P(K = k) = \tfrac{n!}{k!(n-k)!} p^k (1 - p)^{n-k}$$

- What is the probability of getting exactly 3 heads out of 7 coin flips?

$$\begin{aligned} P(K = 3) &= \frac{7!}{3!(7-3)!} .5^3 (.5^{7-3}) \\ &= \frac{7!}{3!4!} (.125)(.0625) = \frac{5040}{6(24)} (.0078) \\ &= \frac{5040}{144} (.0078) = 35(.0078) = .2734 \end{aligned}$$

Using the binomial distribution

$$P(K = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

- What is the probability of rolling a 1 on a six sided die exactly 3 times in 5 rolls?

Note: $p = \frac{1}{6}$ or .167; $n = 5$; $k = 3$.

$$\begin{aligned} P(K = 3) &= \frac{5!}{3!(5-3)!} .167^3 (1 - .167)^{5-3} \\ &= \frac{5!}{3!2!} .005(.694) \\ &= \frac{120}{6(2)} .005(.694) \\ &= \frac{120}{12} .005(.694) = 10(.005)(.694) = .032 \end{aligned}$$

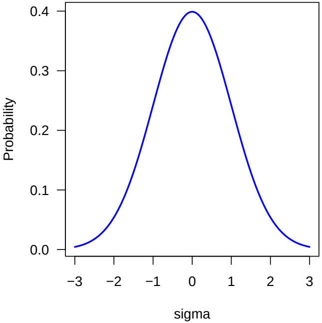
Using the binomial distribution

$$P(K = k) = \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$$

- What is the probability of getting 5 or more heads out of 7 coin flips?

$$\begin{aligned} P(K \geq 5) &= P(K = 5) + P(K = 6) + P(K = 7) \\ &= .164 + .055 + .008 = .227 \end{aligned}$$

The Standard Normal Distribution (SND)



$Y \sim N(\mu, \sigma)$

- in SD units. Mean is at 0.
- symmetric about the mean
- mode = median = mean
- inflection points at $\pm\sigma$
- asymptotes to X axis (Y=0), range is $[-\infty, +\infty]$.
- total area under the curve = 1

Calculating probabilities from the SND

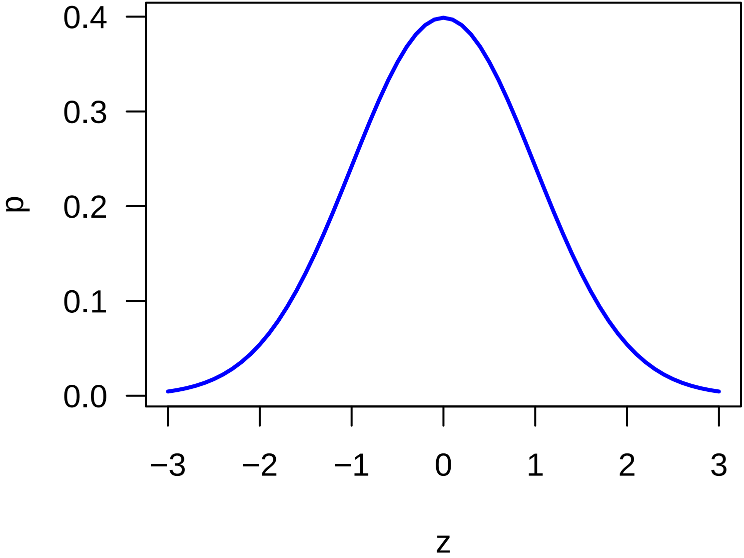
Use z-scores to calculate probabilities of individual scores.

$$z = \frac{X - \bar{X}}{\hat{s}}$$

- Note: z is continuous, not discrete as in binomial distribution. Probability of any specific value of z (e.g., 1.0000000001) is infinitesimally small.
- Therefore, need to calculate intervals over a {*range} of values.*
  - $P(z_L \leq z \leq z_U) = p$
- *Probability = area under the curve for the given interval.*

How “extreme” is an observation?

Consider the standard normal distribution.



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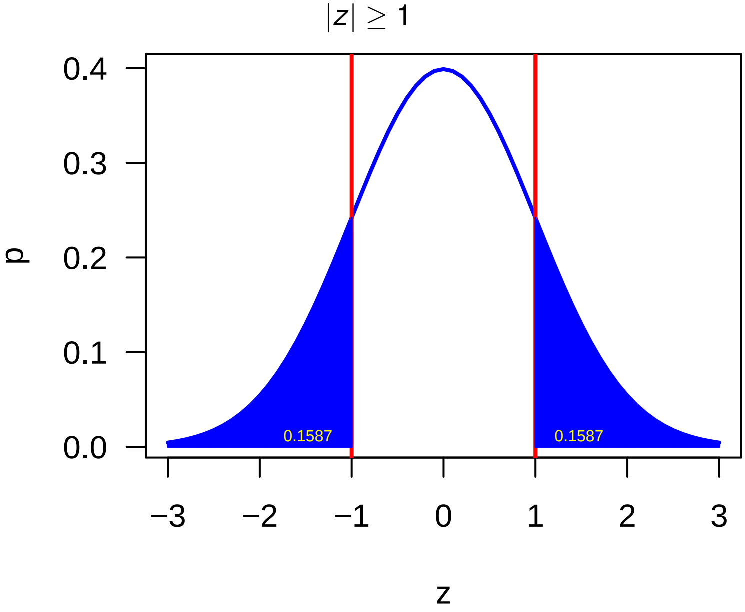

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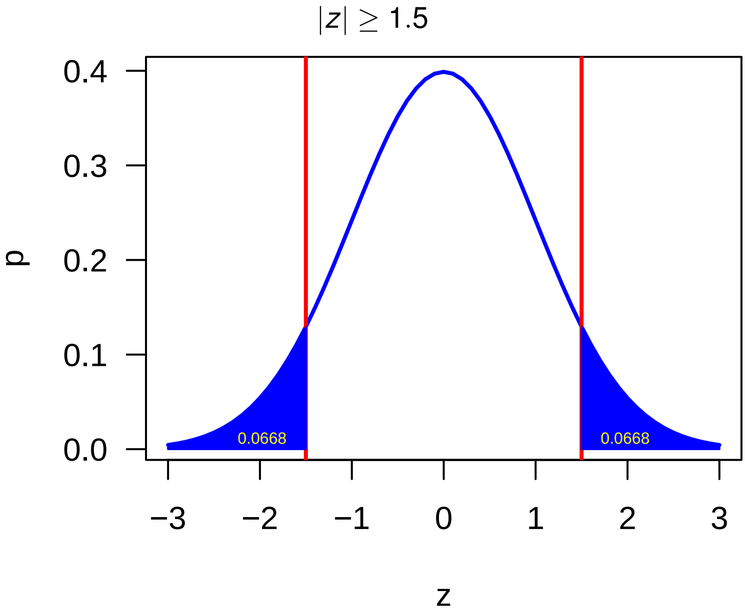

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