MULTIPLE REGRESSION

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MOVING BEYOND SIMPLE REGRESSION

1. introduction
    - estimation and interpretation
2. using multiple regression
    - partial variable plots
    - standardized coefficients
    - model comparison
3. categorical predictors
    - dummy coding schemes
    - one factor ANOVA using regression

MULTIPLE REGRESSION

General model for single-level data with m predictors:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_m X_{mi} + e_i$$

individual Xs can be any combination of continuous and categorical predictors (and their interactions)

Each β_j is the **partial effect of X_j holding all other Xs constant**

(NB: single-level data is rare in psychology)

EXAMPLE

Are lecture attendance and engagement with online materials associated with higher grades in statistics?

Does this relationship hold after controlling for overall GPA?

DATA IMPORT

grades.csv

```
grades <- read_csv("data/grades.csv",
                   col_types = "ddii")

grades
```

```
# A tibble: 100 x 4
   grade   GPA lecture nclicks
   <dbl> <dbl>   <int>   <int>
 1  2.40 1.13        6      88
 2  3.67 0.971       6      96
 3  2.85 3.34        6     123
 4  1.36 2.76        9      99
 5  2.31 1.02        4      66
 6  2.58 0.841       8      99
 7  2.69 4           5      86
 8  3.05 2.29        7     118
 9  3.21 3.39        9      98
10  2.24 3.27       10     115
# … with 90 more rows
```

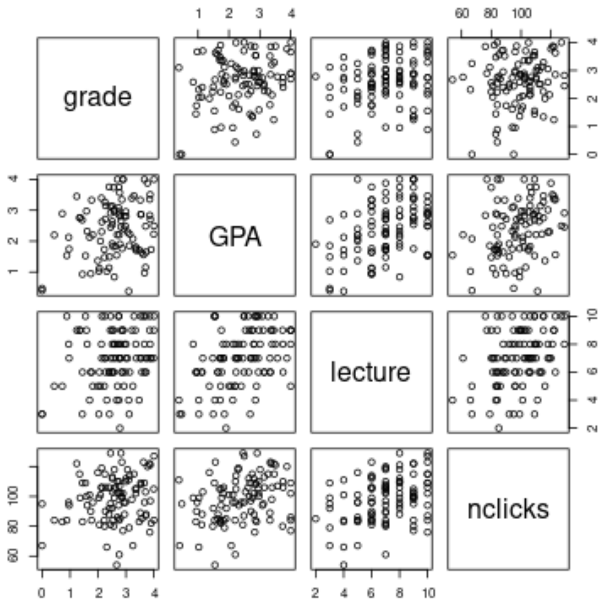
CORRELATIONS

```
library("corrr")

grades %>%
  correlate() %>%
  shave() %>%
  fashion()
```

```
Correlation method: 'pearson'
Missing treated using: 'pairwise.complete.obs'

  rowname grade  GPA lecture nclicks
1   grade
2     GPA   .25
3 lecture   .24  .44
4 nclicks   .16  .30     .36
```

VISUALIZATION

```
grades %>%
  pairs()
```



ESTIMATION AND INTERPRETATION

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_m X_{mi} + e_i$$

```
lm(Y ~ X1 + X2 + ... + Xm, data)
```

```r
my_model <- lm(grade ~ lecture + nclicks, grades)
summary(my_model)
```

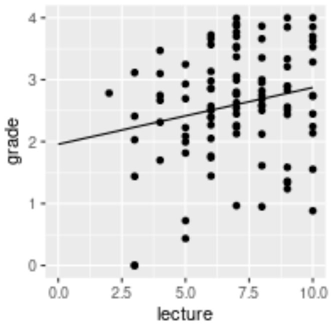
```
Call:
lm(formula = grade ~ lecture + nclicks, data = grades)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.21653 -0.40603  0.02267  0.60720  1.38558 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) 1.462037   0.571124   2.560   0.0120 *
lecture     0.091501   0.045766   1.999   0.0484 *
nclicks     0.005052   0.006051   0.835   0.4058  
---
codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.8692 on 97 degrees of freedom
Multiple R-squared:  0.06543,	Adjusted R-squared:  0.04616 
F-statistic: 3.395 on 2 and 97 DF,  p-value: 0.03756
```

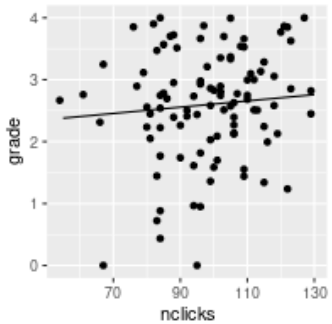
USING MULTIPLE REGRESSION

VISUALIZING PARTIAL EFFECTS

```r
dmean <- grades %>%
  pull(nclicks) %>% mean()

new1 <- crossing(lecture = 0:10,
                 nclicks = dmean)

new2 <- new1 %>%
  mutate(grade = predict(my_model,
                         new1))

ggplot(grades, aes(lecture, grade)) +
  geom_point() +
  geom_line(data = new2)
```

```r
lmean <- grades %>%
  pull(lecture) %>% mean()

new3 <- crossing(nclicks = 55:130,
                 lecture = lmean)

new4 <- new3 %>%
  mutate(grade = predict(my_model,
                         new3))

ggplot(grades, aes(nclicks, grade)) +
  geom_point() +
  geom_line(data = new4)
```





See `?predict.lm()`, `?tidyr::crossing()`

STANDARDIZED COEFFICIENTS

Which predictor matters more?

```
grades2 <- grades %>%
  mutate(lecture_c = (lecture - mean(lecture)) / sd(lecture),
         nclicks_c = (nclicks - mean(nclicks)) / sd(nclicks))

summary(lm(grade ~ lecture_c + nclicks_c, grades2))
```

```

Call:
lm(formula = grade ~ lecture_c + nclicks_c, data = grades2)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.21653 -0.40603  0.02267  0.60720  1.38558 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.59839    0.08692  29.895   <2e-16 ***
lecture_c    0.18734    0.09370   1.999   0.0484 *  
nclicks_c    0.07823    0.09370   0.835   0.4058    
---
codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.8692 on 97 degrees of freedom
Multiple R-squared:  0.06543,	Adjusted R-squared:  0.04616 
F-statistic: 3.395 on 2 and 97 DF,  p-value: 0.03756
```

See `?base::scale()`

STANDARDIZED VS NON-STANDARDIZED

- standardized

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.59839    0.08692  29.895   <2e-16 ***
lecture_c    0.18734    0.09370   1.999   0.0484 *  
nclicks_c    0.07823    0.09370   0.835   0.4058    
```

- not standardized

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) 1.462037   0.571124   2.560   0.0120 *
lecture     0.091501   0.045766   1.999   0.0484 *
nclicks     0.005052   0.006051   0.835   0.4058  
```

MODEL COMPARISON

Is engagement (as measured by lecture attendance and downloads) positively associated with final course grade **above and beyond** student ability (as measured by GPA)?

STRATEGY

Create a "base" model with all control vars and compare to a "bigger" model with all control and focal vars

```
base_model <- lm(grade ~ GPA, grades)
big_model <- lm(grade ~ GPA + lecture + nclicks, grades)

anova(base_model, big_model)
```

```
Analysis of Variance Table

Model 1: grade ~ GPA
Model 2: grade ~ GPA + lecture + nclicks
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     98 73.528
2     96 71.578  2    1.9499 1.3076 0.2752
```

$$F(2, 96) = 1.31, p = .275$$

If $p < \alpha$, bigger model is better.

CATEGORICAL PREDICTORS

DUMMY CODING BINARY VARS

Arbitrarily assign one of the two levels to 0; assign the other to 1.

NB: sign of the variable depends on the coding!

See `?dplyr::if_else()`

FACTORS WITH $k > 2$

Arbitrarily choose one level as “baseline” level.

- $k = 3$

	`A2v1`	`A3v1`
A_1	0	0
A_2	1	0
A_3	0	1

- $k = 4$

	`A2v1`	`A3v1`	`A4v1`
A_1	0	0	0
A_2	1	0	0
A_3	0	1	0
A_4	0	0	1

ONE FACTOR ANOVA

$$Y_{ij} = \mu + A_i + S(A)_{ij}$$

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + e_i$$