INTERACTIONS

STATISTICAL MODELS

PSYCHOLOGY, UNIVERSITY OF GLASGOW

INTERACTIONS

"It depends."

The effect of a predictor variable on the response variable may depend upon the value(s) of one or more other predictor variables.

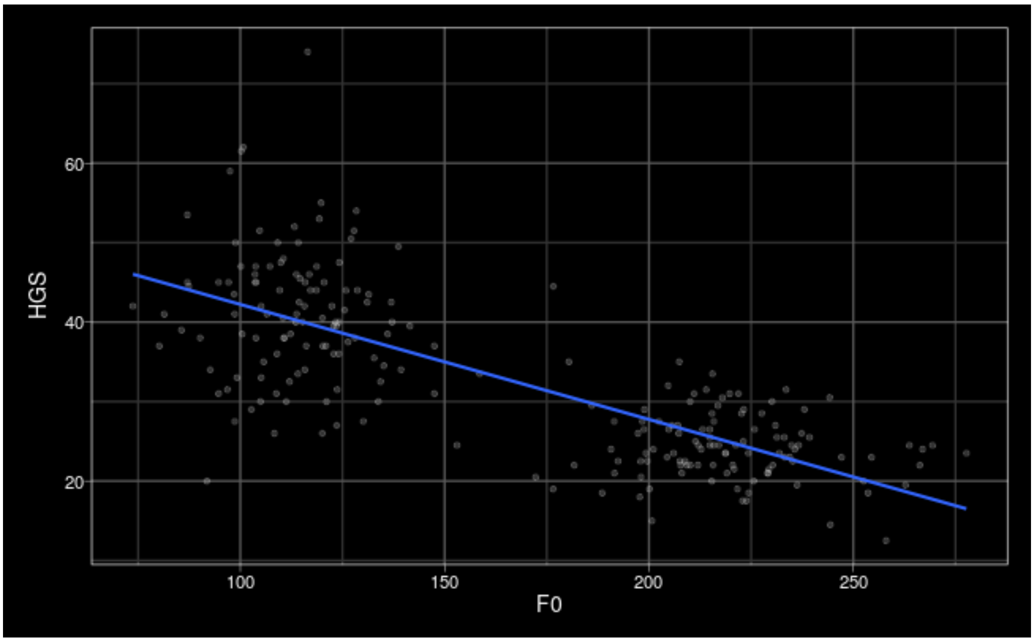
EXAMPLE

Do stronger people have lower voices?

- HGS: Hand grip strength
- F0: voice fundamental frequency

```
# A tibble: 221 x 4
      ID sex      HGS    F0
   <int> <chr> <dbl> <dbl>
 1     4 male   45.5 115. 
 2     7 male   31   147. 
 3     8 male   40   123. 
 4    19 male   37   120. 
 5    21 male   45    94.7
 6    22 male   50    98.8
 7    30 male   31    94.7
 8    31 male   47.5 124. 
 9    35 male   34    92.6
10    36 male   30   111. 
# … with 211 more rows
```

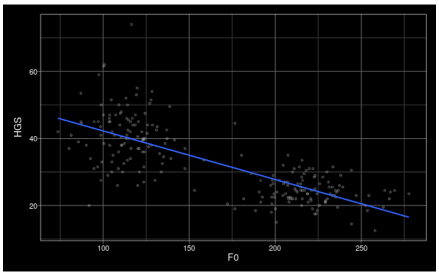
Han, C., Wang, H., Fasolt, V., Hahn, A., Holzleitner, I. J., Lao, J., DeBruine, L., Feinberg, D., Jones, B. C. Open Science Framework, retrieved from https://osf.io/na6be/.



N = 221

GLM

$$HGS_i = \beta_0 + \beta_1 F0_i + e_i$$



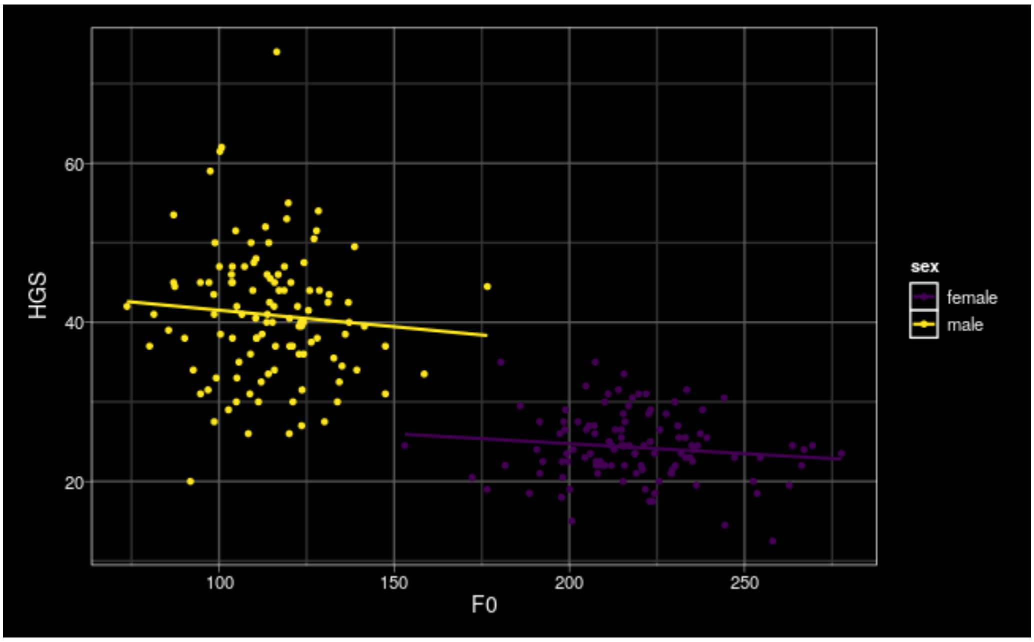

```
Call:
lm(formula = HGS ~ F0, data = hgs)

Residuals:
    Min      1Q  Median      3Q     Max 
-23.408  -4.115  -0.161   4.252  34.157 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 56.699687   1.491239   38.02   <2e-16 ***
F0          -0.144729   0.008509  -17.01   <2e-16 ***
---
codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 7.008 on 219 degrees of freedom
Multiple R-squared:  0.5692,	Adjusted R-squared:  0.5672 
F-statistic: 289.3 on 1 and 219 DF,  p-value: < 2.2e-16
```

```r
ggplot(hgs, aes(F0, HGS, color = sex)) +
  geom_point() +
  geom_smooth(method = "lm", se = FALSE)
```



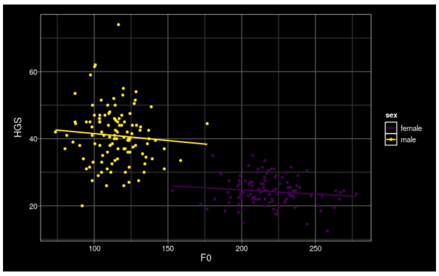
GLM

$$HGS_i = \beta_0 + \beta_1 F0_i + \beta_2 SEX_i + \beta_3 F0_i SEX_i + e_i$$

$$= \beta_0 + \beta_2 SEX_i + (\beta_1 + \beta_3 SEX_i) F0_i + e_i$$

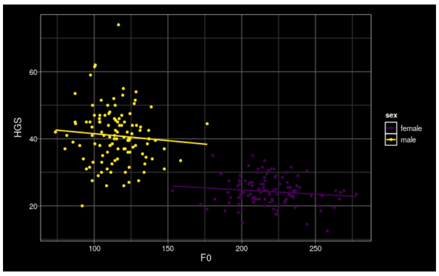
```
HGS ~ F0 + sex + F0:sex
```

```
HGS ~ F0 * sex
```



- SEX: 0 = female, 1 = male
- female: $\beta_0 + \beta_1 F0_i$
- male: $\beta_0 + \beta_2 + (\beta_1 + \beta_3) F0_i$

ANALYSIS




```r
hgs2 <- hgs %>%
  mutate(sex_male = if_else(sex == "male", 1, 0))

lm(HGS ~ sex_male * F0, hgs2) %>% summary()
```

```

Call:
lm(formula = HGS ~ sex_male * F0, data = hgs2)

Residuals:
    Min      1Q  Median      3Q     Max 
-21.859  -3.540  -0.421   3.361  33.163 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 29.75789    6.50985   4.571 8.14e-06 ***
sex_male    15.91254    7.87733   2.020   0.0446 *  
F0          -0.02508    0.02965  -0.846   0.3985    
sex_male:F0 -0.01642    0.04847  -0.339   0.7351    
---
codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.643 on 217 degrees of freedom
Multiple R-squared:  0.6163,	Adjusted R-squared:  0.611 
F-statistic: 116.2 on 3 and 217 DF,  p-value: < 2.2e-16
```

TWO-FACTOR ANOVA

RATIONALE FOR FACTORIAL ANOVA

- Used to address question involving more than one factor that can influence a DV, with each factor acting alone *or in combination with other factors*
  - What are the effects of cognitive therapy and drug therapy on mood in depressed individuals?
  - Do male and female students learn better with male or female teachers?

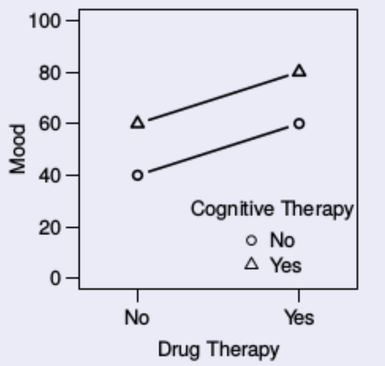
FULL FACTORIAL DESIGNS

- A study has a full factorial design if it has more than one IV and the levels of the IVs are “fully crossed”
- designs are designated using RxC (row-by-column) format
- **cell:** unique combination of the levels of the factors

	Factor B	
	Level B_1	Level B_2
Level A_1	A_1B_1	A_1B_2
Level A_2	A_2B_1	A_2B_2

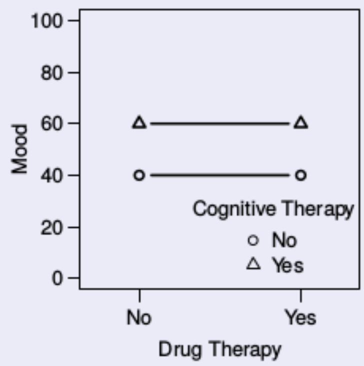
FACTORIAL PLOTS AND INTERPRETATION

Scenario A



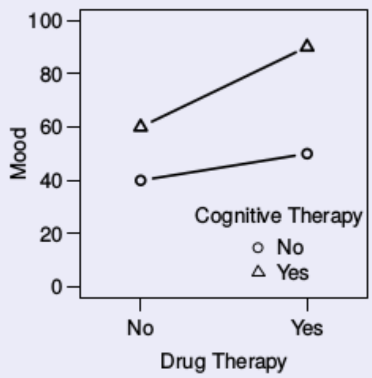
		Drug: No	Drug: Yes	
Cog.	No	40	60	50
	Yes	60	80	70
		50	70	

Scenario B



		Drug: No	Drug: Yes	
Cog.	No	40	40	40
	Yes	60	60	60
		50	50	

Scenario C



		Drug: No	Drug: Yes	
Cog.	No	40	50	45
	Yes	60	90	75
		50	70	

EFFECTS IN FACTORIAL DESIGNS

- Main Effects: tests of *marginal means*
  - $H_0 : \mu_{A_1} = \mu_{A_2}$
  - $H_0 : \mu_{B_1} = \mu_{B_2}$
- Simple Effects: effect of factor at level of other
  - eff of B at $A_1, H_0 : \mu_{A_1 B_1} = \mu_{A_1 B_2}$
  - eff of B at $A_2, H_0 : \mu_{A_2 B_1} = \mu_{A_2 B_2}$
- Interaction: equivalence of simple effects
  - $H_0 : \mu_{A_1 B_2} - \mu_{A_1 B_1} = \mu_{A_2 B_2} - \mu_{A_2 B_1}$

A COMMON FALLACY

"The percentage of neurons showing cue-related activity increased with training in the mutant mice ($p < 0.05$), but not in the control mice ($p > 0.05$)."

- saying the simple effect is significant in one case but not in another does not imply that the simple effects are statistically different

Gelman, A., & Stern, H. (2012). The difference between "significant" and "not significant" is not itself statistically significant. *The American Statistician*, *60*, 328–331.

Nieuwenhuis, S., Forstmann, B. U., & Wagenmakers, E. J. (2011). Erroneous analyses of interactions in neuroscience: a problem of significance. *Nature Neuroscience*, *14*, 1105-1107.

GLM FOR 2-FACTOR ANOVA

$$Y_{ijk} = \mu + A_i + B_j + AB_{ij} + S(AB)_{ijk}$$

	B_1	B_2
A_1	Y_{111}	Y_{121}
	Y_{112}	Y_{122}
	Y_{113}	Y_{123}
A_2	Y_{211}	Y_{221}
	Y_{212}	Y_{222}
	Y_{213}	Y_{223}

Y_{ijk}	DV, sub k in row i col j
μ	grand mean
A_i	effect of A (level i)
B_j	effect of B (level j)
AB_{ij}	interaction (cell ij)
$S(AB)_{ijk}$	error, sub k in cell ij

ESTIMATION EQUATIONS

$$Y_{ijk} = \mu + A_i + B_j + AB_{ij} + S(AB)_{ijk}$$

	B_1	B_2	
A_1	$Y_{11.}$	$Y_{12.}$	$Y_{1..}$
A_2	$Y_{21.}$	$Y_{22.}$	$Y_{2..}$
	$Y_{.1.}$	$Y_{.2.}$	

$$\begin{array}{ll}
\hat{\mu} & Y_{...} \\
\hat{A}_i & Y_{i..} - \hat{\mu} \\
\hat{B}_j & Y_{.j.} - \hat{\mu} \\
\widehat{AB}_{ij} & Y_{ij.} - \hat{\mu} - \hat{A}_i - \hat{B}_j \\
S(\widehat{AB})_{ijk} & Y_{ijk} - \hat{\mu} - \hat{A}_i - \hat{B}_j - \widehat{AB}_{ij}
\end{array}$$

DECOMPOSITION

	B_1	B_2
A_1	Y_{111}	Y_{121}
	Y_{112}	Y_{122}
	Y_{113}	Y_{123}
A_2	Y_{211}	Y_{221}
	Y_{212}	Y_{222}
	Y_{213}	Y_{223}

$$
\begin{aligned}
Y_{ijk} &= \hat{\mu} + \hat{A}_i + \hat{B}_j + \widehat{AB}_{ij} + S(\widehat{AB})_{ijk} \\
\hline
Y_{111} &= \hat{\mu} + \hat{A}_1 + \hat{B}_1 + \widehat{AB}_{11} + S(\widehat{AB})_{111} \\
Y_{112} &= \hat{\mu} + \hat{A}_1 + \hat{B}_1 + \widehat{AB}_{11} + S(\widehat{AB})_{112} \\
Y_{113} &= \hat{\mu} + \hat{A}_1 + \hat{B}_1 + \widehat{AB}_{11} + S(\widehat{AB})_{113} \\
Y_{121} &= \hat{\mu} + \hat{A}_1 + \hat{B}_2 + \widehat{AB}_{12} + S(\widehat{AB})_{121} \\
Y_{122} &= \hat{\mu} + \hat{A}_1 + \hat{B}_2 + \widehat{AB}_{12} + S(\widehat{AB})_{122} \\
Y_{123} &= \hat{\mu} + \hat{A}_1 + \hat{B}_2 + \widehat{AB}_{12} + S(\widehat{AB})_{123} \\
Y_{211} &= \hat{\mu} + \hat{A}_2 + \hat{B}_1 + \widehat{AB}_{21} + S(\widehat{AB})_{211} \\
Y_{212} &= \hat{\mu} + \hat{A}_2 + \hat{B}_1 + \widehat{AB}_{21} + S(\widehat{AB})_{212} \\
Y_{213} &= \hat{\mu} + \hat{A}_2 + \hat{B}_1 + \widehat{AB}_{21} + S(\widehat{AB})_{213} \\
Y_{221} &= \hat{\mu} + \hat{A}_2 + \hat{B}_2 + \widehat{AB}_{22} + S(\widehat{AB})_{221} \\
Y_{222} &= \hat{\mu} + \hat{A}_2 + \hat{B}_2 + \widehat{AB}_{22} + S(\widehat{AB})_{222} \\
Y_{223} &= \hat{\mu} + \hat{A}_2 + \hat{B}_2 + \widehat{AB}_{22} + S(\widehat{AB})_{223}
\end{aligned}
$$

Main Effects and Interactions: 2x2 Factorial

Intercept:

-10 0 10

-10 -8 -6 -4 -2 0 2 4 6 8 10

Main Effect of A:

-20 0 20

-20 -16 -12 -8 -4 0 4 8 12 16 20

Main Effect of B:

-20 0 20

-20 -16 -12 -8 -4 0 4 8 12 16 20

AB Interaction:

-80 0 80

-80 -64 -48 -32 -16 0 16 32 48 64 80

Reset

Cell and Marginal Means

	B1	B2	
A1	0	0	0
A2	0	0	0
	0	0	NA

Decomposition of Cell Means

i	j	Y_ij	intercept	A_i	B_j	AB_ij
1	1	0	0	0	0	0
1	2	0	0	0	0	0
2	1	0	0	0	0	0
2	2	0	0	0	0	0