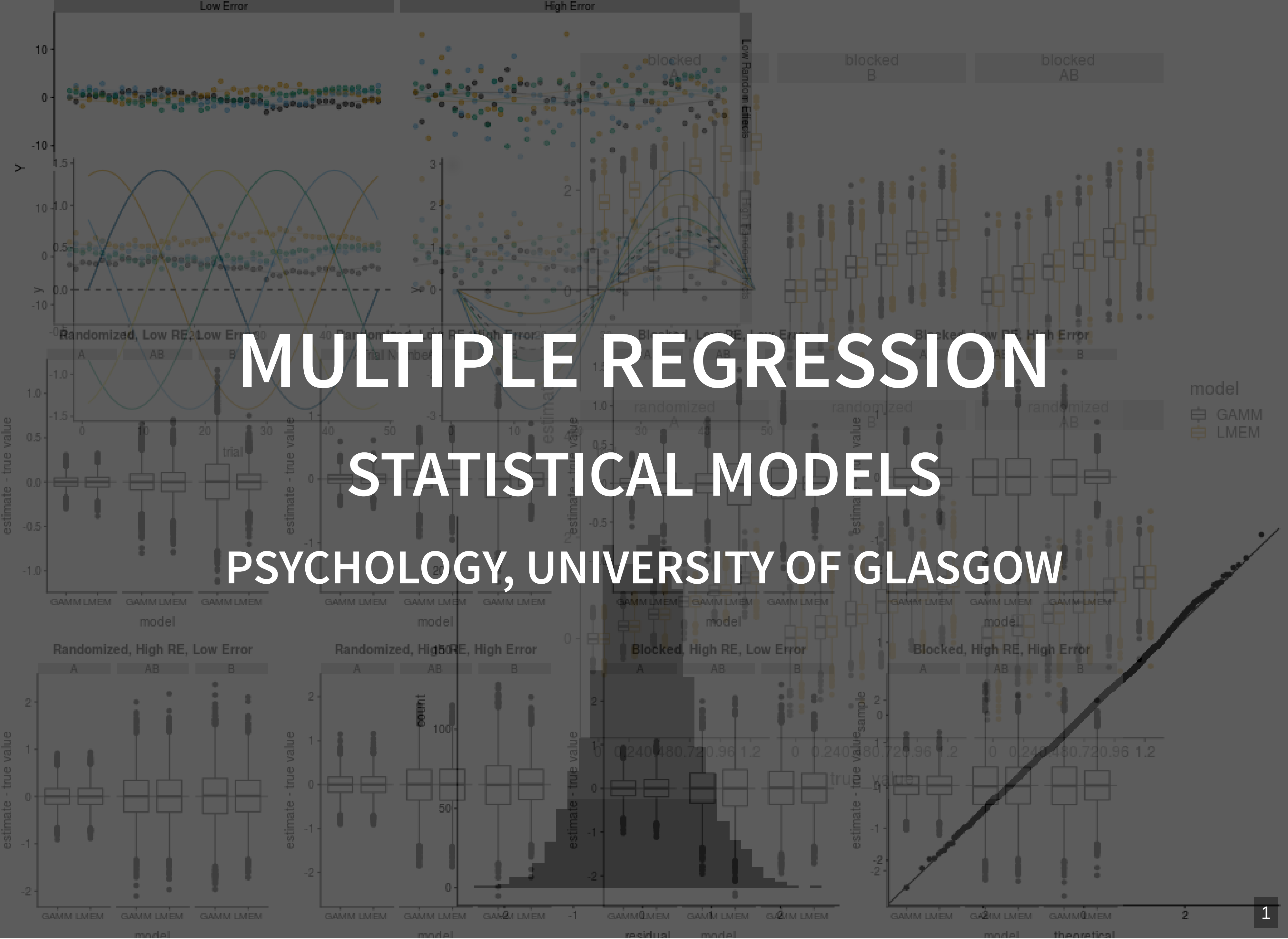


# MULTIPLE REGRESSION

## STATISTICAL MODELS

## PSYCHOLOGY, UNIVERSITY OF GLASGOW

# MOVING BEYOND SIMPLE REGRESSION

- dealing with multiple predictors
- coding continuous predictors
- coding categorical predictors
    - one factor ANOVA using regression
- continuous-by-categorical interactions

# MULTIPLE REGRESSION

General model for single-level data with $m$ predictors:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_m X_{mi} + e_i$$

individual $X$s can be any combination of continuous and categorical predictors (and their interactions)

Each $\beta_j$ is the **partial effect of $X_j$ holding all other $X$s constant**

**(NB: single-level data is rare in psychology)**

# EXAMPLE

Are lecture attendance and engagement with online materials associated with higher grades in statistics?

Does this relationship hold after controlling for overall GPA?

# DATA IMPORT AND VISUALIZATION

## grades.csv

```
grades <- read_csv("data/grades.csv",
                   col_types = "ddii")

grades
```

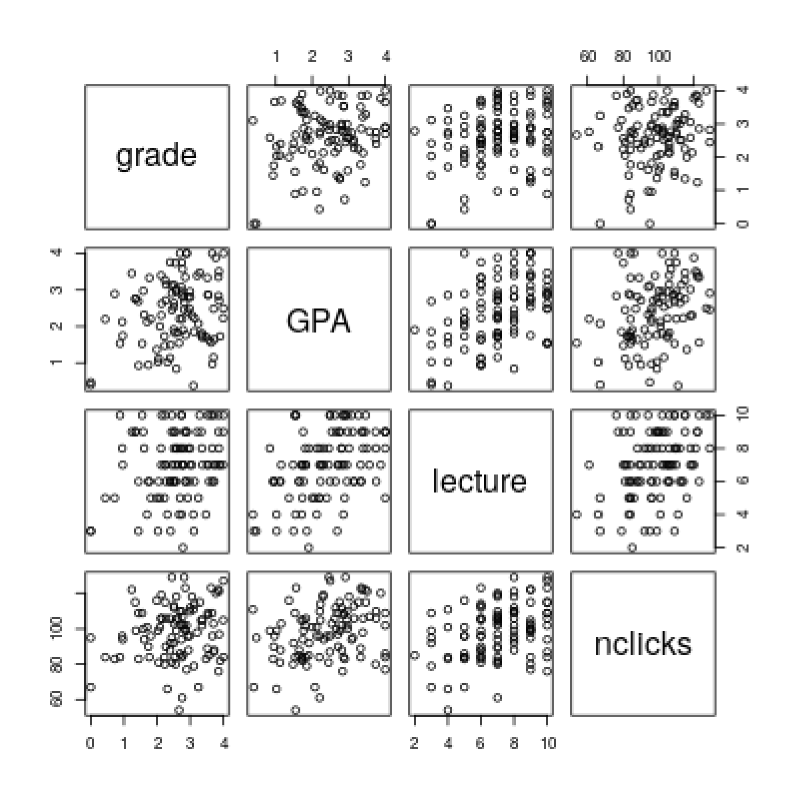
```
# A tibble: 100 x 4
   grade   GPA lecture nclicks
   <dbl> <dbl>   <int>   <int>
 1  2.40 1.13        6      88
 2  3.67 0.971       6      96
 3  2.85 3.34        6     123
 4  1.36 2.76        9      99
 5  2.31 1.02        4      66
 6  2.58 0.841       8      99
 7  2.69 4           5      86
 8  3.05 2.29        7     118
 9  3.21 3.39        9      98
10  2.24 3.27       10     115
# … with 90 more rows
```

```
library("corrr")

grades %>%
  correlate() %>%
  shave() %>%
  fashion()
```

```

Correlation method: 'pearson'
Missing treated using: 'pairwise.complete.obs'

  rowname grade  GPA lecture nclicks
1   grade
2     GPA   .25
3 lecture   .24  .44
4 nclicks   .16  .30     .36
```

# DATA IMPORT AND VISUALIZATION

```r
grades %>%
  pairs()
```



# ESTIMATION AND INTERPRETATION

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_m X_{mi} + e_i$$

```
lm(Y ~ X1 + X2 + ... + Xm, data)
```

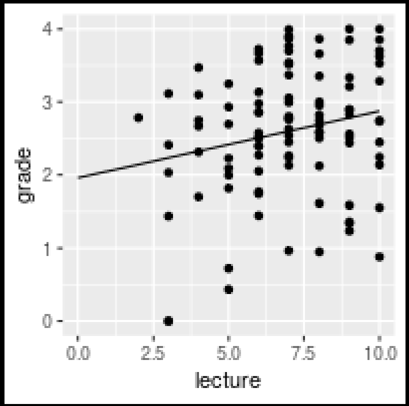
```
my_model <- lm(grade ~ lecture + nclicks, grades)
summary(my_model)
```

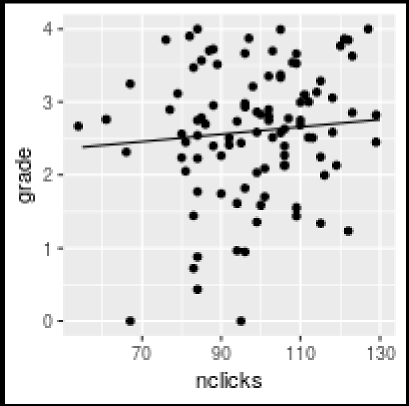
```

Call:
lm(formula = grade ~ lecture + nclicks, data = grades)

Residuals:
     Min       1Q   Median       3Q      Max
-2.21653 -0.40603  0.02267  0.60720  1.38558

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.462037   0.571124   2.560   0.0120 *
lecture     0.091501   0.045766   1.999   0.0484 *
nclicks     0.005052   0.006051   0.835   0.4058
---
codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.8692 on 97 degrees of freedom
Multiple R-squared:  0.06543,   Adjusted R-squared:  0.04616
F-statistic: 3.395 on 2 and 97 DF,  p-value: 0.03756
```

# VISUALIZING PARTIAL EFFECTS

```r
dmean <- grades %>% pull(nclicks) %>% mean()

new1 <- crossing(lecture = 0:10,
                 nclicks = dmean)

new2 <- new1 %>%
  mutate(grade = predict(my_model, new1))

ggplot(grades, aes(lecture, grade)) +
  geom_point() +
  geom_line(data = new2)
```

```r
lmean <- grades %>% pull(lecture) %>% mean()

new3 <- crossing(nclicks = 55:130,
                 lecture = lmean)

new4 <- new3 %>%
  mutate(grade = predict(my_model, new3))

ggplot(grades, aes(nclicks, grade)) +
  geom_point() +
  geom_line(data = new4)
```




See `?predict.lm()`, `?tidyr::crossing()`

# STANDARDIZED COEFFICIENTS

## Which predictor matters more?

```
grades2 <- grades %>%
  mutate(lecture_c = (lecture - mean(lecture)) / sd(lecture),
         nclicks_c = (nclicks - mean(nclicks)) / sd(nclicks))

summary(lm(grade ~ lecture_c + nclicks_c, grades2))
```

```

Call:
lm(formula = grade ~ lecture_c + nclicks_c, data = grades2)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.21653 -0.40603  0.02267  0.60720  1.38558 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.59839    0.08692  29.895   <2e-16 ***
lecture_c    0.18734    0.09370   1.999   0.0484 *  
nclicks_c    0.07823    0.09370   0.835   0.4058    
---
codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.8692 on 97 degrees of freedom
Multiple R-squared:  0.06543,	Adjusted R-squared:  0.04616 
F-statistic: 3.395 on 2 and 97 DF,  p-value: 0.03756
```

# See `?base::scale()`

# MODEL COMPARISON

Is engagement (as measured by lecture attendance and downloads) positively associated with final course grade **above and beyond** student ability (as measured by GPA)?

# STRATEGY

Create a "base" model with all control vars and compare to a "bigger" model with all control and focal vars

```
base_model <- lm(grade ~ GPA, grades)
big_model <- lm(grade ~ GPA + lecture + nclicks, grades)

anova(base_model, big_model)
```

```
Analysis of Variance Table

Model 1: grade ~ GPA
Model 2: grade ~ GPA + lecture + nclicks
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     98 73.528
2     96 71.578  2    1.9499 1.3076 0.2752
```

$$F(2,96) = 1.31, p = .275$$

If $p < \alpha$, bigger model is better.

# DUMMY CODING BINARY VARS

Arbitrarily assign one of the two levels to 0; assign the other to 1.

**NB: sign of the variable depends on the coding!**

See `?dplyr::if_else()`

# FACTORS WITH $k > 2$

Arbitrarily choose one level as "baseline" level.

- $k = 3$

| | `A2v1` | `A3v1` |
|---|---|---|
| $A_1$ | 0 | 0 |
| $A_2$ | 1 | 0 |
| $A_3$ | 0 | 1 |

- $k = 4$

| | `A2v1` | `A3v1` | `A4v1` |
|---|---|---|---|
| $A_1$ | 0 | 0 | 0 |
| $A_2$ | 1 | 0 | 0 |
| $A_3$ | 0 | 1 | 0 |
| $A_4$ | 0 | 0 | 1 |

# ONE FACTOR ANOVA USING REGRESSION

$$Y_{ij} = \mu + A_i + S(A)_{ij}$$

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + e_i$$