Interactions

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“It depends.”

The effect of a predictor variable on the response variable may depend upon the value(s) of one or more other predictor variables.

Continuous-by-Categorical Interactions

Do stronger people tend to have lower voices?

Han, C., Wang, H., Fasolt, V., Hahn, A. C., Holzleitner, I. J., Lao, J., DeBruine, L. M., Feinberg, D. R., & Jones, B. C. (2018). No evidence for correlations between handgrip strength and sexually dimorphic acoustic properties of voices.. *American Journal of Human Biology*, *30*, e23178.

Data: https://osf.io/na6be/

The Data

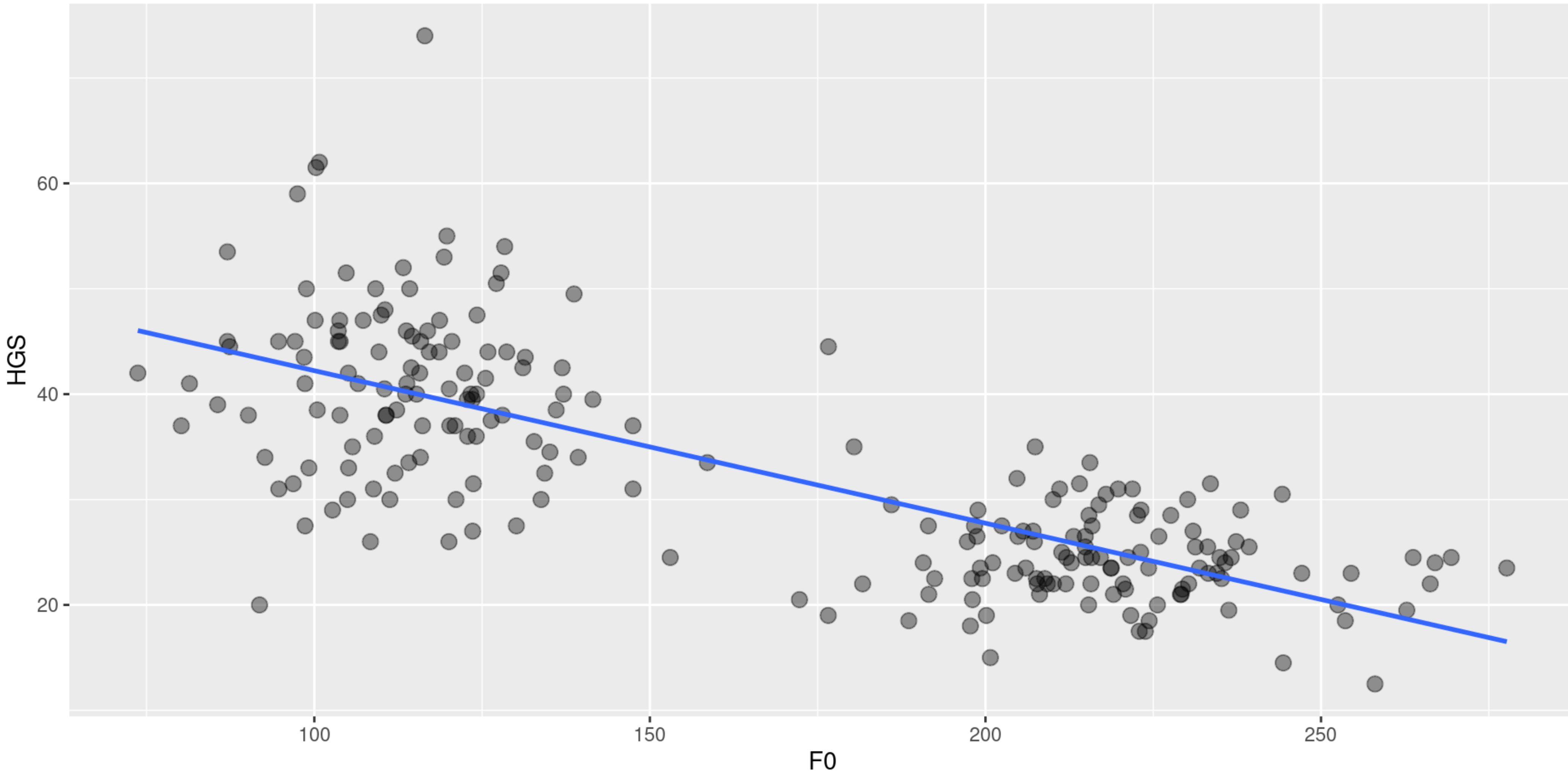
```
hgs <- readxl::read_excel("../data/Han_et_al.xlsx") |>
  mutate(ID = as.integer(ID)) |>
  select(ID, sex, HGS, F0)

hgs
```

```
# A tibble: 221 × 4
      ID sex      HGS    F0
   <int> <chr>  <dbl> <dbl>
 1     4 male    45.5 115.
 2     7 male    31   147.
 3     8 male    40   123.
 4    19 male    37   120.
 5    21 male    45    94.7
 6    22 male    50    98.8
 7    30 male    31    94.7
 8    31 male    47.5 124.
 9    35 male    34    92.6
10    36 male    30   111.
# ℹ 211 more rows
```

HGS: Hand grip strength, **F0**: voice fundamental frequency

```r
ggplot(hgs, aes(F0, HGS)) +
  geom_point(colour = "black", size = 3, alpha = .4) +
  geom_smooth(method = "lm", se = FALSE)
```



```r
lm(HGS ~ F0, hgs) |> summary()
```

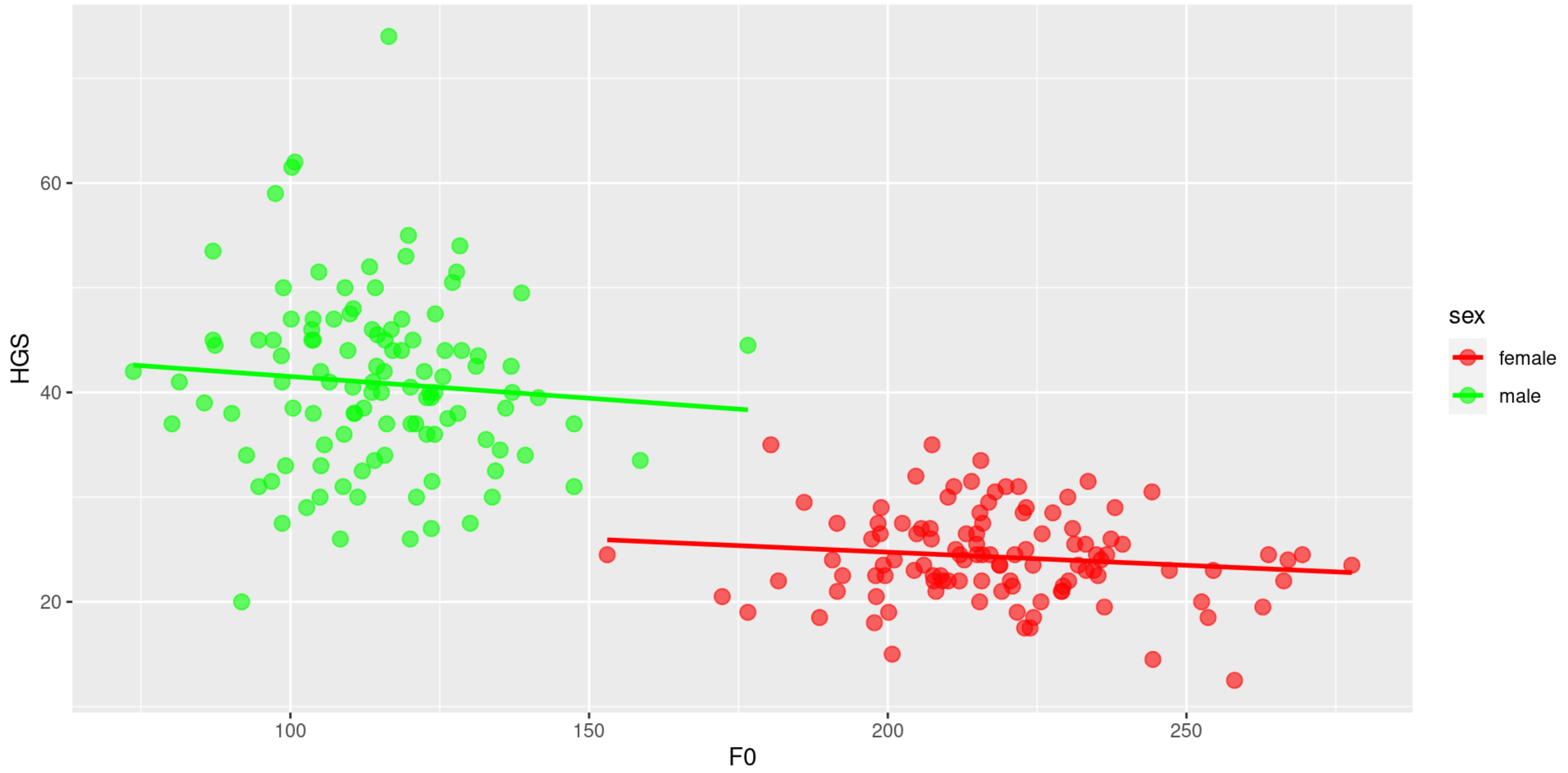
```

Call:
lm(formula = HGS ~ F0, data = hgs)

Residuals:
    Min      1Q  Median      3Q     Max 
-23.408  -4.115  -0.161   4.252  34.157 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 56.699687   1.491239   38.02   <2e-16 ***
F0          -0.144729   0.008509  -17.01   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
ggplot(hgs, aes(F0, HGS, color = sex)) +
  geom_point(size = 3, alpha = .6) +
  geom_smooth(method = "lm", se = FALSE) +
  scale_color_manual(values = c("red", "green"))
```



$$HGS_i = \beta_0 + \beta_1 F0_i + \beta_2 SEX_i + \beta_3 F0_i SEX_i + e_i$$

$$= \beta_0 + \beta_2 SEX_i + (\beta_1 + \beta_3 SEX_i) F0_i + e_i$$

```
HGS ~ F0 + sex + F0:sex
HGS ~ F0 * sex
```

- SEX: 0 = female, 1 = male
- female: $\beta_0 + \beta_1 F0_i$
- male: $\beta_0 + \beta_2 + (\beta_1 + \beta_3) F0_i$

Dummy Coding

```r
hgs2 <- hgs |>
  mutate(sex_male = if_else(sex == "male", 1, 0))

## double check
hgs2 |> distinct(sex, sex_male)
```

```
# A tibble: 2 × 2
  sex    sex_male
  <chr>     <dbl>
1 male          1
2 female        0
```

Analysis

```r
lm(HGS ~ sex_male * F0, hgs2) |> summary()
```

```

Call:
lm(formula = HGS ~ sex_male * F0, data = hgs2)

Residuals:
    Min      1Q  Median      3Q     Max 
-21.859  -3.540  -0.421   3.361  33.163 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 29.75789    6.50985   4.571 8.14e-06 ***
sex_male    15.91254    7.87733   2.020   0.0446 *  
F0          -0.02508    0.02965  -0.846   0.3985    
sex_male:F0 -0.01642    0.04847  -0.339   0.7351    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Categorical-by-Categorical Interactions

Factorial designs

- Used to address a question involving more than one factor that can influence a DV, with each factor acting alone *or in combination with other factors*
  - What are the effects of cognitive therapy and drug therapy on mood in depressed individuals?
  - Do male and female students learn better with male or female teachers?

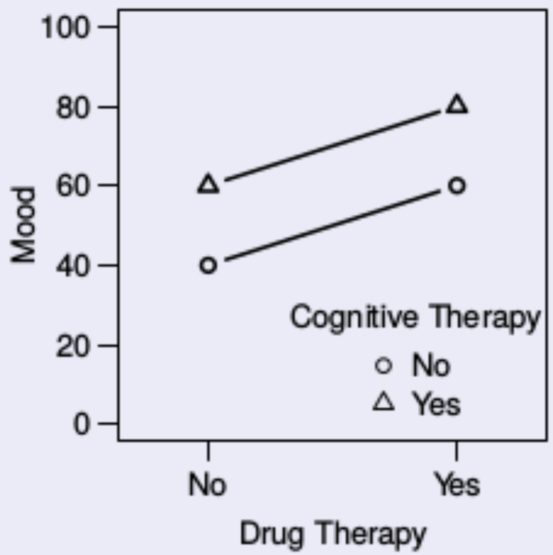
Full Factorial Designs

- A study has a full factorial design if it has more than one IV and the levels of the IVs are "fully crossed"
- designs are designated using RxC (row-by-column) format
- *cell:* unique combination of the levels of the factors

	Factor B	
	Level B_1	Level B_2
Level A_1	$A_1 B_1$	$A_1 B_2$
Level A_2	$A_2 B_1$	$A_2 B_2$

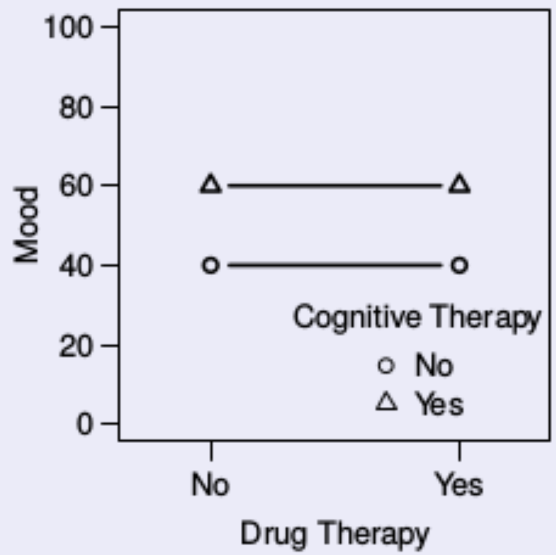
Factorial Plots and Interpretation

Scenario A



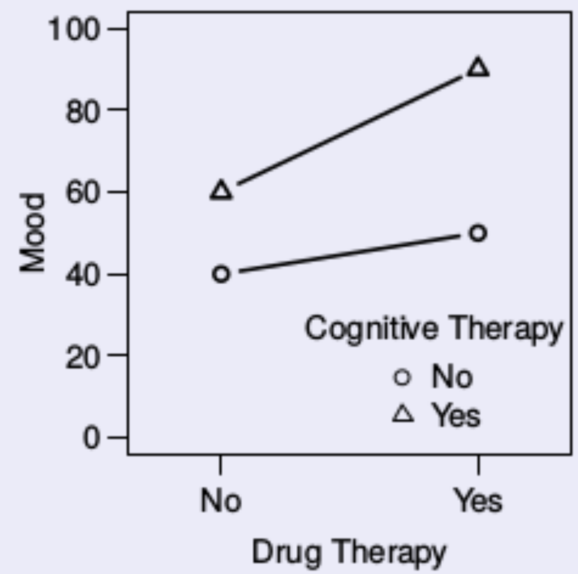
		Drug: No	Drug: Yes	
Cog.	No	40	60	50
	Yes	60	80	70
		50	70	

Scenario B



		Drug: No	Drug: Yes	
Cog.	No	40	40	40
	Yes	60	60	60
		50	50	

Scenario C



		Drug: No	Drug: Yes	
Cog.	No	40	50	45
	Yes	60	90	75
		50	70	

Effects in Factorial Designs

- Main Effects: tests of /marginal means/
  - $H_0 : \mu_{A_1} = \mu_{A_2}$
  - $H_0 : \mu_{B_1} = \mu_{B_2}$
- Simple Effects: effect of factor at level of other
  - eff of B at A_1, $H_0 : \mu_{A_1 B_1} = \mu_{A_1 B_2}$
  - eff of B at A_2, $H_0 : \mu_{A_2 B_1} = \mu_{A_2 B_2}$
- Interaction: equivalence of simple effects
  - $H_0 : \mu_{A_1 B_2} - \mu_{A_1 B_1} = \mu_{A_2 B_2} - \mu_{A_2 B_1}$

A fallacy

> "The percentage of neurons showing cue-related activity increased with training in the mutant mice ($p < 0.05$), but not in the control mice ($p > 0.05$)."

Saying the simple effect is significant in one case but not in another does not imply that the simple effects are statistically different!

Gelman, A., & Stern, H. (2012). The difference between "significant" and "not significant" is not itself statistically significant. *The American Statistician*, *60*, 328–331.

Nieuwenhuis, S., Forstmann, B. U., & Wagenmakers, E. J. (2011). Erroneous analyses of interactions in neuroscience: a problem of significance. *Nature Neuroscience*, *14*, 1105-1107.

Coding categorical predictors

Scheme	A_1	A_2
Treatment (dummy)	0	1
Sum	-1	1
Deviation	$-\frac{1}{2}$	$\frac{1}{2}$

Choice of a coding scheme impacts interpretation of:

1. the intercept term; and
2. the interpretation of the tests for all but the highest-order effects and interactions in a factorial design.

term	treatment	sum	deviatio
μ	$\bar{Y}_{111}$	$\bar{Y}_{...}$	$\bar{Y}_{...}$
A	$\bar{Y}_{211} - \bar{Y}_{111}$	$\frac{(\bar{Y}_{2..} - \bar{Y}_{1..})}{2}$	$\bar{Y}_{2..} - \bar{Y}$
B	$\bar{Y}_{121} - \bar{Y}_{111}$	$\frac{(\bar{Y}_{.2.} - \bar{Y}_{.1.})}{2}$	$\bar{Y}_{.2.} - \bar{Y}$
C	$\bar{Y}_{112} - \bar{Y}_{111}$	$\frac{(\bar{Y}_{..2} - \bar{Y}_{..1})}{2}$	$\bar{Y}_{..2} - \bar{Y}$
AB	$(\bar{Y}_{221} - \bar{Y}_{121}) - (\bar{Y}_{211} - \bar{Y}_{111})$	$\frac{(\bar{Y}_{22.} - \bar{Y}_{12.}) - (\bar{Y}_{21.} - \bar{Y}_{11.})}{4}$	$(\bar{Y}_{22.} - \bar{Y}_{12.}) - ($
AC	$(\bar{Y}_{212} - \bar{Y}_{211}) - (\bar{Y}_{112} - \bar{Y}_{111})$	$\frac{(\bar{Y}_{2.2} - \bar{Y}_{1.2}) - (\bar{Y}_{2.1} - \bar{Y}_{1.1})}{4}$	$(\bar{Y}_{2.2} - \bar{Y}_{1.2}) - ($
BC	$(\bar{Y}_{122} - \bar{Y}_{112}) - (\bar{Y}_{121} - \bar{Y}_{111})$	$\frac{(\bar{Y}_{.22} - \bar{Y}_{.12}) - (\bar{Y}_{.21} - \bar{Y}_{.11})}{4}$	$(\bar{Y}_{.22} - \bar{Y}_{.12}) - ($

More than 2 levels ($k > 2$)?

Arbitrarily choose one as "baseline".

Dummy ($k = 3$) Deviation ($k = 3$)

scheme code

	A2v1	A3v1
A_1	0	0
A_2	1	0
A_3	0	1

Linear Model Formulas in R

```
y ~ a * b * c
```

is shorthand for

```
y ~ a + b + c + a:b + a:c + b:c + a:b:c
```

when factor A has 3 levels:

```
y ~ (a1 + a2) * b
```

is shorthand for

```
y ~ a1 + a2 + b + a1:b + a2:b
```

when A and B have 3 levels:

```
y ~ (a1 + a2) * (b1 + b2)
```

is shorthand for

```
y ~ a1 + a2 + b1 + b2 + a1:b1 + a1:b2 +
a2:b1 + a2:b2
```