Linear Mixed-Effects Models

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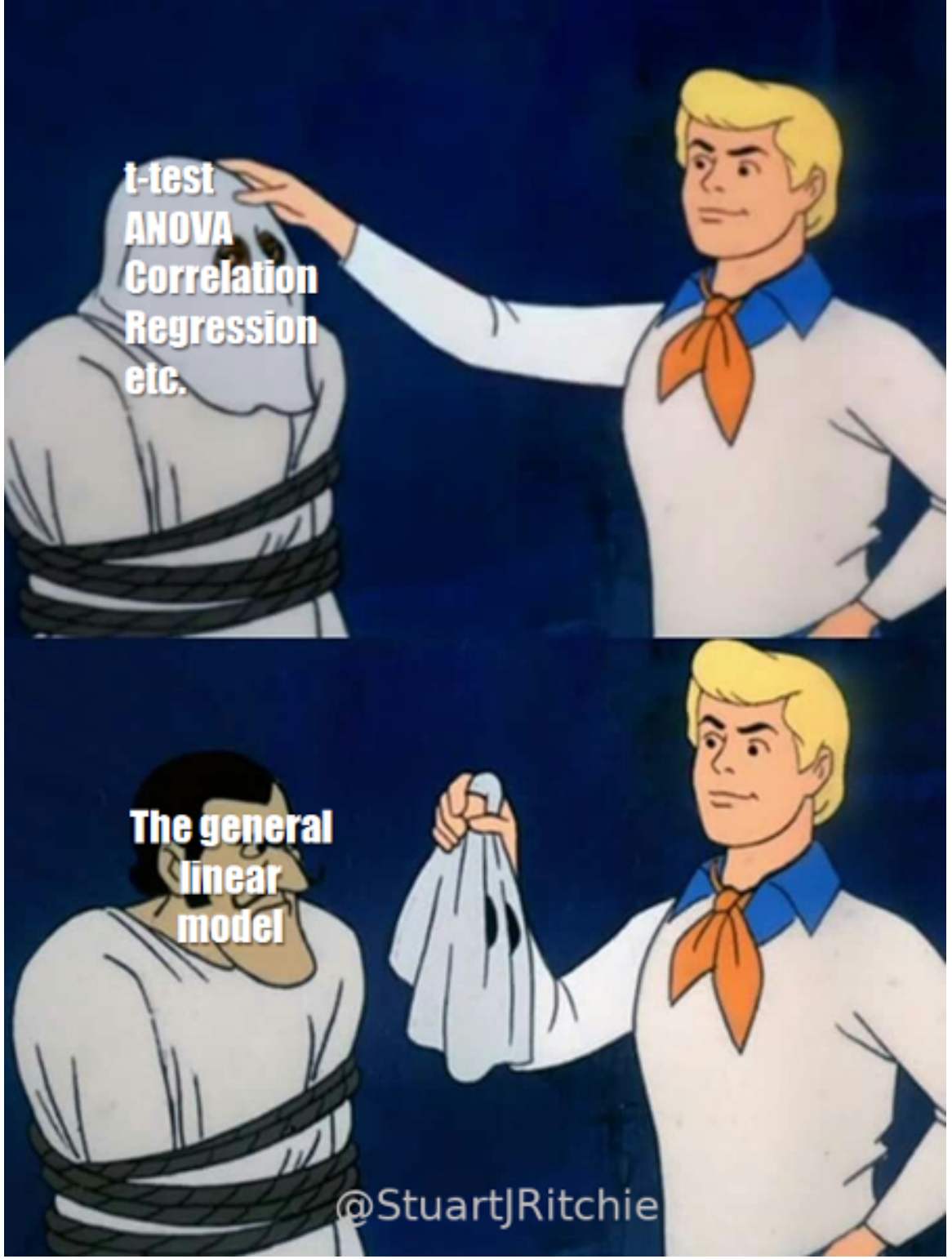
Overview

- introduction to linear mixed-effects modeling
- example: sleepstudy data
  - distinguish complete pooling, no pooling and partial pooling approaches
- understand the DGP behind multi-level data and estimation with `lme4::lmer()`


t-test
ANOVA
Correlation
Regression
etc.
The general linear model
@StuartJRitchie

Multilevel data

- All parametric models assume model residuals are IID ("independently and identically distributed")
- Data often has 'clusters' of correlated observations due to
  - natural clustering
  - multistage sampling

pros and cons of LMEMs

Pros

- powerful and expressive
- modeling of continuous & categorical predictors
- unbalanced/missing data (partial pooling)
- multiple random factors
- discrete DVs and/or non-normal distributions

Cons

- complex
- estimated iteratively and may not converge!

Understanding multi-level modeling

Belenky et al. (2003)

Worked example: Belenky et al. (2003) `sleepstudy` data

Free Access

Patterns of performance degradation and restoration during sleep restriction and subsequent recovery: a sleep dose-response study

Gregory Belenky, Nancy J. Wesensten, David R. Thorne, Maria L. Thomas ... See all authors

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Belenky et al. (2003)

SUMMARY

Daytime performance changes were examined during chronic sleep restriction or augmentation and following subsequent recovery sleep. Sixty-six normal volunteers spent either 3 (*n* = 18), 5 (*n*= 16), 7 (*n* = 16), or 9 h (*n* = 16) daily time in bed (TIB) for 7 days (restriction/augmentation) followed by 3 days with 8 h daily TIB (recovery). In the 3-h group, speed (mean and fastest 10% of responses) on the psychomotor vigilance task (PVT) declined, and PVT lapses (reaction times greater than 500 ms) increased steadily across the 7 days of sleep restriction. In the 7- and 5-h groups speed initially declined, then appeared to stabilize at a reduced level; lapses were increased only in the 5-h group. In the 9-h group, speed and lapses remained at baseline levels. During recovery, PVT speed in the 7- and 5-h groups (and lapses in the 5-h group) remained at the stable, but reduced levels seen during the last days of the experimental phase, with no evidence of recovery. Speed and lapses in the 3-h group recovered rapidly following the first night of recovery sleep; however, recovery was incomplete with speed and lapses stabilizing at a level comparable with the 7- and 5-h groups. Performance in the 9-h group remained at baseline levels during the recovery phase. These results suggest that the brain adapts to chronic sleep restriction. In mild to moderate sleep restriction this adaptation is sufficient to stabilize performance, although at a reduced level. These adaptive changes are hypothesized to restrict brain operational capacity and to persist for several days after normal sleep duration is restored, delaying recovery.

Belenky et al. (2003)

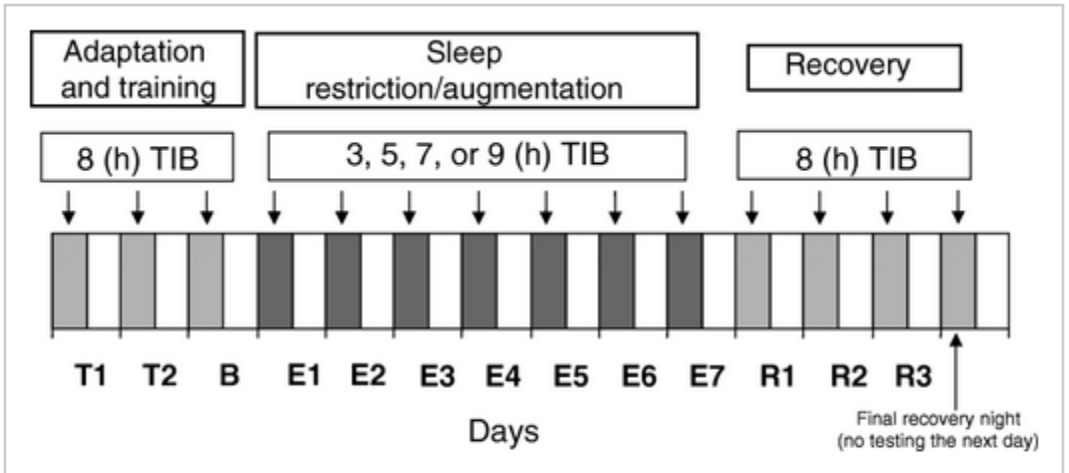


Figure 1 Open in figure viewer | PowerPoint

Study experimental design, showing nightly time in bed across days (adaptation/training, baseline, experimental phase, recovery phase).

TIB: time in bed

Psychomotor vigilance test

Psychomotor vigilance test

The PVT measures simple reaction time to a visual stimulus, presented approximately 10 times/minute (interstimulus interval varied from 2 to 10 s in 2-s increments) for 10 min and implemented in a thumb-operated, hand-held device (Dinges and Powell 1985). Subjects attended to the LED timer display on the device and pressed the response button with the preferred thumb as quickly as possible after the appearance of the visual stimulus. The visual stimulus was the LED timer turning on and incrementing from 0 at 1-ms intervals. In response to the subject's button press, the LED timer display stopped incrementing and displayed the subject's response latency for 0.5 s, providing trial-by-trial performance feedback. At the end of this 0.5-s interval the display turned off for the remainder of the foreperiod preceding the next stimulus. Foreperiods varied randomly from 2 to 10 s. Dependent measures, averaged or summed across the 10-min PVT session, included mean speed (reciprocal of average response latency), number of lapses (lapse = response latency exceeding 500 ms), and mean speed for the fastest 10% of all responses.

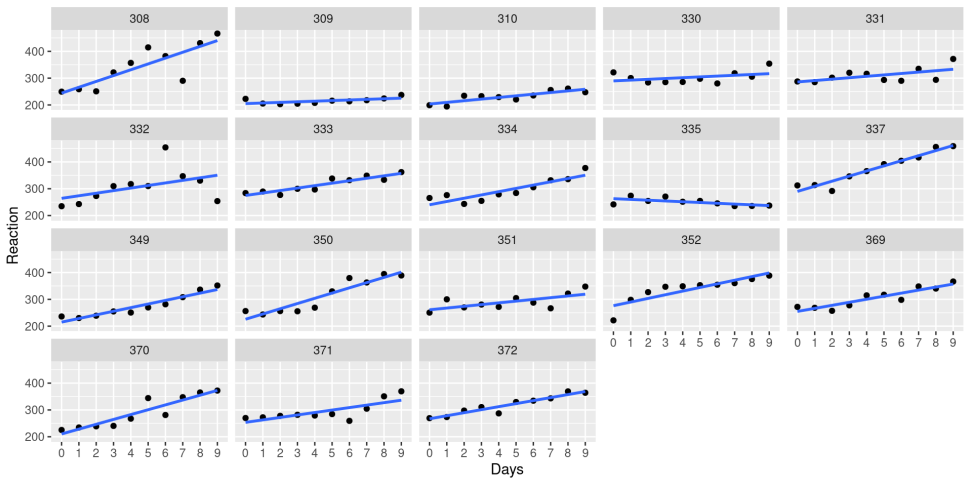
lme4::sleepstudy

$$Y_{ij} = \beta_0 + \beta_1 X_{ij} + e_{ij}$$

but: observations within subject *not* independent

```
library("lme4")

ggplot(sleepstudy, aes(Days, Reaction)) +
  geom_point() +
  geom_smooth(method = "lm", se = FALSE) +
  scale_x_discrete(limits = 0:9) +
  facet_wrap(~Subject)
```



Approaches to ML data

1. complete pooling
    - ignore dependencies in the data
2. no pooling
    - account for dependencies by fitting each subject independently
3. partial pooling
    - account for dependencies by explicitly modeling them
    - fit for each subject informed by the fits for other subjects

Multilevel Simulation

Model Specification

complete pooling

fixed intercept (gamma_00)

225 300 375

225 255 285 315 345 375

random intercept SD (tau_00)

0 15 30

0 3 6 9 12 15 18 21 24 27 30

fixed slope (gamma_10)

-20 -10 20

-20 -12 -8 -4 0 4 8 12 16 20

random slope SD (tau_11)

0 15 30

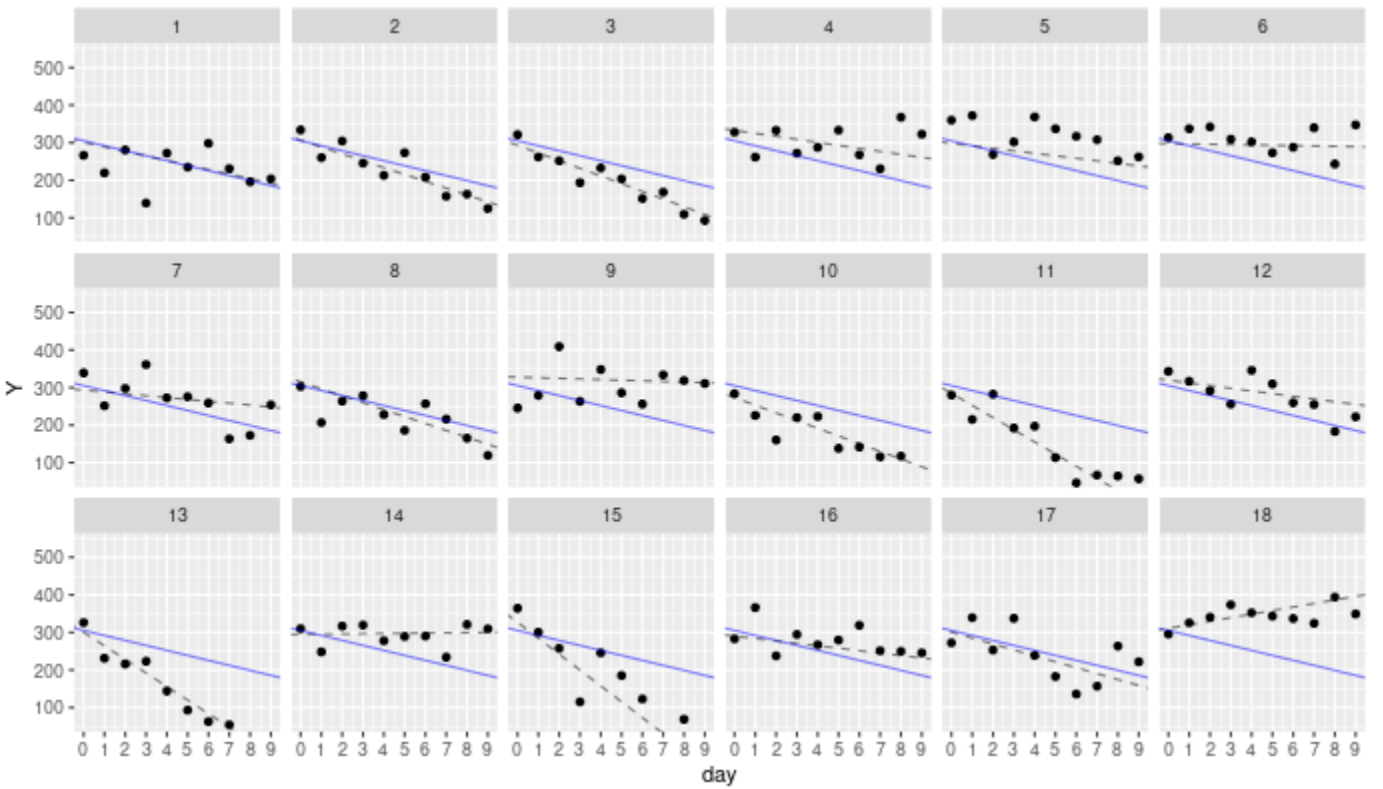
0 3 6 9 12 15 18 21 24 27 30

error SD (sigma)

0 40 80

0 8 16 24 32 40 48 56 64 72 80

plots | data | GLM | estimates | residuals



Zoom

10 50 100

10 20 30 40 50 60 70 80 90 100

DGP and estimation

GLM for sleepstudy

Level 1:

$$Y_{ij} = \beta_0 + \beta_1 X_{ij} + e_{ij}$$

Level 2:

$$\beta_0 = \gamma_{00} + S_{0i}$$

$$\beta_1 = \gamma_{10} + S_{1i}$$

Variance Components

$$\langle S_{0i}, S_{1i} \rangle \sim N(\langle 0, 0 \rangle, \boldsymbol{\Sigma})$$

$$\boldsymbol{\Sigma} = \begin{pmatrix} {\tau_{00}}^2 & \rho\tau_{00}\tau_{11} \\ \rho\tau_{00}\tau_{11} & {\tau_{11}}^2 \end{pmatrix}$$

$$e_{ij} \sim N(0, \sigma^2)$$

Estimation

```
library("lme4")

mod <- lmer(Reaction ~ Days + (Days | Subject),
            data = sleepstudy)

summary(mod)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: Reaction ~ Days + (Days | Subject)
   Data: sleepstudy

REML criterion at convergence: 1743.6

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.9536 -0.4634  0.0231  0.4634  5.1793

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Subject  (Intercept) 612.10   24.741
          Days         35.07    5.922   0.07
 Residual             654.94   25.592
Number of obs: 180, groups:  Subject, 18

Fixed effects:
            Estimate Std. Error t value
(Intercept)  251.405      6.825  36.838
Days          10.467      1.546   6.771
```

model syntax

DV ~ iv1 + iv2 + (iv1 | random_factor)

```
lmer(Reaction ~ Days + (1 | Subject), sleepstudy) # (1) random intercept

lmer(Reaction ~ Days + (1 + Days | Subject), sleepstudy) # (2) random slope mod
lmer(Reaction ~ Days + (Days | Subject), sleepstudy) # (3) identical to (2)

lmer(Reaction ~ Days + (1 | Subject) + (0 + Days | Subject)) # (4) zero-covaria
lmer(Reaction ~ Days + (Days || Subject), sleepstudy) # (5) identical to (4)
```

p-values: model comparison

```
mod1 <- lmer(Reaction ~ Days + (Days | Subject),
             sleepstudy, REML = FALSE)
mod2 <- lmer(Reaction ~ (Days | Subject),
             sleepstudy, REML = FALSE)

## or:
## mod2 <- update(mod1, . ~ . -Days)
anova(mod1, mod2)
```

```
# A tibble: 2 × 8
   npar   AIC   BIC logLik deviance Chisq    Df `Pr(>Chisq)`
  <dbl> <dbl> <dbl>  <dbl>    <dbl> <dbl> <dbl>        <dbl>
1     5 1785. 1801.  -888.    1775.  NA      NA  NA
2     6 1764. 1783.  -876.    1752.  23.5     1   0.00000123
```

NB: use REML=FALSE

p-values: t-as-z

```
mod <- lmer(Reaction ~ Days + (Days | Subject), sleepstudy, REML = FALSE)

stderr <- sqrt(diag(vcov(mod)))
tvals <- fixef(mod) / stderr

2 * (1 - pnorm(abs(tvals)))
```

```
 (Intercept)         Days
0.000000e+00 3.218759e-12
```

often reported as “Wald z”