MANIPULABILITY AND REDUNDANCY CONTROL OF ROBOTIC MECHANISMS

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The concept of manipulability measure of robotic mechanisms is discussed. This is a measure of manipulating ability of robotic mechanisms in positioning and orienting the end-effectors. Some properties of this measure are established and the best postures of various types of manipulators from the viewpoint of this measure are given. A four-joint wrist mechanism is also analyzed with respect to its manipulability, and a control algorithm for this redundant mechanism is developed .

INTRODUCTION

Determination of the mechanism and size of a robot manipulator at the design stage, and determination of the posture of the manipulator in the workspace for performing a given task at the operation stage, have largely been done on the basis of experience and intuition. One of various measures used for these determinations, seems to be the easiness of changing arbitrarily the position and orientation of the end-effector at the tip of the manipulator. It will be beneficial for design and control of robots and for task planning if we have a quantitative measure of manipulating ability of robot arms in positioning and orienting the end-effectors. The concept manipulability measure" has been proposed by the author [1] as one such measure.

Manipulability measure is a generalized concept of the determinant of Jacobian matrix, the latter having been used by Paul and Stevenson [2] as a measure of degeneracy for the analysis of robot wrists. They have shown that any three-joint orienting system (wrist) has two cone regions of degeneracy in which its ability to orient the end-effector is poor. One way to overcome this difficulty is to use a 4 or more degrees of freedom (d.o.f.) wrist mechanism.

In the present paper, some properties of the manipulability measure and its utilization for determining the best postures of various types of manipulators will be discussed. A four-joint wrist mechanism will also be analyzed from the viewpoint of the manipulability measure, and a control algorithm for this redundant mechanism will be given.

MANIPULABILITY MEASURE

In this section the definition of manipulability measure is given, and some properties of this measure are discussed.

We consider a manipulator with n d.o.f. whose joint variables are denoted by θ_i, i=1, 2, ..., n. We assume that the position and/or orientation of its end-effector can be described by m variables r_j, j=1, 2, ..., m ($m \leq n$) with respect to a reference orthogonal coordinate frame, and that the relation between θ_i and r_j is given by

$$r = f(\theta) \tag{1}$$

where $\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T$ is the joint vector, $r = [r_1, r_2, \ldots, r_m]^T$ is the manipulation vector and the superscript ^{T} denotes the transpose. The end-effector velocity v corresponding to r is related to joint velocity $\dot{\theta}$ by

$$v = J(\theta)\dot{\theta} \tag{2}$$

where $v \in R^m$ (m dimensional Euclidian space), $\dot{\theta} = d\theta/dt \in R^n$, and $J(\theta) \in R^{m \times n}$ (the set of all $m \times n$ real matrices). The matrix $J(\theta)$ is called the Jacobian [3].

[Remark 1] Let r and v be given by $r^T = [r_p{}^T \; r_a{}^T]$ and $v^T = [v_p{}^T \; v_a{}^T]$, where r_p and v_p are related to the position of the end-effector and r_a and v_a are related to its orientation. Then we can take $v_p = \dot{r}_p$. But for the orientation, since v_p is usually defined as the angular velocity around the fixed reference axes, v_a is not usually equal to $\dot{r}_a$. If the rotations are around some fixed axes, then we can take $v_a = \dot{r}_a$, and in this case we have $J(\theta) = df(\theta)/d\theta^T$.

We can assume, without any loss of generality that the following condition is satisfied.

$$\max_{\theta} \; \text{rank}\, J(\theta) = m \tag{3}$$

and we say that the degree of redundancy of this manipulator is (n-m). More detailed discussion on the degree of redundancy and a related concept of redundant space can be found in Hanafusa et al. [4].

If for some θ^*,

$$\text{rank}\, J(\theta^*) < m \tag{4}$$

then we say that the manipulator is in a singular state. This state θ^* is not desirable since the manipulation vector r cannot move in a certain direction, meaning that the manipulability is seriously deteriorated.

We now give the following definition.

Definition [1] : A scalar value w given by

$$w = \sqrt{\det J(\theta) J^T(\theta)} \tag{5}$$

is called the **manipulability measure** at state θ with respect to manipulation vector r.

The following three facts have been established in Yoshikawa [1].

(i) Let the singular value decomposition [5] of J be

$$J = U \Sigma V^T \tag{6}$$

where $U \in R^{m \times m}$ and $V \in R^{n \times n}$ are orthogonal matrices and

$$\Sigma = \left[\begin{array}{cccc|c} \sigma_1 & & 0 & & \\ & \sigma_2 & & & \\ & & \ddots & & 0 \\ 0 & & & \sigma_m & \end{array}\right] \in R^{m \times n} \tag{7}$$

with

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_m \geq 0. \tag{8}$$

Then the measure w can be expressed as the product of the singular values $\sigma_1, \sigma_2, \dots, \sigma_m$:

$$w = \sigma_1 \sigma_2 \cdots \sigma_m \tag{9}$$

(ii) We can show that the subset of all realizable velocity v in the space R^m using joint velocity $\dot{\theta}$ such that $\| \dot{\theta} \|^2 = \dot{\theta}_1^2 + \dot{\theta}_2^2 + \cdots + \dot{\theta}_n^2 \leq 1$ is an ellipsoid with principal axes $\sigma_1 u_1, \sigma_2 u_2, \dots, \sigma_m u_m$, where $u_i \in R^m$ is the i-th column vector of U, i.e., $[u_1\ u_2 \cdots u_m] = U$. This ellipsoid can be called the **manipulability ellipsoid** and could be a good means for the analysis, design and control of robot manipulators. The volume v_e of this ellipsoid is given by

$$v_e = d\ w \tag{10a}$$
$$d = \{\pi^{m/2} / \Gamma[(m/2)+1]\} \tag{10b}$$

where $\Gamma(\cdot)$ is the gamma function. Therefore, w is equal to the volume of the manipulability ellipsoid except for the constant coefficient d. The manipulability ellipsoid can be expressed by

$$\{v \mid v^T (JJ^T)^{+} v \leq 1, \text{ and } v \in \mathrm{Im}(J)\} \tag{11}$$

Except at the singular states where the volume of the ellipsoid becomes zero, the ellipsoid can be defined by

$$v^T (JJ^T)^{-1} v \leq 1 \tag{12}$$

instead of (11). The measure can also be regarded as a distance from the singular states since at singular states w takes the minimum value 0.

(iii) When m=n, i.e., when we consider nonredundant manipulators, the measure w reduces to

$$w = |\det J(\theta)| \tag{13}$$

This type of measure has been used by Paul and Stevenson [2] for analysis of robot wrists.

Now we will make clear several new properties of the measure.

(iv) We will discuss the relation between the measure w and the maximum velocities of joints. So far, we have implicitly assumed that the maximum velocities of all joints are equal to one. When this assumption does not hold, the velocities of joints should be normalized. After fixing a set of units for distance, angle, and time (for example, m, rad, sec), we denote the maximum (angular) velocity of joint i by $\dot{\theta}_{i0}$. We also select the desirable maximum (angular) velocity of each manipulation variable v_{j0} taking into consideration the class of tasks which the manipulator is supposed to perform. Then letting

$$\hat{\dot{\theta}} = [\hat{\dot{\theta}}_1, \hat{\dot{\theta}}_2, \dots, \hat{\dot{\theta}}_n]^T, \quad \hat{\dot{\theta}}_i = \dot{\theta}_i / \dot{\theta}_{i0} \tag{14}$$
$$\hat{v} = [\hat{v}_1, \hat{v}_2, \dots, \hat{v}_m]^T, \quad \hat{v}_j = v_j / v_{j0} \tag{15}$$

we obtain

$$\hat{v} = \hat{J}(\theta) \hat{\dot{\theta}} \tag{16}$$

where

$$\hat{J}(\theta) = T_r J(\theta) T_\theta^{-1} \tag{17}$$
$$T_r = \mathrm{diag}[1/v_{j0}] \in R^{m \times m} \tag{18}$$
$$T_\theta = \mathrm{diag}[1/\dot{\theta}_{i0}] \in R^{n \times n} \tag{19}$$

Since $|\hat{\dot{\theta}}_i| \leq 1$ and $|\hat{v}_j| \leq 1$, we can define the manipulability measure w using the normalized Jacobian $\hat{J}(\theta)$. Defining the measure w for $J(\theta) T_\theta^{-1}$ as $\hat{w}_\theta$, and that for $\hat{J}(\theta) = T_r J(\theta) T_\theta^{-1}$ as $\hat{w}$, we have

$$\hat{w} = [\prod_{j=1}^{m} (1/v_{j0})] \hat{w}_\theta \tag{20}$$

and especially when n=m, we have

$$\hat{w} = [\prod_{i=1}^{n} (\dot{\theta}_{i0} / v_{i0})] w \tag{21}$$

Hence the transformation T_r has only the effect of multiplying the scalar value $\prod_{j=1}^{m} (1/v_{j0})$. The relative shape of w as a function of θ is independent of the transformation T_r. Furthermore, when n=m, the relative shape of w is independent of both T_r and T_θ.

(v) Letting h denote the force (and moment) applied to an object by the end-effector and letting τ denote the necessary joint driving force (and torque), we have [2]

$$\tau = J^T h \tag{22}$$

Hence the set of manipulating force h which is realizable by a joint driving force τ such that $\|\tau\| \leq 1$, is given by (except for the singular states)

$$h^T JJ^T h \leq 1 \tag{23}$$

This set is an ellipsoid that can be called the manipulating force ellipsoid. Its principal axes are given by $(1/\sigma_1)u_1, (1/\sigma_2)u_2, \dots, (1/\sigma_m)u_m$, and its volume is d/w. In other words, the volume of the manipulating force ellipsoid is in the inverse proporton to that of the manipulability ellipsoid. Also the length of each principal axis of the manipulability ellipsoid is in inverse proportion to the magnitude of realizable manipulating force in the direction of this axis. This means that the direction in which a large manipulating force can be generated is the one in which the manipulability is poor and vice versa.

[Remark 2] The force ellipsoid defined by Asada [6] is equivalent to the above manipulating force ellipsoid when the motor constants of all motors are the same.

[**Remark 3**] Some other indexes than manipulability measure representing different features of the manipulability ellipsoid will also be useful for detailed evaluation of the manipulation ability of robotic mechanisms. Two examples are the condition number σ_1/σ_m and the minimum singular value σ_m. The former is a measure of directional unifomity of the ellipsoid, or in other words, closeness to the sphere. The latter means the upper bound of the velocity with which the end-effector can be moved in all direction. The condition number has been used by Salisbury and Craig [7] as a measure of workspace quality. Klein [8] has done a primary comparison study of these measures. We feel that, although all these indexes will be necessary for detailed analysis and/or design of robotic mechanisms, it will usually be enough to use only the manipulability measure for control or task planning.

BEST POSTURES OF VARIOUS ROBOTIC MECHANISMS FROM THE VIEWPOINT OF MANIPULABILITY MEASURE

Two-Joint Link Mechanisms

In this section, the manipulability measure is calculated for various robotic mechanisms, and the best postures and the best points in the workspace of these mechanisms from the viewpoint of manipulability measure are determined.

First of all, a two-joint link mechanism shown in Fig. 1, which is the simplest case of multi-joint manipulators, will be considered. When the hand position $[x, y]^T$ is taken as the manipulation vector r, the Jacobian matrix is given by

$$J(\theta) = \begin{bmatrix} \ell_1 c_1 + \ell_2 c_{12} & \ell_2 c_{12} \\ -\ell_1 s_1 - \ell_2 s_{12} & -\ell_2 s_{12} \end{bmatrix} \quad (24)$$

where $c_1 = \cos\theta_1$, $c_{12} = \cos(\theta_1+\theta_2)$, $s_1 = \sin\theta_1$, $s_{12} = \sin(\theta_1+\theta_2)$. Hence the manipulability measure w is

$$w = |\det J(\theta)| = \ell_1 \ell_2 |s_2| \quad (25)$$

✓ Therefore, the manipulator takes its best posture when $\theta_2 = \pm 90°$, for any given values of ℓ_1, ℓ_2, and θ_1. If the lengths ℓ_1 and ℓ_2 can be specified under the condition of constant total length, i.e., $\ell_1 + \ell_2$ = const., the ✓ manipulability measure attains its maximum when $\ell_1 = \ell_2$ for any given θ_1 and θ_2.

Fig. 2(a), (b), and (c) show the manipulability ellipsoid, the manipulability measure, and the manipulating force ellipsoid, respectively, for the case $\ell_1 = \ell_2 = 1$ and $\dot\theta_{10} = \dot\theta_{20} = 1$, taking ℓ_a, the distance between the first joint and the hand, as a parameter. The magnitude and direction of the realizable tip velocity and manipulating force can be easily understood from the figure.

When the human arm is regarded as a two-joint link mechanism by neglecting the d.o.f. of sideward direction at the shoulder and the d.o.f. of the wrist, it approximately satisfies the relation $\ell_1 = \ell_2$. Moreover, when we handle some object by our hands, the elbow angle is usually in the neighborhood of 90°. Hence it could be said that people are unconsciously taking the best arm posture from the viewpoint of manipulability.

SCARA Type Robot Manipulators

Consider the SCARA type manipulators with four d.o.f. shown in Fig. 3. Let $r = [x, y, z, \alpha]^T$, where $[x, y, z]^T$ is the hand position and α is the rotational angle of the hand about z

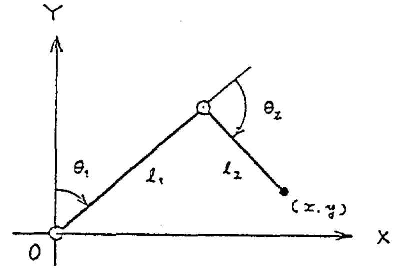
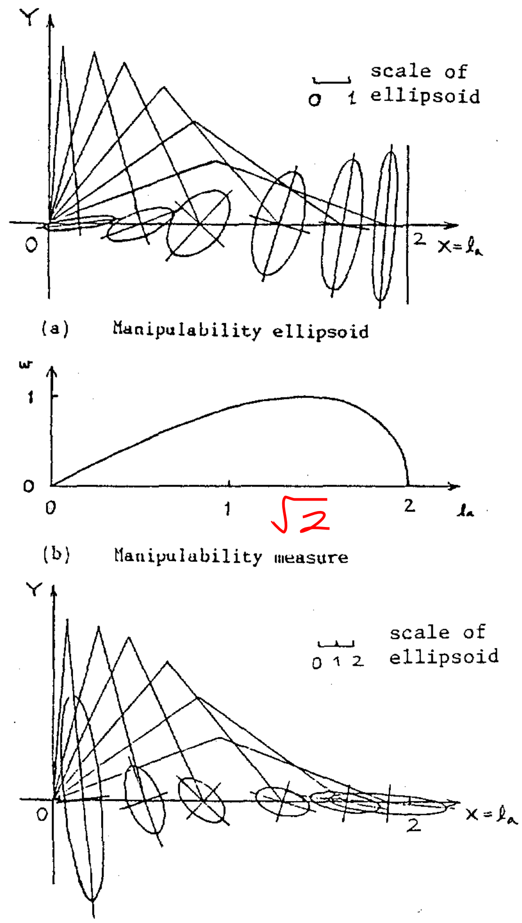


Fig. 1 Two-joint link mechanism



(a) Manipulability ellipsoid

(b) Manipulability measure

(c) Manipulating force ellipsoid

Fig. 2 Manipulability ellipsoid, manipulability measure, and manipulating force ellipsoid of two-joint link mechanism axis. The Jacobian matrix for this case is given by

$$J(\theta) = \begin{bmatrix} \ell_1 c_1 + \ell_2 c_{12} & \ell_2 c_{12} & 0 & 0 \\ -\ell_1 s_1 - \ell_2 s_{12} & -\ell_2 s_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \tag{26}$$

Hence

$$w = \ell_1 \ell_2 |s_2|$$

Therefore as in the case of two-joint link mechanism of Fig. 1, the best posture is given by $\theta_2 = \pm 90°$, for any given values of ℓ_1, ℓ_2, θ_1, θ_3, and θ_4. Also under the constraint of $\ell_1 + \ell_2 =$ const., the manipulability measure attains its maximum when $\ell_1 = \ell_2$, $\theta_2 = \pm 90°$. Notice that there are many commercial SCARA type manipulators satisfying $\ell_1 = \ell_2$.

PUMA Type Robot Manipulators

Most of the PUMA type robot manipulators commercially available today have five or six d.o.f.. Many of them have links with some displacements in the direction of joint axes. However, we consider only the main 3 joints shown in Fig. 4 neglecting the d.o.f. placed at the wrist and neglecting the displacements in the direction of joint axes. The joint vector is $\theta = [\theta_1, \theta_2, \theta_3]^T$. The manipulation vector is taken to be $r = [x, y, z]^T$. Then the Jacobian matrix is

$$J(\theta) = \begin{bmatrix} -s_1(\ell_2 s_2 + \ell_3 s_{23}) & c_1(\ell_2 c_2 + \ell_3 c_{23}) & c_1 \ell_3 c_{23} \\ c_1(\ell_2 s_2 + \ell_3 s_{23}) & s_1(\ell_2 c_2 + \ell_3 c_{23}) & s_1 \ell_3 c_{23} \\ 0 & -(\ell_2 s_2 + \ell_3 s_{23}) & -\ell_3 s_{23} \end{bmatrix} \tag{27}$$

and the manipulability measure is

$$w = \ell_2 \ell_3 |(\ell_2 s_2 + \ell_3 s_{23}) s_3| \tag{28}$$

The best posture for given ℓ_2 and ℓ_3 is obtained as follows. First, θ_1 is not related to w and can take any value. Second, from $\partial w / \partial \theta_2 = 0$ we have

$$\tan\theta_2 = \frac{\ell_2 + \ell_3 c_3}{\ell_2 s_3} \tag{29}$$

This means that the tip of arm should be on the xy plane, i.e., at the same height as the second joint. This can further be interpreted as making maximum the contribution of the angular velocity of the first joint to the manipulability measure.

Substituting (29) into (28) yields

$$w = \ell_2 \ell_3 \sqrt{\ell_2^2 + \ell_3^2 + 2\ell_2 \ell_3 c_3}\, |s_3| \tag{30}$$

The value of θ_3 which maximizes w is given by

$$c_3 = c_3^* = \{\sqrt{(\ell_2^2 + \ell_3^2)^2 + 12\ell_2^2 \ell_3^2} - (\ell_2^2 + \ell_3^2)\} / 6\ell_2 \ell_3 \tag{31}$$

Fig. 5 shows the best postures for the cases $\ell_3 = \tilde{\gamma}\ell_2$, $\tilde{\gamma} = 0.5, 1, 2$ (only those satisfying $0° \leq \theta_2 \leq 90°$ are shown in the figure). If the manipulator is regarded as a two-joint mechanism consisting of θ_2 and θ_3, the optimal angle for θ_3 is 90° from the previous discussion. In the present case, however, the optimal θ_3 is smaller than 90°. The reason for this is that the contribution of $\dot{\theta}_1$ to w can be made larger by placing the tip of arm farther away from the first joint axis. For c_3^* given by (31), w is calculated as

$$w = (\ell_2 \ell_3)^{3/2} (1 - c_3^{*2}) |c_3^*|^{1/2}$$

Hence unde the constraint $\ell_2 + \ell_3 =$ const., the best ratio of ℓ_2 and ℓ_3 is again 1 to 1.

FOUR-JOINT REDUNDANT WRIST MECHANISM

Manipulability Analysis

As mentioned in Introduction, It has been shown in Paul and Stevenson[2] that any three-joint orienting system (wrist) has two cone regions of degeneracy in which its ability to orient the end-effector is poor. One way to overcome this difficulty will be to use a 4 or more d.o.f. wrist mechanism. One such wrist has already been developed by Itoh[9], but without detailed study of manipulating ability or control algorithm.

In this section, a four-joint wrist mechanism whose basic structure is shown in Fig. 6(a), is proposed and analyzed from the viewpoint of manipulability measure. This mechanism can be regarded as a universal joint with two rotational joints at its both ends. The unique figures of this structure are that the hand position is determined uniquely when the hand orientation is given, and that the universal joint part can be rotated freely without changing the hand orientation and position at all. The change of the hand position caused by the change of the hand orientation can usually be compensated by the

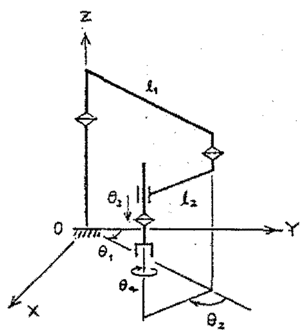


Fig.3 SCARA type robot

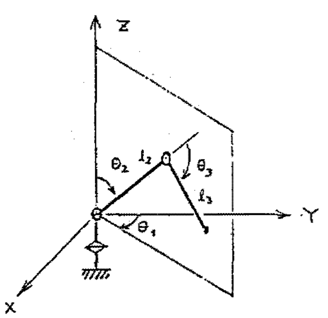


Fig.4 PUMA type robot

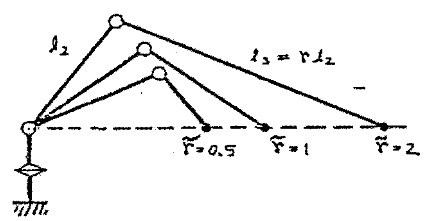


Fig.5 Best arm posture for PUMA type robot

control of arm to which the wrist is attached. Hence, in the following, we assume that the purpose of the wrist mechanism is only to control the orientation of the hand. We also assume that $\dot{\theta}_{i0}=\dot{\theta}_0$, i=1,2,3,4.

By attaching the coordinate frame Σ_i to link i and taking the configuration shown in Fig. 6(b) as the standard state where the joint vector $\theta=[\theta_1, \theta_2, \theta_3, \theta_4]^T=0$, the Jacobian matrix relating the joint velocity $\dot{\theta}$ and the angular velocity $v=[\omega_x, \omega_y, \omega_z]^T$ around the coordinate axes of hand coordinate frame Σ_4, is given by

$$J(\theta)=\begin{bmatrix} c_2s_3s_4-s_2c_4 & c_3s_4 & c_4 & 0 \\ c_2s_3c_4-s_2s_4 & c_3c_4 & -s_4 & 0 \\ c_2c_3 & -s_3 & 0 & 1 \end{bmatrix} \tag{32}$$

The manipulability measure for this case is

$$w=\sqrt{2(1-s_2{}^2s_3{}^2)} \tag{33}$$

Let the hand orientation be expressed by the Euler angles (α, β, γ) with respect to the base coordinate frame Σ_0. Then we have

$$\cos\beta=c_2c_3 \tag{34}$$

and

$$\sqrt{2} \geq w \geq \sqrt{2\{1-(1-|\cos\beta|)^2} \tag{35}$$

Note that β is the angle between the z_4 and z_0 axes as shown in Fig. 7. The maximum value of w, $w_{max}=\sqrt{2}$, is attained when $s_2=0$ or $s_3=0$, and the minimum value $w_{min}=\sqrt{2\{1-(1-|\cos\beta|)^2}$ is attained when $c_2{}^2=c_3{}^2=|\cos\beta|$. Fig. 8 shows the values w_{max} and w_{min} as functions of β in a polar coordinate form. From the figure, it is understood that when $\beta=90°$, the manipulability measure can vary very much depending on the joint angles. But since the maximum value of the measure is $\sqrt{2}$ for any β, the wrist has the possibility of avoiding degradation of manipulability by properly selecting the joint angles.

In designing a wrist with above basic structure, we tried to make the angle limit for θ_2 large in the plus side. This is for enlarging the workspace. When it is necessary to move θ_2 into the minus direction, we can change this motion to that in the θ_2 plus direction by rotating θ_1 for 180°. Fig. 9 shows a fabricated prototype of the wrist mechanism.

Redundancy Control

Now the control problem of this wrist mechanism will be discussed. We specify the task of the wrist as consisting of two subtasks. The subtask with the first priority is to realize the given desirable trajectory of hand orientation. The second subtask, which should be performed using the ability left to the wrist after achieving the first subtask, is to maximize a given performance criterion which implies a compromise of three objectives; (a) to keep the manipulability measure large, (b) to bring the joint angle θ_2 to the plus side, and (c) to keep all joints within their hardware limits.

The general solution of (2) for a given desired velocity v_d is given by

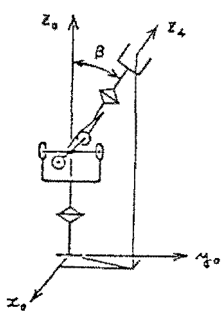


Fig.7 Angle β

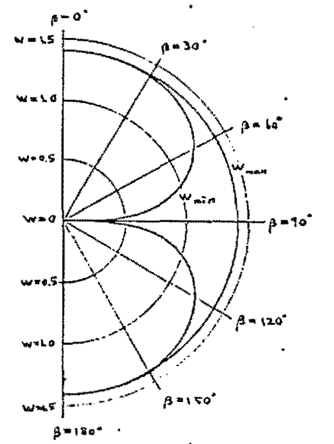


Fig.8 Maximum and minimum values of manipulability measure

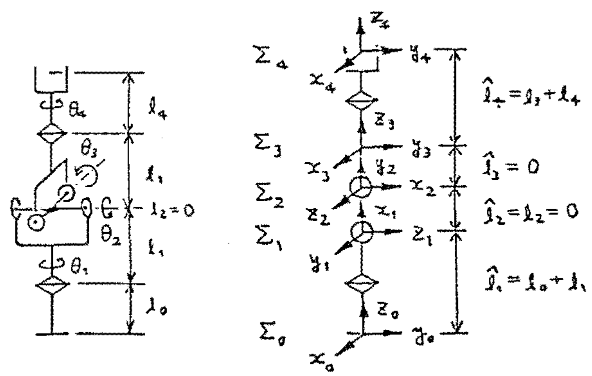


Fig.6 Four-joint wrist mechanism

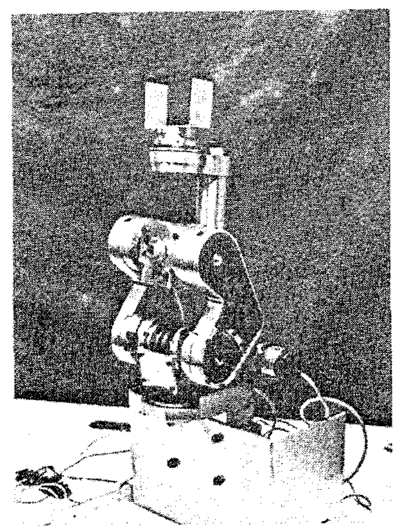


Fig.9 Four-joint wrist prototype

$$\dot{\theta} = J^{+}(\theta) v_d + [I - J^{+}(\theta) J(\theta)] k \tag{36}$$

where $J^{+}(\theta)$ is the pseudoinverse of $J(\theta)$, and k is an arbitrary vector. It is known that, after realizing the desired velocity v_d, one way of making the given performance criterion $q(\theta)$ large is to select the vector k as follows [10,1].

$$k = \xi k_1, \quad \xi = dq(\theta)/d\theta \tag{37}$$

where k_1 is a constant to be determined.

The following criterion has been chosen for the wrist mechanism.

$$q(\theta) = k_a w(\theta) + k_b \theta_2 + \sum_{i=1}^{4} k_{ci} \{ (\theta_i - \bar{\theta}_i)^{-2} + (\theta_i - \underline{\theta}_i)^{-2} \} /2 \tag{38}$$

where k_a, k_b, and k_{ci} are constants to be determined, and $\bar{\theta}_i$, $\underline{\theta}_i$ are upper and lower limits for θ_i.

To verify the effectiveness of the control algorithm given by (36), (37), and (38), computer simulation has been done. For simplicity the dynamics of the wrist has been neglected, and the joint velocity $\dot{\theta}$ calculated by (36) has been assumed to be realized exactly. The sampling period is 0.05 sec, and

$$\bar{\theta}_2 = 140, \quad \underline{\theta}_2 = -110$$

$$\bar{\theta}_3 = -\underline{\theta}_3 = \begin{cases} 180, & 90 \le \theta_2 \le 140 \\ \{3(\theta_2 - 90)/8 + 100\}, & -110 \le \theta_2 \le 90 \end{cases}$$

$$k_1 = 1, \ k_a = 3, \ k_b = 20, \ k_{c2} = k_{c3} = 1, \ k_{c1} = k_{c4} = 0$$

The desired trajectory for the hand orientation has been specified by the Euler angles (α, β, γ). The solid line in Fig. 10(a) shows the time trajectory of the manipulability measure for the case where the wrist is rotated around the z_0 axis with $\dot{\alpha} = 30$, $\dot{\beta} = \dot{\gamma} = 0$ (deg/sec) from the initial state $\theta = [0, 90, -45, 0]^T$ shownin Fig. 10(b). The broken line in the figure is for the algorithm (36) with k=0. It can be seen from the figure that the algorithm (36) with (37) and (38) keeps the wrist away from the singular state.

From this and other simulation results, it is expected that the four-joint wrist mechanism will be controlled satisfactorily by the pseudoinverse control algorithm (36) with (37) and (38).

CONCLUSION

Properties of the manipulability measure, which was proposed in a previous paper as a measure of manipulating ability of robot arms in positioning and orienting the end-effectors, have been studied. Its utilization for determining the best postures of various types of manipulators has been discussed. The obtained best postures have some resemblance to those taken by human arms. As an application of the manipulability measure, a four-joint wrist mechanism has been analyzed with respect to its ability of orienting the end-effector, and a control algorithm has been developed. A simulation study shows the effectiveness of the algorithm in controlling this wrist mechanism.

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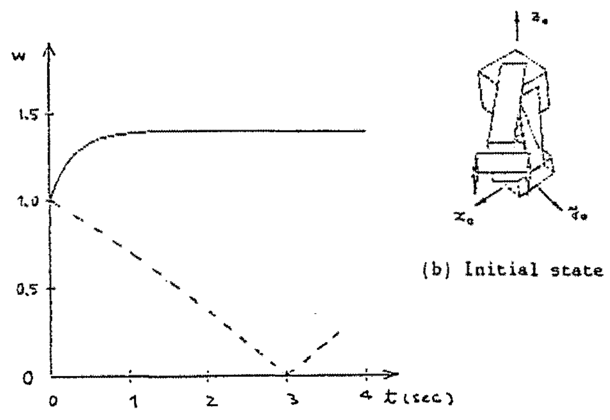

(a) Manipulability measure
—— : control algorithm (36) with (37) and (38)
– – – : control algorithm (36) with k=0

Fig.10 Comparison of control algorithms