Runge-Kutta method

From Rosetta Code

Given the example Differential equation:

$$y'(t) = t \times \sqrt{y(t)}$$

With initial condition:

$t_0 = 0$ and $y_0 = y(t_0) = y(0) = 1$

This equation has an exact solution:

$$y(t) = \tfrac{1}{16}(t^2 + 4)^2$$

Runge-Kutta method
You are encouraged to solve this task according to the task description, using any language you may know.

Task

Demonstrate the commonly used explicit fourth-order Runge–Kutta method to solve the above differential equation.

- Solve the given differential equation over the range $t = 0 \ldots 10$ with a step value of $\delta t = 0.1$ (101 total points, the first being given)
- Print the calculated values of y at whole numbered t's ($0.0, 1.0, \ldots 10.0$) along with error as compared to the exact solution.

Method summary

Starting with a given y_n and t_n calculate:

$$\delta y_1 = \delta t \times y'(t_n, y_n)$$
$$\delta y_2 = \delta t \times y'(t_n + \tfrac{1}{2}\delta t, y_n + \tfrac{1}{2}\delta y_1)$$
$$\delta y_3 = \delta t \times y'(t_n + \tfrac{1}{2}\delta t, y_n + \tfrac{1}{2}\delta y_2)$$
$$\delta y_4 = \delta t \times y'(t_n + \delta t, y_n + \delta y_3)$$

then:

$$y_{n+1} = y_n + \tfrac{1}{6}(\delta y_1 + 2\delta y_2 + 2\delta y_3 + \delta y_4)$$
$$t_{n+1} = t_n + \delta t$$

Contents