

AMALTHEA-based GPU Response Time Analysis for NVidia's Jetson TX2

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Doctor of Philosophy

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*I, AUTHORMNAME confirm that the work presented in this thesis is my own.
Where information has been derived from other sources, I confirm that this
has been indicated in the thesis.*

Abstract

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Acknowledgements

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Abbreviations and Symbols

Abbreviations

API	Application Programming Interface
CE	Copy Engine
CPU	Central Processing Unit
EE	Execution Engine
FIFO	First In First Out
GPU	Graphics Processing Unit
HMP	Heterogeneous Multi-Processor
SM	Streaming Multiprocessor
OS	Operating System

Symbols

τ	Set of tasks or kernels
τ_i	Task or kernel i
i	Period of τ_i
g_i	Grid size of τ_i
b_i	Number of thread per block within g_i
C_i	Execution time workload of a thread within b_i
u_i	Utilization of τ_i
r_i	Release time of τ_i
f_i	Completion time of τ_i

R_i	Response time of τ_i
U_t	Total utilization of τ
b_{max}	Maximum amount of threads in a SM
g_{max}	Maximum amount of blocks that can be allocated in a SM
g_f	Available blocks at some point in time
t_a	Point in time at which a block will be allocated

Chapter 1

Introduction

1.1 Motivation

Car manufactures want to reduce cost in terms of money and time required to develop, test and validate a new piece of software due to a change of supplier. For that reason, centralized end-to-end architectures are the solution they are aiming to, because for car companies such as BWW and Audi the car of future will be similar to a “data center on wheels” [1].

Centralized end-to-end architectures would be the first step stone toward to decoupling software and hardware [2]. This type of architectures not only will take advantage of internet connectivity, cloud computing and powerful heterogeneous processing units, but also will allow scalable, hierarchical and highly integrated system.

In other words, car manufactures prefer now a days low-latency, hierarchical and cost effectiveness of centralized end-to-end architectures, because today requirements of computational power, bandwidth, integration, safety and real-time [3].

However, car manufactures don’t forget that at the end, in centralized end-to-end architectures, different types of software would run on top of an heterogeneous hardware supplied by companies such as NVIDIA, Mobileye or Qualcomm. Thus, it’s important to analyze and understand how software

will behave under those conditions, in order to ensure a predictable and efficient system.

1.2 Industrial challenge WATERS 2019

Predictability is a key property for safety-critical and hard real-time systems [4]. Analyzing time related characteristics is an important step to design predictable embedded systems. However, in multi-core or heterogeneous systems based on centralized end-to-end architectures is harder to satisfy timing constraints due to scheduling, caches, pipelines, out-of-order executions, and different kinds of speculation [5]. Thus, development of timing-analysis methods for these types of architectures has become, nowadays, one of the main focus of research in both industry and academic environment.

Robert Bosch GmbH or Bosch, the German multinational engineering and electronics company, and one of the top leaders in development technology for the automotive industry announces every year *the WATERS Challenge*. The purpose of the WATERS industrial challenge is to share ideas, experiences and solutions to concrete timing verification problems issued from real industrial case studies [6].

This year, 2019, the challenge focuses on timing-analysis for heterogeneous software-hardware systems based on centralized end-to-end architectures. The platform chosen for this purpose is the NVIDIA® Jetson™ TX2 platform which has an heterogeneous architecture equipped with a Quad ARM A57 processor, a Dual Denver processor, 8GB of LPDDR4 memory and 256 CUDA cores of NVIDIA’s Pascal Architecture. For the challenge it is available an Amalthea model for this platform to design a solution, and test it later on real hardware.

1.3 NVIDIA Jetson TX2: Architecture Overview

NVIDIA Jetson TX2 is an embedded system-on-module (SOM). It is ideal for deploying advanced AI to remote field locations with poor or expensive internet connectivity, Robotics, Gaming Devices, Virtual Reality (VR), Augmented Reality (AR) and Portable Medical Devices. In addition, it offers near-real-time responsiveness and minimal latency—key for intelligent machines that need mission-critical autonomy [7].

The main components of the Jetson TX2 are dual-core ARMv8 based NVIDIA Denver2, quad-core ARMv8 Cortex-A57, 8GB 128-bit LPDDR4 and integrated 256-core Pascal NVIDIA GPU. The quad-core Cortex-A57 and dual-core NVIDIA Denver2 can be seen as a cluster of heterogeneous multiprocessors (HMP) [8]. Both HMP and GPU shares a 8GB SRAM memory as shown in Figure 1.1. Hereafter, whenever we use the term **host**, we will refer to HMP, similarly we will use **device** to refer to GPU.

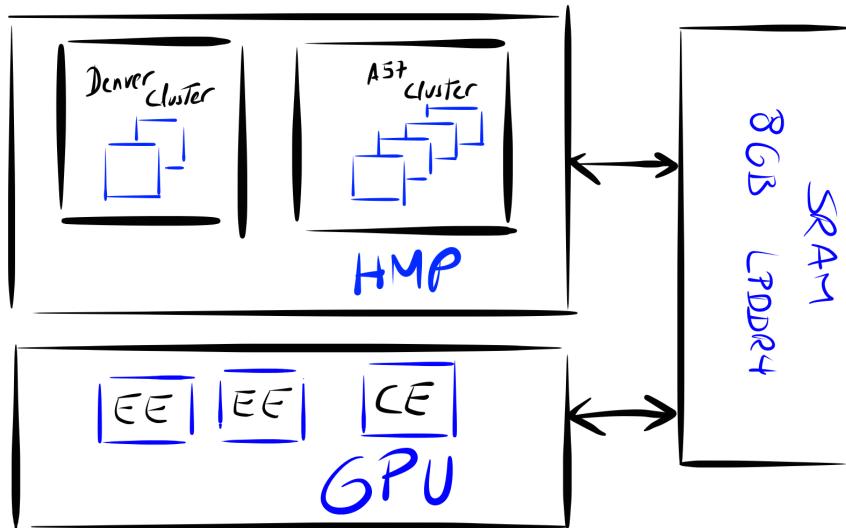


Figure 1.1: Jetson TX2 Architecture Overview

Any NVIDIA GPU has two types of engines, **Copy Engines (CE)** and **Execution Engines (EE)**. The Jetson TX2 has only one CE and two EE also known as **Streaming multiprocessors**. CE is in charge of data transfers from host to device and viceversa. There is, moreover, the possibility that

EE and CE can run concurrently.

The GPU uses **streams** to run applications. The number of streams depends on the GPU resources. An application can run in one or multiple streams, the GPU scheduler, by default, manages how the application will be allocated on streams in order to maximize throughput. In Chapter 2, we will discuss in more detail how the TX2 GPU scheduler behaves in case of multiple applications.

1.4 Jetson TX2 Amalthea Model

AMALTHEA is a platform for engineering multi- and many-core embedded systems. This platform enables the creation and management of complex tool chains including simulation and validation [9]. In the context of WATERS Challenge 2019, Bosch offers an AMALTHEA model of the Jetson TX2. In this model, a CPU runnable will read data from memory, execute some computation (Ticks) and write back data into memory as shown in Figure 1.2.

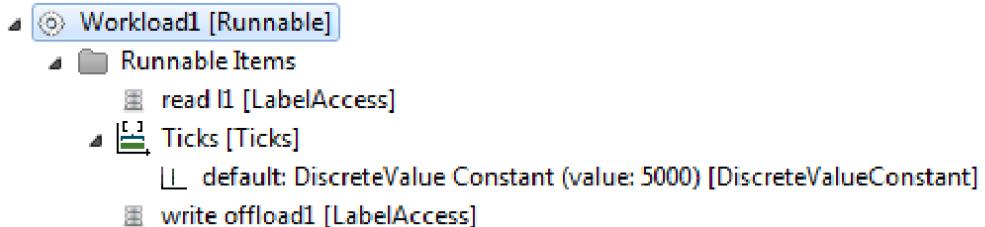


Figure 1.2: Runnable example for a CPU [6]

In the case of GPU modeling, the runnable will follow the same pattern as in the CPU case: read, execution, write back. However, the reading operation is actually to copy memory from host to device, thus it is modeled as *memory reading from host* and then as *memory writing to device*. On the other hand, the writing back operation requires to copy memory from device to host, therefore it is modeled as *memory reading from device* and then as *memory writing to host* as shown in Figure 1.3.

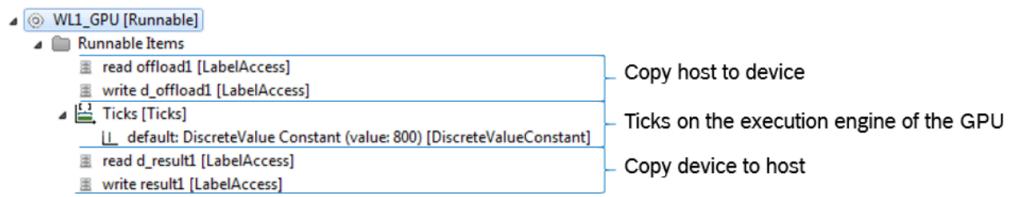


Figure 1.3: Runnable example for a GPU [6]

Chapter 2

CUDA and Jetson TX2

In this chapter...blablabla

2.1 NVIDIA GPU Software Model

Now a days computer applications run on heterogeneous hardware and GPUs are important in order to achieve high performance computing. Since 2006 running software on NVIDIA GPUs are known as a *CUDA application* [10]. A CUDA application will run concurrently multiple instances of special functions called **kernel**s. Each instance runs on a **thread**. Moreover, these threads are arranged in **block**s, and blocks compose **grid**s as shown in Figure 2.1.

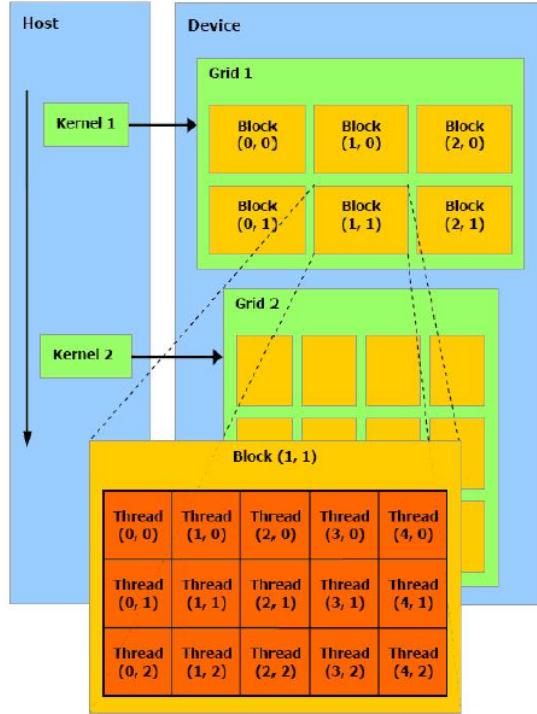


Figure 2.1: Organisation of grids, blocks, threads, and kernels [11].

It's logical to think that there is also a hierarchical memory structure. Threads, blocks and grids have access to different memory spaces as illustrated in Figure 2.2. The types of memory are summarized in Table 2.1.

Table 2.1: Types of memories in a GPU

Memory	Main Characteristics	Scope	Lifetime
Global	R/W, Slow and big	Grid	Application
Texture	ROM, Fast, Optimized for 2D/3D access	Grid	Application
Constant	ROM, Fast, Constants and kernel parameters	Grid	Application
Shared	R/W, Fast, it's on-chip	Block	Block
Local	R/W, Slow as global, when registers are full	Thread	Thread
Registers	R/W, Fast	Thread	Thread

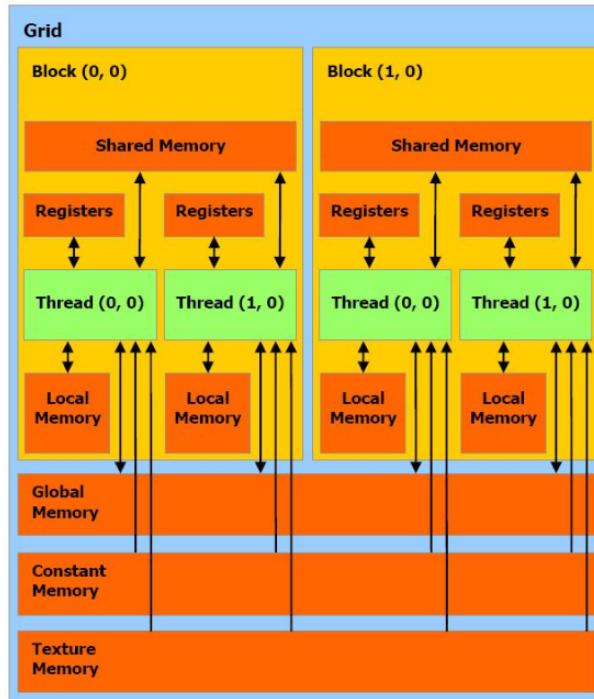


Figure 2.2: Memory hierarchy [11].

In summary, CUDA application solve problems that were modeled based on *divide and conquer* principle. Moreover, CUDA software model not only allow users to achieve high computational performance, but also CUDA application are highly scalable.

2.2 NVIDIA GPU Hardware Model

The CUDA architecture is based on **Streaming Multiprocessors** (SM) which perform the actual computation. Each SM has its own control units, registers, execution pipelines and local memories, but they also have access to global memory as illustrated in Figure 2.3. A **stream** is a queue of CUDA operations, memory copy and kernel launch. We will talk more about streams in following sections.

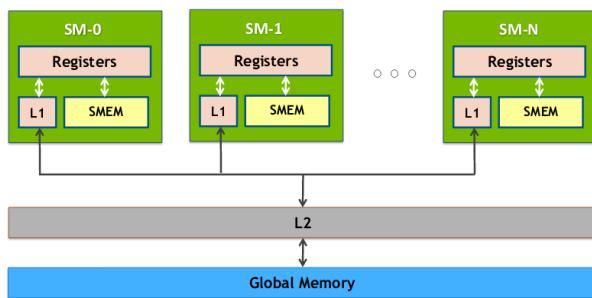


Figure 2.3: Memory hierarchy

When a kernel grid is launched blocks are enumerated and assigned to the SMs. Once the blocks are assigned, threads are managed in **wraps** by the **wrap scheduler**. A wrap is a group of 32 threads that run in parallel. Thus, it's highly recommendable to use block sizes of size $32N$, $N \in \mathbb{N}$, otherwise there would be “inactive” threads. An example is shown in Figure 2.4, where there is a block of 140 threads but since the wrap scheduler works with wraps, 20 threads are wasted and no other block can make use of them.

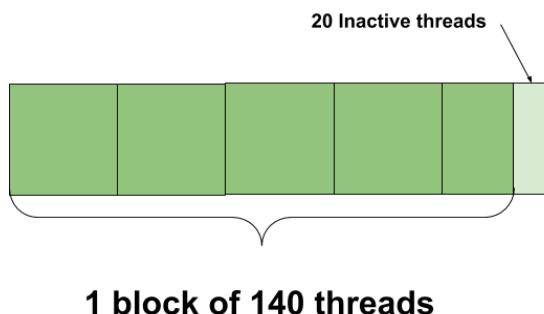


Figure 2.4: Inactive threads

The amount of threads and blocks that can run concurrently per SM depends on the number of 32-bit registers and shared memory within SM, as well as the CUDA computing capability of the GPU. Information related to maximum amount of blocks or threads, as well as the computing capability of the GPU can be displayed executing `deviceQuery` tool. Some information about Jetson TX2 is presented below:

```
CUDA Device Query (Runtime API) version (CUDART static linking)
Detected 1 CUDA Capable device(s)
Device 0: "NVIDIA Tegra X2"
    CUDA Driver Version / Runtime Version      9.0 / 9.0
    CUDA Capability:                          6.2
    Total amount of global memory:            7850 MBytes
    ( 2) SM, (128) CUDA Cores/SM:           256 CUDA Cores
    L2 Cache Size:                           524288 bytes
    Total amount of shared memory per block: 49152 bytes
    Total number of registers per block:    32768
    Warp size:                             32
    Max. number of threads per SM:          2048
    Max. number of threads per block:       1024
    Max dim. size of a thread block (x,y,z): (1024, 1024, 64)
    Max dim. size of a grid size   (x,y,z): (2^31-1, 65535, 65535)
```

2.3 NVIDIA Jetson TX2's GPU Scheduler

It's common to use several kernels in an application. In order to reduce computation time and maximize GPU utilization, it's desire to run multiple kernels in parallel. CUDA uses streams to achieve this goal. As mentioned before, a stream is a queue of CUDA operations, memory copy and kernel launch. Thus, it is possible either to launch multiple kernels within one stream or multiple kernels on multiple streams. Operations within the same stream are managed in FIFO (First In First Out) fashion, thus, we will also use the term **stream queue** when we talk about FIFO queues within a stream. The Jetson TX2's GPU assigns resources to streams using its

internal scheduler.

Predictability is an important characteristic of safety-critical systems. It requires both functional and timing correctness. However, a detailed information about the Jetson TX2’s GPU scheduler behaviour is not publicly available. Without such details, it is impossible to analyze timing constraints. Nevertheless, there are some efforts [12], [13] and [14] aimed at revealing these details through black-box experimentation.

NVIDIA GPU scheduling policies depend on whether the GPU workloads are launched by a CPU executing OS threads or OS processes. We will focus on the first case, because GPU computations launched by OS processes have more unpredictable behaviours, as stated in [12] and [13]. In this section, we will present GPU scheduling policies devired by [12] and use them in an example to clarify their use.

Let’s start by defining some terms. When one block of a kernel has been scheduled for execution on a SM it’s said that the block was **assigned**. Moreover, it’s said a kernel was **dispatched** as soon as one of its blocks were assigned, and **fully dispatched** once all its blocks were assigned. The same applies to copy operations and CE.

There are, in addition, FIFO CE queues used to schedule copy operations, and FIFO EE queues used to schedule kernel launches. Stream queues feed CE and EE queues. Bellow we will present the rules that determine scheduler and queues behaviours.

- **General Scheduling Rules:**

- **G1** A copy operation or kernel is enqueued on the stream queue for its stream when the associated CUDA API function (memory transfer or kernel launch) is invoked.
- **G2** A kernel is enqueued on the EE queue when it reaches the head of its stream queue.
- **G3** A kernel at the head of the EE queue is dequeued from that queue once it becomes fully dispatched.
- **G4** A kernel is dequeued from its stream queue once all of its blocks complete execution.

- **Non-preemptive execution:**
 - **X1** Only blocks of the kernel at the head of the EE queue are eligible to be assigned.
- **Rules governing thread resources:**
 - **R1** A block of the kernel at the head of the EE queue is eligible to be assigned only if its resource constraints are met.
 - **R2** A block of the kernel at the head of the EE queue is eligible to be assigned only if there are sufficient thread resources available on some SM.
- **Rules governing shared-memory resources:**
 - **R3** A block of the kernel at the head of the EE queue is eligible to be assigned only if there are sufficient shared-memory resources available on some SM.
- **Copy operations:**
 - **C1** A copy operation is enqueued on the CE queue when it reaches the head of its stream queue.
 - **C2** A copy operation at the head of the CE queue is eligible to be assigned to the CE.
 - **C3** A copy operation at the head of the CE queue is dequeued from the CE queue once the copy is
 - **C4** A copy operation is dequeued from its stream queue once the CE has completed the copy.
- **Streams with priorities:**
 - **A1** A kernel can only be enqueued on the EE queue matching the priority of its stream.
 - **A2** A block of a kernel at the head of any EE queue is eligible to be assigned only if all higher-priority EE queues (priority-high over priority-low) are empty.

Authors in [12] mentioned that rules related to **registry resources** are expected to have exactly the same impact as threads and shared-memory rules.

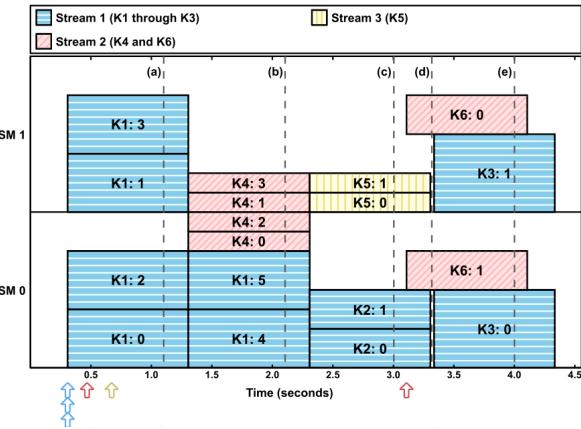


Figure 2.5: Basic GPU scheduling experiment [12]

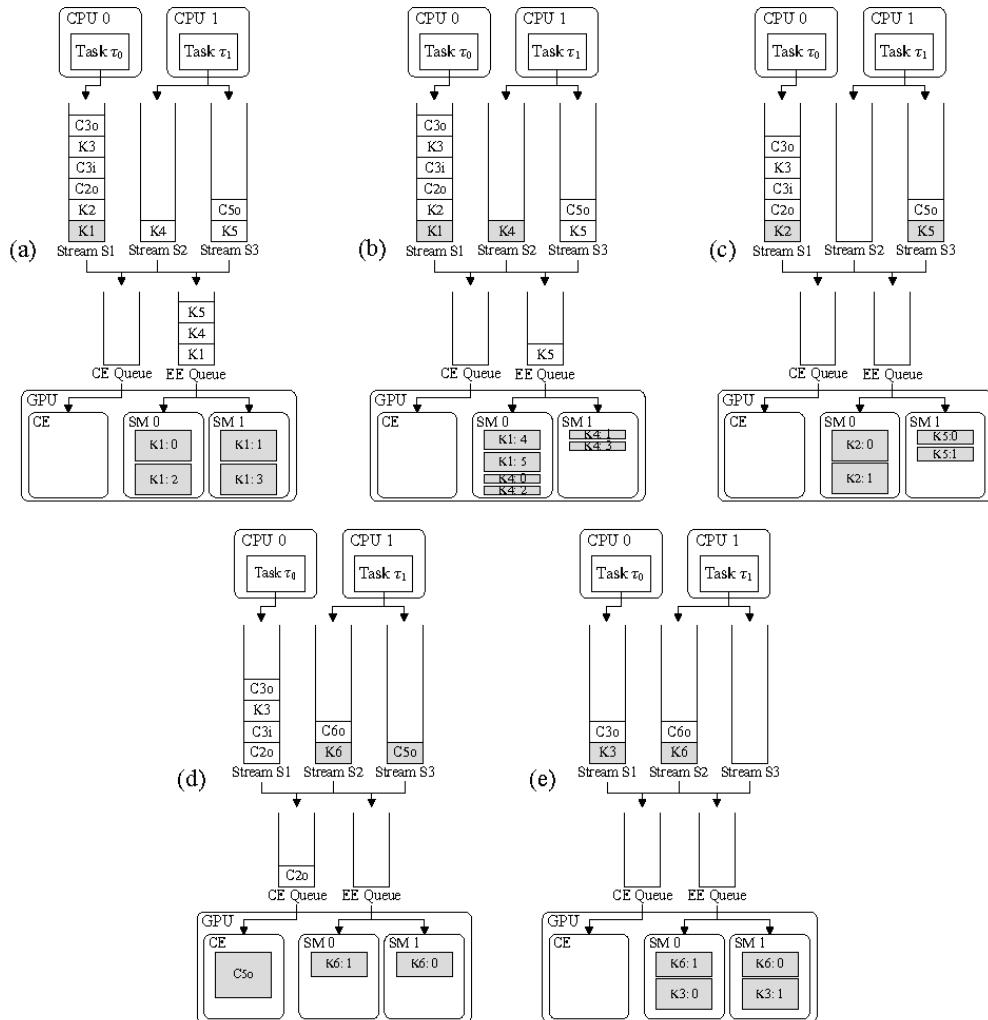


Figure 2.6: Detailed state information at various time points in Fig. 2.5 [12].

Chapter 3

Jetson TX2's GPU scheduler response time analysis

3.1 Introduction

In this chapter, we present our approach to calculate the response time analysis for Jetson TX2's GPU scheduler based on the set of scheduling rules explained in the last chapter. **Need a better introduction**

3.2 Task model

There is a set of tasks or kernels τ of n independent kernels $\{\tau_1, \tau_2, \dots, \tau_n\}$ on a single GPU. Each kernel has a period T_i defined as the separation between two consecutive releases of τ_i , thread execution time workload C_i and a grid of blocks g_i . Each block contains b_i threads.

$$\tau = \{\tau_i\}; \quad i \geq n \wedge n \in \mathbb{N} \quad (3.1)$$

$$\tau_i = \{T_i, C_i, g_i, b_i\} \quad (3.2)$$

Thus each kernel τ_i has a total of $g_i \cdot b_i$ threads, and the total execution time workload of τ_i is $C_i \cdot g_i \cdot b_i$. The utilization of each kernel is defined as the total execution time workload divided by the period, as stated in [15].

$$u_i = \frac{C_i g_i b_i}{T_i} \quad (3.3)$$

In addition, the total utilization of the set of tasks τ is defined as:

$$U_t = \sum_{\tau_i \in \tau} u_i \quad (3.4)$$

For a kernel τ_i we denote the release time as r_i , the completion time as f_i and response time as $R_i = f_i - r_i$. We assume that a kernel τ_i has a deadline equal to its period T_i .

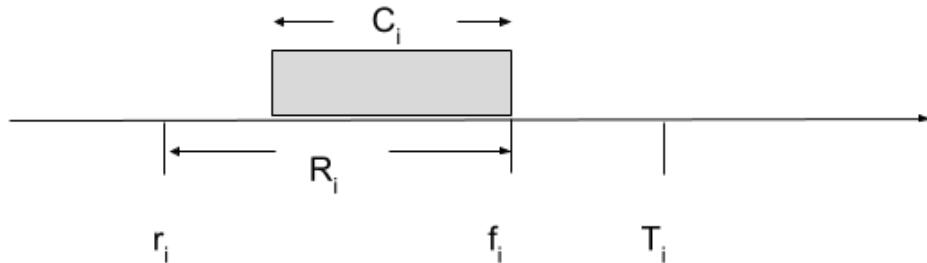


Figure 3.1: Time chart

3.3 Assumptions

For the calculation of the response time we have two assumption:

3.3.1 ALL BLOCKS HAVE THE SAME AMOUNT OF THREADS

The election of the optimum number of threads for a specific kernel is a hard task, for that reason there have been some efforts towards that direction [16], [17], [18], [19]. However, NVIDIA developers recommend, for practical

purposes, on their offical guides [20] and [21] to use block sizes equals to either 128, 256, 512 or 1024, because it has been documented that these values are more likely to take full advantage of the GPU resources. In our case we will assume that all the blocks, regarless the kernel, are the same size.

$$b_i = b, \quad \forall \tau_i \in \tau \quad (3.5)$$

3.3.2 ONE BIG STREAMING MULTIPROCESSOR

This assumption is derived from the previous one. Each streaming multiprocessor in the Jetson TX2 has 2048 available threads and since b_i can be either 128, 256, 512 or 1024 ($2048/b_i = k, k \in \mathbb{N}$), we can think of the two streaming multiprocessors as a big one of 4096 threads.

It means that it could be allocated $2048/b_i$ blocks per SM or $4096/b_i$ blocks in the big SM. Hereafter we will refer the big SM as it were the only SM in the Jetson TX2's GPU. Thus, we defined g_{max} as the maximum number of blocks that can be allocated in the SM at some point in time.

$$g_{max} = \frac{b_{max}}{b}, \quad g_{max} \in \mathbb{N} \quad (3.6)$$

Where b_{max} is the maximum amount of threads in the GPU, in the case of Jetson TX2 is 4096.

3.4 Introduction to GPU Response Time Analysis

In addition to the variables defined in our assumptions we define g_f as the number of blocks that are available at some point in time t , and t_a as the point in time in which a block $b_i \in g_i$ can be allocated.

For example in Figure 3.2a is shown that for a $t = t_1$ the amount of free blocks g_f is lower than g_{max} while in Figure 3.2b for a $t = t_2$, $g_f = g_{max}$.

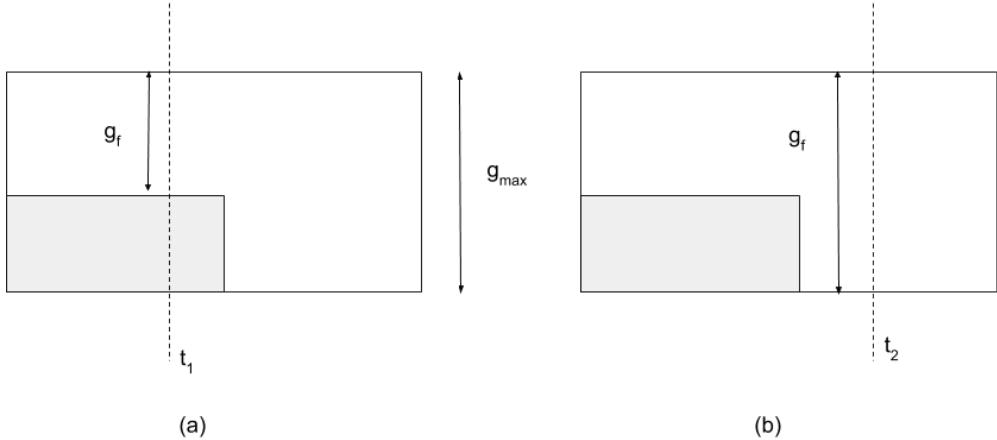


Figure 3.2: Free blocks (a) at $t = t_1$, $g_f < g_{max}$ (b) at $t = t_2$, $g_f = g_{max}$

In Figure 3.3 we present two cases. Let's assume there is a new kernel K4 which wants to allocate a block $b_i \in g_i$. In Figure 3.3a the release time r_4 of the kernel 4 is lower than t_1 , which means that $t_a = t_1$ because $r_4 \leq t_1$ and kernel 3 (K3) was already dequeued. In Figure 3.3b r_4 lies between t_2 and t_3 , in that case $t_a = r_4$, because all previous kernels were already dequeued and there are enough resources.

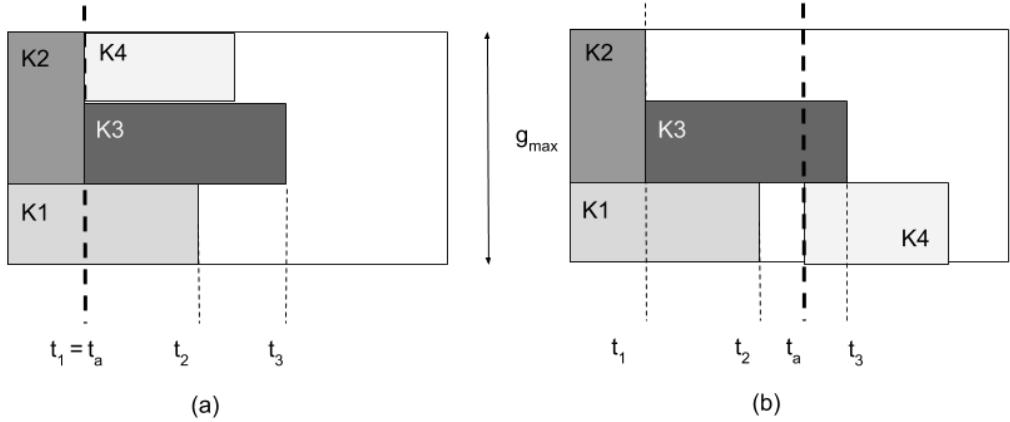


Figure 3.3: (a) $t_a = t_1 \quad \forall r_4 \text{ s.t } r_4 \leq t_1$ (b) $t_a = r_4 \quad \forall r_4 \text{ s.t } t_2 \leq r_4 \leq t_3$

Assuming t_a is known, we would need to calculate how many blocks can be allocated at that point of time. In other words, we need to know the value of g_f at t_a . In Figure 3.4a a new kernel K3 with 6 blocks $g_3 = 6$ is going

to be allocation on the Jetson’s GPU. Each block have 512 threads, which means that $g_{max} = 8$. The GPU is not executing any kernel at $t = t_a$ as shown in Figure 3.4b therefore $g_f = g_{max} = 8$ at $t = t_a$. Given that $g_3 < g_f$ all the blocks of K3 will be allocated at the same time as shown in Figure 3.4c. The completion time f_3 of kernel K3 is t_a plus the thread execution time given by C_3 , $f_3 = C_3 + t_a$. If we assume that the release time r_3 is the same as t_a then the completion time for K3 is the same as the response time R_3 , otherwise $R_3 \geq f_3$.

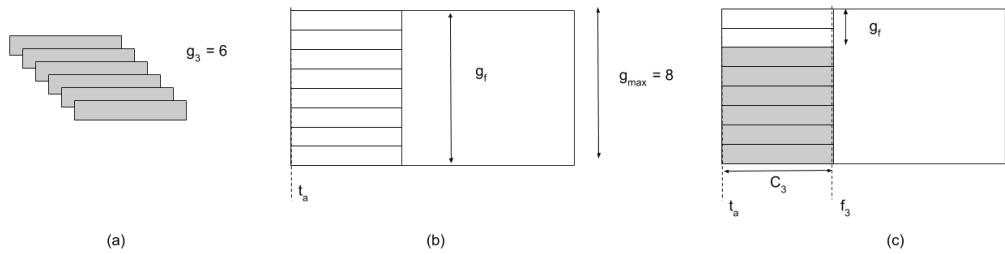


Figure 3.4: (a)New kernel $K3$ with 6 blocks to allocate $g_3 = 6$. (b) State prior to $K3$ of the GPU (c) state after $K3$ allocation

Once f_3 and R_3 are calculated, it’s important to update the values of t_a and g_f , because these values will be used by the following kernel. Let’s start with g_f , it is easy to notice that after K3 allocation there are two free blocks $g_f - g_3 = 2$ as a result the new value of $g_f = 2$. On the other hand, by definition t_a is the point in time in which a block $b_i \in g_i$ can be allocated, therefore t_a will not change because $g_f > 0$.

In Figure 3.5 we analyze another highly probable scenario. We use the same kernel K3 as in the last example ($g_3 = 6$). However, as shown in Figure 3.5b, there were two kernels allocated previously to K3. Kernel K1 with 5 allocated blocks $g_1 = 5$ and K2 with 3 allocated blocks $g_2 = 3$. Note that these kernels have different completion time $f_2 > f_1$. Nevertheless, what matters is not either K1 or K2 completion time but the value of t_a and g_f . In this example, t_a is the same as K1 completion time and g_f has the same value as g_1 , $g_f = 5$. Thus, 5 blocks from K3 will be allocated first as shown in Figure 3.5c.

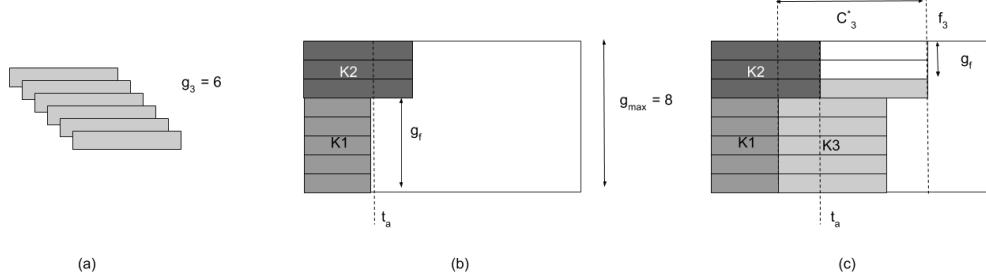


Figure 3.5: (a) New kernel K_3 with 6 blocks to allocate $g_3 = 6$. (b) State prior to K_3 of the GPU. Kernels K_1 and K_2 were previously allocated (c) state after K_3 allocation

The next logic question is where the last block of K_3 should be allocated. The answer again is given by the updated values of t_a and g_f . Looking at Figure 3.5c is easy to get these new values. The new value of t_a is f_2 since $f_2 < (t_a + C_3)$, and for $t = t_a$ the corresponding g_f is g_2 . Thus, the last K_3 block is allocated at $t = t_a = f_2$ and that give us the completion time for K_3 that is $f_3 = f_2 + C_3$ or $f_3 = t_a + C_3$. In Figure 3.5c, we defined a new variable C_3^* as the total amount of time in which K_3 was using GPU resources.

After K_3 allocation, t_a and g_f should be updated again. In this example, the new g_f is the old value of g_f minus the last allocated K_3 blocks $g_f = g_f - 1 = 2$, while t_a remains the same $t_a = f_2$ because the conditions are the same as in the later example where there was only one kernel.

3.5 Response Time Analysis Algorithm

Our algorithm is focused on the calculation of t_a and g_f for each block regardless from which kernel t_i comes. In addition, it is important to notice that t_a and g_f depend on how previous blocks were allocated and on the GPU state at some point in time, as it was described above and illustrated in the Figure 3.4 and Figure 3.5.

The output of our algorithm is a set of release times f_1, f_2, \dots, f_n where n is the length of τ which values f_i depend on t_a and C_i .

$$f_i = f(t_a, C_i) \quad (3.7)$$

A basic version of our algorithm is described in Algorithm 1. This version is derived directly from the examples illustrated in Figure 3.4 and Figure 3.5, in other words, this basic algorithm is a summary of the section above. We have omitted details such as how t_a and g_f are updated in the case that $g_f \geq g_i$, however we still keep the big picture of what is necessary at each step.

```

Input :  $\tau$ 
Output:  $f_1, \dots, f_n$ 

Initialization:  $t_a = 0, g_f = g_{max}, i = 1$ 
while  $i \leq n$  do
    if  $g_f \geq g_i$  then
         $f_i = t_a + C_i;$ 
        Update  $g_f$  and  $t_a$ ;
         $i++$ ; // Next kernel
    else
         $g_i = g_i - g_f;$ 
        Update  $g_f$  and  $t_a$ ;
    end
end

```

Algorithm 1: Basic real time analysis algorithm

In order to analyze a new kernel τ_i and update t_a and g_f we need to track old values of $g_f \quad \forall t \leq t_a$. Fortunately, it is necessary only to track g_f at specific points of time, as it was shown in the previous example described by Figure 3.5, are given by completion times of previous kernels, in other words we must track g_{i-k} and f_{i-k} where $k \in 1, 2, \dots, i-1$, because updated values of g_f and t_a depend as well on these them.

Let's define a set h of pair of values (t_k, g_k) where g_k are the number of free blocks at $t = t_k$ such that $t_k \geq t_a$. In a further example we will show step by step how this array h is filled and updated in order to have a better understanding.

A complete version of our algorithm is presented in Algorithm 2.

```

Input :  $\tau$ 
Output:  $f_1, \dots, f_n$ 

Initialization:  $t_a = 0, g_f = g_{max}, i = 1, h = \{\}$ 
while  $i \leq n$  do
    if  $g_f \geq g_i$  then
         $f_i = t_a + C_i;$ 
         $h = \{h; (f_i, g_i)\};$ 
         $t_a = t_a;$ 
         $g_f = g_f - g_i;$ 
         $i++$ ; // Next kernel
    else
         $g_i = g_i - g_f;$ 
         $h = \{h; (t_a + C_i, g_f)\};$ 
         $[t_a, index] = \min(h[:, 1]);$ 
         $g_f = h[index, 2];$ 
    end
end

```

Algorithm 2: Real time analysis algorithm

Our algorithm is based on three main updates: h , t_a and g_f . The set h can be seen as an array of size $N \times 2$, where N is the number of tracked pairs. For this reason, when $g_f > g_i$ we used MATLAB notation of `min` function `[value, index] = min(A)`, where `index` is the position of the pair or row $(t_k, g_k) \in h$ that has the minimum of all time values saved in h . Once we know which pair has the minimum time, we just assign $t_a = t_k$ and $g_f = g_k$. It is important to mention again that by definition of h , all the tracked times should be greater or equal than the current t_a , meaning that pairs that have tracked times lower than t_a must be removed.

3.6 Example

Let's say there are four kernels we want to allocated, all with the same period $T = 15$ and block size of 512 threads, $b = 512$, which means $g_{max} = 8$. The four tasks are defined as $\tau = \{\tau_1 = \{15, 4, 2, 512\}, \tau_2 = \{15, 6, 7, 512\}, \tau_3 = \{15, 6, 2, 512\}, \tau_4 = \{15, 5, 5, 512\}\}$.

At the beginning $t_a = 0, i = 1, h = \{\}$ and $g_f = g_{max} = 8$. Let's start

with τ_1 . Kernel τ_1 and initial state of GPU are shown in Figure 3.6(a) and Figure 3.6(b) respectively.

- $g_f \geq g_1$? yes, because $g_1 = 2$
- $f_1 = t_a + C_1 = 0 + 4 = 4$
- $h = \{h, (f_1, g_1)\} = \{(4, 2)\}$
- $t_a = 0$
- $g_f = g_f - g_1 = 8 - 2 = 6$
- $i = 2$

After τ_1 allocation the GPU state is as shown in Figure 3.6(c), as it is observed, t_a remains the same but g_f now is 6. Furthermore, that is the initial GPU state when τ_2 arrives.

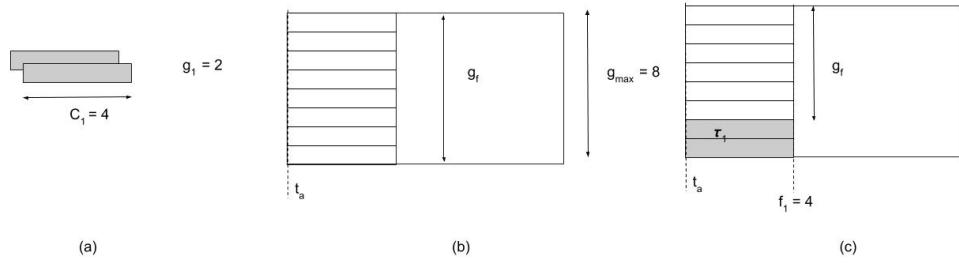


Figure 3.6: (a) Kernel τ_1 (b)GPU state prior to τ_1 allocation (c) GPU state after τ_1 allocation

Since $i = 2$, it's time to analyze τ_2 . Figure 3.7(a) shows the number of blocks that should be allocated for τ_2 . In this case $t_a = 0$ and $g_f = 6$ as shown in Figure 3.7(b).

- $g_f \geq g_2$? no, because $g_i = 7$
- $g_2 = g_2 - g_f = 7 - 6 = 1$
- $h = \{h, (t_a + C_2, g_f)\} = \{(4, 2), (6, 6)\}$
- $[t_a, \text{index}] = \min(h[:, 1]) = \min([4, 6])$
- $[t_a, \text{index}] = [4, 1]$
- $g_f = h[\text{index}, 2] = h[1, 2] = 2$
- $h = h - \{(4, 2)\} = \{(4, 2), (6, 6)\} - \{(4, 2)\}$

- $h = \{(6, 6)\}$

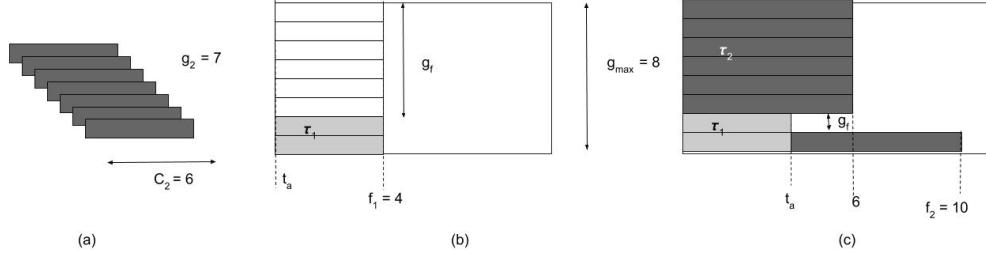


Figure 3.7: (a) Kernel τ_2 (b)GPU state prior to τ_2 allocation (c) GPU state after τ_2 allocation

Values for t_a , g_f , g_2 and h were updated. Notice that current value of t_a is the completion time of τ_1 and g_f is g_1 , that is why, as mention before, it is important to track f_1 and g_1 . However, completion time for τ_2 is not known yet. Let's continue with the analysis.

- $g_f \geq g_2$? yes, because $g_2 = 1$
- $f_2 = t_a + C_2 = 4 + 6 = 10$
- $h = \{h, (f_2, g_2)\} = \{(6, 6), (10, 1)\}$
- $t_a = 4$
- $g_f = g_f - g_2 = 2 - 1 = 1$
- $i = 3$

In Figure 3.7(c) is shown the GPU state, t_a and g_f values after τ_2 allocation. This setup is the starting point for the analysis of τ_3 as observed in Figure 3.8(b). Since we already described step by step analysis for τ_1 and τ_2 , we skip some details in τ_3 analysis, however we will point out something important about h .

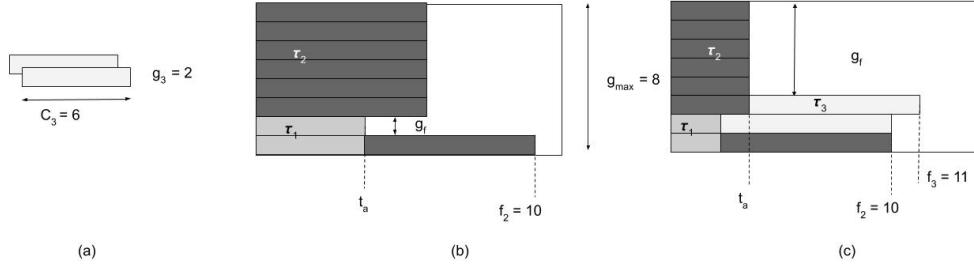


Figure 3.8: (a) Kernel τ_3 (b)GPU state prior to τ_3 allocation (c) GPU state after τ_3 allocation

The initial setup for τ_3 is $t_a = 4$, $g_f = 1$ and $h = \{(6, 6), (10, 1)\}$. The number of blocks and thread execution time of τ_3 is illustrated in Figure 3.8.

- $g_f \geq g_3$? no
- $g_3 = g_2 - g_f = 1$
- $h = \{(6, 6), (10, 1), (10, 1)\}$
- $h = \{(6, 6), (10, 2)\}$
- $[t_a, \text{index}] = [6, 1]$
- $g_f = h[\text{index}, 2] = h[1, 2] = 6$
- $h = \{(10, 2)\}$

As mention before, we performed an *extra* step with h in which h went from having three pairs to having just two. The reason behind it lies on the definition of h . The set h of pair of values (t_k, g_k) where g_k are the number of free blocks at $t = t_k$; notice that at $t = 10$ there are two *free* tracked blocks, one that comes from τ_2 and other from τ_3 as shown in Figure 3.8(c), as well as the results of the following τ_3 analysis.

- $g_f \geq g_3$? yes
- $f_3 = t_a + C_3 = 12$
- $h = \{(10, 2), (12, 1)\}$
- $t_a = 6$
- $g_f = g_f - g_3 = 5$
- $i = 4$

The analysis for τ_4 is straightforward. The blocks and thread execution time for τ_4 , the GPU prior to τ_4 allocation and GPU state after τ_4 allocation are shown in Figure 3.9. It is easy to find out that the completion time f_4 is 11.

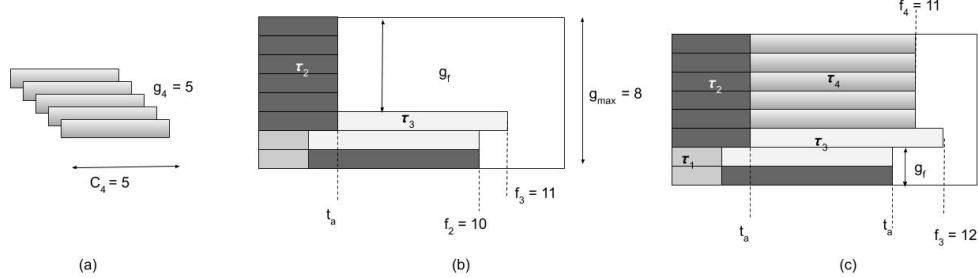


Figure 3.9: (a) Kernel τ_4 (b)GPU state prior to τ_4 allocation (c) GPU state after τ_4 allocation

We have calculated the completion times $f_i \quad \forall i \in \tau, f = \{4, 10, 12, 11\}$. Given the fact that all kernel were scheduled at the same time, the release time for all kernels is 0. Thus, the response time for each kernel is the same as their completion times. Furthermore, we can conclude that all the kernels can be scheduled because $R_i \leq T \quad \forall i \in \tau$.

3.7 A Special case

In this section we demonstrate that if all kernels are released at the same time and also have the same thread execution time, $C_i = C \quad \forall i \in \tau$, our response time analysis has not algorithmic behavior, instead it's a set of three equations.

In this special case, there is no need of h , because t_a and g_f can be calculated directly with two equations. The goal is as always to find t_a , g_f and f_i , however, we will exploit the fact that C_i is the same for all kernels. The case in which $g_i \leq g_f$ is trivial to analyze, therefore we will analyze the other case. If Figure 3.10 is shown the block distribution when $g_i > g_f$, thus in order to find g_i^* and t_a^* , that are the updated values, we must calculate K .

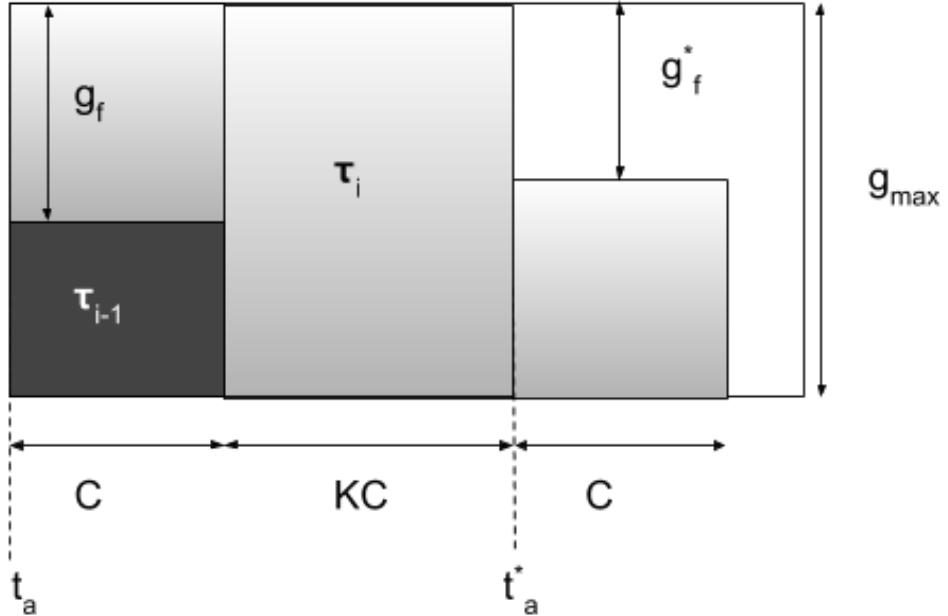


Figure 3.10: New kernel allocation

It is not hard to see that K is given by equation 3.8. The interpretation is that K is the maximum amount of times that g_{max} blocks can be allocated.

$$K = \lfloor \frac{g_i - g_f}{g_{max}} \rfloor \quad (3.8)$$

From K , the value of g_f^* is calculated using equation 3.9. If we observed the blocks in Figure 3.10, we notice that g_f^* can be calculated using geometry. The area of the first block is g_f , of the second one Kg_{max} and the last one is $g_i - g_f - Kg_{max}$.

$$g_f^* = g_{max} - (g_i - g_f - Kg_{max}) \quad (3.9)$$

The value of t_a^* is calculated in similar fashion and is given by equation 3.10. On the other hand, the calculation of f_i remains the same.

$$t_a^* = t_a + C + KC \quad (3.10)$$

We are at a point at which we are able to calculate with formulas the updated values of t_a and g_f , nonetheless, we still have the algorithmic behavior given by the **if** condition. Therefore to no longer use that dependency, we will make use of the absolute value and the signum function. The signum function of a real number x is defined as follows:

$$\text{sgn}(x) = \begin{cases} -1 & \text{if } x < 0, \\ 0 & \text{if } x = 0, \\ 1 & \text{if } x > 0. \end{cases} \quad (3.11)$$

We redefine K with the absolute value using equation 3.12, we use absolute value in 3.12 because we want K to be zero when $g_i \leq g_f$.

$$K = \left\lfloor \frac{\|g_i - g_f\|}{g_{max}} \right\rfloor \quad (3.12)$$

In addition, we define α which value is given by equation 3.13. The value of α is zero when $g_i \leq g_f$ and one otherwise.

$$\alpha = \frac{\text{sgn}(g_i - g_f) + 1}{2} \quad (3.13)$$

With 3.12 and 3.13 we eliminate the **if** condition and at the same time the algorithmic behavior. Therefore, the updated values of t_a and g_f as calculated using 3.14 and 3.15. To emphasize again, when $g_i \leq g_f$ 3.14 and 3.15 are the same steps described in Algorithm 2.

$$t_a = t_a + C\alpha + KC \quad (3.14)$$

$$g_f = g_{max}\alpha - (g_i - g_f - Kg_{max}) \quad (3.15)$$

Chapter 4

Research containing a figure

4.1 Introduction

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4.2 Method

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4.3 Results

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4.4 Discussion

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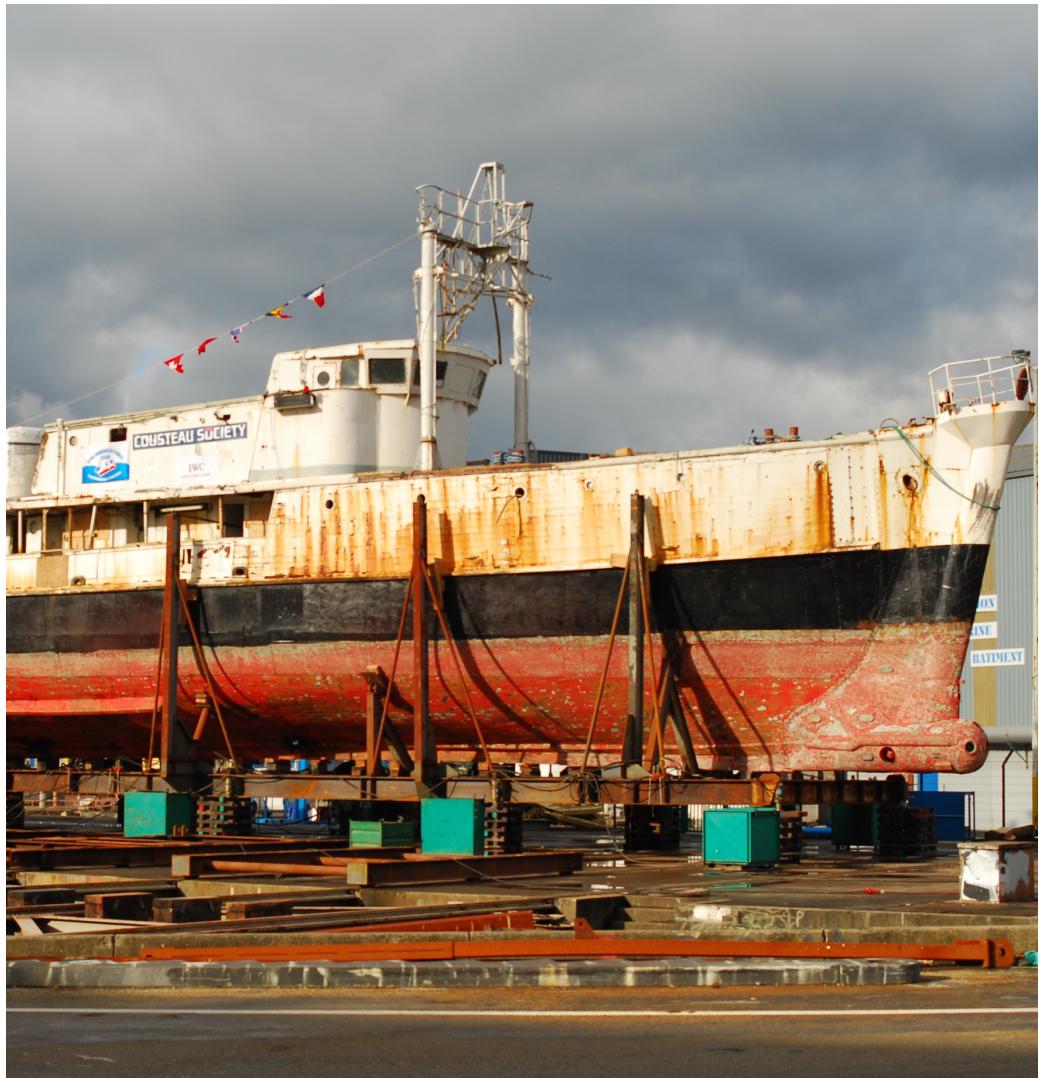


Figure 4.1: RV Calypso is a former British Royal Navy minesweeper converted into a research vessel for the oceanographic researcher Jacques-Yves Cousteau. It was equipped with a mobile laboratory for underwater field research.

4.5 Conclusion

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Chapter 5

Research containing a table

5.1 Introduction

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5.3 Results

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Column 1	Column 2	Column 3
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Row 2	0.3	0.3
Row 3	0.4	0.4
Row 4	0.5	0.6

5.4 Discussion

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5.5 Conclusion

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Chapter 6

Final research study

6.1 Introduction

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6.4 Discussion

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6.5 Conclusion

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Chapter 7

Conclusion

7.1 Thesis summary

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7.2 Future work

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Appendix 1: Some extra stuff

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Appendix 2: Some more extra stuff

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