AMALTHEA-based GPU Response Time Analysis for NVidia's Jetson TX2

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Abstract

The purpose of the thesis is to develop a response time algorithm for AMALTHEA-based models to analyze timing behaviors of CUDA kernels on NVIDIA Jetson TX2's GPU. This work was developed on the context of Bosch WATERS Challenge 2019. The challenge focuses on timing-analysis for heterogeneous software-hardware systems based on centralized end-to-end architectures, and NVIDIA Jetson TX2 was the selected platform for testing. Furthermore, key concepts related to NVIDIA's GPU architecture are presented, as well as a detailed explanation of rules behind platform's GPU scheduler.

Our algorithm is based on these rules and experimental results show the accuracy of our approach to estimate completion times for kernels executed on Jetson TX2 platform. We used in our experiments real timing data from NVIDIA's platform. Moreover, we implemented our algorithm on Eclipse APP4MC, which allows an AMALTHEA-based response time analysis for NVIDIA's Jetson TX2.

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Table of Contents

\mathbf{A}	bstra	act	i
D	eclar	ation of Authorship	ii
Εŀ	ren	wörtliche Erklärung	iii
\mathbf{A}	bbre	viations and Symbols	
		breviations	
1	Inti	roduction	1
	1.1	Motivation	1
	1.2	Industrial challenge WATERS 2019	2
	1.3	NVIDIA Jetson TX2: Architecture Overview	3
	1.4	Jetson TX2 Amalthea Model	4
2	$\mathbf{C}\mathbf{U}$	DA and Jetson TX2	6
	2.1	NVIDIA GPU Software Model	6
	2.2	NVIDIA GPU Hardware Model	9
	2.3	NVIDIA Jetson TX2's GPU Scheduler	11
3	Jets	son TX2's GPU scheduler response time analysis	18
	3.1	Task model	18
	3.2	Assumptions	19
		3.2.1 All blocks have the same amount of threads	20

		3.2.2 One big streaming multiprocessor	20
	3.3	Introduction to GPU Response Time Analysis	21
	3.4	Response Time Analysis Algorithm	24
	3.5	Example	26
	3.6	A Special case	29
	3.7	Computational Complexity	32
4	Exp	erimental Results	36
	4.1	Ground truth generation	36
	4.2	Implementation results	38
	4.3	More results	40
5	Con	clusion	44
	5.1	Thesis summary	44
	5.2	Conclusions	44
	5.3	Future work	45
Aı	ppen	dix 1: Example APP4MC	46
Re	eferei	nces	49

List of Figures

1.1	Jetson TX2 Architecture Overview	3
1.2	Runnable example for a CPU [6]	4
1.3	Runnable example for a GPU [6]	5
2.1	Organisation of grids, blocks, threads, and kernels [11]	7
2.2	Memory hierarchy [11]	8
2.3	Memory hierarchy	Ö
2.4	Inactive threads	10
2.5	Basic GPU scheduling experiment [12]	13
2.6	Detailed state information at various time points in Fig. $2.5\ [12]$.	14
3.1	Time chart	19
3.2	Free blocks (a) at $t = t_1$, $g_f < g_{max}$ (b) at $t = t_2$, $g_f = g_{max}$.	21
3.3	(a) $t_a = t_1 \forall r_4 \text{ s.t } r_4 \leq t_1$ (b) $t_a = r_4 \forall r_4 \text{ s.t } t_2 \leq r_4 \leq t_3$.	22
3.4	(a) New kernel $K3$ with 6 blocks to allocate $g_3=6$. (b) State	
	prior to $K3$ of the GPU (c) state after K3 allocation	22
3.5	(a) New kernel $K3$ with 6 blocks to allocate $g_3=6$. (b) State	
	prior to $K3$ of the GPU. Kernels K1 and K2 were previously	
	allocated (c) state after K3 allocation	23
3.6	(a) Kernel τ_1 (b)GPU state prior to τ_1 allocation (c) GPU state	
	after τ_1 allocation	26
3.7	(a) Kernel τ_2 (b)GPU state prior to τ_2 allocation (c) GPU state	
	after τ_2 allocation	27

3.8	(a) Kernel τ_3 (b)GPU state prior to τ_3 allocation (c) GPU state	
	after τ_3 allocation	28
3.9	(a) Kernel τ_4 (b)GPU state prior to τ_4 allocation (c) GPU state	
	after τ_4 allocation	29
3.10	New kernel allocation	30
3.11	Best fit for computational complexity	33
3.12	Analysis of computational complexity when g_{max} is variable $$.	34
3.13	Analysis of computational complexity when b_{\max} is variable	35
4.1	Output of the CUDA Scheduling Viewer	37
4.2	APP4MC: Scenario 1 - K2,K3,K4,K1	38
4.3	JetsonTX2: Scenario 2 - K2,K4,K1,K3	39
4.4	APP4MC: Scenario 2 - K2,K4,K1,K3	39
4.5	JetsonTX2: Scenario 3 - K2,K1,K3,K4	40
4.6	APP4MC: Scenario 3 - K2,K1,K3,K4	41
4.7	JetsonTX2	42
4.8	APP4MC	42
4.9	JetsonTX2	43
4.10	APP4MC	43

List of Tables

2.1	Types of memories in a GPU	8
2.2	Detailed state information at various time points in Fig	15
2.3	Detailed state information at various time points in Fig	16
2.4	Detailed state information at various time points in Fig	17
3.1	Computational Complexity	32

Abbreviations and Symbols

Abbreviations

API Application Programming Interface

CE Copy Engine

CPU Central Processing Unit

EE Execution Engine
FIFO First In First Out

GPU Graphics Processing Unit

HMP Heterogeneous Multi-Processor

SM Streaming Multiprocessor

OS Operating System

Symbols

au	Set	of	tasks	or	kernels

 au_i Task or kernel i

i Period of τ_i

 $g_i \qquad \qquad \text{Grid size of } \tau_i$

 b_i Number of thread per block within g_i

 C_i Execution time workload of a thread within b_i

 u_i Utilization of τ_i

 r_i Release time of τ_i

f_{i}	Completion time of τ_i
R_{i}	Response time of τ_i
U_t	Total utilization of τ
b_{max}	Maximum amount of threads in a SM
g_{max}	Maximum amount of blocks that can be allocated in a SM
g_f	Available blocks at some point in time
$t_{\tilde{a}}$	Point in ime at which a block will be allocated

Chapter 1

Introduction

1.1 Motivation

Car manufactures want to reduce cost in terms of money and time required to develop, test and validate a new piece of software due to a change of supplier. For that reason, centralized end-to-end architectures are the solution they are aiming to, because for car companies such as BWW and Audi the car of future will be similar to a "data center on wheels" [1].

Centralized end-to-end architectures would be the first step stone toward to decoupling software and hardware [2]. This type of architectures not only will take advantage of internet connectivity, cloud computing and powerful heterogeneous processing units, but also will allow scalable, hierarchical and highly integrated system.

In other words, car manufactures prefer now a days low-latency, hierarchical and cost effectiveness of centralized end-to-end architectures, because today requirements of computational power, bandwidth, integration, safety and real-time [3].

However, car manufactures do not forget at the end, in centralized end-to-end architectures, different types of software would run on top of an heterogeneous

hardware supplied by companies such as NVIDIA, Mobileye or Qualcomm. Thus, it is important to analyze and understand how software will behaves under those conditions, in order to ensure a predictable and efficient system.

1.2 Industrial challenge WATERS 2019

Predictability is a key property for safety-critical and hard real-time systems [4]. Analyzing time related characteristics is an important step to design predictable embedded systems. However, in multi-core or heterogeneous systems based on centralized end-to-end architectures is harder to satisfy timing constrains due to scheduling, caches, pipelines, out-of-order executions, and different kinds of speculation [5]. Thus, development of timing-analysis methods for these types of architectures has become, nowadays, one of the main focus of research in both industry and academic environment.

Robert Bosch GmbH or Bosch, the German multinational engineering and electronics company, and one of the top leaders in development technology for the automotive industry announces every year the WATERS Challenge. The purpose of the WATERS industrial challenge is to share ideas, experiences and solutions to concrete timing verification problems issued from real industrial case studies [6].

This year, 2019, the challenge focuses on timing-analysis for heterogeneous software-hardware systems based on centralized end-to-end architectures. The platform chosen for this purpose is the NVIDIA® JetsonTM TX2 platform which has an heterogeneous architecture equipped with a Quad ARM A57 processor, a Dual Denver processor, 8GB of LPDDR4 memory and 256 CUDA cores of NVIDIA's Pascal Architecture. For the challenge it is available an Amalthea model for this platform to design a solution, and test it later on real hardware.

1.3 NVIDIA Jetson TX2: Architecture Overview

NVIDIA Jetson TX2 is an embedded system-on-module (SOM). It is ideal for deploying advanced AI to remote field locations with poor or expensive internet connectivity, Robotics, Gaming Devices, Virtual Reality (VR), Augmented Reality (AR) and Portable Medical Devices. In addition, it offers near-real-time responsiveness and minimal latency—key for intelligent machines that need mission-critical autonomy [7].

The main components of the Jetson TX2 are dual-core ARMv8 based NVIDIA Denver2, quad-core ARMv8 Cortex-A57, 8GB 128-bit LPDDR4 and integrated 256-core Pascal NVIDIA GPU. The quad-core Cortex-A57 and dual-core NVIDIA Denver2 can be seen as a cluster of heterogeneous multiprocessors (HMP) [8]. Both HMP and GPU shares a 8GB SRAM memory as shown in Figure 1.1. Hereafter, whenever we use the term **host**, we will refer to HMP, similarly we will use **device** to refer to GPU.

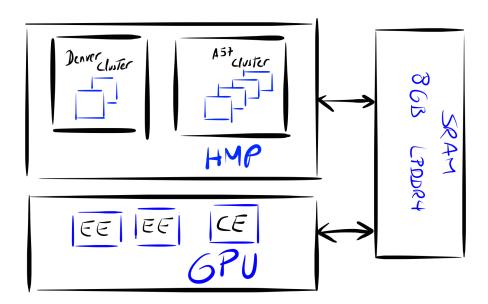


Figure 1.1: Jetson TX2 Architecture Overview

Any NVIDIA GPU has two types of engines, Copy Engines (CE) and Execution Engines (EE). The Jetson TX2 has only one CE and two EE also known

as **Streaming multiprocessors**. CE is in charge of data transfers from host to device and viceversa. There is, moreover, the possibility that EE and CE can run concurrently.

The GPU uses **streams** to run applications. The number of streams depends on the GPU resources. An application can run in one or multiple streams, the GPU scheduler, by default, manages how to application will be allocated on streams in order to maximize throughput. In Chapter 2, we will discuss in more detail how the TX2 GPU scheduler behaves in case of multiple applications.

1.4 Jetson TX2 Amalthea Model

AMALTHEA is a platform for engineering multi- and many-core embedded systems. This platform enables the creation and management of complex tool chains including simulation and validation [9]. In the context of WATERS Challenge 2019, Bosch offers an AMALTHEA model of the Jetson TX2. In this model, a CPU runnable will read data from memory, execute some computation (Ticks) and write back data into memory as shown in Figure 1.2.

Figure 1.2: Runnable example for a CPU [6]

In the case of GPU modeling, the runnable will follow the same pattern as in the CPU case: read, execution, write back. However, the reading operation is actually to copy memory from host to device, thus it is modeled as memory reading from host and then as memory writing to device. On the other hand, the writing back operation requires to copy memory from device to host, therefore it is modeled as memory reading from device and then as memory writing to

host as shown in Figure 1.3.

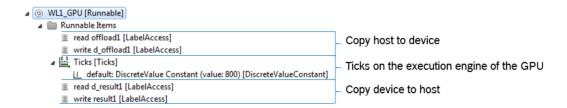


Figure 1.3: Runnable example for a GPU [6]

Chapter 2

CUDA and Jetson TX2

In this chapter we give an overview of the theorical background of NVIDIA GPU software and hardware model. We introduce the concepts of threads, blocks, kernels and streaming multriprocesor, and how they apply to our study case. We discuss about Jetson TX2's memory hierarchy and scheduler. In addition, we describe the rules behind the Jetson TX2's hardware scheduler with an example.

2.1 NVIDIA GPU Software Model

Now a days computer applications run on heterogeneous hardware and GPUs are important in order to achieve high performance computing. Since 2006 running software on NVIDIA GPUs are known as a *CUDA application* [10]. A CUDA application will run concurrently multiple instances of special functions called **kernels**. Each instance runs on a **thread**. Moreover, these threads are arranged in **blocks**, and blocks compose **grids** as shown in Figure 2.1.

It's logical to think that there is also a hierarchical memory structure. Threads, blocks and grids have access to different memory spaces as ilustrated in Figure 2.2. The types of memory are summarized in Table 2.1.

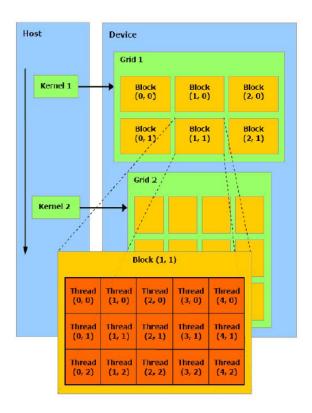


Figure 2.1: Organisation of grids, blocks, threads, and kernels [11].

Table 2.1: Types of memories in a GPU

Memory	Main Characteristics	Scope	Lifetime
Global	R/W, Slow and big	Grid	Application
Texture	ROM, Fast, Optimized for 2D/3D access	Grid	Application
Constant	ROM, Fast, Constants and kernel	Grid	Application
	parameters		
Shared	R/W, Fast, it's on-chip	Block	Block
Local	R/W, Slow as global, when registers are	Thread	Thread
	full		
Registers	R/W, Fast	Thread	Thread

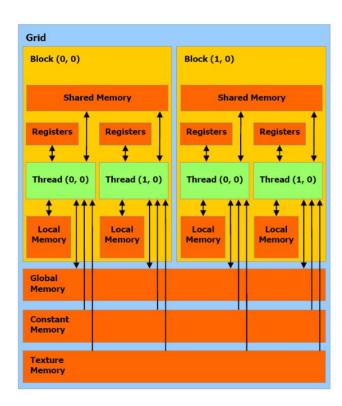


Figure 2.2: Memory hierarchy [11].

In summary, CUDA applications solve problems that were modeled based on divide and conquer principle. Moreover, CUDA software model not only allow

users to achieve high computational performance, but also CUDA application are highly scalable.

2.2 NVIDIA GPU Hardware Model

The CUDA architecture is based on **Streaming Multiprocessors** (SM) which perform the actual computation. Each SM has its own control units, registers, execution pipelines and local memories, but they also have access to global memory as ilustrated in Figure 2.3. A **stream** is a queue of CUDA operations, memory copies and kernel launches. We will talk more about streams in following sections.

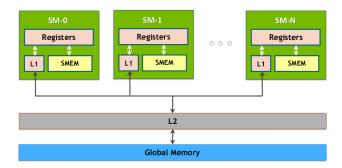
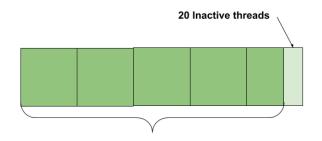


Figure 2.3: Memory hierarchy

When a kernel grid is launched blocks are enumerated and assigned to the SMs. Once the blocks are assigned, threads are managed in **wraps** by the **wrap** scheduler. A wraps is a group of 32 threads that run in parallel. Thus, it is highly recommendable to use block sizes of size $32N, N \in \mathbb{N}$, otherwise there would be "inactive" threads. A example is shown in Figure 2.4, where there is a block of 140 threads but since the wrap scheduler works with wraps, 20 threads are wasted and no other block can make use of them.

The amount of threads and blocks that can run concurrently per SM depends on the number of 32-bit registers and shared memory within SM, as well as the CUDA computing capability of the GPU. Information related to maximum amount of blocks or threads, as well as the computing capability of the GPU



1 block of 140 threads

Figure 2.4: Inactive threads

can be display executing deviceQuery tool. Some information about Jetson TX2 is presented below:

CUDA Device Query (Runtime API) version (CUDART static linking)
Detected 1 CUDA Capable device(s)

Device 0: "NVIDIA Tegra X2"

CUDA Driver Version / Runtime Version 9.0 / 9.0

CUDA Capability: 6.2

Total amount of global memory: 7850 MBytes

(2) SM, (128) CUDA Cores/SM: 256 CUDA Cores

L2 Cache Size: 524288 bytes

Total amount of shared memory per block: 49152 bytes
Total number of registers per block: 32768

Warp size: 32

Max. number of threads per SM: 2048

Max. number of threads per block: 1024

Max dim. size of a thread block (x,y,z): (1024, 1024, 64)

Max dim. size of a grid size (x,y,z): $(2^31-1, 65535, 65535)$

2.3 NVIDIA Jetson TX2's GPU Scheduler

It's common to use several kernels in an application. In order to reduce computation time and maximaze GPU utilization, it is desired to run multiple kernels in parallel. CUDA uses streams to achieve this goal. As mentioned before, a stream is a queue of CUDA operations, memory copies and kernel launches. Thus, it is possible either to launch multiple kernels within one streams or multiple kernels on multiple streams. Operations within the same stream are managed in FIFO (First In First Out) fashion, thus, we will also use the term **stream queue** when we talk about FIFO queues within a stream. The Jeston TX2's GPU assigns resources to streams using its internal scheduler.

Predictability is an important characteristic of safety-critical systems. It requires both functional and timing correctness. However, a detailed information about the Jetson TX2's GPU scheduler behaviour is not publicly available. Without such details, it is imposible to analyze timing constrains. Nevertheless, there are some efforts [12], [13] and [14] aimed at revealing these details through black-box experimentation.

NVIDIA GPU scheduling policies depend on whether the GPU workloads are launched by a CPU executing OS threads or OS processes. We will focus on the first case, because GPU computations launched by OS processes have more unpredictable behaviours, as stated in [12] and [13]. In this section, we will present GPU scheduling policies devired by [12] and use them in an example to clarify their use.

Let us start by defining some terms. When one block of a kernel has been scheduled for execution on a SM it's said that the block was **assigned**. Moreover, it's said a kernel was **dispatched** as soon as one of its blocks were assigned, and **fully dispatched** once all its blocks were assigned. The same applies to copy operations and CE.

There are, in addition, FIFO CE queues used to schedule copy operations, and FIFO EE queues used to schedule kernel launches. Stream queues feed CE and EE queues. Bellow we will present the rules that determine scheduler and

queues behaviours.

• General Scheduling Rules:

- G1 A copy operation or kernel is enqueued on the stream queue for its stream when the associated CUDA API function (memory transfer or kernel launch) is invoked.
- G2 A kernel is enqueued on the EE queue when it reaches the head of its stream queue.
- G3 A kernel at the head of the EE queue is dequeued from that queue once it becomes fully dispatched.
- G4 A kernel is dequeued from its stream queue once all of its blocks complete execution.

Non-preemptive execution:

 X1 Only blocks of the kernel at the head of the EE queue are eligible to be assigned.

• Rules governing thread resources:

- R1 A block of the kernel at the head of the EE queue is eligible to be assigned only if its resource constraints are met.
- R2 A block of the kernel at the head of the EE queue is eligible to be assigned only if there are sufficient thread resources available on some SM.

• Rules governing shared-memory resources:

- R3 A block of the kernel at the head of the EE queue is eligible to be assigned only if there are sufficient shared-memory resources available on some SM.

• Copy operations:

- C1 A copy operation is enqueued on the CE queue when it reaches the head of its stream queue.
- C2 A copy operation at the head of the CE queue is eligible to be assigned to the CE.
- C3 A copy operation at the head of the CE queue is dequeued from the CE queue once the copy is

 C4 A copy operation is dequeued from its stream queue once the CE has completed the copy.

• Streams with priorities:

- A1 A kernel can only be enqueued on the EE queue matching the priority of its stream.
- A2 A block of a kernel at the head of any EE queue is eligible to be assigned only if all higher-priority EE queues (priority-high over priority-low) are empty.

Authors in [12] mentioned that rules related to **registry resources** are expected to have exactly the same impact as threads and shared-memory rules.

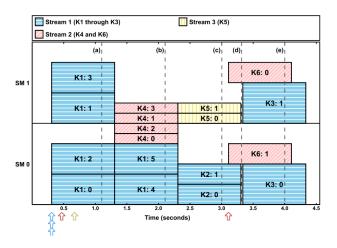


Figure 2.5: Basic GPU scheduling experiment [12]

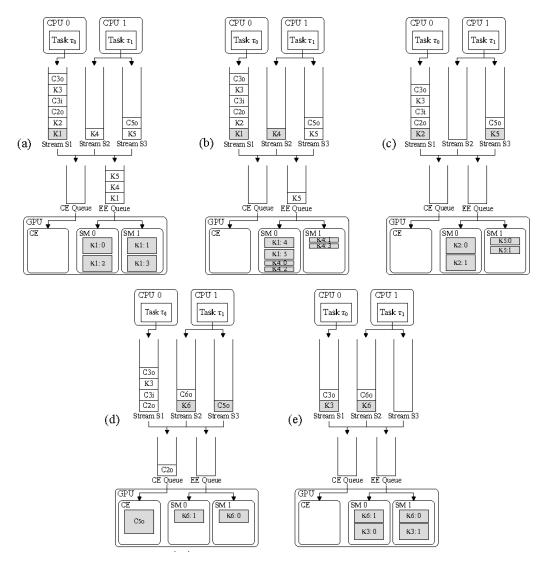


Figure 2.6: Detailed state information at various time points in Fig. 2.5 [12] .

G1 All Kernels except for are not yet enqueued on their streams. K6 is launched at t = 3.2s G2 K1, K4, K5 were enqueued on EE queue. G3 No kernels fullfill this rule. G4 No kernels fullfill this rule. G4 No kernels fullfill this rule. G5 No kernels fullfill this rule. G6 No kernels fullfill this rule. G6 No kernels fullfill this rule. G6 No kernels fullfill this rule. G8 No kernels fullfill this rule. G9 No kernels fullfil	Rules	(a) t=1.0s	(b) t=2.1s	(c) $t=3.0s$	(d) $t=3.4s$
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queues.				from stream	
1 1 2 2 2 2 2 2				queues.	

Table 2.2: Detailed state information at various time points in Fig. $2.6\,$

Rules	(a) t=1.0s	(b) $t=2.1s$	(c) $t=3.0s$	(d) $t=3.4s$
X1	K4 cannot be launched because of this rule, even when there are enough resources (512 threads)	K4 was the next kernel on the EE queue. It was launch because K1 already dispatched it's remaining blocks. K5 is eligible,	K5 blocks became eligible then dispatched. After that K2 blocks became eligible and then dispatched. K5 became el-	K6 blocks became eligible, because K6 was at the head of EE queue.
KI	Applies only to K1.	but check R3	igible. K2 became eligible after K5.	There were enough resources for K6.
R2	Applies only to K1.	K5 is eligible, but check R3	There were enough thread resources for K2 and K5 (1024 threads in SM0, and 1536 threads in SM1).	There were enough thread resources in each SM for K6 (free 512 threads per SM, each K6 block needed 512 threads).
R3	Applies only to K1.	There is not enough shared memory to launch K5. Each K5 block requires 32KB (64KB in total), but K4 blocks are consuming the whole shared memory available per SM (64KB).	There were enough shared memory for K2 and K5 (64KB in each SM).	K6 blocks required no memory shared.

Table 2.3: Detailed state information at various time points in Fig. $2.6\,$

Rules	(a) t=1.0s	(b) t=2.1s	(c) $t=3.0s$	(d) $t=3.4s$
C1	No copy op-	No copy op-	No copy op-	C5o, C2o were
	erations at	erations at	erations at	enqueued on
	the head of	the head of	the head of	CE queue.
	streams.	streams.	streams.	
C2	No avail-	No avail-	No avail-	C5o was as-
	able copy	able copy	able copy	signed to CE.
	operations.	operations.	operations.	
C3	No avail-	No avail-	No avail-	C5o was de-
	able copy	able copy	able copy	queued from
	operations.	operations.	operations.	CE.
C4	No copy op-	No copy op-	No copy op-	C5o is still
	erations at	erations at	erations at	copying. Thus
	the head of	the head of	the head of	it cannot be
	streams.	streams.	streams.	dequeued
				from S3.

Table 2.4: Detailed state information at various time points in Fig. 2.6

Chapter 3

Jetson TX2's GPU scheduler response time analysis

In this chapter, we present our approach to perform a response time analysis for Jetson TX2's GPU scheduler based on the set of scheduling rules explained in the last chapter. We define a task model, declare our assumptions, and give a brief introduction to GPU response time analysis.

In addition we introduce our algorithm, show some examples, describe how our algorithm can change under certain conditions, and discuss its computational complexity.

3.1 Task model

There is a set of tasks or kernels τ of n independent kernels $\{\tau_1, \tau_2, \dots, \tau_n\}$ on a single GPU. Each kernel has a period T_i defined as the separation between two consecutives releases of τ_i , thread execution time workload C_i and a grid of blocks g_i . Each block contains b_i threads.

$$\tau = \{\tau_i\}; \quad i \ge n \land n \in \mathbb{N}$$
 (3.1)

$$\tau_i = \{ T_i, C_i, g_i, b_i \} \tag{3.2}$$

Thus each kernel τ_i has a total of $g_i \cdot b_i$ threads, and the total execution time workload of τ_i is $C_i \cdot g_i \cdot b_i$. The utilization of each kernel is defined as the total execution time workload divided by the period, as stated in [15].

$$u_i = \frac{C_i g_i b_i}{T_i} \tag{3.3}$$

In addition, the total utilization of the set of tasks τ is defined as:

$$U_t = \sum_{\tau_i \in \tau} u_i \tag{3.4}$$

For a kernel τ_i we denote the release time as r_i , the completion time as f_i and response time as $R_i = f_i - r_i$ We assume that a kernel τ_i has a deadline equal to its period T_i .

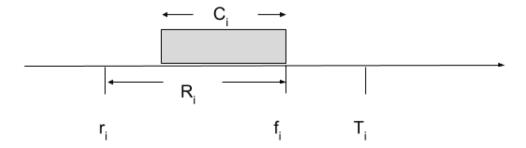


Figure 3.1: Time chart

3.2 Assumptions

For the calculation of response times we have two assumption:

3.2.1 All blocks have the same amount of threads

The election of the optimum number of threads for a specific kernel is a hard task, for that reason there have been some efforts towards that direction [16], [17], [18], [19]. However, NVIDIA developers recommend, for practical purposes, on their offical guides [20] and [21] to use block sizes equals to either 128, 256, 512 or 1024, because it has been documented that these values are more likely to take full advantage of the GPU resources. In our case we will assume that all the blocks, regardless the kernel, are the same size.

$$b_i = b, \quad \forall \tau_i \in \tau \tag{3.5}$$

3.2.2 One big streaming multiprocessor

This assumption is derived from the previous one. Each streaming multiprocessor in the Jetson TX2 has 2048 available threads and since b_i can be either 128, 256, 512 or 1024 (2048/ $b_i = k, k \in \mathbb{N}$), we can think of the two streaming multiprocessors as a big one of 4096 threads.

It means that it could be allocated $2048/b_i$ blocks per SM or $4096/b_i$ blocks in the big SM. Hereafter we will refer the big SM as it were the only SM in the Jetson TX2's GPU. Thus, we defined g_{max} as the maximum number of blocks that can be allocated in the SM at some point in time.

$$g_{max} = \frac{b_{max}}{b}, \quad g_{max} \in \mathbb{N}$$
 (3.6)

Where b_{max} is the maximum amount of threads in the GPU, in the case of Jetson TX2 is 4096.

3.3 Introduction to GPU Response Time Analysis

In addition to the variables defined in our assumptions we define g_f as the number of blocks that are available at some point in time t, and t_a as the point in time in which a block $b_i \in g_i$ can be allocated.

For example in Figure 3.2a is shown that for a $t=t_1$ the amount of free blocks g_f is lower than g_{max} while in Figure 3.2b for a $t=t_2$, $g_f=g_{max}$.

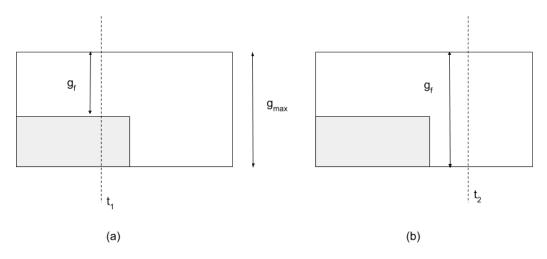


Figure 3.2: Free blocks (a) at $t=t_1$, $g_f < g_{max}$ (b) at $t=t_2$, $g_f = g_{max}$

In Figure 3.3 we present two cases. Let us assume there is a new kernel K4 which wants to allocate a block $b_i \in g_i$. In Figure 3.3a the release time r_4 of the kernel 4 is lower than t_1 , which means that $t_a = t_1$ because $r_4 \le t_1$ and kernel 3 (K3) was already dequeued. In Figure 3.3b r_4 lies between t_2 and t_3 , in that case $t_a = r_4$, because all previous kernels were already dequeued and there are enough resources.

Assuming t_a is known, we would need to calculate how many blocks can be allocated at that point of time. In other words, we need to know the value of g_f at t_a . In Figure 3.4a a new kernel K3 with 6 blocks $g_3 = 6$ is going to be allocation on the Jetson's GPU. Each block have 512 threads, which means that $g_{max} = 8$. The GPU is not executing any kernel at $t = t_a$ as shown in Figure 3.4b therefore $g_f = g_{max} = 8$ at $t = t_a$. Given that $g_3 < g_f$ all the blocks of K3

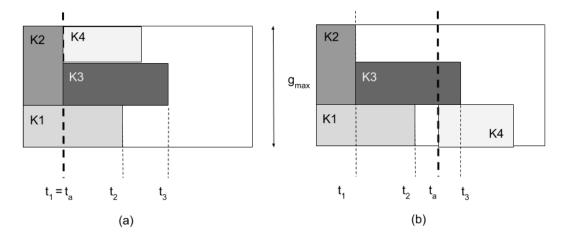


Figure 3.3: (a) $t_a=t_1 \quad \forall r_4 \text{ s.t } r_4 \leq t_1$ (b) $t_a=r_4 \quad \forall r_4 \text{ s.t } t_2 \leq r_4 \leq t_3$

will be allocated at the same time as shown in Figure 3.4c. The completion time f_3 of kernel K3 is t_a plus the thread execution time given by C_3 , $f_3 = C_3 + t_a$. If we assume that the release time r_3 is the same as t_a then the completion time for K3 is the same as the response time R_3 , otherwise $R_3 \ge f_3$.

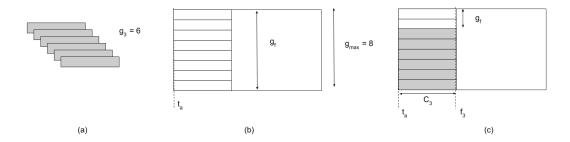


Figure 3.4: (a)New kernel K3 with 6 blocks to allocate $g_3=6$. (b) State prior to K3 of the GPU (c) state after K3 allocation

Once f_3 and R_3 are calculated, it's important to update the values of t_a and g_f , because these values will be used by the following kernel. Let us start with g_f , it is easy to notice that after K3 allocation there are two free blocks $g_f - g_3 = 2$ as a result the new value of $g_f = 2$. On the other hand, by definition t_a is the point in time in which a block $b_i \in g_i$ can be allocated, therefore t_a will not change because $g_f > 0$.

In Figure 3.5 we analyze another highly probable scenario. We use the same kernel K3 as in the last example $(g_3=6)$. However, as shown in Figure 3.5b, there were two kernels allocated previously to K3. Kernel K1 with 5 allocated blocks $g_1=5$ and K2 with 3 allocated blocks $g_2=3$. Note that these kernels have different completion time $f_2>f_1$. Nevertheless, what matters is not either K1 or K2 completion time but the value of t_a and g_f . In this example, t_a is the same as K1 completion time and g_f has the same value as g_1 , $g_f=5$. Thus, 5 blocks from K3 will be allocated first as shown in Figure 3.5c.

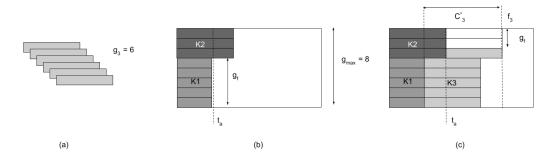


Figure 3.5: (a)New kernel K3 with 6 blocks to allocate $g_3=6$. (b) State prior to K3 of the GPU. Kernels K1 and K2 were previously allocated (c) state after K3 allocation

The next logic question is where the last block of K3 should be allocated. The answer again is given by the updated values of t_a and g_f . Looking at Figure 3.5c is easy to get these new values. The new value of t_a is f_2 since $f_2 < (t_a + C_3)$, and for $t = t_a$ the corresponding g_f is g_2 . Thus, the last K3 block is allocated at $t = t_a = f_2$ and that give us the completion time for K3 that is $f_3 = f_2 + C_3$ or $f_3 = t_a + C_3$. In Figure 3.5c, we defined a new variable C_3^* as the total amount of time in which K3 was using GPU resources.

After K3 allocation, t_a and g_f should be updated again. In this example, the new g_f is the old value of g_f minus the last allocated K3 blocks $g_f = g_f - 1 = 2$, while t_a remains the same $t_a = f_2$ because the conditions are the same as in the later example where there was only one kernel.

3.4 Response Time Analysis Algorithm

Our algorithm is focused on the calculation of t_a and g_f for each block regardless from which kernel τ_i comes. In addition, it is important to notice that t_a and g_f depend on how previous blocks were allocated and on the GPU state at some point in time, as it was described above and illustrated in the Figure 3.4 and Figure 3.5.

The output of our algorithm is a set of release times f_1, f_2, \dots, f_n where n is the length of τ which values f_i depend on t_a and C_i .

$$f_i = f(t_a, C_i) \tag{3.7}$$

A basic version of our algorithm is described in Algorithm 1. This version is derived directly from the examples illustrated in Figure 3.4 and Figure 3.5, in other words, this basic algorithm is a summary of the section above. We have omitted details such as how t_a and g_f are updated in the case that $g_f \geq g_i$, however we still keep the big picture of what is necessary at each step.

```
\begin{array}{l} \textbf{Input} &: \tau \\ \textbf{Output:} \ f_1, \cdots, f_n \\ \textbf{Initialization:} \ t_a = 0, \ g_f = g_{max}, \ i = 1 \\ \textbf{while} \quad i \leq n \ \textbf{do} \\ & | \  \  if \  \  g_f \geq g_i \ \textbf{then} \\ & | \  \  f_i = t_a + C_i; \\ & | \  \  \text{Update} \ g_f \ \text{and} \ t_a; \\ & | \  \  i++\  ; \  // \ \text{Next kernel} \\ & | \  \  else \\ & | \  \  g_i = g_i - g_f; \\ & | \  \  \text{Update} \  g_f \ \text{and} \  t_a; \\ & | \  \  end \\ & | \  \  end \\ \\ & | \  \  end \\ \end{array}
```

Algorithm 1: Basic real time analysis algorithm

In order to analyze a new kernel τ_i and update t_a and g_f we need to track old values of g_f $\forall t \leq t_a$. Fortunately, it is necessary only to track g_f at specific

points of time. Some relevant points of time, as it was shown in the previous example described by Figure 3.5, are given by completion times of previous kernels, in other words we must track g_{i-k} and f_{i-k} where $k \in {1,2,\ldots,i-1}$, because updated values of g_f and t_a depend as well on these them.

Let us define a set h of pair of values (t_k, g_k) where g_k are the number of free blocks at $t = t_k$ such that $t_k \ge t_a$. In a further example we will show step by step how this array h is filled and updated in order to have a better understanding.

A complete version of our algorithm is presented in Algorithm 2.

Algorithm 2: Real time analysis algorithm

Our algorithm is based on three main updates: h, t_a and g_f . The set h can be seen as an array of size Nx2, where N is the number of tracked pairs. For this reason, when $g_f > g_i$ we used MATLAB notation of min function [value, index] = min(A), where index is the position of the pair or row $(t_k, g_k) \in h$ that has the minimum of all time values saved in h. Once we know which pair has the minimum time, we just assign $t_a = t_k$ and $g_f = g_k$. It is important to mention again that by definition of h, all the tracked times should be greater

or equal than the current t_a , meaning that pairs that have tracked times lower than t_a must be removed.

3.5 Example

Let us say there are four kernels we want to allocated, all with the same period T=15 and block size of 512 threads, b=512, which means $g_{max}=8$. The four tasks are defined as $\tau=\{\tau_1=\{15,4,2,512\},\tau_2=\{15,6,7,512\},\tau_3=\{15,6,2,512\},\tau_4=\{15,5,5,512\}\}$.

At the beginning $t_a=0,\,i=1,\,h=\{\}$ and $g_f=g_{max}=8$. Let us start with τ_1 . Kernel τ_1 and inital state of GPU are shown in Figure 3.6(a) and Figure 3.6(b) respectively.

- $g_f \ge g_1$? yes, because $g_1 = 2$
- $f_1 = t_a + C_1 = 0 + 4 = 4$
- $h = \{h, (f_1, g_1)\} = \{(4, 2)\}$
- $t_a = 0$
- $g_f = g_f g_1 = 8 2 = 6$
- i = 2

After τ_1 allocation the GPU state is as shown in Figure 3.6(c), as it is observed, t_a remains the same but g_f now is 6. Furthermore, that is the initial GPU state when τ_2 arrives.

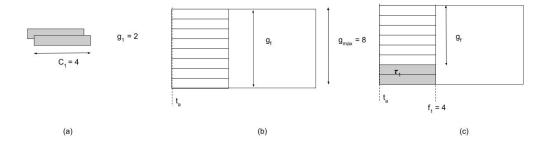


Figure 3.6: (a) Kernel au_1 (b)GPU state prior to au_1 allocation (c) GPU state after au_1 allocation

Since i=2, it's time to analyze τ_2 . Figure 3.7(a) shows the number of blocks that should be allocated for τ_2 . It this case $t_a=0$ and $g_f=6$ as shown in Figure 3.7(b).

- $g_f \ge g_2$? no, because $g_i = 7$
- $g_2 = g_2 g_f = 7 6 = 1$
- $h = \{h, (t_a + C_2, g_f)\} = \{(4, 2), (6, 6)\}$
- $\bullet \ [t_a,\mathrm{index}] = \min(h[:,1]) = \min([4,6])$
- $[t_a, index] = [4, 1]$
- $g_f = h[\text{index}, 2] = h[1, 2] = 2$
- $h = h \{(4,2)\} = \{(4,2), (6,6)\} \{(4,2)\}$
- $h = \{(6,6)\}$

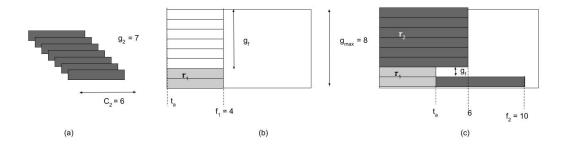


Figure 3.7: (a) Kernel au_2 (b)GPU state prior to au_2 allocation (c) GPU state after au_2 allocation

Values for t_a , g_f , g_2 and h were updated. Notice that current value of t_a is the completion time of τ_1 and g_f is g_1 , that is why, as mention before, it is important to track f_1 and g_1 . However, completion time for τ_2 is not known yet. Let us continue with the analysis.

- $g_f \ge g_2$? yes, because $g_2 = 1$
- $f_2 = t_a + C_2 = 4 + 6 = 10$
- $h = \{h, (f_2, g_2)\} = \{(6, 6), (10, 1)\}$
- $t_a = 4$
- $g_f = g_f g_2 = 2 1 = 1$
- i = 3

In Figure 3.7(c) is shown the GPU state, t_a and g_f values after τ_2 allocation. This setup is the starting point for the analysis of τ_3 as observed in Figure 3.8(b). Since we already described step by step analysis for τ_1 and τ_2 , we skip some details in τ_3 analysis, however we will point out something important about h.

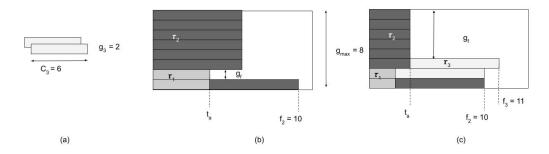


Figure 3.8: (a) Kernel au_3 (b)GPU state prior to au_3 allocation (c) GPU state after au_3 allocation

The initial setup for τ_3 is $t_a = 4$, $g_f = 1$ and $h = \{(6,6), (10,1)\}$. The number of blocks and thread execution time of τ_3 is illustrated in Figure 3.8.

- $g_f \ge g_3$? no
- $g_3 = g_2 g_f = 1$
- $h = \{(6,6), (10,1), (10,1)\}$
- $h = \{(6,6), (10,2)\}$
- $[t_a, index] = [6, 1]$
- $g_f = h[index, 2] = h[1, 2] = 6$
- $h = \{(10, 2)\}$

As mention before, we performed an extra step with h in which h went from having three pairs to having just two. The reason behind it lies on the definition of h. The set h of pair of values (t_k, g_k) where g_k are the number of free blocks at $t = t_k$; notice that at t = 10 there are two free tracked blocks, one that comes from τ_2 and other from τ_3 as shown is Figure 3.8(c), as well as the results of the following τ_3 analysis.

- $g_f \ge g_3$? yes
- $f_3 = t_a + C_3 = 12$

- $h = \{(10, 2), (12, 1)\}$
- $t_a = 6$
- $g_f = g_f g_3 = 5$
- i = 4

The analysis for τ_4 is straightforward. The blocks and thread execution time for τ_4 , the GPU prior to τ_4 allocation and GPU state after τ_4 allocation are shown in Figure 3.9. It is easy to find out that the completion time f_4 is 11.

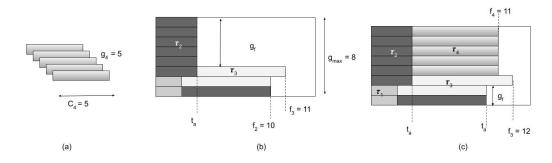


Figure 3.9: (a) Kernel au_4 (b)GPU state prior to au_4 allocation (c) GPU state after au_4 allocation

We have calculated the completion times $f_i \quad \forall i \in \tau, f = \{4, 10, 12, 11\}$. Given the fact that all kernel were scheduled at the same time, the release time for all kernels is 0. Thus, the response time for each kernel is the same as their completion times. Furthermore, we can conclude that all the kernels can be scheduled because $R_i \leq T \quad \forall i \in \tau$.

3.6 A Special case

In this section we demostrate that if all kernels are released at the same time and also have the same thread execution time, $C_i = C \quad \forall i \in \tau$, our response time analysis has not algorithmic behavior, instead it's a set of three equations.

In this special case, there is no need of h, because t_a and g_f can be calculated directly with two equations. The goal is as always to find t_a , g_f and f_i , however, we will exploit the fact that C_i is the same for all kernels. The case in which $g_i \leq g_f$ is trivial to analyze, therefore we will analyze the other case. If Figure

3.10 is shown the block distribution when $g_i > g_f$, thus in order to find g_f^* and t_a^* , that are the updated values, we must calculate K.

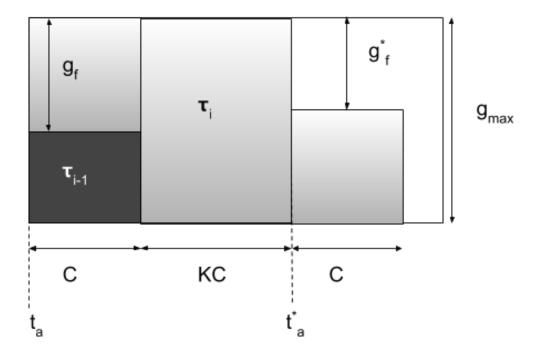


Figure 3.10: New kernel allocation

It is not hard to see that K is given by equation 3.8. The interpretation is that K is the maximum amount of times that g_{max} blocks can be allocated.

$$K = \lfloor \frac{g_i - g_f}{g_{max}} \rfloor \tag{3.8}$$

From K, the value of g_f^* is calculated using equation 3.9. If we observed the blocks in Figure 3.10, we notice that g_f^* can be calculated using geometry. The area of the first block is g_f , of the second one Kg_{max} and the last one is $g_i - g_f - Kg_{max}$.

$$g_f^* = g_{max} - (g_i - g_f - Kg_{max}) (3.9)$$

The value of t_a^* is calculated in similar fashion and is given by equation 3.10. On the other hand, the calculation of f_i remains the same.

$$t_a^* = t_a + C + KC (3.10)$$

We are at a point at which we are able to calculate with formulas the updated values of t_a and g_f , nonetheless, we still have the algoritmic behavior given by the if condition. Therefore to no longer use that dependency, we will make use of the absolute value and the signum function. The signum function of a real number x is defined as follows:

$$\operatorname{sgn}(x) = \begin{cases} -1 & \text{if } x < 0, \\ 0 & \text{if } x = 0, \\ 1 & \text{if } x > 0. \end{cases}$$
 (3.11)

We redefine K with the absolute value using equation 3.12, we use absolute value in 3.12 because we want K to be zero when $g_i \leq g_f$.

$$K = \lfloor \frac{\|g_i - g_f\|}{q_{max}} \rfloor \tag{3.12}$$

In addition, we define α which value is given by equation 3.13. The value of α is zero when $g_i \leq g_f$ and one otherwise.

$$\alpha = \frac{\operatorname{sgn}(g_i - g_f) + 1}{2} \tag{3.13}$$

With 3.12 and 3.13 we eliminate the if condition and at the same time the algoritmic behavior. Therefore, the updated values of t_a and g_f are calculated using 3.14 and 3.15. To emphasize again, when $g_i \leq g_f$ 3.14 and 3.15 are the same steps described in Algorithm 2.

$$t_a = t_a + \alpha C + KC \tag{3.14}$$

$$g_f = \alpha g_{max} - (g_i - g_f - K g_{max}) \tag{3.15}$$

3.7 Computational Complexity

Our algorithm described in Algorithm 2 has two branches, inside the while loop, given by an if conditional. In Table 3.1 is shown the computational complexity of each step of the real time analysis algorithm.

Table 3.1: Computational Complexity

		Average
Step	Type of operation	Cost
$f_i = t_a + C_i$	Sum	O(1)
$h = \{h; (f_i, g_i)\}$	Append	O(1)
$t_a = t_a$	Sum	O(1)
$g_f = g_f - g_i$	Sum	O(1)
i++	Sum	O(1)
$g_i = g_i - g_f$	Sum	O(1)
$h = \{h; (t_a + C_i, g_f)\}$	Append	O(1)
$[t_a, \mathrm{index}] = \min(h[:, 1])$	Min	O(n)
$g_f = h[\mathrm{index}, 2]$	Index	O(1)

The first branch is when $g_f \geq g_i$. The computational complexity, given by big O notation, of that branch is O(1), because all the operations in this branch are O(1).

In the case of the second branch the computational complexity is O(n), because min function is the most costly operation. In the worst case scenario n is the number of kernels we want to allocate, it is important to highlight that only on

the first branch the length of h increases. Thus, the computational complexity of the if statement is O(n).

Let us analyze the outer while loop. The number of iterations depends on number of kernels, their grid sizes g_i and how many blocks can be allocated in total in the GPU or g_{max} . An estimation can be given by $\frac{g}{g_{max}}$, where g is $\sum g_i$, g contains the information about number of kernels and their grid sizes. Thus, computational complexity of our algorithm is $O(\frac{ng}{g_{max}})$.

It can be observed in Figure 3.11 the relationship between computation time and number of kernels and total of allocated blocks. The right side graph shows the polinomial behavior between time and number of kernels. On the other hand, the lelf side graph shows a more linear relation between computation time and number of allocated blocks.

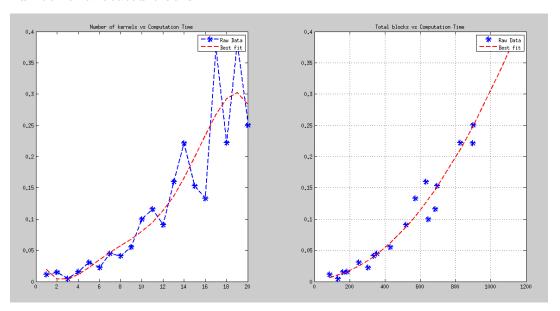


Figure 3.11: Best fit for computational complexity

We analyze in more details the influence of g_{max} and maximum number of allocated blocks on computation time. Let us assume that number of blocks per kernel increase in linear fashion to the number of kernels. In order words, the more kernels we want to launch, the more blocks each kernel will have. As shown in Figure 3.12b. Results are shown in Figure 3.12. The value of g_{max} has

a strong influence on computation times. When $g_{max}=128$ computation times are an order of magnitude lower than when $g_{max}=8$. The inverse relation is consistent with computational complexity. The value of g_{max} tells us how many blocks can be allocated on the GPU at the same time which implies fewer iterations of our algorithm.

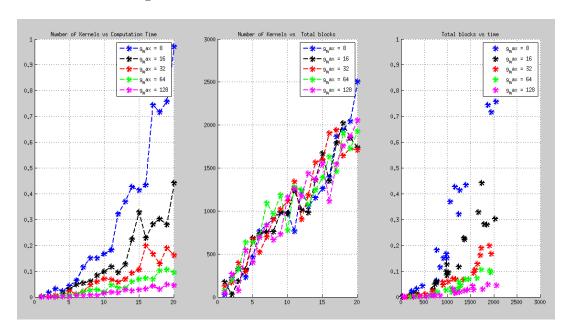


Figure 3.12: Analysis of computational complexity when g_{max} is variable

Let us assume now the value of g_{max} is constant, but the number of of allocated blocks increase as shown in Figure 3.13b. Results are shown in Figure 3.13. In Figure 3.13c, we can see that for the same amount of blocks the algorithm runs faster because there are fewer kernels.

The graphs above tell us that our algorithm perform better when the number of threads per block is low. In addition, the grid size has a minor influence on the algorithm performance as shown in Figure 3.13.

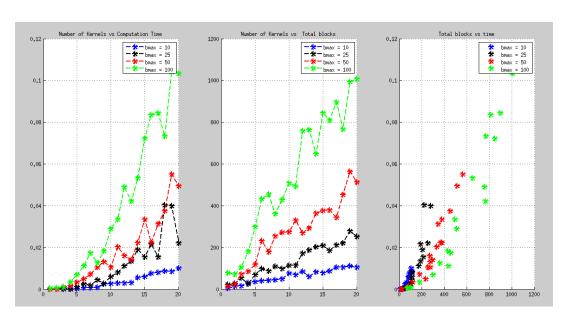


Figure 3.13: Analysis of computational complexity when $\boldsymbol{b}_{\boldsymbol{max}}$ is variable

Chapter 4

Experimental Results

In this chapter we presents our experimental results. A complete example using AMALTHEA models can be found in the Appendix 1. We compare results from Jetson TX2 platfrom againts our APP4MC implementation. The former are used as ground truth to verify our implementation and assumptions.

4.1 Ground truth generation

Amert et. al [12] published their code in github. They developed a CUDA Scheduling Viewer, which is a tool for examining block-level scheduling behavior and co-scheduling performance on CUDA devices. The input are configuration files on the JSON format, and the output can be displayed as figure using a Python script, which is provided as well. An example output is shown in Figure 4.1

Our test scenario was similar to the example presented in the last chapter. We had four kernels we wanted to allocate. The parameters were: block size = 512 threads, and $g_{max} = 8$. The four kernels were defined as $\tau = \{\tau_1 = \{15, 4, 2, 512\}, \tau_2 = \{15, 6, 7, 512\}, \tau_3 = \{15, 6, 2, 512\}, \tau_4 = \{15, 5, 5, 512\}\}$.

An example of a kernel description in the configuration file was as follows:

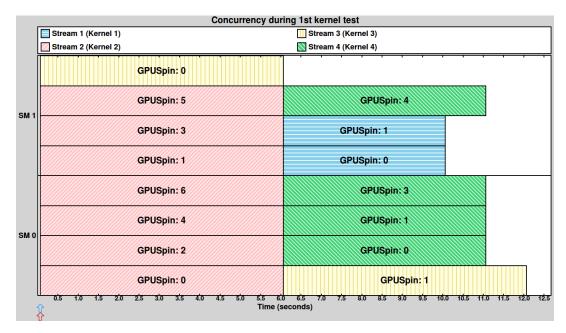


Figure 4.1: Output of the CUDA Scheduling Viewer

```
"filename": "./bin/timer_spin.so",
"log_name": "k3.json",
"label": "Kernel 3",
"thread_count": 512,
"block_count": 2,
"additional_info": 6000000000
```

The filename is the benchmark binary file we used as a kernel. For all the kernels was timer_spin.so. This file defines a bare-bones CUDA benchmark which spins waiting for a user-specified amount of time to complete. The execution time in nanoseconds or C_i was set as additional_info. The log_name was the JSON file that contained metadata and results related to a specified kernel (label). In addition, thread_count and block_count were the values of b_i and g_i respectively.

4.2 Implementation results

We implemented our algorithm in Eclipse APP4MC. The goal was not to test how many kernel the Jetson could manage, instead we focused on verifying our assumptions and therefore our algorithm.

We set up three test scenarios. The four previously described kernels were launched in different order. The first scenario was the one presented in Figure 4.1. The kernels were launched on the following order: K2, K3, K4, K1. As showed in Figure 4.1 the completion times were $f = \{6, 12, 11, 10\}$. The results from APP4MC are shown in Figure 4.2.

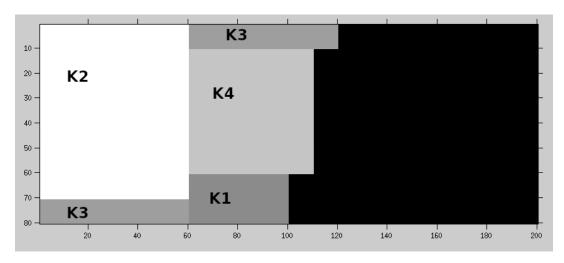


Figure 4.2: APP4MC: Scenario 1 - K2,K3,K4,K1

In the second scenario kernels were launched on the following order: K2, K4, K1, K3. As observed in Figure 4.3 the completion times were $f = \{6, 11, 10, 12\}$. Notice that GPUSping: 5 for kernel 4 should be shown, but there is a bug in the code from [12] in which sometimes the log file doesn't contain all the data. On the other hand, results from APP4MC are shown in Figure 4.4. The block allocation differ from Jetson's allocation because our code follows our assumption described in section 3.3.2.

In the third scenario kernels were launched on the following order: K2, K1, K3, K4. As observed in Figure 4.5 the completion times were $f = \{6, 8, 12, 11\}$.

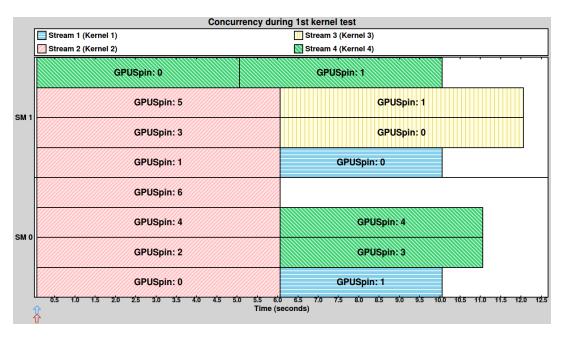


Figure 4.3: JetsonTX2: Scenario 2 - K2,K4,K1,K3

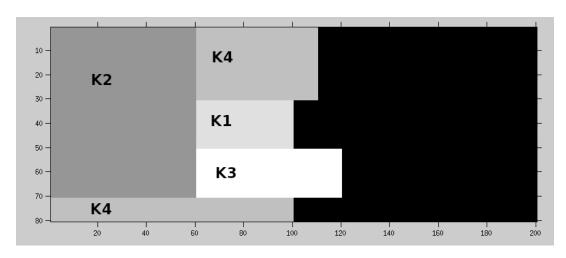


Figure 4.4: APP4MC: Scenario 2 - K2,K4,K1,K3

Notice in this case that *GPUSping:4* from kernel 4 and *GPUSping:1* from kernel 5 overlap in the figure. This is again an error on how the log file was created. We tested [12] C implementation using printf, and the values were correct. Nevertheless, results from APP4MC shown in Figure 4.4 remain congruent with the results of its counterpart.

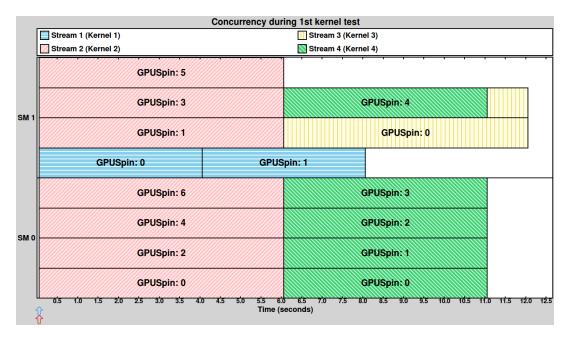


Figure 4.5: JetsonTX2: Scenario 3 - K2,K1,K3,K4

4.3 More results

In this section we present more results. We focused in this section on the interaction between kernels with several blocks, and kernels with long execution time.

In Figure 4.7 can be observed the result after executing five kernels. The kernels were defined as follows:

- $\tau_1 = \{15, 4, 2, 512\}$: small block count, short execution time.
- $\tau_2 = \{15, 7, 1, 512\}$: big block count, short execution time.
- $\tau_3 = \{15, 10, 4, 512\}$: big block count, long execution time.

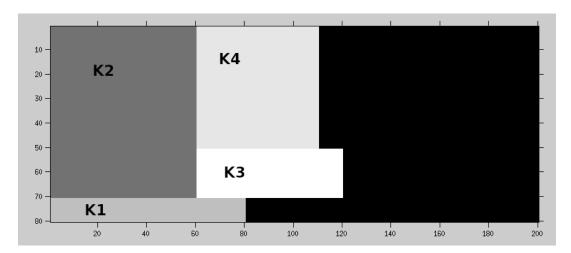


Figure 4.6: APP4MC: Scenario 3 - K2,K1,K3,K4

- $\tau_4 = \{15, 1, 11, 512\}$: small block count, very long execution time.
- $\tau_5 = \{15, 3, 6, 512\}$: big block count, medium execution time.

Our result shown in Figure 4.8 is still consistent with the ground truth. We obtained the same completion time for each kernel.

In this experiment the setup was as follows:

- $\tau_3 = \{15, 7, 1.5, 512\}$: big block count, small execution time.
- $\tau_4 = \{15, 1, 4, 512\}$: very small block count, big execution time.

As expected completion times shown in Figure 4.9 and Figure 4.10 are the same for each kernel.

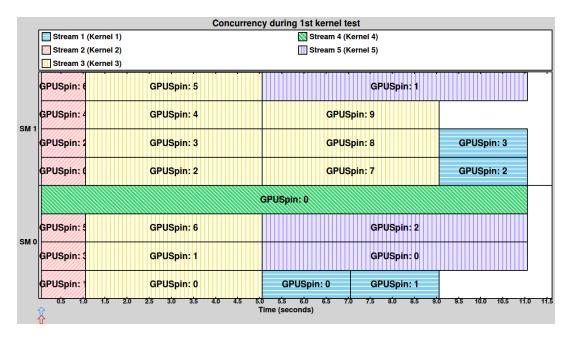


Figure 4.7: JetsonTX2

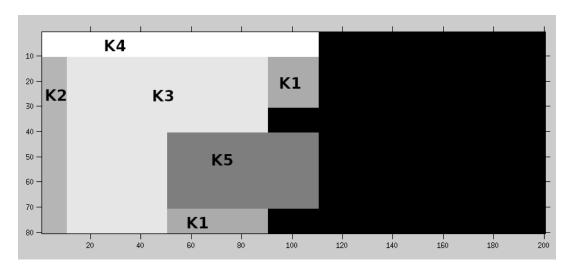


Figure 4.8: APP4MC

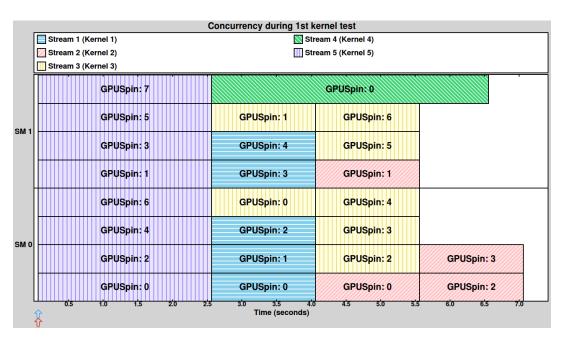


Figure 4.9: JetsonTX2

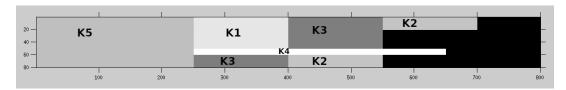


Figure 4.10: APP4MC

Chapter 5

Conclusion

5.1 Thesis summary

We gave an overview in Chapter 1 of the motivation of this work, the Bosch WATERS Challenge 2019, explained the architecture of NVIDIA Jetson TX2 platform and its AMALTHEA model. Furthermore, in Chapter 2 we introduced key concepts related to NVIDIA's GPU such as kernel definition, block and threads, as well as its memory hierarchy. We described in detail Jetson TX2 hardware architecture, and explained the rules behind hardware scheduler. In Chapter 3 we introduced our real time analysis algorithm for Jetson TX2's scheduler. In addition we gave some examples to explain in detail each step. Finally in Chapter 4 we showed our experimental results.

5.2 Conclusions

It has been proved that for purposes of mathematical calculation, the main assumption, due to all blocks have the same thread count we considered all GPU streaming multiprocessors as one big streaming multiprocessor, was correct. Our experimental results show the accuracy of our algorithm to estimate completion times for kernels executed on Jetson TX2 platform. Moreover, we implemented

our algorithm on Eclipse APP4MC, which allows AMALTHEA based response time analysis for NVIDIA's Jetson TX2.

5.3 Future work

There are several potential directions for extending this thesis. First, the possibility to develop a complete model-based development for NVDIA platform using AMALTHEA models, which may include automatic CUDA C code generation for deployment and testing. Second, there is still no full understanding of how Jetson TX2's scheduler decides which kernel should run first. Thus, developers should either consider all possible cases when analyzing completion times or reverse engineering Jetson TX2's scheduler. Finally, we didn't consider memory transaction and other constrains such as amount of shared memory, influence of the *null* stream and priorities within scheduler. Therefore, there is still room to go deeper into this topic.

Appendix 1: Example APP4MC

Here is a complete example of how to use our response time analysis algorithm in APP4MC to analyze AMALTHEA based Jetson TX2's models.

Listing 5.1: Complete example

```
1
2
3
     * Copyright (c) 2018 Robert Bosch GmbH.
4
5
     * This program and the accompanying materials are made
       available under the terms of the Eclipse Public License 2.0
6
 7
      which is available at https://www.eclipse.org/legal/epl-2.0/
 8
9
     * SPDX-License-Identifier: EPL-2.0
10
     * Contributors:
11
12
          Robert Bosch GmbH — initial API and implementation
13
14
15
16
    package app4mc.example.tool.java;
17
    import java.util.ArrayList;
    import java.util.HashMap;
19
20
    import java.util.Map;
21
    import java.util.Random;
23
    import org.eclipse.app4mc.amalthea.model.AmaltheaFactory;
    import org.eclipse.app4mc.amalthea.model.DiscreteValueConstant;
    import org.eclipse.app4mc.amalthea.model.SWModel;
    import org.eclipse.app4mc.amalthea.model.Ticks;
    import org.eclipse.app4mc.amalthea.model.Runnable;
    import org.eclipse.app4mc.amalthea.model.util.*;
    import org.eclipse.emf.common.util.EList;
30
31
    public class rta{
32
33
       public static Long findIndexOfMinValue(Map<Long, Integer> hashMap){
          ArrayList<Long> al = new ArrayList<Long>();
```

```
35
           for (Long m: hashMap.keySet()) {
36
                 al.add(m);
37
           Long minVal = Long.MAX_VALUE;
38
           for (int i=0; i<hashMap.size();i++) {</pre>
39
              if (al.get(i)<minVal) {</pre>
40
41
                 minVal = al.get(i);
42
43
44
45
           return minVal;
46
47
48
       public static Map<Long, Integer> updateH (Map<Long, Integer> h, Long ticks, Integer
         blocks){
49
           if ( h.containsKey(ticks) ){
50
              h.put( ticks , h.get( ticks ) + blocks);
51
52
           else h.put( ticks, blocks );
53
           return h;
54
55
        // Input: swmodel
56
        // Output: Completion times
57
        public static ArrayList<Long> rtaAlgorithm(SWModel swmodel){
58
59
           EList<Runnable> rList = swmodel.getRunnables();
60
61
           ArrayList < Long > c_i = new ArrayList < Long > ();
62
           ArrayList < Integer > g_i = new ArrayList < Integer > ();
63
           ArrayList<Long> f = new ArrayList < Long > ();
64
65
           // Set values c_i, g_i
66
           for (int i = 0; i < rList.size(); i++) {
67
              Runnable rr = rList.get(i);
              c\_i.add(\ ((DiscreteValueConstant)\ SoftwareUtil.getTicks(rr,\ \textbf{null}).get(0).getDefault()).
68
         getValue() );
69
              g_i.add( CustomPropertyUtil.customGetInteger(rr, "GridSize") );
70
71
72
           // Initialization algorithm
73
           Long t_a = (long) 0;
74
           Integer g_max = 8;
75
           Integer g_f = g_max;
           Map< Long, Integer> h = new HashMap<Long, Integer>(); int current_kernel = 0;
76
77
78
           Long minimumRegisteredTicks;
79
80
          // Main loop
           while ( current_kernel < rList.size() )</pre>
81
              if (g_f >= g_i.get(current_kernel))
82
83
                f.add(current\_kernel, t\_a + c\_i.get(current\_kernel));
84
85
                h = updateH(h, f.get(current_kernel), g_i.get(current_kernel));
86
87
                g_f = g_f - g_i.get(current_kernel);
```

```
88
                 current_kernel++;
 89
              else {
 90
 91
                  g_i.set(current_kernel, g_i.get(current_kernel) - g_f);
 92
 93
                  h = updateH(h, t_a + c_i.get(current_kernel), g_f);
 94
                  minimumRegisteredTicks = findIndexOfMinValue(h);
 95
 96
                  g_f = h.get(minimumRegisteredTicks);
 97
                  t_a = minimumRegisteredTicks;
 98
 99
                  h.remove(minimumRegisteredTicks);
100
101
102
103
            return f;
104
105
        public static void main(String[] args) {
106
107
           / Creating a SWModel
108
            SWModel swmodel = AmaltheaFactory.eINSTANCE.createSWModel();
109
            Random rand = new Random();
110
111
            int tick;
            int NumberOfRunnables = 5;
112
113
            int minTicks = 10;
            int maxTicks = 20;
114
115
            int minGridSize = 2;
            int maxGridSize = 20;
116
117
            int gridSize;
118
            for(int i=0; i<NumberOfRunnables; i++) {</pre>
119
             Runnable r = AmaltheaFactory.elNSTANCE.createRunnable();
120
121
             Ticks ticks = AmaltheaFactory.eINSTANCE.createTicks();
122
             DiscreteValueConstant dvc = AmaltheaFactory.eINSTANCE.
          createDiscreteValueConstant();
123
               tick = rand.nextInt((maxTicks - minTicks) + 1) + minTicks;
124
             dvc.setValue(tick);
125
             ticks.setDefault(dvc);
126
             r.getRunnableItems().add(ticks);
127
             gridSize = rand.nextInt((maxGridSize - minGridSize) + 1) + minGridSize;
               CustomPropertyUtil.customPut(r, "GridSize", gridSize);
128
129
             swmodel.getRunnables().add(r);
130
131
            ArrayList < Long > f = new ArrayList < Long > ();
132
133
134
           f = rtaAlgorithm(swmodel);
135
136
            for( int i=0; i<f.size(); i++){
137
               System.out.println("\t Computation time:" + f.get(i));
138
139
140
```

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