UNIVERSITY OF NEW SOUTH WALES

School of Mathematics and Statistics

MATH3041 - Stochastic Modelling

This assignment must be submitted by 5:00 pm on June 5, 2009.

1. Assume the CRR model with $d = .97, u = 1.05, r = 0.02$ and $S_0 = 12$. Consider the European call option written on the stock S with the expiration date $T = 8$.

   (a) Compute the arbitrage price $(C_t)_{t=0}^T$ for the call option using the recursive formula, for $t = 0, 1, \ldots, T-1$,

   $$C_t(S_t) = (1+r)^{-1}\Big(p_* C_{t+1}(uS_t) + (1-p_*)C_{t+1}(dS_t)\Big)$$

   with $C_T(S_T) = (S_T - K)^+$. Consider the strikes $K = 9 + i$ for $i = 1, \ldots, 8$.

   (b) Find the replicating strategy for the option when $K = 14$.

2. Assume the CRR model with $d = 0.95, u = 1.05, r = 0.02$ and $S_0 = 100$. Consider the American put option written on the stock S with the expiration date $T = 10$ and strike $K = 100$.

   (a) Compute the arbitrage price process $(P_t^a)_{t=0}^T$ for the option using the recursive formula, for $t = 0, 1, \ldots, T-1$,

   $$P_t^a(S_t) = \max\Big\{(K - S_t)^+,\, (1+r)^{-1}\Big(p_* P_{t+1}^a(uS_t) + (1-p_*)P_{t+1}^a(dS_t)\Big)\Big\}$$

   with $P_T^a(S_T) = (K - S_T)^+$.

   (b) Find the optimal exercise time τ_0^* for the holder of the option.

3. Assume the CRR model with $d = 0.9, u = 1.1, r = 0.05$ and $S_0 = 25$. Consider the game option with the expiration date $T = 12$ and the payoff functions $h(S_t)$ and $\ell(S_t)$ where

   $$h(S_t) = (K - S_t)^+ + \alpha$$

   and

   $$\ell(S_t) = (K - S_t)^+$$

   where $\alpha = 0.02$ and $K = 27$.

   (a) Find the arbitrage price process $(G_t)_{t=0}^T$ for the option using the recursive formula, for $t = 0, 1, \ldots, T-1$,

   $$G_t(S_t) = \min\Big\{h(S_t),\, \max\Big[\ell(S_t),\, (1+r)^{-1}\Big(p_* G_{t+1}(uS_t) + (1-p_*)G_{t+1}(dS_t)\Big)\Big]\Big\}$$

   with $G_T(S_T) = \ell(S_T)$.

   (b) Find the optimal exercise times τ_0^* and σ_0^* for the holder and the writer of the option.

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