The Adaptive Method of Lines

CDEs Group Presentation

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Motivation

Method of lines

- Discretise in space (ie using finite difference) to generate a system of ODEs.
- Solve the system using some solver such as ode15s.
- Temporal accuracy handled by the ODE solver, however spatial accuracy results from the discretisation used.

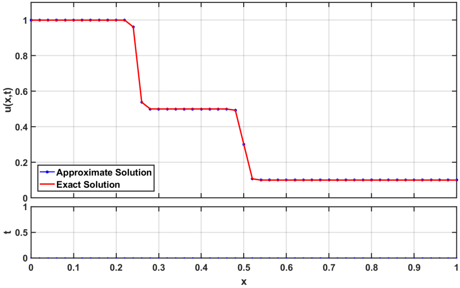
Motivation

Mesh adaptation

- Method of lines discretises the space uniformly.
- Using a high density uniform mesh to achieve high spatial accuracy wastes nodes in regions of low activity.
- Want to adapt the spatial mesh such that nodes are efficiently placed.
  - Static moving mesh.
  - Static grid refinement.
  - Dynamic mesh.

Method of Lines - Burgers' Equation

- content



Equidistribution Principle

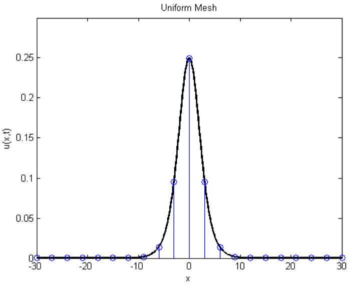
Tracking the action

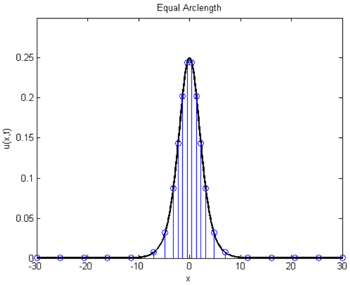
- We want to assign nodes to areas with high activity.
- Use some monitor function $m(x)$ to measure the activity in a region.
- Choose the x_i such that for all i, $\int_{x_{i-1}}^{x^i} m(x)\,\mathrm{d}x = \int_{x_i}^{x^{i+1}} \mathrm{d}x$ (equidistribution).
- In practice equidistribution is considered optimal, but some suboptimal methods may be used to have roughly equal distribution in order to reduce computational effort.

Choice of Monitor Function

- Arc length $m(x) = \sqrt{\alpha + u_x^2(x)}$.
- Local curvature $m(x) = |u_{xx}(x)|$.
- Other options involving many tuning parameters exist.
- Since the true solution is not known, derivatives must be estimated. Can use natural splines to interpolate current solution and use its derivatives as approximation.

Uniform vs Equidistributed Arclengths (n=21)



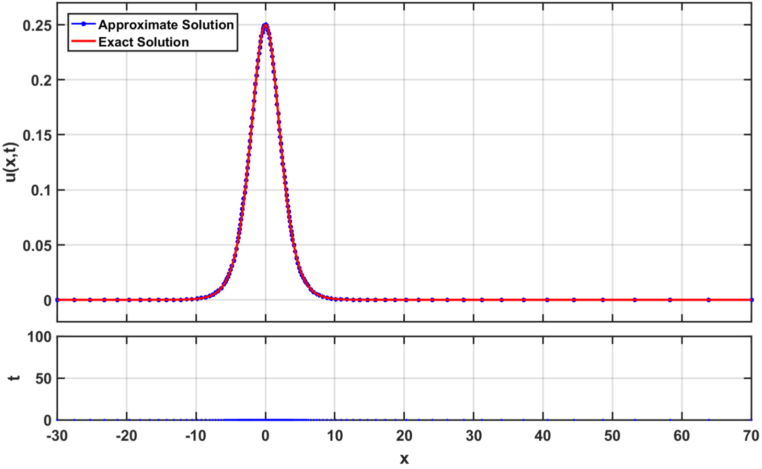


Moving Mesh

- Fix the number of nodes.
- Pause the PDE solver at a set interval to move the mesh nodes to achieve equidistribution.
- Interpolate the solution from old mesh to generate initial conditions for new mesh.

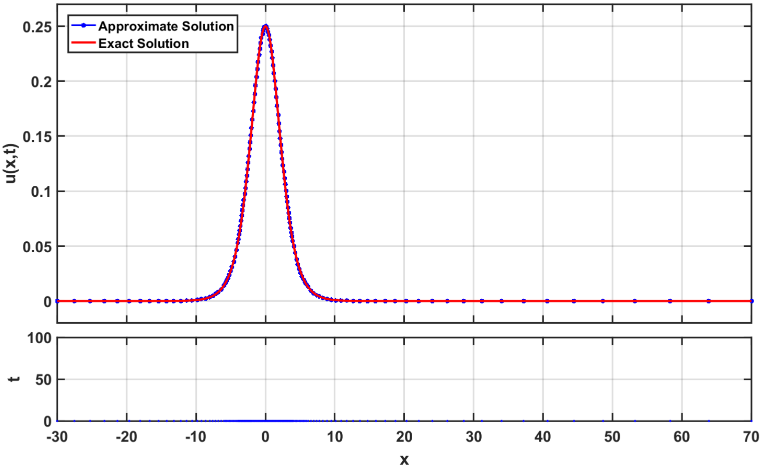
Moving Mesh - KDV Equation Example 1

- Time step = 10



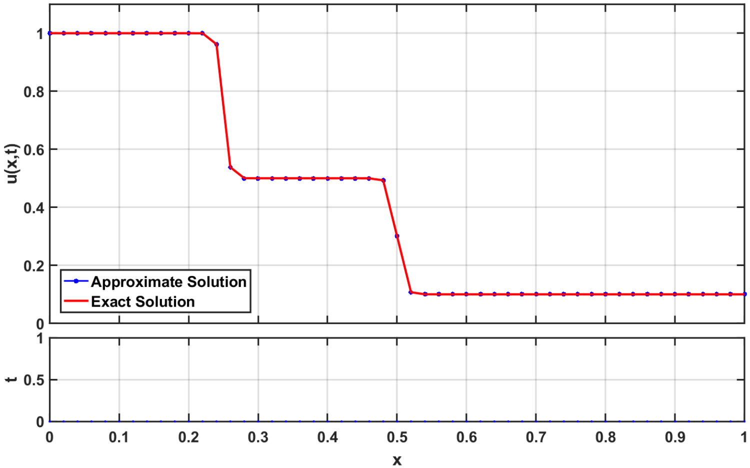
Moving Mesh - KDV Equation Example 2

- Time step = 2



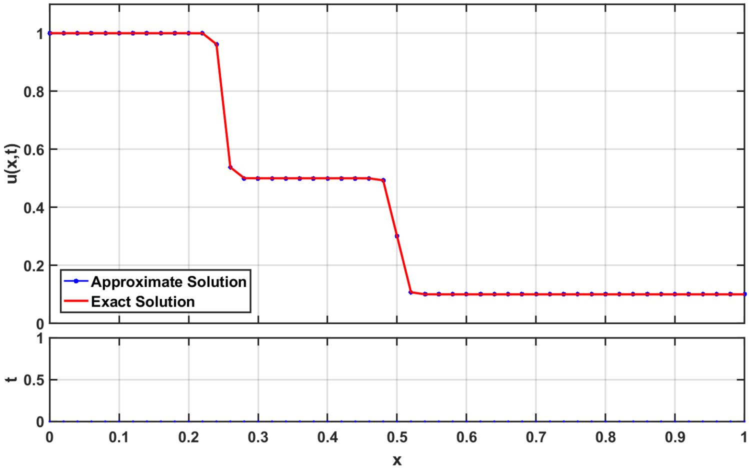
Moving Mesh - Burgers' Equation Example 1

- Time step = 0.1



Moving Mesh - Burgers' Equation Example 2

- Time step $=$ 0.01

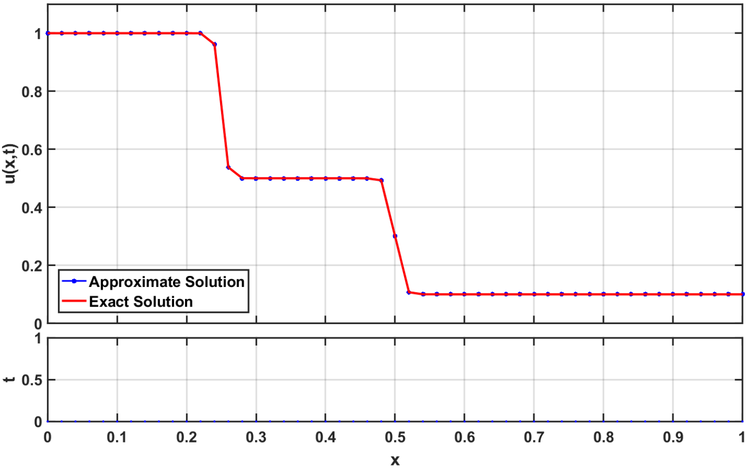


Mesh Refinement

- Just as in moving mesh, the mesh is updated at set intervals.
- Rather than trying to achieve equidistribution. An uniform grid is used as a skeleton, at each pause the monitor function is measured between each node, additional nodes with uniform spacing is added in between nodes that exceed some threshold.
- Interpolate and continue.

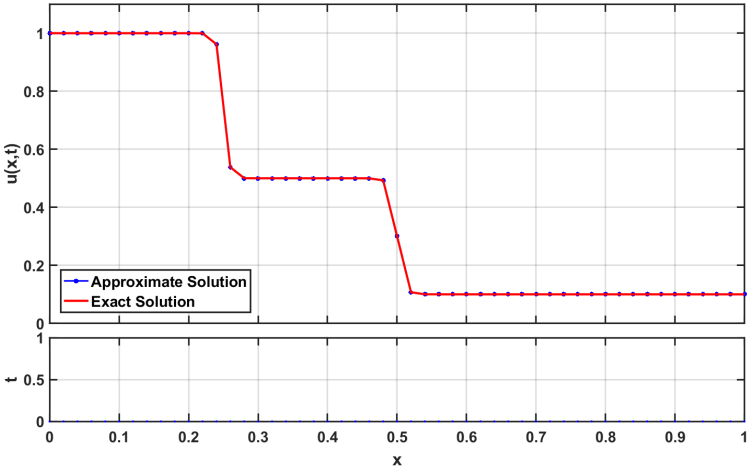
Mesh Refinement - Burgers' Equation Example 1

- Time step = 0.1



Mesh Refinement - Burgers' Equation Example 2

- Time step = 0.01

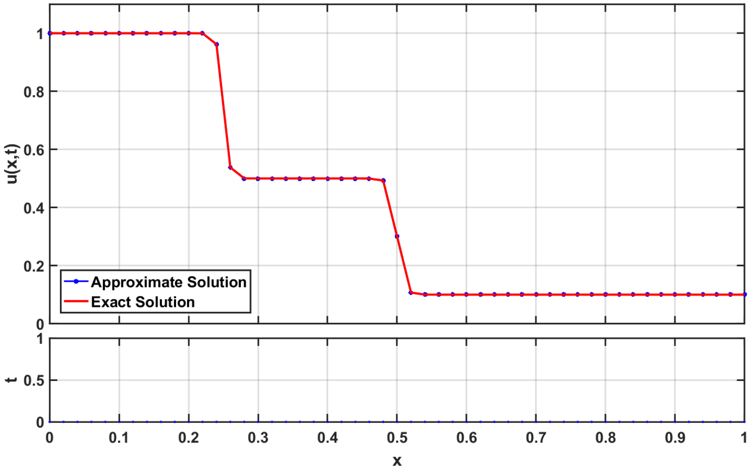


Dynamic Method

- Refining the grid at set time steps can cause the mesh to lag behind the moving solution.
- Refining too often is computationally expensive and negates any benefits from adaptive mesh.
- May be possible to use a priori information to generate functions for the wave to travel along. For example the characteristic curves along which the solution is constant. For a simple moving wave this tracks positions on the wave-front.
- Requires specific a priori knowledge on the problem, not always possible to derive characteristic curves.

Dynamic Method - Burgers' Equation Example

- Number of mesh points = 51



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Any questions?