Letter Vol. 43, No. 11 / 1 June 2018 / Optics Letters 2547

Optics Letters

Designing optical circuits using plasmonic beam splitters

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Received 13 March 2018; revised 27 April 2018; accepted 27 April 2018; posted 30 April 2018 (Doc. ID 325816); published 22 May 2018

Plasmonic optical circuits hold great promise in reducing device footprints by orders of magnitude and enabling high device complexity. The decomposition of an arbitrary linear transformation using unitary beam splitters is well-known. However, because of the inherent lossy nature of plasmonic devices, this decomposition is not useful for the practical design of devices. In this Letter, we provide a method to design an arbitrary unitary transformation using plasmonic beam splitters, which takes into account the inherent lossy nature of plasmonic modes in the decomposition process itself, while preserving the fidelity of the transformation. We do this by selecting the loss in each arm of the beam splitters and the interconnects. We also show how this method can be extended for the case of any linear transformation by extending the singular value decomposition. This method is applicable to plasmonic and waveguide-based lossy beam splitters.

OCIS codes: (130.0130) Integrated optics; (310.6628) Subwavelength structures, nanostructures; (230.1360) Beam splitters; (200.4650) Optical interconnects; (200.4660) Optical logic.

https://doi.org/10.1364/OL.43.002547

Integrated optical circuits are interesting because of multiple benefits such as inherent stability, scalability, and the possibility of low power consumptions for computing applications. Various classical applications such as optical computation [1,2], frequency filtering [3–6], and beam shaping [7,8], to quantum applications for both discrete [9–11] and continuous variable methods [12] have been implemented using integrated optical devices. A general algorithm was proposed by Reck *et al.* [13] using linear optical elements such as two-port beam splitters and phase shifters which could be used to design complex linear multiport devices. With the increasing complexity of devices, the need to come up with techniques to add large numbers of functional devices in a region with a small footprint is of great interest.

Plasmonics offers particular advantages in this regard. Its well-known benefits of being able to confine light to subdiffraction limited regimes have been exploited for various applications such as integrated signal processing systems, nanoresolution optical imaging [14–17], and sensing [18,19]. In spite of being inherently lossy, plasmonic beam splitters have been used to demonstrate various quantum functionalities such as the smallest implementation of quantum C-NOT gate [20] and the anti-coalescence of bosons in a plasmonic beam splitter [21]. However, with plasmonic systems, the consideration of losses in the design process is very important. In this Letter, we consider an algorithm which could be used to design a general linear transformation using plasmonic circuits. The methods used for decomposition, in general, are not resistant to losses, and the fidelity of the transformation to be implemented drops drastically with increasing loss. Reference [22] proposes a decomposition method where the circuits are much more robust against loss; however, with increasing loss, the fidelity for this method also drops rapidly. In this Letter, we discuss a method that may be used to design unitary circuits using lossy beam splitters and interconnects, and retain their fidelity for any amount of loss. We also then extend this method for any linear transformation using singular (SVD) [23].

In this Letter, we consider lossy beam splitters to be of the form as shown in Fig. 1. This form may be obtained using both plasmonic beam splitters and waveguide-based splitters with losses at the bends. Basically, the form consists of attenuation factors for the input electric fields, followed by unitary exchange further followed by loss for the propagating output fields. For a two-port beam splitter, it is given by the following equation:

$$\begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix}\begin{bmatrix} t_1 & -r_1 \\ r_1 & t_1 \end{bmatrix}\begin{bmatrix} s_3 & 0 \\ 0 & s_4 \end{bmatrix}. \quad (1)$$

Here s_1, s_2 are the transmittance of ports 1 and 2 before the unitary matrix, while s_3, s_4 is the transmittance of ports 1 and 2 after the unitary matrix. The central matrix in Eq. (1) represents a unitary beam splitter with t_1, r_1 as its transmittance and reflectance, respectively. We illustrate the effect of lossy beam splitters on the fidelity by considering an example of a 3×3 unitary transformation using the decomposition proposed in Ref. [22]. Consider the unitary matrix given by the following decomposition:

$$U = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \times \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

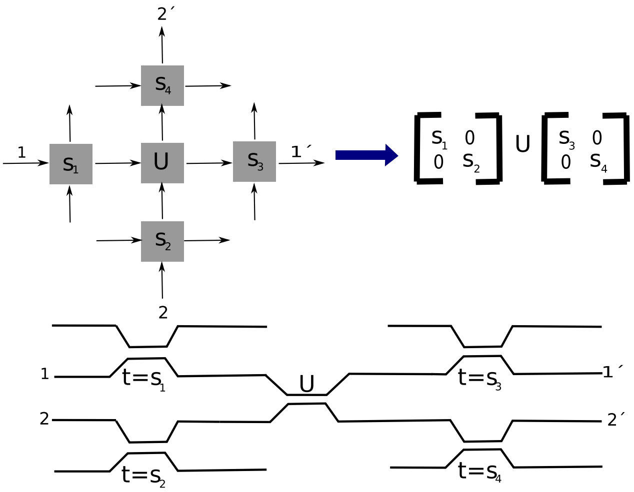

Fig. 1. Schematic for the lossy beam splitters considered in this Letter. The figure corresponds to the transformation shown in Eq. (1). We also show an equivalent representation with directional couplers. We see the transmittance t of the directional couplers which are used to model losses in ports 1 and 2, and are given by s_1, s_2 before U and s_3, s_4 after U. The matrix U represents the matrix $[t_1, -r_1; r_1, t_1]$ in Eq. (1).

Each of the three matrices in the above equation represents a two-port unitary beam splitter as the matrix $\begin{bmatrix} t_1 & -r_1 \\ r_1 & t_1 \end{bmatrix}$ in Eq. (1), with now $t_1 = r_1 = 1/\sqrt{2}$. In the presence of equal losses in all the input and output ports, these get modified to

$$U_{\text{loss}} = \begin{bmatrix} s_1^2/\sqrt{2} & -s_1^2/\sqrt{2} & 0 \\ s_1^2/\sqrt{2} & s_1^2/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & s_1^2/\sqrt{2} & -s_1^2/\sqrt{2} \\ 0 & s_1^2/\sqrt{2} & s_1^2/\sqrt{2} \end{bmatrix} \times \begin{bmatrix} s_1^2/\sqrt{2} & -s_1^2/\sqrt{2} & 0 \\ s_1^2/\sqrt{2} & s_1^2/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

where each 2×2 beam splitter has been modified as in Eq. (1) with $s_1 = s_2 = s_3 = s_4$.

To characterize the effect of losses on the unitary transformation, we use fidelity as the figure of merit. This fidelity for a $N \times N$ matrix is defined as [22]

$$F(U, U_{\text{loss}}) = \left| \frac{\text{tr}(U^\dagger U_{\text{loss}})}{\sqrt{N \text{tr}(U_{\text{loss}}^\dagger U_{\text{loss}})}} \right|^2. \tag{4}$$

The normalization makes sure that the effect of a constant scaling of all the elements of the unitary matrix does not change its fidelity. Figure 2 shows how this fidelity changes with different values for scaling factor s_1. We see that already at 50% loss the fidelity drops to 85% and drops rapidly below. We now show how a unitary matrix scaled down by a constant factor can be implemented. A uniformly scaled unitary matrix can be written as

$$U_{N,N,\text{loss}} = U_{N,N} \times \text{diag}(l, \ldots l)_N, \tag{5}$$

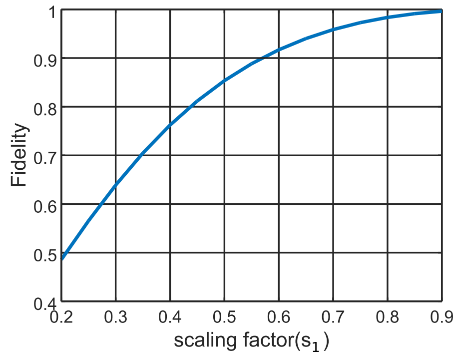

Fig. 2. Chart showing how the fidelity for 3×3 unitary transformation changes with s_1, according to Eq. (3).

where $\text{diag}(l, \ldots l)_N$ is a $N \times N$ diagonal matrix with scaling factor $l \leq 1$. We first decompose $U_{N,N}$ into constituent beam splitter transformations $T_{m,n}$ where

$$\begin{bmatrix} 1 & .. & 0 & .. & 0 & .. & 0 \\ . & .. & 1 & .. & 0 & .. & .. \\ 0 & .. & t_{m,m} & .. & t_{m,n} & .. & 0 \\ . & .. & t_{n,m} & .. & t_{n,n} & .. & 0 \\ . & .. & 0 & .. & 0 & .. & .. \\ 0 & .. & 0 & .. & 0 & .. & 1 \end{bmatrix}. \tag{6}$$

Such a matrix, given by Eq. (5), maintains its fidelity, according to the definition in Eq. (4). We then use the following algorithm: suppose decomposition of $U_{N,N}$ leads to a product of matrices $T_{m1,n1}, T_{m2,n2}, .., T_{mN_{tot},nN_{tot}}$ with $N_{\text{tot}} = N(N-1)/2$, then we can write $U_{N,N,\text{loss}}$ as:

$$U_{N,N,\text{loss}} = T_{m1,n1} T_{m2,n2} .. T_{mN_{\text{tot}},nN_{\text{tot}}} \times \text{diag}(l, ..l)_N. \tag{7}$$

Now using

$$\begin{aligned} &T_{i,j} \times \text{diag}(1, ..1_i, .., 1_j..l_k, ..1) \\ &\quad = \text{diag}(1, ..1_i, .., 1_j..l_k, ..1) \times T_{i,j}, \end{aligned} \tag{8}$$

and

$$\begin{aligned} T_{i,j} \times \text{diag}(1, ..l_i, .., l_j..1, ..1) = \text{diag}(1, ..f, .., f..1, ..1) \times T_{i,j} \\ \times \text{diag}(1, ..l_i/f, .., l_j/f..1, ..1), \end{aligned} \tag{9}$$

where $l_i = l_j = l$ and f are scaling factors, we can then show that matrix $U_{3,3,\text{loss}}$ can be written as the following decomposition:

$$\begin{aligned} U_{3,3,\text{loss}} &= T_{12} T_{23} T_{12} \times \text{diag}(s_1^6, s_1^6, s_1^6) \\ &= \text{diag}(1, 1, s_1^2) \times \text{diag}(s_1, s_1, 1) \times T_{12} \times \text{diag}(s_1^3, s_1, 1) \\ &\quad \times \text{diag}(1, s_1, s_1) \times T_{23} \times \text{diag}(1, s_1, s_1^3) \times \text{diag}(s_1, s_1, 1) \\ &\quad \times T_{12} \times \text{diag}(s_1, s_1, 1). \end{aligned} \tag{10}$$

We see that each of the matrices T_{12}, T_{23} has a scaling matrix before and after it and, thus, is in the form discussed in Eq. (1). Here $(1 - |s_1|^2)$ corresponds to the smallest amount of unavoidable loss in any beam splitter arm. We see here that these scaling factors will also be useful in designing interconnect lengths in every arm; as in the case of plasmonic splitters, the

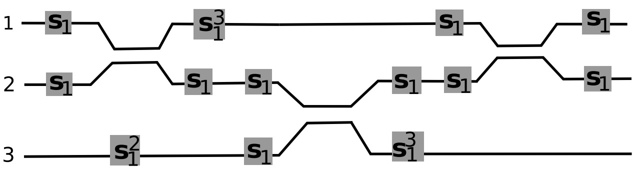

Fig. 3. Decomposition of different matrices, according to Eq. (10).

interconnects themselves can be lossy. In addition, once designed, these loss factors are the same for any unitary matrix of order N. Figure 3 shows the implementation of this method for a unitary matrix of order N. Thus, we see that a unitary matrix scaled by a constant factor may be implemented by a proper combination of lossy beam splitters without loss in fidelity.

This method also provides some advantages for unbalanced losses. Unbalanced losses are caused mainly because of the fact that different paths pass through different numbers of beam splitters and, thus, may experience different losses. However, that is not an issue here as losses are not only because of the beam splitters, but also because of the interconnects themselves. By decomposing Eq. (5) and distributing losses over the entire circuit (this might mean adjusting the length of the interconnects), we make sure all paths experience the same loss. This method may also be made more robust to fabrication imperfections by using a more loss-resistant decomposition of $U_{N,N}$, as discussed in Ref. [22]. The method discussed in this Letter also allows us to take into account unequal losses in different beam splitters, as well as unequal losses in different arms of a beam splitter. This is because the decomposition in Eq. (10) is not unique and gives more freedom in adjusting losses in different ports of a beam splitter. Suppose we consider decomposition of $U_{3,3,\mathrm{loss}}$:

$$\begin{aligned} U_{3,3,\mathrm{loss}} &= T_{12}T_{23}T_{12} \times \mathrm{diag}(s_1^6, s_1^6, s_1^6) \\ &= T_{12}T_{23} \times \mathrm{diag}(1, 1, s_1^6) \times \mathrm{diag}(s_1^5, s_1^5, 1) \\ &\quad \times T_{12} \times \mathrm{diag}(s_1, s_1, 1) \\ &= T_{12}T_{23} \times \mathrm{diag}(1, 1, s_1^6) \times \mathrm{diag}(s_1^3, s_1^2, 1) \\ &\quad \times \mathrm{diag}(s_1^2, s_1^3, 1) \times T_{12} \times \mathrm{diag}(s_1, s_1, 1). \end{aligned} \tag{11}$$

The last three terms could be used to design a beam splitter for ports 1 and 2 as in Eq. (1), now with unequal losses in the input ports (s_1^2 and s_1^3 in ports 1 and port 2, respectively). The term $\mathrm{diag}(s_1^3, s_1^2, 1)$ can be propagated further and used in the later stages of the decomposition.

This method can be extended for any linear transformation by using SVD. Any $N \times N$ matrix can be implemented by decomposing it into a product of unitary matrices along with a scaling matrix:

$$A = U_{1,N,N} \times \mathrm{diag}(f_1, f_2, ..f_N) \times U_{2,N,N}, \tag{12}$$

where we assume that A has been appropriately scaled so that at least one scaling factor in the diagonal matrix is 1, and all others are ≤ 1. Note that the scaling matrix $\mathrm{diag}(f_1, f_2, ..f_N)$ is inherent to the decomposition of A and not because of losses in beam splitters. Similar to the above discussion for unitary matrices, the matrix implemented by lossy beam splitters can be written as

$$\begin{aligned} A_{\mathrm{loss}} &= U_{1,N,N} \times \mathrm{diag}(l, l\ldots l) \times \mathrm{diag}(f_1, f_2, \ldots, f_N) \\ &\quad \times \mathrm{diag}(l, l, ..l) \times U_{2,N,N}, \end{aligned} \tag{13}$$

where we have used the fact that a diagonal matrix with equal entries commutes with any matrix. For symmetry reasons, this can be written as

$$\begin{aligned} A_{\mathrm{loss}} &= U_{1,N,N} \times \mathrm{diag}(l, l, ..l) \times \mathrm{diag}\left(\sqrt{f_1}, \sqrt{f_2}, .., \sqrt{f_N}\right) \\ &\quad \times \mathrm{diag}\left(\sqrt{f_1}, \sqrt{f_2}, .., \sqrt{f_N}\right) \times \mathrm{diag}(l, l, ..l) \times U_{2,N,N}. \end{aligned} \tag{14}$$

Using Eqs. (8) and (9), in addition to using

$$\begin{aligned} \mathrm{diag}(1, ..l_i, .., l_j..1, ..1) \times T_{i,j} &= \mathrm{diag}(1, ..f, .., f..1, ..1) \times T_{i,j} \\ &\quad \times \mathrm{diag}(1, ..l_i/f, .., l_j/f..1, ..1), \end{aligned} \tag{15}$$

matrix A_{loss} can be similarly implemented. The difference here is that now the scaling matrix is no longer a constant scaling matrix, but it is given by the matrix product of $\mathrm{diag}(l, l, ..l) \times \mathrm{diag}(\sqrt{f_1}, \sqrt{f_2}, ..\sqrt{f_N})$.

Factoring the diagonal matrix $\mathrm{diag}(f_1, f_2, ..f_N)$ into the beam splitter losses of the two unitary matrices of the SVD decomposition can also help increase overall count rates. This can be seen as follows: suppose we implement Eq. (13) in three stages. In stage 1, we implement $U_{1,N,N} \times \mathrm{diag}(l, l, ..l)$, in stage 2 we implement $\mathrm{diag}(f_1, f_2, \ldots, f_N)$ and, in stage 3, we implement $\mathrm{diag}(l, l, ..l) \times U_{2,N,N}$. This is how SVD is usually implemented in hardware. However, because of the lossy nature of interconnects and beam splitters, stage 2 which consists of $\mathrm{diag}(f_1, f_2, \ldots, f_N)$ with at least one entry equal to 1, would be impossible to implement. This means that in an actual implementation the transformation which would be implemented would be $U_{1,N,N} \times \mathrm{diag}(l, l\ldots l) \times \mathrm{diag}(f_1, f_2, \ldots, f_N) \times S \times \mathrm{diag}(l, l, ..l) \times U_{2,N,N}$, where S is some scaling matrix that would affect stage 2. This means the overall count rate would be reduced. However, by using the method proposed, by including $\mathrm{diag}(f_1, f_2, \ldots, f_N)$ in the losses of stage 1 and stage 3, as shown in Eq. (14), we can actually implement Eq. (13) without any reduction in the overall count rate.

To conclude, we have first proposed a method for decomposition and implementation of a unitary transformation using lossy beam splitters up to a constant scaling factor without loss in fidelity, by suitably dividing the losses in different arms of the beam splitters. This division of losses is independent of which unitary transformation is being used and, thus, remains fixed. We then extended this method to a general linear transformation using SVD decomposition. In addition to increasing the count rate, this method includes loss in each arm of every beam splitter as a design parameter. This makes this method especially useful for plasmonic beam splitters where interconnects can also be lossy.

Acknowledgment. The author thanks Akshatha Mohan for helpful discussions and critical assessment of the Letter.

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