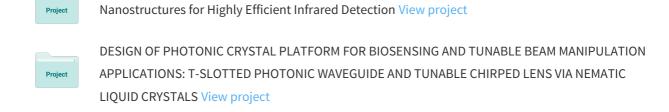
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# Photonic Crystal Sub-Wavelength $\lambda$ /5 Focusing Lens Design Using Optimization Method

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## **ABSTRACT**

In this paper, differential evolution (DE) optimization algorithm is applied to design sub-wavelength focusing PCs structure for the first time to the best of our knowledge. For this purpose, the transverse cross section of the input beam is spatially shaped by optimized PC structure. Strong focusing of beam with FWHM equal to  $\lambda/5$  and with suppressed magnitude of side lobe intensities is achieved. In addition, microwave experiments to verify the numerical results are conducted. The numerical and experimental results given in the paper are in good agreement with each other.

**Keywords**: photonic crystals, sub-wavelength focusing, differential evolution algorithm.

# 1. INTRODUCTION

Coherent interaction of light with photonic structures possessing wavelength scale refractive index variation results in unique optical properties in the electromagnetic spectrum [1, 2]. Conventional optical materials and structures manipulate light with limited capability. It is possible to improve light controlling efficiency by introducing artificial spatial periodicity to the dielectric medium. These artificial dielectric structures known as photonic crystals (PC) that have periodical modulation of refractive indexes are first introduced in 1987 by pioneering work of Yablonovitch [2]. Since that time to control light efficiently, PCs have been heavily investigated by designing different devices including PC waveguides [3], cavities [4], bio-chemical sensors [5] and lenses [6]. Considering all of the early previous research works on PC structures, we can declare that PCs are highly symmetric and periodic structures. It is because of the fact that high periodicity in refractive index form is the main governing mechanism for the presence of photonic band gap phenomena [7]. Later, the domain of structural forms has widened thanks to the study in the field of quasi-crystals [8]. While studies on periodic and quasi-periodic photonic structures continue, aperiodic disordered/random structures have become an intense research area for engineers and scientist [9].

Recent researches on the light manipulation structures such as focusing apparatus, i.e. lenses, introduce new approaches in the field of optics. Focusing of light can be accomplished by using either mirrors or lenses with curved surfaces [1]. Although light focusing with curved optical lenses enables large freedom of light manipulation, their resolutions are limited due to the diffraction nature of the light. Thus, it becomes difficult to control the light at sub-wavelength scale. For this reason, recent research works are focused on overcoming the diffraction limit or obtaining sub-wavelength focusing/imaging by creating artificial structures known as superlenses [10]. The importance of sub-wavelength focusing covers different applications including imaging [11-13], nanofabrication [12, 13], manipulation [12-16] and nano-focusing [17]. Obtaining tightly focused light beams allows us to manipulate small particles. Thus, many studies have been performed aiming to contribute these goals [18, 19].

Design of photonic structures with desired optical and structural characteristics based on theoretical knowledge and educated intuition is generally limited. For this reason, a variety of optimization algorithms are used to design efficient photonic integrated structures. The optimization algorithms enable to design aperiodic and disordered structures that possess rich optical properties. For instance, optimization algorithms such as genetic algorithm, topology optimization, convex optimization and nonlinear search algorithm have been utilized to design PC spot size converter [20], waveguide [21], compact wavelength demultiplexer [22] and polarization beam splitter [23].

# 2. SUB-WAVELENGTH FOCUSING LENS: DESIGN AND ANALYSIS

The wave nature of light has limitation to focus light below a certain spot size which is smaller than half of the wavelength of propagating light. The main objective of the proposed paper is to find desired PC design that results in subwavelength focusing of the illuminating light wave. The size of the focal spot can be characterized by FWHM value and for subwavelength focusing it should be smaller than the value of  $\lambda/2$  in air. In addition, alongside with strong focusing the emerged magnitudes of side lobes are also very important. Therefore, subwavelength focusing optimization process can be considered as a multi-objective optimization problem, i.e. minimization of both FWHM and side lobe magnitude values.

To obtain strong focusing of light the transverse filling ratio/effective index of the PC structure is optimized by using DE algorithm [24]. We have recently proposed the detailed analysis of an optimized PC structure design for strong focusing utilizing the DE algorithm [25]. ]. Transverse filling ratio modulation is performed by optimizing the lateral size of PC unit cells while keeping the material type and rod radii the same. We consider a 2D PC lens in air background that consists of dielectric (n = 3.13) alumina rods with radius r = 0.2a where "a" is the lattice constant of the PC structure. The minimum and maximum lateral sizes of the PC unit cells in transverse direction are defined as 0.44a and 2a, respectively. Corresponding variations of cell sizes are depicted in Fig. 1(a) as an inset. PC lens lattice periodicity along the propagation direction is kept intact and is equal to the a lattice constant. Exploiting defined structural parameters, we calculated the dispersion diagrams of PC unit cells with maximum and minimum lateral sizes with the help of plane wave expansion (PWE) method [26]. Extracted dispersion relations of transverse magnetic (TM) bands are shown in Fig. 1(a). Regarding group indices of each band  $(n_e)$  are also extracted and illustrated in Fig. 1(b). The calculated group indices within the normalized frequencies  $a/\lambda = [0.11-014]$ , cover values between 1.40 and 2.20. As it can be seen in Fig. 1(a), there is a linear region of the group refractive index values for the normalized frequencies between  $a/\lambda=0.11$  to  $a/\lambda = 0.14$ . Within that frequency region (long wavelengths) where  $n_e$  stays linear, the intended design of PC structure can be considered as an effective homogeneous medium.

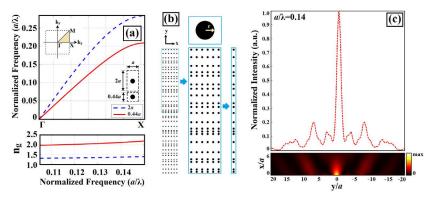


Figure 1: (a) Band diagram (top) and group refractive index diagram (bottom); (b) Optimized PC lens; (c) Transverse intensity cross section at focal point and electric field distribution at the back surface of PC lens.

After determining design frequencies, i.e. staying in homogenization region, DE can be applied to generate compact and feasible PC lens. Numerically modelling of the optimized PC lens and its time-domain analysis are realized by the help of finite-difference time-domain (FDTD) method [27]. Continuous source with a Gaussian profile is utilized to calculate spatial intensity distribution of the focusing effect. Furthermore, to see the effect of longitudinal length of the optimized PC structure on focusing ability we applied DE for different structural lengths (different size of periods) which are varying from 4a to 9a.

As specified before, DE is applied to design different structures for different normalized frequency values  $(a/\lambda=[0.11,0.12,0.13,0.14])$ . For each normalized frequency, different numbers of columns are chosen to design structures. The schematic view of the optimized best PC structure is presented in Fig. 2(c). Throughout the optimization process, we search for the sub-wavelength focusing at the outside of the back surface of the lens in the air. In addition to sub-wavelength focusing, we also suppressed the maximum side lobe (MSL) value of the cross section of normalized light intensity at the focal point. In our design, we set the conditions for FWHM and MSL as  $0.20\lambda$  and under 0.20, respectively.

As a result of application of DE, subwavelength focusing with FWHM =  $0.19\lambda$  is achieved from the PC lens which is designed at frequency of  $a/\lambda = 0.14$ , having structural length equal to 5a. The electric field distribution and transverse intensity cross section at the focal point of the best structure in terms of focusing ability is shown in Fig. 2(c). As can be seen from Fig. 2(c), sub-wavelength focusing is achieved with accompanied side lobes and using optimization method we strictly push the level of the side lobes to stay below a certain value. Thus, the majority of the energy is taken by the main lobe. One can deduce that there are two physical mechanisms that govern the strong focusing of light. The first one is the rather irregularly placed dielectric rods providing index gradient which is nonlinear. The other mechanism is based on the interference of light where at certain places the structure has wide distance between rods. As a result of these two phenomena, strong focusing of beam at the outside of the structure occurs.

# 3. EVALUATION OF EXPERIMENTAL RESULTS

In this section, we share the results of an experimental verification of subwavelength focusing by DE optimized PC lens at the microwave regime. Numerically optimized PC lens is constructed using dielectric alumina rods for experiment realization. Figure 2(a) shows the schematic of the experimental setup where a vector network analyzer (Agilent E5071C ENA) was used to generate a wave source and record the intensity field of the focused

wave. Excitation of the PC lens and measuring of the steady-state intensity distributions are achieved by using the horn and monopole antennas (operating range: 4GHz - 6GHz).

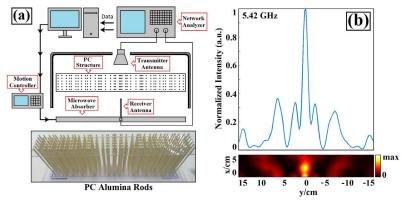


Figure 2: (a) schematic of experimental setup (top) and picture of PC lens structure (bottom); (b) cross section at focal point (top) and electric field distribution behind the back surface of lens (bottom).

Steady state intensity field at the back focal plane was measured by moving the monopole antenna 30 cm in y-and 2.5 cm x-directions with spatial steps equal to 2.5 mm. Figure 2(b) shows experimental data of the electric intensity field profile of the focused wave and its cross-sectional intensity profile at the operating frequency of 5.42 GHz (coincides to the normalized frequency of  $a/\lambda = 0.14$ ). Measured FWHM value is equal to  $0.21\lambda$  and as can be seen from Fig. 2(b) MSL magnitudes are staying below the normalized intensity value of 0.4. If one compares the measured and numerically calculated FWHM values it can be realized that there is a negligible mismatch between them, i.e. difference between calculated and measured value is  $0.02\lambda$ .

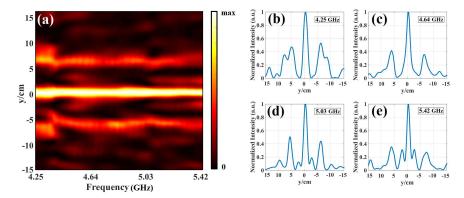


Figure 3: (a) Experimental plot of the transverse cross section intensity map at the back focal plane at a distance of x=0.50 mm after the PC lens; Transverse cross-sectional profiles taken at (b) 4.25 GHz, (c) 4.64 GHz, (d) 5.03 GHz and (e) 5.42 GHz that have FWHM values of 0.234 $\lambda$ , 0.230 $\lambda$ , 0.225 $\lambda$  and 0.210 $\lambda$ , respectively.

In order to show broadband operation of the designed PC lens as a subwavelength focusing lens we also perform same experimental steps for the frequency range of  $4.25\,\mathrm{GHz} - 5.42\,\mathrm{GHz}$  ( $a/\lambda = 0.11 - 0.14$ ). Figure 3(a) shows the transverse cross-sectional intensity map at the focal plane, plotted as a function of the incident wave frequency. Inspecting the intensity map shown in Fig. 3(a), at four defined normalized frequencies of: (b)  $4.25\,\mathrm{GHz}$  ( $a/\lambda = 0.11$ ), (c)  $4.64\,\mathrm{GHz}$  ( $a/\lambda = 0.12$ ), (d)  $5.03\,\mathrm{GHz}$  ( $a/\lambda = 0.13$ ), and (e)  $5.42\,\mathrm{GHz}$  ( $a/\lambda = 0.14$ ), we observe the strong focusing effect with FWHM values of  $0.234\lambda$ ,  $0.230\lambda$ ,  $0.225\lambda$ , and  $0.190\lambda$ , respectively. Besides the numerical calculated FWHM values for regarding four normalized frequencies are equal to  $0.205\lambda$ ,  $0.200\lambda$ ,  $0.199\lambda$ , and  $0.190\lambda$ , respectively. As can be concluded from the results one can deduce that experimental and numerical results are in perfect agreement with each other. When we inspect the FWHM values of numerical and experimental results at the four operating frequencies as summarized in Table 1, we see broadband nature of the designed photonic structure as sub-wavelength focusing lens.

Table 1. Numerical results of designed PC lens for different normalized frequency values.

Normalized	Microwave	FWHM (λ)	FWHM (λ)
Frequency $(a/\lambda)$	frequency(GHz)	Numerical	Experimental
0.11	4.25	0.205	0.234
0.12	4.64	0.200	0.230
0.13	5.03	0.195	0.225
0.14	5.42	0.190	0.190

## 4. CONCLUSIONS

In this paper, we propose an optimization algorithm known as differential evolution (DE) to design a photonic medium made of lossless dielectric circular elements. The aim is to find a photonic structure that provides strong beam focusing. The algorithm is integrated into the finite-difference time-domain method and multi-objectives such as small full-width at half-maximum (FWHM) and low side lobes at focal point are defined inside the objective function. FWHM values less than  $0.25\lambda$  in air with low intensity side lobes is obtained and demonstrated both numerically and experimentally. The sub-wavelength focusing of light is linked with interference effect and irregular index gradient. The proposed DE algorithm can also be applied to design other photonic structures for additional light manipulation scenarios.

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