

# Resonant Mode Analysis of the Nanoscale Surface Plasmon Polariton Waveguide Filter with Rectangle Cavity

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**Abstract** The resonant mode characteristics of the nanoscale surface plasmon polaritons (SPP) waveguide filter with rectangle cavity are studied theoretically. By using the finite difference time domain method, both the band-stop- and band-pass-type rectangle SPP filters are analyzed. The results show that the whispering gallery mode (WGM) and the Fabry–Perot (FP) mode can be supported by the rectangle SPP resonator. Furthermore, both traveling-wave mode and standing-wave mode can be realized by the WGM, while only standing-wave mode can be introduced by the FP mode. The traveling-wave mode can only be realized by the square-shaped SPP resonator, and the traveling-wave mode is splitted into two standing-wave modes by transforming the cavity shape from square to rectangle. Also, the effects of the cavity shape, cavity size, and coupling gap size on the transmission spectra of the SPP resonators are analyzed in detail. This simple SPP waveguide filter is very promising for the high-density SPP waveguide integrations.

**Keywords** Surface plasmon polariton · Resonator · Rectangle · Filter

## Introduction

Surface plasmon polaritons (SPPs) are electromagnetic waves coherently coupled to electron oscillations and

propagating at the interface between a dielectric and a metal, with evanescently decaying fields in both sides [1, 2]. With the strong confinement at the dielectric/metal interface, the SPP waveguide can realize subwavelength waveguiding, which is very promising for the high optical integration. There are two basic SPP waveguide configurations, which are the insulator–metal–insulator (IMI) [3, 4] and metal–insulator–metal (MIM) structures [5, 6]. Recently, the MIM SPP waveguide has received plenty of attention because of its nanoscale-waveguiding capability, which cannot be realized by the dielectric waveguide. The SPP waveguide filter is one kind of the important components in the high-density SPP waveguide integration platform. And some kinds of resonance cavity filters based on the MIM SPP waveguides have been proposed, including the circular ring resonators [7–10], rectangle ring resonators [11–13], tooth-shaped resonators [14–17], Fabry–Perot resonators [18–20], nanodisk resonators [21, 22], rectangle cavity resonators [23, 24], and nano-capillary resonators [25]. Among them, the SPP ring resonators and nanodisk resonators have been well-studied, while there are few researches about the rectangle cavity SPP resonators. Comparing to the circular ring SPP resonator, the SPP filter with rectangle cavity is easier for fabrication. But according to the complex geometry than that of the nanodisk SPP resonator, the resonance modes of the rectangle cavity SPP resonator are more geometry-dependent. In Ref. [24], only the first-order resonance mode is used to realize the gain-induced switching. Although the first and second modes are analyzed in Ref. [23], the types of the modes and the mode characteristics of different cavity geometries have not been well-studied. In this article, the resonance mode characteristics of SPP resonator with different rectangle cavity geometries are analyzed in detail. The results show that

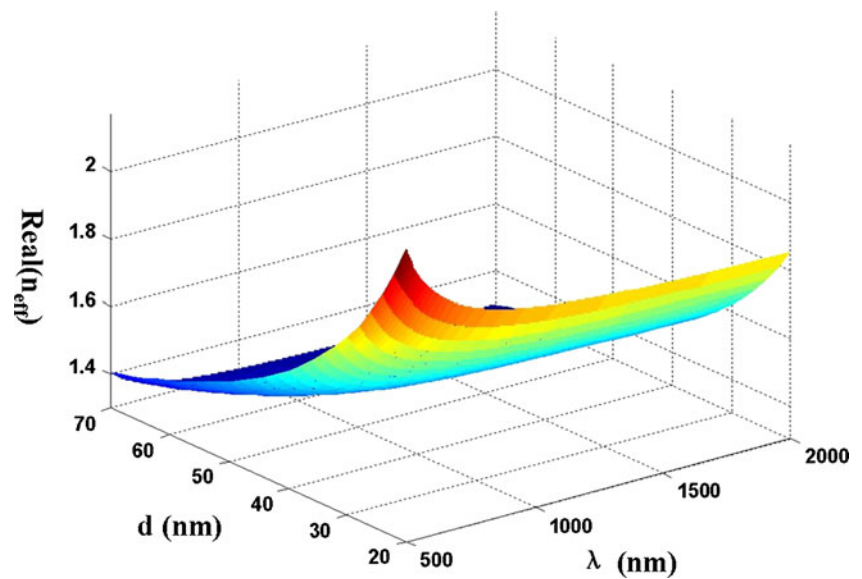
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**Fig. 1** The dependence of Real ( $n_{\text{eff}}$ ) on the fundamental TM mode on the wavelength of incident light  $\lambda$  and the width  $d$



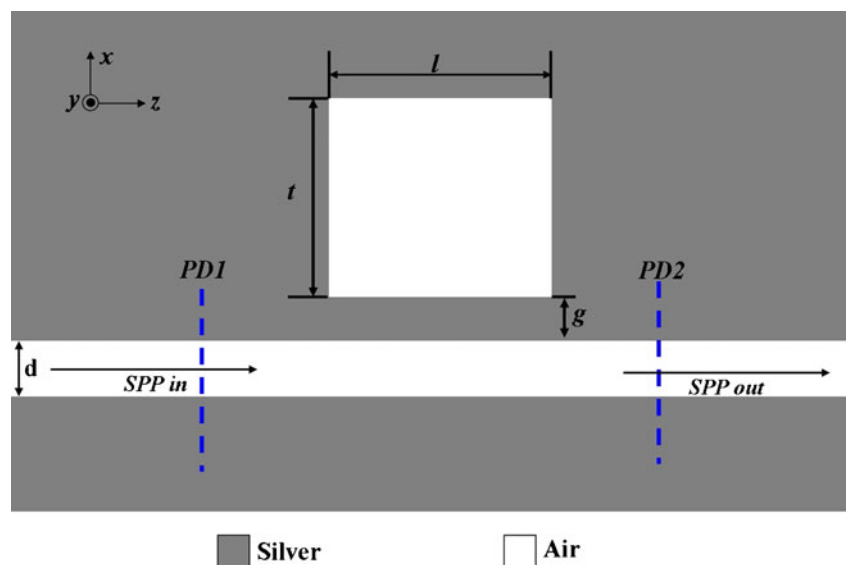
both the whispering gallery mode (WGM) and the Fabry–Perot (FP) mode can be supported by the rectangle SPP resonator. Furthermore, both traveling-wave mode and standing-wave mode can be realized by the WGM, while only standing-wave mode can be introduced by the FP mode. The traveling-wave mode can only be realized with the square-shaped SPP resonator, and the traveling-wave mode is splitted into two standing-wave modes by transforming the cavity shape from square to rectangle. Also, the effects of the cavity shape, cavity size, and coupling gap size on the transmission spectra of the SPP resonators are analyzed in detail.

#### Dispersion of MIM SPP Waveguide and the FDTD Method

In this article, the MIM SPP waveguide with 50-nm gap size is used as the input/output SPP waveguide, which couples the SPP into a rectangle resonance cavity. With this gap size, only the fundamental transverse magnetic (TM) mode is supported and its dispersion relation is given by:

$$\varepsilon_{\text{in}} k_{z2} + \varepsilon_m k_{z1} \coth\left(-\frac{ik_{z1}}{2}d\right) = 0 \quad (1)$$

**Fig. 2** The band-stop filter based on the side-coupled rectangle SPP resonator



and  $k_{z1}$  and  $k_{z2}$  are:

$$k_{z1}^2 = \varepsilon_{\text{in}} k_0^2 - \beta^2, k_{z2}^2 = \varepsilon_m k_0^2 - \beta^2 \quad (2)$$

where  $\varepsilon_{\text{in}}$  and  $\varepsilon_m$  are the dielectric constants of the insulator and the metal, and  $k_0 = 2\pi/\lambda$  is the free-space wave vector. And the dielectric constant of the metal silver is characterized by the Lorentz–Drude model [10, 18]:

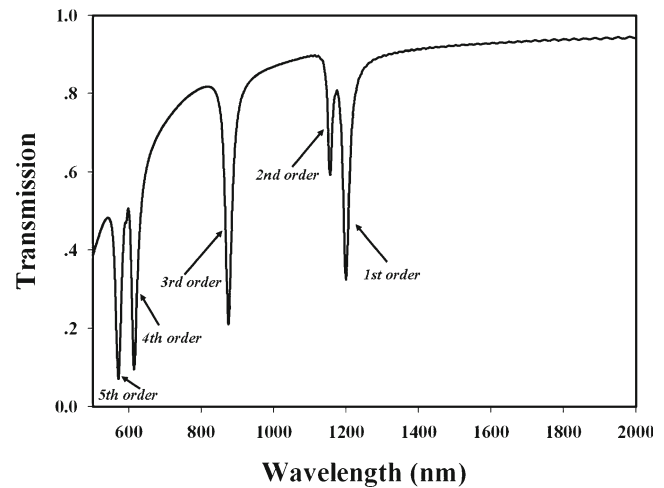
$$\varepsilon_m = \varepsilon_\infty - \sum_{m=0}^5 \frac{G_m \Omega_m^2}{\omega_m^2 - \omega^2 + i\omega\Gamma_m} \quad (3)$$

where  $\varepsilon_\infty$  is the relative permittivity at the infinity frequency,  $G_m$  is the oscillator strength,  $\Omega_m$  is the plasma frequency, and  $\omega$  is the angular frequency of incident light; all these parameters used in the simulations are listed in Ref. [26]. According to Eqs. (1) and (2), the real part of the effective index of the fundamental TM mode ( $n_{\text{eff}} = \beta/k_0$ ) as a function of the slit width  $w$  and the incident light wavelength  $\lambda$  is obtained and shown in Fig. 1. The  $n_{\text{eff}}$  decreases as  $d$  increases and decreases relatively slow with an increasing  $\lambda$ .

The 2D FDTD method provided by the Rsoft fullwave package ([www.rsoftdesign.com](http://www.rsoftdesign.com)) with perfect matched layer (PML) boundary condition is used to simulate the transmission spectra of the SPP filters. The perfect matched layer is an artificial absorbing material which effectively absorbs field energy that propagates through the PML layers, allowing it to completely leave the domain almost without back reflections. The fundamental TM mode of the MIM waveguide is excited by a dipole source and the mesh grid size is set to 1 nm in order to keep convergence. Two monitors PD1 and PD2 (as shown in Fig. 2) are set to detect the incident power  $A_1$  (without the resonator cavity for reference) and the transmission power  $A_2$  (with the resonator cavity). And the transmittance is defined as  $T = A_2/A_1$  [10].

### Mode Characteristics of the Band-stop Filter Based on Rectangle SPP Resonator

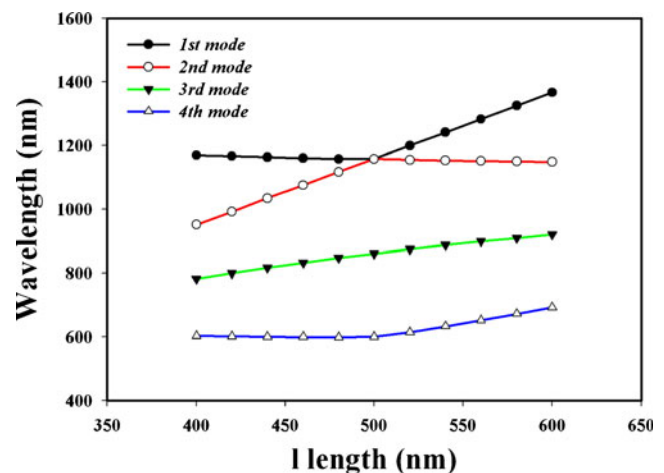
The structure of the band-stop filter based on the side-coupled rectangle SPP resonator is shown in Fig. 2. The side lengths of the rectangle cavity are  $l$  and  $t$ , and  $d$  and  $g$  are the width and the gap size of the MIM waveguide of the coupled region, respectively. The SPP wave is side-coupled into/out the rectangle resonator by the MIM SPP waveguide. The typical transmission spectrum of the band-stop SPP filter with rectangle cavity ( $l=520$  nm,  $t=500$  nm) is shown in Fig. 3. There are five transmission dips according to the resonant modes in the wavelength range of 500–2,000 nm. In order to analysis the effect of the geometry shape of the rectangle resonant cavity, the resonance wavelengths of the



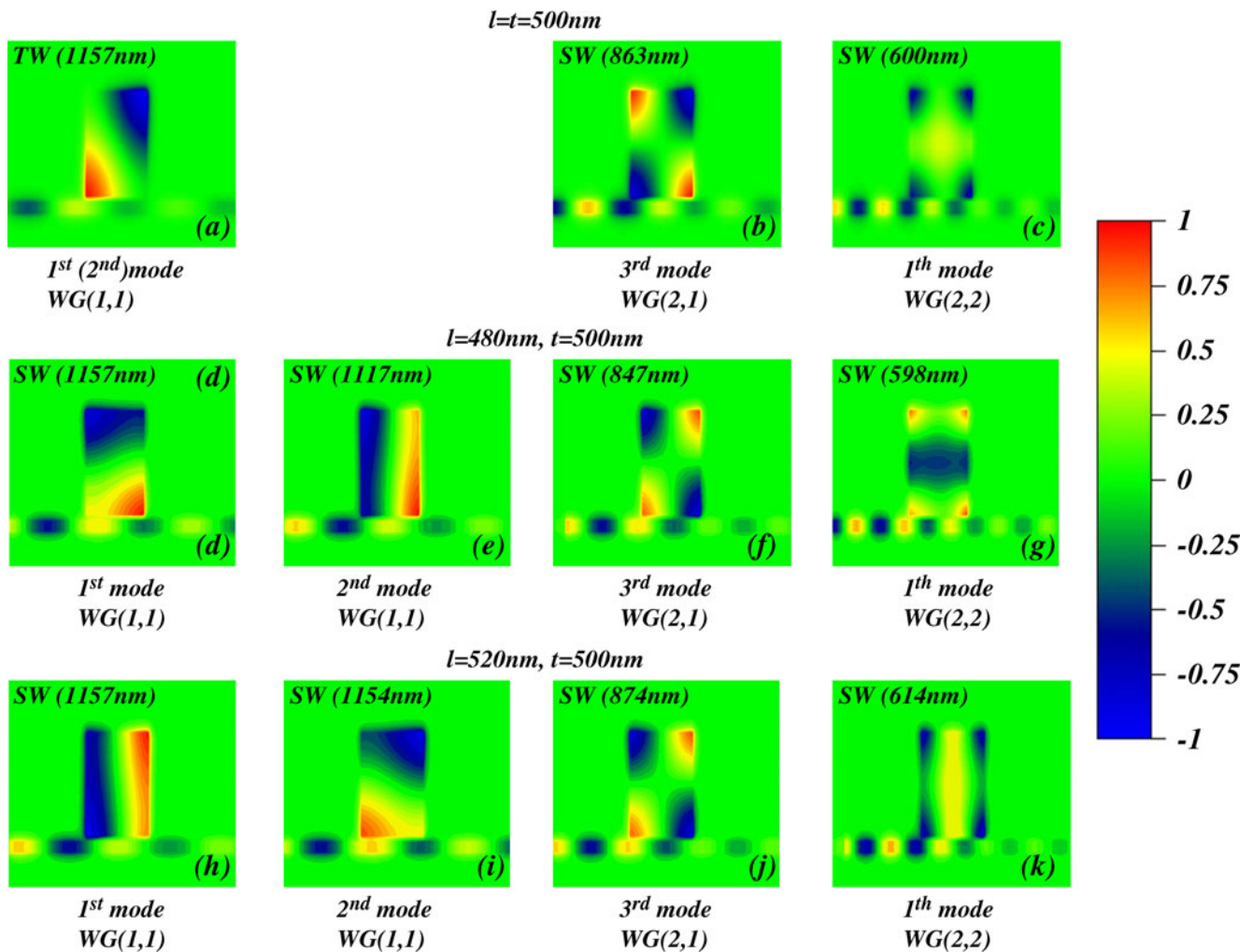
**Fig. 3** The transmission spectrum of rectangle cavity ( $l=520$  nm,  $t=500$  nm,  $g=30$  nm)

first four resonance dips are simulated with a different side length  $l$ , while the side length is kept at  $t=500$  nm. The results are shown in Fig. 4. From Fig. 4, it is clear that the resonance wavelength of the first mode is decreased with an increasing  $l$  when  $l < t$ , while the opposite trend is observed when  $l > t$ . And the second mode has a reversed trend compared to that of the first mode. Also, the resonance wavelengths of the first and second modes intersect at  $l=t=500$  nm, which means that the first mode and the second mode are degenerated when the shape of the resonance cavity is changed from rectangle to square, while the resonance wavelength of the third mode has a linear relationship with the side length  $l$  and increases with increasing  $l$ . Also, the fourth mode has the same trend as the first mode.

In order to explain the interesting phenomenon of the different modes, the  $H_y$  fields of the first four resonant



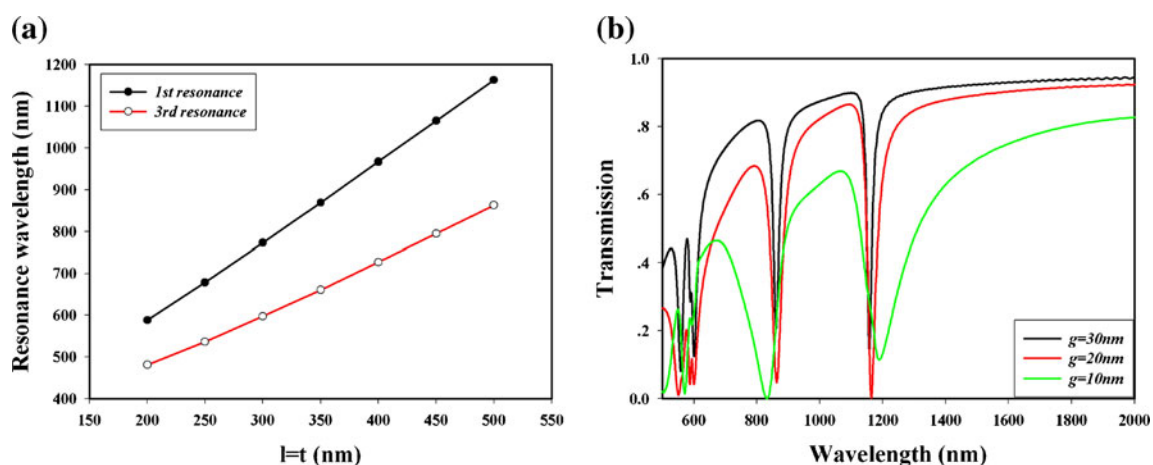
**Fig. 4** Resonance wavelengths of rectangle resonator with different  $l$  ( $t=500$  nm,  $g=30$  nm)



**Fig. 5** *a–k* The  $H_y$  fields of the resonant modes with different geometries

modes are simulated and shown in Fig. 5. The results show that both traveling-wave (TW) and standing-wave

(SW) can be realized in the square ( $l=t$ ) resonant cavity, while only SW can be excited in the rectangle ( $l \neq t$ ) resonant

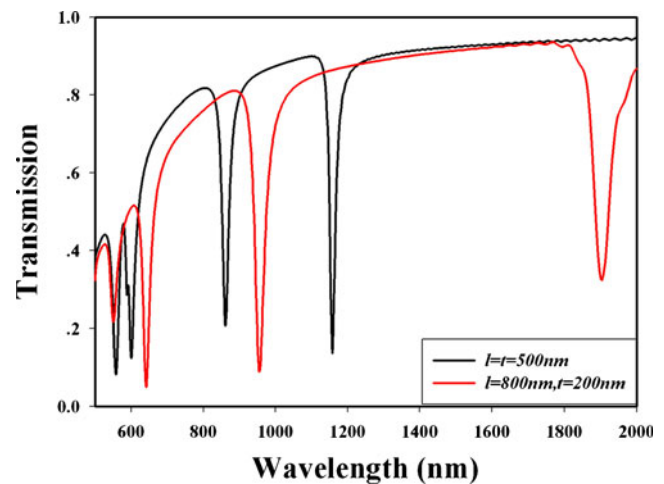


**Fig. 6** **a** The resonance wavelength of the first and third mode of square cavity with different cavity sizes ( $g=30\text{ nm}$ ). **b** The transmission spectra of square resonant cavity with different coupling gap sizes ( $l=t=500\text{ nm}$ )



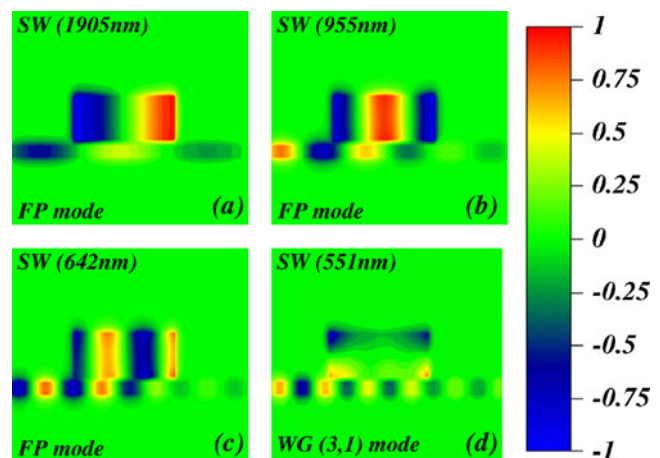
cavity. It is clear that all the modes are WGM modes, which are formed by the interferences of the SPP waves at the Ag/air interface. The first mode (WG(1,1) mode) of the square resonant cavity ( $l=t=500$  nm) is a TW mode, which splits into two SW modes when the resonant cavity changes from square to rectangle geometry. And the  $H_y$  fields of the two modes are distributed on the cavity wall along the  $x$  and  $z$  directions, respectively. Also, the  $H_y$  field distribution of the splitting second mode of the rectangle resonant cavity ( $l=480$  nm,  $t=500$  nm) is like that of the first mode of the rectangle resonant cavity ( $l=520$ ,  $t=500$  nm). The same trends are observed for the second mode and first mode of the rectangle resonant cavities with  $l=480$  nm and  $l=520$  nm, respectively. So, there is an intersection between the wavelengths of the first and second mode in Fig. 4 and the first mode with  $l<500$  nm and the second mode with  $l>500$  nm can be connected as a linear line. The third modes (WG (2,1) mode) of the square and rectangle resonant cavities are both SW modes, so the resonance wavelength only has a linear relation with the length  $l$  and increases with an increasing  $l$  just as a scaling effect of the cavity length. From Fig. 4, the fourth mode (WG (2,2) mode) has the same trend as the first mode, while the fourth mode is a SW mode. And when the geometry of the resonator changes from square to rectangle, the fourth mode of the square resonator is also degenerated into two SW modes (WG (2,2) mode), whose center  $H_y$  fields are coupled to the border of the cavity along  $x$  and  $z$  directions, respectively.

The above results show that there is an intersection between the resonance wavelengths of the first and second modes of the different rectangle resonant cavity sizes because of the mode degeneration. In order to show cavity size effects on the resonance wavelengths of the square resonant cavity, the resonance wavelengths of first and third modes according to different square cavity sizes are simulated and shown in Fig. 6a. It is obvious that the resonance wavelengths of the first and third modes of the square resonant cavity show linear relations with respect to the cavity sizes and increase with increasing cavity sizes, which is different from the rectangle resonant cavities. So, the filter wavelength can be linearly changed by altering the size of the square resonator. Then, the effects of the coupling gap size  $g$  on the transmission spectra are studied and the results are shown in Fig. 6b. The results show that the resonance bandwidth decreases by increasing the coupling gap size; in other words, the larger the coupling gap size, the larger the quality factor of the resonator can be realized. This is because by increasing the coupling gap size, coupling strength is reduced and more SPP energy can be stored in the resonant cavity. Also, it is clear that the resonance wavelengths are almost same except for a little shift due to the effective index perturbations of the coupling region.



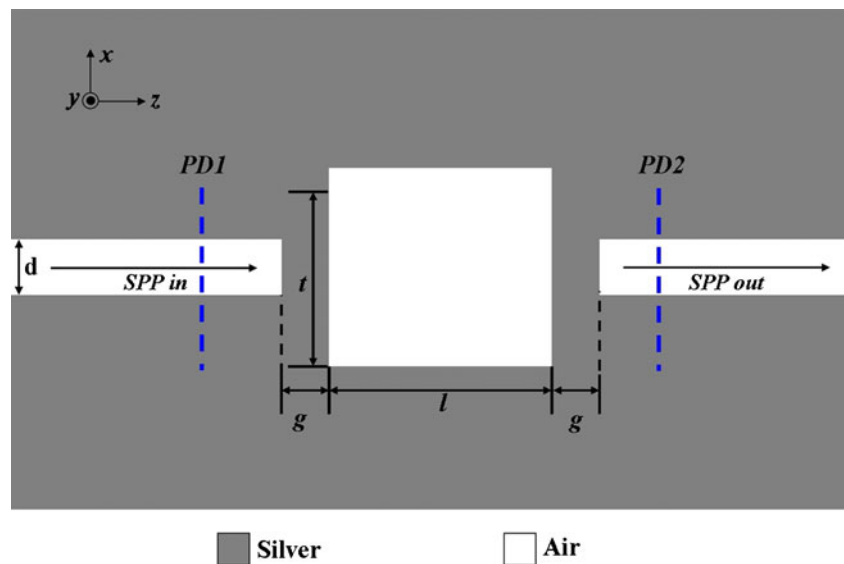
**Fig. 7** Transmission spectra of rectangle resonant cavity ( $l=t=500$  nm vs  $l=800$  nm,  $t=200$  nm)

All the modes in the above rectangle resonant cavities are the formed by the WGM because at the resonance wavelengths, the SPP fields at the silver/air interfaces in the cavity are independent and not coupled with each other very much, which can be clearly seen from the  $H_y$  field patterns. But when the width  $t$  or the length  $l$  is changed to a relative small value, the SPP fields at the opposite sides of the rectangle cavity are strongly coupled and the rectangle cavity can be treated as an MIM waveguide. Then, the Fabry–Perot (FP) modes can be formed in the rectangle resonance cavity [18], which are different from the WGM. In order to exclude the size effect of the resonant cavity, the perimeter of the rectangle resonant cavity ( $l=800$  nm,  $t=200$  nm) is kept the same as that of the square cavity ( $l=t=500$  nm). The simulated transmission spectrum of the rectangle resonant cavity is compared with that of the square resonant cavity and is shown in Fig. 7. There



**Fig. 8** a–d The  $H_y$  fields of the resonant modes of rectangle cavity ( $l=800$  nm,  $t=200$  nm)

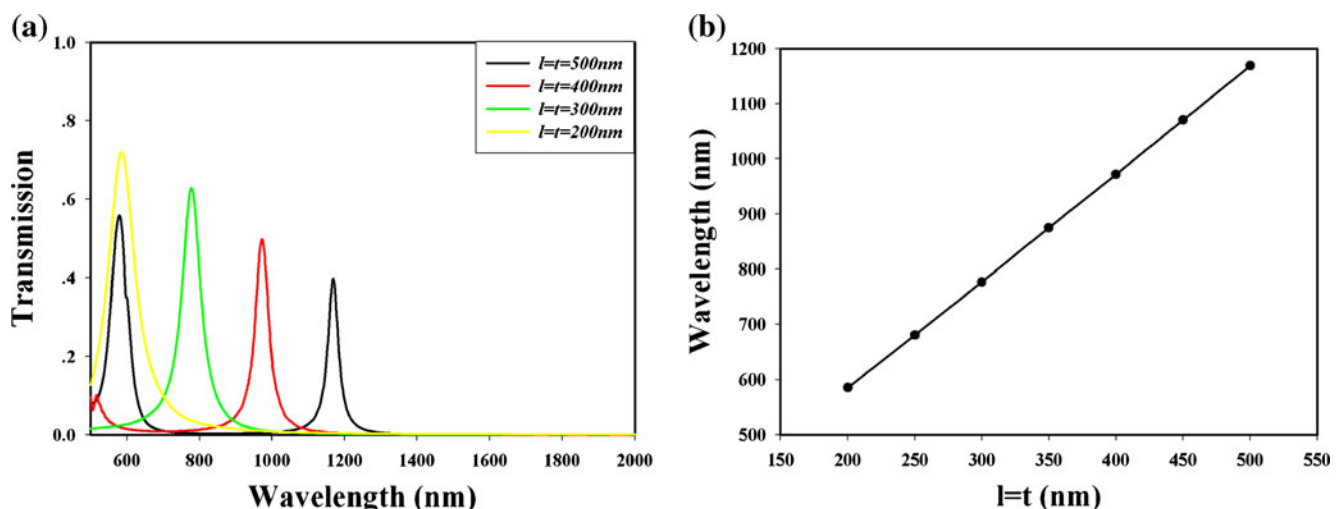
**Fig. 9** The band-pass filter based on the direct-coupled rectangle SPP resonator



are also four resonance dips in the wavelength range but only the fourth mode resonance wavelength is consistent with that of the square cavity. Also, the  $H_y$  fields of the four resonant modes are shown in Fig. 8. From the  $H_y$  field distributions, we can conclude that the first three are the FP modes, while the last resonant mode with smallest resonance wavelength is the WGM because with this small wavelength, the SPP waves at the opposite silver/air interfaces are weakly coupled. So, in Fig. 7, only the resonance wavelength of the WGM mode of the rectangle cavity is accorded with that of the square cavity, while the other resonance wavelengths are different because of the different resonant mode types.

### Mode Characteristics of the Band-pass Filter Based on Rectangle SPP Resonator

The band-stop-type SPP filter can be changed to a band-pass-type filter simply by changing from the side coupling to the direct coupling as shown in Fig. 9. The typical transmission spectra of the resonators with different square cavity sizes are shown in Fig. 10a. It can be concluded that there are fewer modes in the direct-coupled resonators than the side-coupled ones, which are caused by the different coupling conditions. Figure 10b shows that the resonance wavelength of the first mode of the square resonator follows the linear relation with the cavity size, which is the same as the side-coupled resonator. So, the SPP filter wavelength

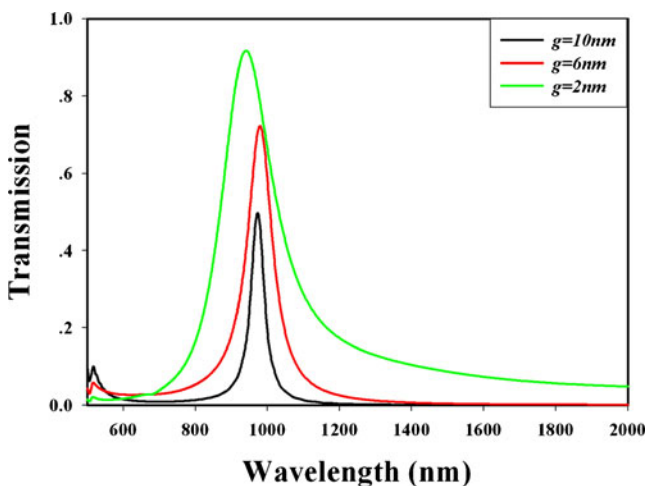


**Fig. 10** **a** The transmission spectra of square resonator with different cavity size ( $g=20$  nm). **b** The relation between the resonance wavelength of first mode and cavity size ( $g=20$  nm)

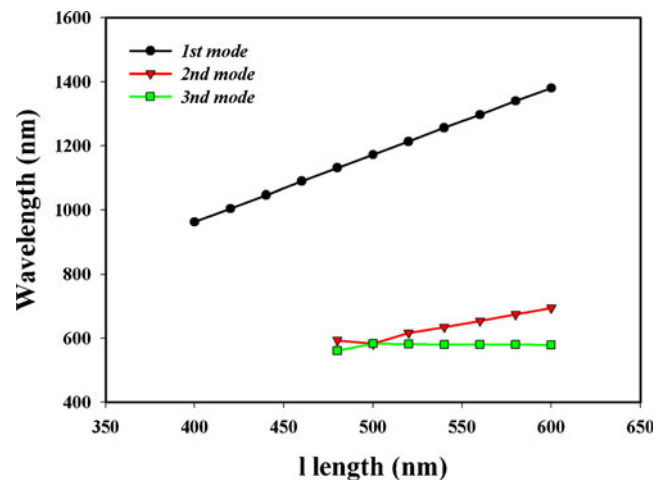
can also be easily changed by altering the cavity size, which is very useful in the SPP integration applications. Also, the transmission spectra of the square resonator with different coupling gap sizes are shown in Fig. 11. It is obvious that the bandwidth of the resonance increases with a decreasing gap size just as the side-coupled resonator. The transmission peak value is increased with a decreasing gap size. So, there is a contradiction between the transmission and the quality factor. Also, there are small resonance peak wavelength shifts for different coupling gap sizes due to the small perturbation of the SPP effective index in the coupling region.

Finally, the resonance wavelengths of the different resonant modes of the direct-coupled rectangle cavities are shown in Fig. 12. The results show that the resonance wavelength of the first mode increases linearly with an increasing length  $l$ , while the resonance wavelengths of the second and third modes are intersected when the cavity is square in shape ( $l=t=500$  nm). This phenomenon is like that of the first and second modes of side-coupled resonators in Fig. 4. Here, the second mode of the square resonator is degenerated to two modes (second and third) in the rectangle resonator. The corresponding  $H_y$  fields are also shown in Fig. 13. Both the SW and the TW modes are supported by the direct-coupled resonator, which just is like the side-coupled resonators. Besides, the hybrid modes of the SW and the TW modes also exist, which are not supported by the side-coupled resonators. These hybrid modes maybe caused by the perturbations of the coupling MIM waveguides.

According to the above results, the rectangle SPP resonators are highly geometric-dependent and can support both the traveling-wave modes and standing-wave modes, which are different from that of the classical



**Fig. 11** The transmission spectra of the square resonant cavity with different coupling gap sizes ( $l=t=400$  nm)



**Fig. 12** Resonance wavelengths of rectangle resonator with different  $l$  ( $t=500$  nm,  $g=10$  nm)

Fabry–Perot resonator, where only the standing-wave modes can be realized. For the side-coupled band-stop SPP rectangle filter, the traveling-wave mode can only be realized by the WG mode with the square-shaped SPP resonator, while the standing-wave mode can be realized by both the WG mode and the FP mode. And the traveling-wave mode is splitted into two standing-wave modes by transforming the cavity shape from square to rectangle. Also, for the direct-coupled band-pass SPP rectangle filter, both the SW and TW modes are supported, like the side-coupled resonators. Besides, the hybrid modes of SW and TW modes also exist, which are not supported by the side-coupled resonators. By comparing to the classical dielectric waveguide approach, the mode size of SPP waveguide can be greatly reduced because of its strong bonding capability at the dielectric/metal interface, which can greatly reduce the component size. Also, with this very strong bonding capability, the bending loss at the  $90^\circ$  corner in the rectangle cavity is greatly reduced and can be neglected, while for the class dielectric waveguide, very high bending loss will be introduced, which is an inherent defect of the dielectric waveguide.

The rectangle SPP resonator can be fabricated by the focused ion beam method [27] on a silver film. In order to analyze the effect of the fabrication uncertainties on the SPP square resonator ( $l=t$ ) which has the traveling-wave mode, the variation of the side length  $l$  relative to the side length  $t$  is taken into account. Figure 14 shows the transmission spectrum of the side-coupled rectangle SPP resonator ( $l=495$  nm,  $t=500$  nm), where the difference of the two side lengths is 5 nm. It is obvious that the first and the second standing-wave modes shown in Fig. 3 are merged into one traveling mode, which is the characteristic of the square resonator. So, in order to realize the traveling-wave mode of the square resonator,

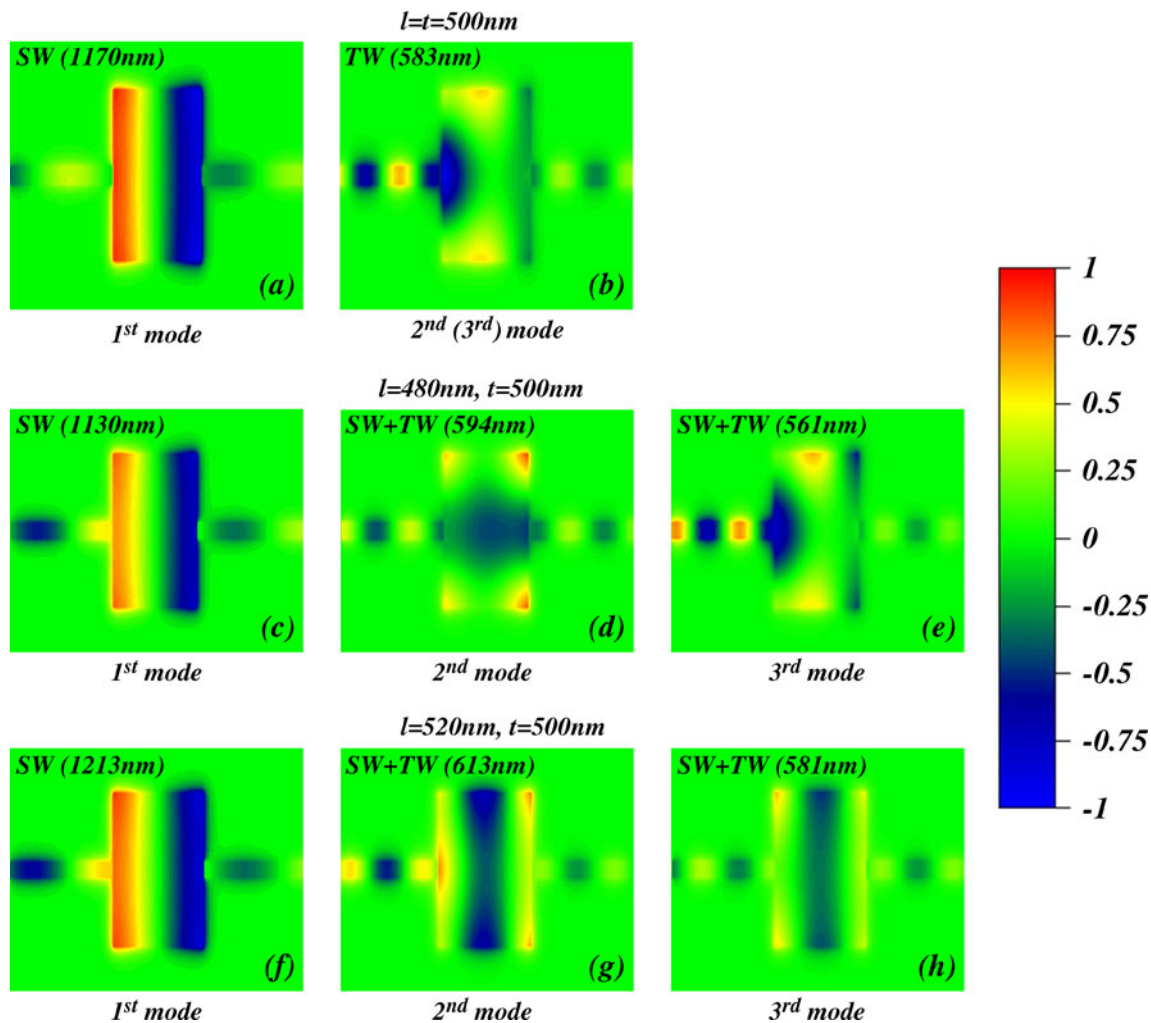


Fig. 13 a–h The  $H_y$  fields of the resonant modes with different geometries

the side length difference of the resonator should not be larger than  $\pm 5$  nm.

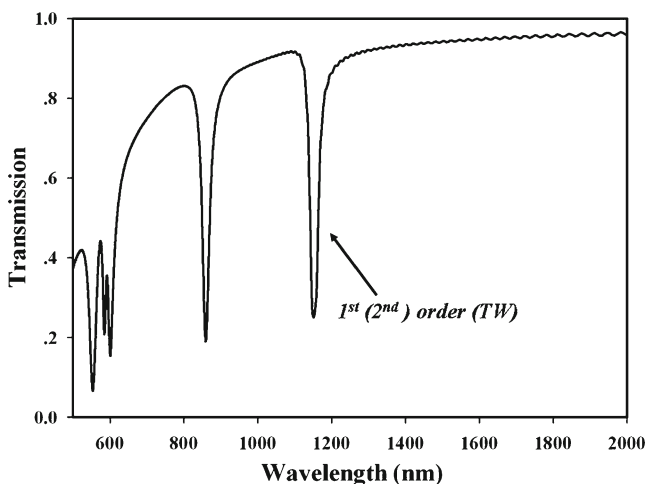


Fig. 14 The transmission spectrum of rectangle cavity ( $l=495$  nm,  $t=500$  nm,  $g=30$  nm)

## Conclusion

In this article, the characteristics of the resonant modes in the side-coupled (band-stop filter) and direct-coupled (band-pass filter) SPP rectangle cavities are studied and analyzed in detail. The whispering gallery mode and the Fabry–Perot mode can be supported by the rectangle SPP resonator. Furthermore, both traveling-wave mode and standing-wave mode can be realized by the WGM, while only standing-wave mode can be introduced by the FP mode. For the side-coupled resonator, the traveling-wave mode can be degenerated into two standing-wave modes by changing the resonator cavity from square to rectangle geometry, while for the direct-coupled resonator, the traveling-wave mode degenerates into two hybrid modes by changing the resonator cavity from square to rectangle geometry. And for the resonators with square cavities, the resonance wavelengths change linearly according to the cavity sizes.



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