Optimization using NLOPT

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1 Definition of the BEM problem

BEM Matrix M

The first meeting reviewed the definition and size of the matrices (and vectors) that constitute the BEM problem. The surface current in response to an input field is obtained from:

$$\underline{\underline{\mathbf{M}}}\,\underline{\mathbf{C}} = \underline{\mathbf{F}}, \tag{1a}$$

$$\underline{\mathbf{C}} = \underline{\underline{\mathbf{M}}}^{-1}\underline{\mathbf{F}}. \tag{1b}$$

where the surface current $\underline{\mathbf{C}}$ and input field $\underline{\mathbf{F}}$ are vectors of length 2N, where N is the number of edges in the mesh. $\underline{\underline{\mathbf{M}}} = \underline{\underline{\mathbf{M}}}^T$. is the symmetric BEM matrix. (computed using SCUFF) *Note: Avoid using matrix inversion whenever possible. For example, avoid* $\underline{\underline{\mathbf{M}}}^{-1}$.

Power/Force/Torque matrix Q

For power, force, and torque, BEM computes them via the PFT matrix $\underline{\underline{\mathbf{Q}}}$, which obeys

$$\underline{\underline{\mathbf{Q}}} = \underline{\underline{\mathbf{Q}}}^{\dagger},$$

where $\underline{\underline{\mathbf{Q}}}^{\dagger} = (\overline{\underline{\underline{\mathbf{Q}}}})^T$ denotes complex conjugate. $\underline{\underline{\mathbf{M}}}$ and $\underline{\underline{\mathbf{Q}}}_{OPFT}$ are independent of the incident field.

2 FOM

Definition

The second meeting reviewed how to obtain the scalar value *Figure of Merit* (FOM) and its gradient. The FOM is obtained from $\underline{\underline{\mathbf{Q}}}$ as:

$$FOM = \underline{\mathbf{C}}^{\dagger}\underline{\underline{\mathbf{Q}}}\,\underline{\mathbf{C}}, \tag{2}$$

where the $\underline{\mathbf{C}}$ is obtained from eq. 1a.

Differential of FOM

The differential (δ, also called 'grad') of FOM can be obtained from applying the gradient to eq. 1a:

$$\delta(\underline{\underline{\mathbf{M}}}\,\underline{\mathbf{C}} = \delta\underline{\mathbf{F}}) \tag{3a}$$

$$\underline{\underline{\mathbf{M}}}\,\delta\underline{\mathbf{C}} = \delta\underline{\mathbf{F}} \tag{3b}$$

Using $\delta(\underline{\mathbf{C}}^{\dagger}) = (\delta\underline{\mathbf{C}})^{\dagger}$,

$$\begin{aligned}\delta(FOM) &= \delta(\underline{\mathbf{C}}^{\dagger}\underline{\underline{\mathbf{Q}}}\,\underline{\mathbf{C}})\\ &= \delta\underline{\mathbf{C}}^{\dagger}\underline{\underline{\mathbf{Q}}}\,\underline{\mathbf{C}} + \underline{\mathbf{C}}^{\dagger}\underline{\underline{\mathbf{Q}}}\,\delta(\underline{\mathbf{C}})\\ &= \delta\underline{\mathbf{F}}^{\dagger}(\underline{\underline{\mathbf{M}}}^{-1})^{\dagger}\underline{\underline{\mathbf{Q}}}\,\underline{\mathbf{C}} + \underline{\mathbf{C}}^{\dagger}\underline{\underline{\mathbf{Q}}}\,\underline{\underline{\mathbf{M}}}^{-1}\delta\underline{\mathbf{F}}\end{aligned}$$

$$\therefore\ \delta(FOM) = \delta\underline{\mathbf{F}}^{\dagger}[(\underline{\underline{\mathbf{M}}}^{-1})^{\dagger}\underline{\underline{\mathbf{Q}}}\,\underline{\mathbf{C}}] + [\underline{\mathbf{C}}^{\dagger}\underline{\underline{\mathbf{Q}}}\,\underline{\underline{\mathbf{M}}}^{-1}]\delta\underline{\mathbf{F}}. \tag{3c}$$

3 Adjoint Current

From eq. 3c, The terms inside the brackets can be grouped together and computed in terms of the adjoint current $\underline{\mathbf{C}}_A$ to avoid matrix inversion. Let us write:

$$\underline{\mathbf{C}}_A^T = [\underline{\mathbf{C}}^{\dagger}\underline{\underline{\mathbf{Q}}}\,\underline{\underline{\mathbf{M}}}^{-1}] \tag{4a}$$

$$\underline{\mathbf{C}}_A = [\underline{\mathbf{C}}^{\dagger}\underline{\underline{\mathbf{Q}}}\,\underline{\underline{\mathbf{M}}}^{-1}]^T = \underline{\underline{\mathbf{M}}}^{-1}\overline{\underline{\underline{\mathbf{Q}}}}\,\overline{\underline{\mathbf{C}}} \tag{4b}$$

$$\overline{\underline{\mathbf{C}}_A} = (\overline{\underline{\underline{\mathbf{M}}}^{-1}})\underline{\underline{\mathbf{Q}}}\,\underline{\mathbf{C}} = (\underline{\underline{\mathbf{M}}}^{-1})^{\dagger}\underline{\underline{\mathbf{Q}}}\,\underline{\mathbf{C}} \tag{4c}$$

$$\therefore\ \boxed{\underline{\underline{\mathbf{M}}}\,\underline{\mathbf{C}}_A = \overline{\underline{\underline{\mathbf{Q}}}}\,\overline{\underline{\mathbf{C}}}} \tag{4d}$$

Notice the similarity between eq. 4d and $\underline{\underline{\mathbf{M}}}\,\underline{\mathbf{C}} = \underline{\mathbf{F}}$. Both can be solved with LU factorization. eq. 3c now becomes:

$$\delta(FOM) = \delta\underline{\mathbf{F}}^{\dagger}\,[\,\overline{\underline{\mathbf{C}}_A}\,] + [\,\underline{\mathbf{C}}_A^T\,]\,\delta\underline{\mathbf{F}} \tag{5a}$$

$$\therefore \quad \boxed{\delta(FOM) = 2 \cdot Re\left[\, \underline{\mathbf{C}}_A^T \, \delta \underline{\mathbf{F}} \,\right].} \tag{5b}$$

Using the complex vectors $\underline{\mathbf{F}}$ and $\underline{\mathbf{F}}^{\dagger}$ separately, we get two differential equations for FOM.

$$\left(\frac{\delta(FOM)}{\delta(\underline{\mathbf{F}})} = \underline{\mathbf{C}}_A^T \right) \tag{6a}$$

$$\left(\frac{\delta(FOM)}{\delta(\underline{\mathbf{F}}^{\dagger})} = \overline{\underline{\mathbf{C}}_A} \right) \tag{6b}$$

4 Input Field Decomposition

Construction of $\underline{\mathbf{F}}$

The total incident field vector $\underline{\mathbf{F}}$ is computed inside BEM in the following way:

$$F_i = \left\langle \boldsymbol{\phi}^{inc}, \boldsymbol{\beta}_i \right\rangle. \tag{7a}$$

In integral form, this is equivalent to:

$$\boxed{F_i = \int_{S_i} \boldsymbol{\phi}^{inc} \cdot \boldsymbol{\beta}_i \, \mathrm{d}A, \quad (i = 1..2\mathrm{N})} \tag{7b}$$

$$\boxed{\underline{\mathbf{F}} = \int_{S} \boldsymbol{\phi}^{inc} \cdot \underline{\boldsymbol{\beta}} \, \mathrm{d}A,} \tag{7c}$$

where $\boldsymbol{\phi}^{inc}$ is the total incident field in (x, y, z), and $\boldsymbol{\beta}_i$ is the unit normal for surface current in (x, y, z), per edge $i = 1..2\mathrm{N}$. Keep in mind that $\boldsymbol{\beta}_i$ and $\boldsymbol{\phi}^{inc}$ are vectors in 3D, while $\underline{\mathbf{F}}$ is a 2N-dimensional vector, and $\underline{\boldsymbol{\beta}}$ is a (2N×3) matrix.

Decomposition of $\boldsymbol{\phi}^{inc}$

The incident field $\boldsymbol{\phi}^{inc}$ can be expanded into any orthonormal basis $\mathbf{Y}^{lm}$.

$$\boldsymbol{\phi}^{inc} = \sum c_{lm} \; \mathbf{Y}^{lm} \tag{8}$$

Both $\boldsymbol{\phi}$ and $\mathbf{Y}$ are spatial vectors in 3D.

Decomposition of $\underline{\mathbf{F}}$

The 2N-dimensional incident field vector $\underline{\mathbf{F}}$ is a superposition of the $\underline{\mathbf{F}}^{lm}$ vectors.

$$\underline{\mathbf{F}} = \sum c_{lm} \; \underline{\mathbf{F}}^{lm}, \tag{9}$$

Each $\underline{\mathbf{F}}^{lm}$ is separately computed using the same integration method of eq. 7b:

$$F_i^{lm} = \left\langle \mathbf{Y}^{lm}, \boldsymbol{\beta}_i \right\rangle. \tag{10a}$$

In integral form, this is equivalent to:

$$\boxed{F_i^{lm} = \int \mathbf{Y}^{lm} \cdot \boldsymbol{\beta}_i \, \mathrm{d}A, \quad (i = 1..2\mathrm{N})} \tag{10b}$$

$$\boxed{\underline{\mathbf{F}}^{lm} = \int \mathbf{Y}^{lm} \cdot \underline{\boldsymbol{\beta}} \, \mathrm{d}A.} \tag{10c}$$

Decomposition of FOM

$\delta(FOM)$ can now be computed with respect to the change in each orthonormal mode $\mathbf{Y}^{lm}$.

$$\begin{aligned}
\delta(FOM) &= 2 \cdot Re\left[\, \underline{\mathbf{C}}_A^T \, \delta \underline{\mathbf{F}} \,\right] \\
&= 2 \cdot Re\left[\underline{\mathbf{C}}_A^T \sum \delta \left(c_{lm} \underline{\mathbf{F}}^{lm} \right) \right] \\
&= 2 \cdot Re\left[\, \delta c_{lm} \left(\sum \underline{\mathbf{C}}_A^T \, \underline{\mathbf{F}}^{lm} \right) \, \right] \\
&= 2 \cdot Re\left[\, \delta c_{lm} \left(\sum \underline{\mathbf{C}}_A^T \int \mathbf{Y}^{lm} \cdot \underline{\boldsymbol{\beta}} \, \mathrm{d}A \right) \, \right]
\end{aligned}$$

Therefore,

$$\frac{\delta(FOM)}{\delta c_{lm}} = 2 \cdot Re\left[\, \underline{\mathbf{C}}_A^T \, \underline{\mathbf{F}}^{lm} \, \right], \tag{11a}$$

$$\frac{\delta(FOM)}{\delta c_{lm}} = 2 \cdot Re\left[\, \underline{\mathbf{C}}_A^T \int \mathbf{Y}^{lm} \cdot \underline{\boldsymbol{\beta}} \, \mathrm{d}A \, \right]. \tag{11b}$$

Recall that the surface current vector $\underline{\mathbf{C}}$ and the adjoint current vector $\underline{\mathbf{C}}_A$ depend on the incident field. For the computation of FOM, we use $\underline{\mathbf{C}}$ and $\underline{\mathbf{C}}_A$ computed using the total incident field vector $\underline{\mathbf{F}}$.

5 Input to NLOPT

Steven suggested on 2015.03.16 that the following function be constructed:

```
function objective (int n, double *c)
        compute RHS
        sol=A\rhs
        return FOM
```

where *c is an array of coefficients c_{lp}. (Steven said, why don't you start with 25 coeff.s, 5 by 5? You would need real and imaginary parts. There is no meaning in trying to make sense of different force/torque from modes. go straight to optimizing.)

5.1 Implementation details