Thousands of plots

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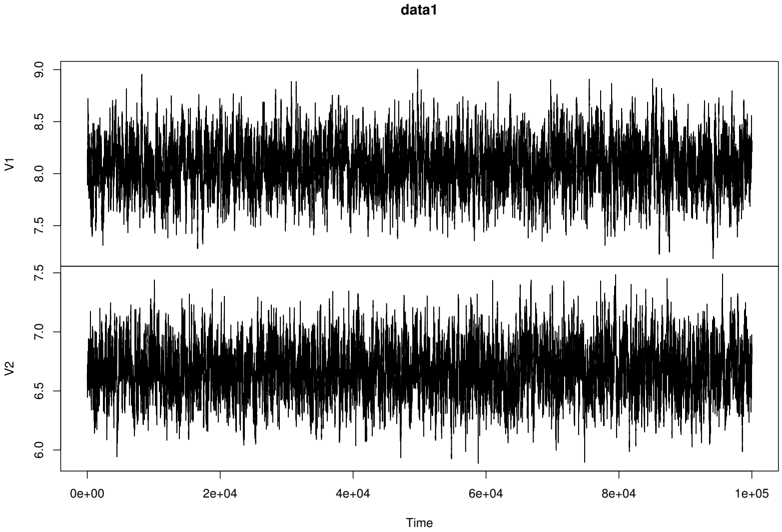
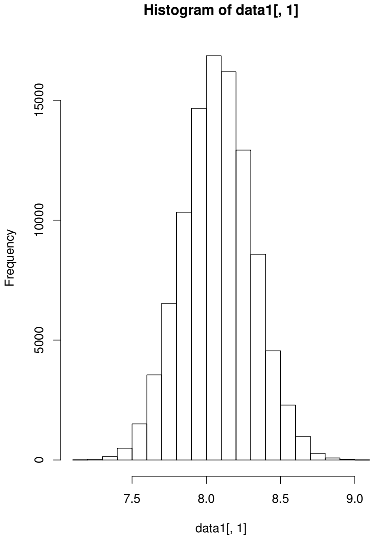
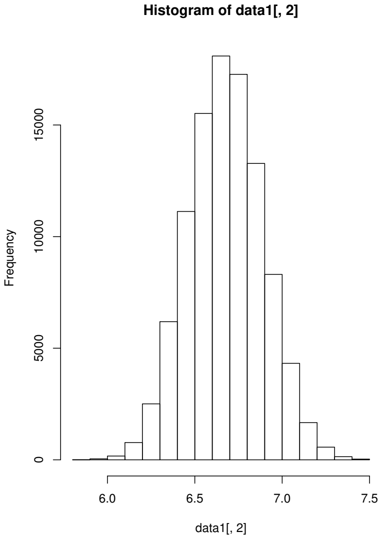
We found many folders containing cryptic .txt files. We tried plotting the numbers in one of them as a time series:



Let's look at the distributions of values for the two variables:





It looks a bit Gaussian. What say Kolmogorov and Smirnov?
For variable 1, normality is not very probable, but not excluded:

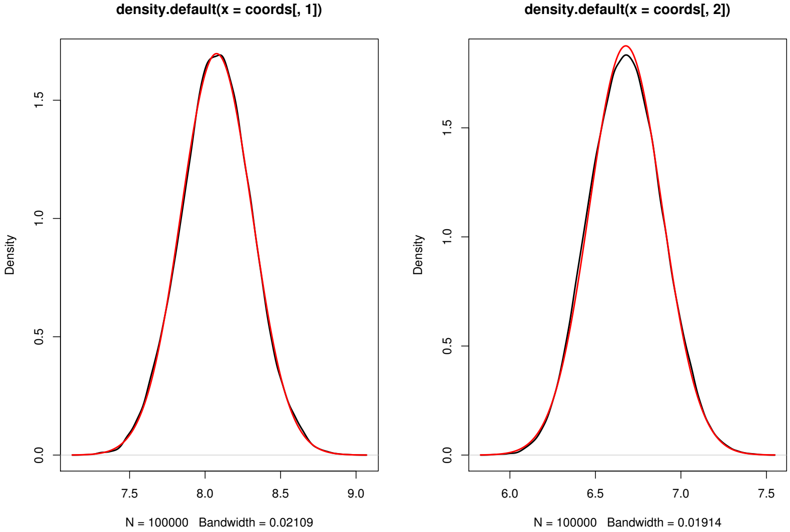
$$\text{p-value} = 0.08741$$

For variable 2, it's very abnormal:

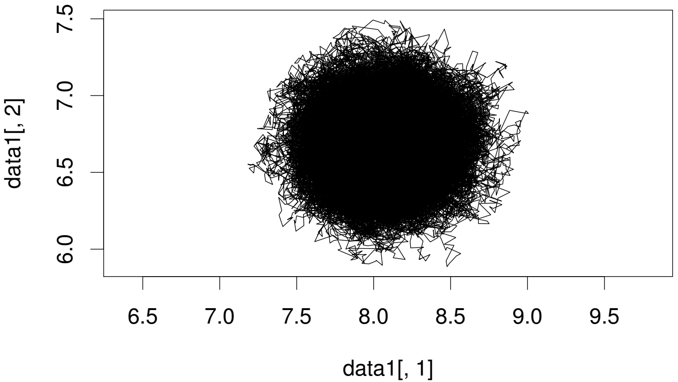
$$\text{p-value} = 1.972e - 05$$

Let's look at the density anyway...

Pretty close, we'll roll with it.



Wait, what about correlation between the two variables? Let's do a line plot:



Not much relation, but it's not completely random either. Let's compare. . .

Figure: Left: First 2000 pairs. Right: 2000 random pairs

The jumps between the consecutive points are small compared to actual random variables. Hints of dynamics. . .

If there are dynamics, we should see the later values depending on the earlier ones. Can we measure it somehow?

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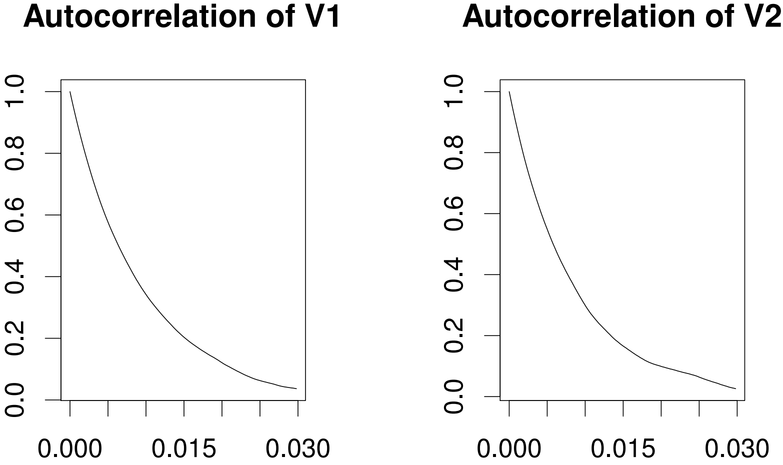
Yes!

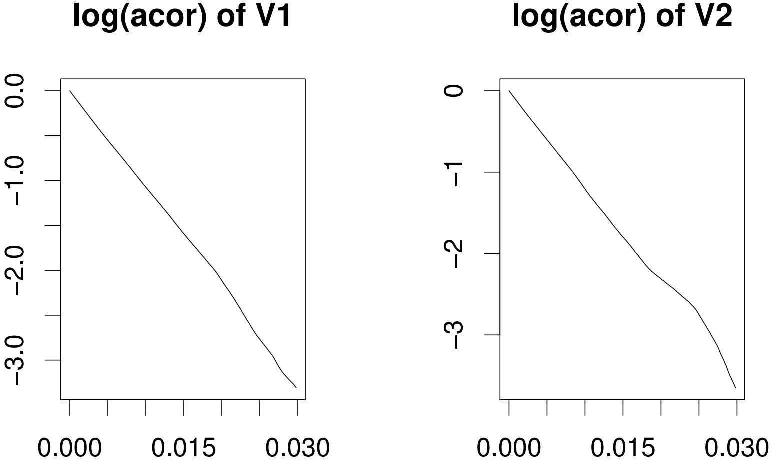
For a variable X with mean μ and variance σ^2, we can define the autocorrelation as a function of time as

$$\mathrm{acor}(\tau) = \frac{E[(X_t - \mu)(X_{t+\tau} - \mu)]}{\sigma^2}.$$

This tells us the dependence between values of X that are time τ apart. The filename mentions a frequency of 5kHz, so the time difference between the points is probably 0.2 ms.



Noise would only have a peak at 0, so we have something! Let's check these in log scale...



The dependence between values seems to decay exponentially
(for a while...)

Since the data is Gaussian, the only stochastic process (dynamic) which produces this decay is the *Ornstein-Uhlenbeck process*,

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -kx(t) + \eta(t).$$

Here

- k is the stiffness of a harmonic force
- η is white Gaussian noise

The solution of the equation is

$$x(t) = \int_{-\infty}^{t} e^{-k(t-s)} \mathrm{d}B(s),$$

with B denoting Brownian motion ($\mathrm{d}B(s) = \eta(s)\mathrm{d}s$).
We find the autocorrelation for this process to be

$$\begin{aligned}
\mathrm{acor}(x(\tau)) &= \frac{E[x(0)x(\tau)]}{\sigma^2} \\
&= \ldots \\
&= e^{-kt}.
\end{aligned}$$

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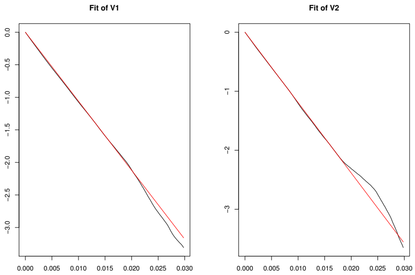
We can fit this to the data to determine k!



The fit gives us the values of k for variables V1 and V2

$$k_{V1} = 106.00,$$
$$k_{V2} = 119.25.$$

How do we know if the k we find is right? *Modelling!*
We can integrate the Ornstein-Uhlenbeck process between two consecutive points Δt apart to derive a formula for simulation:

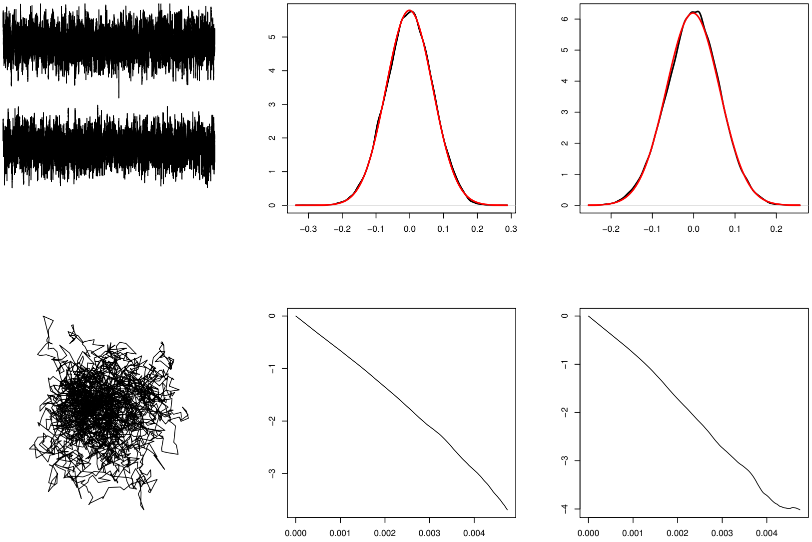
$$x_{n+1} = \alpha x_n + z_n,$$

where

$$\alpha = e^{-k\Delta t}$$

and z_n is a white Gaussian noise with variance $(1-\alpha^2)\sigma^2$.

Let's calculate everything from the model using the k values we found:



For the simulated data, the linear fit results in the k values

$$k_{V1} = 109.37,$$
$$k_{V2} = 138.61,$$

which mean that for the first and second variables we are within 4% and 17% of the actual values in the simulation, respectively.

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Conclusion:

The data can be modeled quite well as an Ornstein-Uhlenbeck process!