A Theory of Saving under Risk Preference Dynamics

Ma, Song & Toda (2025) - Reading Group Presentation

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Paper Overview

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Central Questions:

- What explains the high saving rates among wealthy households?
- How can macroeconomic models generate empirically realistic wealth distributions?

Key Innovation: Time-varying risk aversion.

Main Finding:

- Zero asymptotic MPCs arise naturally.
- No need for complex return processes.

Outline

The Empirical Puzzle

Stylized Facts

- Wealthy households have substantially higher saving rates.
- Top 1% exhibit markedly lower MPC than median households.
- This pattern persists across countries and time periods.

Theoretical Challenge

Existing models require restrictive assumptions to yield zero asymptotic MPCs:

- Ma & Toda (2021): Need stringent conditions on return risk.
- Benhabib et al. (2015): Require specific capital income risk structure.
- Carroll and Shanker (2025): $(\frac{R}{\beta})^{1/\gamma} \geq R$.

Relevance of Studying Asymptotic MPCs

The literature lacks consensus on the mechanisms underlying high saving rates among affluent households.

Theoretical Importance:

- Requires a unified consumption theory explaining behavior across the wealth distribution.
- Critical for policy design with heterogeneous agents and impulse response analysis.

Potential Limitations:

- Unclear whether the core issue concerns MPCs, distributional tails, risk perceptions, or market frictions.
- The inability of standard models to generate zero asymptotic MPCs may not constitute sufficient grounds for model rejection.

Paper's Key Contribution

Introduces stochastic risk aversion into the standard income fluctuation problem.

Methodological Innovations:

- Risk preferences exhibit state-dependent and temporal variation.
- Incorporates documented empirical evidence of preference heterogeneity.
- Eliminates reliance on complex return processes or restrictive parameter conditions.

Principal Finding

Zero asymptotic MPCs emerge endogenously when agents face potential transitions to lower risk aversion states.

The Optimal Savings Problem

Agent's Dynamic Problem

$$\max_{\{c_t, w_t\}} \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\prod_{i=0}^{t} \beta_i \right) u(c_t, Z_t) \right]$$

$$\text{s.t.} \quad w_{t+1} = R_{t+1}(w_t - c_t) + Y_{t+1}$$

$$0 \leq c_t \leq w_t$$

Key State Variables:

- Z_t: Markov chain with preference shocks.
- β_t: Stochastic discount factor.
- R_t: Stochastic returns.
- Y_t: Non-financial income.

Novel feature: Z_t affects risk aversion $\gamma(Z_t)$ directly.

State-Dependent Risk Aversion

Utility specification

$$u(c,z) = \begin{cases} \frac{c^{1-\gamma(z)}}{1-\gamma(z)} & \text{if } \gamma(z) > 0, \gamma(z) \neq 1 \\ \log c & \text{if } \gamma(z) = 1 \end{cases}$$

Key feature: $\gamma(z)$ varies with state z.

- State decomposition: $Z_t = (\bar{Z}_t, \tilde{Z}_t)$.
- Risk aversion driven by $\bar{Z}_t \in \{\bar{z}_1, \dots, \bar{z}_N\}$.
- Ordering: $0 < \gamma(\bar{z}_1) < \cdots < \gamma(\bar{z}_N)$.
- Transition matrix: $\bar{P} = (\bar{p}_{ij})_{1 \leq i,j \leq N}$.

Optimality Conditions

Euler Equation

The optimal consumption function $c^*(w,z)$ satisfies:

$$u'(c^*(w,z),z) = \max\left\{\mathbb{E}_z[\hat{\beta}\hat{R}u'(c^*(\hat{w},\hat{Z}),\hat{Z})], u'(w,z)\right\}$$

where $\hat{w} = \hat{R}(w - c^*(w,z)) + \hat{Y}$

Key Assumption for Main Results

Assumption (2.2: Returns and Discounting)

1. *For all $z \in Z$:*
    - $\mathbb{E}_z[u_c(\hat{Y}, \hat{Z})] < \infty$
    - $\mathbb{E}_z[\hat{\beta}\hat{R} \cdot u_c(\hat{Y}, \hat{Z})] < \infty$
2. *Spectral radius condition:* $r(K(1)) < 1$

   where the matrix $K(\theta)$ is defined by:

$$K_{zz'}(\theta) = P(z, z') \int \beta(z, z', \varepsilon) R(z, z', \varepsilon)^{\theta} \pi(d\varepsilon)$$

Economic Interpretation

The condition $r(K(1)) < 1$ ensures convergence of wealth in present value terms
Generalizes the standard transversality condition $\beta R < 1$

Existence and Uniqueness

Theorem (2.1: Existence and Uniqueness)

If Assumptions 2.1 and 2.2 hold, then:

1. *The time iteration operator $T : \mathcal{C} \to \mathcal{C}$ has a unique fixed point c^*.*
2. *For any $c \in \mathcal{C}$, $\rho(T^k c, c^*) \to 0$ as $k \to \infty$.*

Approach:

- Work in space $\mathcal{C}$ of continuous consumption functions.
- Use marginal utility metric:
  $\rho(c_1, c_2) := \sup_{(w,z)} |u'(c_1(w,z), z) - u'(c_2(w,z), z)|$.
- Time iteration operator contracts in this metric.

Zero Asymptotic MPCs

Theorem (3.2: Positive Transition Matrix)

If every entry of $\bar{P}$ is strictly positive and $\Pr_{z,z'}(\hat{\beta}\hat{R} > 0) > 0$ for all $(z, z') \in Z^2$, then

$$\lim_{w \to \infty} \frac{c^*(w, z_{ij})}{w} = 0 \quad \textit{for all states with } i \neq 1.$$

Key Insight (generalized from Theorem 3.1)

Under a fully mixing risk aversion Markov chain, zero asymptotic MPCs obtain at all non-minimal risk aversion states.

Economic Intuition

What mechanism links potential risk aversion reduction to enhanced savings behavior?

Theoretical Mechanism

1. Agents anticipate potential transitions to lower risk aversion states.
2. Reduced future risk aversion $\Rightarrow$ Increased marginal utility of future consumption.
3. Generates persistent precautionary saving incentives.
4. Precautionary motive remains operative across all wealth levels.

Comparison with Existing Theory

Departure from Existing Literature:

- Ma & Toda (2021): Zero MPC requires knife-edge parameter restrictions.
- Present framework: Zero MPC emerges as a generic equilibrium outcome.

Result obtains under simplified conditions:

- Deterministic returns ($R_t \equiv R$).
- Absence of income uncertainty.
- Time-invariant discount factor.

Contrasting Result: Upward Transitions

Theorem (3.4: Strictly Increasing Risk Aversion)

If risk aversion strictly increases next period from state i and R is bounded below by $m > 0$, then

$$\lim_{w \to \infty} \frac{c^*(w, z_{ij})}{w} = 1 \quad \textit{for all } j.$$

(consumption function is nonconcave).

Upward vs Downward Transitions

Theoretical Dichotomy

- Theorem 3.2: Downward risk aversion transitions $\Rightarrow \bar{c} = 0$ (complete wealth retention).
- Theorem 3.4: Upward risk aversion transitions $\Rightarrow \bar{c} = 1$ (complete wealth depletion).

Economic Intuition: Anticipated increases in future risk aversion induce intertemporal substitution toward present consumption.

When MPCs Can Be Positive

Proposition (3.1: No Downward Transitions Required)

If $\bar{c}(z_{ij}) > 0$ for some i, j, then:

$$\bar{p}_{i1} = \cdots = \bar{p}_{i,i-1} = 0.$$

(Zero probability of moving to lower risk aversion).

Summary of Main Theorems

Central Implications:

- Zero MPCs constitute a generic outcome when risk aversion permits downward transitions.
- Strictly positive MPCs require restrictive conditions that preclude transitions to lower risk aversion states.

Comparison with Existing Literature

Theoretical Significance:

- Provides a parsimonious mechanism independent of growth rate assumptions.
- Generates robust theoretical predictions across parameter spaces.

Outstanding Questions:

- Is zero limiting MPC necessary? What specific empirical regularities require explanation?
- Does the mechanism operate through risk channels or intertemporal substitution?
- The literature now contains multiple model-specific results with disparate growth conditions.
- External validity: Does preference dynamics satisfy the "Kath Day-Knight Test"?

Kath Day-Night Test

Question: Why is your MPC limiting to zero?