A Theory of Saving under Risk Preference Dynamics

Ma, Song & Toda (2025) - Reading Group Presentation

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Paper Overview

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Central Question:

- Why do wealthy households save so much?
- How to generate realistic wealth distributions in macro models?

Key Innovation: Time-varying risk aversion.

Main Finding:

- Zero asymptotic MPCs arise naturally.
- No need for complex return processes.

Outline

The Empirical Puzzle

Stylized Facts

- Wealthy households have substantially higher saving rates.
- Top 1% exhibit markedly lower MPC than median households.
- This pattern persists across countries and time periods.

Theoretical Challenge

Existing models require restrictive assumptions to yield zero asymptotic MPCs:

- Ma & Toda (2021): Need stringent conditions on return risk.
- Benhabib et al. (2015): Require specific capital income risk structure.
- Carroll and Shanker (2025): $(\frac{R}{\beta})^{1/\gamma} \geq R$.

Relevance of Studying Asymptotic MPCs

Overall, we seem to have a poor understanding of why the rich are thrifty.

Why it matters:

- Need a theory of consumption that explains behaviour across agent types.
- Matters for policy design with heterogeneous agents and generating IRFs.

Why it (may) not matter:

- Is the question about MPCs, tails, risk perceptions or frictions?
- We do not really know, but saying asymptotic MPCs cannot be zero in a standard model may not be enough motivation to reject it.

Paper's Key Contribution

Incorporate stochastic risk aversion in an otherwise vanilla income fluctuation problem.

Key Features:

- Risk preferences vary across states and over time.
- Captures empirical evidence of preference heterogeneity.
- No need for complex return processes/edge case conditions on returns and discounting.

Main Result

Zero asymptotic MPCs arise when agents can become less risk averse in the future.

The Optimal Savings Problem

Agent's Dynamic Problem

$$\max_{\{c_t, w_t\}} \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\prod_{i=0}^{t} \beta_i \right) u(c_t, Z_t) \right]$$

$$\text{s.t.} \quad w_{t+1} = R_{t+1}(w_t - c_t) + Y_{t+1}$$

$$0 \leq c_t \leq w_t$$

Key State Variables:

- Z_t: Markov chain with preference shocks.
- β_t: Stochastic discount factor.
- R_t: Stochastic returns.
- Y_t: Non-financial income.

Novel feature: Z_t affects risk aversion $\gamma(Z_t)$ directly.

State-Dependent Risk Aversion

Utility specification

$$u(c,z) = \begin{cases} \frac{c^{1-\gamma(z)}}{1-\gamma(z)} & \text{if } \gamma(z) > 0, \gamma(z) \neq 1 \\ \log c & \text{if } \gamma(z) = 1 \end{cases}$$

Key feature: $\gamma(z)$ varies with state z.

- State decomposition: $Z_t = (\bar{Z}_t, \tilde{Z}_t)$.
- Risk aversion driven by $\bar{Z}_t \in \{\bar{z}_1, \ldots, \bar{z}_N\}$.
- Ordering: $0 < \gamma(\bar{z}_1) < \cdots < \gamma(\bar{z}_N)$.
- Transition matrix: $\bar{P} = (\bar{p}_{ij})_{1 \leq i,j \leq N}$.

Optimality Conditions

Euler Equation

The optimal consumption function $c^*(w,z)$ satisfies:

$$u'(c^*(w,z),z) = \max\left\{\mathbb{E}_z[\hat{\beta}\hat{R}u'(c^*(\hat{w},\hat{Z}),\hat{Z})], u'(w,z)\right\}$$

where $\hat{w} = \hat{R}(w - c^*(w,z)) + \hat{Y}$

Key Assumption for Main Results

Assumption (2.2: Returns and Discounting)

1. *For all $z \in Z$:*
   - $\mathbb{E}_z[u_c(\hat{Y}, \hat{Z})] < \infty$
   - $\mathbb{E}_z[\hat{\beta}\hat{R} \cdot u_c(\hat{Y}, \hat{Z})] < \infty$
2. *Spectral radius condition:* $r(K(1)) < 1$

   where the matrix $K(\theta)$ is defined by:

$$K_{zz'}(\theta) = P(z, z') \int \beta(z, z', \varepsilon) R(z, z', \varepsilon)^{\theta} \pi(d\varepsilon)$$

Interpretation

$r(K(1)) < 1$ ensures wealth doesn't explode in present value terms

Generalizes the standard condition $\beta R < 1$

Existence and Uniqueness

Theorem (2.1: Existence and Uniqueness)

If Assumptions 2.1 and 2.2 hold, then:

1. *The time iteration operator $T : \mathcal{C} \to \mathcal{C}$ has a unique fixed point c^*.*
2. *For any $c \in \mathcal{C}$, $\rho(T^k c, c^*) \to 0$ as $k \to \infty$.*

Approach:

- Work in space $\mathcal{C}$ of continuous consumption functions.
- Use marginal utility metric:
  $\rho(c_1, c_2) := \sup_{(w,z)} |u'(c_1(w,z),z) - u'(c_2(w,z),z)|$.
- Time iteration operator contracts in this metric.

Zero Asymptotic MPCs

Theorem (3.2: Positive Transition Matrix)

If every entry of $\bar{P}$ is strictly positive and $\Pr_{z,z'}(\hat{\beta}\hat{R} > 0) > 0$ for all $(z, z') \in Z^2$, then

$$\lim_{w \to \infty} \frac{c^*(w, z_{ij})}{w} = 0 \quad \text{for all states with } i \neq 1.$$

Key Insight (from more general result)

With fully mixing risk aversion Markov chain, zero asymptotic MPCs arise at all or almost all states.

Economic Intuition

Why does the possibility of lower risk aversion drive saving?

The Mechanism

1. Agent anticipates possible decrease in risk aversion.
2. Lower future risk aversion $\Rightarrow$ Higher future consumption desire.
3. Creates perpetual precautionary saving motive.
4. Saving motive persists regardless of wealth level.

Comparison with Existing Theory

Key difference from existing theory:

- Ma & Toda (2021): Zero MPC is knife-edge case.
- This result: Zero MPC is generic feature.

Result holds even with:

- Constant returns ($R_t \equiv R$).
- No income risk.
- Constant discount factor.

Contrasting Result: Upward Transitions

Theorem (3.4: Strictly Increasing Risk Aversion)

If risk aversion strictly increases next period from state i and R is bounded below by $m > 0$, then

$$\lim_{w \to \infty} \frac{c^*(w, z_{ij})}{w} = 1 \quad \textit{for all } j.$$

(consumption function is nonconcave).

Upward vs Downward Transitions

Key Contrast

- Theorem 3.2: Downward transitions $\Rightarrow \bar{c} = 0$ (save everything).
- Theorem 3.4: Upward transitions $\Rightarrow \bar{c} = 1$ (consume everything).

Intuition: When agents expect to become more risk averse in the future, they consume aggressively today before their preferences change.

When MPCs Can Be Positive

Proposition (3.1: No Downward Transitions Required)

If $\bar{c}(z_{ij}) > 0$ for some i, j, then:

$$\bar{p}_{i1} = \cdots = \bar{p}_{i,i-1} = 0.$$

(Zero probability of moving to lower risk aversion).

Summary of Main Theorems

Main takeaway:

- Zero MPCs are generic when γ can decrease.
- Strictly positive MPCs require restrictive conditions disallowing downward transitions.

Comparison with Existing Literature

Why this matters:

- Single, clear and simple mechanism, does not rely on growth rates.
- Robust theoretical result.

Why I want to know more:

- Do we need limiting MPCs to be zero? What specific empirical fact are we trying to explain?
- Is this a story about risk or inter-temporal substitution?
- Overall, we are now left with lots of results on MPCs and tails, with fruit salad of ad hoc growth conditions.
- Does risk aversion dynamics pass the "Kath Day-Night Test"?

Kath Day-Night Test

Question: Why is your MPC limiting to zero?