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**Technical University of Crete**  
**School of Electrical and Computer Engineering**  
Course: **Convex Optimization**

Exercise 3

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50/1000 points

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In this exercise, we shall solve simple (but important) optimization problems by explicitly solving the KKT conditions. **In all cases, start with a drawing of the problem.**

1. (10) Compute the projection of  $\mathbf{x}_0 \in \mathbb{R}^n$  onto the set  $\mathbf{B}(\mathbf{0}, r) := \{\mathbf{x} \in \mathbb{R}^n \mid \|\mathbf{x}\|_2 \leq r\}$ .
  - (a) Draw a scheme of the problem.
  - (b) Write down the optimization problem you must solve, in terms of **differentiable** functions.
  - (c) Write down the KKT conditions, in terms of the optimal parameters  $\mathbf{x}_*$  and  $\lambda_*$ .
  - (d) Consider the case  $\lambda_* > 0$ . What is the conclusion?
  - (e) Consider the case  $\lambda_* = 0$ . What is the conclusion?
2. (10) Repeat the steps of the previous question and compute the projection of  $\mathbf{x}_0 \in \mathbb{R}^n$  onto the set  $\mathbf{B}(\mathbf{y}, r) := \{\mathbf{x} \in \mathbb{R}^n \mid \|\mathbf{x} - \mathbf{y}\|_2 \leq r\}$  (for given  $\mathbf{y} \in \mathbb{R}^n$  and  $r \in \mathbb{R}_{++}$ ).
3. (10) Let  $\mathbf{a} \in \mathbb{R}^n$ . Compute the projection of  $\mathbf{x}_0 \in \mathbb{R}^n$  onto set  $\mathbb{S} := \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{a} \leq \mathbf{x}\}$ .
4. (10) Let  $\mathbf{0} \neq \mathbf{a} \in \mathbb{R}^n$ ,  $b \in \mathbb{R}$ . Solve the problem

$$\min_{\mathbf{x}} \frac{1}{2} \|\mathbf{x}\|_2^2, \text{ subject to } \mathbf{a}^T \mathbf{x} = b. \quad (1)$$

5. (10) Let  $\mathbf{A} \in \mathbb{R}^{p \times n}$ , with  $\text{rank}(\mathbf{A}) = p$ , and  $\mathbf{b} \in \mathbb{R}^p$ . Find the distance of a point  $\mathbf{x}_0 \in \mathbb{R}^n$  from the set  $\mathbb{S} := \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{A}\mathbf{x} = \mathbf{b}\}$ .