Technical University of Crete
School of Electrical and Computer Engineering
Course: **Convex Optimization**
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Exercise 4 (100/1000)
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1. (100) Consider the problem

$$\begin{array}{ll} \min_{\mathbf{x}\in\mathbb{R}^n} & f(\mathbf{x}) = -\sum_{i=1}^n \log(x_i) \\ \text{subject to} & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{array} \tag{1}$$

where $\mathbf{A} \in \mathbb{R}_+^{p\times n}$, $\mathrm{rank}(\mathbf{A}) = p$, $\mathbf{b} \in \mathbb{R}_+^p$. (In order to focus on the algorithmic aspects of the problem, we shall generate feasible problems).

   1. Data generation: In order to guarantee feasibility, we generate $\mathbf{A}$ with i.i.d. elements, with $A_{i,j} \sim \mathcal{U}[0,1]$, and $\mathbf{x}$ with i.i.d. elements, with $x_i \sim \mathcal{U}[0,1]$, and set $\mathbf{b} = \mathbf{A}\mathbf{x}$.
   2. (10) Solution 1: solve problem (1) via `cvx`.
   3. (40) Solution 2: solve problem (1) via Newton, starting from a feasible point, as follows:
      i. compute a feasible point $\mathbf{x}_0$ via `cvx` ($\mathbf{x}_0 > \mathbf{0}$, $\mathbf{A}\mathbf{x}_0 = \mathbf{b}$);
      ii. implement the Newton algorithm, starting from $\mathbf{x}_0$.
      iii. in order to understand how the algorithm progresses, in every algorithm iteration, depict in a common plot the cvx solution and the solution estimate, $\mathbf{x}_k$.
   4. (30) Solution 3: solve problem (1) via the primal-dual algorithm, starting from point $\mathbf{x}_0 = \mathbf{1}$ (in order to understand how the algorithm progresses, in every algorithm iteration, depict in a common plot the cvx solution and the solution estimate, $\mathbf{x}_k$).
   5. (10) Solution 4: derive the problem dual to (1) and solve it via `cvx`, and then find the primal solution via the solution of the dual.
   6. (10) For the case with $p = 1$ and $n = 2$, plot the convergence of the algorithm towards the solution.