

Σετ ασκήσεων 1

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Θέμα 1

Assume that T is a binary search tree with n nodes, and height h . Prove that a range query returning k elements, has cost $O(h + k)$. Give pseudo-code for such a range search query.

N.B.: A range query is defined by two keys, say a and b , and returns all keys x in the tree, where $a \leq x \leq b$.

Θέμα 2

Describe an algorithm that takes as input two line segments, defined by the pairs of endpoints (p_1, p_2) and (q_1, q_2) respectively, where $p_1, p_2, q_1, q_2 \in \mathbb{R}^2$. The algorithm returns TRUE if the two segments intersect, and FALSE otherwise.

Θέμα 3

Describe an $O(n \log n)$ -time algorithm for the following problem:

INPUT: A collection S of n vertical planar segments, described as triples $(a_i, b_i, c_i) \in \mathbb{R}^3$ such that $b_i < c_i$, corresponding to the vertical segment with endpoints $\begin{bmatrix} a_i \\ b_i \end{bmatrix}$ and $\begin{bmatrix} a_i \\ c_i \end{bmatrix}$.

OUTPUT: TRUE if there exists a line $y = px + q$, intersecting every vertical segment (i.e., for all i , $b_i < pa_i + q < c_i$, and FALSE otherwise.

Hint: Hyperplane separation theorem

Θέμα 4

Prove that the convex hull of a finite set of points in \mathbb{R}^2 is the minimum-perimeter simple polygon containing the set.

Θέμα 5

Let $p_i \in \mathbb{R}^3$, $i = 1, \dots, 3$ be the vertices of a 3-d triangle. Consider the ray (half-line) $a + \lambda D$, where $a, D \in \mathbb{R}^3$ and $\lambda \geq 0$. With p_i, a, D given, write a predicate to compute whether the ray intersects the interior of the triangle. Also, write a formula that returns the intersection point (when it exists and is unique).