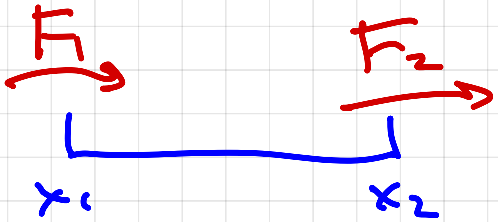
Conservation laws

$$\underset{\sim}{u}_t + \nabla \cdot F(\underset{\sim}{u}) = 0$$

$\underset{\sim}{u}$ = conserved quantity

Example



let $q(x,t)$ = concentration

then $\int_{x_1}^{x_2} q(x,t)\,dx$ = total mass in $[x_1, x_2]$ at "t"

→ mass only changes due to flux at the ends

$$\frac{d}{dt}\int_{x_1}^{x_2} q(x,t)\,dx = F_1(t) - F_2(t)$$

Fluid? $F = u(x,t) \cdot q(x,t)$

velocity $\frac{m}{s}$ density $\frac{kg}{m}$

$= \bar{u}\, q(x,t)$ for constant velocity

$$\rightarrow \frac{d}{dt}\int_{x_1}^{x_2} q(x,t)\,dx = f(q(x_1,t)) - f(q(x_2,t))$$

$$= -f(q(x,t))\Big|_{x_1}^{x_2}$$

$$= \int_{x_1}^{x_2} \frac{\partial f(q(x,t))}{\partial x}\,dx$$

$$\rightarrow \int_{x_1}^{x_2} \frac{\partial q(x,t)}{\partial t} + \frac{\partial f(q(x,t))}{\partial x}\,dx = 0$$

$$q_t + (f(q))_x = 0$$

$$u_t + \nabla \cdot \left(\frac{u^2}{2}\right) = 0$$

$$\underset{\partial_x}{\uparrow}$$

$$u_t + \nabla \cdot (a u) = 0$$

$$\int (u_t + \nabla \cdot F) v = 0$$

$$= \int u_t v + \int_{\partial\Omega} n \cdot F v - \int_{\Omega} F \cdot \nabla v = 0$$

PINN

main challenges:

1. nothing PDE specific
   → how to enforce conservation, e.g.?
2. cost
   → everything is monolithic

Conservative physics-informed neural networks on discrete domains for conservation laws: Applications to forward and inverse problems

Ameya D. Jagtap[a], Ehsan Kharazmi[a], George Em Karniadakis[a,b,*]

2020 → cPINN

physics constrained NN

Extended physics-informed neural networks (XPINNs) : A generalized space-time domain decomposition based deep learning framework for nonlinear partial differential equations

Ameya D. Jagtap* [1]
George Em Karniadakis [1]

2021 → generalizes cPINN

XPINN

D3M: A Deep Domain Decomposition Method for Partial Differential Equations

KE LI[1,2,3], (Student Member, IEEE), KEJUN TANG[1,2,3], TIANFAN WU[4], AND QIFENG LIAO[1]

2019 → D3M

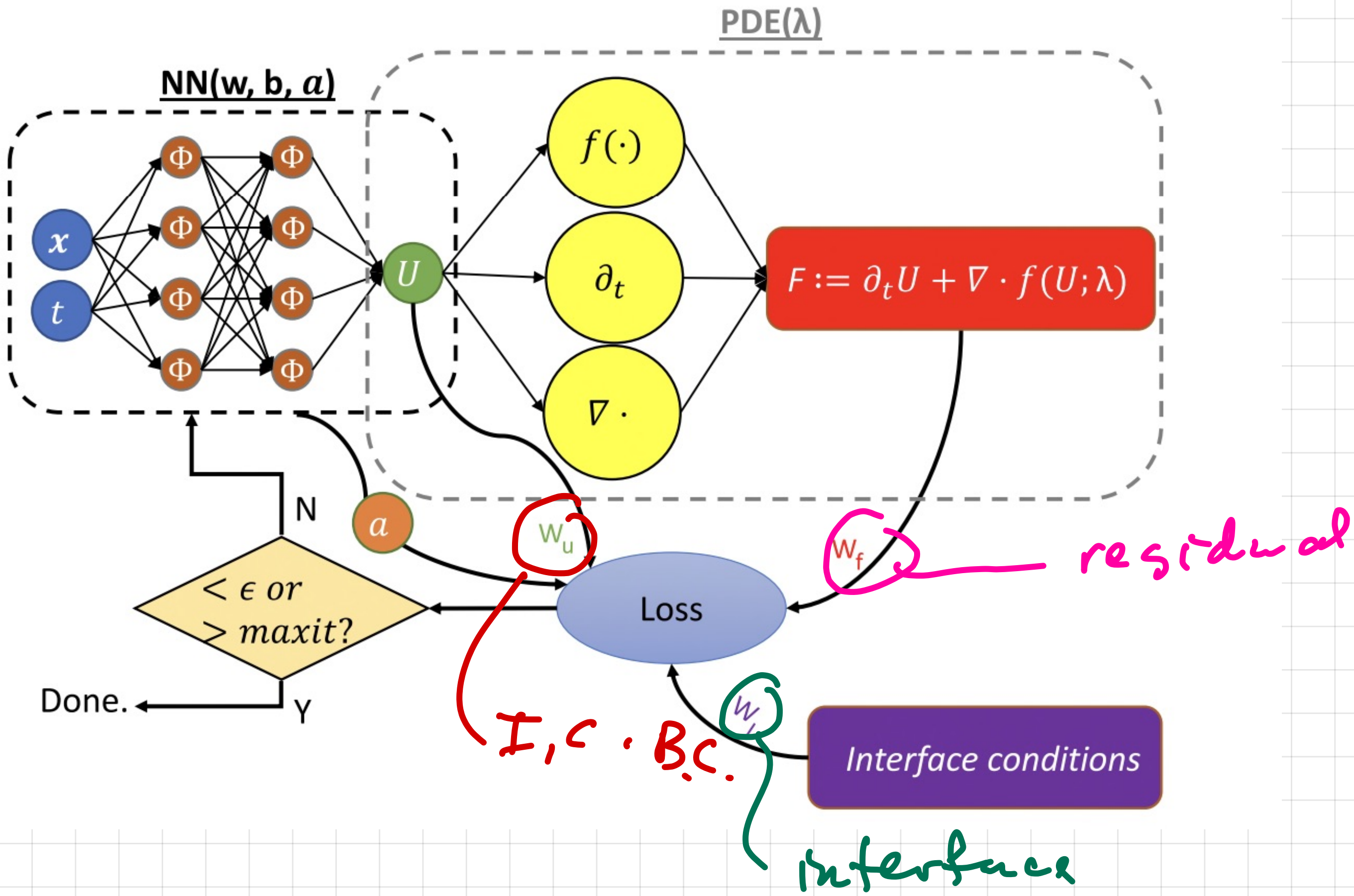
Parallel Physics-Informed Neural Networks via Domain Decomposition

Khemraj Shukla, Ameya D. Jagtap, George Em Karniadakis*

2021?

cPINN

- use domain decomp to enforce locality
- provide information on <u>fluxes</u> at interfaces
- per domain NN selection
- adaptive activations



PDE(λ)
NN(w, b, a)
x
t
Φ
U
$f(\cdot)$
∂_t
$\nabla \cdot$
$F := \partial_t U + \nabla \cdot f(U;\lambda)$
a
N
$< \epsilon$ or $> maxit?$
Done.
Y
W_u
Loss
W_f
residual
I, C . B.C.
W_I
Interface conditions
interface