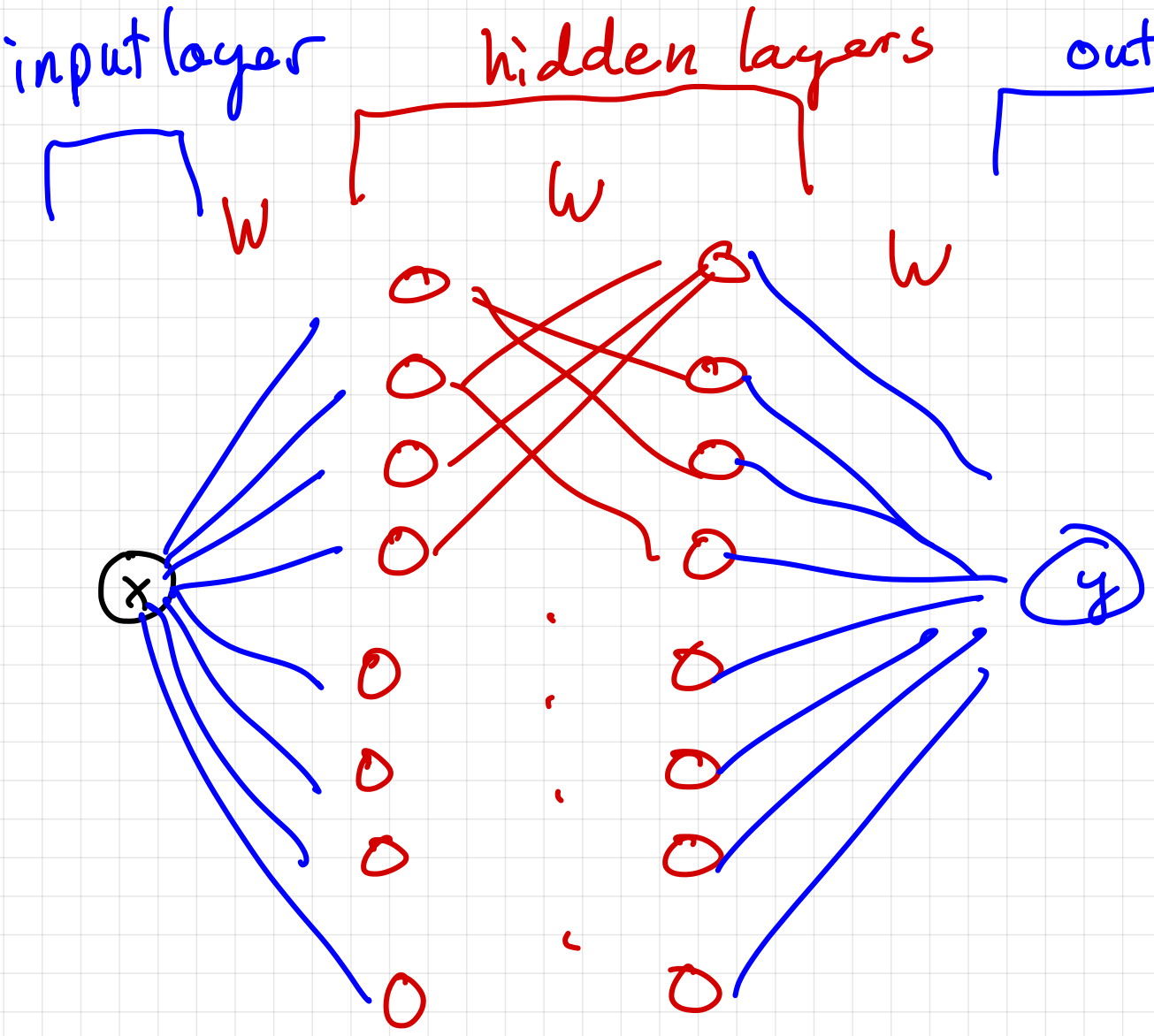


$$\text{loss} = \frac{1}{N} \sum_{i=1}^{N_{\text{samples}}} \left| y_i^{\text{train}} - y_i \right|^2$$

Note: add derivatives

Consider: $\frac{\partial u}{\partial t} + N(u, \lambda) = 0$

λ — unknown or parameter

u — unknown or data

N — nonlinear differential op.

$x \in \Omega$

$t \in [0, T]$

Example

$u_t + u u_x = 0$ or $\Rightarrow u_{xx}$

$u(x, 0) = -\sin(x)$ I.C.

$u(-1, t) = u(1, t) = 0$ B.C.

PINN:

① For some λ, can we find u?

② For some data, which λ?

Find u, f such that

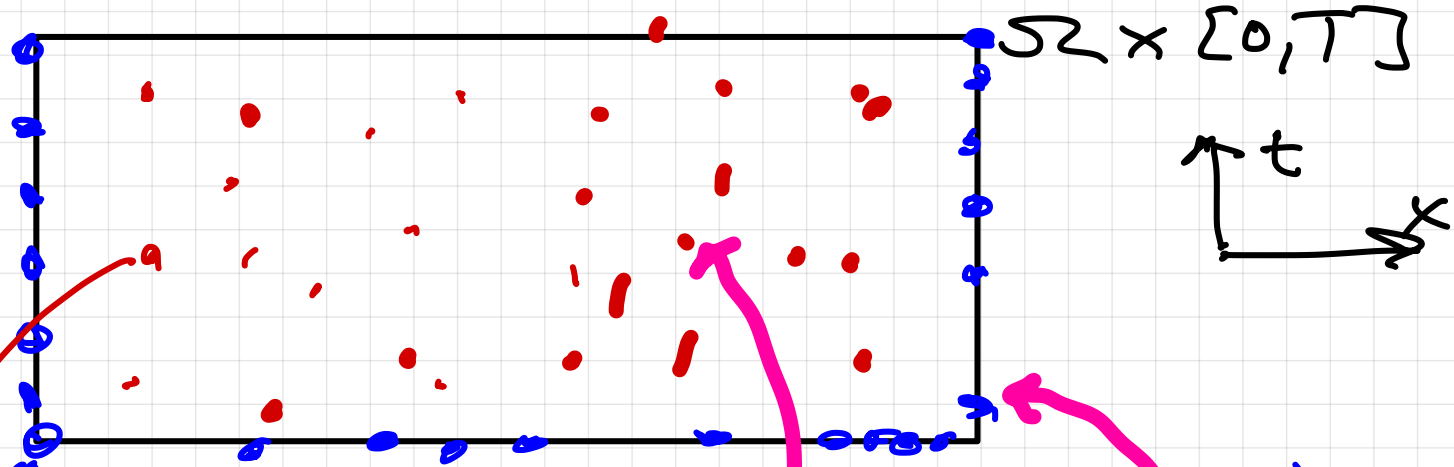
$$u(-1,t) = u(1,t) = 0 \qquad \text{B.C.}$$

$$u(x,0) = -\sin(x) \qquad \text{I.C.}$$

and

$$f = u_t + uu_x = 0$$

New loss:

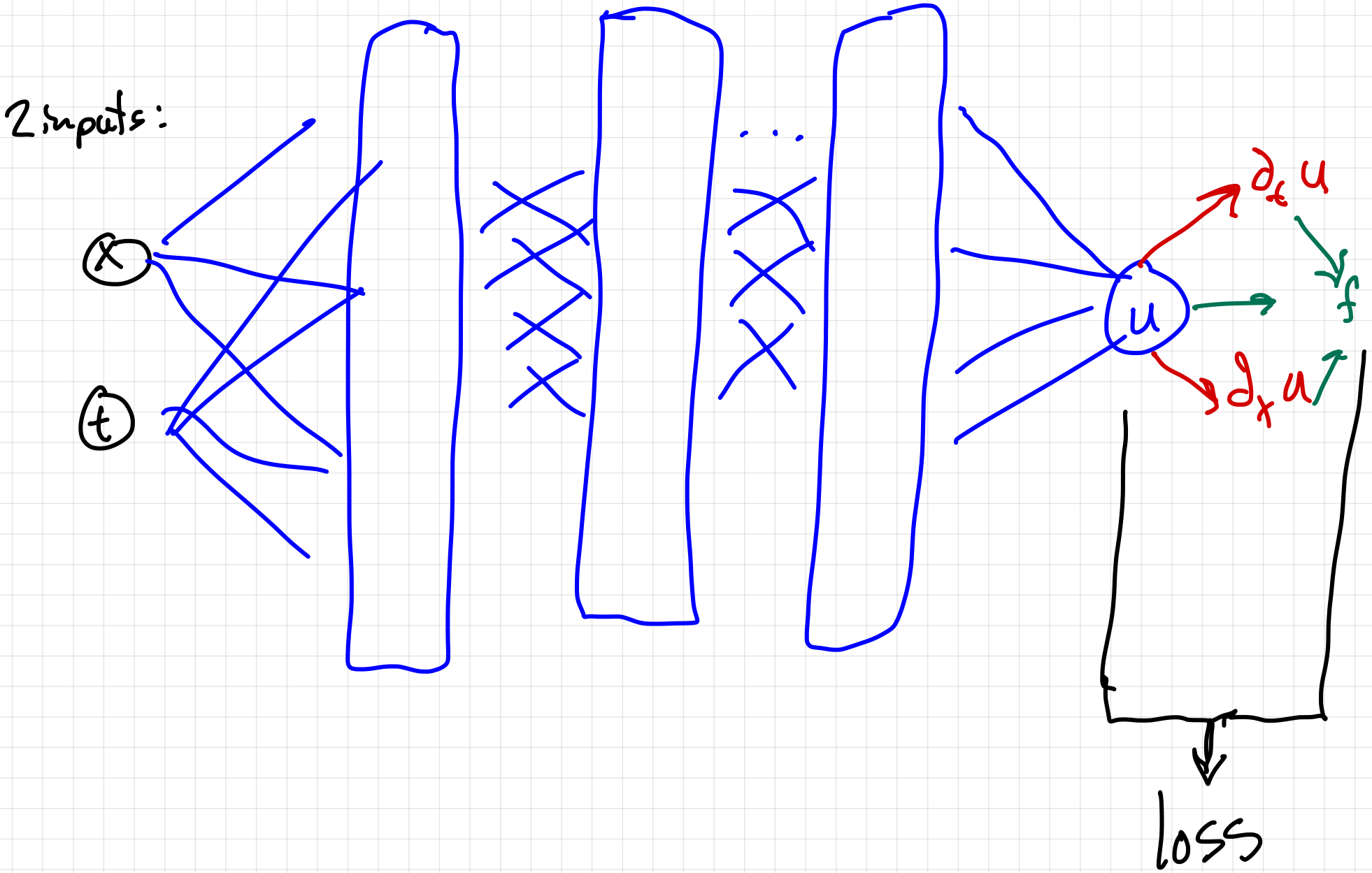


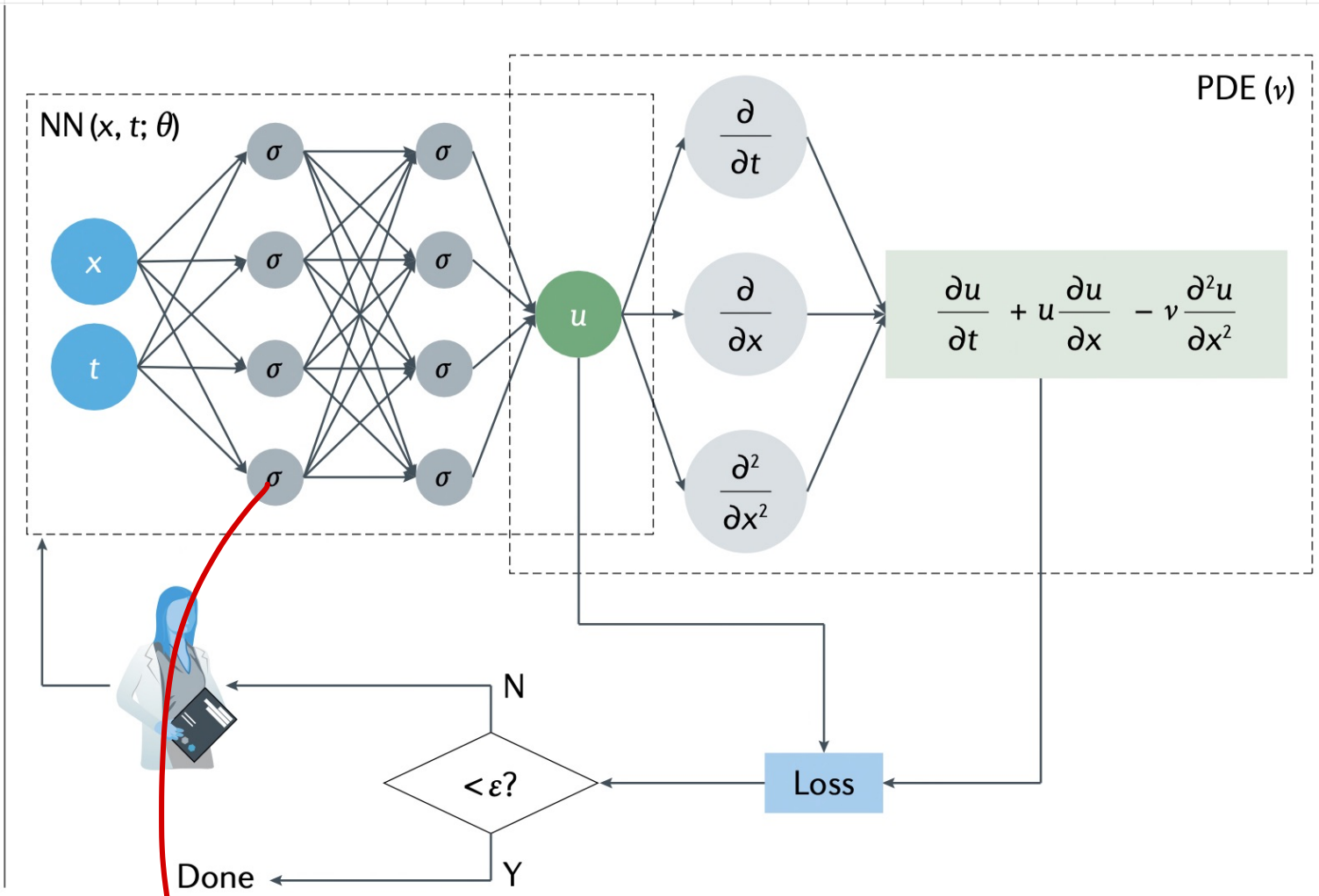
N_u samples on the t-x space b dy.

N_f samples in $\Omega \times [0,T]$

$$\text{loss} = \frac{1}{N_u} \sum_{i=1}^{N_u} \left| u^i_{\text{model}} - u(t^i_u, x^i_u) \right|^2 + \frac{1}{N_f} \sum_{i=1}^{N_f} \left| f(t^i_f, u^i_f) \right|^2$$

NN





$\sigma = \tanh$

$$u_t + u\,v = 0$$

$$u_x - v = 0$$

$u_t + uu_x = 0$ $\quad u_t + \left(\frac{u^2}{2}\right)_x = 0$

$u(x,0) = -\sin(x)$

$u(-1,t) = u(1,t) = 0$