Exercise P1: Consider the following performance function

$$G\left(X_1, X_2\right) = 1 - \frac{(X_1 + X_2 - 5)^2}{30} - \frac{(X_1 - X_2 - 12)^2}{120}$$

where $X_1 \sim$ N (3.5, 0.5), $X_2 \sim$ N (3.8, 0.5^2).

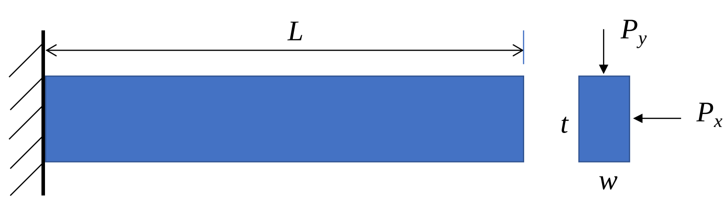
Conduct the probability of failure analysis for PoF = Pr (G > 0), using

(1) MCS method with N = 1,000,000 samples.
(2) FORM with the HL-RF method.

Exercise P2: Considering the design of a cantilever beam shown in the figure below. One of the failure modes of the beam is that the tip displacement may exceed the allowable threshold, D_0. The performance function is defined as the difference between D_0 and the tip displacement given by the function as

$$G(P_x, P_y) = \frac{4L^3}{Ewt}\sqrt{\left(\frac{P_x}{w^2}\right)^2 + \left(\frac{P_y}{t^2}\right)^2} - D_0 \tag{1}$$

where the allowable tip displacement $D_0 = 2.5''$, E = 2.9e7 psi is the modulus of elasticity, $L = 100''$ is the length, $w = 2''$ and $t = 4''$ are the width and height of the cross section, respectively. Px and Py are the external forces with normal distributions, $P_x \sim N\,(500, 100^2)$ lb and $P_y \sim N\,(1000, 100^2)$lb.



(a) Please use the Monte Carlo simulation (MCS) with 1000,000 sample points to determine the reliability of the beam, defined as

$$PoF = \Pr\left\{G\left(P_x, P_y\right) > 0\right\}$$

(b) Please use FORM with the HL_RF method to calculate the PoF