Physics-informed neural networks to solve the compressible Euler equations

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Overview

- Approximate solution of the Euler equations using a neural network (forward method)

1-D Compressible Euler equations

$$\frac{\partial U}{\partial t} + A\frac{\partial U}{\partial x} = 0$$

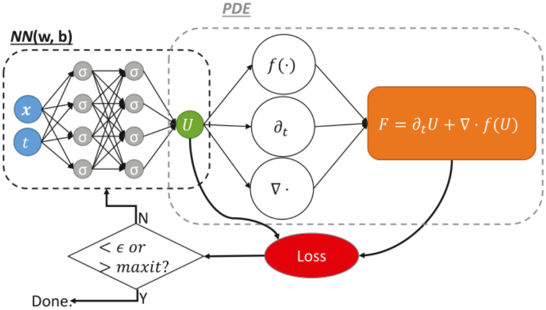
where,

$$U = (\rho, u, p)^T$$

$$A = \begin{bmatrix} u & \rho & 0 \\ 0 & u & \frac{1}{\rho} \\ 0 & \rho a^2 & u \end{bmatrix}$$

$$a = \sqrt{\gamma p/\rho}$$

- 2 inputs $(x,\ t)$
- 3 states outputs $(\rho,\ u,\ p)$



Loss function

- In general, the loss function is defined as follows:

$$L(\theta) = \frac{1}{N_f} \left| \frac{\partial \tilde{U}}{\partial t}(x,t,\theta) + \tilde{A} \frac{\partial \tilde{U}}{\partial x}(x,t,\theta) \right|^2_{\Omega \times (0,T]} +$$
$$\frac{1}{N_{IC}} \left| \tilde{U}(x,0,\theta) - U(x,0) \right|^2_{\Omega} + \frac{1}{N_{BC}} \left| \tilde{U}(x,t,\omega) - U(x,t) \right|^2_{\partial\Omega \times (o,T]}$$

- Problem-specific $\rightarrow$ Sod shock tube problem

$$\begin{bmatrix} \rho_L \\ u_L \\ P_L \end{bmatrix} = \begin{bmatrix} 1.0 \\ 0.0 \\ 1.0 \end{bmatrix} \qquad \begin{bmatrix} \rho_L \\ u_L \\ P_L \end{bmatrix} = \begin{bmatrix} 0.125 \\ 0.0 \\ 0.1 \end{bmatrix}$$

Advances

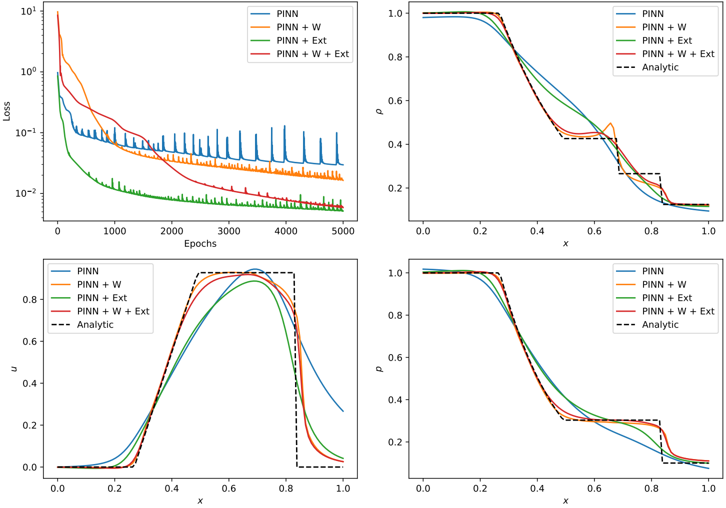
A public repository link was setup to host the models implemented

Four different models were trained by using a neural network (30-neuron width & 7 hidden layers), a learning rate of 0.0005 and the classical Adam optimizer. The epochs for these cases were set to 5000

- Space domain unaltered ($x \in [0, 1]$), no weights for losses
- Space domain extended ($x \in [-1.5, 3.125]$), no weights for losses
- Space domain unaltered ($x \in [0, 1]$), loss weights added (0.1 for PDE loss and 10 for IC loss)
- Space domain extended ($x \in [-1.5, 3.125]$), loss weights added (0.1 for PDE loss and 10 for IC loss)

Preliminary results

- The outcomes from the mentioned trained models are shown below.
- A comparison has been made only the exact solution at the desired time $(t = 0.2)$



Difficulties & Next steps

- **Difficulties**
  - None of the models trained is able to completely capture the physics at the discontinuity (shock)
  - Some models are presenting oscillations, while others present dissipation
  - So far, training the models using cpu-only machines requires significant time (for 5000 epoch $\approx 15[\mathrm{min}]$)

- **Next steps**
  - Train the model using any GPU-accelerated device (HAL, local) and compare training times
  - Implement a formal comparison with the exact solution and a high-order numerical method (WENO scheme)
  - Define a set of neural networks parameters (width, hidden layers, optimizers, mini batches) and train the model with their combination.
  - For the previous goal, a shell script will be created to set the aforesaid parameters and run the jobs automatically in a remote computer
  - Implement either a 2-D compressible flow case or solve the inverse problem for a given set of density gradients (mimicking a Schlieren image)

References

1. Mao, Z., Jagtap, A. D., & Karniadakis, G. E. (2020). Physics-informed neural networks for high-speed flows. Computer Methods in Applied Mechanics and Engineering, 360, 112789
2. Papados, A. Solving hydrodynamic shock-tube problems using weighted physics-informed neural networks with domain extension.