1 KDA-based and fitted ATLAS CSZs

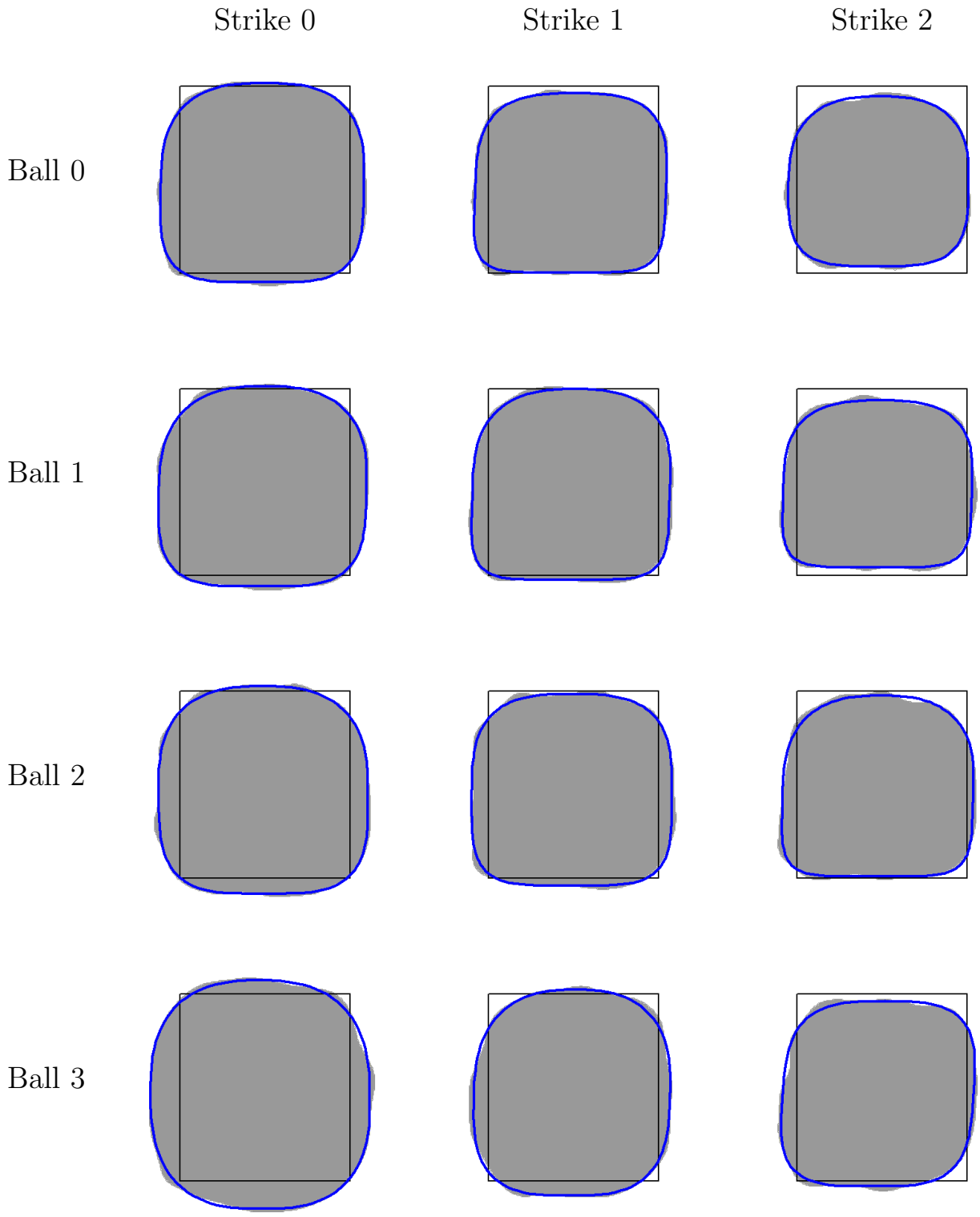

Figure 1: Called strike zones for the 12 Ball-Strike combinations for RHP-RHB-Home.

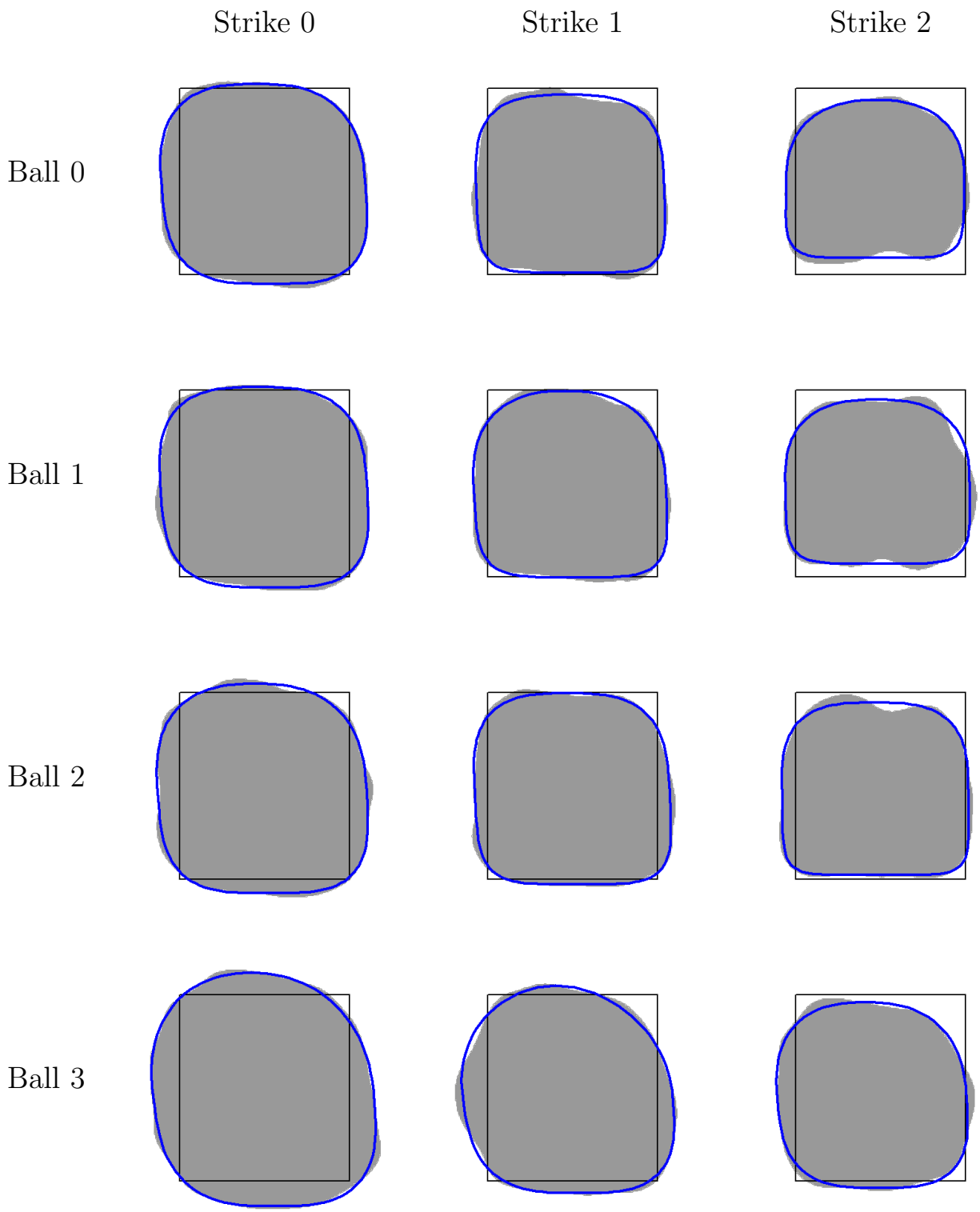

Figure 2: Called strike zones for the 12 Ball-Strike combinations for LHP-RHB-Home.

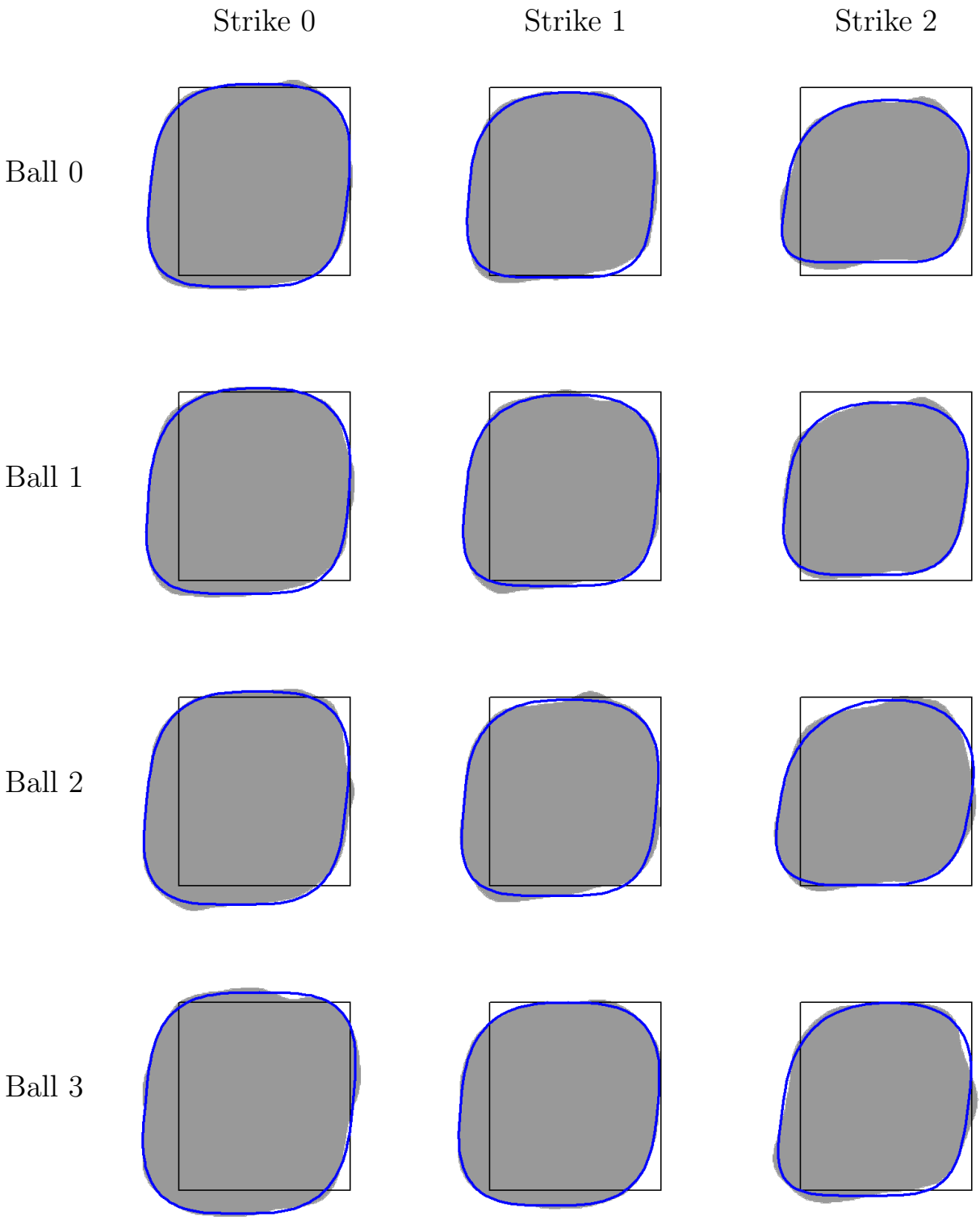

Figure 3: Called strike zones for the 12 Ball-Strike combinations for RHP-LHB-Home.

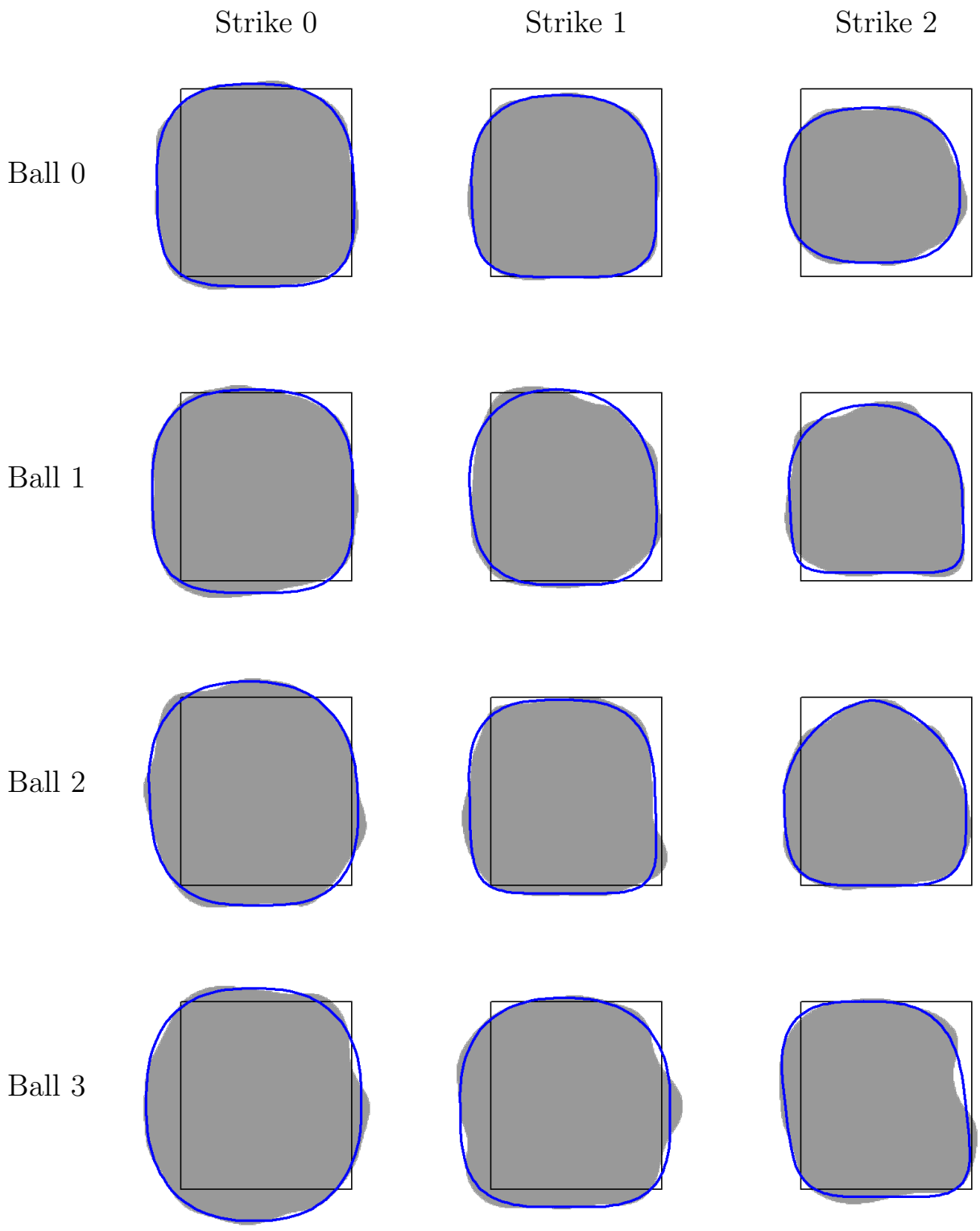

Figure 4: Called strike zones for the 12 Ball-Strike combinations for LHP-LHB-Home.

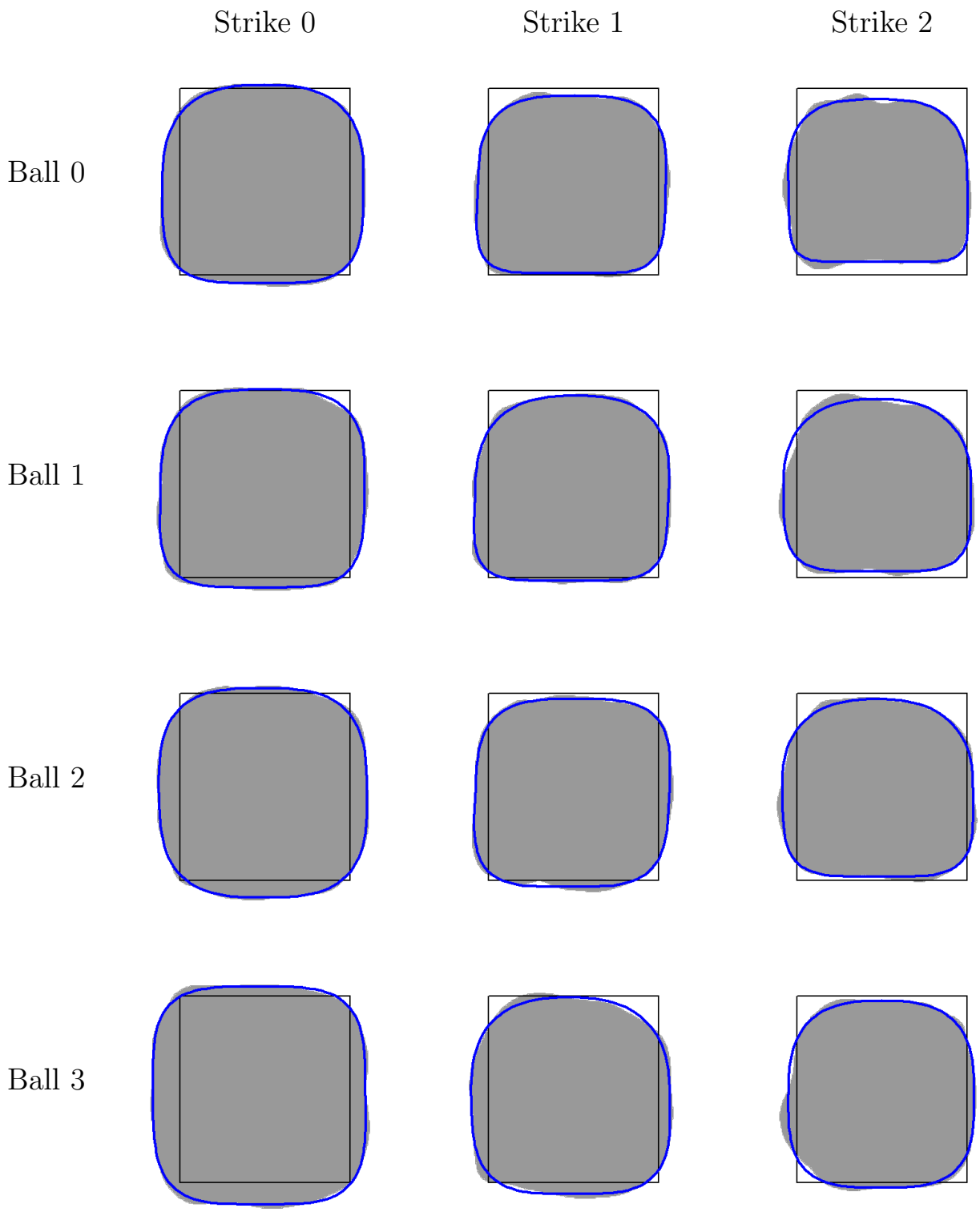

Figure 5: Called strike zones for the 12 Ball-Strike combinations for RHP-RHB-Away.

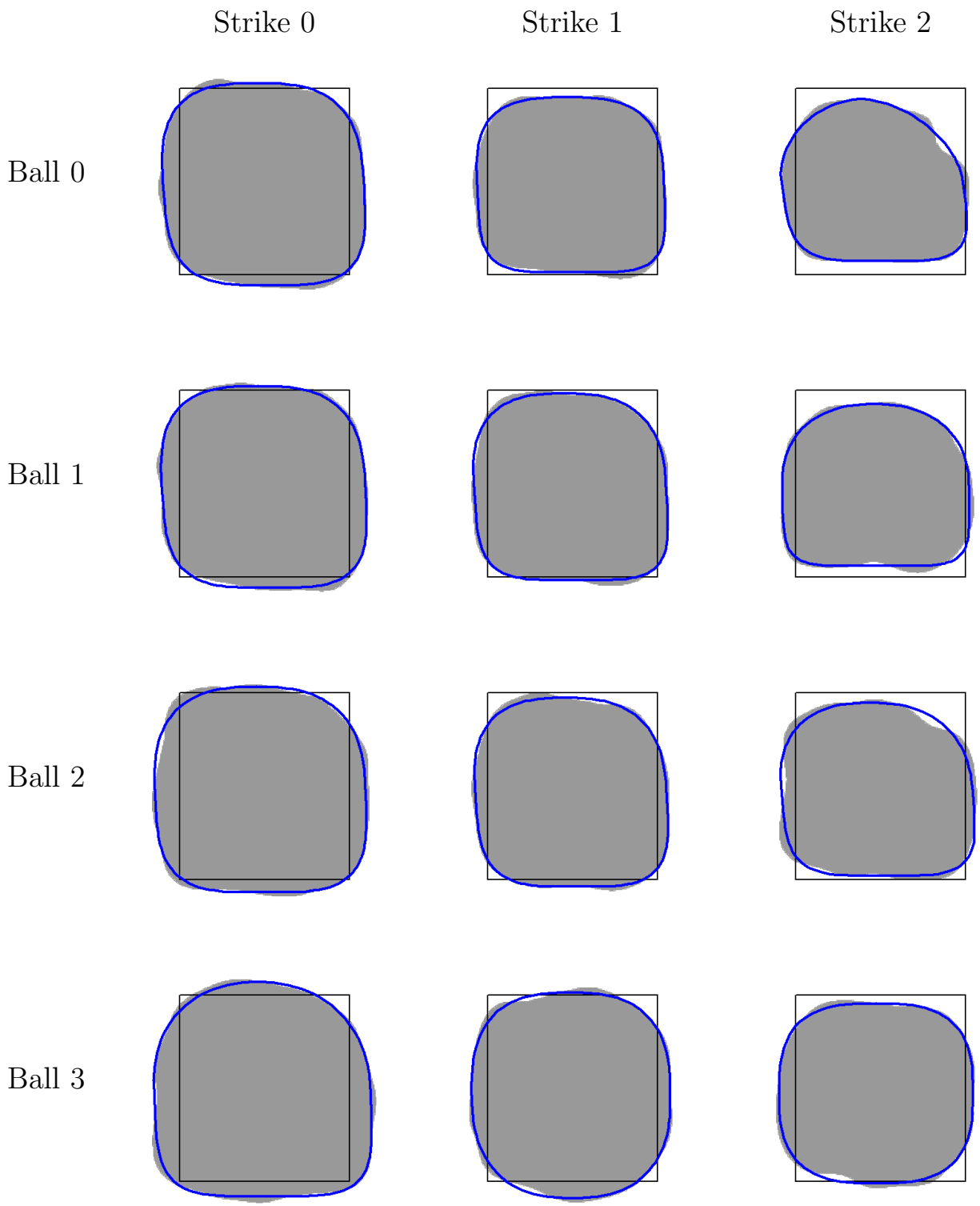

Figure 6: Called strike zones for the 12 Ball-Strike combinations for LHP-RHB-Away.

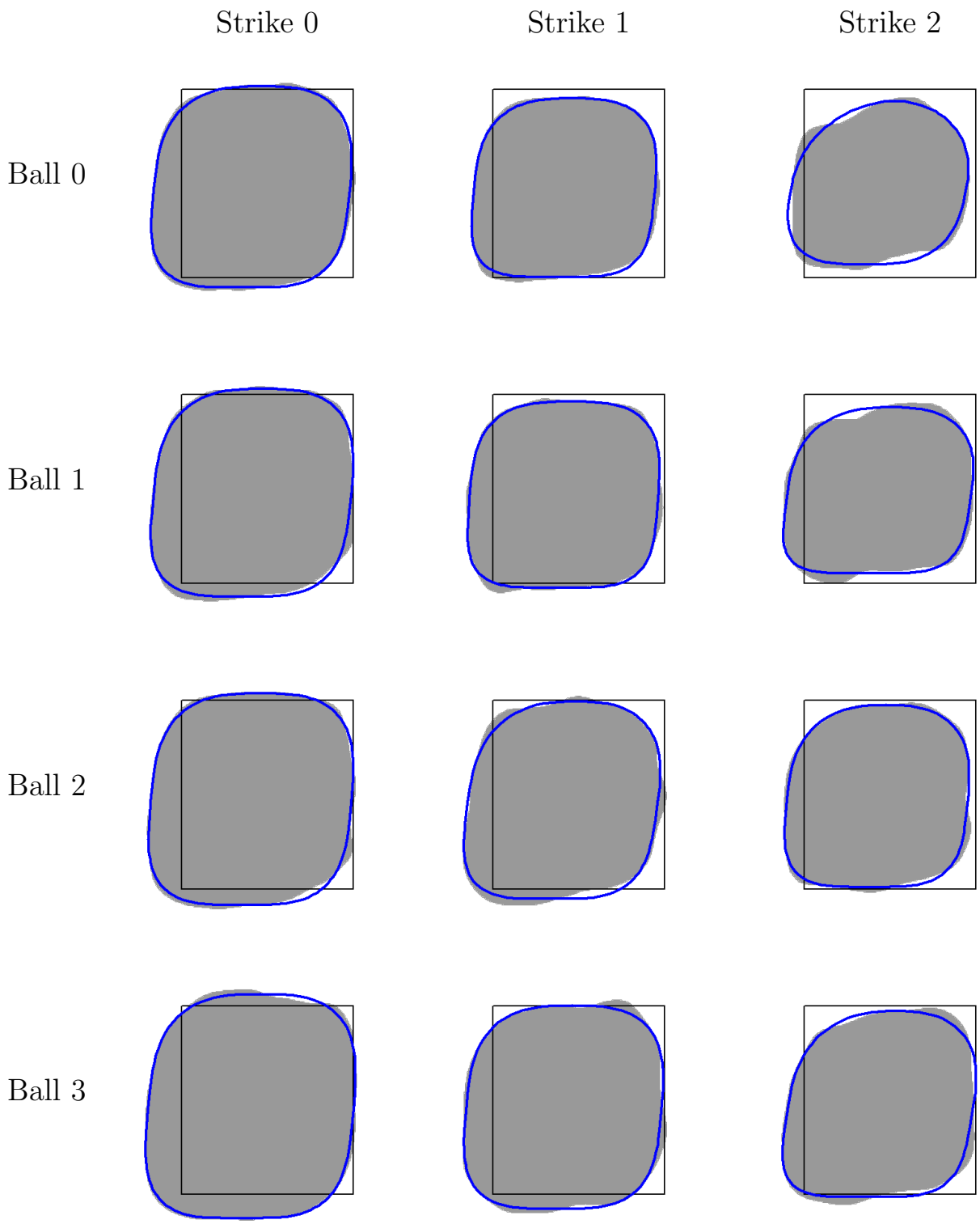

Figure 7: Called strike zones for the 12 Ball-Strike combinations for RHP-LHB-Away.

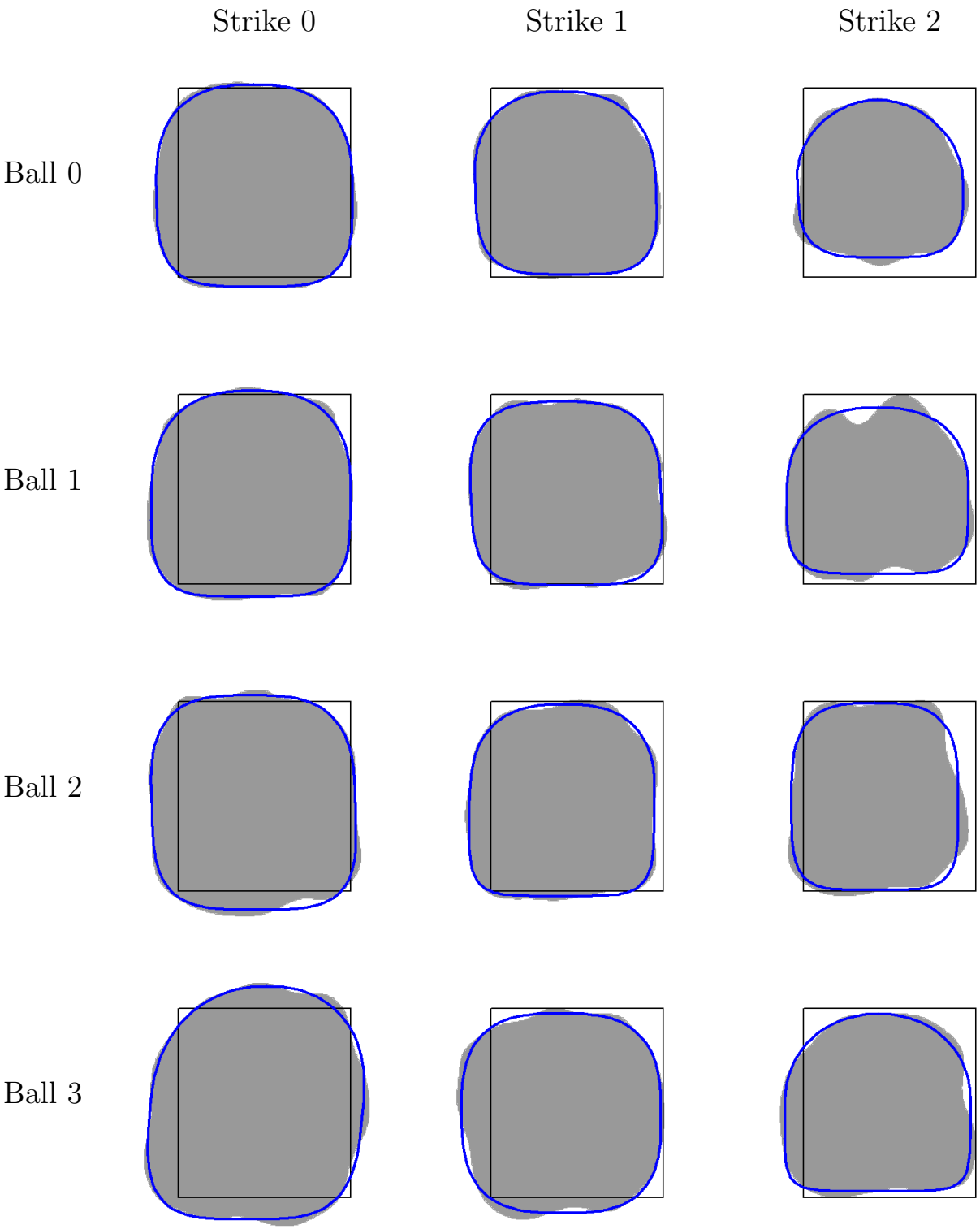

Figure 8: Called strike zones for the 12 Ball-Strike combinations for LHP-LHB-Away.

2 Estimated ATLAS coefficients corresponding to the 96 factor combinations

	npts	nobs	$\hat{x}_0$	$\hat{y}_0$	$\hat{a}$	$\hat{b}$	$\hat{A}$	$\hat{E}$	$\hat{r}_1$	$\hat{r}_2$	$\hat{s}$
group1	510	76414	-0.027	2.486	0.993	0.974	3.474	1.019	1.391	1.999	-0.022
group2	492	28736	-0.028	2.484	0.932	0.877	3.015	1.063	1.545	2.668	-0.041
group3	437	14151	-0.035	2.500	0.880	0.831	2.592	1.059	1.327	1.811	-0.019
group4	515	22038	-0.023	2.474	1.015	0.982	3.536	1.033	1.316	1.868	-0.025
group5	507	18857	-0.022	2.493	0.966	0.935	3.280	1.033	1.358	2.608	-0.034
group6	468	16916	-0.043	2.499	0.923	0.819	2.766	1.127	1.443	2.597	-0.030
group7	507	6758	-0.018	2.463	1.023	1.014	3.683	1.009	1.346	1.796	0.007
group8	510	7878	-0.017	2.464	0.978	0.936	3.332	1.044	1.517	2.151	-0.007
group9	471	11108	-0.039	2.503	0.931	0.884	2.981	1.053	1.295	2.718	-0.029
group10	520	3091	-0.048	2.445	1.072	1.118	4.108	0.959	1.301	1.334	0.012
group11	464	3145	-0.016	2.466	0.960	1.007	3.364	0.953	1.282	1.548	-0.031
group12	482	4754	-0.039	2.454	0.932	0.905	3.056	1.031	1.644	1.795	-0.072
group13	475	37635	-0.013	2.492	0.998	0.982	3.456	1.017	1.355	1.654	0.075
group14	484	13426	-0.025	2.494	0.920	0.874	2.958	1.053	1.528	2.617	0.033
group15	417	6393	-0.055	2.540	0.871	0.774	2.425	1.125	1.195	2.935	-0.009
group16	510	11911	-0.009	2.478	1.011	0.981	3.539	1.030	1.477	1.695	0.060
group17	480	9305	-0.030	2.508	0.940	0.912	3.057	1.031	1.222	2.295	0.050
group18	429	7913	-0.032	2.532	0.901	0.803	2.617	1.123	1.300	2.632	0.014
group19	504	3961	-0.027	2.488	1.024	1.025	3.686	0.999	1.287	1.704	0.063
group20	502	4331	-0.007	2.487	0.953	0.935	3.255	1.020	1.511	2.305	0.049
group21	475	5549	-0.054	2.486	0.912	0.845	2.853	1.079	1.507	3.184	0.013
group22	525	1943	-0.018	2.500	1.080	1.146	4.191	0.943	1.090	1.494	0.101
group23	454	1850	-0.057	2.499	1.027	1.019	3.482	1.009	0.970	1.536	0.107
group24	448	2550	-0.087	2.445	0.926	0.911	2.927	1.017	1.239	1.569	0.072
group25	493	71827	-0.149	2.476	0.959	0.987	3.386	0.971	1.466	1.760	-0.094
group26	437	25285	-0.133	2.479	0.894	0.900	2.866	0.994	1.311	1.967	-0.081
group27	414	11461	-0.084	2.516	0.870	0.790	2.456	1.101	1.201	2.563	-0.147
group28	493	23687	-0.152	2.470	0.971	0.998	3.441	0.973	1.332	1.826	-0.087
group29	461	18133	-0.133	2.474	0.928	0.927	3.069	1.001	1.322	1.988	-0.090
group30	405	14706	-0.094	2.491	0.868	0.837	2.579	1.036	1.321	1.852	-0.128
group31	516	8008	-0.173	2.451	0.968	1.034	3.587	0.937	1.508	1.734	-0.100
group32	475	8448	-0.144	2.454	0.935	0.951	3.176	0.983	1.405	1.823	-0.092
group33	429	10414	-0.092	2.504	0.922	0.899	2.863	1.026	1.106	1.814	-0.164
group34	515	3956	-0.152	2.450	1.006	1.074	3.887	0.937	1.723	1.563	-0.094
group35	477	3908	-0.157	2.439	0.957	0.987	3.336	0.969	1.441	1.560	-0.085
group36	458	4934	-0.097	2.485	0.899	0.938	3.000	0.959	1.270	2.058	-0.140
group37	498	15914	-0.108	2.492	0.958	0.988	3.366	0.970	1.335	1.871	0.019
group38	436	5905	-0.122	2.481	0.897	0.887	2.826	1.012	1.264	2.029	0.021
group39	350	2877	-0.138	2.491	0.852	0.754	2.198	1.129	1.240	1.385	0.027
group40	484	4891	-0.132	2.471	0.979	0.988	3.421	0.990	1.369	1.691	0.006
group41	456	3987	-0.135	2.512	0.902	0.951	2.929	0.949	1.088	1.627	0.065
group42	427	3530	-0.108	2.494	0.839	0.817	2.435	1.027	1.085	3.146	0.048
group43	494	1544	-0.128	2.493	1.013	1.087	3.777	0.932	1.220	1.431	0.040
group44	488	1788	-0.138	2.463	0.905	0.941	3.123	0.962	1.488	2.596	0.033
group45	413	2330	-0.114	2.498	0.882	0.900	2.630	0.980	0.836	2.078	0.029
group46	488	710	-0.123	2.427	1.047	1.133	3.997	0.924	1.281	1.190	-0.001
group47	507	781	-0.110	2.446	1.021	1.020	3.635	1.001	1.214	1.727	0.008
group48	456	1081	-0.114	2.479	0.877	0.954	3.055	0.920	1.499	2.295	0.116

	npts	nobs	$\hat{x}_0$	$\hat{y}_0$	$\hat{a}$	$\hat{b}$	$\hat{A}$	$\hat{E}$	$\hat{r}_1$	$\hat{r}_2$	$\hat{s}$
group49	502	73752	-0.021	2.491	0.983	0.972	3.412	1.011	1.372	1.906	-0.010
group50	468	27057	-0.019	2.487	0.920	0.872	2.950	1.055	1.554	2.471	-0.033
group51	427	13031	-0.043	2.525	0.875	0.799	2.579	1.095	1.429	3.240	0.020
group52	514	22322	-0.026	2.473	0.999	0.968	3.519	1.031	1.540	2.070	-0.024
group53	488	18143	-0.016	2.474	0.947	0.904	3.093	1.048	1.342	2.373	-0.038
group54	444	15805	-0.048	2.503	0.916	0.844	2.744	1.085	1.221	2.149	0.009
group55	512	7199	-0.021	2.458	1.016	1.022	3.683	0.994	1.554	1.504	0.011
group56	475	8022	-0.014	2.456	0.949	0.918	3.178	1.033	1.714	1.864	-0.045
group57	461	10564	-0.043	2.505	0.931	0.867	2.877	1.074	1.254	2.133	0.014
group58	564	3299	-0.057	2.455	1.039	1.068	4.080	0.973	1.910	1.859	0.010
group59	487	3386	-0.030	2.452	0.971	0.964	3.296	1.008	1.286	1.739	0.019
group60	469	4612	-0.008	2.467	0.907	0.916	2.942	0.990	1.404	1.651	-0.007
group61	502	36817	-0.012	2.488	0.982	0.993	3.483	0.990	1.461	1.740	0.061
group62	475	13372	-0.021	2.483	0.915	0.860	2.873	1.065	1.518	2.238	0.040
group63	406	6034	-0.086	2.529	0.887	0.794	2.380	1.118	0.907	2.126	0.135
group64	513	11570	-0.010	2.482	0.997	0.987	3.521	1.009	1.445	1.802	0.056
group65	481	9290	-0.027	2.485	0.941	0.915	3.094	1.028	1.347	2.151	0.063
group66	438	7652	-0.050	2.501	0.913	0.794	2.579	1.150	1.143	2.687	0.018
group67	537	4044	-0.044	2.482	1.031	1.006	3.680	1.024	1.327	1.831	0.040
group68	454	4257	-0.019	2.455	0.937	0.925	3.086	1.013	1.352	1.871	0.063
group69	455	5473	-0.038	2.485	0.935	0.847	2.802	1.104	1.201	2.122	0.083
group70	522	2045	-0.026	2.505	1.059	1.052	3.906	1.008	1.105	2.223	0.041
group71	465	1936	-0.016	2.445	0.972	1.008	3.347	0.964	1.370	1.236	0.012
group72	447	2558	-0.046	2.464	0.948	0.883	2.935	1.075	1.467	1.429	0.001
group73	485	68701	-0.150	2.483	0.950	0.978	3.310	0.971	1.409	1.775	-0.095
group74	446	23820	-0.138	2.474	0.875	0.868	2.737	1.008	1.439	1.976	-0.086
group75	384	10631	-0.101	2.522	0.849	0.793	2.250	1.071	1.000	1.512	-0.184
group76	512	23214	-0.146	2.478	0.957	1.007	3.444	0.950	1.451	1.763	-0.099
group77	470	17393	-0.144	2.459	0.913	0.902	2.987	1.012	1.524	2.019	-0.073
group78	430	13937	-0.101	2.502	0.890	0.805	2.559	1.105	1.284	2.128	-0.139
group79	521	8004	-0.159	2.472	0.971	1.026	3.565	0.947	1.491	1.715	-0.087
group80	454	8305	-0.154	2.462	0.920	0.957	3.116	0.962	1.317	1.798	-0.129
group81	445	10108	-0.121	2.502	0.876	0.881	2.737	0.994	1.304	1.874	-0.095
group82	539	4095	-0.160	2.456	0.995	1.087	3.878	0.915	1.489	1.761	-0.085
group83	505	3958	-0.145	2.446	0.946	0.986	3.334	0.959	1.473	1.714	-0.083
group84	459	4844	-0.088	2.475	0.896	0.903	2.875	0.993	1.264	2.028	-0.156
group85	475	15699	-0.099	2.485	0.946	0.978	3.271	0.967	1.287	1.845	0.012
group86	447	5661	-0.113	2.512	0.873	0.884	2.710	0.987	1.203	1.877	0.038
group87	351	2574	-0.096	2.553	0.798	0.763	2.058	1.046	0.969	1.818	0.029
group88	476	5006	-0.131	2.474	0.960	0.994	3.391	0.966	1.269	2.025	-0.013
group89	442	4045	-0.108	2.475	0.917	0.885	2.937	1.036	1.517	1.974	0.041
group90	421	3216	-0.117	2.501	0.875	0.802	2.500	1.092	1.213	2.326	-0.001
group91	509	1622	-0.108	2.457	0.984	1.034	3.674	0.951	1.539	1.853	0.030
group92	488	1849	-0.146	2.477	0.893	0.924	2.985	0.966	1.323	2.558	-0.015
group93	462	2282	-0.143	2.513	0.803	0.901	2.653	0.892	1.576	2.260	0.000
group94	503	781	-0.074	2.516	1.027	1.126	4.001	0.911	1.127	1.771	-0.087
group95	466	804	-0.153	2.417	0.962	0.965	3.256	0.996	1.439	1.468	0.008
group96	448	1073	-0.116	2.520	0.895	0.857	2.711	1.045	1.057	3.088	-0.008

Table 1: Number of points on the outline (npts), number of called pitches (nobs), and estimated ATLAS coefficients (plus area and eccentricity) for the 96 combinations of player attribute and game situation factors, for the combined 2014-2016 data.

3 MANOVA table, ANOVA tables and standard errors in Section 6

3.1 MANOVA table

```
manova <- manova(cbind(Xo, Yo, a, b, A, E, r1, r2, s) ~ batter + pitcher + ball + strike +
                 inning + batter * pitcher + batter * ball + batter * strike +
                 batter * inning + pitcher * ball + pitcher * strike + pitcher * inning +
                 ball * strike + ball * inning + strike * inning,  weights = nobs ^ (2 / 3),
                 data = atlas_96)
summary(manova, test = "Wilks")
              Df   Wilks approx F num Df den Df    Pr(>F)
batter         1 0.01688   356.02      9  55.00 < 2.2e-16 ***
pitcher        1 0.10118    54.29      9  55.00 < 2.2e-16 ***
ball           3 0.06146     9.55     27 161.27 < 2.2e-16 ***
strike         2 0.00325   101.13     18 110.00 < 2.2e-16 ***
inning         1 0.48488     6.49      9  55.00 3.081e-06 ***
batter:pitcher 1 0.50108     6.08      9  55.00 6.857e-06 ***
batter:ball    3 0.61692     1.07     27 161.27    0.3765
batter:strike  2 0.16673     8.86     18 110.00 2.596e-14 ***
batter:inning  1 0.86906     0.92      9  55.00    0.5144
pitcher:ball   3 0.60268     1.13     27 161.27    0.3111
pitcher:strike 2 0.34730     4.26     18 110.00 9.148e-07 ***
pitcher:inning 1 0.85442     1.04      9  55.00    0.4201
ball:strike    6 0.08411     3.30     54 285.04 5.119e-11 ***
ball:inning    3 0.70554     0.76     27 161.27    0.7994
strike:inning  2 0.65417     1.44     18 110.00    0.1253
Residuals     63
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

3.2 ANOVA tables

```
## Xo--------------------------------------------------------------------------------------
anova_Xo <- anova(lm(Xo ~ batter + pitcher + ball + strike + inning + batter * pitcher +
                     batter * ball + batter * strike + batter * inning + pitcher * ball +
                     pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                     strike * inning,  weights = nobs ^ (2 / 3), data = atlas_96))
anova_Xo
              Df  Sum Sq Mean Sq   F value    Pr(>F)
batter         1 113.105 113.105 2021.8609 < 2.2e-16 ***
pitcher        1   0.433   0.433    7.7423 0.0071093 **
ball           3   0.196   0.065    1.1696 0.3284491
strike         2   0.204   0.102    1.8223 0.1700771
inning         1   0.000   0.000    0.0000 0.9978622
batter:pitcher 1   0.676   0.676   12.0801 0.0009283 ***
batter:ball    3   0.217   0.072    1.2936 0.2844149
batter:strike  2   7.623   3.812   68.1367 < 2.2e-16 ***
batter:inning  1   0.010   0.010    0.1789 0.6737954
pitcher:ball   3   0.218   0.073    1.3013 0.2818665
pitcher:strike 2   2.269   1.135   20.2831 1.584e-07 ***
pitcher:inning 1   0.022   0.022    0.3853 0.5370298
ball:strike    6   0.301   0.050    0.8976 0.5023474
ball:inning    3   0.149   0.050    0.8905 0.4510283
strike:inning  2   0.100   0.050    0.8904 0.4155763
Residuals     63   3.524   0.056
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## Yo--------------------------------------------------------------------------------
anova_Yo <- anova(lm(Yo ~ batter + pitcher + ball + strike + inning + batter * pitcher +
                     batter * ball + batter * strike + batter * inning + pitcher * ball +
                     pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                     strike * inning,  weights = nobs ^ (2 / 3), data = atlas_96))
anova_Yo
               Df Sum Sq Mean Sq F value    Pr(>F)
batter          1 0.3463  0.3463  5.9044 0.0179643 *
pitcher         1 0.7005  0.7005 11.9429 0.0009874 ***
ball            3 3.2119  1.0706 18.2532 1.233e-08 ***
strike          2 6.4068  3.2034 54.6153 1.745e-14 ***
inning          1 0.0014  0.0014  0.0242 0.8768151
batter:pitcher  1 0.0058  0.0058  0.0983 0.7549609
batter:ball     3 0.1053  0.0351  0.5984 0.6184404
batter:strike   2 0.1974  0.0987  1.6825 0.1941460
batter:inning   1 0.1601  0.1601  2.7301 0.1034474
pitcher:ball    3 0.0650  0.0217  0.3696 0.7752177
pitcher:strike  2 0.0677  0.0338  0.5767 0.5646680
pitcher:inning  1 0.0769  0.0769  1.3110 0.2565476
ball:strike     6 0.7883  0.1314  2.2401 0.0505946 .
ball:inning     3 0.2093  0.0698  1.1897 0.3208998
strike:inning   2 0.4280  0.2140  3.6483 0.0316821 *
Residuals      63 3.6952  0.0587
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## a---------------------------------------------------------------------------------
anova_a <- anova(lm(a ~ batter + pitcher + ball + strike + inning + batter * pitcher +
                    batter * ball + batter * strike + batter * inning + pitcher * ball +
                    pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                    strike * inning,  weights = nobs ^ (2 / 3), data = atlas_96))
anova_a
               Df Sum Sq Mean Sq  F value    Pr(>F)
batter          1 13.721  13.721 218.7018 < 2.2e-16 ***
pitcher         1  0.146   0.146   2.3202  0.132707
ball            3  4.300   1.433  22.8447 3.987e-10 ***
strike          2 75.599  37.799 602.5111 < 2.2e-16 ***
inning          1  1.261   1.261  20.1073 3.165e-05 ***
batter:pitcher  1  0.057   0.057   0.9039  0.345384
batter:ball     3  0.097   0.032   0.5138  0.674281
batter:strike   2  0.071   0.036   0.5690  0.568984
batter:inning   1  0.039   0.039   0.6178  0.434809
pitcher:ball    3  0.815   0.272   4.3321  0.007708 **
pitcher:strike  2  0.626   0.313   4.9921  0.009716 **
pitcher:inning  1  0.015   0.015   0.2438  0.623191
ball:strike     6  1.059   0.176   2.8124  0.017286 *
ball:inning     3  0.128   0.043   0.6800  0.567540
strike:inning   2  0.163   0.081   1.2964  0.280717
Residuals      63  3.952   0.063
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## b---------------------------------------------------------------------------------
anova_b <- anova(lm(b ~ batter + pitcher + ball + strike + inning + batter * pitcher +
                    batter * ball + batter * strike + batter * inning + pitcher * ball +
                    pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                    strike * inning,  weights = nobs ^ (2 / 3), data = atlas_96))
```

```
anova_b
               Df  Sum Sq Mean Sq   F value    Pr(>F)
batter          1   1.022   1.022   11.7530 0.0010757 **
pitcher         1   0.068   0.068    0.7800 0.3805060
ball            3  23.625   7.875   90.5910 < 2.2e-16 ***
strike          2 210.562 105.281 1211.1239 < 2.2e-16 ***
inning          1   1.146   1.146   13.1825 0.0005684 ***
batter:pitcher  1   0.007   0.007    0.0855 0.7709712
batter:ball     3   0.618   0.206    2.3694 0.0789849 .
batter:strike   2   0.184   0.092    1.0589 0.3529111
batter:inning   1   0.104   0.104    1.1925 0.2789899
pitcher:ball    3   0.187   0.062    0.7158 0.5462030
pitcher:strike  2   1.557   0.779    8.9582 0.0003768 ***
pitcher:inning  1   0.003   0.003    0.0389 0.8442119
ball:strike     6   2.656   0.443    5.0932 0.0002564 ***
ball:inning     3   0.378   0.126    1.4484 0.2372055
strike:inning   2   0.295   0.148    1.6971 0.1914748
Residuals      63   5.476   0.087
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## A-----------------------------------------------------------------------------------------
anova_A <- anova(lm(A ~ batter + pitcher + ball + strike + inning + batter * pitcher +
                    batter * ball + batter * strike + batter * inning + pitcher * ball +
                    pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                    strike * inning,  weights = nobs ^ (2 / 3), data = atlas_96))
anova_A
               Df Sum Sq Mean Sq   F value    Pr(>F)
batter          1  138.6  138.56  137.3724 < 2.2e-16 ***
pitcher         1   24.5   24.48   24.2668 6.383e-06 ***
ball            3  405.5  135.15  133.9920 < 2.2e-16 ***
strike          2 5907.5 2953.77 2928.3977 < 2.2e-16 ***
inning          1   46.2   46.16   45.7627 5.127e-09 ***
batter:pitcher  1    0.0    0.00    0.0002   0.98902
batter:ball     3    6.0    2.00    1.9844   0.12534
batter:strike   2    5.9    2.96    2.9369   0.06033 .
batter:inning   1    1.1    1.11    1.1032   0.29757
pitcher:ball    3    6.4    2.14    2.1193   0.10664
pitcher:strike  2   37.3   18.67   18.5125 4.740e-07 ***
pitcher:inning  1    0.7    0.67    0.6684   0.41669
ball:strike     6   61.4   10.24   10.1496 7.840e-08 ***
ball:inning     3    4.6    1.52    1.5038   0.22218
strike:inning   2    6.8    3.42    3.3912   0.03993 *
Residuals      63   63.5    1.01
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## E-----------------------------------------------------------------------------------------
anova_E <- anova(lm(E ~ batter + pitcher + ball + strike + inning + batter * pitcher +
                    batter * ball + batter * strike + batter * inning +  pitcher * ball +
                    pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                    strike * inning, weights = nobs ^ (2 / 3), data = atlas_96))
anova_E
               Df Sum Sq Mean Sq  F value    Pr(>F)
batter          1 25.425 25.4254 111.0077 1.569e-15 ***
pitcher         1  0.000  0.0002   0.0008   0.97729
ball            3 11.224  3.7414  16.3348 5.806e-08 ***
strike          2 48.174 24.0872 105.1649 < 2.2e-16 ***
```

```
inning          1  0.000  0.0002   0.0009   0.97636
batter:pitcher  1  0.183  0.1833   0.8001   0.37445
batter:ball     3  1.418  0.4727   2.0638   0.11397
batter:strike   2  0.102  0.0511   0.2231   0.80067
batter:inning   1  0.019  0.0190   0.0828   0.77453
pitcher:ball    3  0.347  0.1156   0.5046   0.68050
pitcher:strike  2  0.565  0.2823   1.2324   0.29854
pitcher:inning  1  0.003  0.0029   0.0127   0.91071
ball:strike     6  3.231  0.5385   2.3511   0.04113 *
ball:inning     3  0.557  0.1856   0.8103   0.49293
strike:inning   2  0.338  0.1688   0.7370   0.48261
Residuals      63 14.430  0.2290
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## r1--------------------------------------------------------------------------------
anova_r1 <- anova(lm(r1 ~ batter + pitcher + ball + strike + inning + batter * pitcher +
                     batter * ball + batter * strike + batter * inning + pitcher * ball +
                     pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                     strike * inning,  weights = nobs ^ (2 / 3), data = atlas_96))
anova_r1
```

```
               Df Sum Sq Mean Sq F value    Pr(>F)
batter          1  22.22  22.219  3.0538   0.08542 .
pitcher         1  67.44  67.436  9.2683   0.00340 **
ball            3   5.53   1.843  0.2533   0.85868
strike          2 186.40  93.201 12.8095 2.148e-05 ***
inning          1   4.83   4.825  0.6631   0.41852
batter:pitcher  1   1.75   1.746  0.2400   0.62590
batter:ball     3  12.63   4.209  0.5785   0.63126
batter:strike   2  31.47  15.737  2.1628   0.12346
batter:inning   1   0.07   0.070  0.0096   0.92224
pitcher:ball    3  21.45   7.149  0.9825   0.40678
pitcher:strike  2   0.46   0.229  0.0315   0.96903
pitcher:inning  1   0.49   0.491  0.0675   0.79583
ball:strike     6  70.36  11.727  1.6117   0.15866
ball:inning     3  11.25   3.748  0.5152   0.67332
strike:inning   2  35.23  17.614  2.4209   0.09706 .
Residuals      63 458.38   7.276
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## r2--------------------------------------------------------------------------------
anova_r2 <- anova(lm(r2 ~ batter + pitcher + ball + strike + inning + batter * pitcher +
                     batter * ball + batter * strike + batter * inning + pitcher * ball +
                     pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                     strike * inning,  weights = nobs ^ (2 / 3), data = atlas_96))
anova_r2
```

```
               Df  Sum Sq Mean Sq F value    Pr(>F)
batter          1  534.76  534.76 14.3733 0.0003381 ***
pitcher         1    0.68    0.68  0.0182 0.8931279
ball            3  478.97  159.66  4.2913 0.0080821 **
strike          2 1757.27  878.64 23.6161 2.221e-08 ***
inning          1   23.75   23.75  0.6384 0.4272796
batter:pitcher  1   94.57   94.57  2.5418 0.1158715
batter:ball     3  242.42   80.81  2.1720 0.1001053
batter:strike   2  276.21  138.11  3.7121 0.0299229 *
batter:inning   1   14.94   14.94  0.4016 0.5285675
pitcher:ball    3  145.43   48.48  1.3030 0.2813213
```

```
pitcher:strike  2   88.82   44.41  1.1937 0.3098550
pitcher:inning  1    2.17    2.17  0.0585 0.8097401
ball:strike     6  199.33   33.22  0.8930 0.5056415
ball:inning     3  101.99   34.00  0.9137 0.4394868
strike:inning   2   80.99   40.50  1.0884 0.3429900
Residuals      63 2343.91   37.20
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## s-------------------------------------------------------------------------------
anova_s <- anova(lm(s ~ batter + pitcher + ball + strike + inning + batter * pitcher +
                    batter * ball + batter * strike + batter * inning + pitcher * ball +
                    pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                    strike * inning,  weights = nobs ^ (2 / 3), data = atlas_96))
anova_s
               Df Sum Sq Mean Sq  F value    Pr(>F)
batter          1 70.215  70.215 257.4879 < 2.2e-16 ***
pitcher         1 82.801  82.801 303.6427 < 2.2e-16 ***
ball            3  0.109   0.036   0.1332  0.939940
strike          2  2.271   1.136   4.1644  0.020016 *
inning          1  0.113   0.113   0.4140  0.522294
batter:pitcher  1  6.753   6.753  24.7637 5.302e-06 ***
batter:ball     3  0.659   0.220   0.8058  0.495336
batter:strike   2  3.730   1.865   6.8389  0.002052 **
batter:inning   1  0.839   0.839   3.0775  0.084243 .
pitcher:ball    3  0.102   0.034   0.1243  0.945415
pitcher:strike  2  0.342   0.171   0.6266  0.537687
pitcher:inning  1  0.450   0.450   1.6512  0.203504
ball:strike     6  0.944   0.157   0.5769  0.747248
ball:inning     3  0.459   0.153   0.5612  0.642584
strike:inning   2  1.345   0.673   2.4669  0.093006 .
Residuals      63 17.180   0.273
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

3.3 Standard errors of weighted level means

Factor	Levels	$\hat{x}_0$	$\hat{y}_0$	$\hat{a}$	$\hat{b}$	$\hat{A}$	$\hat{E}$	$\hat{r}_1$	$\hat{r}_2$	$\hat{s}$
Pitcher	RHP	0.001	0.001	0.001	0.002	0.006	0.003	0.016	0.036	0.003
	LHP	0.002	0.002	0.002	0.002	0.008	0.004	0.022	0.050	0.004
Batter	RHB	0.002	0.002	0.002	0.002	0.006	0.003	0.017	0.039	0.003
	LHB	0.002	0.002	0.002	0.002	0.007	0.003	0.019	0.043	0.004
Venue	Home	0.002	0.002	0.002	0.002	0.007	0.003	0.018	0.041	0.004
	Away	0.002	0.002	0.002	0.002	0.007	0.003	0.018	0.041	0.004
Ball	0	0.002	0.002	0.002	0.002	0.007	0.003	0.020	0.044	0.004
	1	0.002	0.002	0.002	0.003	0.009	0.004	0.024	0.054	0.005
	2	0.003	0.003	0.003	0.003	0.011	0.005	0.031	0.070	0.006
	3	0.004	0.004	0.004	0.004	0.015	0.007	0.040	0.090	0.008
Strike	0	0.002	0.002	0.002	0.002	0.007	0.003	0.020	0.044	0.004
	1	0.002	0.002	0.002	0.003	0.009	0.004	0.023	0.053	0.005
	2	0.002	0.002	0.002	0.003	0.009	0.004	0.025	0.057	0.005

Batter×Strike:	Batter	Strike	$\hat{x}_0$
	RHB	0	0.002
	RHB	1	0.003
	RHB	2	0.003
	LHB	0	0.003
	LHB	1	0.003
	LHB	2	0.003

Batter×Pitcher:	Batter	Pitcher	$\hat{s}$
	RHB	RHP	0.004
	RHB	LHP	0.005
	LHB	RHP	0.004
	LHB	LHP	0.007

Table 2: Standard errors associated with weighted level means of estimated ATLAS coefficients, plus area and eccentricity, corresponding to each player attribute and game situation factor and selected two-factor combinations, for the combined 2014–2016 data.

4 Similar results for a model that includes year

4.1 MANOVA table

```
manova1 <- manova(cbind(Xo, Yo, a, b, A, E, r1, r2, s) ~ batter + pitcher + ball + strike +
                  inning + year + batter * pitcher + batter * ball + batter * strike +
                  batter * inning + pitcher * ball + pitcher * strike + pitcher * inning +
                  ball * strike + ball * inning + strike * inning + year * batter +
                  year * pitcher + year * ball + year * strike + year * inning,
                  weights = nobs ^ (2 / 3), data = atlas_3yr)
summary(manova1, test = "Wilks")
               Df   Wilks approx F num Df  den Df    Pr(>F)
batter          1 0.07723  304.030      9  229.00 < 2.2e-16 ***
pitcher         1 0.38807   40.123      9  229.00 < 2.2e-16 ***
ball            3 0.24406   15.391     27  669.44 < 2.2e-16 ***
strike          2 0.02434  137.639     18  458.00 < 2.2e-16 ***
inning          1 0.83469    5.039      9  229.00 3.288e-06 ***
year            2 0.57990    7.969     18  458.00 < 2.2e-16 ***
batter:pitcher  1 0.84228    4.765      9  229.00 7.937e-06 ***
batter:ball     3 0.86603    1.252     27  669.44    0.1786
batter:strike   2 0.48065   11.257     18  458.00 < 2.2e-16 ***
batter:inning   1 0.97245    0.721      9  229.00    0.6896
pitcher:ball    3 0.87189    1.192     27  669.44    0.2318
pitcher:strike  2 0.64535    6.229     18  458.00 9.479e-14 ***
pitcher:inning  1 0.95422    1.221      9  229.00    0.2830
ball:strike     6 0.43085    3.898     54 1172.27 < 2.2e-16 ***
```

```
ball:inning      3 0.92082    0.710     27  669.44    0.8604
strike:inning    2 0.92652    0.990     18  458.00    0.4702
batter:year      2 0.79273    3.133     18  458.00 1.766e-05 ***
pitcher:year     2 0.89527    1.447     18  458.00    0.1050
ball:year        6 0.88993    0.502     54 1172.27    0.9990
strike:year      4 0.82015    1.298     36  859.91    0.1151
inning:year      2 0.95806    0.551     18  458.00    0.9325
Residuals      237
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

4.2 ANOVA tables

```
## Xo--------------------------------------------------------------------------------------------
anova_Xo <- anova(lm(Xo ~ batter + pitcher + ball + strike + inning + year + batter * pitcher +
                         batter * ball + batter * strike + batter * inning + pitcher * ball +
                         pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                         strike * inning + year * batter + year * pitcher + year * ball +
                         year * strike + year * inning,  weights = nobs ^ (2 / 3),
                         data = atlas_3yr))
anova_Xo
               Df  Sum Sq Mean Sq   F value    Pr(>F)
batter          1 163.680 163.680 1839.3926 < 2.2e-16 ***
pitcher         1   0.025   0.025    0.2843  0.594375
ball            3   0.343   0.114    1.2852  0.280124
strike          2   0.181   0.091    1.0173  0.363126
inning          1   0.001   0.001    0.0113  0.915354
year            2   2.334   1.167   13.1123 3.972e-06 ***
batter:pitcher  1   0.919   0.919   10.3288  0.001492 **
batter:ball     3   0.385   0.128    1.4416  0.231334
batter:strike   2  13.041   6.520   73.2756 < 2.2e-16 ***
batter:inning   1   0.006   0.006    0.0687  0.793511
pitcher:ball    3   0.470   0.157    1.7589  0.155725
pitcher:strike  2   5.162   2.581   29.0043 5.398e-12 ***
pitcher:inning  1   0.053   0.053    0.5996  0.439493
ball:strike     6   0.367   0.061    0.6880  0.659505
ball:inning     3   0.218   0.073    0.8158  0.486221
strike:inning   2   0.070   0.035    0.3917  0.676349
batter:year     2   2.403   1.201   13.5013 2.800e-06 ***
pitcher:year    2   0.125   0.062    0.7021  0.496569
ball:year       6   0.224   0.037    0.4202  0.865211
strike:year     4   0.187   0.047    0.5244  0.717863
inning:year     2   0.267   0.133    1.4983  0.225610
Residuals     237  21.090   0.089
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Yo--------------------------------------------------------------------------------------------
anova_Yo <- anova(lm(Yo ~ batter + pitcher + ball + strike + inning + year + batter * pitcher +
                         batter * ball + batter * strike + batter * inning + pitcher * ball +
                         pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                         strike * inning + year * batter + year * pitcher + year * ball +
                         year * strike + year * inning,  weights = nobs ^ (2 / 3),
                         data = atlas_3yr))
anova_Yo
               Df Sum Sq Mean Sq F value    Pr(>F)
batter          1  0.505  0.5051  3.5511  0.060728 .
pitcher         1  1.327  1.3268  9.3287  0.002514 **
```

```
ball             3  4.144  1.3813  9.7120 4.543e-06 ***
strike           2  7.036  3.5178 24.7343 1.754e-10 ***
inning           1  0.033  0.0326  0.2291  0.632612
year             2  6.078  3.0391 21.3683 2.937e-09 ***
batter:pitcher   1  0.034  0.0338  0.2379  0.626170
batter:ball      3  0.087  0.0289  0.2032  0.894140
batter:strike    2  0.359  0.1793  1.2606  0.285379
batter:inning    1  0.127  0.1265  0.8896  0.346545
pitcher:ball     3  0.142  0.0472  0.3318  0.802390
pitcher:strike   2  0.062  0.0312  0.2192  0.803315
pitcher:inning   1  0.202  0.2017  1.4184  0.234862
ball:strike      6  1.108  0.1847  1.2988  0.258414
ball:inning      3  0.294  0.0980  0.6894  0.559339
strike:inning    2  0.598  0.2991  2.1032  0.124333
batter:year      2  0.068  0.0342  0.2407  0.786248
pitcher:year     2  0.012  0.0058  0.0410  0.959848
ball:year        6  0.295  0.0492  0.3460  0.911775
strike:year      4  0.143  0.0357  0.2510  0.908847
inning:year      2  0.075  0.0373  0.2621  0.769657
Residuals      237 33.707  0.1422
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## a ------------------------------------------------------------------------------------------
anova_a <- anova(lm(a ~ batter + pitcher + ball + strike + inning + year + batter * pitcher +
                      batter * ball + batter * strike + batter * inning + pitcher * ball +
                      pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                      strike * inning + year * batter + year * pitcher + year * ball +
                      year * strike + year * inning,  weights = nobs ^ (2 / 3),
                      data = atlas_3yr))
anova_a
               Df  Sum Sq Mean Sq  F value    Pr(>F)
batter          1  21.200  21.200 195.7706 < 2.2e-16 ***
pitcher         1   0.252   0.252   2.3250 0.1286427
ball            3  10.553   3.518  32.4853 < 2.2e-16 ***
strike          2 187.892  93.946 867.5606 < 2.2e-16 ***
inning          1   2.279   2.279  21.0480 7.262e-06 ***
year            2   4.823   2.412  22.2695 1.372e-09 ***
batter:pitcher  1   0.131   0.131   1.2131 0.2718450
batter:ball     3   0.032   0.011   0.0970 0.9616060
batter:strike   2   0.215   0.107   0.9906 0.3728957
batter:inning   1   0.080   0.080   0.7366 0.3916283
pitcher:ball    3   1.775   0.592   5.4636 0.0011989 **
pitcher:strike  2   2.674   1.337  12.3465 7.931e-06 ***
pitcher:inning  1   0.003   0.003   0.0254 0.8735979
ball:strike     6   2.913   0.486   4.4835 0.0002539 ***
ball:inning     3   0.269   0.090   0.8288 0.4791648
strike:inning   2   0.060   0.030   0.2773 0.7580356
batter:year     2   0.875   0.438   4.0408 0.0188096 *
pitcher:year    2   0.682   0.341   3.1479 0.0447412 *
ball:year       6   0.419   0.070   0.6447 0.6943486
strike:year     4   0.717   0.179   1.6564 0.1609142
inning:year     2   0.010   0.005   0.0464 0.9546650
Residuals     237  25.664   0.108
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## b ------------------------------------------------------------------------------------------
```

```
anova_b <- anova(lm(b ~ batter + pitcher + ball + strike + inning + year + batter * pitcher +
                        batter * ball + batter * strike + batter * inning + pitcher * ball +
                        pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                        strike * inning + year * batter + year * pitcher + year * ball +
                        year * strike + year * inning,  weights = nobs ^ (2 / 3),
                        data = atlas_3yr))
anova_b
               Df Sum Sq Mean Sq   F value    Pr(>F)
batter          1   1.82   1.821   10.1260 0.0016574 **
pitcher         1   0.00   0.001    0.0028 0.9578764
ball            3  47.26  15.752   87.6046 < 2.2e-16 ***
strike          2 425.69 212.846 1183.7218 < 2.2e-16 ***
inning          1   2.65   2.649   14.7347 0.0001588 ***
year            2   1.71   0.856    4.7617 0.0093862 **
batter:pitcher  1   0.39   0.389    2.1643 0.1425711
batter:ball     3   1.27   0.422    2.3496 0.0731667 .
batter:strike   2   0.76   0.379    2.1077 0.1237894
batter:inning   1   0.17   0.170    0.9446 0.3320773
pitcher:ball    3   0.66   0.219    1.2180 0.3038231
pitcher:strike  2   2.70   1.352    7.5197 0.0006818 ***
pitcher:inning  1   0.00   0.003    0.0147 0.9036013
ball:strike     6   5.78   0.963    5.3584 3.299e-05 ***
ball:inning     3   0.67   0.224    1.2451 0.2940702
strike:inning   2   0.44   0.218    1.2123 0.2993590
batter:year     2   0.22   0.109    0.6039 0.5474967
pitcher:year    2   0.70   0.350    1.9465 0.1450540
ball:year       6   0.60   0.101    0.5592 0.7625296
strike:year     4   0.12   0.031    0.1723 0.9524035
inning:year     2   0.48   0.242    1.3446 0.2626360
Residuals     237  42.62   0.180
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## A ---------------------------------------------------------------------------------------
anova_A <- anova(lm(A ~ batter + pitcher + ball + strike + inning + year + batter * pitcher +
                        batter * ball + batter * strike + batter * inning + pitcher * ball +
                        pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                        strike * inning + year * batter + year * pitcher + year * ball +
                        year * strike + year * inning,  weights = nobs ^ (2 / 3),
                        data = atlas_3yr))
anova_A
               Df  Sum Sq Mean Sq   F value    Pr(>F)
batter          1   211.8   211.8  114.1298 < 2.2e-16 ***
pitcher         1    48.3    48.3   26.0044 6.973e-07 ***
ball            3   866.0   288.7  155.5275 < 2.2e-16 ***
strike          2 13017.3  6508.6 3506.5677 < 2.2e-16 ***
inning          1    76.1    76.1   41.0131 8.048e-10 ***
year            2     6.3     3.2    1.6989  0.185103
batter:pitcher  1     8.4     8.4    4.5110  0.034713 *
batter:ball     3    11.6     3.9    2.0847  0.102904
batter:strike   2    18.8     9.4    5.0641  0.007021 **
batter:inning   1     2.0     2.0    1.0685  0.302331
pitcher:ball    3    20.7     6.9    3.7205  0.012113 *
pitcher:strike  2   118.2    59.1   31.8340 5.680e-13 ***
pitcher:inning  1     1.0     1.0    0.5363  0.464681
ball:strike     6   184.5    30.8   16.5703 6.134e-16 ***
ball:inning     3    12.3     4.1    2.2016  0.088563 .
strike:inning   2     9.8     4.9    2.6429  0.073251 .
```

```
batter:year      2    15.7     7.9    4.2340  0.015606 *
pitcher:year     2    17.4     8.7    4.6894  0.010062 *
ball:year        6     5.9     1.0    0.5297  0.785467
strike:year      4     1.0     0.3    0.1401  0.967188
inning:year      2     1.7     0.8    0.4475  0.639750
Residuals      237   439.9     1.9
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## E ---------------------------------------------------------------------------------------
anova_E <- anova(lm(E ~ batter + pitcher + ball + strike + inning + year + batter * pitcher +
                        batter * ball + batter * strike + batter * inning + pitcher * ball +
                        pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                        strike * inning + year * batter + year * pitcher + year * ball +
                        year * strike + year * inning, weights = nobs ^ (2 / 3),
                        data = atlas_3yr))
anova_E
               Df Sum Sq Mean Sq  F value    Pr(>F)
batter          1 38.849  38.849 101.3601 < 2.2e-16 ***
pitcher         1  0.116   0.116   0.3027  0.582737
ball            3 17.097   5.699  14.8688 6.683e-09 ***
strike          2 73.946  36.973  96.4648 < 2.2e-16 ***
inning          1  0.037   0.037   0.0973  0.755309
year            2 13.662   6.831  17.8218 6.161e-08 ***
batter:pitcher  1  0.068   0.068   0.1779  0.673575
batter:ball     3  1.834   0.611   1.5952  0.191218
batter:strike   2  0.448   0.224   0.5840  0.558445
batter:inning   1  0.011   0.011   0.0300  0.862705
pitcher:ball    3  0.626   0.209   0.5442  0.652505
pitcher:strike  2  0.328   0.164   0.4283  0.652099
pitcher:inning  1  0.005   0.005   0.0123  0.911701
ball:strike     6  6.935   1.156   3.0158  0.007362 **
ball:inning     3  0.814   0.271   0.7081  0.548036
strike:inning   2  0.694   0.347   0.9060  0.405550
batter:year     2  0.357   0.178   0.4652  0.628591
pitcher:year    2  0.381   0.191   0.4973  0.608814
ball:year       6  0.831   0.139   0.3615  0.902704
strike:year     4  1.026   0.256   0.6690  0.614088
inning:year     2  0.588   0.294   0.7669  0.465589
Residuals     237 90.838   0.383
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## r1--------------------------------------------------------------------------------------
anova_r1 <- anova(lm(r1 ~ batter + pitcher + ball + strike + inning + year + batter * pitcher +
                          batter * ball + batter * strike + batter * inning + pitcher * ball +
                          pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                          strike * inning + year * batter + year * pitcher + year * ball +
                          year * strike + year * inning,  weights = nobs ^ {2 / 3},
                          data = atlas_3yr))
anova_r1
               Df  Sum Sq Mean Sq F value    Pr(>F)
batter          1   10.95  10.947  0.8805 0.3490188
pitcher         1  121.82 121.823  9.7989 0.0019653 **
ball            3    1.86   0.620  0.0499 0.9852313
strike          2  202.13 101.064  8.1292 0.0003849 ***
inning          1   21.37  21.374  1.7192 0.1910588
year            2    8.15   4.076  0.3279 0.7207677
```

```
batter:pitcher   1    0.33   0.330  0.0265 0.8707051
batter:ball      3   19.49   6.496  0.5225 0.6671861
batter:strike    2   32.39  16.197  1.3028 0.2737001
batter:inning    1    0.00   0.003  0.0003 0.9867333
pitcher:ball     3   37.96  12.652  1.0177 0.3855220
pitcher:strike   2    1.94   0.970  0.0780 0.9249868
pitcher:inning   1    0.02   0.024  0.0020 0.9646523
ball:strike      6  106.12  17.687  1.4227 0.2065512
ball:inning      3   12.80   4.268  0.3433 0.7940371
strike:inning    2   16.12   8.059  0.6482 0.5238897
batter:year      2   17.50   8.752  0.7040 0.4956436
pitcher:year     2   32.39  16.195  1.3026 0.2737512
ball:year        6   27.30   4.551  0.3660 0.8999605
strike:year      4   10.37   2.592  0.2085 0.9335431
inning:year      2   19.48   9.739  0.7834 0.4580285
Residuals      237 2946.44  12.432
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## r2--------------------------------------------------------------------------------------------
anova_r2 <- anova(lm(r2 ~ batter + pitcher + ball + strike + inning + year + batter * pitcher +
                         batter * ball + batter * strike + batter * inning + pitcher * ball +
                         pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                         strike * inning + year * batter + year * pitcher + year * ball +
                         year * strike + year * inning,  weights = nobs ^ (2 / 3),
                         data = atlas_3yr))
anova_r2
               Df  Sum Sq Mean Sq F value    Pr(>F)
batter          1   805.6  805.58  8.7745  0.003366 **
pitcher         1    33.8   33.75  0.3676  0.544873
ball            3   598.0  199.34  2.1712  0.092092 .
strike          2  3604.6 1802.31 19.6310 1.292e-08 ***
inning          1     6.0    5.99  0.0652  0.798607
year            2    31.3   15.63  0.1702  0.843561
batter:pitcher  1    73.1   73.09  0.7961  0.373154
batter:ball     3   374.6  124.87  1.3601  0.255681
batter:strike   2   658.7  329.33  3.5872  0.029190 *
batter:inning   1    30.2   30.19  0.3289  0.566877
pitcher:ball    3   160.0   53.32  0.5808  0.628163
pitcher:strike  2   406.5  203.27  2.2141  0.111511
pitcher:inning  1     0.8    0.76  0.0082  0.927728
ball:strike     6   437.1   72.85  0.7935  0.575827
ball:inning     3   127.3   42.44  0.4623  0.708864
strike:inning   2    95.5   47.76  0.5203  0.595045
batter:year     2     1.4    0.71  0.0078  0.992261
pitcher:year    2   425.0  212.50  2.3146  0.101035
ball:year       6   210.9   35.14  0.3828  0.889625
strike:year     4   548.8  137.21  1.4945  0.204549
inning:year     2   113.6   56.82  0.6189  0.539395
Residuals      237 21758.7   91.81
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## s ---------------------------------------------------------------------------------------------
anova_s <- anova(lm(s ~ batter + pitcher + ball + strike + inning + year + batter * pitcher +
                        batter * ball + batter * strike + batter * inning + pitcher * ball +
                        pitcher * strike + pitcher * inning + ball * strike + ball * inning +
                        strike * inning + year * batter + year * pitcher + year * ball +
```

```
                    year * strike + year * inning,  weights = nobs ^ (2 / 3),
                    data = atlas_3yr))
anova_s
               Df  Sum Sq Mean Sq  F value    Pr(>F)
batter          1 101.675 101.675 204.7124 < 2.2e-16 ***
pitcher         1 143.915 143.915 289.7566 < 2.2e-16 ***
ball            3   0.824   0.275   0.5533   0.64639
strike          2   2.663   1.332   2.6813   0.07055 .
inning          1   0.227   0.227   0.4571   0.49966
year            2   1.015   0.508   1.0220   0.36144
batter:pitcher  1  10.362  10.362  20.8631 7.935e-06 ***
batter:ball     3   1.292   0.431   0.8673   0.45871
batter:strike   2   4.745   2.373   4.7769   0.00925 **
batter:inning   1   1.285   1.285   2.5874   0.10905
pitcher:ball    3   0.209   0.070   0.1400   0.93594
pitcher:strike  2   2.137   1.069   2.1514   0.11859
pitcher:inning  1   0.909   0.909   1.8296   0.17747
ball:strike     6   3.000   0.500   1.0067   0.42158
ball:inning     3   0.345   0.115   0.2313   0.87454
strike:inning   2   1.431   0.716   1.4407   0.23882
batter:year     2   3.159   1.579   3.1801   0.04336 *
pitcher:year    2   2.338   1.169   2.3541   0.09720 .
ball:year       6   2.963   0.494   0.9944   0.42969
strike:year     4   2.630   0.657   1.3236   0.26182
inning:year     2   1.077   0.538   1.0838   0.33999
Residuals     237 117.712   0.497
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

4.3 Weighted level means

Factor	Levels	$\hat{x}_0$	$\hat{y}_0$	$\hat{a}$	$\hat{b}$	$\hat{A}$	$\hat{E}$	$\hat{r}_1$	$\hat{r}_2$	$\hat{s}$
Pitcher	RHP	-0.079	2.482	0.940	0.930	3.123	1.014	1.378	1.969	-0.065
	LHP	-0.064	2.492	0.940	0.929	3.081	1.018	1.289	2.048	0.046
Batter	RHB	-0.028	2.488	0.956	0.925	3.161	1.038	1.360	2.098	0.008
	LHB	-0.131	2.482	0.920	0.936	3.045	0.988	1.333	1.871	-0.072
Venue	Home	-0.075	2.486	0.945	0.935	3.140	1.015	1.329	2.007	-0.030
	Away	-0.074	2.484	0.934	0.924	3.078	1.016	1.367	1.984	-0.026
Ball	0	-0.072	2.491	0.932	0.918	3.057	1.020	1.351	2.020	-0.024
	1	-0.073	2.486	0.936	0.912	3.047	1.032	1.341	2.059	-0.031
	2	-0.078	2.481	0.943	0.944	3.148	1.003	1.356	2.000	-0.030
	3	-0.081	2.464	0.975	1.003	3.434	0.975	1.344	1.713	-0.034
Strike	0	-0.076	2.481	0.993	1.007	3.541	0.987	1.384	1.762	-0.022
	1	-0.074	2.478	0.925	0.914	3.016	1.013	1.376	2.061	-0.026
	2	-0.072	2.500	0.870	0.820	2.504	1.065	1.256	2.306	-0.039
Year	2014	-0.084	2.480	0.950	0.922	3.105	1.035	1.335	1.963	-0.026
	2015	-0.072	2.476	0.940	0.933	3.122	1.013	1.345	2.017	-0.024
	2016	-0.068	2.499	0.930	0.934	3.101	1.000	1.364	2.008	-0.034

Batter × Strike		$\hat{x}_0$	Batter × Year		$\hat{x}_0$	Batter × Pitcher		$\hat{s}$
RHB	0	-0.022	RHB	2014	-0.032	RHB	RHP	-0.023
RHB	1	-0.021	RHB	2015	-0.022	RHB	LHP	0.057
RHB	2	-0.048	RHB	2016	-0.030	LHB	RHP	-0.109
LHB	0	-0.142	LHB	2014	-0.144	LHB	LHP	0.026
LHB	1	-0.139	LHB	2015	-0.132			
LHB	2	-0.102	LHB	2016	-0.115			

Table 3: Weighted level means of estimated ATLAS coefficients, plus area and eccentricity, corresponding to each player attribute and game situation factor, year and selected two-factor combinations. Units are identical to those in Table 3.

4.4 Standard errors

Factor	Levels	$\hat{x}_0$	$\hat{y}_0$	$\hat{a}$	$\hat{b}$	$\hat{A}$	$\hat{E}$	$\hat{r}_1$	$\hat{r}_2$	$\hat{s}$
Pitcher	RHP	0.001	0.002	0.002	0.002	0.007	0.003	0.017	0.047	0.003
	LHP	0.002	0.003	0.002	0.003	0.009	0.004	0.024	0.066	0.005
Batter	RHB	0.002	0.002	0.002	0.002	0.007	0.003	0.019	0.051	0.004
	LHB	0.002	0.002	0.002	0.003	0.008	0.004	0.021	0.057	0.004
Venue	Home	0.002	0.002	0.002	0.002	0.008	0.003	0.020	0.054	0.004
	Away	0.002	0.002	0.002	0.002	0.008	0.003	0.020	0.054	0.004
Ball	0	0.002	0.002	0.002	0.003	0.008	0.004	0.021	0.058	0.004
	1	0.002	0.003	0.002	0.003	0.010	0.005	0.026	0.071	0.005
	2	0.003	0.004	0.003	0.004	0.013	0.006	0.034	0.091	0.007
	3	0.004	0.005	0.004	0.005	0.017	0.008	0.044	0.118	0.009
Strike	0	0.002	0.002	0.002	0.003	0.008	0.004	0.021	0.058	0.004
	1	0.002	0.003	0.002	0.003	0.010	0.004	0.025	0.069	0.005
	2	0.002	0.003	0.003	0.003	0.011	0.005	0.027	0.074	0.005
Year	2014	0.002	0.003	0.002	0.003	0.009	0.004	0.024	0.066	0.005
	2015	0.002	0.003	0.002	0.003	0.009	0.004	0.024	0.066	0.005
	2016	0.002	0.003	0.002	0.003	0.009	0.004	0.024	0.066	0.005

Batter × Strike		$\hat{x}_0$	Batter × Year		$\hat{x}_0$	Batter × Pitcher		$\hat{s}$
RHB	0	0.002	RHB	2014	0.003	RHB	RHP	0.005
RHB	1	0.003	RHB	2015	0.003	RHB	LHP	0.006
RHB	2	0.003	RHB	2016	0.003	LHB	RHP	0.005
LHB	0	0.003	LHB	2014	0.003	LHB	LHP	0.008
LHB	1	0.003	LHB	2015	0.003			
LHB	2	0.003	LHB	2016	0.003			

Table 4: Standard errors associated with weighted level means of estimated ATLAS coefficients, plus area and eccentricity, corresponding to each player attribute and game situation factor, year and selected two-factor combinations. Units are identical to those in Table 3.