Three physics model equations

1 Neutronics

1.1 Model

One-dimensional reactor with zero-flux boundary condition at the top and bottom. Cross sections are piece-wise constant as a function of z (height) and they represent homogenized fuel assembly properties.

1.2 Old version of the neutronics solver

The 1-D 2-g neutron diffusion equation is solved (do I really have to write it down?).

In **steady-state**, I used to the neutronics eigenproblem in order to get the initial values for a transient run. The multi-physics coupling was performed as followed

1. initialize fuel temperature field T_{fuel} and moderator densities ρ_m
2. solve the neutronics eigenproblem
3. compute the power and normalize it so that it is equal to the specified value at $t = 0$ (initial power value)
4. pass the power to the TH and solve for T_{fuel} and ρ_m
5. if the new values of T_{fuel} and ρ_m and k_{eff} have converged, exit; otherwise go back to 2

Of course, k_{eff} may not be equal to 1 necessary (we are doing a made up test case, so why would $k_{eff} = 1$ anyway), so I renormalize the $\nu\Sigma_f$ value for k_{eff} so that the initial configuration is truly critical.

Then, in **transient** mode, the time-dependent neutron diffusion equation, supplemented with the precursors ODEs are solved.

The **boundary conditions** are always Dirichlet zero flux at the top and bottom of the 1-D domain.

1.3 New version of the neutronics solver

Cristian asked to first focus on steady state and to let the power be a free parameter, determined by the nonlinear solve of the initial steady state equations (neutronics+TH). Therefore, one may write the nonlinear residual pertaining to the neutronics as follows:

$$F_N(\phi, T_{eff}, \rho_m) = 0 = A_N(T_{eff}, \rho_m)\phi \tag{1}$$

where A_N is the discretized operator for

$$\vec{\nabla}\cdot D(T_{eff},\rho_m)\vec{\nabla}\cdot+(\Sigma_a-\nu\Sigma_f)\cdot \tag{2}$$

I have written the 1-g form but you get the picture and it is easy to write the multigroup form …Basically, D and the Σ's depends on T_{eff} and ρ_m.
Do note that I have purposefully used T_{eff} and not T_{fuel}. T_{eff} is defined as

$$T_{eff}=\alpha T_{fuel}(0)+(1-\alpha)T_{fuel}(R_f) \tag{3}$$

with $T_{fuel}(0)$ the fuel centerline temperature and $T_{fuel}(R_f)$ the fuel surface temperature (R_f = fuel pellet radius).

1.4 Cross-sections

They are in tabular form, parameterized as a function of T_{eff} and ρ_m. They are taken form the MSLB OECD benchmark (PRESSURISED WATER REACTOR MAIN STEAM LINE BREAK (MSLB) BENCHMARK, NEA/NSC/DOC(99)8). Again, T_{eff} is computed with the knowledge of the temperature radial distribution in the pin, see Eq. (3).

1.5 Power

With the knowledge of the fission cross sections (and the TH fields to interpolate them) and the multigroup fluxes, the neutronic power is computed. In the old version of the solver, this power was renormalized so that the power integrated over the domain would yield a specified value (in Watts) at the initial steady state (during transient, the power would naturally evolve from this value).
In the new solver, the power is not renormalized and an initial steady state equilibrium is reached (feedback is negative, remember!). However, to help int he numerics, there is still a multiplicative constant (a scaling factor if you prefer)

2 Pin heat transfer

2.1 Model

A single (average) pin is considered. Axial heat conduction is neglected. Only radial heat transfers are accounted for. For a given axial node of the neutronic domain, we compute a radial temperature distribution in the pin element.

2.2 Equations

Here again, nothing fancy …

$$\rho_f C_{p,f}\frac{\partial T}{\partial t} - \vec{\nabla}\cdot k_f(T)\vec{\nabla}T = q''' \tag{4}$$

(remove time derivative for steady state) where the volumetric heat density q''' is determined by the neutronic power in the axial node under consideration. There is obviously a conversion factor to go from Watts in the axial node to Watt/cm^3 for q'''.

Boundary conditions:

1. at $r = 0$, symmetry $\frac{\partial T}{\partial r} = 0$
2. at the clad outer radius ($r = R_c$), the heat flux $\varphi = -k\partial_n T = h_{conv}\Big(T_m - T(R_c)\Big)$

T_m is the moderator bulk temperature at that axial node. h_{conv} is the heat transfer coefficient. For single-phase forced convection, I use the Dittus-Boelter correlation (which requires the Nusselt number of the fluid, which require a lot of fluid properties; they are all coded but I will not type then in this short note).

The above heat conduction equation is valid in the fuel and the clad. At the fuel-clad interface, a simple relation relates the heat flux leaving the last fuel ring and the first clad ring:

$$\varphi = h_{gap}\Big(T_{fuel}(R_f) - T_{clad}(R_f)\Big) \tag{5}$$

where h_{gap} is the so-called gap conductance.

The nonlinear residual for the discretized temperature equation can be put as

$$F_T(\phi, T, T_m) = 0 = A_T(T, T_m)T - S(\phi) \tag{6}$$

For the neutronic residual, I used ρ_m and not T_m in the residual expression but that's OK. The pressure in the core is assumed to be fixed (155 bars) and, with the single phase assumption, there is a one-to-one relation between the fluid density, temperature, enthalpy, ...

3 Fluid model

3.1 Model

The same axial 1-D nodalization as for neutronics is employed. The inlet enthalpy (H^{in}) is known and the pressure is fixed and constant throughout the domain. Since we are using an average pin, we are also using an average channel for that pin and we compute a representative hydraulic area, S_{hy}, for that pin.

3.2 Enthalpy balance

We do a simple enthalpy balance:

$$\dot{m}\Delta H = \Delta z 2\pi R_c \varphi \tag{7}$$

where $\dot{m} = \rho v S_{hy}$ is the mass flow rate. In steady state, one can actually show that $\Delta z 2\pi R_c \varphi$ is exactly the power deposited in the pin for the axial node under consideration but the expression given is more general and works for transients(provided that you add the appropriate time derivative term in the enthalpy balance equation).

The nonlinear residual for the discretized temperature equation can be put as

$$F_H(H, T) = 0 = A_H(H, T)T - S(T_c) \tag{8}$$

where, again, I use indifferently H, ρ_m, or T_m for the fluid unknown.

4 Solver

Putting all the nonlinear residual together, we have

$$F(X) = 0 \tag{9}$$

where

$$X = [\phi, T, T_m]^T \tag{10}$$

(with the freedom to swap T_m for any other fluid unknown due to a one-to-one relationship) and

$$F(X) = \begin{bmatrix} F_N \\ F_T \\ F_H \end{bmatrix} \tag{11}$$

This is solved using Newton's method. The Jacobian needed for this solve is obtain using finite differences

$$J_{i,j} = \frac{F_j(X + \epsilon\delta_{i,j}) - F_j(X)}{\epsilon} \tag{12}$$

As an example, the numerical Jacobian is plotted in Fig. 1.

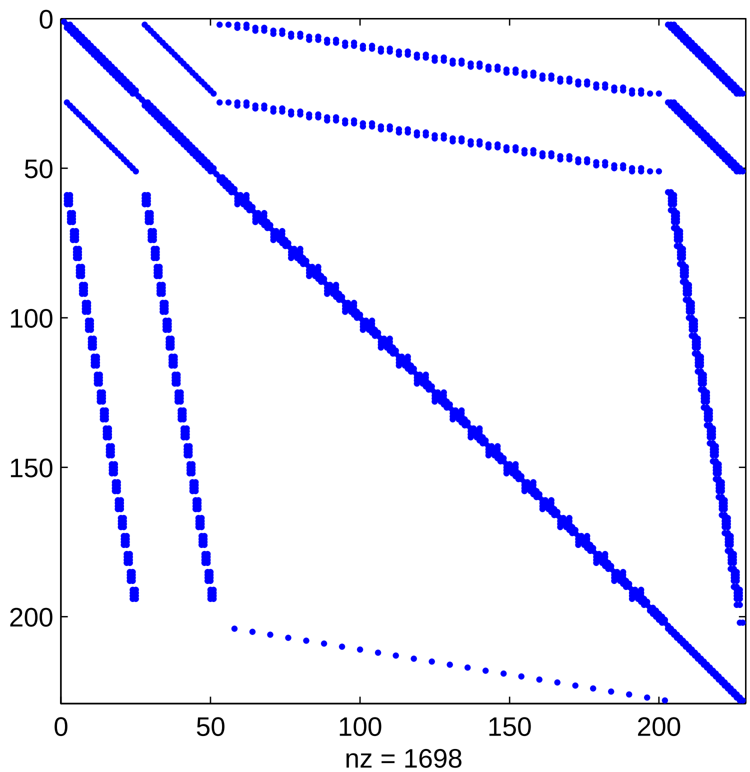

Figure 1 num jac