

First Partial for MATE-1214 Cálculo Integral con Ecuaciones Diferenciales

28/08/2018

1. Compute the following integral

$$\int x \tan^{-1} x \, dx.$$

2. Decompose into partial fractions the following integral:

$$\int \frac{1}{x^4 - 3x^2 - 4} \, dx.$$

3. Let $f: [0, \infty) \rightarrow \mathbb{R}$ is a continuous function such that $f(x) \geq 0$ for all $x \in [0, \infty)$. Show that if

$$\int_0^\infty f(x) \, dx$$

is convergent, then the same holds for

$$\int_0^\infty e^{-x} f(x) \, dx.$$

4. The figure eight curve called *Gerono lemniscate* is given by the equation $x^4 = 9(x^2 - y^2)$.

- (a) Write down the integral to compute the area bounded by it;
(b) Evaluate that integral.

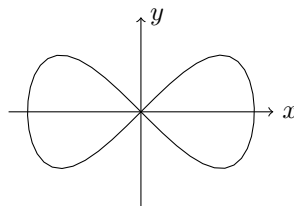


Figure 1: Gerono lemniscate.

5. Compute the length of the curve $C: y = 1 - e^{-x}$ between the points $(0, 0)$ and $(1, 1 - e^{-1})$.

[Hint: Use substitution.]