Whale Population Simulation

The blue whale and fin whale are two similar species that inhabit the same areas. Hence they are thought to compete. The intrinsic growth rate of each species is estimated at 5 % per year for the blue whale and 8 % per year for the fin whale. The environmental carrying capacity (the maximum number of whales that the environment can support) is estimated at 150,000 blue whales and 400,000 fin whales. The extent to which the whales compete is unknown. In the last 100 years intense harvesting has reduced the whale population to around 3000 blues and 15000 fins. Firstly, we will create a mathematical model representing the population of whales. Using the mathematical model, we will determine if both species of whales grow back, or will one or both become extinct.

We will model the problem as a dynamical system, with the population of each whale species representing a state variable.
B = number of blue whales
F = number of fin whales
gB = growth rate for blue whales (per year)
gF = growth rate for fin whales (per year)
cB = effect of competition on blue whales (whales per year) cF = effect of competition on fin whales (whales per year)

Assumptions:
gB =0.05B(1–B/150,000)
gF =0.08F(1–F/400,000)
cB = cF = αBF,
B ≥ 0, F ≥ 0, α > 0

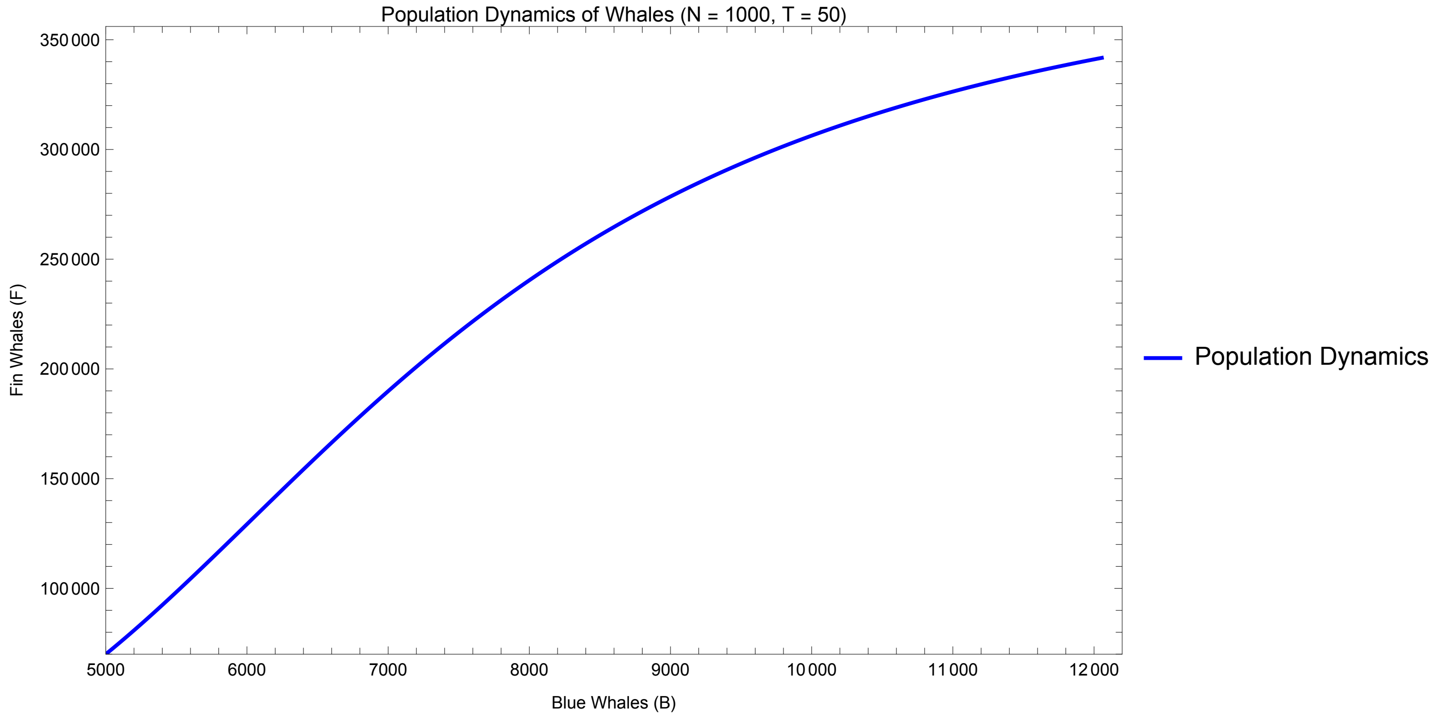
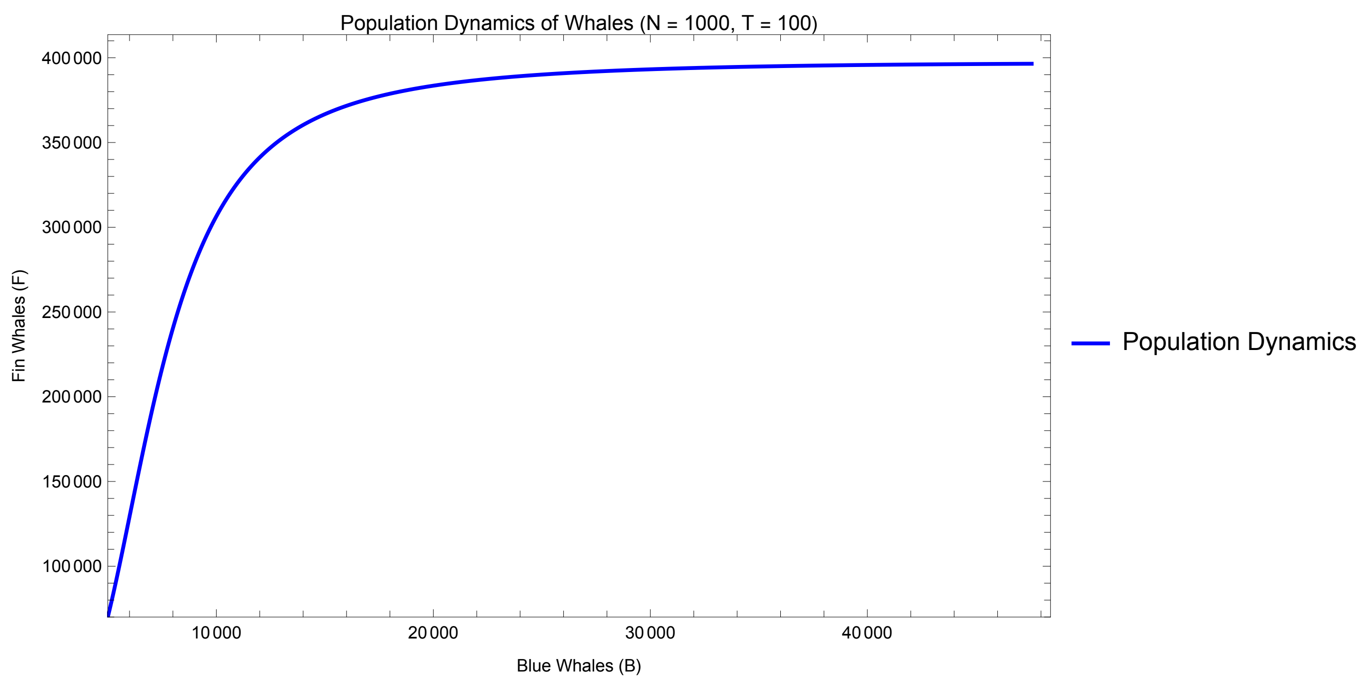
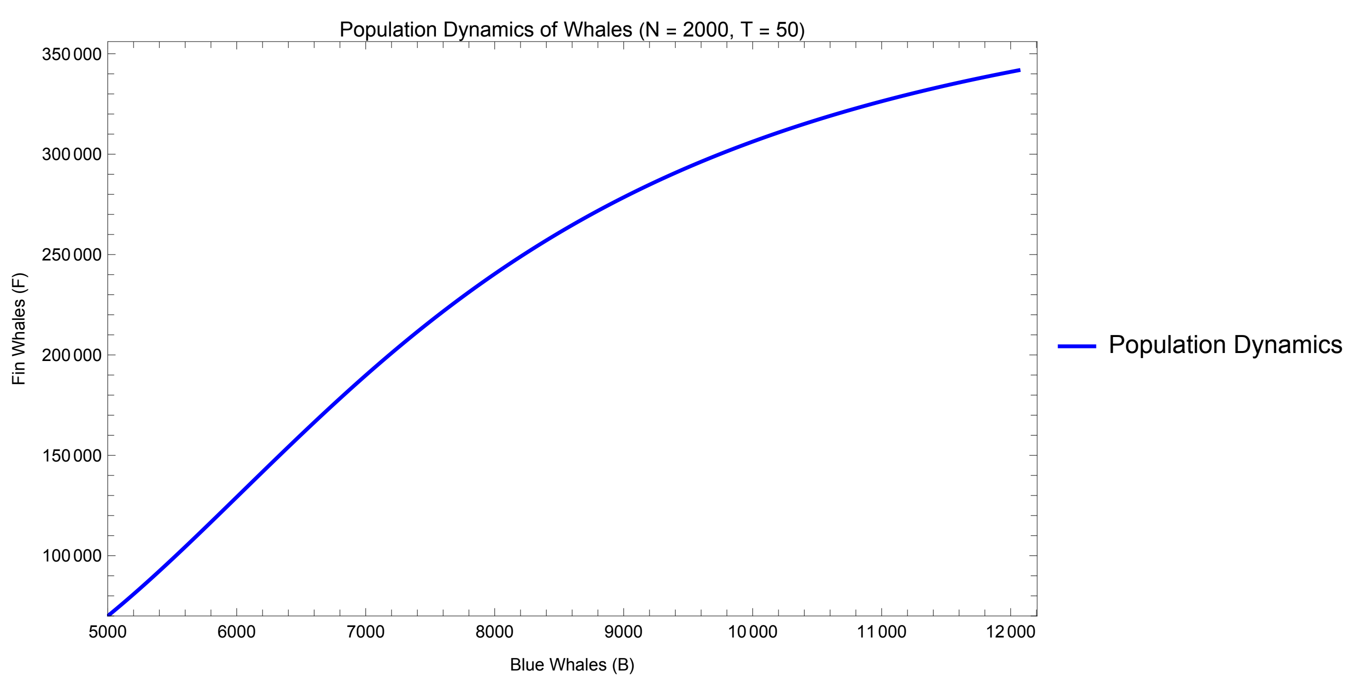
We can now build the model using the assumptions. The populations B and F are functions of the independent variable t.

```
dBdt[B_, F_] := 0.05*B*((B - 3000)/(B + 3000))*(1 - B/150000) - 10^-8*B*F
dFdt[B_, F_] := 0.08*F*((F - 15000)/(F + 15000))*(1 - F/400000) - 10^-8*B*F
```

Using a computer implementation of Euler's Method we will simulate the behaviour of the model starting with initial conditions $B(0) = 5,000$ and $F(0) = 70,000$. We will perform a sensitivity analysis on both T and N to determine if both species of whales grow back, or one or both will become extinct and about how long it will take.

```
(* Define the system of differential equations *)
dBdt[B_, F_] := 0.05*B*((B - 3000)/(B + 3000))*(1 - B/150000) - 10^-8*B*F
dFdt[B_, F_] := 0.08*F*((F - 15000)/(F + 15000))*(1 - F/400000) - 10^-8*B*F

(* Implement Euler's method *)
eulerMethod[{B0_, F0_}, T_, numSteps_] := Module[{h, tValues, BValues, FValues, dB, dF, B, F}, h = T/numSteps;
  tValues = Table[t, {t, 0, T, h}];
  BValues = FValues = Table[0, {numSteps + 1}];
  BValues⟦1⟧ = B0;
  FValues⟦1⟧ = F0;
  Do[
   {dB, dF} = {dBdt[B, F], dFdt[B, F]} /. {B → BValues⟦i⟧, F → FValues⟦i⟧};
   BValues⟦i + 1⟧ = BValues⟦i⟧ + h*dB;
   FValues⟦i + 1⟧ = FValues⟦i⟧ + h*dF;
   , {i, 1, numSteps}];
  {tValues, BValues, FValues}
]

(* Define initial conditions *)
initialConditions = {5000, 70000};

(* Define parameters for simulations *)
params = {{1000, 50}, {1000, 100}, {2000, 50}};

(* Perform simulations and plot results *)
plots = Table[
  {numSteps, T} = params⟦i⟧;
  {tValues, BValues, FValues} = eulerMethod[initialConditions, T, numSteps];
  ListLinePlot[{Transpose[{BValues, FValues}]},
   PlotLegends → {"Population Dynamics"}, PlotStyle → Blue,
   Frame → True, FrameLabel → {"Blue Whales (B)", "Fin Whales (F)"},
   PlotRange → {{5000, Automatic}, {70000, Automatic}},
   PlotLabel → Row[{"Population Dynamics of Whales (N = ", numSteps, ", T = ", T, ")"}]],
  {i, Length[params]}]

(* Display the plots *)
Show[plots]
```

Out[ ]=

{

Population Dynamics of Whales (N = 1000, T = 50)
Fin Whales (F)
Blue Whales (B)
350 000
300 000
250 000
200 000
150 000
100 000
5000
6000
7000
8000
9000
10 000
11 000
12 000
Population Dynamics
,

Population Dynamics of Whales (N = 1000, T = 100)
Fin Whales (F)
Blue Whales (B)
400 000
350 000
300 000
250 000
200 000
150 000
100 000
10 000
20 000
30 000
40 000
Population Dynamics
,

Population Dynamics of Whales (N = 2000, T = 50)
Fin Whales (F)
Blue Whales (B)
350 000
300 000
250 000
200 000
150 000
100 000
5000
6000
7000
8000
9000
10 000
11 000
12 000
Population Dynamics
}

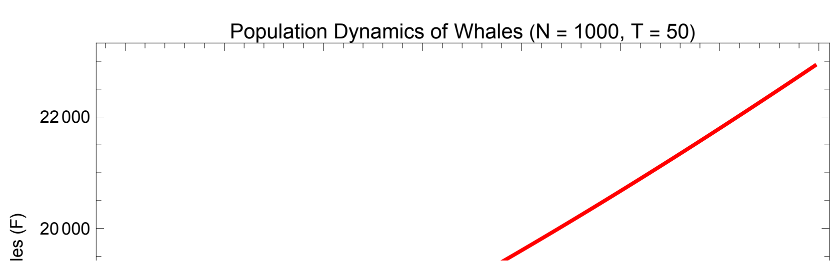
Both species grow back!! Levelling off in both populations at $T > 200$. It should take around 200 years for the species to grow back to a coexistence state.

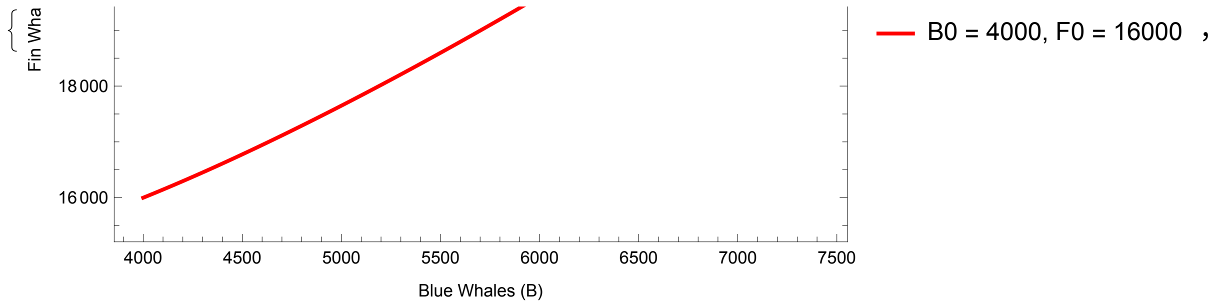
Now we will determine for different initial conditions, what happens to the species of whales over long term period.
Initial conditions:
$(B(0), F(0))$ = (4000,16000)
$(B(0), F(0))$ = (3000,16000)
$(B(0), F(0))$ = (4000,15000)
$(B(0), F(0))$ = (3000,15000)

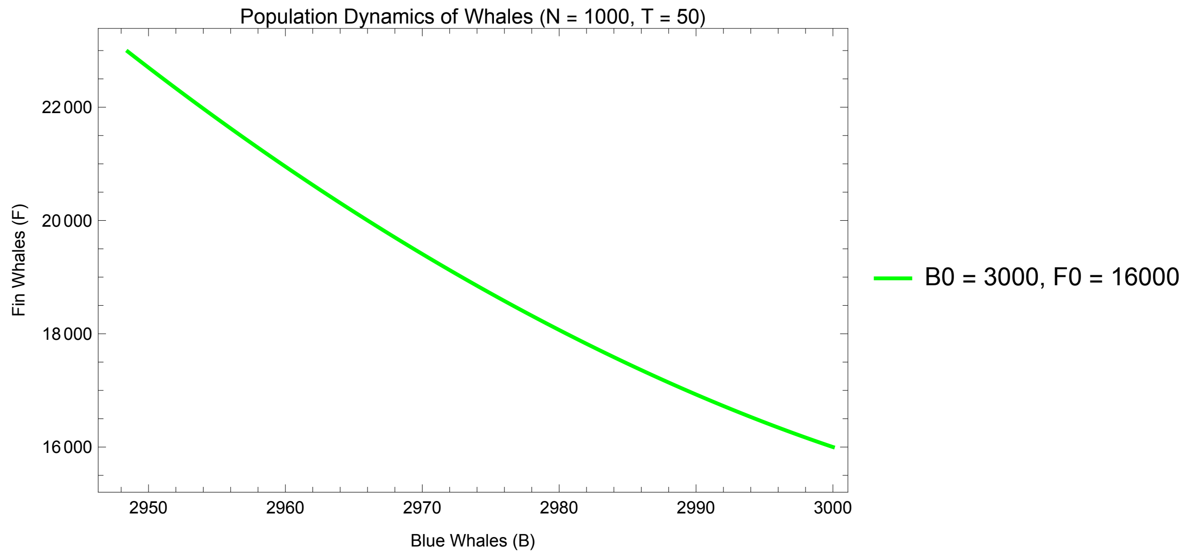
In[ ]:=

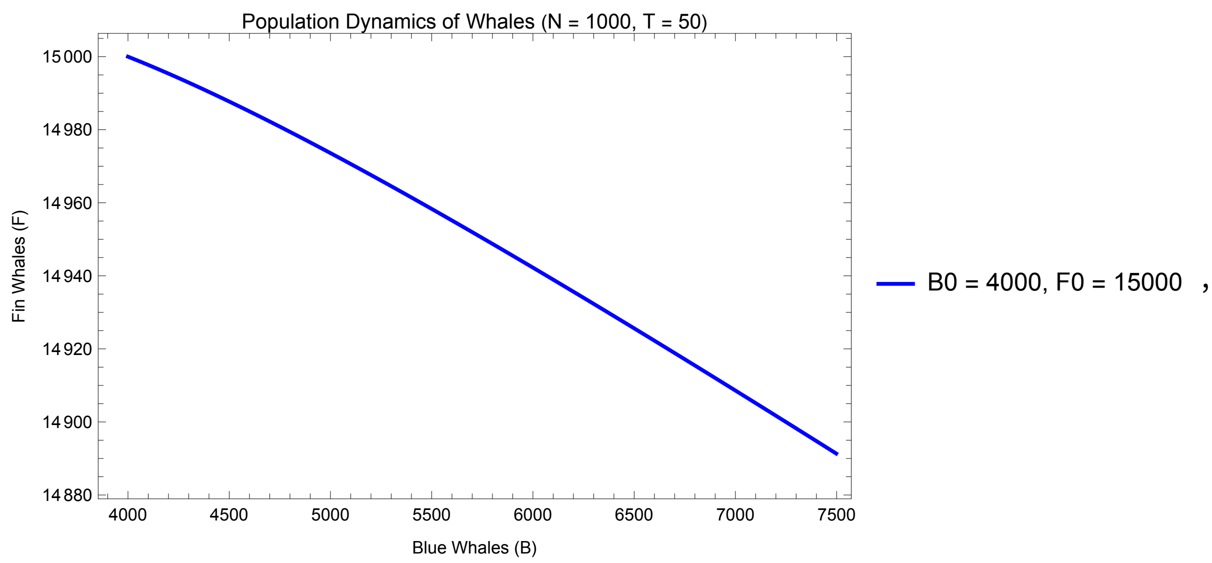
```
(* Define the system of differential equations *)
dBdt[B_, F_] := 0.05*B*((B - 3000)/(B + 3000))*(1 - B/150000) - 10^-8*B*F
dFdt[B_, F_] := 0.08*F*((F - 15000)/(F + 15000))*(1 - F/400000) - 10^-8*B*F

(* Implement Euler's method *)
eulerMethod[{B0_, F0_}, T_, numSteps_] := Module[{h, tValues, BValues, FValues, dB, dF, B, F},
  h = T/numSteps;
  tValues = Table[t, {t, 0, T, h}];
  BValues = FValues = Table[0, {numSteps + 1}];
  BValues[[1]] = B0;
  FValues[[1]] = F0;
  Do[
   {dB, dF} = {dBdt[B, F], dFdt[B, F]} /. {B → BValues[[i]], F → FValues[[i]]};
   BValues[[i + 1]] = BValues[[i]] + h*dB;
   FValues[[i + 1]] = FValues[[i]] + h*dF;
   , {i, 1, numSteps}];
  {tValues, BValues, FValues}
]

(* Define parameters for simulations *)
params = {{{4000, 16000}, {3000, 16000}, {4000, 15000}, {3000, 15000}}, {1000, 50}};

(* Specify colors *)
colors = {Red, Green, Blue, Orange};

(* Perform simulations and plot results *)
plots = Table[
  initialConditions = params[[1, j]];
  {numSteps, T} = params[[2]];
  {tValues, BValues, FValues} = eulerMethod[initialConditions, T, numSteps];
  ListLinePlot[{Transpose[{BValues, FValues}]},
   PlotLegends → {"B0 = " <> ToString[initialConditions[[1]]] <> ", F0 = " <> ToString[initialConditions[[2]]]},
   PlotStyle → colors[[j]], Frame → True,
   FrameLabel → {"Blue Whales (B)", "Fin Whales (F)"},
   PlotLabel → Row[{"Population Dynamics of Whales (N = ", numSteps, ", T = ", T, ")"}]],
  {j, Length[params[[1]]]}]

(* Display the plots *)
Show[plots]

(* Perform simulations and collect results *)
data = Table[
  initialConditions = params[[1, j]];
  {numSteps, T} = params[[2]];
  {tValues, BValues, FValues} = eulerMethod[initialConditions, T, numSteps];
  Transpose[{BValues, FValues}],
  {j, Length[params[[1]]]}]

(* Plot results *)
ListLinePlot[data, PlotLegends → {"B0 = (4000, 16000)", "B0 = (3000, 16000)", "B0 = (4000, 15000)", "B0 = (3000, 15000)
 Frame → True, FrameLabel → {"Blue Whales (B)", "Fin Whales (F)"},
 PlotLabel → "Population Dynamics of Whales"]
```

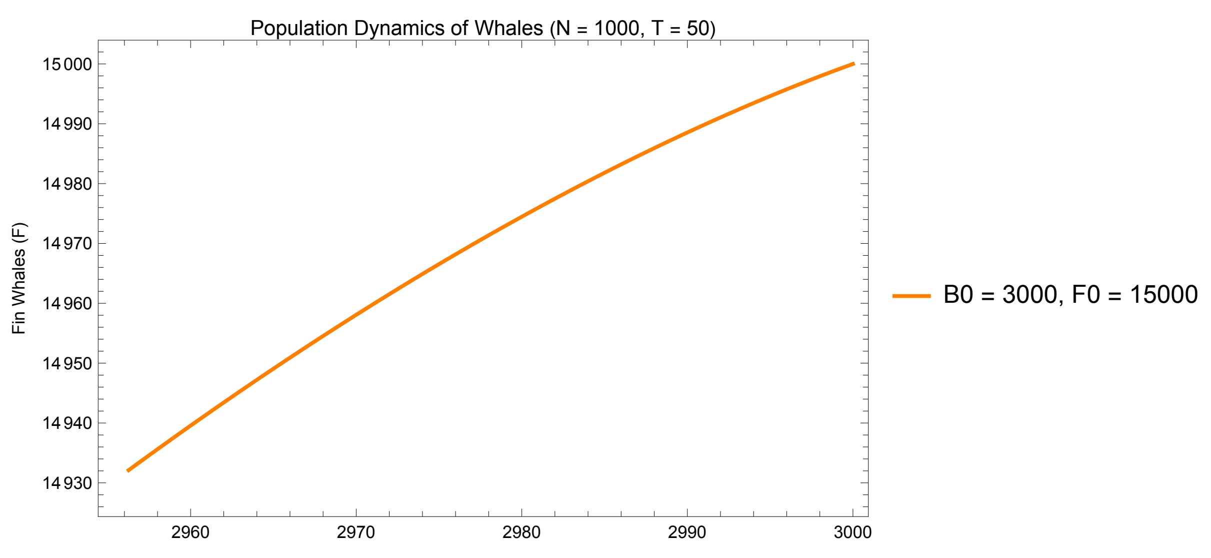
Out[ ]=




Fin Wha
18 000
16 000
4000
4500
5000
5500
6000
6500
7000
7500
Blue Whales (B)
B0 = 4000, F0 = 16000 ,


Population Dynamics of Whales (N = 1000, T = 50)
Fin Whales (F)
22 000
20 000
18 000
16 000
2950
2960
2970
2980
2990
3000
Blue Whales (B)
B0 = 3000, F0 = 16000 ,


Population Dynamics of Whales (N = 1000, T = 50)
Fin Whales (F)
15 000
14 980
14 960
14 940
14 920
14 900
14 880
4000
4500
5000
5500
6000
6500
7000
7500
Blue Whales (B)
B0 = 4000, F0 = 15000 ,

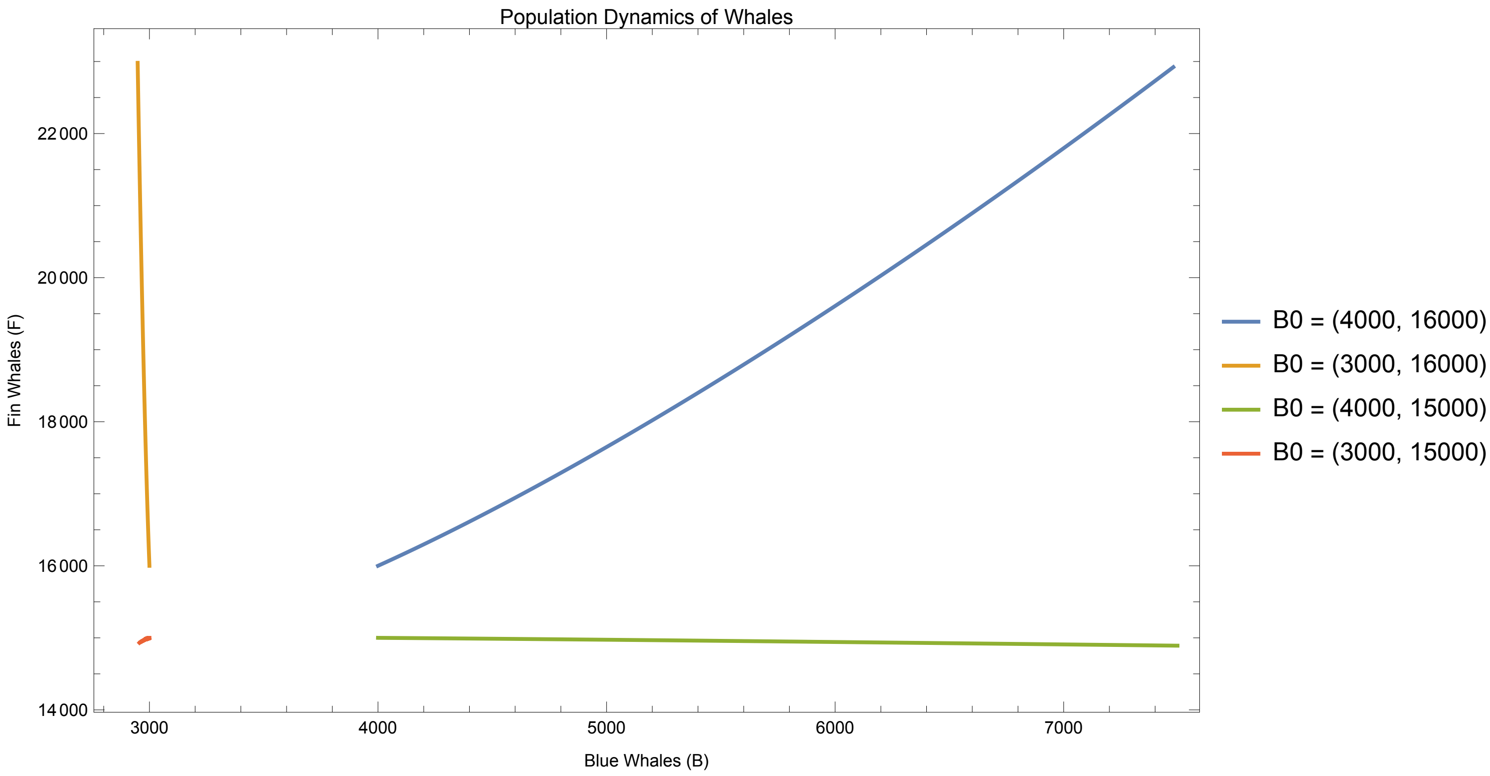

Population Dynamics of Whales (N = 1000, T = 50)
Fin Whales (F)
15 000
14 990
14 980
14 970
14 960
14 950
14 940
14 930
2960
2970
2980
2990
3000
Blue Whales (B)
B0 = 3000, F0 = 15000

Out[ ]=

```
{{{4000, 16 000}, {4001.36, 16 001.9}, {4002.72, 16 003.9}, {4004.08, 16 005.9}, {4005.45, 16 007.8}, {4006.81, 16 009.8},
  {4008.18, 16 011.8}, {4009.55, 16 013.7}, {4010.93, 16 015.7}, {4012.3, 16 017.7}, {4013.68, 16 019.7}, {4015.06, 16 021.7},
  {4016.44, 16 023.7}, {4017.82, 16 025.7}, … 973 …, {7383.34, 22 692.1}, {7390.66, 22 709.5}, {7398., 22 726.9}, {7405.35, 22 744.4},
  {7412.72, 22 761.9}, {7420.1, 22 779.5}, {7427.5, 22 797.1}, {7434.91, 22 814.7}, {7442.33, 22 832.4}, {7449.77, 22 850.2},
  {7457.22, 22 868.}, {7464.69, 22 885.8}, {7472.17, 22 903.7}, {7479.66, 22 921.6}}, … 2 …, {{3000, 15 000}, … 999 …, {… 1 …}}}
```

Full expression not available (original memory size: 385.2 kB)

Out[ ]=



□ From the graph, we can see that
(4000, 16 000) both grow back
(3000, 16 000) blue whales go extinct, fin whales approach carrying capacity
(4000, 15 000) fin whales become extinct, blue whales approach carrying capacity
(3000, 15 000) both go extinct

Out[ ]=


```
{{{4000, 16 000}, {4001.36, 16 001.9}, {4002.72, 16 003.9}, {4004.08, 16 005.9}, {4005.45, 16 007.8}, {4006.81, 16 009.8},
  {4008.18, 16 011.8}, {4009.55, 16 013.7}, {4010.93, 16 015.7}, {4012.3, 16 017.7}, {4013.68, 16 019.7}, {4015.06, 16 021.7},
  {4016.44, 16 023.7}, {4017.82, 16 025.7}, … 973 …, {7383.34, 22 692.1}, {7390.66, 22 709.5}, {7398., 22 726.9}, {7405.35, 22 744.4},
  {7412.72, 22 761.9}, {7420.1, 22 779.5}, {7427.5, 22 797.1}, {7434.91, 22 814.7}, {7442.33, 22 832.4}, {7449.77, 22 850.2},
  {7457.22, 22 868.}, {7464.69, 22 885.8}, {7472.17, 22 903.7}, {7479.66, 22 921.6}}, … 2 …, {{3000, 15 000}, … 999 …, {… 1 …}}}
```

Full expression not available (original memory size: 385.2 kB)

□

In[ ]:=