UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	CALCULUS I
COURSE CODE	:	MAT183
EXAMINATION	:	FEBRUARY 2023
TIME	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of five (5) questions
2. Answer ALL questions in the Answer Booklet. Start each answer on a new page.
3. Do not bring any material into the examination room unless permission is given by the invigilator.
4. Please check to make sure that this examination pack consists of :
   i) the Question Paper
   ii) a two – page Appendix 1
   iii) an Answer Booklet – provided by the Faculty
5. Answer ALL questions in English.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of 6 printed pages

QUESTION 1

a) Evaluate each of the following limits:

i) $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 1}$

(3 marks)

ii) $\lim\limits_{x \to 2} \dfrac{\sqrt{x+2} - 2}{3x - 6}$

(4 marks)

iii) $\lim\limits_{x \to 0} \dfrac{2\sin 5x}{3x}$

(4 marks)

b) The function $g(x)$ is defined as follows:

$$g(x) = \begin{cases} 4m + x \quad , & x < -1 \\ \dfrac{x^2 - 9}{x - 3} \quad , & -1 \le x < 3 \\ 3x + 2 \quad , & x \ge 3 \end{cases}$$

i) Find $\lim\limits_{x \to 1} g(x)$.

(2 marks)

ii) Find the value of m if $g(x)$ is continuous at $x = -1$.

(3 marks)

iii) Determine whether the function $g(x)$ is continuous at $x = 3$.

(4 marks)

QUESTION 2

a) Given $f(x) = \dfrac{2x}{3 - x}$.

i) Use definition of derivative to find $f'(x)$.

(5 marks)

ii) Find the equation of a tangent line to the function at point $\left(1, \dfrac{3}{2}\right)$.

(3 marks)

b) Given the function $e^{y^2-4}-\frac{x}{y^3}=1-\cos(5x)$. Find $\frac{dy}{dx}$ for the function using implicit differentiation.

(5 marks)

c) Use differentials to estimate the value of $\sqrt[4]{15.98}-\left(\frac{15.98}{4}\right)^2$ correct to four decimal places.

(5 marks)

QUESTION 3

a) Water is poured into a conical tank at the rate of 9 m^3s^{-1}. The radius of the tank is 4 m and the height is 12 m. At what rate is the water level rising when the water reached 4 m deep.

[Hint: $v_{cone}=\frac{1}{3}\pi r^2 h$]

(6 marks)

b) Given a function $f(x)=\frac{1}{3}x^2(x-6)$.

i) Find the *x*-intercept(s) and *y*-intercept.

(2 marks)

ii) Find the critical points of *f*.

(3 marks)

iii) Find the interval(s) where *f* is increasing or decreasing. Hence, determine the relative extremum of *f*.

(4 marks)

iv) Find the interval(s) where *f* is concave up or concave down. Hence, determine the inflection point of *f*.

(4 marks)

v) Sketch the graph of *f* using the above information.

(2 marks)

c) A factory plans to manufacture opened cylindrical steel cases that can hold 450cm^3. Let *r* and *h* be the radius and height of the cylinder respectively.

i) Express *h* in terms of *r*.

(2 marks)

ii) Show that the surface area of the cylindrical steel cases is $A=\pi r^2+\frac{900}{r}$.

(3 marks)

QUESTION 4

a) If $\int_{1}^{4} f(x)dx = 2$, evaluate $\int_{1}^{4} \left[1+3f(x)\right]dx$ using the properties of integral.

(3 marks)

b) Evaluate each of the following integrals using appropriate substitution.

i) $\int \frac{3x^2}{x-2}\,dx$

(4 marks)

ii) $\int \frac{\sin x}{(2+\cos x)^4}\,dx$

(5 marks)

c) The Mean Value Theorem for differentiation states that if f(x) is differentiable on (a,b) and continuous on [a,b], then there is at least one point c in (a,b) where

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

If $f(x) = 3x^2 - x^3$, find the value of c in the interval (1,3) that satisfies the above theorem.

(4 marks)

d) Given $F(x) = \int_{0}^{\sin(2x)} (3\cos(t) - 5t^2)dt$. Use the Second Fundamental Theorem of Calculus to find:

i) $F'(x)$.

(3 marks)

ii) $F'(0)$.

(1 mark)

QUESTION 5

a) Figure 1 below shows a shaded region **R** bounded by curves $y = x^2 - 4x$ and $y = -x^2 + 2x + 8$.

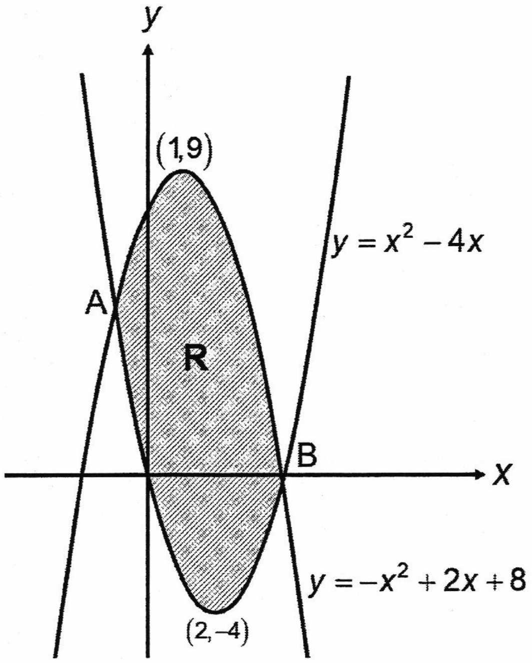

Figure 1

i) Find the coordinates of A and B.

(2 marks)

ii) Find the area of the shaded region **R**.

(4 marks)

b) Figure 2 below shows the shaded region **S** bounded by the curves $x = y^2 - 4y$ and $y = \sqrt{-x}$.

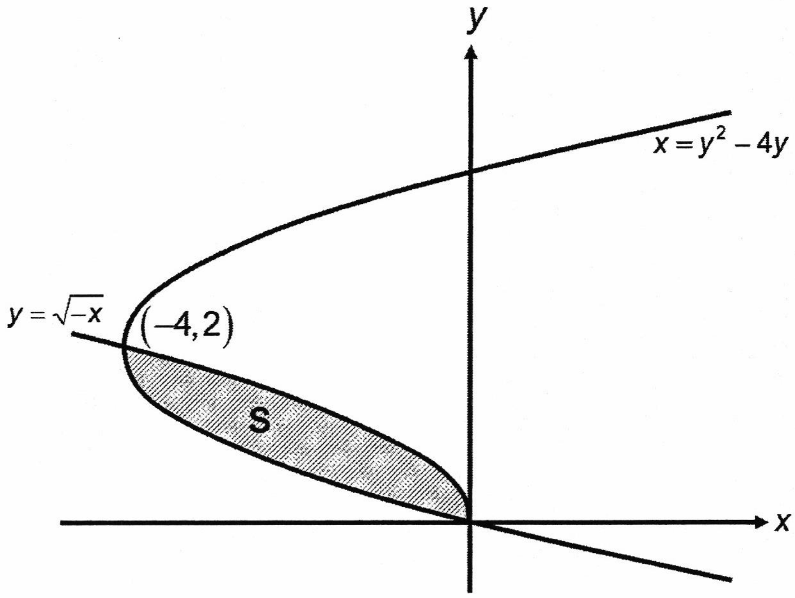

Figure 2

i) Find the volume of the solid generated when the shaded region **S** is revolved about the line $x = 0$ using the **Washer Method**.

(5 marks)

ii) Find the volume of the solid generated when the shaded region **S** is revolved about the line $y = 2$ using **Shell Method.**

(5 marks)

END OF QUESTION PAPER

RULES OF DIFFERENTIATION

1. Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

2. Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

3. Power Rule

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x)$$

LIST OF INTEGRALS

1. $$\int (ax+b)^n\,dx = \begin{cases} (ax+b)^{n+1} + C; & n \neq -1 \\ \dfrac{1}{a}\ln|ax+b| + C & n = 1 \end{cases}$$

2. $$\int \frac{1}{x}\,dx = \ln|x| + C$$

3. $$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$

4. $$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$$

5. $$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C$$

6. $$\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax + C$$

THE SQUEEZING THEOREM

1. $$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

2. $$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

DEFINITION OF DIFFERENTIATION

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

LINEAR APPROXIMATION

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

THE SECOND FUNDAMENTAL THEOREM OF CALCULUS

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)dt\right] = f[g(x)]g'(x)$$

