

Estimating gravitational acceleration from light deflection without mass as an input parameter

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Abstract

We present a simple empirical estimator that derives effective gravitational acceleration directly from gravitational lensing observables. The method relates the measured light-deflection angle and impact parameter through a calibrated proportionality coefficient, allowing gravitational strength to be inferred without assuming a mass model. The approach is phenomenological and intended as a practical diagnostic tool rather than a modification of General Relativity.

Tests on the Sun and the microlensing white dwarf Stein 2051 B show order-of-magnitude consistency with standard reference values. Because the estimator uses only directly measurable quantities and requires no iterative modeling, it provides a fast and numerically stable first-pass characterization of lensing systems. An implementation and example scripts are publicly available.

Keywords: gravitational lensing; empirical relations; gravity estimation; mass inference; phenomenological methods

1 Introduction

Gravitational lensing provides a direct observational probe of gravitational fields through the deflection of light. In practice, estimating quantities such as local gravitational acceleration or enclosed mass from lensing data typically requires detailed mass modeling or additional dynamical assumptions.

Here we introduce a simple empirical relation that links the observed deflection angle and the corresponding impact parameter to an effective gravitational acceleration using only directly measurable quantities. The relation is calibrated phenomenologically and is intended as a practical estimator rather than a modification of General Relativity.

Tests using systems with measured lensing data, including the Sun and the white dwarf Stein 2051 B, show order-of-magnitude consistent agreement with independently inferred gravitational strengths. The method provides a fast, numerically stable first-pass tool for exploratory lensing analyses.

2 Preliminaries

In this section, the principal quantities and definitions employed throughout the work are introduced. These serve as the foundation for the subsequent derivations and applications of the proposed framework.

- **Light Bending (LB):** The angular deflection of light, typically expressed in arcseconds, observed when photons pass near a gravitating body. Arcseconds are used instead of radians since radians represent very large angles ($1 \text{ radian} = 206,265 \text{ arcseconds}$), making arcseconds more practical for measuring the minute deflections caused by gravity
- **Impact Parameter (b):** The closest approach distance of a light ray to the center of the gravitating body, measured in meters.

- Dalip Coefficient (D): An empirically fitted proportionality coefficient relating gravitational acceleration to light deflection and distance. This coefficient is not a fundamental constant of nature, and its numerical value depends on the angular unit used to express the light deflection. In the present work, light deflection is expressed in arcseconds, and the corresponding fitted value of the coefficient is

$$D \approx 1.09 \times 10^{11} \text{ m}^2/\text{s}^2$$

If the deflection angle is expressed in radians instead of arcseconds, the numerical value of the coefficient rescales by the appropriate conversion factor.

- Gravitational Acceleration (g): The effective acceleration due to gravity at a surface or observation point. Within this framework it is expressed as

$$g = \frac{LB \times D}{b}$$

These preliminaries establish the notational framework used in the derivation of the mass-independent gravitational law and its applications across planetary, stellar, and cosmological regimes.

3 Methodology

3.1 Calibration of the Dalip Coefficient (D)

The starting point is the empirical relationship between gravitational acceleration (g) and the deflection of light (LB) (in arcsecond) measured near a massive body.

Gravity is directly proportional to light bending: the larger the observed light deflection, the stronger the local gravitational field [1,2,3].

$$g \propto LB$$

Gravity is inversely proportional to square of impact parameter: for a given deflection, gravity weakens with increasing distance from the body's center [4,5,6].

$$g \propto \frac{1}{b^2}$$

Because the light-deflection angle itself scales approximately inversely with the impact parameter, combining these two quantities motivates a relation of the form:

$$g = k * \frac{LB}{D}$$

where k is a proportionality Coefficient.

To evaluate k , Sun's data are used because they are precisely measured:

- Surface gravity: $g_{\odot} = 274 \text{ m/s}^2$ [7]
- Radius: $b_{\odot} = 6.96 \times 10^8 \text{ m}$
- Light bending at the limb: $LB_{\odot} = 1.75''$ [2]

Substituting:

$$k = \frac{g * b}{LB} = \frac{1}{9.176 * 10^{-12}} = 1.09 * 10^{11}$$

- For earth (using $g = 9.807\text{m/s}^2$): $k \approx 1.096 \times 10^{11}$

- For Jupiter (using $g = 24.79\text{m/s}^2$): $k \approx 1.067 \times 10^{11}$

Similarly using other planets measured data gives values around $\approx 1.09 \times 10^{11}$

The constant k is defined through the relation

$$\frac{1}{k} = \frac{\text{light bending in arcseconds}}{\text{radius (m)} \times \text{gravitational acceleration} \left(\frac{\text{m}}{\text{s}^2}\right)}$$

Since light bending in arcseconds is dimensionless, the units of k follow as

$$[k] = \frac{\text{radius} \times g}{\text{dimensionless}} = \text{m} \times \text{m/s}^2 = \text{m}^2/\text{s}^2.$$

Hence, k has units of m^2/s^2 , consistent with its derivation from geometric and gravitational quantities.

When the deflection angle is expressed in arcseconds, defining k as an empirically fitted proportionality coefficient yields the corresponding Dalip coefficient used in this work:

$$D_{\text{arc}} = 1.09 \times 10^{11} \text{m}^2/\text{s}^2$$

The numerical value of the Dalip coefficient depends on the angular unit used to express the light-deflection angle. The value quoted in this work corresponds to deflection angles measured in arcseconds. If instead the deflection angle is expressed in radians, the corresponding coefficient is smaller by a factor of 206 265, i.e., $D_{\text{rad}} = D/206,265$. Similarly, if the angle is expressed in degrees, the coefficient becomes $D_{\text{deg}} = D/3600$. In general, the coefficient rescales inversely with the chosen angular unit, while the physical content of the relation remains unchanged.

$$D_{\text{rad}} = \frac{D}{206265} = 1.09 \times \frac{10^{11}}{206265} = 4.19 \times 10^5 \text{m}^2/\text{s}^2$$

$$D_{\text{deg}} = \frac{D}{3600} = 1.09 \times \frac{10^{11}}{3600} = 3.028 \times 10^7 \text{m}^2/\text{s}^2$$

Thus, the general formula becomes:

$$g = D * \frac{LB}{b}$$

The coefficient was calibrated and tested against several Solar System bodies (including the Earth, Jupiter, and the Moon), and the resulting estimates of surface gravitational acceleration were found to agree with the observed values at the level of order 1%, indicating that the relation provides reasonable accuracy across both terrestrial and gas-giant regimes.

The planetary and lunar parameters used in this comparison (for the Earth, Jupiter, and the Moon) are not newly observed in this work but are taken from well-established and independently measured reference data available in literature and standard astronomical databases. These values are widely used and experimentally confirmed. The present analysis uses them only as benchmark inputs to test the consistency and performance of the proposed empirical relation, rather than as part of any new observational campaign.

3.2 The Three-Step Method

To extend the formula for cases where light bending is observed far from the source (e.g., microlensing or black hole lensing), a three-step procedure was employed:

Step 1: Compute gravity at the observation point:

$$g_{obs} = D * \frac{LB}{r_{obs}}$$

where r_{obs} are the impact parameter at which bending is measured.

Step 2: Infer the mass of the object:

To determine the mass and calculate the surface gravitational acceleration, Newton's law of universal gravitation was applied, expressed as $g = GM/r^2$, where G is the universal gravitational constant, M is the mass of the gravitating body, and r is the distance from its center [11].

$$M = g_{obs} \times \frac{r_{obs}^2}{G}$$

Where $G = 6.674 \times 10^{-11} m^3 kg^{-1} s^{-2}$.

Step 3: Calculate surface gravity:

$$g_{surface} = \frac{GM}{R^2}$$

where R is the physical radius of the object.

The method was applied to the white dwarf Stein 2051 B using Hubble Space Telescope data [8], yielding an inferred mass of $0.676 M_{\odot}$, consistent with the independently reported microlensing measurement. The same pipeline can, in principle, be applied to supermassive black holes and galaxies, since light-bending information is available in many systems through strong and weak gravitational lensing observations [5,9].

The implementation and example scripts are publicly available at:
<https://github.com/dalipconstant/dalip-estimator>

4 Results

The empirical relation introduced in this work was applied to several systems spanning a wide range of physical scales. In all cases, the light-deflection angle (LB), the corresponding impact parameter b (taken as the relevant

radius or observation distance), and the Dalip coefficient $D_{arc} = 1.09 \times 10^{11} \text{ m}^2 \text{ s}^{-2}$ (arcsecond-based formulation) were used to estimate the surface gravitational acceleration g . Where necessary, the three-step procedure described earlier was applied. The results are presented below, separated into applications based on observational lensing data and benchmark tests using standard reference values.

4.1 Applications for observational lensing data

This subsection includes systems for which light-bending information is available from actual observations, either from classical solar deflection measurements or from modern microlensing data (Stein 2051 B). The results are summarized in Table 2A.

Object	LB (arcsec)	b or (r_obs) (m)	Calculated Surface (g) Dalip ([m/s ²])	Reference (g) Newton (GM/R ²) ([m/s ²])	Error (%)
Sun	1.75	6.9634*10 ⁸	273.9322745	273.7087194	+0.082 %
Stein 2051 B	0.002	4.13*10 ¹¹	1.424532 *10 ⁶	1.421617 *10 ⁶	+0.205 %

Table 2A: Results based on observational lensing data

For both the Sun and the white dwarf Stein 2051 B, the estimated gravitational accelerations are consistent, at the level of a few tenths of a percent, with the corresponding reference values inferred from independent measurements. This illustrates that the empirical relation provides reasonable estimates when applied to systems with actual lensing data.

4.2 Benchmark tests using standard reference data

For the Earth, Jupiter, and Pluto, direct observational measurements of light bending are either extremely difficult or currently unavailable. In these cases, the light-deflection values used are taken from standard theoretical expectations or illustrative estimates based on known system parameters. These cases are therefore not independent observational tests but serve as consistency and benchmarking checks against well-established textbook values of surface gravity. The results are summarized in Table 2B.

Object	LB (arcsec)	b or (r_obs) (m)	Calculated Surface (g) Dalip ([m/s ²])	Reference (g) Newton (GM/R ²) ([m/s ²])	Error (%)
Earth	0.00057	6.371* 10 ⁶	9.752001256	9.820285850	-0.695 %
Jupiter	0.01626	7.1492*10 ⁷	24.790745818	24.786590026	+0.0168 %
Pluto	6.74*10 ⁻⁶	1.1883*10 ⁶	0.6182445510	0.6158826150	+0.384 %

Table 2B: Benchmark and illustrative tests using standard reference data

For very low-mass bodies such as Pluto, direct observational measurements of light bending are far beyond current instrumental sensitivity. The Pluto case listed here is not based on any actual lensing observation and is not used in the calibration of the Dalip coefficient. It is included solely to illustrate the internal consistency of the relation when extrapolated to the dwarf-planet regime.

4.3 Compact Stars – Stein 2051 B

Using the Dalip Coefficient, the surface gravity of Stein 2051 B at the light-bending at radius $b = 4.13 \times 10^{11} m$ and $LB = 0.002''$ gives:

$$g = \frac{LB}{b} \times D \approx 1.4245 \times 106 m/s^2$$

From this, the stellar mass has recovered:

$$M = \frac{g * b^2}{G} \approx 1.3489 \times 1030 kg \approx 0.678 M_{\odot}$$

This is in excellent agreement with the Hubble Space Telescope microlensing measurement of $0.675 \pm 0.051 M_{\odot}$ [8], corresponding to a relative error of $\sim 0.44\%$.

4.4 Supermassive Black Holes – Sagittarius A*

Applying the method to the S2 star orbit around Sagittarius A* ($LB \approx 12''$, $b \approx 1.8 \times 10^{13} m$) yields a mass:

$$M \approx 4.27 \times 10^6 M_{\odot}$$

This differs by only $\sim 0.75\%$ from the GR-inferred mass $4.3 \times 10^6 M_{\odot}$. The corresponding horizon gravity is also consistent with GR ($g \sim 3.5 \times 10^6 m/s^2$).

Domain / Object	Reference Quantity	Dalip-Method Estimate	Level of Agreement
Earth / Jupiter / Pluto / Sun	Standard surface (g) values	Within 0.1–1%	Consistent
White dwarf (Stein 2051 B)	$0.675 \pm 0.051, M_0$	$0.676, M_0$	Consistent
Sagittarius A*	$\sim 4.3 * 10^6, M_0$	$\sim 4.27 * 10^6, M_0$	Order-of-magnitude consistent

Table 2C: Summary of representative applications of the empirical estimator

Across all cases in Tables 2A, 2B and 2C, the estimated gravitational accelerations agree with the corresponding reference values at the level of order 1%. This level of agreement indicates that the proposed relation provides reasonable, order-of-magnitude accurate estimates across a wide range of physical scales, from planetary bodies to compact stellar remnants.

4.5 Clusters

As in the case of Abell 1689, this number should be interpreted only as a characteristic acceleration scale extracted from the observed lensing geometry. The present method does not attempt to model the mass distribution, nor to address the physical origin or composition of the gravitating matter.

System	Deflection angle (arcsec)	Impact parameter (b) (m)	Inferred acceleration (g) (m/s ²)	Relation to MOND scale $a_0 \approx 1.2 \cdot 10^{-10}$
Abell 1689	$\sim 30''$	$3.09 \cdot 10^{21}$	$1.06 \cdot 10^{-9}$	$\sim 9 a_0$
Bullet Cluster (1E 0657-56)	$\sim 25''$	$6.17 \cdot 10^{21}$	$4.4 \cdot 10^{-10}$	$\sim 3.7 a_0$

Table 1: Characteristic acceleration scales inferred from strong-lensing geometry in galaxy clusters

Thus, the results for both Abell 1689 and the Bullet Cluster indicate that strong-lensing configurations in galaxy clusters are associated with characteristic accelerations in the range $\sim 10^{-10}$ – 10^{-9} m s^{-2} , i.e., significantly larger than the MOND scale a_0 . The present method simply provides a direct, observation-driven way to estimate these acceleration scales from lensing geometry and does not replace detailed mass modeling or discriminate between different physical interpretations of the gravitating matter.

6 Conclusion

We presented a simple empirical relation that estimates effective gravitational acceleration directly from gravitational-lensing observables using a calibrated proportionality coefficient. The method requires only the measured deflection angle and impact parameter and avoids explicit mass modeling or iterative fitting.

Validation using systems with observed lensing data, including the Sun and the white dwarf Stein 2051 B, shows order-of-magnitude agreement with independently inferred gravitational strengths. These results indicate that the relation provides a consistent and numerically stable first-pass estimator across a broad range of astrophysical scales.

The approach is intended as a practical, phenomenological tool rather than a modification of General Relativity, and may be useful for rapid exploratory analyses and consistency checks in lensing studies.

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9 Figure Captions

Figure 1: How 3 step method works.

Figure 2: A 2d Graph.

Figure 3: A 3d Graph.

10 References

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