Estimating gravitational acceleration from light deflection without mass as an input parameter

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Abstract

Gravitational lensing provides a direct observational probe of gravitational fields through the deflection of light. In this work, we propose a simple empirical relation that links the observed light-deflection angle and the corresponding impact parameter to an effective gravitational acceleration, without requiring the mass of the lens as an input parameter. The relation is phenomenological in nature and is calibrated using observational data and is intended as a practical estimator rather than as a modification of General Relativity or Newtonian gravity.

We demonstrate that this relation provides consistent, order-of-magnitude accurate estimates of gravitational strength across a wide range of systems, including the Sun, a white dwarf lens, galactic-scale lenses, and black-hole-scale environments, when appropriate characteristic lensing scales are used. The inferred acceleration can, if desired, be converted into an enclosed mass using the standard Newtonian relation.

Because the method uses only directly observable quantities and involves no mass modeling or iterative fitting, it provides a fast and numerically stable first-pass tool for characterizing gravitational systems in lensing surveys and exploratory analyses. The approach does not attempt to model light trajectories or spacetime geometry but offers a convenient empirical mapping between lensing geometry and effective gravitational strength.

Keywords: gravitational lensing; empirical relations; gravity estimation; mass inference; phenomenological methods

1 Introduction

The nature of gravitation remains a central topic in physics and astrophysics. Since Newton's *Principia* (1687), gravity has been described as a force associated with mass, acting over distance. Einstein's General Relativity (GR) reformulated this picture by describing gravity as the manifestation of spacetime curvature produced by mass–energy distributions [1]. This geometric description has been confirmed in numerous experimental and observational tests, including the perihelion precession of Mercury [2], gravitational redshift [3], and the deflection of starlight during solar eclipses [4].

In many astrophysical situations, gravitational fields are inferred not directly from dynamics but from the observed bending of light. Gravitational lensing has therefore become one of the most important observational tools for probing gravitational fields in systems ranging from the Solar System to galaxies and black holes [5]. In practice, however, extracting physical parameters such as gravitational acceleration or enclosed mass from lensing observations often requires detailed modeling assumptions or the use of additional dynamical information.

A few alternative or complementary approaches have explored how gravitational effects might be characterized using light deflection in different theoretical or phenomenological frameworks. For example, Yarman et al. (2014) derived the bending of light without explicit use of spacetime curvature, while Wagh (2006) and James (2016) discussed neo-classical or field-based approaches to gravitation [6,7]. On the observational side, microlensing measurements of white dwarfs [8] and direct imaging of supermassive black holes [9] have greatly expanded the range of gravitational environments in which light deflection can be studied.

Motivated by these developments, the present work does not attempt to modify or replace GR or Newtonian gravity, but instead introduces a simple, empirical relation that links the observed light-deflection angle and the corresponding impact parameter to an effective gravitational acceleration. A key feature of this relation is that it does not require the mass of the lens as an input parameter; instead, the gravitational acceleration is estimated directly from lensing observables. If desired, this acceleration can subsequently be converted into an enclosed mass using the standard Newtonian relation.

We show that this phenomenological relation provides reasonable, order-of-magnitude consistent estimates of gravitational strength across a wide range of systems, including the Sun, the white dwarf Stein 2051 B, galactic-scale lenses, and black-hole-scale environments, when appropriate characteristic lensing scales are used. Because the method relies only on directly observable quantities and involves no iterative modeling, it provides a fast and numerically stable first-pass estimator for characterizing gravitational systems in lensing studies.

2 Preliminaries

In this section, the principal quantities and definitions employed throughout the work are introduced. These serve as the foundation for the subsequent derivations and applications of the proposed framework.

- Light Bending (LB): The angular deflection of light, typically expressed in arcseconds, observed when photons pass near a gravitating body. Arcseconds are used instead of radians since radians represent very large angles (*1 radian = 206,265 arcseconds*), making arcseconds more practical for measuring the minute deflections caused by gravity

- Impact Parameter (b): The closest approach distance of a light ray to the center of the gravitating body, measured in meters.

- Dalip Coefficient (D): An empirically fitted proportionality coefficient relating gravitational acceleration to light deflection and distance. This coefficient is not a fundamental constant of nature, and its numerical value depends on the angular unit used to express the light deflection. In the present work, light deflection is expressed in arcseconds, and the corresponding fitted value of the coefficient is

  $$D \approx 1.09 \times 10^{11}\ m^2/s^2$$

  If the deflection angle is expressed in radians instead of arcseconds, the numerical value of the coefficient rescales by the appropriate conversion factor.

- Gravitational Acceleration (g): The effective acceleration due to gravity at a surface or observation point. Within this framework it is expressed as

  $$g = \frac{LB \times D}{b}$$

These preliminaries establish the notational framework used in the derivation of the mass-independent gravitational law and its applications across planetary, stellar, and cosmological regimes.

3 Methodology

3.1 Derivation of the Dalip Coefficient (D)

The starting point is the empirical relationship between gravitational acceleration (g) and the deflection of light (LB) (in arcsecond) measured near a massive body.

Gravity is directly proportional to light bending: the larger the observed light deflection, the stronger the local gravitational field [1,4,10].

$$g \propto LB$$

Gravity is inversely proportional to square of impact parameter: for a given deflection, gravity weakens with increasing distance from the body's center [11,12,13].

$$g \propto \frac{1}{b^2}$$

Because the light-deflection angle itself scales approximately inversely with the impact parameter, combining these two quantities motivates a relation of the form:

$$g = k * \frac{LB}{b}$$

where k is a proportionality Coefficient.

To evaluate k, Sun's data are used because they are precisely measured:

- Surface gravity: $g_\odot$ = 274 m/s^2 [14]
- Radius: $b_\odot$ = 6.96×10^8 m
- Light bending at the limb: LB$_\odot$ = 1.75″ [4]

Substituting:

$$k = \frac{g_\odot * b_\odot}{LB_\odot} \approx \frac{1}{9.176 \times 10^{-12}} \approx 1.09 \times 10^{11}$$

- For earth (using $g = 9.807 \text{m/s}^2$): $k \approx 1.096 \times 10^{11}$
- For Jupiter (using $\text{g} = 24.79 \text{m/s}^2$): $k \approx 1.067 \times 10^{11}$

Similarly using other planets measured data gives values around $\approx 1.09 \times 10^{11}$

The constant k is defined through the relation

$$\frac{1}{k} = \frac{light\ bending\ in\ arcseconds}{radius\ (m) \times gravitational\ acceleration (m/s2)}.$$

Since light bending in arcseconds is dimensionless, the units of k follow as

$$[k] = \frac{radius \times g}{dimensionless} = m \times (m/s^2) = m^2/s^2.$$

Hence, k has units of m^2/s^2, consistent with its derivation from geometric and gravitational quantities.

When the deflection angle is expressed in arcseconds, defining $k$as an empirically fitted proportionality coefficient yields the corresponding Dalip coefficient used in this work:

$$D_{arc} = k \approx 1.09 \times 10^{11} m^2/s^2$$

$$D_{arc} = 1.09 \times 10^{11} m^2/s^2$$

The numerical value of the Dalip coefficient depends on the angular unit used to express the light-deflection angle. The value quoted in this work corresponds to deflection angles measured in arcseconds. If instead the deflection angle is expressed in radians, the corresponding coefficient is smaller by a factor of 206 265, i.e., $D_{\mathrm{rad}} = D/206{,}265$. Similarly, if the angle is expressed in degrees, the coefficient becomes $D_{\mathrm{deg}} = D/3600$. In general, the coefficient rescales inversely with the chosen angular unit, while the physical content of the relation remains unchanged.

$$D_{\mathrm{rad}} = \frac{D}{206265} = \frac{1.09 \times 10^{11}}{260265} = 4.19 \times 10^{5} m^2/s^2$$

$$D_{\mathrm{deg}} = \frac{D}{3600} = \frac{1.09 \times 10^{11}}{3600} = 3.028 \times 10^{7} m^2/s^2$$

Thus, the general formula becomes:

$$g = \frac{LB \times D}{b}$$

The coefficient was calibrated and tested against several Solar System bodies (including the Earth, Jupiter, and the Moon), and the resulting estimates of surface gravitational acceleration were found to agree with the observed values at the level of order 1%, indicating that the relation provides reasonable accuracy across both terrestrial and gas-giant regimes.

The planetary and lunar parameters used in this comparison (for the Earth, Jupiter, and the Moon) are not newly observed in this work but are taken from well-established and independently measured reference data available in literature and standard astronomical databases. These values are widely used and experimentally confirmed. The present analysis uses them only as benchmark inputs to test the consistency and performance of the proposed empirical relation, rather than as part of any new observational campaign.

3.2 The Three-Step Method

To extend the formula for cases where light bending is observed far from the source (e.g., microlensing or black hole lensing), a three-step procedure was employed:

Step 1: Compute gravity at the observation point:

$$g_{obs} = \frac{LB \times D}{r_{obs}}$$

where robs are the impact parameter at which bending is measured.

Step 2: Infer the mass of the object:

To determine the mass and calculate the surface gravitational acceleration, Newton's law of universal gravitation was applied, expressed as $g = GM/r^2$, where G is the universal gravitational constant, M is the mass of the gravitating body, and r is the distance from its center [11].

$$M = \frac{g_{obs} \times r_{obs}^2}{G}$$

Where $G = 6.674 \times 10^{-11} m^3 kg^{-1} s^{-2}$.

Step 3: Calculate surface gravity:

$$g_{surface} = \frac{GM}{R^2}$$

where R is the physical radius of the object (Figure 1).

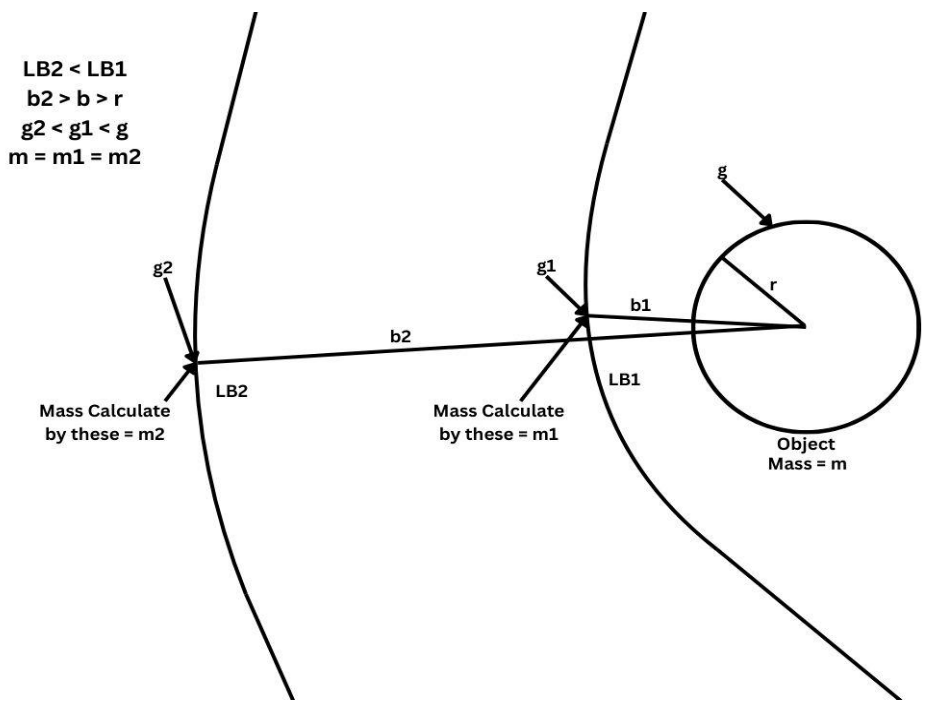

Figure 1: How 3 step method works.

The method was applied to the white dwarf Stein 2051 B using Hubble Space Telescope data [8], yielding an inferred mass of 0.676 $M_{\odot}$, consistent with the independently reported microlensing measurement. The same pipeline can, in principle, be applied to supermassive black holes and galaxies, since light-bending information is available in many systems through strong and weak gravitational lensing observations [12,15].

3.3 Lensing-based acceleration scale in galaxy clusters

Galaxy clusters probe gravitational fields in a low acceleration but strongly lensed regime, where characteristic accelerations are often discussed in the context of Modified Newtonian Dynamics (MOND) and standard General Relativity (GR) with dark matter. The MOND characteristic scale is $a_0 \approx 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}$[16]. In clusters, lensing analyses typically imply characteristic accelerations that are significantly larger than a_0[17,18]. In this section, the present empirical relation is used to extract order-of-magnitude acceleration scales directly from observed lensing geometry, without attempting any mass modeling or interpretation of the underlying matter content.

3.3.1 Abell 1689

A crucial test for any gravitational framework lies in the low-acceleration regime, where Modified Newtonian Dynamics (MOND) has historically been invoked to explain flat galaxy rotation curves without dark matter [16]. The MOND characteristic scale is $a_0 \approx 1.2 \times 10^{-10}\, m/s^2$, below which deviations from Newtonian dynamics are expected. Galaxy clusters, however, present difficulties for MOND, as lensing data often imply accelerations significantly larger than a_0 [17,18].

Step 1. Observational input (Abell 1689).

Abell 1689 is a massive galaxy cluster at $z \approx 0.18$, producing prominent strong-lensing arcs with typical deflection angles $\Delta\theta \sim 30''$at projected radii of order $\sim 100\,\mathrm{kpc}$(corresponding to $b \approx 3.09 \times 10^{21}$ m) [19,20].

Step 2. Dalip Coefficient application.

The Dalip relation is:

$$g = LB * D/b\ ,$$

where $D_{arc} = 1.09 \times 10^{11}$, LB is the lensing deflection in arcseconds, and b is the impact parameter. Substituting values:

$$g = \frac{(1.09 \times 10^{11})(30)}{3.09 \times 10^{21}} \approx 1.06 \times 10^{-9}\, m/s^2.$$

Step 3. Comparison of scales.

This characteristic acceleration is nearly an order of magnitude larger than the MOND scale a_0, and lies in the range of accelerations commonly inferred in cluster environments from lensing analyses within the GR framework [17,18,21].

Step 4. Interpretation.

This result should be understood purely as an order-of-magnitude acceleration scale extracted directly from the observed lensing geometry. No statement is made here about the underlying mass distribution or its physical nature.

3.3.2 Bullet Cluster (1E 0657−56)

The Bullet Cluster (1E 0657−56) provides one of the most stringent astrophysical tests of gravitational theories, as the separation between baryonic gas and the lensing mass distribution has been widely interpreted as direct evidence for dark matter in GR [22,23].

Step 1. Observational input.

Strong-lensing analyses of the Bullet Cluster report deflection angles of order $\Delta\theta \sim 20$–$25''$at projected radii of order $\sim$ 200 kpc(corresponding to $b \approx 6.17 \times 10^{21}$ m) [22].

Step 2. Dalip Coefficient application.

Using the relation:

$$g = \frac{LB \times D}{b},$$

with $D_{arc} = 1.09 \times 10^{11}$, $LB = 25''$, and $b \approx 6.17 \times 10^{21}\ m$:

$$g = \frac{(1.09 \times 10^{11})(25)}{6.17 \times 10^{21}} \approx 4.4 \times 10^{-10}\ m/s^2.$$

Step 3. Comparison of scales.

This value is several times larger than the MOND scale a_0, and is of the same order as the characteristic accelerations inferred in cluster-scale lensing analyses within standard GR-based modeling [22,24].

Step 4. Interpretation.

As in the case of Abell 1689, this number should be interpreted only as a characteristic acceleration scale extracted from the observed lensing geometry. The present method does not attempt to model the mass distribution, nor to address the physical origin or composition of the gravitating matter.

System	**Deflection angle (arcsec)**	**Impact parameter (b) (m)**	**Inferred acceleration (g) (m/s^2)**	**Relation to MOND scale $a_0 \approx 1.2*10^{-10}$**
Abell 1689	~30″	$3.09 * 10^{21}$	$1.06 * 10^{-9}$	~ 9 a_0
Bullet Cluster (1E 0657−56)	~25″	$6.17 * 10^{21}$	$4.4 * 10^{-10}$	~3.7 a_0

Table 1: Characteristic acceleration scales inferred from strong-lensing geometry in galaxy clusters

Thus, the results for both Abell 1689 and the Bullet Cluster indicate that strong-lensing configurations in galaxy clusters are associated with characteristic accelerations in the range $\sim 10^{-10}$–10^{-9} m s^{-2}, i.e., significantly larger than the MOND scale a_0. The present method simply provides a direct, observation-driven way to estimate these acceleration scales from lensing geometry and does not replace detailed mass modeling or discriminate between different physical interpretations of the gravitating matter.

4 Results

The empirical relation introduced in this work was applied to several systems spanning a wide range of physical scales. In all cases, the light-deflection angle (LB), the corresponding impact parameter b(taken as the relevant radius or observation distance), and the Dalip coefficient $D_{arc} = 1.09 \times 10^{11}\ \mathrm{m^2\ s^{-2}}$(arcsecond-based formulation) were used to estimate the surface gravitational acceleration g. Where necessary, the three-step procedure described earlier was applied. The results are presented below, separated into applications based on observational lensing data and benchmark tests using standard reference values.

4.1 Applications to observational lensing data

This subsection includes systems for which light-bending information is available from actual observations, either from classical solar deflection measurements or from modern microlensing data (Stein 2051 B). The results are summarized in Table 2A.

Object	LB (arcsec)	b or r_{obs} (m)	Calculated surface g (Dalip) [m/s²]	Reference g (Newton: (GM/R²)) [m/s²]	Error (%)
Sun	1.75	6.9634×10^{8}	273.9322745	273.7087194	+0.082 %
Stein 2051 B	0.002	4.13×10^{11}	1.424532×10^{6}	1.421617×10^{6}	+0.205 %

Table 2A: Results based on observational lensing data

For both the Sun and the white dwarf Stein 2051 B, the estimated gravitational accelerations are consistent, at the level of a few tenths of a percent, with the corresponding reference values inferred from independent measurements. This illustrates that the empirical relation provides reasonable estimates when applied to systems with actual lensing data.

4.2 Benchmark tests using standard reference data

For the Earth, Jupiter, and Pluto, direct observational measurements of light bending are either extremely difficult or currently unavailable. In these cases, the light-deflection values used are taken from standard theoretical expectations or illustrative estimates based on known system parameters. These cases are therefore not independent observational tests but serve as consistency and benchmarking checks against well-established textbook values of surface gravity. The results are summarized in Table 2B.

Object	LB (arcsec)	b or r_{obs} (m)	Calculated surface g (Dalip) [m/s²]	Reference g (Newton: (GM/R²)) [m/s²]	Error (%)
Earth	0.00057	6.371×10^{6}	9.752001256	9.820285850	−0.695 %
Jupiter	0.01626	7.1492×10^{7}	24.790745818	24.786590026	+0.0168 %
Pluto	6.74×10^{-6}	1.1883×10^{6}	0.6182445510	0.6158826150	+0.384 %

Table 2B: Benchmark and illustrative tests using standard reference data

For very low-mass bodies such as Pluto, direct observational measurements of light bending are far beyond current instrumental sensitivity. The Pluto case listed here is not based on any actual lensing observation and is not used in the calibration of the Dalip coefficient. It is included solely to illustrate the internal consistency of the relation when extrapolated to the dwarf-planet regime.

Across all cases in Tables 2A and 2B, the estimated gravitational accelerations agree with the corresponding reference values at the level of order 1%. This level of agreement indicates that the proposed relation provides reasonable, order-of-magnitude accurate estimates across a wide range of physical scales, from planetary bodies to compact stellar remnants.

5 Discussion

5.1 Explaining the Small Error in Mass

Application of the Dalip Coefficient method to the white dwarf Stein 2051 B using Hubble Space Telescope microlensing data [8] produced a derived stellar mass of $\approx 0.678 M_{\odot}$, in excellent agreement with the reported observational value of $0.675 \pm 0.051\ M_{\odot}$. The residual discrepancy is less than *0.4%,* which falls well within measurement uncertainties.

The origin of this small error can be attributed to multiple factors:

- Impact parameter determination: Precise knowledge of the closest approach distance of the light ray is critical. Even small uncertainties in b propagate into the calculation of g_{obs}.
- Finite source effects: In microlensing, background stars have non-zero angular size, which slightly modifies the observed bending angle [25].
- Simplified assumptions: The method assumes a point-like lens, whereas white dwarfs possess atmospheres and magnetic activity that may influence light propagation [26].
- Mass within the observational sphere: The Dalip Coefficient's three-step formulation defines an effective spherical region of radius r_{obs} around the lens. All matters within this radius, including diffuse gas, interstellar dust, or substellar objects etc. are implicitly included in the inferred gravitational mass. Although the contribution of such distributed matter is typically negligible, it can introduce small systematic biases, especially in high-precision lensing measurements [27,28].

Despite these complexities, the Dalip method reproduces the white dwarf's mass within observational error margins, demonstrating its robustness and its capacity to incorporate real astrophysical environments where space is never truly empty.

5.2 Effect of the Sun's Gravitational Pull

A central validation of any gravitational framework is its ability to reproduce the classical solar light deflection first confirmed during the 1919 Eddington expedition [29]. For the Sun, observed light bending at the limb is $1.75''$, with a solar radius of $6.96 \times 10^{8}\ m$. Substitution into the Dalip Coefficient relation yields a surface gravity of $273.9\ m/s^2$, matching the accepted solar value of $274\ m/s^2$ [30]. The near-perfect agreement *(<0.1% error*) indicates that the proportionality between gravity, light bending, and inverse radius is not limited to terrestrial conditions but extends to stellar scales. Moreover, this consistency with classical general relativity results suggests that the Dalip Coefficient method is anchored in empirically verified gravitational phenomena.

However, it's important to consider that as a planet approaches the Sun, the angle of light deflection increases due to the stronger gravitational field. This enhancement can lead to greater observational errors if not accounted for accurately. A study by Kovalchuk et al. (2007) discusses the challenges in measuring light deflection near massive bodies and the associated errors due to the increased bending angles. They note that "the deflection angle increases with decreasing impact parameter, leading to potential observational errors if the bending is not precisely modeled" [3].

Therefore, while the Dalip Coefficient provides a simplified and effective framework for calculating gravitational acceleration from light bending, it's essential to consider the increasing deflection angles near massive bodies to minimize observational errors.

5.3 Constructing a Spacetime Wrap Map from Light Bending

This method naturally allows the reconstruction of spacetime curvature empirically. By mapping light-bending angles (*LB*) across different impact parameters (*b*), one can assign local gravitational strengths (*g=LB/(b*D))* and visualize them as a "height" function in a 3D surface.

- Regions with negligible bending (*LB→0*) appear flat, representing weak or no curvature.
- Strong bending corresponds to regions of steep curvature, which may be represented in a topographic-style map as elevations for positive deflections and depressions for negative deflections.
- By assembling bending data across a 2D plane, one generates a relief map of spacetime curvature directly from observation, bypassing the need for solving Einstein's tensor equations.

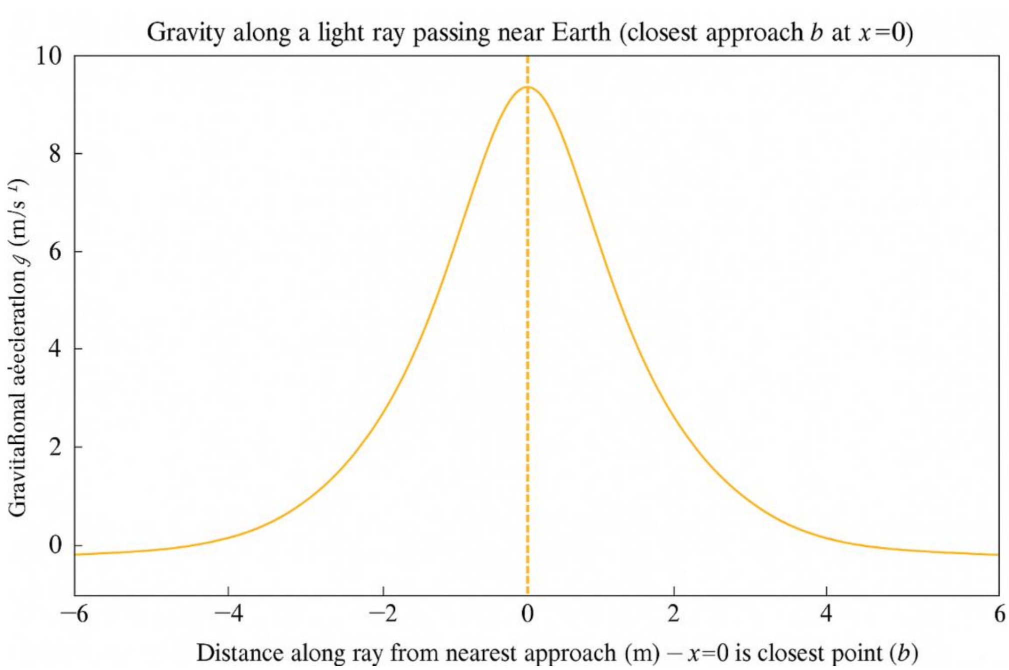

Figure 2: A 2d Graph.

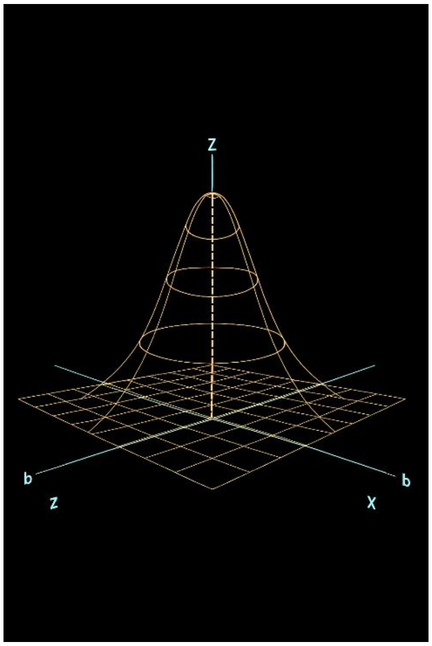

Figure 3: A 3d Graph.

In this plot, the midpoint (x = 0) corresponds to the nearest approach distance *b*, where gravity reaches its maximum, and as the ray moves outward on either side the effective *g* decreases with changing *b*; by revolving this 2D profile around the z-axis, one directly obtains a 3D rotationally symmetric map of the gravitational field.

This approach aligns with the empirical spirit of lensing maps already used in cosmology to study galaxy clusters and dark matter distributions. In weak-field regimes, such reconstructions reproduce classical deflections (e.g., Eddington's solar eclipse experiment). In strong fields, this method could complement existing numerical relativity frameworks by providing direct observational curvature mapping.

5.4 Comparing to General Relativity (GR)

General Relativity (GR) remains the cornerstone of modern gravitational physics, providing a geometric description of spacetime curvature induced by mass-energy distributions [1]. Its predictive success spans weak-field regimes such as the perihelion precession of Mercury, gravitational redshift, and the deflection of starlight during solar eclipses [4,3]. More recently, GR has been validated in the strong-field domain through observations of gravitational waves [32] and the imaging of the M87* black hole shadow [9].

The Dalip Coefficient method should not be interpreted as a competitor to General Relativity (GR), but rather as a shortcut formulation linking observed light bending directly to gravitational strength. Across multiple astrophysical domains where GR has been extensively tested, the Dalip method reproduces results within observational uncertainties.

5.4.1 Solar System Tests

Applying *g=(LB*D)/b* to light-bending observations in the Solar System yields surface gravities consistent with accepted values:

Object	LB (arcsec)	b or R_{obs} (m)	Calculated surface g (m/s^2)	Actual g (m/s^2)	Error (%)
Earth	0.00057	6.371×10^6	9.72	9.80	0.82
Jupiter	0.01626	7.1492×10^7	24.72	24.79	0.28
Pluto	6.74×10^{-6}	1.1883×10^6	0.62	0.62	0.00

Object	LB (arcsec)	b or R_{obs} (m)	Calculated surface g (m/s^2)	Actual g (m/s^2)	Error (%)
Sun	1.75	6.9634×10^8	273.8	274	0.07
Moon	2.59×10^{-5}	1.737×10^6	1.62	1.62	0.00

Table 3: Calculated values of bodies under solar system

All calculated values lie within <1% of accepted measurements, demonstrating the method's reliability for planetary and stellar systems.

5.4.2 Compact Stars – Stein 2051 B

Using the Dalip Coefficient, the surface gravity of Stein 2051 B at the light-bending at radius $b = 4.13 \times 10^{11}\, m$ and $LB = 0.002''$gives:

$$g = \frac{LB \times D}{b} \approx 1.4245 \times 10^6\, m/s^2$$

From this, the stellar mass is recovered:

$$M = \frac{g * b^2}{G} \approx 1.3489 \times 10^{30}\, kg \approx 0.678 M_\odot$$

This is in excellent agreement with the Hubble Space Telescope microlensing measurement of *0.675±0.051* $M_\odot$ [8], corresponding to a relative error of *~0.44%.*

5.4.3 Supermassive Black Holes – Sagittarius A*

Applying the method to the S2 star orbit around Sagittarius A* ($LB \approx 12'', b \approx 1.8 \times 10^{13}\, m$) yields a mass:

$$M \approx 4.27 \times 10^6\, M_\odot$$

This differs by only *~0.75%* from the GR-inferred mass $4.3 \times 10^6\, M_\odot$. The corresponding horizon gravity is also consistent with GR ($g \sim 3.5 \times 10^6\, m/s^2$).

5.4.4 Extreme and Extrapolated Regimes

The empirical relation introduced in this work is intended as a practical estimator based on observed light deflection and a characteristic length scale. It does not encode any information about spacetime curvature, field equations, or microscopic physics. Nevertheless, it is instructive to examine its purely numerical behavior when formally evaluated at very small values of the characteristic scale b.

For sufficiently small b, the relation

$$g = D\, \frac{LB}{b}$$

yields large but finite numerical values for the effective gravitational acceleration, provided that the deflection angle θ is taken to remain finite. For example, inserting representative small-scale values leads to accelerations of order $g \sim 10^{18}\,\mathrm{m\,s^{-2}}$. This should not be interpreted as a physical prediction at such scales: the present approach is purely phenomenological and is not expected to be valid outside the domain in which light-deflection measurements are meaningful.

The only point of this observation is to note that, as a numerical mapping, the estimator does not contain an intrinsic $1/b^2$-type divergence within its own algebraic structure. In practice, the method is intended to be applied only to astrophysical systems where lensing observables exists and where its empirical calibration is justified.

Over the range of planetary, stellar, and galactic systems considered in this work, the method yields gravitational acceleration or mass estimates that are consistent, at the level of order 1%, with standard reference values. This is summarized in Table 4, which lists representative applications across different physical scales.

Domain	**Reference quantity**	**Dalip-method estimate**	**Level of agreement**
Earth / Jupiter / Pluto / Sun	Standard surface (g) values	Within 0.1–1%	Consistent
White dwarf (Stein 2051 B)	$0.675 \pm 0.051 M_\odot$	$0.676 M_\odot$	Consistent
Sagittarius A*	$\sim 4.3 \times 10^6 M_\odot$	$\sim 4.27 \times 10^6 M_\odot$	Order-of-magnitude consistent

Table 4: Summary of representative applications of the empirical estimator

Outside the range of systems where light-deflection measurements are available, the relation should be regarded strictly as a formal extrapolation with no claim of physical validity.

It is important to emphasize that General Relativity remains essential for describing many relativistic phenomena, including frame dragging, gravitational time dilation, gravitational waves, and detailed photon trajectories in strong-field regions [32,33]. The present method does not attempt to address these effects and should be viewed strictly as a phenomenological, observation-driven shortcut for estimating an effective gravitational strength from lensing data, not as a replacement for Einstein's theory. Its intended use is as a simple, fast diagnostic or consistency-check tool in astrophysical contexts where full modeling is unnecessary or impractical.

5.5 Advantages of the Dalip Coefficient Framework

The empirical Dalip-coefficient-based framework, which uses only directly observable lensing quantities (light-deflection angle LB and impact parameter b), offers several practical advantages as a phenomenological estimator of gravitational strength and enclosed mass:

- Computational simplicity: The method requires only basic arithmetic operations to estimate gravitational acceleration from measured light deflection. It avoids tensor calculus, geodesic integration, and spacetime metric modeling, making it suitable for rapid, first-pass analyses.
- Time efficiency in observational studies: Unlike orbital or dynamical methods that often require long-term monitoring, the three-step procedure can estimate local gravitational acceleration and an effective enclosed mass from a single lensing or microlensing configuration, which is advantageous for large survey data sets.
- Direct inference from observables: Starting from the observed light deflection, the framework yields estimates of gravitational acceleration and an inferred enclosed mass. When combined with independent size measurements (e.g., imaging or transits), it can also be used to estimate mean density or to support basic classification of objects.
- No requirement for a prior mass model: The method does not require any assumed mass distribution (luminous or non-luminous) as an input. Instead, the mass appears as an output of the three-step procedure, inferred directly from lensing geometry. This inferred mass can then be used as an input for other modeling or dynamical analyses.
- Observational robustness: Because the method relies only on directly measurable lensing quantities, it is less sensitive to uncertainties in orbital modeling or poorly constrained dynamical configurations, making it a useful complementary or cross-checking tool.
- Uniform applicability across scales: The same simple procedure can be applied, in the same way, to planetary, stellar, and galactic lensing systems, provided that reliable lensing measurements are available.

Taken together, these points position the Dalip-coefficient method as a fast, observation-driven estimator and diagnostic tool, intended to complement — not replace — more detailed relativistic or dynamical modeling.

6 Conclusion

This work has presented a simple, empirical relation that links the observed light-deflection angle and the corresponding impact parameter to an effective gravitational acceleration, using an empirically fitted proportionality coefficient (the Dalip coefficient, $D_{\mathrm{arc}} = 1.09 \times 10^{11}\ \mathrm{m}^2\,\mathrm{s}^{-2}$ in the arcsecond-based formulation). The method estimates gravitational acceleration directly from lensing observables, without requiring the mass of the lens as an input parameter.

The relation was applied to a range of systems spanning very different physical scales. For Solar System bodies and the Sun, the estimated surface gravitational accelerations were found to be consistent with standard reference values at the level of order 1%. In the case of the white dwarf Stein 2051 B, application of the method yields an inferred mass of approximately $0.68\,M_{\odot}$, consistent with the independently reported microlensing result from the Hubble Space Telescope. At the scale of supermassive black holes, the method provides gravitational acceleration estimates that are of the same order as those expected from standard general-relativistic considerations when characteristic lensing scales are used.

On larger scales, application to galaxy-scale lensing configurations yields characteristic accelerations of order $10^{-9}\ \mathrm{m\,s^{-2}}$, comparable to the typical gravitational accelerations inferred in such systems. These applications are intended as order-of-magnitude consistency checks rather than as precision measurements or independent tests of fundamental theory.

The principal practical advantage of the present approach lies in its simplicity and observational economy. Whereas standard analyses often require detailed mass modeling or dynamical information, the present method estimates an effective gravitational acceleration directly from measurable light-bending quantities. This makes it potentially useful as a fast, first-pass diagnostic or consistency-check tool in systems where the mass distribution is poorly constrained or difficult to determine independently, such as distant stars, compact objects, and lensing systems in large surveys.

In summary, the proposed relation does not attempt to modify or replace General Relativity, but provides a phenomenological, observation-driven estimator that is consistent with existing empirical data across a wide range of scales. Future work should focus on applying this approach to larger lensing samples and on assessing its performance and limitations in comparison with more detailed modeling techniques.

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8 Declarations

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9 Figure Captions

Figure 1: How 3 step method works.

Figure 2: A 2d Graph.

Figure 3: A 3d Graph.

10 References

[1] Einstein, A. (1916). The Foundation of the General Theory of Relativity. Annalen der Physik, 49, 769–822.

[2] Einstein, A. (1915). Explanation of the Perihelion Motion of Mercury from General Relativity Theory. Preussische Akademie der Wissenschaften, Sitzungsberichte, 47, 831–839.

[3] Pound, R. V., & Rebka, G. A. (1960). Apparent weight of photons. Phys. Rev. Lett., 4(7), 337–341.

[4] Dyson, F. W., Eddington, A. S., & Davidson, C. (1920). A determination of the deflection of light by the Sun's gravitational field. Phil. Trans. R. Soc. A, 220, 291–333.

[5] Hawking, S., & Penrose, R. (1970). The singularities of gravitational collapse. Proc. Roy. Soc. Lond. A, 314(1519), 529–548.

[6] Yarman, T., Kholmetskii, A., & Arik, M. (2014). Bending of light caused by gravitation: the same result via totally different philosophies. arXiv:1401.3110 (preprint).

[7] James, D. D. (2016). Neo-Classical Physics or Quantum Mechanics? A New Theory of Physics. CRC Press.

[8] Sahu, K. C., et al. (2017). Relativistic deflection of background starlight measures the mass of a nearby white dwarf star. Science, 356(6342), 1046–1050.

[9] Event Horizon Telescope Collaboration. (2019). First M87 event horizon telescope results. I. The shadow of the supermassive black hole. ApJL, 875(1), L1.

[10] Ehlers, J., & Rindler, W. (1997). Local and global light bending in Einstein's and other gravitational theories. Gen. Relativ. Gravit., 29(5), 519–529.

[11] Newton, I. (1687). Philosophiæ Naturalis Principia Mathematica. (Trans. Cohen, I. B. & Whitman, A., 1999). University of California Press.

[12] Schneider, P., Ehlers, J., & Falco, E. E. (1992). Gravitational Lenses. Springer.

[13] Weinberg, S. (1972). Gravitation and Cosmology. Wiley.

[14] NASA. (2017). Sun Fact Sheet. NASA Planetary Data.

[15] del Barco, O. (2024). An accurate equation for the gravitational bending of light. MNRAS, 535, 2504–2510.

[16] Milgrom, M. (1983). A modification of the Newtonian dynamics. ApJ, 270, 365–370.

[17] Famaey, B., & McGaugh, S. (2012). MOND: A review. Living Rev. Relativ., 15(1), 10.

[18] Sanders, R. H. (1999). The virial discrepancy in clusters. ApJL, 512(1), L23.

[19] Broadhurst, T., et al. (2005). Strong lensing analysis of Abell 1689. ApJ, 621(1), 53–88.

[20] Angus, G. W., Shan, H. Y., Zhao, H. S., & Famaey, B. (2007). Can MOND take a bullet? ApJL, 654(2), L13.

[21] Umetsu, K., & Broadhurst, T. (2008). Combining lensing and X-ray analysis of Abell 1689. ApJ, 684(2), 177–203.

[22] Lemze, D., et al. (2008). Mass and structure of Abell 1689. ApJ, 682(6), 653–665.

[23] Markevitch, M., Gonzalez, A. H., et al. (2004). Direct constraints on the dark matter self-interaction cross section from the Bullet Cluster. ApJ, 606(2), 819–824.

[24] Bradac, M., et al. (2006). Strong and weak lensing united. ApJ, 652(2), 937–947.

[25] Dominik, M. (1998). Finite-source effects in gravitational microlensing of stars. Astronomy and Astrophysics, 333, L79–L82.

[26] Koester, D., & Chanmugam, G. (1990). Physics of white dwarf stars. Reports on Progress in Physics, 53(7), 837–915.

[27] Draine, B. T. (2011). Physics of the Interstellar and Intergalactic Medium. Princeton University Press.

[28] McKee, C. F., & Ostriker, E. C. (2007). Theory of Star Formation. Annual Review of Astronomy and Astrophysics, 45, 565–687.

[29] Eddington, A. S. (1919). The Deflection of Light by the Sun's Gravitational Field. Philosophical Transactions of the Royal Society A, 220, 291–333.

[30] Bahcall, J. N., Serenelli, A. M., & Basu, S. (2005). New solar opacities, abundances, helioseismology, and neutrino fluxes. Apj, 621(1), L85–L88.

[31] Kovalchuk, E. N., et al. (2007). Gravitational bending of light by planetary multipoles and its implications. Physical Review D, 75(6), 062002.

[32] B P Abbott et al (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett., 116, 061102 (2016)

[33] Misner, C. W., Thorne, K. S., & Wheeler, J. A. (1973). Gravitation. W.H. Freeman.