ORIE 5530: Modeling Under Uncertainty

Lecture 1 (Introduction)

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Adapted from Professor Itai Gurvich's original notes

We dedicated the first class mostly to motivation and to get us thinking about probability – think of our first meeting as a warmup. The purpose of the warmup is to have us start thinking (and carefully so) in terms of mathematical models of uncertainty. We will embark on our analytical journey starting next class.

Below is a bullet-point list of some examples I discussed informally in this first class. These are examples for motivation and to get us thinking. I do not go into any of these in much detail but I will return to these examples (or variants thereof) once we can ground our answers in models.

1. What is probability?
   (a) Frequency-based approach (how many Heads out of 100 tosses of a coin) and models. Say I just tossed a coin a 100 times. What is the likelihood that the 101st toss comes out Heads? Well, depends on what I know about the coin ... . Coin tossing is a basic probability model. Rolling a die is another basic probability model. Approach can provide nice/helpful intuition.
   (b) Axiomatic approach and models. Addresses some disadvantages of frequency-based approach, forming basis of modern probability.
2. Birthday problem and exploiting $\mathbb{P}[A] = 1 - \mathbb{P}[A^c]$, where event A^c is the complement of event A.
3. Single-order, multi-period inventory model as an illustration of stochastic optimization. Distributions are the simplest of probabilistic models but are very useful. It is important to be familiar with a basic set that is widely applicable.
4. Dynamic inventory as an example of a dynamical system.
5. Queueing (waiting line) models and queueing network models.
6. Workforce management models.
7. Portfolio example: This is intended to emphasize that the details of the model matter but how much they matter depends also on our objective; see more details below. We will revisit this example in a future class once we cover *correlations* and multi-dimensional decisions.
8. Conditional probabilities
   (a) Simple example: $\mathbb{P}[A] = 30\%$, where $A = \{\text{baseball batter gets a hit}\}$. $\mathbb{P}[A|L] = 40\%$ and $\mathbb{P}[A|R] = 20\%$, where $L = \{\text{left-handed pitcher}\}$ and $R = \{\text{right-handed pitcher}\}$.
   (b) Monty Hall problem (donkey and car): conditional probabilities and information updating.
9. Emergency room: An informal illustration of using Markov chains in a realistic setting. Evaluation for given cost, benefit and pricing optimization.

Portfolio optimization: An example. Say you have a \$100 that you want to invest in one of two investments A or B. Each will give you, in expectation, a 5% return in a year. This means that if you put your 100 on A, you will have (in expectation) \$5 in a year from now. Same for B.

Where would you put your money? It depends on two things:

1. Do you care about expected return or also about risk? If you only care about expected return you should not care. However, things are different if you care about risk. It could be that A gives you 5% for sure while B gives you 10% return with a 50% chance but 0 otherwise. So you might get 10% but you also might end up getting nothing.

2. If you care about risk, what do you know about the correlation between the two investment products? You could utilize this correlation to your advantage (which is what finance models do).