In this case *A and A′ are equivalent*. We leave it to the reader to verify these claims. The process must always terminate because all pairs in column 1 are distinct pairs and there are only finitely many distinct pairs of the vertices of A and A'.

Q.E.D.

EXAMPLE 1-7

1. Consider the following two finite automata A and A' over $\Sigma = \{a, b\}$ described in Fig. 1-1.

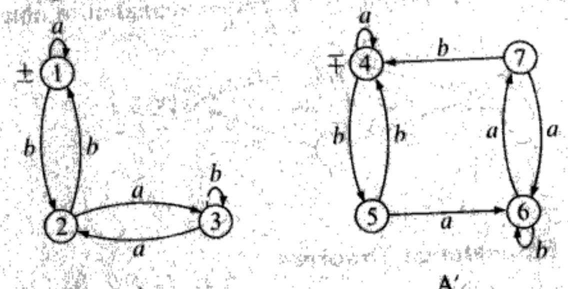

Figure 1-1 The finite automata A and A'.

They are equivalent because the corresponding comparison table is

(v, v')	(v_a, v'_a)	(v_b, v'_b)
(1, 4)	(1, 4)	(2, 5)
(2, 5)	(3, 6)	(1, 4)
(3, 6)	(2, 7)	(3, 6)
(2, 7)	(3, 6)	(1, 4)

There are no pairs in columns 2 and 3 that do not occur in column 1.

2. Consider the two finite automata B and B' described in Fig. 1-2.

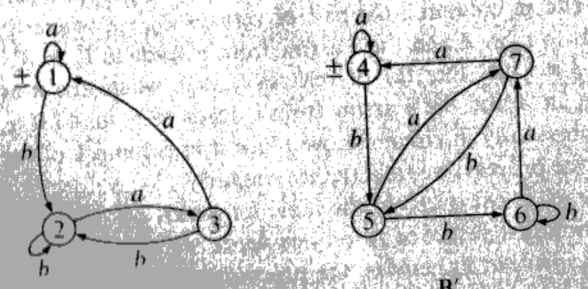

Figure 1-2 The finite automata B and B'.

They are equivalent because the corresponding comparison table is

(v, v')	(v_a, v'_a)	(v_b, v'_b)
(1, 4)	(1, 4)	(2, 5)
(2, 5)	(3, 7)	(2, 6)
(3, 7)	(1, 4)	(2, 5)
(2, 6)	(3, 7)	(2, 6)

Again, there are no pairs in columns 2 and 3 that do not occur in column 1.

3. Consider the two finite automata A and B' described in Fig. 1-3.

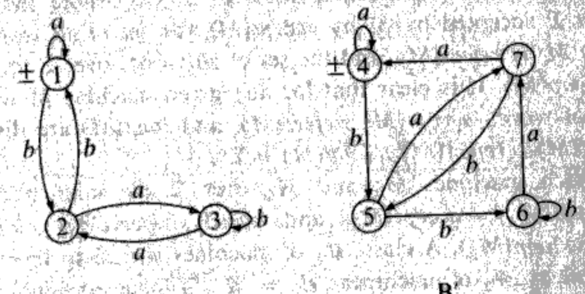

Figure 1-3 The finite automata A and B'.

They are *not* equivalent because the corresponding comparison table is

(v, v')	(v_a, v'_a)	(v_b, v'_b)
(1, 4)	(1, 4)	(2, 5)
(2, 5)	(3, 7)	(1, 6)

Note that 1 is a final vertex of A while 6 is a nonfinal vertex of B'. Since this pair was obtained by applying the letter b twice, the table actually shows us, not only that A and B' are not equivalent, but also that an appropriate counterexample is the word bb.

It can be shown that A accepts the set of all words which are the binary representation of natural numbers divisible by 3, where a stands for 0 and b stands for 1. (Such numbers with leading 0's on the left are also accepted.) The corresponding sets S_1, S_2, and S_3 contain all words representing binary numbers which, after division by 3, yield remainders 0, 1, and 2, respectively. The automaton B' accepts the set of all words which are the binary representation of natural numbers divisible by 4. The