Indian Institute of Technology Dharwad

CS 202: Assignment 1

Instructions:

Submission Deadline : **22:00 hours, 3 January 2022**.

You will submit a single pdf (scanned copy of all your solutions) named *rollno.pdf*.

Submission should be made only on **Moodle** portal CS 202 course from your own login.

Marks will be given for what you have written, not for what you meant.

Late submission policy is applicable.

Copying is fatal.

Total Marks: 50.

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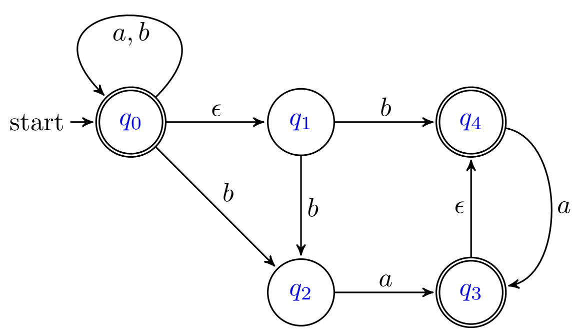
1. Construct DFA equivalent to the NFA shown below and explicitly mention $(Q, \Sigma, \delta, q_0, F)$.

[4 marks]



2. Given the formal description of DFA defined over $\Sigma = \{a, b\}$ that accepts the following languages and explicitly mention $(Q, \Sigma, \delta, q_0, F)$:

   (a) all strings containing **exactly** 3 a's

   (b) all strings containing **atmost** 3 a's

   (c) all strings containing **atleast** 3 a's

[$3 \times 2 = 6$ marks]

3. Give the formal description of a DFA that accepts the set of all strings of 0's and 1's such that the 4th symbol from the right is 1 (explicitly mention $(Q, \Sigma, \delta, q_0, F)$).

[5 marks]

4. Let $\Sigma = \{0, 1\}$.

   (a) Describe a DFA D $= (Q, \Sigma, \delta, q_0, F)$ defined over Σ that recognizes words that **do not** contain 101 as a contiguous subword.

   (b) Prove/disprove that the machine D has 3 states.

[$2 + 3 = 5$ marks]

5. During Thursday's class (23/12/21), we discussed the algorithm to eliminate ϵ- transitions in an $\epsilon - NFA$. Give the proof of correctness for this algorithm.

[5 marks]

6. Design a DFA which accepts the language L=$\{w \in \{0, 1\}^* \mid |w|_0$ =0 mod 3 and $|w|_1$ =0 mod 2$\}$. [5 marks]

7. For any set of strings A, define the set firstHalves(A)=$\{x \mid \exists_y |y| = |x|$ and $xy \in A\}$. For example, firstHalves($\{a, ab, bab, bbab\}$)=$\{a, bb\}$. Show that if A is regular, then so is firstHalves(A). [5 marks]

8. (a) Write an algorithm which takes an ϵ-NFA as input and computes ϵ-closure of all its states in Q in the given ϵ-NFA. Your algorithm should work for any given δ.

   Input: For any given δ represented as Table1, Assume $\Sigma = \{a, b, c\}$
   Output: A matrix E of size $\mid Q \mid \times \mid Q \mid$. The matrix entry at $E(q_i, q_j) = 1$ if state q_j is in the $\epsilon-$closure of state q_i and 0 otherwise.

   (b) What is the running time for this algorithm (in big-O notation)?

[$4 + 1 = 5$ marks]

	a	b	c	ϵ
q_0	q_0	-	-	q_1
q_1	-	q_1	-	q_2
q_2	-	-	q_2	ϵ

Table 1: δ

9. Let A be a DFA and q be a particular state of A, such that δ(q,a)=q for all input symbols a. Show by induction on the length of the input that for all input strings w, $\hat{\delta}$(q,w) =q [5 marks]

10. Construct DFAs equivalent to the NFAs :

    (a) $(\{p, q, r, s\}, \{0, 1\}, \delta_1, p, \{s\})$

    (b) $(\{p, q, r, s\}, \{0, 1\}, \delta_2, p, \{q, s\})$

    where δ_1 and δ_2 are given in Table2 and Table3.

[5 marks]

	0	1
p	p,q	p
q	r	r
r	s	-
s	s	s

Table 2: δ_1

	0	1
p	q,s	q
q	r	q,r
r	s	p
s	-	p

Table 3: δ_2