

Module: Logic
CS 203: Discrete Structures
Course Instructor : Prof. Prabuchandran K J

INSTRUCTIONS: You have to give clear and detailed solution for each of the questions. **Send one single pdf file containing solutions to all problems. Take a clear picture and upload the hand written solutions in Moodle(event-assignment2) by 7/9/2021, 10 pm. Name your pdf with your *name_rollno.pdf*.** For example *harrithha_200010018.pdf*. Late submissions will not be graded. Students can discuss but must write their solutions based on their understanding independently. Do not use web resources or answers from your peers to obtain solutions. If anyone is involved in malpractice of any sort, then suitable disciplinary action will be taken. If required, there would be a viva to selected set of students.

1. Create a logical annotation with the help of the following predicates to demonstrate the given statement using first order logic.

- (a) $G(x)$: x is made of gold. Here, the domain of discourse is ornament.
- (b) $S(x)$: x is made of silver. Here, the domain of discourse is ornament.
- (c) $Like(y,x)$: y likes x . Here, the domain of discourse for y is people and for x it is ornament.

Using the functions given above construct the given statement.

Statement: If an ornament is made of gold or made of silver then the ornament is liked by all people. (2)

2. Which of the following is/are tautology

- (a) $a \vee b \rightarrow b \wedge c$
- (b) $a \wedge b \rightarrow b \vee c$
- (c) $a \vee b \rightarrow (b \rightarrow c)$
- (d) $(a \rightarrow b) \rightarrow (b \rightarrow c)$

(2)

3. Given

- $F(x,y)$ means that x is the father of y
- $M(x,y)$ that x is the mother of y
- $H(x,y)$ that x is the husband of y
- $S(x,y)$ that x is the sister of y
- $B(x,y)$ that x is the brother of y
- Siblings are individuals that have a common parent

Use these predicate symbols to translate the following sentences into predicate logic.

- (a) Everybody has a mother
- (b) Whoever has a mother has a father
- (c) Ram is a grandfather
- (d) All brothers are siblings

(5)

4. Using implication and negation can we construct all n variable Boolean functions? Justify. (2)

5. Consider the following logical inferences :

I_1 : If it rains then the cricket match will not be played.

The cricket match was played.

Inference : There was no rain.

I_2 : If it rains then the cricket match will not be played.

It did not rain.

Inference : The cricket match was played.

Which of the above inference is correct ? Justify

(1)

6. For the following argument, explain which rules of inference are used for each step. “Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job.” (1)

7. If a and b are two propositions. Which of the following logical expressions are equivalent? [\sim denotes the negation.]

(a) $(a \vee \sim b)$.

(b) $(a \wedge b) \vee (a \wedge \sim b) \vee (\sim a \wedge \sim b)$.

(c) $(a \wedge b) \vee (a \wedge \sim b) \vee (\sim a \wedge b)$.

(d) $\sim(\sim a \wedge b)$.

(1)

8. Check whether $(\sim(a \leftrightarrow b) \wedge c) \vee (a \wedge b \wedge \sim c)$ is true or false if any two of a , b , or c are true? [\sim denotes the negation.] (2)

9. There are two rooms: A and B. Each room has a sign.

- Sign at room A: “There is a lady in room A, and a tiger in room B”.
- Sign at room B: “There is a lady in one room, and a tiger in another room”.

Assume that exactly one sign is true and another sign is false. Further assume, exactly one thing (lady or tiger) in each room. Determine which room contains what? (1)

10. Five friends Ajay, Harish, Kote, Raghu and Vijay have access to an on-line chat room. We know the following are true.

- Either K or H or both are chatting.
- Either R or V but not both are chatting.
- If A is chatting, then R is chatting.
- V is chatting if and only if K is chatting.
- If H is chatting, then both A and K are chatting.

Determine who is chatting using logical inference. Give a clear argument. (3)