Module: SET THEORY
CS 203: Discrete Structures
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INSTRUCTIONS: The following are the practice problems to improve your understanding of the concepts in SET module. Try to solve all problems. You do not have to submit the solutions.

1. What is the cardinality of each of these sets?
   (a) $\emptyset$
   (b) $\{\emptyset\}$
   (c) $\{\emptyset, \{\emptyset\}\}$
   (d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$
2. State true/false
   (a) $\emptyset \in \{\emptyset\}$
   (b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
   (c) $\{\emptyset\} \in \{\emptyset\}$
   (d) $\{\emptyset\} \in \{\{\emptyset\}\}$
   (e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
   (f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
   (g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$
3. If the cardinality of a set $|S|$=n then, what will be the cardinaity of power set of S ?
4. In a class of 120 students numbered 1 to 140, all odd numbered students opt for Discrete math, those whose numbers are divisible by 4 opt for $C++$ and those whose numbers are divisible by 7 opt for data structure. How many opt for none of the three subjects?
5. In a school, there are 30 teachers who teach Mathematics or Physics. Of these, 15 teach Mathematics and 5 teach both Physics and Mathematics. How many teach Physics only?
6. Prove that the total number of 5 digit numbers using only $0, 1$ and the number of subsets of $\{1, 2, 3, 4, 5\}$ are equal.
7. $A = \{\text{x} \mid \text{x is divisible by 3 but not 5}\}$. The universe $U = \{\text{x} \mid \text{x is divisible by 3 or x is divisible by 5 or both }\}$. Then what will be $\bar{A} =$?
8. Consider a set $\bar{A} = \{\{3, 4\}, \{5\}, \{\emptyset\}, 7\}$ then denote if the following statements are true or false.
   a) $\{3, 4\} \subseteq A$ b) $\{3, 4\} \epsilon A$ c)$7 \subseteq A$ d)$7 \epsilon A$ e)$\{\emptyset\} \epsilon A$ f)$\emptyset \subseteq A$
9. If P(A) has 7 non-empty sets as members of P(A) where P(A) is defined as the power set of A. Comment on the cardinality of set A.

10. State true or false
    a) Every subset of an uncountable set is uncountable
    b) Every subset of a countable set is countable
    c) If A is uncountable and B is uncountable set then $A \cap B$ is uncountable
    d)If A is uncountable and B is uncountable set then $A \cup B$ is uncountable

11. Given A and B are different sets then prove the following
    a) $A - B = (A \cup B) - (A \cap B)$
    b) $A \cap B = ((A \cup B) - (A - B)) - (B - A)$
    Based on the above proof is Only(A)=B-A ? Prove it.

12. Cardinality of set S which includes all the numbers which are divisible by 3 or 7 but not by both.

13. In a room containing 30 people, there are 10 who speak German, 12 who speak English and 22 people who speak Bengali. 9 people speak both German and English, 11 people speak both English and Bengali whereas 13 people speak both Bengali and German. How many people speak all 3?

14. State if correct or not. Provide evidence to support your answer
    i) There exists infinite sets A,B,C such that $A \cap (B \cup C)$ is finite
    ii) There exists two irrational numbers x,y such that their sum is rational
    iii) There exists two irrational numbers whose product is rational?

15. Let A, B, C be three sets. Using the identities of Table 1 of section 2.2.2 of the book prove that $\overline{(A \cup B)} \cap \overline{(B \cup C)} \cap \overline{(A \cup C)} = \overline{A} \cap \overline{B} \cap \overline{C}$.

16. Let A, B, C be three sets. Use the identity $A - B = A \cap \overline{B}$ and the identities of Table 1 of section 2.2.2 of the book to prove that $(A - B) \cap (B - C) \cap (A - C) = \emptyset$

17. Let A and B be subsets of a universal set U. Show that A is a proper subset of B if and only if $\overline{B}$ is a proper subset of $\overline{A}$

18. If there are exactly 81 functions from set A to set B, then which of the following statements is not true ?

    (a) $|A| = 4,\ |B| = 3$
    (b) $|A| = 2,\ |B| = 9$
    (c) $|A| = 1,\ |B| = 81$
    (d) $|A| = 9,\ |B| = 9$

19. If $|A| = n$, $|B| = m$, how many relations are there from A to B which are not functions ?

20. If there are exactly 120 one-one functions possible from A to B, then which of the following is not true ?

    (a) $|A| = 5,\ |B| = 5$
    (b) $|A| = 4,\ |B| = 5$

(c) $|A| = 3,\ |B| = 6$

(d) $|A| = 5,\ |B| = 4$

21. If $|A| = 5,\ |B| = 5$, then how many bijections are there from A to B ?

22. What is the domain of $f(x) = \frac{1}{\sqrt{|x|-x}}$?

23. Let $f : \mathbb{N} \to \mathbb{N}$ such that $f(n) = n + 15$. Then f is

    (a) Bijective

    (b) One-one

    (c) Onto

    (d) Not a function

    Justify.

24. A is an uncountable set and B is a countable set. What can you tell about the cardinality of $A - B$. Justify.

25. give an example of two uncountable sets A and B such that $A \cap B$ is

    (a) finite

    (b) countably finite

    (c) uncountable

26. Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

    (a) all bit strings not containing the bit 0

    (b) the real numbers not containing 0 in their decimal representation

    (c) the real numbers containing only a finite number of 1s in their decimal representation

27. Prove that the set $A = (0, 1]$ has the same cardinality as the set $B = (-\infty, 0)$

28. Consider the following functions on the set of all integers :
    $f(x) = x^2,\ g(x) = x^3$.
    Which of the following statements is correct ?

    (a) f is one-one

    (b) f is onto

    (c) g is one-one

    (d) g is onto

29. If f and g are two functions such that $f(g(x)) = x$ and $g(f(x)) = x$, then which of the following statements is true ?

(a) f and g are inverse to each other.
(b) f and g are one-one functions.
(c) f and g are onto functions.
(d) Cannot say.

30. Show that if A is an infinite set, it contains a countably infinite subset.

31. Show that the set of all finite bit strings is countable.

32. Show that the union of a countable number of countable sets is countable.