Module: Mathematical Logic
CS 203: Discrete Structures
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INSTRUCTIONS: The following are the practice problems to improve your understanding of the concepts in Logic module. Try to solve all problems. You do not have to submit the solutions.

1. Formula $(a \wedge b \wedge c) \rightarrow (c \vee a)$ is a tautology or not?

2. Show that proposition $(a \rightarrow b) \wedge (b \rightarrow a)$ is a contingency.

3. Write the propositional logic formula for "if a, then b unless c".

4. Show the following equivalence:
$((a \rightarrow b) \wedge (c \rightarrow b)) \Leftrightarrow ((a \vee c) \rightarrow b)$.

5. Show the following implication:
$(a \rightarrow (b \rightarrow c)) \rightarrow ((a \rightarrow b) \rightarrow (a \rightarrow c))$.

6. Construct the truth table for $((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$

7. Let p stand for the proposition "I bought a lottery ticket" and q for "I won the jackpot". Translate the following into English:

   (a) $\neg p$
   (b) $p \vee q$
   (c) $p \wedge q$
   (d) $p \implies q$
   (e) $\neg p \implies \neg q$
   (f) $\neg p \vee (p \wedge q)$

8. Use truth tables to determine which of the following are equivalent to each other:

   (a) $(P \wedge Q) \vee (\neg P \wedge \neg Q)$
   (b) $\neg P \vee Q$
   (c) $(P \vee \neg Q) \wedge (Q \vee \neg P)$
   (d) $\neg(P \vee Q)$
   (e) $(Q \wedge P) \vee \neg P$

9. A logician puts four cards on the table in front of you. Each card has a number on one side and a letter on the other. On the uppermost faces, you can see E, K, 4 and 7. He claims that if a card has a vowel on one side, then it has an even number on the other. How many cards do you have to turn over to check this?

10. **Knight, Knave and Spy**

    - Knights always tell the truth

- Knaves always lie.
- Spy can lie or tell the truth.

There is one spy, one knight, and one knave. A says that C is a knave. B says that A is a knight. C says "I am the spy." Which one is the spy, which one is the knight, which one is the knave?

Hint:Reason this out using Rules of Inference.

11. Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book."

12. Construct the truth table for the following formulas :

    (a) $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$

    (b) $\neg(P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$

13. For each of these arguments, explain which rules of inference are used for each step.

    (a) "Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."

    (b) "Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution."

    (c) "Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program."

    (d) "Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean."

14. Assume that "For all positive integers n, if n is greater than 4, then n^2 is less than 2^n is true. Use universal modus ponens to show that $100^2 < 2^{100}$.

15. Show that the truth value of the following formula is independent of its components : $(P \Leftrightarrow Q) \Leftrightarrow ((P \wedge Q) \vee (\neg P \wedge \neg Q))$.

16. Consider the following logical inferences :
    I_1 : If it rains then the cricket match will not be played.
    The cricket match was played.
    Inference : There was no rain.
    I_2 : If it rains then the cricket match will not be played.
    It did not rain.
    Inference : The cricket match was played.
    Which of the above inference is correct ?