

**Module: Counting**  
**CS 203: Discrete Structures**  
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**INSTRUCTIONS:** The following are the practice problems to improve your understanding of the concepts in counting module. Try to solve all problems. You do not have to submit the solutions.

1. In a  $2 \times 2$  matrix the elements  $(a_{ij})$  are either 0 or 1. Find the number of symmetric matrices possible.
2. Find the number of binary strings  $S$  of length  $2n$  with equal number of 1's and 0's.
3. Mr. Francis is a resident of Bengaluru. He has an appointment in Kolkata. On checking the certain travel agency website, he found that there are no direct flights to Kolkata. Assume there is only one way to get to Kolkata via Hyderabad. There are 6 flights from Bengaluru to Hyderabad and 5 connecting flights from Hyderabad to Kolkata. Assume all connecting flights departure time from Hyderabad is after the arrival time of each flight(Bengaluru to Hyderabad) in Hyderabad. How many ways can he take to reach Kolkata.
4. How many  $n$ -digit even numbers have all digits distinct?
5. Give a combinatorial proof of the following:
  - $\sum_{t=0}^n \binom{t}{k} = \binom{n+1}{k+1}$ .
  - $\binom{n}{m} \binom{m}{k} = \binom{n}{k} \binom{n-k}{m-k}$
  - $\binom{m+n}{r} = \sum_{i=0}^{m+n} \binom{m}{i} \binom{n}{r-i}$
6. How many sequences  $x_k, k \geq 0$  non-negative integers satisfy  $x_1 + x_2 + \dots + x_k = n$ ?
7. How many paths are totally present from top left to bottom right of a  $m \times n$  matrix?
8. How many paths are there from  $(0,0)$  to  $(7,7)$  without crossing the diagonal?
9. Suppose you have  $n$  pairs of parentheses and you would like to form valid groupings of them, where "valid" means that each open parenthesis has a matching closed parenthesis. For example, " $((()))$ " is valid, but " $()()()$ " is not. How many groupings are there for each value of  $n$ ?
10. Prove that one can choose evenly many objects from a collection of  $n$  objects in  $2^{n-1}$  ways.
11. Six boxes are numbered 1 through 6. How many ways are there to put 20 identical balls into these boxes so that
  - (a) none of them is empty ?
  - (b) some of them can be empty ?

12. A train with  $m$  passengers must make  $n$  stops. How many ways are there for passengers to get off the train at stops.
13. In how many ways can three people divide among themselves six identical apples, one banana, one orange and one mango ?
14. There are 14 books on a shelf. How many ways are there to choose five of them so that no two of the chosen books stand next to each other ?
15. You have to pick one factor from 8100 and one factor from 1024. But they should be different. How many ways you can do that ?
16. How many unique ways are there to arrange the letters of the word "PRIOR" ?
17. How many ways are there to divide 10 boys into two basketball teams of 5 boys each?
18. how many ways you can arrange the letters of the word EXECUTION such that vowels occur together but they occur between two consonants?