Quiz 3
CS 203: Discrete Structures

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INSTRUCTIONS: Answer all questions. You have to give clear answers with proper justification. Submit one single pdf file containing solutions to all problems. Take clear pictures and convert to a single pdf file. Name your file as *rollno.pdf*. For example, 200010018.*pdf*. An activity named quiz3 is added in classroom. Submit it there. Late submissions will not be graded. Do not use web resources or answers from your peers to obtain solutions. If anyone is involved in malpractice of any sort, then suitable disciplinary action will be taken. If required, there would be a viva to selected set of students. The exam is from 11.45 am to 12.45 pm. You can submit your answer up to 1.00 pm.

1. Consider the sequence of real numbers defined by the relations $x_1 = 1$ and $x_{n+1} = \sqrt{1 + 2x_n}$ for $n \geq 1$. Use the principle of Mathematical Induction to show that $x_n < 4 \ \forall n \geq 1$. (1)

2. Prove that given 5 points inside a square of side length 2, it is always possible to find two of them whose distance apart is at most $\sqrt{2}$. (1)

3. Define Equivalence and POSET relations? Give an example for each relation. (1)

4. Which of the following is true ?

   (a) Union of two equivalence relations is an equivalence relation.

   (b) Intersection of two equivalence relations is an equivalence relation.

   (2)

5. If the generating function of a sequence is $(1 + x)/(1 - x)^3$, then what will be the value of $a_3 - a_0$? (2)

6. If $|A| = n$, then on $A \times A$, what is the cardinality of the smallest reflexive, symmetric, antisymmetric and transitive relation ? (1)

7. There exists two powers of two which differ by multiple of

   (a) 2^{1987}

   (b) 1987

   (c) Both of the above

   (d) None of the above

   Justify. (2)