End Sem Exam
CS 203: Discrete Structures

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INSTRUCTIONS: Answer all questions. You have to give clear answers with proper justification. Submit one single pdf file containing solutions to all problems. Take clear pictures and convert to a single pdf file. Name your pdf with your *name_rollno*.pdf. For example *harrithha_200010018.pdf*. Maximum mark is 25. 1 mark is bonus. If anybody gets 25 or above, it will be considered that he/she has got 25. Late submissions will not be graded. Do not use web resources or answers from your peers to obtain solutions. If anyone is involved in malpractice of any sort, then suitable disciplinary action will be taken. If required, there would be a viva to selected set of students. The exam is from 9 am to 12 pm. You can submit upto 12.15 pm.

1. Prove or disprove the following statements:
   (a) A tree is a bipartite graph.
   (b) Prove that if the degree of every vertex in a graph is even, then the graph is Eulerian
   (c) Give an example of a graph which is both Hamiltonian and Eulerian

   (1+1+1)

2. Let (G,*) be a finite cyclic group with generator a of order 12. Then
   (a) What is the number of the generators in G?
   (b) Which of the following is a generator of G? Justify.
      i. a^2
      ii. a^3
      iii. a^4
      iv. a^5

   (1+1)

3. Use generating functions to find the number of r-combinations from a set with n elements when repetition of elements is allowed. (2)

4. Rakesh is working on a mathematical logic problem which says "Some boys in the class are taller than all girls". After sometime he has come up with a first order logic as $(\exists x)(boy(x) \wedge (\forall y)(girl(y) \wedge taller(x, y)$. Rectify the error in the first order logic statement if any.
   Note: taller(x,y) is true when person x is taller than person y where x and y is defined over the set of all boys and set of all girls.
   boy(x) is a function defined over the set of all boys and 'x' is a boy
   girl(z) is a function defined over the set of all girls and 'z' is a girl (1)

5. Find out the remainder, if 3^{5555} is divided by 11. (1)

6. ScamTel has won a state government contract to connect 17 cities by high-speed fibre optic links. Each link will connect a pair of cities so that the entire network is connected there is a path from each city to every other city. The contract requires the network to remain connected if any single link fails. What is the minimum number of links that ScamTel needs to set up? Give detailed explanation. (1)

7. There are 12 books on a shelf. How many ways are there to choose five of them so that no two of the chosen books stand next to each other?

   (a) $\binom{7}{5}$

   (b) $\binom{8}{5}$

   (c) $\binom{16}{4}$

   (d) $\binom{11}{4}$

   (2)

8. $K_{2,2}$ is isomorphic to C_4–True or false? Justify. (1)

9. Find the number of vertices of a non-planar graph G which has the least number of edges. (1)

10. Given the following statements are true.

    - p
    - $p \to \neg q$
    - $r \to q$

    What can you infer from the above true statements?

    (a) $\neg p$ is true

    (b) r is true

    (c) q is true

    (d) $\neg r$ is true.

    Justify. (1)

11. Imagine an email system which delivers emails to people who join a group. It sends emails in the following way: it sends no mail to the first person, one mail to the second person, 2 mails to the third person, and so on. What is the recurrence relation which represents the total number of emails sent when the nth person joins the group? Solve this recurrence using generating function method (2)

12. - Define the normal subgroup of a group G. The centre $Z(G)$ of a group G is defined as
    $$Z(G) = \{z \in G | \ \forall g \in G, z.g = g.z\},$$
    Is it a normal subgroup of G? Justify by verifying the subgroup property and normal property.

- Give an example of a group G and subgroup H such that H is not normal subgroup of G.

$(1 + 2 + 1)$

13. Given 8 different natural numbers, none greater than 15, show that at least three pairs of them have the same positive difference. (1)

14. Let A be a set of n elements. The number of ordered pairs in the largest and the smallest equivalence relations on A are:

    (a) n, n

    (b) $n^2, 0$

    (c) n^2, n

    (d) $n, 1$

    (1)

15. For the workshops organized at IIT Dharwad, students can participate in more than one workshop saying that all are organized in different timings. 150 students are enrolled in total for one or more than one workshop. Registration details are as follows : 99 students are enrolled for Cyber Security. 56 students are enrolled for Solar Energy, 89 students are enrolled for Artificial Intelligence. 25 students for both Cyber Security and Solar Energy. 65 students for both Cyber Security and Artificial Intelligence and 18 students for both Solar Energy and Artificial Intelligence. How many students are participating in all three workshops ? (1)

16. In how many ways can an interview panel of 3 members be formed from 3 engineers, 2 psychologists and 3 managers if at least 1 engineer must be included? (1)

17. State one useful fact learnt from this course? (1)