Midsem
CS 203: Discrete Structures
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INSTRUCTIONS:

- Answer all questions. You have to give clear answers with justification.
- It is **mandatory** to join the Google Meet (https://meet.google.com/duo-ggid-gic) and keep your video on during throughout the exam, with your face clearly visible onscreen.
- Do not use web resources or answers from your peers to obtain solutions. If anyone is involved in malpractice of any sort, then suitable disciplinary action will be taken. If required, there would be a viva to selected set of students.
- Submit to Google Classroom MidSem activity, a single pdf file containing clear images/solutions to all problems. Name your file as *rollNo.pdf* (example: 200010018.*pdf*). **Only Google Classroom submissions will be graded.** Late submissions will not be graded.
- **Total Marks 27. Bonus Question Mark 2. Midsem will be evaluated for 25 marks.**
- Total duration 2hrs, 10 am - 12 pm.

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1. Set A comprises all multiples of 4 less than 500. Set B comprises all odd multiples of 7 less than 500, Set C comprises all multiples of 6 less than 500. How many elements are present in $(A \cup B \cup C)$? (1)

2. Let $Graph(x)$ be a predicate which denotes that x is a graph. Let $Connected(x)$ be a predicate which denotes that x is connected. Which of the following first order logic sentences DOES **NOT** represent the statement: "Not every graph is connected"?

   (a) $\neg\ \forall x(Graph(x) \implies Connected(x))$
   (b) $\exists x(Graph(x) \wedge \neg Connected(x))$
   (c) $\neg\ \forall x(\neg Graph(x) \vee Connected(x))$
   (d) $\forall x(Graph(x) \implies \neg Connected(x))$

   (2)

3. In how many ways can we distribute 5 distinct balls, $B_1, B_2, \ldots, B_5$ in 5 distinct cells, $C_1, C_2, \ldots, C_5$ such that Ball B_i is not in cell C_i, $\forall i \in \{1, 2, \ldots, 5\}$ and each cell contains exactly one ball? (1)

4. Find the coefficient of x^6y^3 in the expansion of $(x + 2y)^9$. (1)

5. The number of 10 digit numbers which is greater than 1987654321 is (a)*(b!). What is 'a' and 'b'? (1)

6. Which of the following statement is the negation of the below statement.
   Statement: All pentagons are polygon.

(a) All pentagon are not polygon

(b) There is some pentagon which is not polygon

(c) All polygons are pentagons

(d) There is some pentagon which is polygon

(1)

7. Explain, without using a truth table

   (a) Why $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when p, q, and r have the same truth value and it is false otherwise.

   (b) Why $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.

(2)

8. Using the predicates given below, formalize the following sentences in first order logic:

   - InBox(x): x is in the box
   - Red(x): x is red
   - Animal(x) : x is an animal
   - Cat(x) : x is a cat
   - Dog(x) : x is a Dog

   (a) All red things are in the box

   (b) Only red things are in the box

   (c) No animal is both a cat and a dog

   Note: Your solution must be strictly according to first order logic syntax (3)

9.
   - Let S_1={Bit strings of '0s','1s', and '2s' containing only a finite number '2s' in their decimal representation}.
   - Let S_2={Bit strings of '0s','1s', and '2s' containing only a finite number of '1s' and '2s', possibly infinite no of 0s in their decimal representation}.

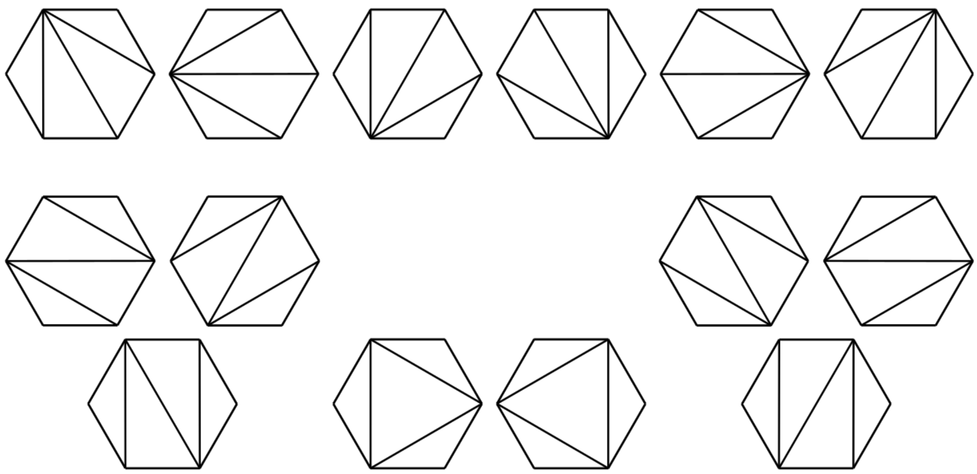
   Are the sets S_1 and S_2 countable or uncountable? Justify. (2)

10. How many n-length binary strings do not have the pattern '100' any where in the string? Write a recurrence. You do not have to solve the recurrence. Verify your recurrence for n=2,3,4. (2)

11. **Bonus Question**: If you write a one variable recursion to the above problem. (2)

12. How many permutation of the 26 letters of the English alphabets do not contain the strings 'gem', 'road', and 'bus'? (2)

13. In how many ways can a polygon with n-sides ($n \geq 3$) be cut into triangles by connecting vertices with non-crossing line segments (polygon triangulation). Justify your answer. For n=3, you get one triangulation and for n=6, you get the following all possible triangulations.



(2)

14. Give combinatorial proof of the following:

    (a) $\sum_{k=0}^{n}(-1)^k\binom{n}{k} = 0$

    (b) $\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$ and deduce $\sum_{k=0}^{m}\binom{n}{k}\binom{n-k}{m-k} = 2^m\binom{n}{m}$

    (c) Using (b), show that $\binom{2n}{2k}\binom{2n-2k}{n-k}\binom{2k}{k} = \binom{2n}{n}\binom{n}{k}^2$.

    You will not get any credits for (a) and (b) if you expand and prove it. For (c), you can use the result of (b) and facts about $\binom{n}{k}$. (1+2+2)