Quiz-Logic
CS 203: Discrete Structures
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9-09-2021

INSTRUCTIONS:Answer all questions. You have to give a clear answers with proper justification. Submit one single pdf file containing solutions to all problems. Take clear pictures and convert to a single pdf file. Name your file as *rollno.pdf*. For example, 200010018.*pdf*. An activity named quiz 2 is added in moodle under assignment section. Submit it there. Late submissions will not be graded. Do not use web resources or answers from your peers to obtain solutions. If anyone is involved in malpractice of any sort, then suitable disciplinary action will be taken. If required, there would be a viva to selected set of students. The exam is from 11.45 am to 12.45 pm. You can submit your answer up to 1.00 pm. If you are unable to submit within 1.00 pm, mail your answer sheets to all the TAs(Sagartanu-183061001, Tephilla-183061002, Ravi-191061001, Sourav-211011002). You should send a single mail addressed to all four of us. After you have emailed your solution, you **must** upload your solution in moodle before 1.30 pm. Only moodle submission will be considered. Mail version will not be evaluated however will be cross verified.

1. Find a statement involving only $\land, \lor$ and $\neg$ that is equivalent to $s \leftrightarrow t$. Try to ensure that your statement has the fewest possible symbols. (1)

2. Each expression below represents a statement about the integers. Using $p(x)$ for x is prime, $q(x, y)$ for $x = y^2, s(x, y, z)$ for $z = xy, t(x, y)$ for $x = y$. Determine which expressions represent true statements and which represent false statements.

   - $\forall x \in \mathbb{Z}(\exists y \in \mathbb{Z}(q(x, y) \lor p(x)))$
   - $\forall x \in \mathbb{Z}(\forall y \in \mathbb{Z}(s(x, x, y) \leftrightarrow q(x, y)))$
   - $\forall y \in \mathbb{Z}(\exists x \in \mathbb{Z}(q(y, x)))$
   - $\exists z \in \mathbb{Z}(\exists z \in \mathbb{Z}(p(x) \land p(y)) \land \neg t(x, y)))$

   (2)

3. Consider the following statements:
   P: Good mobile phones are not cheap
   Q: Cheap mobile phones are not good

   $L : P \rightarrow Q$
   $M : Q \rightarrow P$
   $N : P \leftrightarrow Q$ Then choose the correct statements

   (a) only L is true.

   (b) only M is true.

   (c) All (L, M, N) are true.

   (2)

4. Consider the statement

   "Not all the glitters are gold"

   Predicate glitters(x) is true if x glitters and predicate gold(x) is true if x is gold.
   Which one of the following logical formula represents the above statement?

   (a) $\forall x : glitters(x) \rightarrow \neg gold(x)$

   (b) $\forall x : gold(x) \rightarrow glitters(x)$

   (c) $\exists x : gold(x) \land \neg glitters(x)$

   (d) $\exists x : glitters(x) \land \neg gold(x)$

   (1)

5. Find the correct option:

   p : If Humpty sits on a wall, then he will fall.
   q : If Humpty falls, then he was sitting on a wall.
   r : If Humpty does not fall, then he was not sitting on the wall.
   s : If Humpty does not sit on a wall, then he does not fall.

   (a) $p \equiv s$ and $q \equiv r$

   (b) $p \equiv q$ and $r \equiv s$

   (c) $p \equiv s \equiv r$

   (d) $p \equiv r$ and $q \equiv s$

   Justify your answer. (1)

6. Using the predicates given, formalize the following sentences in first order logic:

   - Boy(x): x is a boy
   - Prize(x): x is a prize
   - Won(x, y): x won y

   (a) Every prize was won by a boy.
   (b) A boy won every prize.

   (2)

7. Are these two symbols $\equiv$ and $\leftrightarrow$ same or different? Justify with an example. (1)