Assignment 3
CS 203: Discrete Structures

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INSTRUCTIONS: You have to give clear and detailed solution for each of the questions. **Make one single pdf file containing solutions to all problems. Take a clear picture and upload the hand written solutions in Classroom(event-assignment3) by 14/10/2021, 10 am. Name your pdf with your** *name_rollno*.**pdf.** For example *harrithha_*200010018.*pdf*. Late submissions will not be graded. Students can discuss but must write their solutions based on their understanding independently. Do not use web resources or answers from your peers to obtain solutions. If anyone is involved in malpractice of any sort, then suitable disciplinary action will be taken. If required, there would be a viva to selected set of students.

1. We can choose a subset of a set of ten given integers, such that their sum is divisible by 10. Find the correct option:

   (a) True

   (b) False

   Justify. (2)

2. Show that 1/(1 - x - x^2 - x^3 - x^4 - x^5 - x^6) is the generating function for the number of ways that the sum n can be obtained when a die is rolled repeatedly and the order of the rolls matters. (1)

3. A binary relation R on $\mathbb{N} \times \mathbb{N}$ is defined as: $(a, b)\ R\ (c, d)$ if $a \leq c$ or $b \leq d$ then check whether following statements are true or false.

   (a) R is reflexive.

   (b) R is transitive.

   (c) R is symmetric

   (d) R is anti-symmetric

   (2)

4. Prove that for any prime number $P > 5$ there exists K such that $111 \cdots 1$ (K times 1 or sequence of K ones) is divisible by P (2)

5. Solve the recurrence $a_n = 5a_{n-1} - 6a_{n-2}$ using the method of generating function and verify if your solution is correct by directly applying the general solution for recurrence taught in the class when we have distinct real roots. (3)

6. Solve the recurrence $a_n = 6a_{n-1} - 9a_{n-2}$ using the method of generating function and verify if your solution is correct by directly applying the general solution for recurrence taught in the class when we have repeated real roots. (3)

7. You are provided with an infinite supply of L-shaped triomino(see Figure.1). A triomino is a collection of 3 blocks arranged in a particular order as shown below. You are provided with a chess-board of dimension $2^{n+1} \times 2^{n+1}$. One block is removed from the same leaving a defective chessboard (see Figure.2) with $2^{n+1} \times 2^{n+1} - 1$ blocks. Prove using mathematical induction any such defective chess board can be tiled using L-shaped triominos. (3)

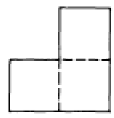

Figure 1: An L-shaped triomino

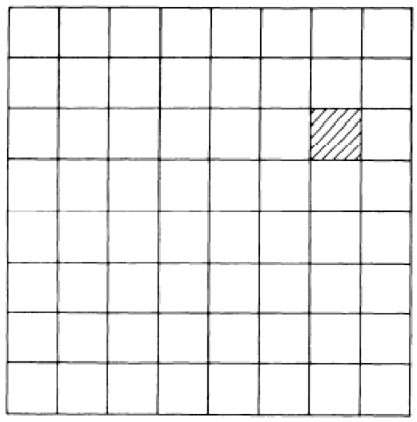

Figure 2: Image of a defective chessboard

8. Let $\{C_n\}$ be the sequence of Catalan numbers, that is, the solution to the recurrence relation $C_n = \Sigma_{k=0}^{n-1} C_k.C_{n-k-1}$ with $C_0 = C_1 = 1$

   (a) Show that if G(x) is the generating function for the sequence of Catalan Numbers, then $xG(x)^2 - G(x) + 1 = 0$. Conclude (using the initial conditions) that $G(x) = (1 - \sqrt{1-4x})/(2x)$.

   (b) Using extended binomial if coefiicient of $(1-4x)^{-1/2}$ is $C(2n, n)$ conclude what will be G(x)=?

      i. $\Sigma_{n=0}^{\infty} \frac{1}{n+1} C(2n, n)x^n$

      ii. $\Sigma_{n=0}^{\infty} \frac{1}{2n+1} C(2n, n)x^n$

      iii. $\Sigma_{n=0}^{\infty} \frac{n}{n+1} C(2n, n-1)x^n$

      iv. $\Sigma_{n=0}^{\infty} \frac{1}{n+1} C(2n, n+1)x^n$

   (c) Show that $C_n \geq 2^{n-1}$ for all positive integers n. (1.5+1.5+1)