Module: SET THEORY
CS 203: Discrete Structures
Course Instructor : Prof. Prabuchandran K J

INSTRUCTIONS: You have to give clear and detailed solution for each of the questions. **Send one single pdf file containing solutions to all problems. Take a clear picture and upload the hand written solutions in Moodle(event-assignment1) by 24/08/2021, 10 pm. Name your pdf with your** *name_rollno*.**pdf.** For example *harrithha_*200010018.*pdf*. Late submissions will not be graded. Students can discuss but must write their solutions based on their understanding independently. Do not use web resources or answers from your peers to obtain solutions. If anyone is involved in malpractice of any sort, then suitable disciplinary action will be taken. If required, there would be a viva to selected set of students.

1. Is p/q countable provided p/q is under the following domains (2.5)
   a) $p \in \mathbb{Q}$ and $q \in \mathbb{Q} - \{0\}$
   b) $p \in \mathbb{N}$ and $q \in \mathbb{N} - \{0\}$
   c) $p \in \mathbb{Z}$ and $q \in \mathbb{Z} - \{0\}$
   d) $p = 1$ and $q \in \mathbb{Z} - \{0\}$
   e) $p = \mathbb{R}$ and $q \in \mathbb{Z} - \{0\}$

2. A hypothetical hotel named Infinite hotel has infinite number of rooms in Dracula city (Bran, Romania) with a (countably) infinite number of floors and rooms.. It was peak season for visitors and so the hotel is full all the time. On this particular night, the hotel is completely full. Late in the evening, Ira arrive at the hotel and inquire about a room. Although there is no vacancy the hotel manager tells you that since this is an infinite hotel she can easily make room for you! Ira was delighted, but confused. If the infinite hotel is completely filled with an infinite number of guests, how does the manager go about securing a room for you? Explain what the manager would do if there were 3 more guests like Ira looking for room so that everyone gets a room? $(2 + 0.5)$

3. Which of the following become **true** when $\in$ is inserted in place of the blank? Which become true when $\subseteq$ is inserted? Justify.

   (a) $\{\emptyset\}$____$\{\emptyset, \{\emptyset\}\}$
   (b) $\{\emptyset\}$____$\{\emptyset, \{\{\emptyset\}\}\}$
   (c) $\{\{\emptyset\}\}$____$\{\emptyset, \{\emptyset\}\}$
   (d) $\{\{\emptyset\}\}$____$\{\emptyset, \{\{\emptyset\}\}\}$
   (e) $\{\{\emptyset\}\}$____$\{\emptyset, \{\emptyset, \{\emptyset\}\}\}$

(3.0)

4. (a) If A is a uncountable set, and B is a countable set, must $A - B$ be uncountable? Justify.
   (b) Given that

i. $(A \cap C) \subseteq (B \cap C)$

ii. $(A \cap \overline{C}) \subseteq (B \cap \overline{C})$

Find the relationship between A and B with proper justification.

(c) Let A, B, C be three sets. Write proof for the following identity:
$(A - B) \cap (B - C) \cap (A - C) = \phi$. $(2 + 2 + 1)$

5. Is $\{5, 6\} \times \mathbb{N}$ is uncountable or countably infinite? Justify. (1)

6. Which set is bigger $\mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q}$ or $(0, 0.000001)$? Justify. (1)

7.
   - $f(A) \triangleq \{f(x) \in Y : \quad x \in A\}$.
   - $f^{-1}(B) \triangleq \{x \in X : \quad f(x) \in B\}$.

   State for each of the following statements 'True' or 'False'? Justify.

   - For any set $A \subset X$, $(f(A))^c = f(A^c)$, where c in the superscript denotes complement of the set.
   - For any two sets $A_1, A_2 \subset X$, $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$
   - For any set $B \subset Y$, $(f^{-1}(B))^c = f^{-1}(B^c)$, where c in the superscript denotes complement of the set.
   - For any two sets $B_1, B_2 \subset Y$, $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$ (0.5+0.5+0.5+0.5)

8. Let $\{A_n,\ n \geq 0\}$ be sequence of sets. Define $\liminf_{n\to\infty} A_n = \cup_{n\geq 1} \cap_{j\geq n} A_j$ $\limsup_{n\to\infty} A_n = \cap_{n\geq 1} \cup_{j\geq n} A_j$. Prove that $\liminf_{n\to\infty} A_n \subset \limsup_{n\to\infty} A_n$. Give an example of sequence of sets where $\liminf_{n\to\infty} A_n$ is a proper subset of $\limsup_{n\to\infty} A_n$. (1.5+1.5)