Solution to Selected Questions: CHAPTER 6
PRODUCTION

2. Suppose a chair manufacturer is producing in the short run (with its existing plant and equipment). The manufacturer has observed the following levels of production corresponding to different numbers of workers:

Number of chairs	Number of workers
1	10
2	18
3	24
4	28
5	30
6	28
7	25

a. Calculate the marginal and average product of labor for this production function.

The average product of labor, AP_L, is equal to $\frac{q}{L}$. The marginal product of labor, MP_L, is equal to $\frac{\Delta q}{\Delta L}$, the change in output divided by the change in labor input. For this production process we have:

L	q	AP_L	MP_L
0	0		
1	10	10	10
2	18	9	8
3	24	8	6
4	28	7	4
5	30	6	2
6	28	4.7	–2
7	25	3.6	–3

b. Does this production function exhibit diminishing returns to labor? Explain.

Yes, this production process exhibits diminishing returns to labor. The marginal product of labor, the extra output produced by each additional worker,

diminishes as workers are added, and this starts to occur with the second unit of labor.

c. **Explain intuitively what might cause the marginal product of labor to become negative.**

Labor's negative marginal product for $L > 5$ may arise from congestion in the chair manufacturer's factory. Since more laborers are using the same fixed amount of capital, it is possible that they could get in each other's way, decreasing efficiency and the amount of output. Firms also have to control the quality of their output, and the high congestion of labor may produce products that are not of a high enough quality to be offered for sale, which can contribute to a negative marginal product.

3. Fill in the gaps in the table below.

Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	0	–	–
1	225		
2			300
3		300	
4	1140		
5		225	
6			225

Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	0		
1	225	225	225
2	600	375	300
3	900	300	300
4	1140	240	285
5	1365	225	273
6	1350	–15	225

5. For each of the following examples, draw a representative isoquant. What can you say about the marginal rate of technical substitution in each case?

a. **A firm can hire only full-time employees to produce its output, or it can hire some combination of full-time and part-time employees. For each full-time worker let go, the firm must hire an increasing number of temporary employees to maintain the same level of output.**

Place part-time workers on the vertical axis and full-time workers on the horizontal. The slope of the isoquant measures the number of part-time workers that can be exchanged for a full-time worker while still maintaining output. At the bottom end of the isoquant, at point A, the isoquant hits the full-time axis because it is possible to produce with full-time workers only and no part-timers. As we move up the isoquant and give up full-time workers, we must hire more and more part-time workers to replace each full-time worker. The slope increases (in absolute value) as we move up the isoquant. The isoquant is therefore convex and there is a diminishing marginal rate of technical substitution.
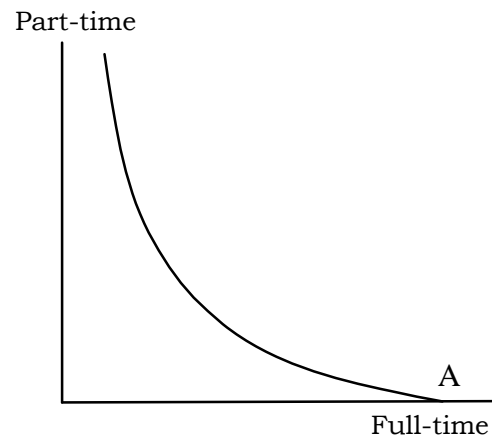
b. **A firm finds that it can always trade two units of labor for one unit of capital and still keep output constant.**

The marginal rate of technical substitution measures the number of units of capital that can be exchanged for a unit of labor while still maintaining output. If the firm can always trade two units of labor for one unit of capital then the MRTS of labor for capital is constant and equal to 1/2, and the isoquant is linear.

c. **A firm requires exactly two full-time workers to operate each piece of machinery in the factory**

This firm operates under a fixed proportions technology, and the isoquants are L-shaped. The firm cannot substitute any labor for capital and still maintain output because it must maintain a fixed 2:1 ratio of labor to capital. The MRTS is infinite (or undefined) along the vertical part of the isoquant and zero on the horizontal part.

6. **A firm has a production process in which the inputs to production are perfectly substitutable in the long run. Can you tell whether the marginal rate of technical substitution is high or low, or is further information necessary? Discuss.**

Further information is necessary. The marginal rate of technical substitution, *MRTS,* is the absolute value of the slope of an isoquant. If the inputs are perfect substitutes, the isoquants will be linear. To calculate the slope of the isoquant, and hence the *MRTS*, we need to know the rate at which one input may be substituted for the other. In this case, we do not know whether the MRTS is high or low. All we know is that it is a constant number. We need to know the marginal product of each input to determine the MRTS.

9. **The production function for the personal computers of DISK, Inc., is given by $q = 10K^{0.5}L^{0.5}$, where q is the number of computers produced per day, K is hours of machine time, and L is hours of labor input. DISK's competitor, FLOPPY, Inc., is using the production function**
   $q = 10K^{0.6}L^{0.4}.$

   a. **If both companies use the same amounts of capital and labor, which will generate more output?**

   Let q_1 be the output of DISK, Inc., q_2, be the output of FLOPPY, Inc., and X be the same equal amounts of capital and labor for the two firms. Then according to their production functions,

   $$q_1 = 10X^{0.5}X^{0.5} = 10X^{(0.5 + 0.5)} = 10X$$

   and

   $$q_2 = 10X^{0.6}X^{0.4} = 10X^{(0.6 + 0.4)} = 10X.$$

   Because $q_1 = q_2$, both firms generate the same output with the same inputs. Note that if the two firms both used the same amount of capital and the same amount of labor, but the amount of capital was not equal to the amount of labor, then the two firms would not produce the same levels of output. In fact, if $K > L$ then $q_2 > q_1$, and if $L > K$ then $q_1 > q_2$.

   b. **Assume that capital is limited to 9 machine hours, but labor is unlimited in supply. In which company is the marginal product of labor greater? Explain.**

   With capital limited to 9 machine hours, the production functions become $q_1 = 30L^{0.5}$ and $q_2 = 37.37L^{0.4}$. To determine the production function with the highest marginal productivity of labor, consider the following table:

L	q Firm 1	MP_L Firm 1	q Firm 2	MP_L Firm 2
0	0.0	—	0.00	—
1	30.00	30.00	37.37	37.37
2	42.43	12.43	49.31	11.94
3	51.96	9.53	57.99	8.68
4	60.00	8.04	65.06	7.07

   For each unit of labor above 1, the marginal productivity of labor is greater for the first firm, DISK, Inc.