2900111 (Section 1)
Economics I: Microeconomics

Chairat Aemkulwat
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Solution to Selected Questions: CHAPTER 12 MONOPOLISTIC COMPETITION AND OLIGOPOLY

1. Suppose all firms in a monopolistically competitive industry were merged into one large firm. Would that new firm produce as many different brands? Would it produce only a single brand? Explain.

Monopolistic competition is defined by product differentiation. Each firm earns economic profit by distinguishing its brand from all other brands. This distinction can arise from underlying differences in the product or from differences in advertising. If these competitors merge into a single firm, the resulting monopolist would not produce as many brands, since too much brand competition is internecine (mutually destructive). However, it is unlikely that only one brand would be produced after the merger. Producing several brands with different prices and characteristics is one method of splitting the market into sets of customers with different price elasticities. The monopolist can sell to more consumers and maximize overall profit by producing multiple brands and practicing a form of price discrimination.

- 3. A monopolist can produce at a constant average (and marginal) cost of AC = MC = \$5. It faces a market demand curve given by Q = 53 P.
 - a. Calculate the profit-maximizing price and quantity for this monopolist. Also calculate its profits.

First solve for the inverse demand curve, P = 53 - Q. Then the marginal revenue curve has the same intercept and twice the slope:

$$MR = 53 - 2Q$$
.

Marginal cost is a constant \$5. Setting MR = MC, find the optimal quantity:

$$53 - 2Q = 5$$
, or $Q = 24$.

Substitute Q = 24 into the demand function to find price:

$$P = 53 - 24 = $29.$$

Assuming fixed costs are zero, profits are equal to

$$\pi = TR - TC = (29)(24) - (5)(24) = $576.$$

b. Suppose a second firm enters the market. Let \mathbf{Q}_1 be the output of the first firm and \mathbf{Q}_2 be the output of the second. Market demand is now given by

$$Q_1 + Q_2 = 53 - P.$$

Assuming that this second firm has the same costs as the first, write the profits of each firm as functions of Q_1 and Q_2 .

When the second firm enters, price can be written as a function of the output of both firms: $P = 53 - Q_1 - Q_2$. We may write the profit functions for the two firms:

$$\pi_1 = PQ_1 - C(Q_1) = (53 - Q_1 - Q_2)Q_1 - 5Q_1$$
, or $\pi_1 = 48Q_1 - Q_1^2 - Q_1Q_2$

and

$$\pi_2 = PQ_2 - C(Q_2) = (53 - Q_1 - Q_2)Q_2 - 5Q_2$$
, or $\pi_2 = 48Q_2 - Q_2^2 - Q_1Q_2$.

c. Suppose (as in the Cournot model) that each firm chooses its profit-maximizing level of output on the assumption that its competitor's output is fixed. Find each firm's "reaction curve" (i.e., the rule that gives its desired output in terms of its competitor's output).

Under the Cournot assumption, each firm treats the output of the other firm as a constant in its maximization calculations. Therefore, Firm 1 chooses Q_1 to maximize π_1 in part (b) with Q_2 being treated as a constant. The change in π_1 with respect to a change in Q_1 is

$$\frac{\partial \pi_1}{\partial Q_1} = 48 - 2Q_1 - Q_2 = 0$$
, or $Q_1 = 24 - \frac{Q_2}{2}$.

This equation is the reaction function for Firm 1, which generates the profit-maximizing level of output, given the output of Firm 2. Because the problem is symmetric, the reaction function for Firm 2 is

$$Q_2 = 24 - \frac{Q_1}{2} \ .$$

d. Calculate the Cournot equilibrium (i.e., the values of Q_1 and Q_2 for which each firm is doing as well as it can given its competitor's output). What are the resulting market price and profits of each firm?

Solve for the values of Q_1 and Q_2 that satisfy both reaction functions by substituting Firm 2's reaction function into the function for Firm 1:

$$Q_1 = 24 - \left(\frac{1}{2}\right)\left(24 - \frac{Q_1}{2}\right)$$
, or $Q_1 = 16$.

By symmetry, $Q_2 = 16$.

To determine the price, substitute Q_1 and Q_2 into the demand equation:

$$P = 53 - 16 - 16 = $21.$$

Profit for Firm 1 is therefore

$$\pi_i = PQ_i - C(Q_i) = \pi_i = (21)(16) - (5)(16) = $256.$$

Firm 2's profit is the same, so total industry profit is $\pi_1 + \pi_2 = \$256 + \$256 = \$512$.

e. Suppose there are N firms in the industry, all with the same constant marginal cost, MC = \$5. Find the Cournot equilibrium. How much will each firm produce, what will be the market price, and how much profit will each firm earn? Also, show that as N becomes large, the market price approaches the price that would prevail under perfect competition.

If there are N identical firms, then the price in the market will be

$$P = 53 - (Q_1 + Q_2 + \dots + Q_N).$$

Profits for the i^{th} firm are given by

$$\pi_i = PQ_i - C(Q_i),$$

$$\pi_i = 53Q_i - Q_1Q_i - Q_2Q_i - \dots - Q_i^2 - \dots - Q_NQ_i - 5Q_i.$$

Differentiating to obtain the necessary first-order condition for profit maximization,

$$\frac{\partial \pi_i}{\partial Q_i} = 53 - Q_1 - Q_2 - \dots - 2Q_i - \dots - Q_N - 5 = 0.$$

Solving for Q_i ,

$$Q_i = 24 - \frac{1}{2}(Q_1 + \dots + Q_{i-1} + Q_{i+1} + \dots + Q_N).$$

If all firms face the same costs, they will all produce the same level of output, i.e., $Q_i = Q^*$. Therefore,

$$Q^* = 24 - \frac{1}{2}(N-1)Q^*$$
, or $2Q^* = 48 - (N-1)Q^*$, or $(N+1)Q^* = 48$, or $Q^* = \frac{48}{(N+1)}$.

Now substitute $Q = NQ^*$ for total output in the demand function:

$$P = 53 - N\left(\frac{48}{N+1}\right).$$

Total profits are

$$\pi_T = PQ - C(Q) = P(NQ^*) - 5(NQ^*)$$

or

$$\pi_T = \left[53 - N \left(\frac{48}{N+1} \right) \right] (N) \left(\frac{48}{N+1} \right) - 5N \left(\frac{48}{N+1} \right) \text{ or }$$

$$\pi_T = \left[48 - (N) \left(\frac{48}{N+1} \right) \right] (N) \left(\frac{48}{N+1} \right)$$

or

$$\pi_T = (48) \left(\frac{N+1-N}{N+1} \right) (48) \left(\frac{N}{N+1} \right) = (2,304) \left(\frac{N}{(N+1)^2} \right).$$

Notice that with N firms

$$Q = 48 \left(\frac{N}{N+1} \right)$$

and that, as N increases $(N \to \infty)$

$$Q = 48$$
.

Similarly, with

$$P = 53 - 48 \left(\frac{N}{N+1}\right),$$

as $N \to \infty$,

$$P = 53 - 48 = 5$$
.

Finally,

$$\pi_T = 2{,}304\left(\frac{N}{\left(N+1\right)^2}\right),$$

so as $N \to \infty$,

$$\pi_T = \$0.$$

In perfect competition, we know that profits are zero and price equals marginal cost. Here, $\pi_T = \$0$ and P = MC = 5. Thus, when N approaches infinity, this market approaches a perfectly competitive one.

5. Two firms compete in selling identical widgets. They choose their output levels Q_1 and Q_2 simultaneously and face the demand curve

$$P = 30 - Q$$

where $Q = Q_1 + Q_2$. Until recently, both firms had zero marginal costs. Recent environmental regulations have increased Firm 2's marginal cost to \$15. Firm 1's marginal cost remains constant at zero. True or false: As a result, the market price will rise to the monopoly level.

Surprisingly, this is true. However, it occurs only because the marginal cost for Firm 2 is \$15 or more. If the market were monopolized before the environmental regulations, the marginal revenue for the monopolist would be

$$MR = 30 - 2Q$$
.

Profit maximization implies MR = MC, or 30 - 2Q = 0. Therefore, Q = 15, and (using the demand curve) P = \$15.

The situation after the environmental regulations is a Cournot game where Firm 1's marginal costs are zero and Firm 2's marginal costs are \$15. We need to find the best response functions:

Firm 1's revenue is

$$PQ_1 = (30 - Q_1 - Q_2)Q_1 = 30Q_1 - Q_1^2 - Q_1Q_2,$$

and its marginal revenue is given by:

$$MR_1 = 30 - 2Q_1 - Q_2$$
.

Profit maximization implies $MR_1 = MC_1$ or

$$30 - 2Q_1 - Q_2 = 0 \Rightarrow Q_1 = 15 - \frac{Q_2}{2}$$
,

which is Firm 1's best response function.

Firm 2's revenue function is symmetric to that of Firm 1 and hence

$$MR_2 = 30 - Q_1 - 2Q_2$$
.

Profit maximization implies $MR_2 = MC_2$, or

$$30 - 2Q_2 - Q_1 = 15 \Rightarrow Q_2 = 7.5 - \frac{Q_1}{2}$$

which is Firm 2's best response function.

Cournot equilibrium occurs at the intersection of the best response functions. Substituting for Q_1 in the response function for Firm 2 yields:

$$Q_2 = 7.5 - 0.5(15 - \frac{Q_2}{2}).$$

Thus $Q_2 = 0$ and $Q_1 = 15$. $P = 30 - Q_1 - Q_2 = 15 , which is the monopoly price.

- 7. Suppose that two competing firms, A and B, produce a homogeneous good. Both firms have a marginal cost of MC = \$50. Describe what would happen to output and price in each of the following situations if the firms are at (i) Cournot equilibrium, (ii) collusive equilibrium, and (iii) Bertrand equilibrium.
 - a. Because Firm A must increase wages, its MC increases to \$80.
 - (i) In a Cournot equilibrium you must think about the effect on the reaction functions, as illustrated in Figure 12.5 of the text. When Firm *A* experiences an increase in marginal cost, its reaction function will shift inward. The quantity produced by Firm *A* will decrease and the quantity produced by Firm *B* will increase. Total quantity produced will decrease and price will increase.
 - (ii) In a collusive equilibrium, the two firms will collectively act like a monopolist. When the marginal cost of Firm A increases, Firm A will reduce its production to zero, because Firm B can produce at a lower marginal cost. Because Firm B can produce the entire industry output at a marginal cost of \$50, there will be no change in output or price. However, the firms will have to come to some agreement on how to share the profit earned by B.
 - (iii) Before the increase in Firm A's costs, both firms would charge a price equal to marginal cost (P = \$50) because the good is homogeneous. After Firm A's marginal cost increases, Firm B will raise its price to \$79.99 (or some price just below \$80) and take all sales away from Firm A. Firm A would lose money on each unit sold at any price below its marginal cost of \$80, so it will produce nothing.
 - b. The marginal cost of both firms increases.
 - (i) Again refer to Figure 12.5. The increase in the marginal cost of both firms will shift both reaction functions inward. Both firms will decrease quantity produced and price will increase.
 - (ii) When marginal cost increases, both firms will produce less and price will increase, as in the monopoly case.
 - (iii) Price will increase to the new level of marginal cost and quantity will decrease.
 - c. The demand curve shifts to the right.
 - (i) This is the opposite of the case in part b. In this situation, both reaction functions will shift outward and both will produce a higher quantity. Price will tend to increase.
 - (ii) Both firms will increase the quantity produced as demand and marginal revenue increase. Price will also tend to increase.
 - (iii) Both firms will supply more output. Given that marginal cost remains the same, the price will not change.

11. Two firms compete by choosing price. Their demand functions are

$$Q_1 = 20 - P_1 + P_2$$
 and $Q_2 = 20 + P_1 - P_2$

where P_1 and P_2 are the prices charged by each firm, respectively, and Q_1 and Q_2 are the resulting demands. Note that the demand for each good depends only on the difference in prices; if the two firms colluded and set the same price, they could make that price as high as they wanted, and earn infinite profits. Marginal costs are zero.

a. Suppose the two firms set their prices at the *same time*. Find the resulting Nash equilibrium. What price will each firm charge, how much will it sell, and what will its profit be? (*Hint*: Maximize the profit of each firm with respect to its price.)

To determine the Nash equilibrium in prices, first calculate the reaction function for each firm, then solve for price. With zero marginal cost, profit for Firm 1 is:

$$\pi_1 = P_1Q_1 = P_1(20 - P_1 + P_2) = 20P_1 - P_1^2 + P_2P_1.$$

Marginal revenue is the slope of the total revenue function (here it is the derivative of the profit function with respect to P_1 because total cost is zero):

$$MR_1 = 20 - 2P_1 + P_2$$
.

At the profit-maximizing price, $MR_1 = 0$. Therefore,

$$P_1 = \frac{20 + P_2}{2}.$$

This is Firm 1's reaction function. Because Firm 2 is symmetric to Firm 1, its reaction function is $P_2 = \frac{20 + P_1}{2}$. Substituting Firm 2's reaction function into that of Firm 1:

$$P_1 = \frac{20 + \frac{20 + P_1}{2}}{2} = 10 + 5 + \frac{P_1}{4}$$
, so $P_1 = 20

By symmetry, $P_2 = 20 .

To determine the quantity produced by each firm, substitute P_1 and P_2 into the demand functions:

$$Q_1 = 20 - 20 + 20 = 20$$
 and

$$Q_2 = 20 + 20 - 20 = 20$$
.

Profits for Firm 1 are P_1Q_1 = \$400, and, by symmetry, profits for Firm 2 are also \$400.

b. Suppose Firm 1 sets its price *first* and then Firm 2 sets its price. What price will each firm charge, how much will it sell, and what will its profit be?

If Firm 1 sets its price first, it takes Firm 2's reaction function into account. Firm 1's profit function is:

$$\pi_1 = P_1 \left(20 - P_1 + \frac{20 + P_1}{2} \right) = 30P_1 - \frac{P_1^2}{2}.$$

To determine the profit-maximizing price, find the change in profit with respect to a change in price:

6

$$\frac{d\pi_1}{dP_1} = 30 - P_1.$$

Set this expression equal to zero to find the profit-maximizing price:

$$30 - P_1 = 0$$
, or $P_1 = 30 .

Substitute P_1 in Firm 2's reaction function to find P_2 :

$$P_2 = \frac{20+30}{2} = $25.$$

At these prices,

$$Q_1 = 20 - 30 + 25 = 15$$
 and

$$Q_2 = 20 + 30 - 25 = 25$$
.

Profits are

$$\pi_1 = (30)(15) = $450$$
 and

$$\pi_2 = (25)(25) = $625.$$

If Firm 1 must set its price first, Firm 2 is able to undercut Firm 1 and gain a larger market share. However, both firms make greater profits than they did in part (a), where they chose prices simultaneously.

c. Suppose you are one of these firms and that there are three ways you could play the game: (i) Both firms set price at the same time; (ii) You set price first; or (iii) Your competitor sets price first. If you could choose among these options, which would you prefer? Explain why.

Your first choice should be (iii), and your second choice should be (ii). (Compare the Nash profits in part (a), \$400, with profits in part (b), \$450 and \$625.) From the reaction functions, we know that the price leader provokes a price increase in the follower. By being able to move second, however, the follower increases price by less than the leader, and hence undercuts the leader. Both firms enjoy increased profits, but the follower does better.