

HS 402: Assignment 2

1. Jane receives utility from days spent traveling on vacation domestically (D) and days spent traveling on vacation in a foreign country (F), as given by the utility function $U(D,F) = 10DF$. In addition, the price of a day spent traveling domestically is \$100, the price of a day spent traveling in a foreign country is \$400, and Jane's annual travel budget is \$4000.
 - (a) Illustrate the indifference curve associated with a utility of 800 and the indifference curve associated with a utility of 1200.
 - (b) Graph Jane's budget line on the same graph.
 - (c) Can Jane afford any of the bundles that give her a utility of 800? What about a utility of 1200?
 - (d) Find Jane's utility maximizing choice of days spent traveling domestically and days spent in a foreign country.
2. Show that the two utility functions given below generate the identical demand functions for goods X and Y :
 - (a) $U(X, Y) = \log(X) + \log(Y)$
 - (b) $U(X, Y) = (XY)^{0.5}$
3. Sharon has the following utility function: $U(X, Y) = \sqrt{X} + \sqrt{Y}$ where X is her consumption of candy bars, with price $P_X = \$1$, and Y is her consumption of espressos, with $P_Y = \$3$.
 - (a) Derive Sharon's demand for candy bars and espressos.
 - (b) Assume that her income $I = \$100$. How many candy bars and how many espressos will Sharon consume?
 - (c) What is the marginal utility of income?
4. Do the following functions exhibit increasing, constant, or decreasing returns to scale? What happens to the marginal product of each individual factor as that factor is increased and the other factor held constant?
 - (a) $q = 3L + 2K$
 - (b) $q = (2L + 2K)^{\frac{1}{2}}$
 - (c) $q = 3LK^2$
 - (d) $q = 4L^{\frac{1}{2}} + 4K$
5. The production function for the personal computers of DISK, Inc., is given by $q = 10K^{0.5}L^{0.5}$, where q is the number of computers produced per day, K is hours of machine time, and L is hours of labor input. DISK's competitor, FLOPPY, Inc., is using the production function $q = 10K^{0.6}L^{0.4}$.
 - (a) If both companies use the same amounts of capital and labor, which will generate more output?

- (b) Assume that capital is limited to 9 machine hours, but labor is unlimited in supply. In which company is the marginal product of labor greater? Explain.
6. Suppose that a firm's production function is $q = 10L^{0.5}K^{0.5}$. The cost of a unit of labor is \$20 and the cost of a unit of capital is \$80.
- The firm is currently producing 100 units of output and has determined that the cost-minimizing optimal quantities of labor and capital are 20 and 5, respectively. Graphically illustrate this using isoquants and isocost lines.
 - The firm now wants to increase output to 140 units. If capital is fixed in the short run, how much labor will the firm require? Illustrate this point graphically and find the firm's new total cost.
 - Graphically identify the optimal cost-minimizing level of capital and labor in the long run if the firm wants to produce 140 units.
 - If the marginal rate of technical substitution is K/L , find the optimal level of capital and labor required to produce the 140 units of output.
7. Joe quits his computer programming job, where he was earning a salary of \$50,000 per year, to start . He opens his own computer software business store in a building that he owns and was previously renting out for \$24,000 per year. In his first year of business he has the following expenses: mortgage \$18,000, salary paid to himself, \$40,000; rent, \$0; other expenses, \$25,000. Find the accounting cost and the economic cost associated with Joe's computer software business.
8. A firm has a fixed production costs of \$5,000 and a constant marginal cost of production of equal to \$500 per unit produced.
- What is the firm's total cost function? Average cost?
 - If the firm wanted to minimize the average total cost, would it choose to be very large or very small? Explain.
9. A chair manufacturer hires its assembly-line labor for \$30 an hour and calculates that the rental cost of its machinery is \$15 per hour. Suppose that a chair can be produced using 4 hours of labor or machinery in any combination. If the firm is currently using 3 hours of labor for each hour of machine time, is it minimizing its costs of production? If so, why? If not, how can it improve the situation? Graphically illustrate the isoquant and the two isocost lines for the current combination of labor and capital and for the optimal combination of labor and capital.
10. Suppose the process of producing lightweight parkas by Polly's Parkas is described by the function $q = 10K^{0.8}(L-40)^{0.2}$ where q is the number of parkas produced, K the number of computerized stitching machine hours, and L the number of person-hours of labor. In addition to capital and labor, \$10 worth of raw materials is used in the production of each parka.
- By minimizing cost subject to the production function, derive the cost-minimizing demands for K and L as a function of output (q), wage rates (w), and rental rates on machines (r). Use these results to derive the total cost function: that is, costs as a function of q, r, w , and the constant \$10 per unit materials cost.
 - This process requires skilled workers, who earn \$32 per hour. The rental rate on the machines used in the process is \$64 per hour. At these factor prices, what are total costs as a function of q ? Does this technology exhibit decreasing, constant, or increasing returns to scale?

- (c) Polly's Parkas plans to produce 2000 parkas per week. At the factor prices given above, how many workers should the firm hire (at 40 hours per week) and how many machines should it rent (at 40 machine-hours per week)? What are the marginal and average costs at this level of production?
11. Suppose the same firm's cost function is $C(q) = 4q^2 + 16$.
- Find variable cost, fixed cost, average cost, average variable cost, and average fixed cost.
 - Show the average cost, marginal cost, and average variable cost curves on a graph.
 - Find the output that minimizes average cost.
 - At what range of prices will the firm produce a positive output?
 - At what range of prices will the firm earn a negative profit?
 - At what range of prices will the firm earn a positive profit?
12. Suppose that a competitive firm has a total cost function $C(q) = 450 + 15q + 2q^2$ and a marginal cost function $MC(q) = 15 + 4q$. If the market price is $P = \$115$ per unit, find the level of output produced by the firm. Find the level of profit and the level of producer surplus.
13. In 2007, Americans smoked 19.2 billion packs of cigarettes. They paid an average retail price of \$4.50 per pack.
- Given that the elasticity of supply is 0.5 and the elasticity of demand is -0.4 , derive linear demand and supply curves for cigarettes.
 - Cigarettes are subject to a federal tax, which was about 40 cents per pack in 2007. What does this tax do to the market-clearing price and quantity?
 - How much of the federal tax will consumers pay? What part will producers pay?