MA 201 Complex Analysis
Quiz 1
IIT Dharwad (Autumn 2021)

Total Marks: 15 **Date & Time:** 21 Aug 2021, 04:00 pm to 04:30 p.m.

(1) **[2M]** (a) Find all the 4th roots of ci where c is the last two digits of your roll number (for example, if your roll number is 190010023, then c=23).
(b) Find all the 4th roots of $-ci$ where c is the last two digits of your roll number (for example, if your roll number is 190010023, then c=23).

(2) **[4M]** (a) Determine the set of all the limit points, the boundary points and the interior points of the set

$$S := \{z \in \mathbb{C} \setminus \{0\} : |z| \le 1 \text{ and } 0 \le \operatorname{Arg}(z) < \pi/4\} \cup \{z \in \mathbb{C} : |z| > 1 \text{ and } \pi/4 < \operatorname{Arg}(z) < 2\pi\}$$

where $\operatorname{Arg}(z)$ is the principal argument of z that lies in $[0, 2\pi)$. Also, find the $\overline{S}$, the closure of the set S.
(b) Determine the set of all the limit points, the boundary points and the interior points of the set

$$S := \{z \in \mathbb{C} \setminus \{0\} : |z| \le 1 \text{ and } \pi/4 < \operatorname{Arg}(z) < 2\pi\} \cup \{z \in \mathbb{C} : |z| < 1 \text{ and } 0 \le \operatorname{Arg}(z) < \pi/4\}$$

where $\operatorname{Arg}(z)$ is the principal argument of z that lies in $[0, 2\pi)$. Also, find the $\overline{S}$, the closure of the set S.

(3) **[1M]** (a) Determine whether the set

$$\{z \in \mathbb{C} : |z| \le 1 \text{ and } 0 \le \operatorname{Arg}(z) \le \pi/4\} \cup \{0\}$$

is compact where $\operatorname{Arg}(z)$ is the principal argument of z that lies in $[0, 2\pi)$. Justify your answer.
(b) Determine whether the set

$$\{z \in \mathbb{C} : |z| \le 1 \text{ and } 0 < \operatorname{Arg}(z) \le \pi/4\} \cup \{0\}$$

is compact where $\operatorname{Arg}(z)$ is the principal argument of z that lies in $[0, 2\pi)$. Justify your answer.

(4) **[3M]** (a) Determine whether the function defined by $f(z) = \begin{cases} \frac{|z|}{\operatorname{Im}(z)} & \text{if } \operatorname{Im}(z) \neq 0 \\ 0 & \text{if } \operatorname{Im}(z) = 0 \end{cases}$ is continuous at 0. Justify your answer (in case the limit exists, prove using $\epsilon - delta$ definition).
(b) Determine whether the function defined by $f(z) = \begin{cases} \frac{|z|}{\operatorname{Re}(z)} & \text{if } \operatorname{Re}(z) \neq 0 \\ 0 & \text{if } \operatorname{Re}(z) = 0 \end{cases}$ is continuous at 0. Justify your answer (in case the limit exists, prove using $\epsilon - delta$ definition).

(5) **[3M]** (a) Consider the function $f(z) = z\operatorname{Re}(z)$ defined on $\mathbb{C}$. Determine where the function is complex differentiable and find the derivative $f'(z)$ at those points. Justify your answer.

(b) Consider the function $f(z) = z\,\mathrm{Im}(z)$ defined on $\mathbb{C}$. Determine where the function is complex differentiable and find the derivative $f'(z)$ at those points. Justify your answer.

(6) **[2M]** (a) Let c be the last two digits of your roll number (for example, if your roll number is 190010023, then c=23). Determine whether the sequence (x_n) converges where $x_n = c + i\frac{(-1)^n}{n}$. If so, find its limit. Justify your answer.

(b) Let c be the last two digits of your roll number (for example, if your roll number is 190010023, then c=23). Determine whether the sequence (x_n) converges where $x_n = \frac{(-1)^n}{n} + ic$. If so, find its limit. Justify your answer.