

201081001

$$3) \frac{d^2 y}{dt^2} + 6 \frac{dy}{dt} + 9y(t) = a_2 x(t) \quad \Rightarrow a_2 = 8.$$

$$D^2 + 6D + 9 = 0$$

$$(D+3)^2 = 0$$

$$D = -3, -3$$

$$\therefore y_h(t) = (c_1 + c_2 t) e^{-3t}$$

$$y_p(t) = k t^2 e^{-3t} \quad \left[\text{As for } k e^{-3t} \text{ \& } k t e^{-3t} \text{ the LHS vanishes} \right]$$

$$y_p'(t) = 2k t e^{-3t} - 3k t^2 e^{-3t}$$

$$y_p''(t) = 2k e^{-3t} - 6k t e^{-3t} - 6k t e^{-3t} + 9k t^2 e^{-3t}$$

Substituting in D.E we get

$$2k e^{-3t} = a_2 e^{-3t}$$

$$\Rightarrow \boxed{k = \frac{a_2}{2}} \Rightarrow k = \frac{8}{2} = 4.$$

$$\boxed{y = y_h + y_p}$$

$$y = (c_1 + c_2 t) e^{-3t} + 4t^2 e^{-3t}$$

$$y'(0) = a_1 = 2, \quad y''(0) = a_2 = 8$$

Use initial conditions to estimate c_1 & c_2 .

→ Time invariance:- As the homogeneous part tends to 0 as $t \rightarrow \infty$ in

→ ~~BIBO Stable~~ As the $y(t) \rightarrow 0$ as $t \rightarrow \infty$ for

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$$x(t) = e^{-3t} u(t) \\ = e^{-3t} \quad t > 0$$

$$|x(t)| < \infty$$

$$y(t) = (c_1 + c_2 t + 4t^2) e^{-3t} \quad t > 0$$

$$|y(t)| < \infty$$

(BIBO stable)

\therefore Bound output for bounded input
BIBO Stable system

Substituting in D.E we get

$$y''' - 3y'' = 0$$

$$\Rightarrow y = 0 \Rightarrow \boxed{y = 0}$$

$$y'' + 3y' = 0$$

$$y = (c_1 + c_2 t) e^{-3t} + 4t^2 e^{-3t}$$

$$y(0) = c_1, \quad y'(0) = c_2$$

Use initial conditions to determine constants c_1 and c_2 .
The invariance of the homogeneous part
forces to 0 as $t \rightarrow \infty$

At the end of the day...