Indian Institute of Technology Dharwad
भारतीय प्रौद्योगिकी संस्थान धारवाड़

EE101 Spring 2021 Homework 5

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Due Sep 12, 2020, 23.59h via Moodle

Instructions:

1. Submit your solutions as a *single PDF file* file through Moodle. Submission via other means will not be accepted. Moodle has file size limits as well as bandwidth limits so please do not leave the task of scanning and uploading to the last minute.
2. You may create the PDF either through LaTeX, Word etc. or scan a clearly / legibly written sheet of paper. Answers that are not legible / readable will marked zero. Please view/check the scanned PDF before you submit it.
3. Please attempt and submit the homework by yourself except where instructions specify group work. If you have questions, comments, doubts about any of the questions please reach out to the TAs or instructor. Do not discuss it with other students until the submission deadline. This will help regulate the pace and content of the course.
4. If any data are missing, make reasonable assumptions and state the same with justification.
5. Points for each question are indicated in square brackets in the right margin.
6. There are 2 questions, for a total of 10 points and 0 bonus points.

1. For the circuit in Figure 1: Find inductor current $i(t)$ and plot it, when $v(t) = u(t) - u(t-5)$ (in Volts), where $u(t)$ is the unit step function. Assume $i(0^-) = 0$. [5]

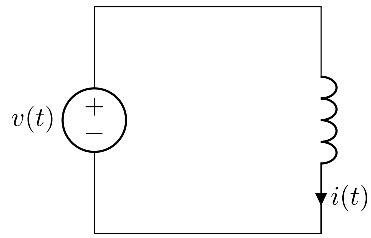
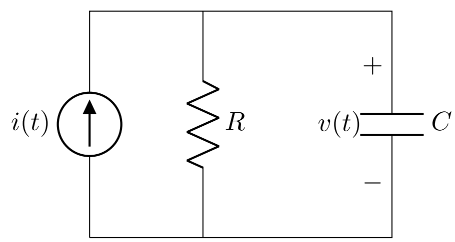


Figure 1: A simple L circuit

2. For the circuit in Figure 2: Find $v(t)$ and plot it, when $i(t) = u(t) - u(t-1)$ (Amps). Assume $v(0^-) = 0$ and R=10$k\Omega$ and C=100μF. [5]



. Figure 2: A simple RC circuit

1. For the circuit in Figure 1: Find inductor current $i(t)$ and plot it, when $v(t) = u(t) - u(t-5)$ (in Volts), where $u(t)$ is the unit step function. Assume $i(0^-) = 0$.

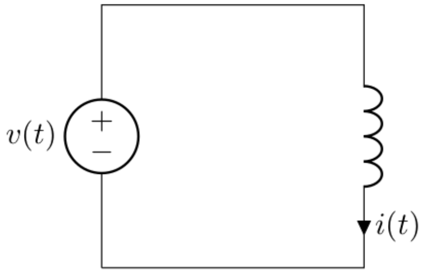
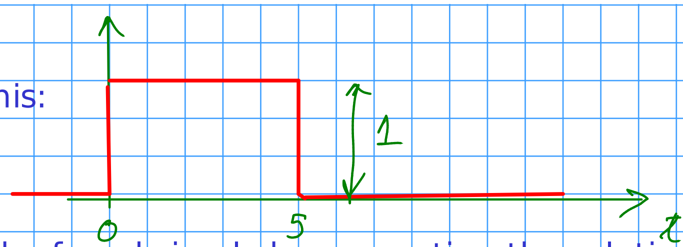


Figure 1: A simple L circuit

Solution:

The input x(t) looks like this:



So the total solution can be found simply by computing the solution from t=-inf to 0, then 0 to 5, and finally 5 to +inf. Since this is a simple L circuit, we don't really have any ODE to solve, rather i(t) is just:

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t) = \frac{1}{L}t + C$$

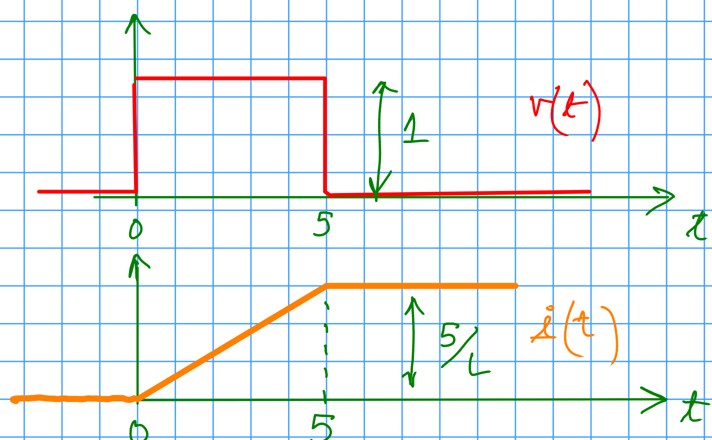
So all we need to do it figure out the correct initial conditions. Since i(0-)=0, for 0<t<5 we can put i(0+)=0 and so i(t)=(t/L).
At t=5, i(5)=5/L.

This will now be the initial condition for second part of the solution, i.e. i(t>5):
So now we have:

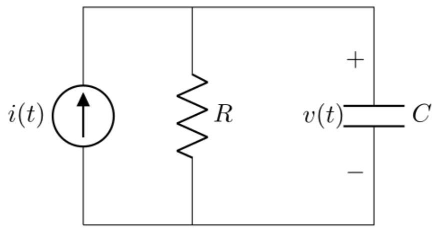
$$i(t) = \frac{1}{L}\int v(t) = \frac{1}{L}[0] + C$$

Since i(5)=(5/L), we know that C=5, i.e. i(t>5)=(5/L) i.e. constant.

Thus the final graph should look like this:



2. For the circuit in Figure 2: Find $v(t)$ and plot it, when $i(t) = u(t) - u(t-1)$ (Amps). Assume $v(0^-) = 0$ and R=10$k\Omega$ and C=100μF.



This problem is marginally harder, since the relationship between i(t) and v(t) is given by an ODE which needs to be solved. The ODE for the circuit is:

$$C\frac{dv}{dt} + \frac{v}{R} = i(t)$$

Lets look at that i(t) looks like. As in the previous problem, we can sketch i(t):

So essentially we can solve this entire problem in two parts: one for [0<t<1] and then for [t>1], taking i(t) as a constant in both cases. We just need to use the correct boundary conditions for both cases.

We have solved enough problems of this form to be able to write the general form of the solution directly:

$$v(t) = Ae^{\frac{-t}{\tau}} + B$$

Here the exponential term is the natural response yn(t) and the B is the term due to the constant input: yf(t). If the forcing function were something other than a constant we would have a function here instead of B.

To figure out B we can use the fact that y(f) must satisfy the ODE, and therefore:

$$C\frac{dB}{dt} + \frac{B}{R} = i(t) = \mathbf{I} \implies B = R\mathbf{I}$$

Now we can substitute B to get:

$$v(t) = Ae^{\frac{-t}{\tau}} + R\mathbf{I}$$

Now to find A we use the initial conditions and value of constant input I
For the first interval, i.e. 0<t<1 have v(0-)=0. Since v(t) has a finite value between t=0- and t=0+ we can safely say that v(0+) is the same as v(0-) i.e. 0.

$$v(0) = 0 = A + R\mathbf{I} \implies A = -R\mathbf{I}$$

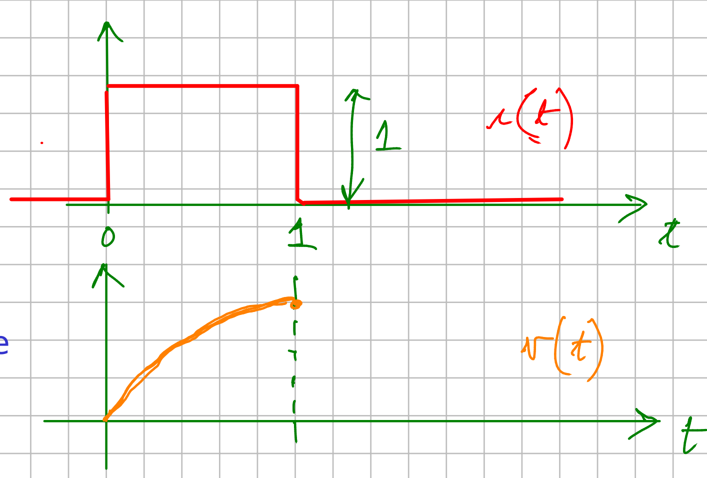
So now to v(t) for 0<t<1 is:

$$v(t) = R\mathbf{I}e^{\frac{-t}{\tau}} + R\mathbf{I} = R\mathbf{I}\left[1 - e^{\frac{-t}{\tau}}\right]$$

And therefore at t=1:

$$v(1) = R\mathbf{I}\left[1 - e^{\frac{-1}{\tau}}\right]$$

Plug in the values of R and Cto get the numeric value for v(1).



Now, finally, we need to find v(t) for t>1. To do that we simply take the solution that we already have for the system and plug in the new input value and new initial condition.

$$v(t) = A_1 e^{\frac{-t}{\tau}} + B_1$$

I have changed the constants to A1 and B1 to highlight the fact that initial conditions and input have both changed, and therefore both constants need to be recalculated.
The equation does not change because the circuit has not changed. If there were switches that changed the *Circuit topology* - then we MUST re-write KVL/KCL and get the new ODEs. So to get the complete solution, first we need to find B1, which in this case is simply 0. (Why? It depends on the input, which is zero - yf must satisfy the oiginal ODE, blah blah blah... so:

$$v(t) = A_1 e^{\frac{-t}{\tau}}$$

All we need now is to find A1, which we can do with the boundary condition that v(1) is known from the previous section. So

$$v(1) = A_1 e^{\frac{-1}{\tau}} = R\mathbf{I}\left[1 - e^{\frac{1}{\tau}}\right]$$

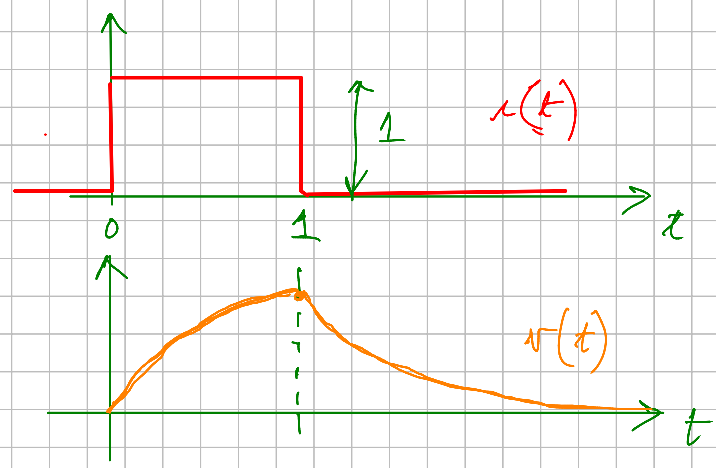
i.e.

$$A_1 = R\mathbf{I}\left[e^{\frac{1}{\tau}} - 1\right]$$

And so, put it all together:

$$v(t) = R\mathbf{I}\left[e^{\frac{1}{\tau}} - 1\right] e^{\frac{-t}{\tau}}$$

And now we can plot it all together:



If needed we can write the function in a piecewise definition as well:

$$v(t) = \begin{cases} R\mathbf{I}\left[1 - e^{\frac{-t}{\tau}}\right] \text{ for } 0 < t < 1 \\ R\mathbf{I}\left[e^{\frac{1}{\tau}} - 1\right] e^{\frac{-t}{\tau}} \text{ for } 1 < t < \infty \end{cases}$$