## 4. Sturm-Liouville BVP

1. For which values of $\lambda$ does the boundary value problem (BVP)

$$y'' - 2y' + (1+\lambda)y = 0,\ y(0) = 0,\ y(1) = 0$$

has non-trivial solutions?

2. Write the following ODEs in the form, $\big(r(x)y'\big)' + \big(q(x) + \lambda p(x)\big)y = 0$

(a) $xy'' + (1-x)y' + x^2y + \lambda y = 0$

(b) $(1-x^2)y'' - 2xy' + n(n+1)y = 0$

(c) $x^2y'' - 2x^3y + y\sin x + \lambda y\cos x = 0.$

(d) $(\sin^2 x)y'' + (\cos x)y' + g(x)y = 0.$

3. Determine whether the given boundary value problem is self-adjoint or not.

(a) $y'' + y' + 2y = 0,\ y(0) = 0,\ y(1) = 0$

(b) $y'' + y = \lambda y,\ y(0) - y'(1) = 0,\ y'(0) - y(1) = 0$

(c) $(1+x^2)y'' + 2xy' + y = 0,\ y'(0) = 0,\ y(1) + 2y'(1) = 0$

(d) $(1+x^2)y'' + 2xy' + y = \lambda(1+x^2)y,\ y(0) - y'(1) = 0,\ y'(0) + 2y(1) = 0$

(e) $y'' + \lambda y = 0,\ y(0) = 0,\ y(\pi) + y'(\pi) = 0.$

$\big($Hint: Try to write the above ODEs in $L(y) + \lambda p(x)y = 0$, where $L(y) = \big(r(x)y'\big)' + q(x)y$ and use Lagrange's Identity with the help of the given boundary conditions $\big)$

4. Find the eigenvalues and eigenfunctions of the following boundary value problems:

(a) $(e^{2x}y)' + e^{2x}(1+\lambda)y = 0,\ y(0) = 0,\ y(\pi) = 0$

(b) $y'' + \lambda y = 0,\ y(0) = 0,\ y'(1) = 0$

(c) $x^2 y'' + xy' + \lambda y = 0,\ y(1) = 0,\ y(\ell) = 0$

5. Determine the normalised eigenfunctions of the BVP

$$y'' + \lambda y = 0,\ y(0) = 0,\ y(1) = 0.$$