Indian Institute of Technology Dharwad
भारतीय प्रौद्योगिकी संस्थान धारवाड़

EE101 Spring 2021 Homework 4

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Due Sep 12, 2020, 23.59h via Moodle

Instructions:

1. Submit your solutions as a *single PDF file* file through Moodle. Submission via other means will not be accepted. Moodle has file size limits as well as bandwidth limits so please do not leave the task of scanning and uploading to the last minute.
2. You may create the PDF either through LaTeX, Word etc. or scan a clearly / legibly written sheet of paper. Answers that are not legible / readable will marked zero. Please view/check the scanned PDF before you submit it.
3. Please attempt and submit the homework by yourself except where instructions specify group work. If you have questions, comments, doubts about any of the questions please reach out to the TAs or instructor. Do not discuss it with other students until the submission deadline. This will help regulate the pace and content of the course.
4. If any data are missing, make reasonable assumptions and state the same with justification.
5. Points for each question are indicated in square brackets in the right margin.
6. There are 5 questions, for a total of 69 points and 0 bonus points.

Some of the following tasks require the use of a software package like Matlab or Octave[1]. If you have it installed locally that is ideal; otherwise a web-version is available for both, available at MATLAB Online and Octave Online. An example program/script to plot some exponential functions is given here to get you started.

Example MATLAB / Octave Script:

```
>> t = [0:1e-3:100]; %Create a vector for time variable
>> tau = 0.5; %Set value of time-constant tau
>> y =  (1 - exp(-t /tau ) ); %Compute the value of exponential function "y"
>> plot(t, y) %Plot the curve. Note the x variable is supplied first
>> hold on %"Hold" the current graph; subsequent plots are made over the existing plot
>>
>> %Plot the curve for a different values of tau
>> tau = 1; y1 =  (1 - exp(-t /tau ) ); plot(t,y1);
>> tau = 5; y2 =  (1 - exp(-t /tau ) ); plot(t,y2);
>> tau = 10; y3 =  (1 - exp(-t /tau ) ); plot(t,y3);
>>
>>
>> tau = 100; y4 =  (1 - exp(-t /tau ) ); plot(t,y4);
>> legend
```

[1] Octave is free and open source, MATLAB is not

1. (a) Find the solution $y(t)$ of the differential equation [5]

$$2\dot{y} + 10y = 2$$

using the methods we have used in class (i.e. find characteristic equation, its roots, etc etc.). Take $y(0) = 0$

(b) Identify the natural response y_n and response due to forcing function y_f where $y_n + y_f = y$ [6]

(c) Use Matlab or Octave to plot y_n, y_f and y [5]

(d) What is the steady state value of y [2]

(e) At what time does y reach the steady state value? [2]

2. On a single graph sheet, sketch the following. Use different colors or line-types for each plot and show an appropriate legend. You may verify your plot with Matlab/Octave **after** you have made at least one attempt to plot by hand.

(a) $y_1 = e^t$ [2]

(b) $y_2 = e^{-t}$ [2]

(c) $y_3 = 2e^{-t}$ [2]

(d) $y_4 = e^{-2t}$ [2]

(e) $y_5 = 1 - e^{-t}$ [2]

3. All answers for this question must be in terms of y_0 or τ:

(a) The solution of a first order differential equation is [5]

$$y(t) = y_0(1 - e^{-\frac{t}{\tau}})$$

Write the differential equation in the form

$$k_1 \frac{dy}{dt} + k_2 y = k_3$$

where $k_1, k_2, \cdots$ are constants and $y(0) = 0$.

(b) At what time does $y(t)$ reach half its steady state value? [1]

(c) Calculate the rise time of $y(t)$, i.e. the time it takes to go from 10% to 90% of its final value? [1]

(d) What is the value of $y(\tau)$? [1]

(e) What is the slope of $y(t)$ at $t = 0$? [1]

4. For the circuit in Figure 1:

(a) Use circuit laws to describe this system as an ordinary differential equation in one variable. [5]

(b) Write the characteristic equation and list its roots [5]

(c) Write the natural response portion of the solution of your ODE. [5]

5. For the circuit in Figure 2:

(a) Use circuit laws to describe this system as an ordinary differential equation in one variable. [5]

(b) Write the characteristic equation and list its roots [5]

(c) Write the natural response portion of the solution of your ODE. [5]

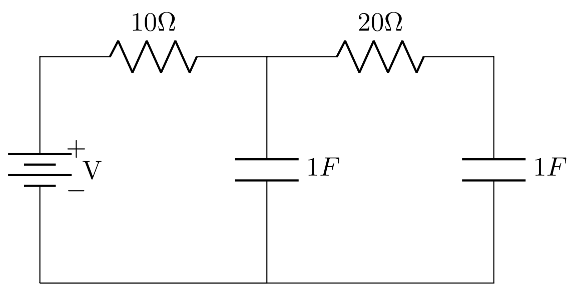

Figure 1: A simple RC Circuit. You may assume that both capacitors have some finite charge at t=0

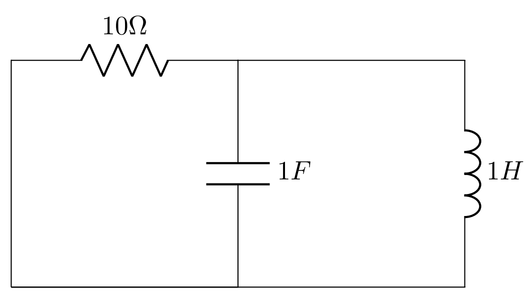

Figure 2: A simple RLC Circuit. You may assume that at t=0 the capacitor has some finite charge and the inductor has some finite current.