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3) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y(t) = a_2 x(t)$ $\Rightarrow a_2 = 8.$

$D^2 + 6D + 9 = 0$

$(D+3)^2 = 0$

$D = -3, -3$

$\Rightarrow y_h(t) = (c_1 + c_2 t) e^{-3t}.$

$y_p(t) = kt^2 e^{-3t}$ [As for ke^{-3t} & kte^{-3t} the LHS vanishes)

$y_p'(t) = 2kte^{-3t} - 3kt^2 e^{-3t}$

$y''_p(t) = 2ke^{-3t} - 6kte^{-3t} - 6kte^{-3t} + 9kt^2 e^{-3t}.$

Substituting in D·E we get

$2ke^{-3t} = a_2 e^{-3t}.$

$\Rightarrow \boxed{k = a_2/2}$ $\Rightarrow k = 8/2 = 4.$

$\boxed{y = y_h + y_p}$

$y = (c_1 + c_2 t)e^{-3t} + 4t^2 e^{-3t}.$

$y'(0) = a_1 = 2$, $y''(0) = a_2 = 8$

Use initial conditions to estimate c_1 & c_2.

→ Time invariance :- ~~as~~ As the homogenous part tends to 0 as $t \to \infty$ ~~as~~

→ ~~BIBO~~ Stable: ~~are~~ ~~it~~ As ~~the~~ ~~$y(t) \to 0$~~ are $t \to \infty$ for

$x(t) = e^{-3t}\, u(t).$

$= e^{-3t} \quad t > 0$

$|x(t)| < \infty$

$y(t) = (c_1 + c_2 t + 4t^2) e^{-3t}, \quad t > 0$

$|y(t)| < \infty$

$\therefore$ Bound output for Boundell input

$\therefore$ BIBO Stable system.