1)

$y[n] = a_1 x[n] + a_2 x[n-1] + a_3 x[n-2]$.

To compute impulse response,

$x[n] = \delta[n]$.

$\Rightarrow h[n] = a_1 \delta[n] + a_2 \delta[n-1] + a_3 \delta[n-3]$.

$\Rightarrow h[0] = a_1 \cdot 1 + 0 + 0 = a_1$

$\Rightarrow h[1] = a_1 \cdot 0 + a_2 + a_3 \cdot 0 = a_2$

$\Rightarrow h[2] = a_1 \cdot 0 + a_2 \cdot 0 + a_3 = a_3$.

$\Rightarrow h[3] = 0$

$\vdots$

$\Rightarrow h[n] = \{a_1, a_2, a_3\}$.

2)

$y[n] = a_1 x[n] + a_2 x[n-1] + a_3 x[n-2]$.

$x[n] = [1, 1, 1]$.

$y[0] = a_1 x[0] + a_2 x[-1] + a_3 x[-2] = a_1$

$y[1] = a_1 x[1] + a_2 x[0] + a_3 x[-1] = a_1 + a_2$.

$y[2] = a_1 x[2] + a_2 x[1] + a_3 x[0] = a_1 + a_2 + a_3$.

$y[3] = a_1 x[3] + a_2 x[2] + a_3 x[1] = a_2 + a_3$.

$y[4] = a_1 x[4] + a_2 x[3] + a_3 x[2] = a_3$.

$y[5] = a_1 x[5] + a_2 x[4] + a_3 x[3] = 0$

$\vdots$

$y[n] = \{a_1, a_1 + a_2, a_1 + a_2 + a_3, a_2 + a_3, a_3\}$.

3) $\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 9y(t) = a_2 x(t)$.

For natural response no input is applied:-

let $\frac{d}{dt} = D$.

$\Rightarrow (D^2 + 6D + 9)y_n(t) = 0$

The characteristic equation:-

$D^2 + 6D + 9 = 0$

$(D+3)^2 = 0$

$D = -3, -3$.

$\Rightarrow \quad y_n(t) = (c_1 + c_2 t)e^{-3t}$.

Now, using initial condition to get c_1 & c_2.

$y_n(0) = c_2 = a_1$

$y'_n(n) = -3tc_1 e^{-3t} + c_2(-3t^2)e^{-3t} + c_2 e^{-3t}$

$y'_n(0) = c_2 = a_2$.

$$\boxed{\Rightarrow \quad y_n(t) = (a_1 + a_2 t)e^{-3t}.}$$

Forced response is taken in the form of input.

$y_f(t) = Ke^{-3t}, t>0$, K is a constant.

$$\underbrace{\frac{d^2y_f(t)}{dt} + 6\frac{d y_f(t)}{dt} + 9y_f(t)}_{} = a_2 e^{-3t}.$$

$9Ke^{-3t} - 18Ke^{-3t} + 9Ke^{-3t} = 0$

$\Rightarrow \boxed{y_f(t) = 0}$

$$y(t) = y_{zi}(t) + y_f(t).$$

$$\boxed{y(t) = (a_1 + a_2 t)e^{-3t}.}$$

→ To check BIBO stability.

$$x(t) = e^{-3t}u(t).$$

$|x(t)| < \infty$ [Bounded input].

$$y(t) = (a_1 + a_2 t)e^{-3t}.$$

$$|y(t)| = |a_1 + a_2 t||e^{-3t}|$$

Decay of $|e^{-3t}|$ is faster ~~than~~ than rise of $|a_1 + a_2 t|$. Therefore $|y(t)| < \infty$.

Hence, BIBO stable.

→ To check Time invariance.

Using property: The coefficients of differential equation are constant. Therefore, time invariant.

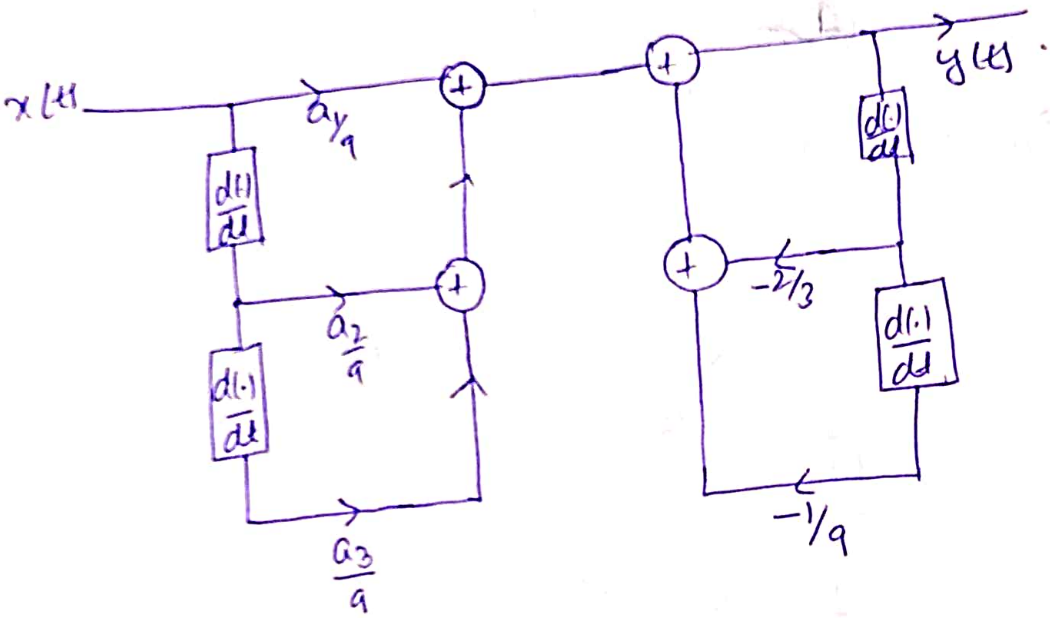
Q4 $y(t) + 6\int y(t) + 9\iint y(t) = a_1 \iint x(t) + a_2 \int x(t) + a_3 x(t).$

Converting to differential equation.

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y(t) = a_1 x(t) + a_2\frac{dx}{dt} + a_3\frac{d^2x}{dt^2}$$

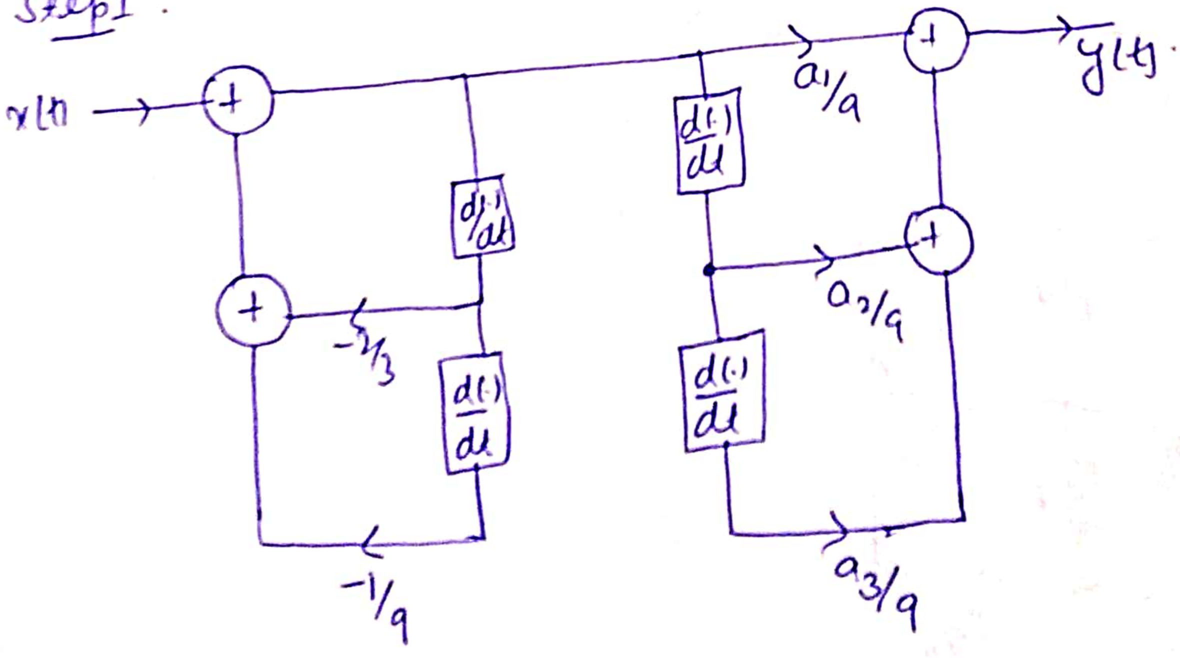
$$\Rightarrow y(t) = -\frac{2}{3}\frac{dy}{dt} - \frac{1}{9}\frac{d^2y}{dt^2} + \frac{a_1}{9}x(t) + \frac{a_2}{9}\frac{dx(t)}{dt} + \frac{a_3}{9}\frac{d^2x(t)}{dt^2}.$$

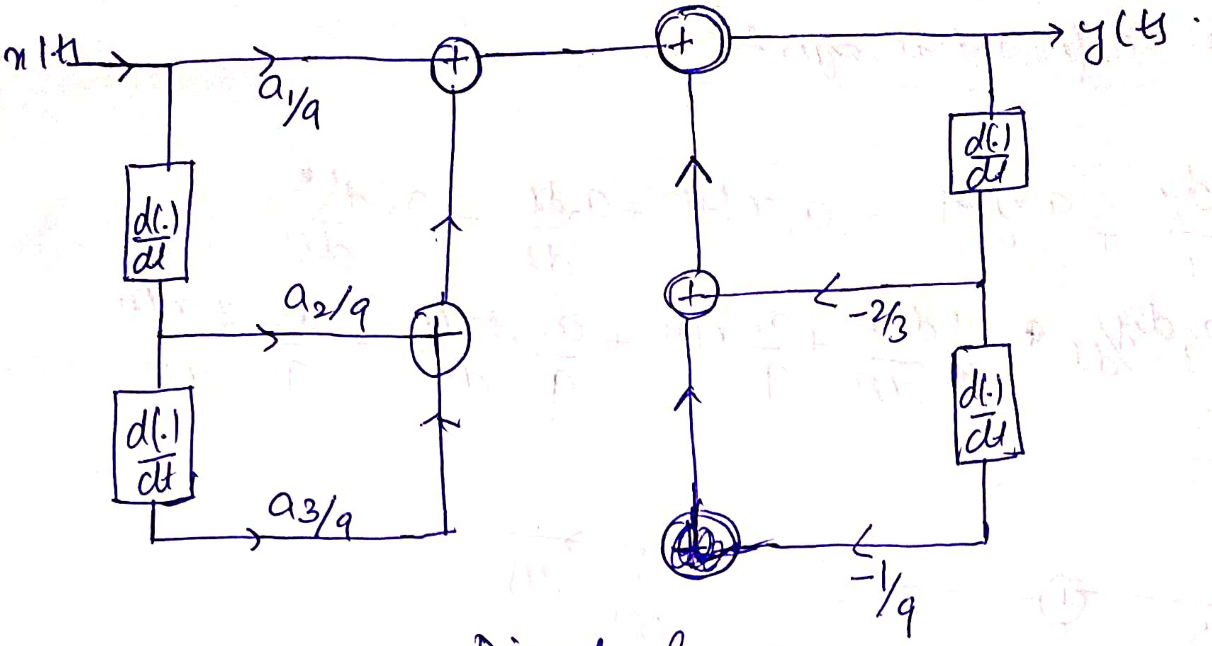
Direct form I



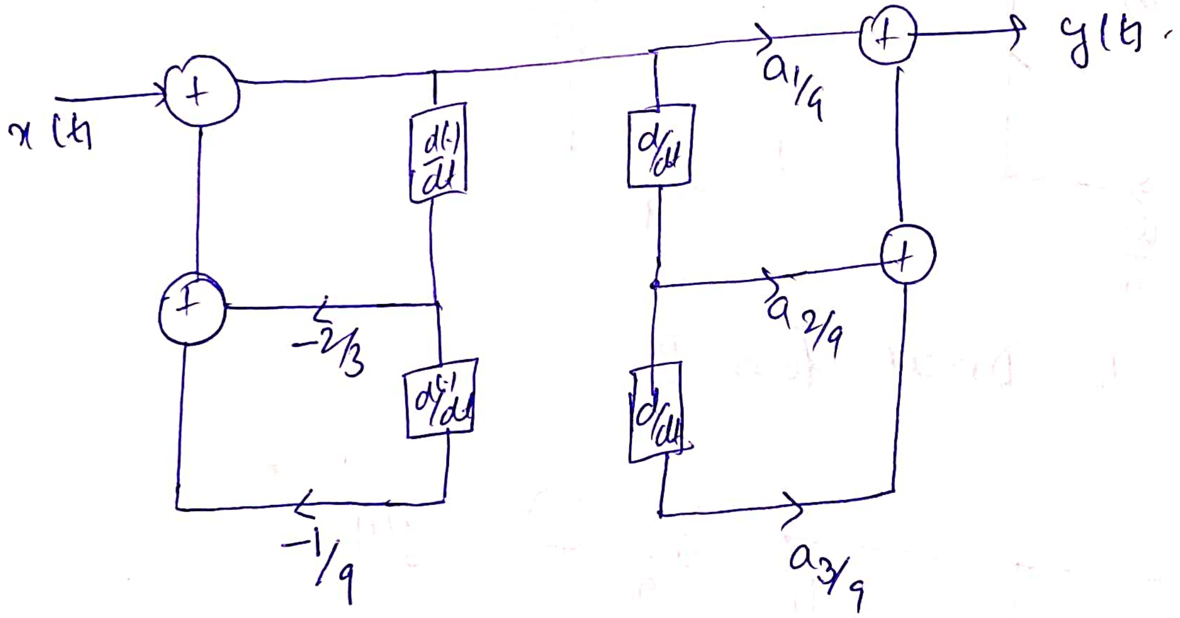
Converting to Direct form II

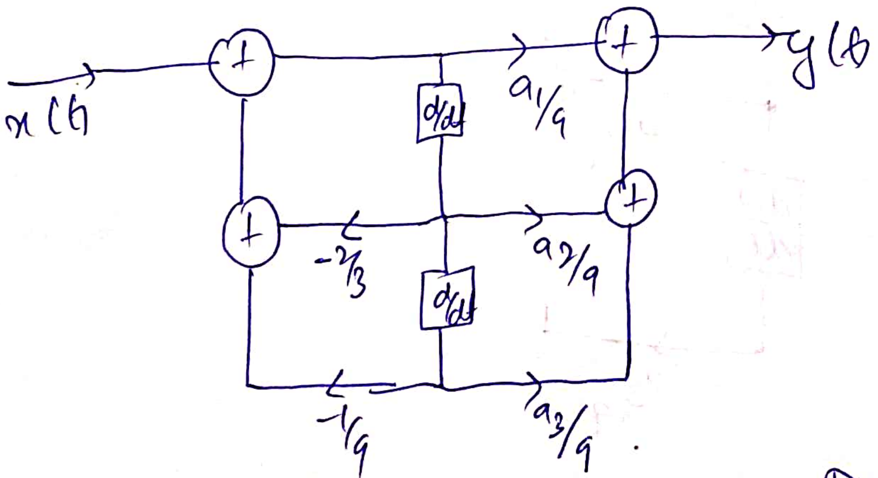
Step I.





Direct form I

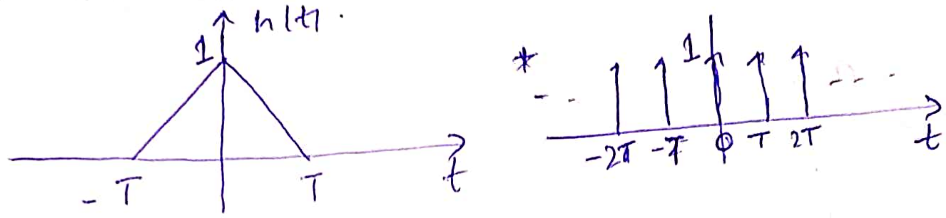




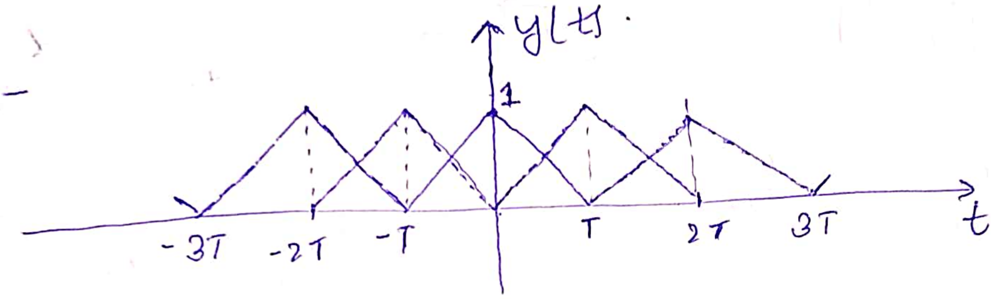
Direct form -II

Find whether ... (cut off)

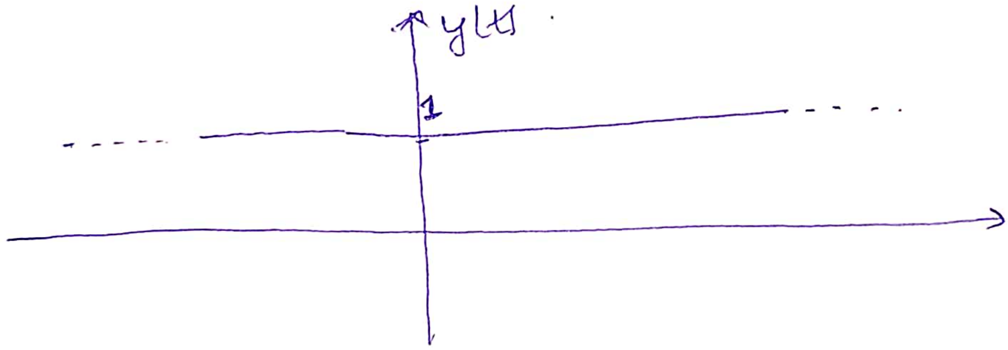
5)



Step 1:-



Step 2:-



6) a)

$$x_1(t) = e^{-(a_1 + j a_2)t}$$

$$= e^{-a_1 t} \cdot (e^{-j a_2 t})$$

$$= \underbrace{e^{-a_1 t}}_{\text{aperiodic}} \underbrace{(\cos a_2 t - j \sin a_2 t)}_{\text{periodic}}.$$

$\Rightarrow$ $x_1(t)$ is aperiodic.

6) (b) Find whether $x_2(t)$ is periodic. If yes find F_0

$x_2(t) = 2\cos(a_1 t + 1) - \sin(a_2 t - 1)$

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$a_1 = 2$
$a_2 = 8$
$a_3 = 1$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{a_1} = \frac{2\pi}{2} = \pi$

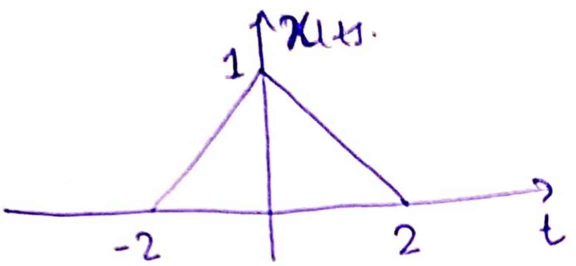
$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{a_2} = \frac{2\pi}{8} = \frac{\pi}{4}$.

$\boxed{\text{Lcm}(T_1, T_2) = \pi}$

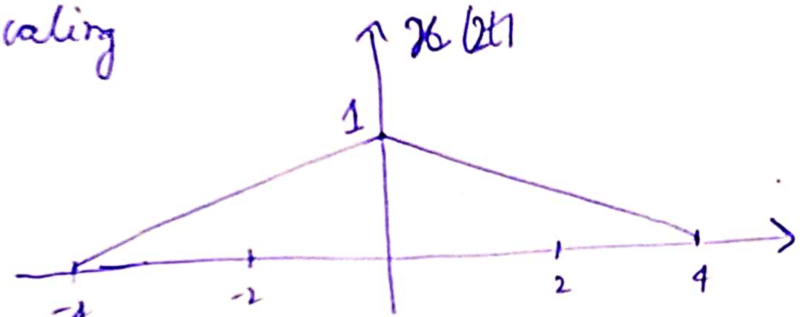
7) Sketch $y(t) = x(a_1 t - a_2) = x(2t-8)$

$a_1 = 2$

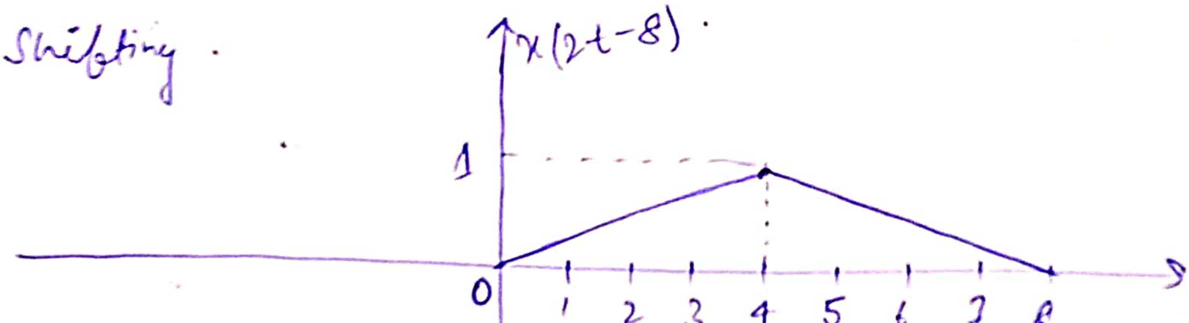
$a_2 = 8$



Step 1 :- Scaling



Step 2 :- Shifting



8)

$x(t) * x(t)$.

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$a_1 = 2$

$a_2 = 8$.