MA-403 Assignment 2 (2022-23 Autumn)
Due date: 15 Sep. 2022 N.S.N. SASTRY
Total: 50 marks

1 (a) For each $a \in \mathbb{Z}_{11}$, find $b \in \mathbb{Z}_{11}$ such that $ab = 1$ in $\mathbb{Z}_{11}$. Is there a unique such b in $\mathbb{Z}_{11}$. (5)

(b) Find all elements $a \in \mathbb{Z}_{16}$ such that there is an element $b \in \mathbb{Z}_{16}$ such that $ab = 1$ in $\mathbb{Z}_{16}$. Find such a b in each case. (5)

2. Show that $X^2 = 6Y^2 = Z^2$ has no nontrivial integer solutions. (Hint: If $a, b, c \in \mathbb{Z}$ is a solution, we may assume $(a, b, c) = 1$ and derive a contradiction) (10)

3. Find all integers x and y such that $x + \sqrt{y} = 11$ and $\sqrt{x} + y = 7$.
(Hint: use Fundamental theorem of Arithmetic) (10)

4. Find all integers x such that $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 4$, $x \equiv 3 \pmod 5$ (6)

5. (a) Prove that $5n^3 + 7n^5 = 0 \pmod{12}$ for all integers $n \geq 1$. (7)

(b) Prove that $n^4 + 4$ is a composite number for all integers n. (7)