MA-403 (2021-22), Autumn term)

IIT, Dharwad N.S.N. Sastry

Assignment-3 Due on: Oct 21, 2022

Note: 1) Write your name and the registration number CLEARLY

2) Submit hard copy only to Dr. Amlan Barua by Oct 21, 2022

1a) Define the order of an element of $\mathbb{Z}_n^*$ and primitive n^{th} root of unity Precisely. (2+2)

b) Determine all the n^{th} roots of unity in $\mathbb{Z}_n$ for $n=18$, $n=23$, $n=16$. (6+6+4)

2) Find all cyclic subgroups of

a) $\mathbb{Z}_{17}$ (b) $\mathbb{Z}_{12}$ (3+7)

3) ~~Write~~ Define the index ind_r^a relative to a primitive root r in $\mathbb{Z}_n^*$.

Find ind_3^a for all elements of $\mathbb{Z}_{17}^*$.

(This requires also verifying that 3 is a 17^{th} root of unity.) (2+6+2)

4) Using the concept of ind_r^a in $\mathbb{Z}_n^*$, Solve the following congruences:

(a) $x^8 \equiv 2 \pmod{17}$

(b) $4^x \equiv 3 \pmod{17}$ (5+5)

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