Indian Institute of Technology Dharwad, Dharwad, Karnataka, India

EE 221 / EE 201: Introduction to Probability / Data Analysis (first half)
Mid Semester Exam / Quiz-1 (Autumn 2021)
Date: 24/08/2021 Time: 90 minutes

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1. (5 points) **Expecting expectations, here you go:** Let X be a non-negative continuous random variable. By default, its expected value $\mathbb{E}[X]$ is determined using a formula that uses the PDF, $f_X(x)$. Derive a formula to express its expectation in terms of its CDF, i.e., $F_X(x)$. Please indicate all the steps clearly.

2. (5 points) **Testing your limits:** Let X be a discrete random variable following the Binomial distribution with parameters p and n. Show that you can approximate X using a Poisson distribution if n is large and p is small. What is the mean of this Poisson distribution?

3. (10 points) **One to test your memory:** Let X be a non-zero, non-negative discrete random variable that takes only integer values. Given that $\mathrm{P}(X > m + n \mid X > m) = \mathrm{P}(X > n)$.
   (a) (5 points) Find the PMF of X considering $\mathrm{P}(X = 1) = p$.
   (b) (5 points) Let $Y = \min\{X_1, X_2, \cdots X_N\}$, where each X_i has the same PMF as that of X. Find the PMF of Y.

4. (10 points) **Numbers game:** Let's say that there is a game involving numbers - $1, 2, 3, \cdots N$. You have formed a group of exactly K players and each one picks a number. The probability that each number is picked is equally likely (uniform distribution) and the players cannot communicate with each other. If each member in your group picks a different number, you win the game. However, if at least two players pick the same number, then you lose the game.
   (a) (3 points) What is probability of winning the game?
   (b) (7 points) Consider a scenario where the number of players (K) varies according to a Poisson distribution with mean λ. Then, what is the average probability of winning the game?

5. (10 points) **Better be insured:** Company "Covinsure" is my start-up whihch offers COVID insurance policies. Currently, I have 1 lakh customers who have each purchased a policy worth Rs. 500. I have conducted a case study and found that the death rate due to COVID is 0.1%. In case of death, my company pays the family Rs. 2 lakhs. What is the probability that
   (a) (5 points) my company suffers a loss?
   (b) (5 points) I make a profit of at least 2.5 crores?

   Note: Please simplify the answers and express the results in percentage.