# Monte Carlo Sampling to Find Probability That Center of Circle is in Triangle

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## 1 Monte Carlo Sampling to Find Probability That Center of Circle is in Triangle

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Sample three random points on the circumference of a circle, and use these to draw a triangle. What is the probability that this triangle includes the center of the circle? Base on the following site:

http://math.stackexchange.com/questions/268635/what-is-the-probability-that-the-center-of-the-circle-is-contained-within-the-tr

the answer is 0.25. The Monte Carlo sampling below confirms this probability.

The first step is to sample three points between 0 degrees and 360 degrees from a random uniform distribution.

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        from matplotlib.patches import Arc, Polygon
        import matplotlib.patches as mpatches
        import math
        from scipy.spatial import distance

        s = np.sort(np.random.uniform(0,360,3))

        print(s)
```

```
[  67.27150413   73.25323343  278.30255756]
```

The next step is to devise an algorithm that will determine if the centre of the circle is on the triangle's surface area. One way is to start with the two first sample points: A and B. For A, extend a line through the center of the circle to the opposite side to point \$ A^{'} \$,forming a diameter. Repeat for B to get \$ B^{'} \$. Points \$ A^{'} \$ and \$ B^{'} \$ delineate an arc on the circle. One criteria for determining if the center of the circle is in the sampled triangle is as follows: (1) it is true if the third sample point in on the arc formed by \$ A^{'} \$ and \$ B^{'}\$; (2) false otherwise.

```
In [2]: #plot function
        def plot(s):
            aPrime = (s[0] + 180) % 360

            bPrime = (s[1] + 180) % 360

            isIn = IsInArc(aPrime,bPrime,s[2])

            aRad = math.radians(s[0])
```

```
bRad = math.radians(s[1])
cRad = math.radians(s[2])
aPrimeRad = math.radians(aPrime)
bPrimeRad = math.radians(bPrime)


xa = math.cos(aRad)
ya = math.sin(aRad)
xb = math.cos(bRad)
yb = math.sin(bRad)
xc = math.cos(cRad)
yc = math.sin(cRad)

xaPrime = math.cos(aPrimeRad)
yaPrime = math.sin(aPrimeRad)
xbPrime = math.cos(bPrimeRad)
ybPrime = math.sin(bPrimeRad)

xArray = np.array([0,xa,xaPrime,xb,xbPrime,xc])
yArray = np.array([0,ya,yaPrime,yb,ybPrime,yc])


triangleArray = np.array([[xa,ya],[xb,yb],[xc,yc]])

#plot circle with center at 0,0
fig = plt.figure(1)

fig.suptitle('Is Center of Circle in Triangle? ' + str(isIn),fontsize=20)
plt.axis([-1.1,1.1,-1.1,1.1])
ax=fig.add_subplot(1,1,1)
circle = plt.Circle((0.0,0.0),radius=1,fill=False,color='blue')
ax.add_patch(circle)


plt.plot(xArray,yArray,'ro')
plt.plot(xArray[1:3],yArray[1:3])
plt.plot(xArray[3:5],yArray[3:5])
plt.annotate("A",xy = (xArray[1],yArray[1]))
plt.annotate("A'",xy = (xArray[2],yArray[2]))
plt.annotate("A",xy = (xArray[1],yArray[1]))
plt.annotate("B",xy = (xArray[3],yArray[3]))
plt.annotate("B'",xy = (xArray[4],yArray[4]))
plt.annotate("C",xy = (xArray[5],yArray[5]))
triangle = Polygon(triangleArray,alpha=0.4)
ax.add_patch(triangle)

thetaArray = np.sort(np.array([aPrime,bPrime]))

angelDiff = thetaArray[1] - thetaArray[0]

if angelDiff < 180:
   thetaArray = thetaArray[::-1]

ellipse = Arc([0,0],2,2,angle=0,theta1=thetaArray[0],theta2=thetaArray[1],color='green',lin
            ax.add_patch(ellipse)
            ax.set_aspect("equal")
            plt.show()


In [3]: def IsInArc(A,B,C):
            primeArray = np.sort(np.array([A,B]))
            diffP = primeArray[1] - primeArray[0]

            if diffP < 180:
                isIn = s[2] >= primeArray[0] and s[2] <= primeArray[1]
            else:
                isIn = s[2] <= primeArray[0] or s[2] >= primeArray[1]

            return isIn

In [5]: #Run Monte Carlo
        count = 0
        numberOfPaths = 100000

        for i in range(0,numberOfPaths):

            s = np.sort(np.random.uniform(0,360,3))

            aPrime = (s[0] + 180) % 360
            bPrime = (s[1] + 180) % 360

            #if aPrime < bPrime:
            #    isIn = aPrime <= s[2] and s[2] <= bPrime
            #else:
            #    isIn = aPrime >= s[2] and s[2] >= bPrime


            if IsInArc(aPrime,bPrime,s[2]):
                count = count + 1

        print("Probability is " + str(round(count/numberOfPaths,2)))

Probability is 0.25

In [6]: #this will return the type of triangle

        def TriangleType(s):
            aRad = math.radians(s[0])
            bRad = math.radians(s[1])
            cRad = math.radians(s[2])

            xa = math.cos(aRad)
            ya = math.sin(aRad)
            xb = math.cos(bRad)
            yb = math.sin(bRad)
            xc = math.cos(cRad)
            yc = math.sin(cRad)

            ab = distance.euclidean((xa,ya),(xb,yb))
```

```
    bc = distance.euclidean((xb,yb),(xc,yc))
    ca = distance.euclidean((xc,yc),(xa,ya))

    triangleSides = np.sort(np.array([ab,bc,ca]))

    pyThagDiff = triangleSides[2]**2 - (triangleSides[0]**2 + triangleSides[1]**2)

    type = "Acute"

    if (pyThagDiff > 0.0):
        type = "Obtuse"
    elif (pyThagDiff == 0.0):
        type = "Square"

    return type
```

```
In [116]: #This section to do hand testing
          %matplotlib inline
          s = np.sort(np.random.uniform(0,360,3))
          #s[0] = 23.89034348
          #s[1] = 227.50068777
          #s[2] = 273.45969593

          #s[0] = 129.62364966
          #s[1] = 223.57256605
          #s[2] = 298.63178767

          #s[0] = 133.69378608
          #s[1] = 312.4282278
          #s[2] = 337.47042175

          aPrime = (s[0] + 180) % 360
          bPrime = (s[1] + 180) % 360
          #print(s)
          #print(aPrime)
          #print(bPrime)

          primeA = np.sort(np.array([aPrime,bPrime]))
          #print(primeA)
          diffP = primeA[1] - primeA[0]
          #print(diffP)


          #if aPrime < bPrime:
          #   isIn = aPrime <= s[2] and s[2] <= bPrime
          #else:
          #   print("FALSE")
          #   isIn = s[2] >= aPrime and s[2] >= bPrime

          #print(IsInArc(aPrime,bPrime,s[2]))
          #print(TriangleType(s))
          #print(s)
          #plot(s)
          #s = np.sort(np.random.uniform(0,360,3))
```

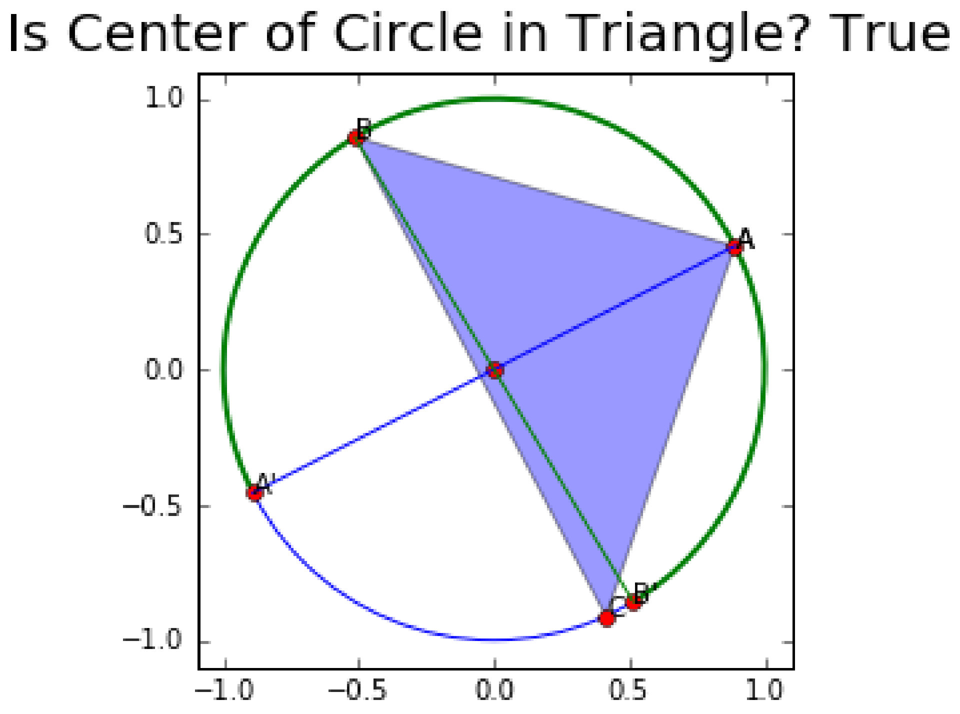
```
#print(s)
#plot(s)
```

Show a couple of plots. The arc of the circle colored in blue is a criteria neighborhood. If the third point is in this neighborhood, then the center of the circle is in the triangle.

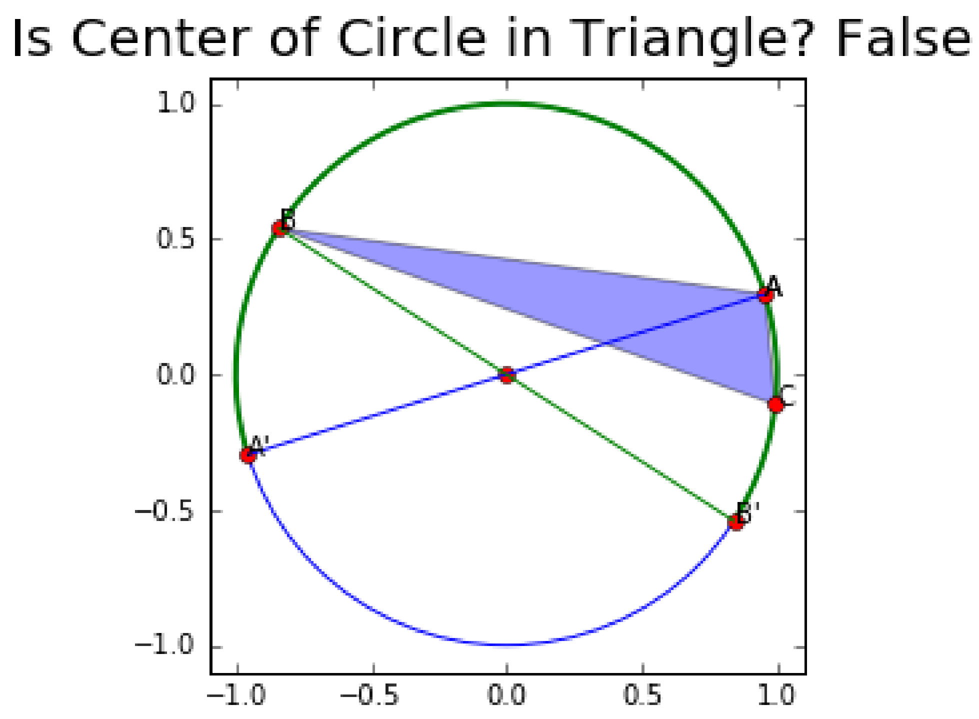
```
In [118]: s = np.sort(np.random.uniform(0,360,3))
          print(s)
          plot(s)

          s = np.sort(np.random.uniform(0,360,3))
          print(s)
          plot(s)
```

```
[  27.10582281  120.95212508  294.43131231]
```



```
[  17.24464385  147.28068957  353.64324721]
```



Try a different criteria here. If the triangle is acute, then it contains its center circle.

```
In [7]: #perform monte carlo using acute test

        count = 0
        numberOfPaths = 100000

        for i in range(0,numberOfPaths):

            s = np.sort(np.random.uniform(0,360,3))

            if TriangleType(s) == "Acute":
                count = count + 1

        print("Probability is " + str(round(count/numberOfPaths,2)))
```

```
Probability is 0.25
```

```
In [ ]:
```