

Signals and Systems | (2nd Edition)

Problem

Compute the Fourier transform of each of the following signals:

(a) $[e^{-\alpha t} \cos \omega_0 t]u(t), \alpha > 0$ (b)

(c) $x(t) = \begin{cases} 1 + \cos \pi t, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$ (d)

(e) $[te^{-2t} \sin 4t]u(t)$ (f)

(g) $x(t)$ as shown in Figure P4.21(a) (h)

(i) $x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$ (j)

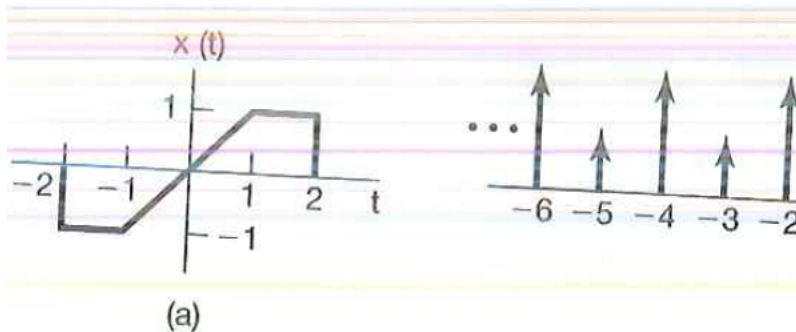


Figure P4.21

Determine the Fourier transform of each signal.

Step-by-step solution

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(a)

The signal, $x(t) = [e^{-\alpha t} \cos \omega_0 t]u(t), \alpha > 0$

Compute the Fourier transform of this signal.

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$$= \int_0^{\infty} e^{-\alpha t} \left(\frac{e^{j(\omega-\omega_0)t} + e^{-j(\omega-\omega_0)t}}{2} \right) e^{-j\omega t} dt$$

$$= \frac{1}{2} \left[\int_0^{\infty} e^{(-\alpha + j(\omega-\omega_0))t} dt + \int_0^{\infty} e^{(-\alpha + j(\omega_0-\omega))t} dt \right]$$

Simplify further.

$$X(j\omega) = \frac{1}{2} \left[\frac{-e^{(-\alpha + j(\omega-\omega_0))t}}{\alpha + j(\omega-\omega_0)} \right]_0^{\infty} + \frac{1}{2} \left[\frac{-e^{(-\alpha + j(\omega_0-\omega))t}}{\alpha + j(\omega_0-\omega)} \right]_0^{\infty}$$

$$= \frac{1}{2} \left[-0 - \left(\frac{-1}{\alpha + j(\omega-\omega_0)} \right) \right] + \frac{1}{2} \left[-0 - \left(\frac{-1}{\alpha + j(\omega_0-\omega)} \right) \right]$$

$$= \frac{1}{2} \left[\frac{1}{\alpha + j(\omega-\omega_0)} + \frac{1}{\alpha + j(\omega_0-\omega)} \right]$$

$$= \frac{1}{2} \left[\frac{1}{\alpha - j(\omega_0-\omega)} + \frac{1}{\alpha + j(\omega_0-\omega)} \right]$$

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Simplify further.

$$X(j\omega) = \frac{1}{2} \left[\frac{1}{\alpha - j(\omega_0-\omega)} + \frac{1}{\alpha + j(\omega_0-\omega)} \right]$$

$$= \frac{0.5}{\alpha - j(\omega_0-\omega)} + \frac{0.5}{\alpha + j(\omega_0-\omega)}$$

Thus, the Fourier transform of the signal $x(t)$ is $\boxed{\frac{0.5}{\alpha - j(\omega_0-\omega)} + \frac{0.5}{\alpha + j(\omega_0-\omega)}}$.



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(b)

The signal, $x(t) = e^{-3|t|} \sin 2t$

Compute the Fourier transform of this signal.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} (e^{-3|t|} \sin 2t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^0 e^{3t} \left(\frac{e^{j2t} - e^{-j2t}}{2j} \right) e^{-j\omega t} dt + \int_0^{\infty} e^{-3t} \left(\frac{e^{j2t} - e^{-j2t}}{2j} \right) e^{-j\omega t} dt$$

$$= \frac{1}{2j} \left[\int_{-\infty}^0 (e^{(3+2-j\omega)t} - e^{(3-2-j\omega)t}) dt \right] + \frac{1}{2j} \left[\int_0^{\infty} (e^{-(3-2+j\omega)t} - e^{-(3+2+j\omega)t}) dt \right]$$

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Simplify further.

$$= \frac{1}{2j} \left\{ \left(\frac{1}{3+j(2-\omega)} \right) - \left(\frac{1}{3-j(2+\omega)} \right) + \left(\frac{1}{(3-2j+j\omega)} \right) \right. \\ \left. - \left(\frac{1}{(3+2j+j\omega)} \right) \right\}$$

Simplify further.

$$X(j\omega) = \frac{1}{2j} \left\{ \frac{1}{3+j(2-\omega)} - \frac{1}{3-j(2+\omega)} + \frac{1}{(3-j(2-\omega))} - \frac{1}{(3+j(2+\omega))} \right\}$$

$$= \frac{1}{2j} \left\{ \left[\frac{1}{3+j(2-\omega)} + \frac{1}{(3-j(2-\omega))} \right] - \left[\frac{1}{3-j(2+\omega)} + \frac{1}{(3+j(2+\omega))} \right] \right\}$$

$$= \frac{1}{2j} \left\{ \frac{6}{9+(2-\omega)^2} \right\} + \frac{1}{2j} \left\{ \frac{-6}{9+(2+\omega)^2} \right\}$$

$$= \frac{-3j}{9+(2-\omega)^2} + \frac{3j}{9+(2+\omega)^2}$$

Thus, the Fourier transform of the signal $x(t)$ is $\boxed{\frac{3j}{9+(2+\omega)^2} - \frac{3j}{9+(2-\omega)^2}}.$

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(c)

The signal, $x(t) = \begin{cases} 1 + \cos \pi t, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$

Compute the Fourier transform of this signal.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-1}^1 (1 + \cos \pi t) e^{-j\omega t} dt$$

$$= \int_{-1}^1 e^{-j\omega t} dt + \int_{-1}^1 \left(\frac{e^{j\pi t} + e^{-j\pi t}}{2} \right) e^{-j\omega t} dt$$

$$= \int_{-1}^1 e^{-j\omega t} dt + \frac{1}{2} \left\{ \int_{-1}^1 e^{j(\pi-\omega)t} dt + \int_{-1}^1 e^{-j(\pi+\omega)t} dt \right\}$$

Simplify further.

$$X(j\omega) = \frac{e^{-j\omega t}}{-j\omega} \Big|_{-1}^1 + \frac{1}{2} \left\{ \frac{e^{j(\pi-\omega)t}}{j(\pi-\omega)} \Big|_{-1}^1 + \frac{e^{-j(\pi+\omega)t}}{-j(\pi+\omega)} \Big|_{-1}^1 \right\}$$

$$= \frac{e^{-j\omega} - e^{j\omega}}{-j\omega} + \frac{1}{2} \left\{ \frac{e^{j(\pi-\omega)} - e^{-j(\pi-\omega)}}{j(\pi-\omega)} - \frac{e^{-j(\pi+\omega)} - e^{j(\pi+\omega)}}{j(\pi+\omega)} \right\}$$

$$= \frac{-2j \sin \omega}{-j\omega} + \frac{2j \sin(\pi-\omega)}{2j(\pi-\omega)} + \frac{2j \sin(\pi+\omega)}{2j(\pi+\omega)}$$

$$= \frac{2 \sin \omega}{\omega} + \frac{\sin(\pi-\omega)}{\pi-\omega} + \frac{\sin(\pi+\omega)}{\pi+\omega}$$

Simplify further.

$$X(j\omega) = \frac{2\sin\omega}{\omega} + \frac{\sin(\pi-\omega)}{\pi-\omega} + \frac{\sin(\pi+\omega)}{\pi+\omega}$$

Note: $\sin(\pi-\omega) = \sin\omega$ and $\sin(\pi+\omega) = -\sin\omega$

Hence,

$$X(j\omega) = \frac{2\sin\omega}{\omega} + \frac{\sin\omega}{\pi-\omega} - \frac{\sin\omega}{\pi+\omega}$$

Thus, the Fourier transform of the signal $x(t)$ is $\boxed{\frac{2\sin\omega}{\omega} + \frac{\sin\omega}{\pi-\omega} - \frac{\sin\omega}{\pi+\omega}}$.

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(d)

The signal, $x(t) = \sum_{k=0}^{\infty} \alpha^k \delta(t-kT), |\alpha| < 1$

Compute the Fourier transform of this signal.

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} \sum_{k=0}^{\infty} \alpha^k \delta(t-kT) e^{-j\omega t} dt \\ &= \sum_{k=0}^{\infty} \alpha^k \int_{-\infty}^{\infty} \delta(t-kT) e^{-j\omega t} dt \\ &= \sum_{k=0}^{\infty} \alpha^k \left(e^{-j\omega t} \Big|_{t=kT} \right) \end{aligned}$$

Simplify further.

$$\begin{aligned} X(j\omega) &= \sum_{k=0}^{\infty} \alpha^k e^{-j\omega kT} \\ &= \sum_{k=0}^{\infty} (\alpha e^{-j\omega T})^k \\ &= \frac{1}{1 - \alpha e^{-j\omega T}} \end{aligned}$$

Thus, the Fourier transform of the signal $x(t)$ is $\boxed{\frac{1}{1 - \alpha e^{-j\omega T}}}$.

(e)

The signal, $x(t) = [te^{-2t} \sin 4t] u(t)$

Compute the Fourier transform of this signal.

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} [te^{-2t} \sin 4t] u(t) e^{-j\omega t} dt \\ &= \int_0^{\infty} te^{-2t} \left(\frac{e^{4jt} - e^{-4jt}}{2j} \right) e^{-j\omega t} dt \\ &= \frac{1}{2j} \left[\int_0^{\infty} \left\{ te^{-(2-4j+j\omega)t} - te^{-(2+4j+j\omega)t} \right\} dt \right] \end{aligned}$$

Simplify further.

$$\begin{aligned}
 &= \frac{1}{2j} \left\{ (0-0) + \int_0^{\infty} \frac{e^{-(2-4j+j\omega)t}}{(2-4j+j\omega)} dt - (0-0) - \int_0^{\infty} \frac{e^{-(2+4j+j\omega)t}}{(2+4j+j\omega)} dt \right\} \\
 &= \frac{1}{2j} \left\{ \int_0^{\infty} \frac{e^{-(2-4j+j\omega)t}}{(2-4j+j\omega)} dt - \int_0^{\infty} \frac{e^{-(2+4j+j\omega)t}}{(2+4j+j\omega)} dt \right\} \\
 &= \frac{1}{2j} \left\{ \left(\frac{1}{(2-4j+j\omega)} \right) \left(\frac{-e^{-(2-4j+j\omega)t}}{(2-4j+j\omega)} \right) \Big|_0^{\infty} - \left(\frac{1}{(2+4j+j\omega)} \right) \left(\frac{-e^{-(2+4j+j\omega)t}}{(2+4j+j\omega)} \right) \Big|_0^{\infty} \right\}
 \end{aligned}$$

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Simplify further.

$$\begin{aligned}
 X(j\omega) &= \frac{1}{2j} \left\{ \left(\frac{1}{(2-4j+j\omega)} \right) \left(\frac{1}{(2-4j+j\omega)} \right) - \left(\frac{1}{(2+4j+j\omega)} \right) \left(\frac{1}{(2+4j+j\omega)} \right) \right\} \\
 &= \frac{1}{2j} \left\{ \frac{1}{(2-4j+j\omega)^2} - \frac{1}{(2+4j+j\omega)^2} \right\}
 \end{aligned}$$

Thus, the Fourier transform of the signal $x(t)$ is

$$\frac{1}{2j} \left\{ \frac{1}{(2-4j+j\omega)^2} - \frac{1}{(2+4j+j\omega)^2} \right\}.$$

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(f)

The signal, $x(t) = \left[\frac{\sin \pi t}{\pi t} \right] \left[\frac{\sin 2\pi(t-1)}{\pi(t-1)} \right]$

Compute the Fourier transform of this signal.

Assume,

$$\begin{aligned}
 x_1(t) &= \frac{\sin \pi t}{\pi t} \\
 x_2(t) &= \frac{\sin 2\pi(t-1)}{\pi(t-1)}
 \end{aligned}$$

Write the Fourier transform of $x_1(t) = \frac{\sin \pi t}{\pi t}$

$$X_1(j\omega) = \begin{cases} 1, & |\omega| < \pi \\ 0, & \text{otherwise} \end{cases}$$

Write the Fourier transform of $x_2(t) = \frac{\sin 2\pi(t-1)}{\pi(t-1)}$

$$X_2(j\omega) = \begin{cases} e^{-2j\omega} & |\omega| < 2\pi \\ 0 & \text{otherwise} \end{cases}$$

Write the multiplication property in Fourier domain.

$$x(t) = x_1(t)x_2(t) \xrightarrow{F.T.} X(j\omega) = \frac{1}{2\pi} \{X_1(j\omega) * X_2(j\omega)\}$$

Thus, the Fourier transform of the signal $x(t)$ is

$$X(j\omega) = \begin{cases} e^{-j\omega} & |\omega| < \pi \\ \left(\frac{1}{2\pi}\right)(3\pi + \omega)e^{-j\omega} & -3\pi < \omega < -\pi \\ \left(\frac{1}{2\pi}\right)(3\pi - \omega)e^{-j\omega} & \pi < \omega < 3\pi \\ 0 & \text{otherwise} \end{cases}.$$

(g)

Consider the following figure:

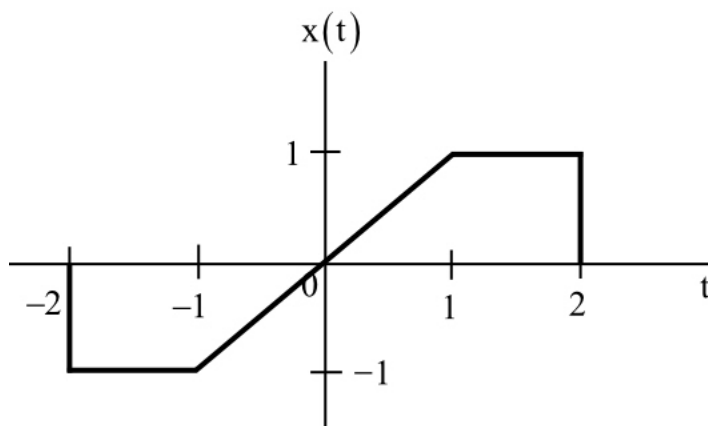


Figure 1

Write the mathematical representation of the signal in Figure 1.

$$x(t) = \begin{cases} -1, & -2 < t < -1 \\ t, & -1 < t < 1 \\ 1, & 1 < t < 2 \end{cases}$$

Compute the Fourier transform of this signal.

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt \\ &= \int_{-2}^{-1} (-1)e^{-j\omega t} dt + \int_{-1}^1 te^{-j\omega t} dt + \int_1^2 (1)e^{-j\omega t} dt \\ &= \left[\frac{-e^{-j\omega t}}{-j\omega} \right]_{-2}^{-1} + \left[t \frac{e^{-j\omega t}}{-j\omega} - \int \frac{e^{-j\omega t}}{-j\omega} dt + \frac{e^{-j\omega t}}{-j\omega} \right]_{-1}^1 \\ &= \left[\frac{e^{j\omega} - e^{2j\omega}}{j\omega} \right] - \left[\frac{e^{-j\omega} - (-1)e^{j\omega}}{j\omega} \right] - \left[\frac{e^{-j\omega t}}{(-j\omega)^2} \right]_{-1}^1 - \left[\frac{e^{-2j\omega} - e^{-j\omega}}{j\omega} \right] \end{aligned}$$

[Comments \(1\)](#)

Simplify further.

$$= -\left(\frac{1}{j\omega}\right) - \left(\frac{1}{\omega^2}\right)$$

$$= \frac{-2\cos 2\omega}{j\omega} - \frac{2j\sin \omega}{\omega^2}$$

Simplify further.

$$X(j\omega) = \frac{-2j\cos 2\omega}{j \cdot j\omega} - \frac{2j\sin \omega}{\omega^2}$$

$$= \frac{2j\cos 2\omega}{\omega} - \frac{2j\sin \omega}{\omega^2}$$

Thus, the Fourier transform of the signal $x(t)$ is $\boxed{\frac{2j\cos 2\omega}{\omega} - \frac{2j\sin \omega}{\omega^2}}$.

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(h)

Consider the following figure:

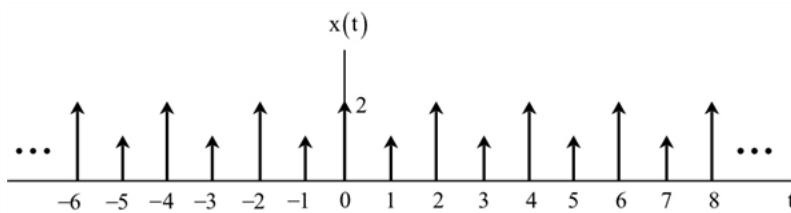


Figure 2

Write the mathematical representation of the signal in Figure 2.

Assume,

$$x_1(t) = \sum_{k=-\infty}^{\infty} \delta(t - 2k)$$

Write the Fourier transform of this signal.

$$X_1(j\omega) = \frac{2\pi}{2} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{2}\right)$$

Clearly from Figure 2, the input signal is,

$$x(t) = 2x_1(t) + x_1(t-1)$$

Compute the Fourier transform of this signal.

$$X(j\omega) = 2X_1(j\omega) + e^{-j\omega} X_1(j\omega)$$

$$= X_1(j\omega) [2 + e^{-j\omega}]$$

$$= \frac{2\pi}{2} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{2}\right) [2 + e^{-j\omega}]$$

$$= \pi \sum_{k=-\infty}^{\infty} \delta(\omega - \pi k) (2 + e^{-j\omega})$$

Simplify further.

$$X(j\omega) = \pi \sum_{k=-\infty}^{\infty} \delta(\omega - \pi k) (2 + e^{-j\omega})$$

$$= \pi \sum_{k=-\infty}^{\infty} \delta(\omega - \pi k) [2 + e^{-j\pi k}]$$

$$= \pi \sum_{k=-\infty}^{\infty} \delta(\omega - \pi k) [2 + (-1)^k]$$

$$= \pi \sum_{k=-\infty}^{\infty} \delta(\omega - \pi k) [2 + (-1)^k]$$

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(i)

The signal, $x(t) = \begin{cases} 1-t^2 & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$

Compute the Fourier transform of this signal.

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \\ &= \int_0^1 (1-t^2) e^{-j\omega t} dt \\ &= \int_0^1 e^{-j\omega t} dt - \int_0^1 t^2 e^{-j\omega t} dt \\ &= \left[\frac{e^{-j\omega t}}{-j\omega} \right]_0^1 - \left[t^2 \frac{e^{-j\omega t}}{-j\omega} \right]_0^1 + \int_0^1 2t \frac{e^{-j\omega t}}{-j\omega} dt \end{aligned}$$

Simplify further.

$$\begin{aligned} X(j\omega) &= \frac{e^{-j\omega} - 1}{-j\omega} + \frac{e^{-j\omega}}{j\omega} + 2 \left[\frac{te^{-j\omega t}}{(-j\omega)^2} \right]_0^1 - \int_0^1 \frac{e^{-j\omega t}}{(-j\omega)^2} dt \\ &= \frac{1 - e^{-j\omega} + e^{-j\omega}}{j\omega} + \frac{2e^{-j\omega}}{(-j\omega)^2} - \frac{2e^{-j\omega}}{(-j\omega)^3} \bigg|_0^1 \\ &= \frac{1}{j\omega} - \frac{2e^{-j\omega}}{-\omega^2} - \frac{2e^{-j\omega}}{(-j\omega)^3} + \frac{2}{(-j\omega)^3} \\ &= \frac{1}{j\omega} - \frac{2e^{-j\omega}}{-\omega^2} + \frac{2-2e^{-j\omega}}{(-j\omega)^3} \end{aligned}$$

Thus, the Fourier transform of the signal $x(t)$ is $\boxed{\frac{1}{j\omega} - \frac{2e^{-j\omega}}{-\omega^2} + \frac{2-2e^{-j\omega}}{(-j\omega)^3}}$.

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(j)

The signal, $x(t) = \sum_{n=-\infty}^{\infty} e^{-|t-2n|}$

Compute the Fourier transform of this signal.

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-|t-2n|} e^{-j\omega t} dt \\ &= \int_{-\infty}^0 \sum_{n=-\infty}^0 e^{t-2n} e^{-j\omega t} dt + \int_0^{\infty} \sum_{n=0}^{\infty} e^{-t+2n} e^{-j\omega t} dt \\ &= \sum_{n=-\infty}^0 e^{-2n} \int_{-\infty}^0 e^t e^{-j\omega t} dt + \sum_{n=0}^{\infty} e^{2n} \int_0^{\infty} e^{-t} e^{-j\omega t} dt \end{aligned}$$

Simplify further.

$$= \left(\frac{1}{1-e^2} \right) \left[\frac{e^{(1-j\omega)t}}{(1-j\omega)} \Big|_{-\infty}^{\infty} - \frac{e^{-(1+j\omega)t}}{(1+j\omega)} \Big|_0^{\infty} \right]$$

$$= \left(\frac{1}{1-e^2} \right) \left[\frac{1}{1-j\omega} + \frac{1}{1+j\omega} \right]$$

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Simplify further.

$$X(j\omega) = \left(\frac{1}{1-e^2} \right) \left[\frac{1}{1-j\omega} + \frac{1}{1+j\omega} \right]$$

$$= \left(\frac{1}{1-e^2} \right) \left(\frac{2}{1+\omega^2} \right)$$

Thus, the Fourier transform of the signal $x(t)$ is $\boxed{\frac{2}{(1-e^2)(1+\omega^2)}}$.

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